

Article

Frequency-Domain Proper Orthogonal Decomposition for Asynchronously Sampled Unsteady Flow Fields

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Abstract

The snapshot proper orthogonal decomposition (POD) method relies on synchronously sampled datasets, significantly limiting its utility for analyzing asynchronous measurements in unsteady flow studies. This paper proposes a frequency-domain proper orthogonal decomposition (FDPOD) method tailored for mode extraction and flow field reconstruction from asynchronously sampled data. The FDPOD framework integrates three key components: frequency-domain transformation to decouple phase discrepancies inherent in asynchronous sampling, power spectral density (PSD) analysis combined with segmented ensemble averaging to suppress spectral leakage errors, and eigenvalue decomposition of energy-ranked frequency components to identify dominant coherent structures. Validated through numerical simulations of a subsonic jet and experimental measurements from a low-speed mixed-flow fan, the method demonstrates exceptional performance under asynchronous conditions: cumulative energy errors are reduced to 0.3% across the first 50 modes, while flow field reconstruction achieves 99.5% accuracy. Dominant mode structures exhibit remarkable consistency with those derived from synchronous conditions, with hot-wire measurement errors remaining below 0.03% for both asynchronous and temporally shuffled datasets. These results position FDPOD as a robust and practical tool for analyzing complex unsteady flows where synchronous data acquisition proves impractical, particularly in large-scale or spatially distributed measurement systems.

Keywords: frequency-domain proper orthogonal decomposition; asynchronous sampling; power spectral density; flow field reconstruction; hot-wire anemometry



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1. Introduction

Various reduced-order models have been proposed to capture the dominant spatial-temporal features of complex, high-dimensional, nonlinear, and unsteady flows by mapping large-scale flow field data into lower-dimensional spaces [1–3]. In recent decades, data-driven techniques such as proper orthogonal decomposition (POD) and dynamic mode decomposition (DMD) have emerged as leading methods. POD performs eigenvalue decomposition on instantaneous flow snapshots to reduce system degrees of freedom while retaining most energy [4,5], while DMD linearly approximates nonlinear system dynamics and excels at preserving periodic or oscillatory flow structures [6].

POD elucidates flow mechanisms in various applications, from cylinder wakes to bio-inspired flow control, by quantifying modal energy distributions. Delville [7] and

Tutkun and George [8] applied POD to investigate the turbulent boundary layers based on the experimental data simultaneously acquired by hot-wire rakes. Deep et al. [9] and Chen et al. [10] used POD with numerical simulations and particle image velocimetry (PIV), respectively, to show that splitter plates and biomimetic whisker-shaped cylinders suppress vortex shedding, reduce drag, and lower noise, with the first six modes capturing over 95% of vorticity. Freund and Colonius [11] further demonstrated via direct numerical simulation (DNS) to show that kinetic energy norm-based POD modes dominate near-field turbulence, while acoustic power norms correlate with far-field radiation, highlighting the role of POD in instability analysis and passive flow control design.

In turbomachinery, POD proves invaluable for multi-scale modeling of flow separation, noise prediction, and energy transfer. Premaratne et al. [12], Bian et al. [13], and Liu et al. [14] applied POD to wind turbine wakes, turbine blades, and fan-outlet guide vane (OGV) systems, respectively, linking the first 5–10 modes to vortex breakdown, 80% entropy variation, and turbulence reconstruction, with 30% fewer modes than Fourier methods, enhancing efficiency in turbomachinery analysis. Similarly, in fundamental studies of jet mixing and boundary-layer transition, Ivashchenko et al. [15], Lengani et al. [16], and Dennis and Siddiqui [17] used POD to connect gas mixing, Kelvin–Helmholtz instabilities, and sub-Taylor scale motions to energy redistribution and dissipation up to three orders of magnitude higher in higher-order modes, bridging microscopic and macroscopic turbulence properties.

Compared to the energy-optimal characteristics of POD, DMD exhibits unique advantages in identifying periodic flow structures and analyzing flow-acoustic coupling due to its ability to approximate system dynamics linearly. Sarmast et al. [18], Schmid et al. [19], and Deng et al. [20] applied DMD to wind turbine wakes, jets, and airfoil noise, respectively, revealing stability differences, dominant structures with just two modes, and frequency-specific acoustic sources, highlighting its utility in dynamic reduction and multi-physics analysis.

Table 1 highlights recent representative applications of POD and DMD in fluid mechanics, ranging from flow control and noise reduction of cylinders [9–11,20], complex turbomachinery flows [12–14,18], and fundamental jet and turbulence studies [15–17,19]. These examples underscore the versatility of data-driven reduced-order methods, with POD excelling at efficient dimensionality reduction via dominant energy modes and DMD predicting dynamic features such as frequencies and spatiotemporal patterns, both proving valuable for practical modeling.

Table 1. Representative studies of POD and DMD in unsteady flow fields.

Author	Year	Research Subject	Methods	Data Source
Delville [7]	1994	Turbulent boundary layer	POD	Hot-wire rakes
Tutkun and George [8]	2017	Turbulent boundary layer	POD	Hot-wire rakes
Deep et al. [9]	2022	Wake behind a circular cylinder with a splitter plate	POD	URANS
Chen et al. [10]	2025	Flow structures and noise of a cylinder	POD	PIV
Freund and Colonius [11]	2009	Jet flow and associated acoustic field	POD	LES
Premaratne et al. [12]	2022	Wake characteristics behind a wind turbine model	POD	PIV
Bian et al. [13]	2020	Shock waves with the boundary layer and wakes in a NGV	POD	DDES
Liu et al. [14]	2021	Fan–OGV interaction broadband noise	POD	RANS

Table 1. Cont.

Author	Year	Research Subject	Methods	Data Source
Ivashchenko et al. [15]	2021	Near field of a turbulent circular jet	POD	Spectral element method (SEM)
Lengani et al. [16]	2014	Unsteady behavior of a laminar separation bubble	POD	PIV
Dennis and Siddiqui [17]	2022	Multi-modal turbulent velocity behavior in the boundary layer	POD	PIV
Sarmast et al. [18]	2014	Tip vortices behind a wind turbine	DMD	LES
Schmid et al. [19]	2012	Transitional water jet	DMD	PIV
Deng et al. [20]	2023	Unsteady flow and noise of airfoil	DMD	LES

However, a critical limitation emerges: both snapshot POD and DMD rely on sampled data synchronously or simultaneously obtained from experimental measurements and numerical simulations, making them inapplicable to asynchronous datasets such as those from hot wires without synchronous sampling. In addition, while snapshot POD effectively extracts large-scale structures, reconstructing complex multi-scale flows often requires tens to hundreds of modes, significantly increasing computational cost [11,17,21,22].

These limitations are relevant to both jet and turbomachinery flows. Although the two types of flows differ significantly in terms of geometry, physical mechanisms, and engineering background, they face a common challenge in modal analysis: the flow fields are high-dimensional and strongly unsteady. For the jet case, high-resolution unsteady data are available, allowing synchronous and asynchronous datasets to be constructed in a controlled manner. In the fan case, hot-wire measurements are performed sequentially, which also leads to asynchronous sampling naturally. Therefore, a modal analysis method that is insensitive to asynchronous sampling is required to extract energy-ranked coherent structures from spatially distributed data and reconstruct unsteady flow fields.

To address these challenges, this study introduces the FDPOD method for analyzing asynchronously sampled flow field data. Recently proposed by Gu et al. [23] and tested on synchronous jet, cylinder wake, and fan flow simulations, FDPOD improves on snapshot POD by requiring fewer modes for flow field reconstruction while retaining energy-ordered modes that evolve from global to local scales, similar to structural vibration modes. It should be noted that the present study differs from the authors' previous work. Gu et al. [23] proposed the FDPOD method and demonstrated its capability for the efficient reconstruction of unsteady flow fields dominated by components of stochastic fluctuations, which are usually synchronously sampled. The present study emphasizes the applicability of FDPOD to asynchronously sampled unsteady flow-field data.

This paper validates the ability of the FDPOD to overcome the limitations of synchronous sampling and to expand its engineering applications. The paper is organized as follows: Section 2 details the FDPOD theoretical framework; Section 3 evaluates its accuracy in processing asynchronous jet data obtained from numerical simulations; Section 4 evaluates its effectiveness with hot-wire fan flow data for complex mode extraction and reconstruction; and Section 5 presents the main conclusions.

2. Methodology

In this study, asynchronous sampling is described using a spatially dependent time offset. For the sampled flow-field signal $q(x_j, t_i)$ measured at a spatial location x_j , each spatial position in asynchronous sampling is associated with a time offset corresponding to its recorded time series:

$$q_j[i] = q(x_j, t_i + \Delta t_j), i = 1, 2, \dots, M, j = 1, 2, \dots, N \quad (1)$$

where, $t_i = t_0 + (i - 1)\Delta t_j$, with Δt_j representing the relative time offset between different spatial locations. Synchronous sampling is a special case in which all $\Delta t_j = 0$; for asynchronous sampling, at least two spatial locations have different time offsets.

To overcome the inability of snapshot POD to process asynchronously sampled data, this study introduces the frequency-domain proper orthogonal decomposition (FDPOD) method. As shown in Figure 1, the main process of this method is described as follows

- (1) Construction of the spatiotemporal data matrix: First, obtain the asynchronous sampled experimental data sequences from various spatial measurement points in the transient flow field, and assemble them into a spatiotemporal data matrix Q . The matrix rows correspond to instantaneous flow field data at specific spatial points, while the columns represent data across different spatial locations. The matrix is defined as follows:

$$Q = \begin{bmatrix} q(x_1, t_1) & q(x_1, t_2) & \cdots & q(x_1, t_N) \\ q(x_2, t_1 + \Delta t) & q(x_2, t_2 + \Delta t) & \cdots & q(x_2, t_N + \Delta t) \\ \vdots & \vdots & \vdots & \vdots \\ q(x_M, t_1 + (M-1)\Delta t) & q(x_M, t_2 + (M-1)\Delta t) & \cdots & q(x_M, t_N + (M-1)\Delta t) \end{bmatrix} \quad (2)$$

where x_j represents the data vector at the j -th spatial location, describing the flow field data acquisition position. t_i refers to the i -th sampling time step, and Δt is the time interval between each asynchronous sample.

- (2) Data block segmentation: Using the Welch periodogram method [24], the data matrix Q is divided into several blocks, with each block containing a certain number of time-series data. The result matrix of the n -th data block is as follows:

$$Q^{(n)} = \begin{bmatrix} q_1^{(n)} & q_2^{(n)} & \cdots & q_{N_f}^{(n)} \end{bmatrix} \quad (3)$$

- (3) Discrete Fourier Transform (DFT): A discrete Fourier transform (DFT) is applied to each block, converting the time domain data to the frequency domain. To reduce the error in the spectral magnitude of each block, the average power spectrum of each block is calculated. The specific mathematical expression is as follows:

$$\hat{Q}^{(n)} = FFT(Q^{(n)}) = \begin{bmatrix} \hat{q}_1^{(n)} & \hat{q}_2^{(n)} & \cdots & \hat{q}_{N_f}^{(n)} \end{bmatrix} \quad (4)$$

$$\hat{Q}^{(mean)}(x, f) = \frac{1}{N_b} \sum_{n=1}^{N_b} \hat{Q}^{(n)}(x, f) \quad (5)$$

where N_f is the number of time steps in each block, and N_b is the total number of blocks.

- (4) Eigenvalue decomposition: Eigenvalue decomposition is performed on the fluctuating part of the power spectral density matrix, which is obtained by ensemble averaging the power spectral density (PSD) and subtracting the mean energy value at all frequencies. The eigenvectors and corresponding eigenvalues are obtained. These eigenvectors

represent the dominant spatiotemporal structures, while the eigenvalues reflect the proportion of energy contributed by these structures in the flow. The fluctuating value of the power spectrum $\hat{Q}'(x, f)$ is computed as:

$$\hat{Q}'(x, f) = \hat{Q}^{(mean)}(x, f) - \bar{Q}(x) \quad (6)$$

where $\bar{Q}(x) = \frac{1}{N_f} \sum_{i=1}^{N_f} Q'(x, f_i)$ is the average energy spectrum at each spatial point across all frequencies.

- (5) Mode reconstruction: The POD modes are ranked according to their eigenvalues, and the top modes with the highest energy contribution are selected for flow field reconstruction. Using the selected modes and their corresponding coefficients, the flow field can be approximated and reconstructed as follows:

$$\hat{Q}(x, f) = \bar{Q}(x) + \sum_{j=1}^N a_j(f) \varphi_j(x) \quad (7)$$

where $\varphi_j(x)$ are orders of the FDPOD basis modes and $a_j(f)$ are the corresponding mode coefficients.

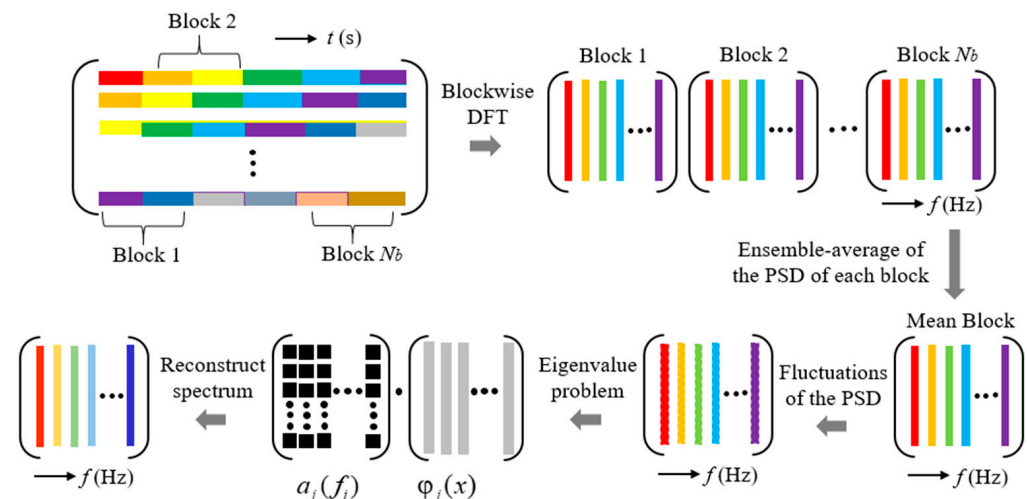


Figure 1. Frequency domain POD algorithm principle diagram.

The FDPOD method transforms time-domain signals into frequency-domain data, combines PSD ensemble averaging to reduce random errors, and performs eigenvalue decomposition on the averaged frequency spectrum to achieve flow field reconstruction. Its key advantage lies in the combination of frequency domain transformation and ensemble averaging of spectral magnitudes, which eliminates the phase-difference effects of asynchronous sampling and significantly reduces data dimensionality. In contrast, the snapshot POD method performs a linear decomposition of a snapshot of a pulsating flow field, which usually requires a large number of modes to capture complex random perturbation features.

The main difference between the FDPOD method and the spectral proper orthogonal decomposition (SPOD) method [25,26] is that SPOD constructs and decomposes the cross-spectral density matrix at each frequency, thereby obtaining frequency-specific coherent modes that retain both amplitude and phase relationships. Therefore, SPOD preserves the phase relationships among spatial locations and is suitable for identifying coherent structures associated with specific frequencies. In contrast, FDPOD first performs block segmentation and ensemble averaging of the power spectral density (PSD),

followed by eigenvalue decomposition of the global spectral energy matrix. FDPOD ranks the modes according to their global spectral energy contribution, rather than solving independent eigenvalue problems at each frequency. Although inspired by frequency-domain methods, FDPOD fundamentally differs from SPOD in implementation and application to asynchronous datasets. It should be noted that SPOD focuses on extracting frequency-resolved coherent structures by decomposing the cross-spectral density matrix, whereas FDPOD focuses on globally energy-ranked modal reconstruction by decomposing the averaged spectral energy matrix. The unique contribution of FDPOD lies in its ability to provide stable modal extraction when sequentially sampled time series are available and the cross-spectral phase relationships among spatial points cannot be reliably determined.

3. Analysis of Jet Flow Field

3.1. Research Object and Data Acquisition

This study uses a subsonic jet as a case study to verify the applicability of the FDPOD method in processing asynchronously sampled data. In this section, snapshot POD is used as the reference method for evaluating FDPOD because both methods aim to obtain energy-ranked modes and reconstruct the flow field using a reduced number of modes. Under asynchronous sampling, the transient matrix assembled from different spatial locations no longer corresponds to a physically meaningful instantaneous flow field. Therefore, the comparison between snapshot POD and FDPOD directly reveals the influence of asynchronous sampling on energy-based modal extraction and reconstruction.

The numerical data is sourced from the study by Schmidt et al. [27], based on large eddy simulation (LES), with an incoming Mach number of 0.9. As shown in Figure 2, the computational domain has axial and radial lengths of $60D$ and $12D$, respectively, where D represents the nozzle radius. The entire computational domain is discretized into 656, 128, and 138 grid nodes along the axial, circumferential, and radial directions, respectively. The dimensionless time step is $c_\infty \cdot \Delta t / D = 0.2$, where c_∞ is the speed of sound. Frequency is expressed in terms of the Strouhal number (St), where $St = f \cdot D / U$, U is the incoming flow velocity. A total of 5 s of high-resolution snapshots, amounting to 5000 snapshots, were generated to fully capture the complex structural characteristics of the turbulent flow field.

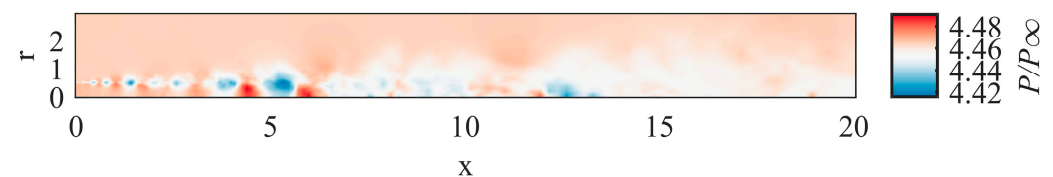


Figure 2. Schematic of subsonic jet flow simulation.

To evaluate the influence of temporal misalignment in a controlled manner, a deterministic time-shifted asynchronous dataset was constructed from the jet data. Specifically, different time windows with prescribed offsets were assigned to different spatial locations. This approach provides a reproducible benchmark for comparing the sensitivity of snapshot POD and FDPOD to asynchronous sampling. It should be noted that this controlled time-shifted dataset is not intended to represent all possible random asynchronous triggering conditions, but rather to isolate the effect of temporal misalignment while keeping the underlying flow data unchanged.

Two methods were used to obtain synchronized and the time-shifted asynchronous sampled flow field data. The time-shifted synchronized dataset refers to the data from all grid nodes in the flow field, which are sampled synchronously within the 0 to 4 s time

period. The method for obtaining the asynchronous sampled dataset is described as follows. The first grid node uses data from 0 to 4 s, the second grid node uses data from 0.1 to 4.1 s, the third grid node uses data from 0.2 to 4.2 s, and so on, until the eleventh grid node uses data from 1 to 5 s. The data sampling for the remaining grid nodes cyclically follows this pattern. Before performing the FDPOD analysis on the flow field, increasing the number of blocks, N_b , helps reduce the uncertainty of the PSD, i.e., random errors. The downside is the increased cost of acquiring the dataset. Based on the recommendation in [23], $N_b = 8$ is chosen in this case study to balance analysis accuracy and cost.

3.2. Comparison of Snapshot POD Results from Different Datasets

The conventional snapshot POD method is widely used to extract dominant coherent structures from unsteady flow fields by performing eigenvalue decomposition on synchronously sampled flow snapshots. However, its effectiveness heavily depends on the availability of synchronized data.

Figure 3 illustrates the relationship between the mode number N and the cumulative energy ratio (CER) of snapshot POD modes for the subsonic jet under synchronous and asynchronous sampling conditions. The CER is defined as follows:

$$CER = \frac{\sum_{j=1}^N \lambda_j}{\sum_{j=1}^M \lambda_j} \quad (8)$$

where λ_j is the eigenvalue of the j -th mode, and M is the total number of modes. CER represents the percentage of total energy captured by the first N modes.

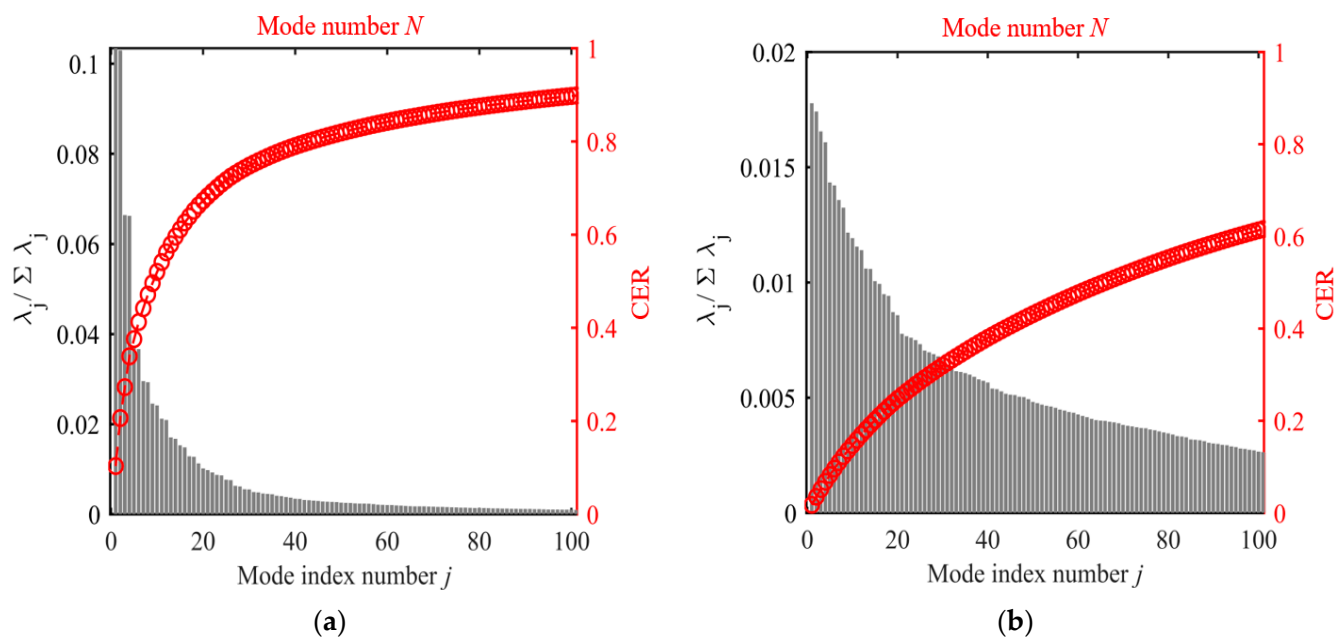


Figure 3. Energy distribution of snapshot POD modes for the subsonic jet: (a) synchronous; (b) asynchronous.

For the synchronous dataset, Figure 3a shows a relatively rapid convergence of energy with the first few modes capturing a significant fraction of the total energy, followed by a gradual decay across higher-order modes. In contrast, Figure 3b reveals a markedly slower energy convergence for the asynchronous dataset, with the energy distributed more diffusely across a larger number of modes. The observed discrepancy underscores that the snapshot POD method inherently requires temporally synchronized data acquisition

across spatially distributed measurement points, and failure to satisfy this synchronization criterion may induce the spurious artifacts demonstrated in Figure 3b.

Table 2 quantifies this disparity by comparing the CER for the synchronous and asynchronous datasets across different numbers of modes. For the synchronous case, the first 5 modes capture 37.6% of the total energy, increasing to 52.0% for 10 modes, 67.3% for 20 modes, and 81.9% for 50 modes. However, for the asynchronous case, these values drop significantly to 8.2%, 14.7%, 24.8%, and 43.1%, respectively. The comparative results presented in Table 2 substantiate that snapshot POD fundamentally mandates strict temporal synchronization of acquired datasets, thereby imposing intrinsic constraints on its applicability to experimental measurements involving hot-wire anemometry.

Table 2. Comparison of cumulative energy ratios for synchronized and asynchronous sampling data.

Number of Modes	5 Modes	10 Modes	20 Modes	50 Modes
Synchronous	0.376	0.520	0.673	0.819
Asynchronous	0.082	0.147	0.248	0.431

Figure 4 further illustrates the effect of asynchronous sampling on the spatial coherence of the extracted modes. Figure 4a shows the first few snapshot POD modes for the synchronous dataset, which exhibit well-defined, large-scale coherent structures consistent with the dominant flow features of the jet, such as shear-layer instabilities and vortex roll-up. In contrast, Figure 4b depicts the modes derived from the asynchronous dataset, which appear fragmented and lack the clear spatial organization seen in the synchronous case. The phase discrepancies introduced by asynchronous sampling disrupt the temporal alignment of flow snapshots, causing snapshot POD to misinterpret the underlying dynamics and produce modes that fail to represent physically meaningful structures.

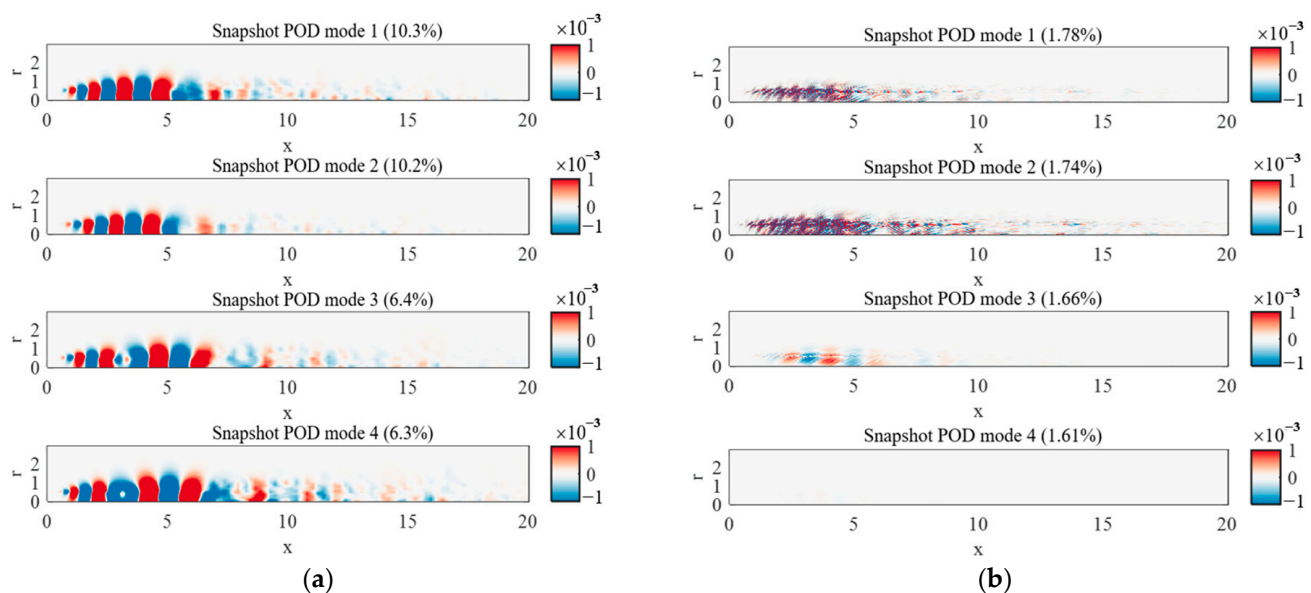


Figure 4. Snapshot POD modes for the subsonic jet: (a) synchronous; (b) asynchronous.

These analyses validate two critical limitations of the conventional snapshot POD method. First, its reliance on synchronous sampling renders it ineffective for datasets where temporal alignment is not guaranteed, such as those obtained from distributed sensors or hot-wire anemometry with sequential measurements. Second, even with synchronous data, snapshot POD often necessitates a large number of modes to reconstruct multi-scale flow fields adequately, as evidenced by the slow energy convergence in Figure 3a and Table 2.

3.3. Comparison of FDPOD Results from Different Datasets

Figure 5 compares the mode numbers N and CER distributions for synchronized and asynchronous sampled data. The results demonstrate that FDPOD accurately extracts the dominant mode energy distribution from asynchronous data, closely matching the outcomes of synchronous sampling.

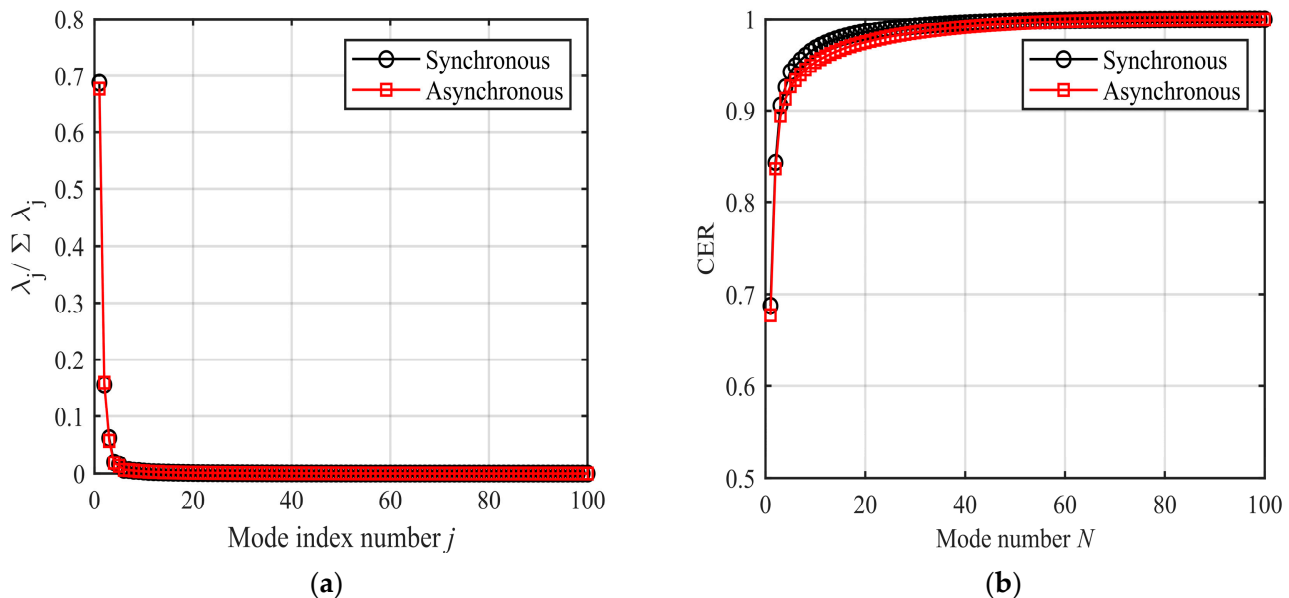


Figure 5. Energy distribution of FDPOD modes for the subsonic jet: (a) energy contribution of each mode; (b) cumulative energy ratio of the first N modes.

Table 3 compares the cumulative energy ratio and relative error for different numbers of modes under synchronous and asynchronous sampling. For synchronous sampling, the cumulative energy ratios for 5, 10, 20, and 50 modes are 0.942, 0.968, 0.986, and 0.998, respectively. For asynchronous sampling, these values are 0.927, 0.953, 0.975, and 0.995, with a maximum relative error of 1.6%.

Table 3. Comparison of cumulative energy ratios for synchronized and asynchronous sampling data.

Number of Modes	5 Modes	10 Modes	20 Modes	50 Modes
Synchronous	0.942	0.968	0.986	0.998
Asynchronous	0.927	0.953	0.975	0.995
Relative error	1.6%	1.5%	1.1%	0.3%

To further illustrate the superiority of FDPOD over conventional snapshot POD, a direct comparison is made with the results from Table 2. For synchronous snapshot POD, 50 modes are required to capture approximately 82% of the energy, whereas FDPOD achieves a comparable energy level with only 10 modes under asynchronous conditions. This indicates that FDPOD requires approximately 80% fewer modes to capture a similar energy content. This modal sparsity significantly improves computational efficiency, as fewer modes result in reduced dimensionality and lower computational cost, which is particularly beneficial for multi-scale flow analysis. It should be noted that FDPOD is not claimed to be generally superior to snapshot POD under synchronized sampling conditions. When synchronized full-field snapshots are dominated by periodic fluctuations, snapshot POD remains the standard and optimal energy-based decomposition method in the time domain. However, as demonstrated by Gu et al. [23], FDPOD offers higher computational efficiency for flows characterized by stochastic fluctuations. Moreover,

the advantage of FDPOD lies in its robustness under asynchronous sampling conditions, whereas snapshot POD loses the physical consistency of the snapshot matrix under asynchronous sampling.

It should be noted that these errors are mainly due to the differences in the PSD energy spectrum within each data block for asynchronously sampled data. Theoretically, if the PSD values within each data block are the same, FDPOD will produce no error. Therefore, by performing multi-segment repeated sampling of the data for any spatial point, the error introduced by asynchronous sampling can be effectively reduced. Especially for hot-wire experimental data, because hot-wire has higher time resolution and longer sampling time, the asynchronous sampling error is much smaller compared to numerical simulations.

3.4. Jet Flow Field Mode Analysis

Figure 6a,b show the first four FDPOD modes of the jet flow field under synchronous and asynchronous sampling conditions, respectively. Although there are slight differences in the local features of the FDPOD mode, the main mode characteristics are consistent, and both effectively capture the primary energy characteristics of the flow field. The spatial structure of the first FDPOD mode is similar to the root mean square (RMS) distribution of the pressure, as shown in Figure 6c, indicating that the first mode captures the primary global perturbation features in the flow field.

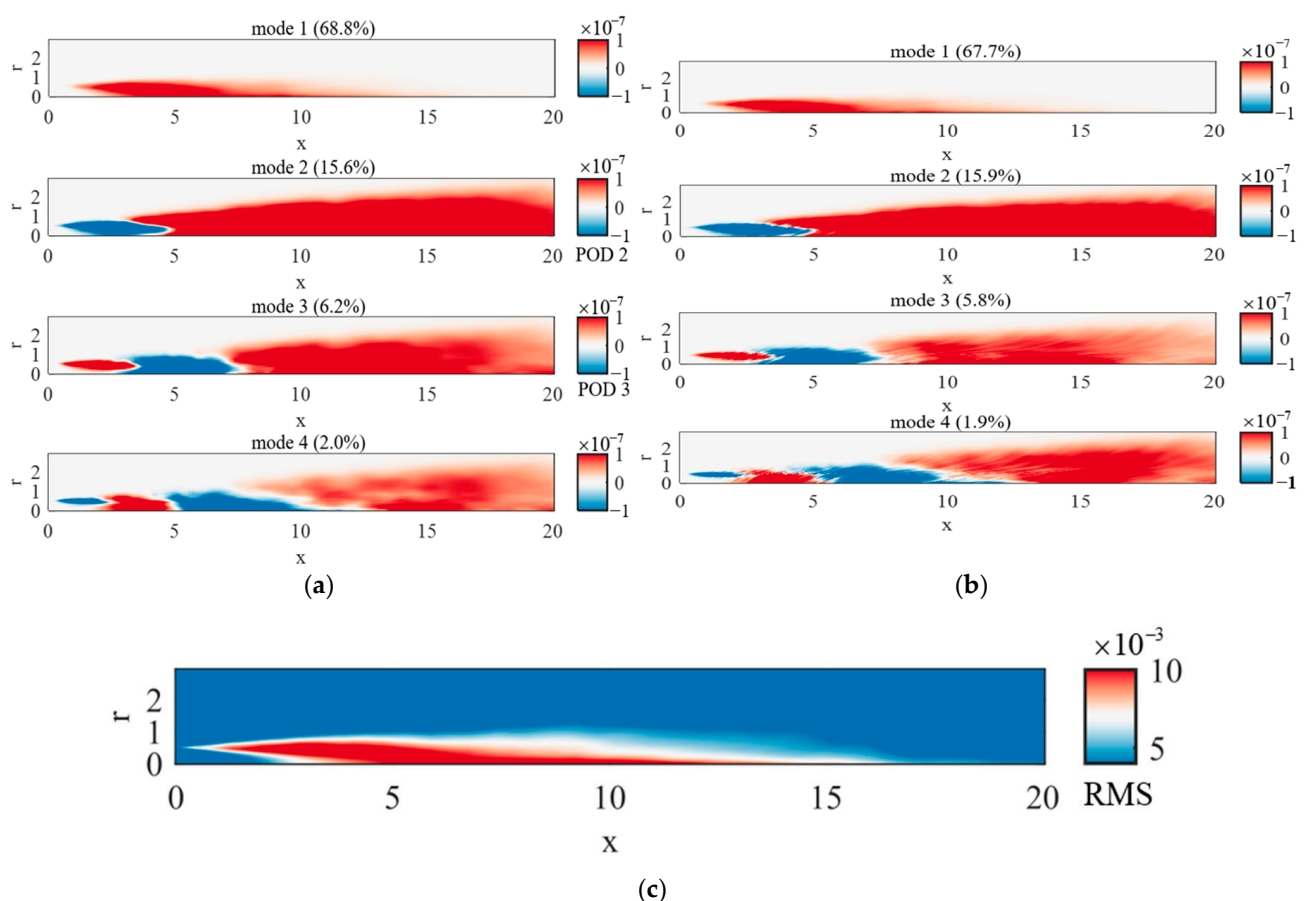


Figure 6. FDPOD modes: (a) synchronized sampling; (b) asynchronous sampling; (c) RMS pressure distribution.

As the mode order increases, the spatial length scale of the FDPOD modes decreases and the proportion of disturbance energy in the flow field is significantly reduced. Lower-order modes primarily reflect the spatial distribution characteristics of the global energy, while higher-order modes reveal the local perturbation features with lower energy propor-

tions. This indicates that the FDPOD modes not only sequentially characterize the spatial distribution of energy from high to low, but also reveal the variation in energy across spatial length scales in the flow field.

Table 4 quantitatively compares the differences in the energy distribution of the first four modes between synchronous and asynchronous sampling using the FDPOD method. The first mode captures the majority of the flow field energy with an absolute error of only 1.1%. As the mode order increases, the energy fraction for each mode decreases, and the absolute error between synchronous and asynchronous sampling gradually decreases, with the error for the fourth mode being only 0.1%.

Table 4. Comparison of energy proportions for individual modes in the jet flow under synchronized and asynchronous sampling data.

Mode Order	Mode 1	Mode 2	Mode 3	Mode 4
Synchronous	68.8%	15.6%	6.2%	2.0%
Asynchronous	67.7%	15.9%	5.8%	1.9%
Absolute error	1.1%	0.3%	0.4%	0.1%

In conclusion, the differences between the results obtained from synchronous and asynchronous sampling using the FDPOD method are minimal. The energy distributions and modes are highly consistent, confirming the effectiveness and reliability of the FDPOD method in processing asynchronous sampling data.

4. Analysis of Fan Flow Field

4.1. Experimental Object and Data Acquisition

The fan studied in this research consists of an upstream mixed-flow rotor, a downstream axial-flow stator, a hub, and a casing. Both the upstream rotor and the downstream stator feature a series blade structure, with 5 + 5 rotating blades for the rotor and 11 + 11 stationary vanes for the stator. The specific structure is shown in Figure 7a,b.

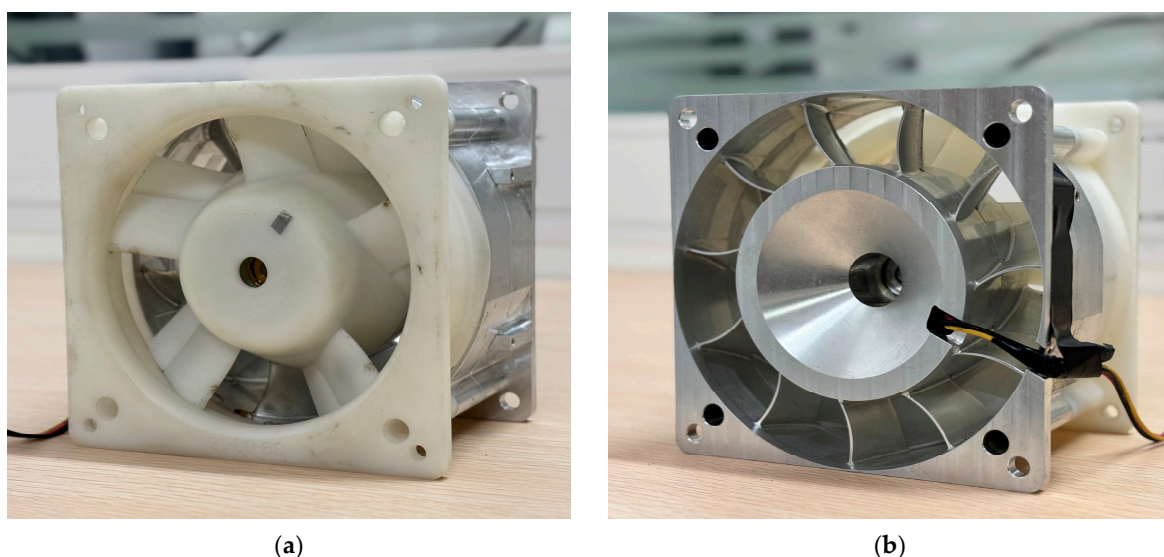


Figure 7. The main structural features of the fan: (a) rotating blades; (b) stationary vanes.

The main structural and operational design parameters of the fan are presented in Table 5. Under the design operating conditions, the fan has an inlet volumetric flow rate of 270 m³/h and a rotational speed of 9500 rpm, corresponding to a tip Mach number of 0.12 for the rotating blades. Numerical studies of unsteady flow simulation inside this

mixed-flow fan can be found in [28], and identification of the aeroacoustic sources can be found in [29]. It should be noted that the mixed-flow fan used in the present study is the same as that investigated by Mao et al. [28], but the research objectives are different. Mao et al. [28] focused on analyzing the unsteady flow mechanisms of the fan through triple decomposition of unsteady pressure and velocity obtained from flow simulation, whereas the present study uses this fan as an experimental case of asynchronous sampling. The measured data are treated as an asynchronous dataset to validate the modal extraction and reconstruction capabilities of FDPOD.

Table 5. The main structural design parameters of the mixed-flow fan.

Design Factors	Values
Number of rotating blades	5
Number of stationary vanes	11
Hub-shroud ratio	0.64
Fan rotation speed	9500 rpm
Incoming flow Mach number	0.03
Reynolds number based on the rotor midspan chord	5.6×10^4

The velocity measurement at the fan outlet flow field is performed using an X-wire probe (55P61, Dantec Dynamics) with a wire diameter of 5 μm and a length of 1.25 mm. For clarity, the coordinate system is defined as follows: the z-axis aligns with the axial direction of the fan, directed from the inlet to the outlet. The origin is located at the lower left corner of the fan out plane, as shown in Figure 8b. The X-wire probe measures velocity components U_x (x-direction, transverse velocity) and U_z (z-direction, axial velocity) to characterize the two-dimensional velocity distribution at the outlet. The probe is mounted on the Dantec 22H26 bracket and attached to the Dantec three-dimensional positioning system. The measurement plane is located 5 mm downstream of the fan outlet, as shown in Figure 8b, where the fan outlet diameter is 120 mm. The sampling points are arranged in a uniform 25×25 grid, totaling 625 points, with each point spaced 5 mm apart. At each sampling point, the hot-wire probe collects data at a sampling frequency of 40 kHz, with a sampling duration of 5 s per point. It should be noted that the data for these 625 points are collected sequentially, meaning the probe completes the 5 s sampling at one point before moving to the next. Based on 625 measurement points and a sampling duration of 5 s at each point, the minimum time required to complete one full scan of the fan outlet plane is approximately 3125 s, excluding the time needed for probe movement and flow stabilization. Therefore, the nominal starting-time offset between two adjacent sequentially measured points is at least 5 s. In practice, this offset is slightly larger due to probe movement, positioning, and stabilization. This estimate is added to clarify that the measured outlet-plane data cannot be regarded as synchronized instantaneous full-field data, which further motivates the use of FDPOD for asynchronous flow-field analysis.

Before measurements, the hot-wire velocity is calibrated using a Prandtl tube with a calibration range of 0 m/s to 40 m/s and 20 logarithmically spaced velocity points. Calibration errors are approximately $\pm 0.5\%$, mainly due to the accuracy of the differential pressure sensor (± 1.0 Pa). In addition, directional calibration is performed using a THORLABS motorized rotary stand (K10CR1A2/M, with an absolute accuracy of $\pm 0.14^\circ$) at 5° intervals within a range of $\pm 45^\circ$. Data acquisition is performed using the Dantec StreamLine Pro CTA system, Dantec StreamWare software, version 6.0, and a National Instruments 9215 A/D converter (National Instruments, Austin, TX, USA), ensuring high accuracy and reliability.

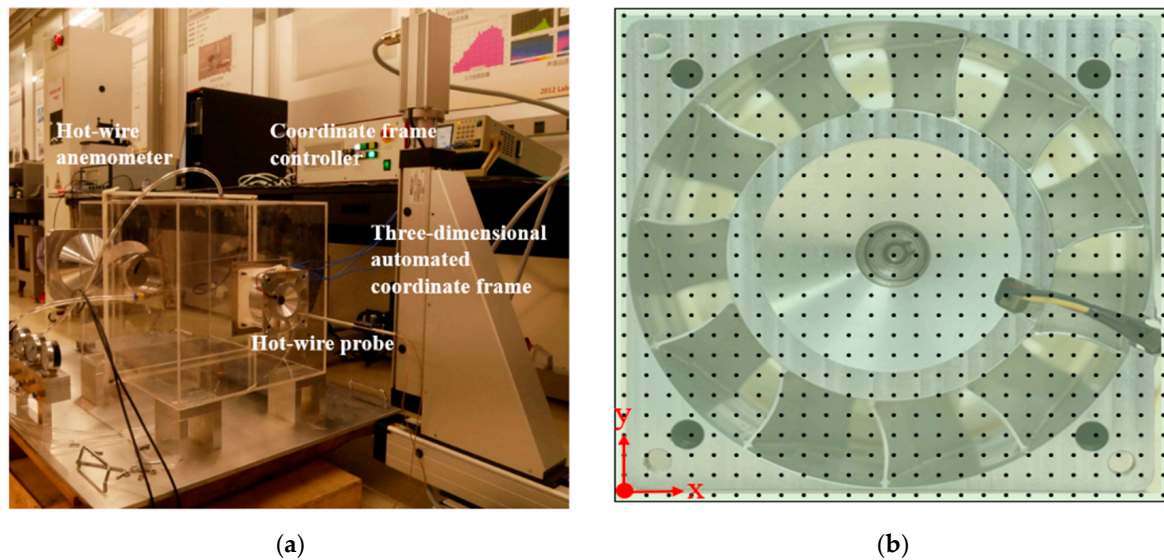


Figure 8. Hot-wire experimental equipment and measurement region: (a) hot-wire experimental setup; (b) fan outlet measurement locations.

In order to verify the effectiveness of the FDPOD method in processing asynchronous sampling data and reduce the uncertainty errors introduced by different sampling times, in this study, five independent repeated measurements were performed at each spatial sampling point under identical operating conditions. Each repeated measurement had a sampling duration of 5 s and a sampling frequency of 40 kHz. The PSD was calculated separately for each repeated measurement, and the ensemble-averaged PSD and standard uncertainty were then obtained from the five PSD results.

Figure 9 shows the PSD distribution results for five independent experimental data blocks at a single spatial sampling point. The PSD curves of each group show high consistency throughout the frequency range, indicating that the flow field characteristics captured by the different segmented experimental datasets are highly repeatable. This result confirms the reliability of the experimental data. Meanwhile, the ensemble-averaged PSD values for the five datasets smooth out the subtle disturbances within each data block, further improving the accuracy of the data.

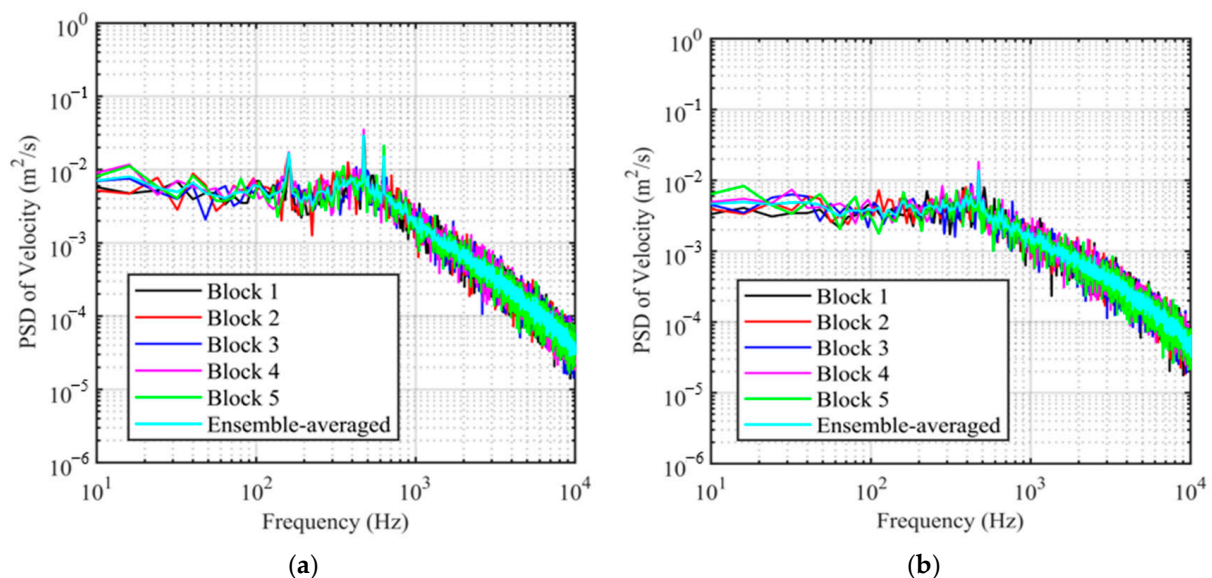


Figure 9. Comparison of PSD block processing results and ensemble average values: (a) U_z ; (b) U_x .

During the measurement process, random uncertainties are primarily due to unpredictable environmental factors such as fluctuations in atmospheric pressure, temperature, and humidity. In this experiment, five sets of measurements were made for each spatial sampling point and a statistical analysis of the PSD uncertainty was performed over all the measurement points. The standard uncertainty is calculated using the formula below:

$$u_{PSD}(\mathbf{x}) = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^n (PSD_i(\mathbf{x}) - \overline{PSD}(\mathbf{x}))^2} \quad (9)$$

where n is the number of measurement sets, $PSD_i(\mathbf{x})$ is the PSD value for the i -th set, and $\overline{PSD}(\mathbf{x})$ is the mean PSD value. The specific calculation method is $\overline{PSD}(\mathbf{x}) = \frac{1}{N_f} \sum_{i=1}^{N_f} PSD(\mathbf{x}, f_i)$.

Figure 10 shows the uncertainty analysis of the PSD values for the velocity components in both directions (U_z and U_x) for all spatial measurement points. The results show that the uncertainty for U_z is less than 1×10^{-5} for most of the measurement points, with a maximum value of 3.5×10^{-5} in a few localized regions. These higher uncertainties may be caused by momentary disturbances or measurement errors. However, the overall distribution is uniform and the values remain relatively small, further confirming the reliability and precision of the experimental data collection. Figure 11 shows the probability distribution of the uncertainty for the velocity components in both directions. The results show that the uncertainty for U_z is below 1×10^{-5} with a probability of 81.28%, while for U_x , the probability is 98.88%. This high consistency and low uncertainty across the dataset provide a robust foundation for subsequent flow field analysis.

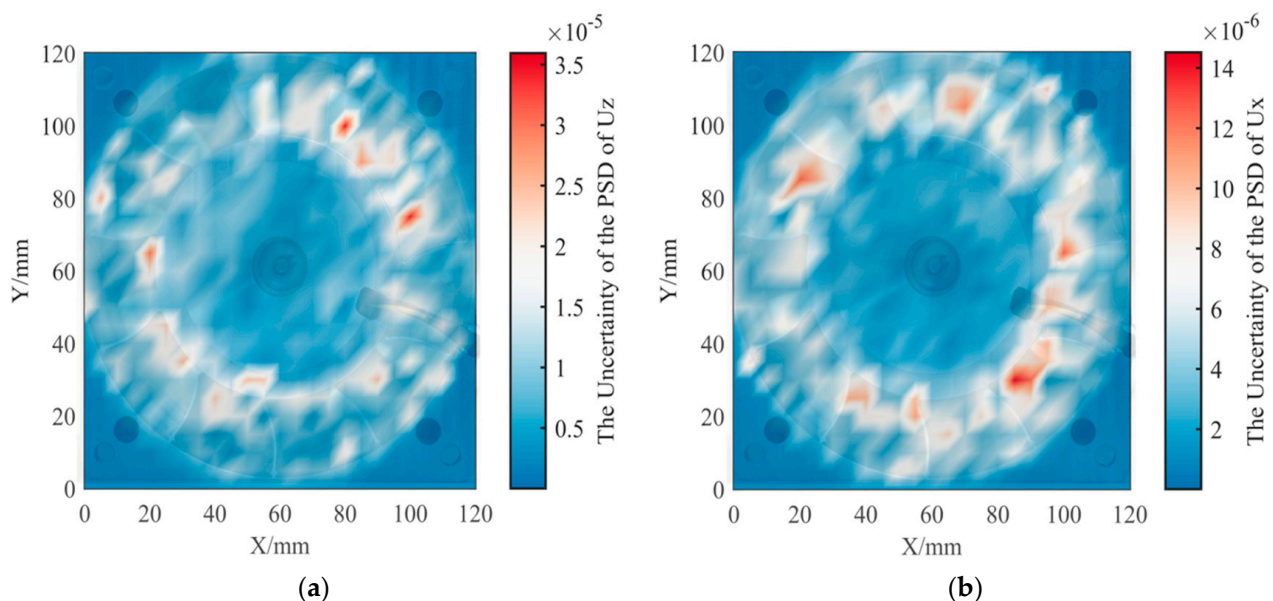


Figure 10. PSD uncertainty analysis for spatial measurement points: (a) U_z ; (b) U_x .

With the reliability of the experimental data established, further insight into the flow field characteristics can be gained by examining the PSD of the velocity. Figure 12 shows the distribution characteristics of the PSD of the velocity at different spatial positions. Figure 12a shows the PSD of the U_z , where the energy in the frequency range lower than 1000 Hz is mainly concentrated around the rotational frequency (fr) and its harmonics, indicating that this frequency range is closely related to the fan speed and periodic disturbances. In the frequency range higher than 1000 Hz, the turbulence exhibits typical inertial subrange characteristics with a spectral slope approaching $-5/3$. After entering the viscous dissipation region, the energy decays rapidly. In contrast, at point P4 near the axis,

influenced by complex vortices and secondary flows, the inertial subrange characteristics shift to lower frequencies, and the high-frequency energy decays more rapidly with a slope of about -5 , reflecting more pronounced viscous dissipation characteristics. Figure 12b shows a similar trend for the PSD of U_x , but there are differences in the characteristics at different measurement points. In particular, point P1 shows a significant energy peak at the BPF, reflecting the strong influence of periodic disturbances on the circumferential velocity near the blade tip.

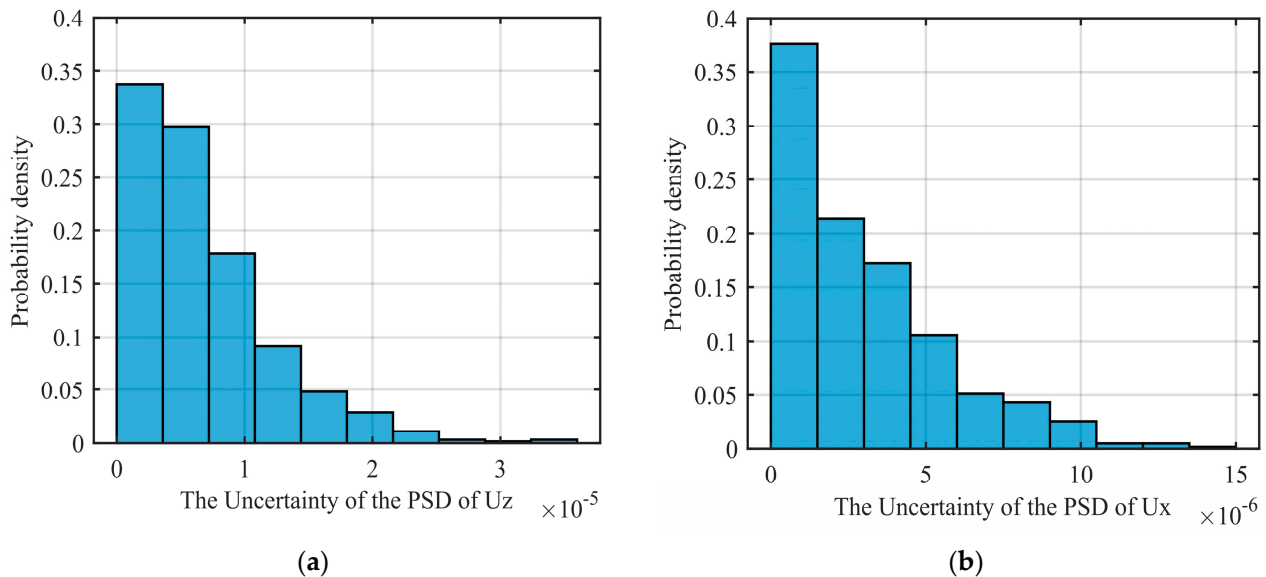


Figure 11. Probability distribution of PSD uncertainty for spatial measurement points: (a) U_z ; (b) U_x .

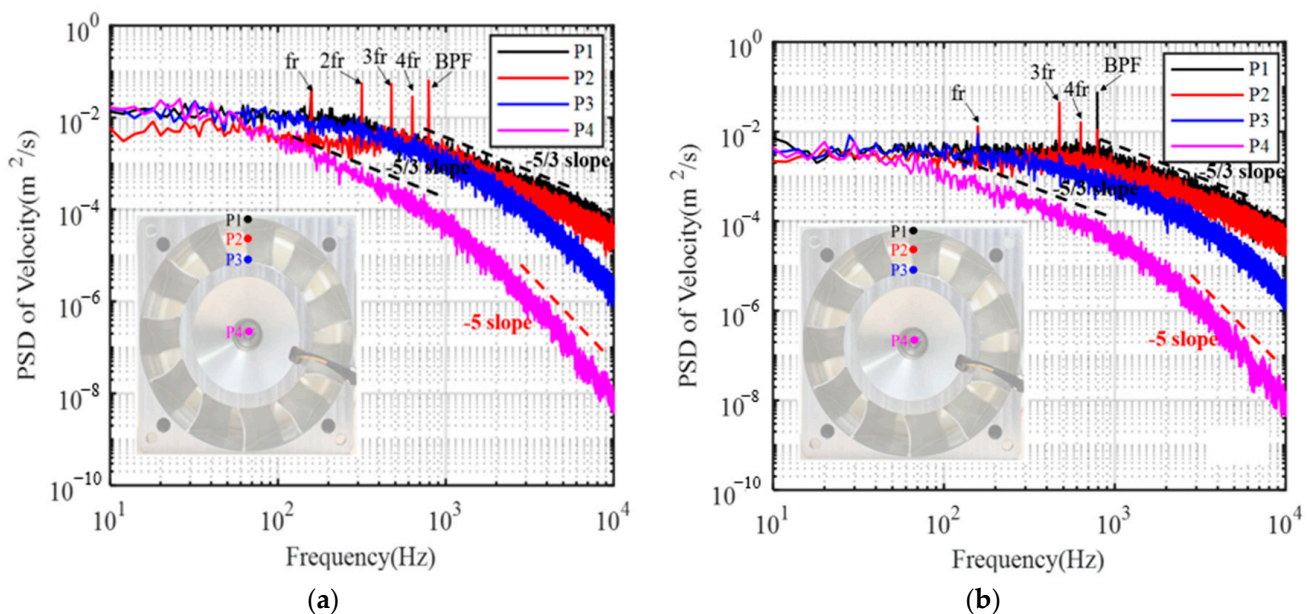


Figure 12. PSD of the velocity at different spatial positions: (a) U_z ; (b) U_x .

4.2. Comparison of FDPOD Results from Different Datasets

To evaluate the effect of asynchronous sampling on FDPOD mode analysis, hot-wire data were processed as follows: A total sampling duration of 5 s was selected, and the first 4 s of data were used for analysis. Specifically, the first spatial sampling point used data from 0 to 4 s, the second point used data from 0.1 to 4.1 s, the third point used data from 0.2 to 4.2 s, and so on. The 11th point used data from 1 to 5 s. This pattern

was then repeated to create a shuffled asynchronous sampling experimental dataset. It should be noted that this dataset is a deterministically time-shifted asynchronous dataset with prescribed temporal offsets, and it is not intended to represent all possible random asynchronous triggering conditions. This procedure did not change the sampling frequency, sampling interval, number of samples, or duration of the analyzed signals. It only changed the starting position of the selected time window among the spatial measurement points. Therefore, the comparison evaluates the sensitivity of FDPOD to deterministic inter-point temporal offsets under a constant sampling frequency, rather than to random variations in the sampling process.

Figures 13 and 14 show the mode energy distributions of U_z and U_x at the fan outlet under asynchronous sampling and manually shuffled sampling conditions. The results show that the FDPOD method can accurately extract the mode energy distribution characteristics of the primary modes with high consistency, regardless of whether the data are shuffled. The mode energy distribution is highly concentrated, with the first 5 modes capturing nearly 95% of the flow field energy.

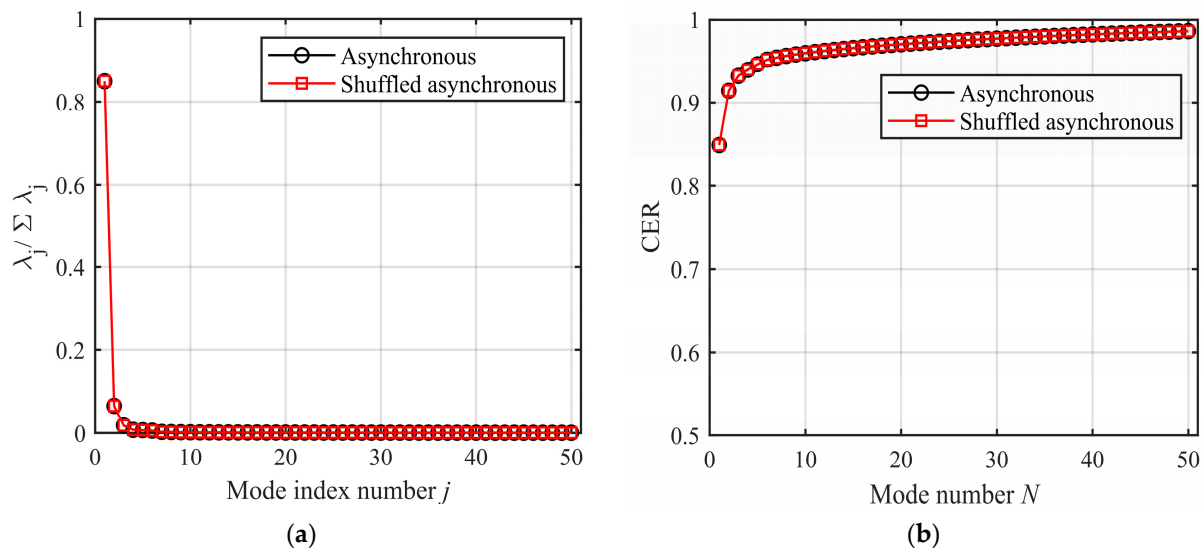


Figure 13. FDPOD mode energy distribution and cumulative energy ratio for U_z : (a) energy contribution of each mode; (b) cumulative energy ratio of the first N modes.

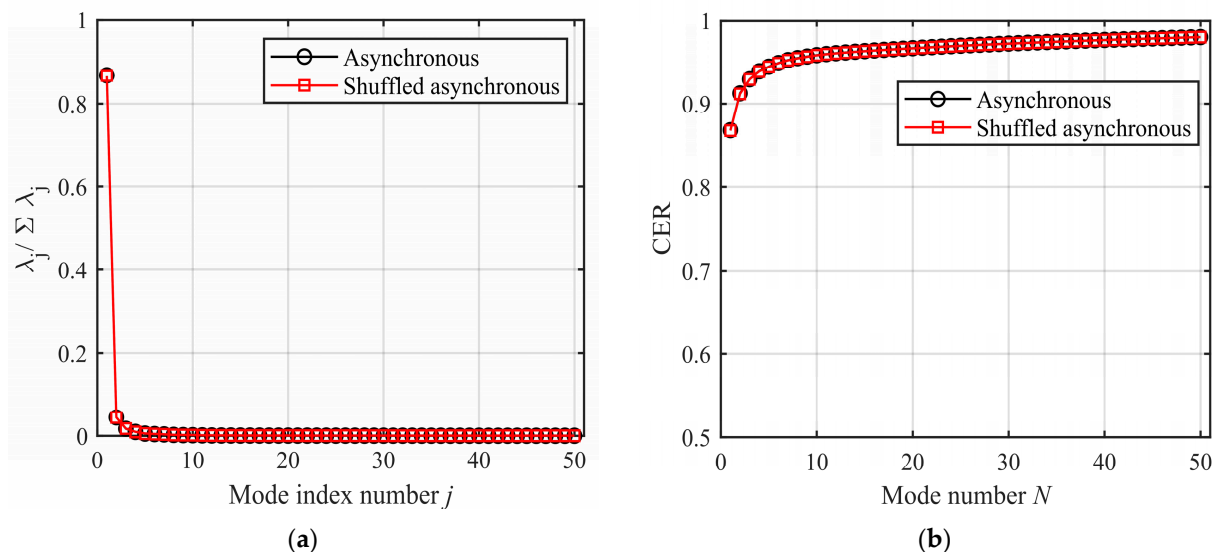


Figure 14. FDPOD mode energy distribution and cumulative energy ratio for U_x : (a) energy contribution of each mode; (b) cumulative energy ratio of the first N modes.

Tables 6 and 7 quantify the cumulative energy ratio and absolute error of the modes under the two sampling datasets. For the U_z , the cumulative energy ratios are almost identical for all modes, and the maximum relative error is only 0.02%. For the U_x , the maximum error is also only 0.03% and continues to decrease as the number of modes increases. These results indicate that the FDPOD method achieves high accuracy and consistency in processing hot-wire experimental data, without the limitation of synchronous sampling used in the snapshot POD.

Table 6. Comparison of cumulative mode energy ratios for U_z under two sampling datasets.

Number of Modes	5 Modes	10 Modes	20 Modes	50 Modes
Asynchronous	0.9461	0.9595	0.9701	0.9858
Shuffled				
asynchronous	0.9462	0.9597	0.9703	0.9858
Absolute error	0.01%	0.02%	0.02%	0%

Table 7. Comparison of cumulative mode energy ratios for U_x under two sampling datasets.

Number of Modes	5 Modes	10 Modes	20 Modes	50 Modes
Asynchronous	0.9446	0.9584	0.9667	0.9801
Shuffled				
asynchronous	0.9443	0.9582	0.9666	0.9800
Absolute error	0.03%	0.02%	0.01%	0.01%

Since hot-wire experiments have high time resolution and long sampling time at each point, the error after PSD ensemble averaging is not larger than 0.03% and can be considered negligible. Therefore, the subsequent analysis of the FDPOD mode characteristics will focus on the asynchronous sampling data extracted from the experimental measurements to further reveal the flow field characteristics.

4.3. Fan Flow Field Mode Analysis

Figure 15 shows the RMS distribution of the velocity field at the fan outlet, with separate plots for the RMS values of the U_z and the U_x . Figure 15a shows the RMS distribution of the U_z at the fan outlet, reflecting the intensity of the U_z perturbations in the flow field. The RMS values are higher near the fan blade root and hub areas, indicating stronger U_z disturbances in these regions. Figure 15b shows the RMS distribution of the U_x , which reflects the intensity of the U_x disturbances. The RMS distribution of the U_x shows similar spatial characteristics to the U_z , but with slight differences in the local details of the energy distribution.

Figure 16 shows the spatial distribution characteristics of the first four modes of the U_z . The first mode shows the main energy distribution of the flow field, with most of the energy concentrated near the blade root and hub region, which aligns closely with the RMS distribution of the U_z shown in Figure 15a. The second and third FDPOD modes represent the local energy distribution in the flow field, with most of the energy located at the trailing edge of the stationary vanes and near the blade root, respectively. The fourth mode describes smaller-scale flow characteristics with significantly reduced energy contributions, mainly near the blade tip, reflecting the presence of small-scale energy structures in the flow field. Overall, from the first to the fourth mode, the energy gradually decreases and the spatial structure of the energy distribution transitions from global to local, showing multi-scale characteristics.

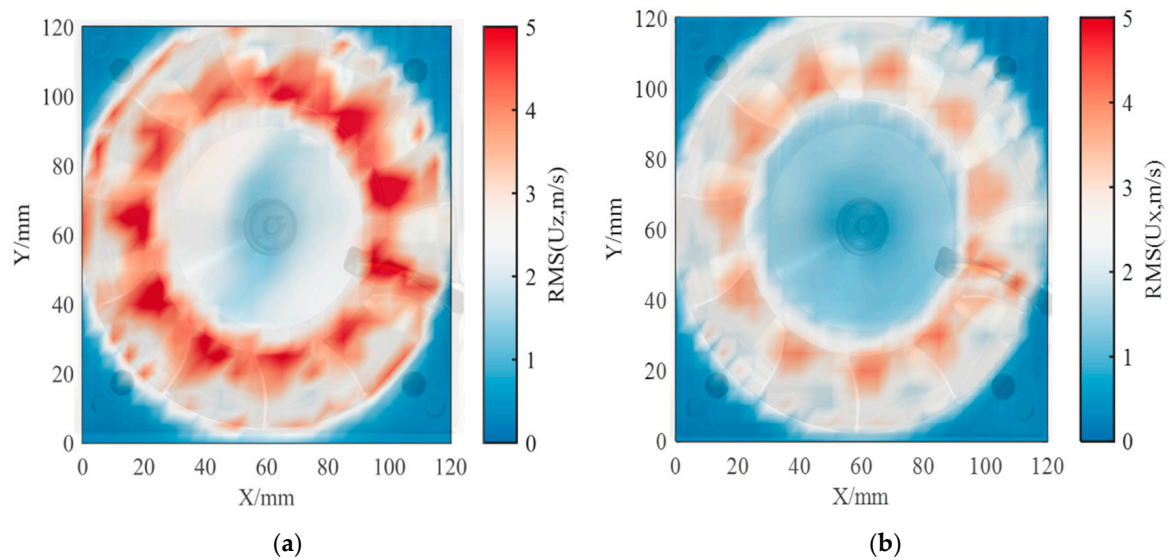


Figure 15. Velocity field RMS distribution: (a) U_z ; (b) U_x .

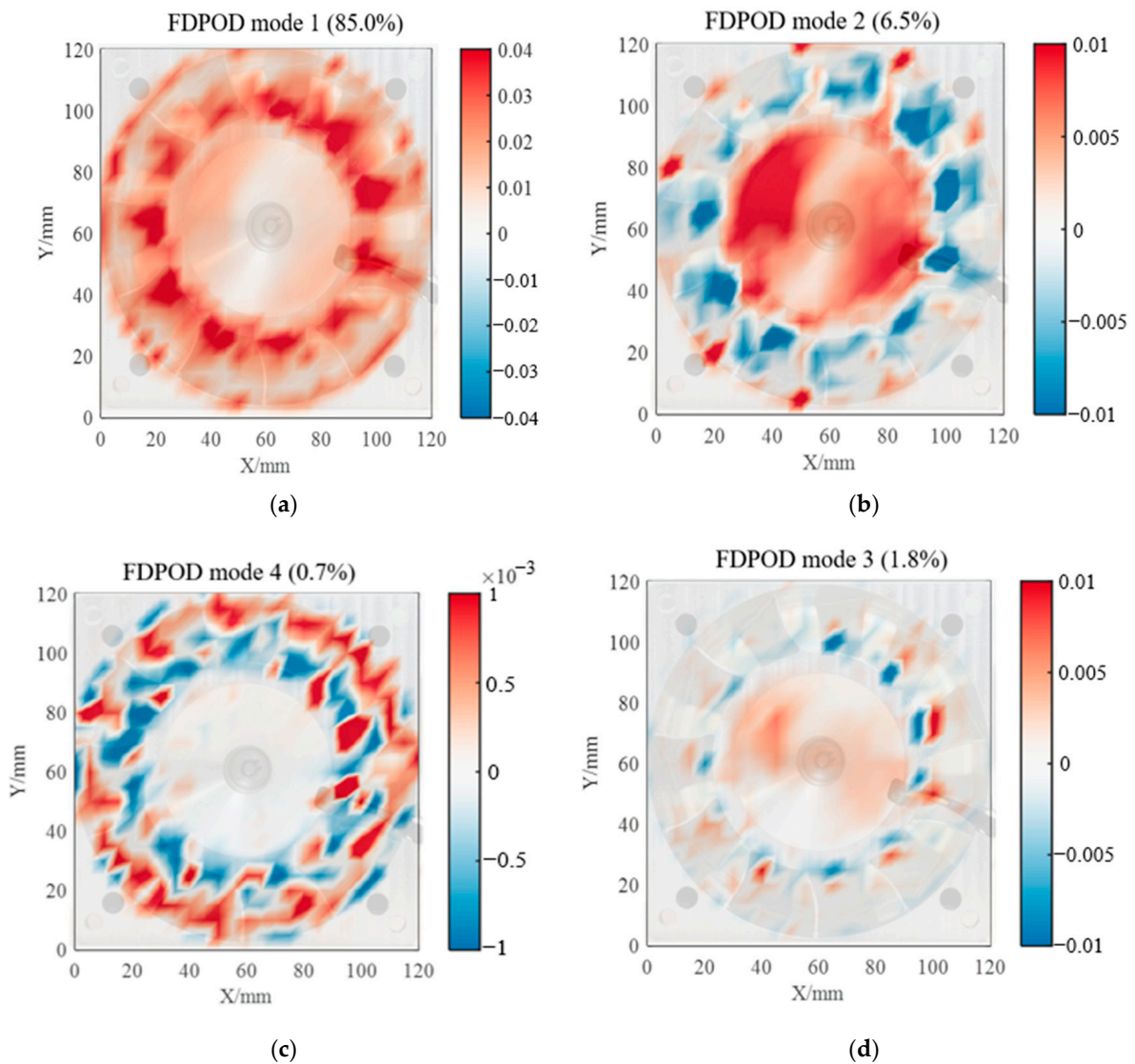


Figure 16. First four modes of U_z distribution: (a) mode 1; (b) mode 2; (c) mode 3; (d) mode 4.

Figure 17 shows the spatial distribution characteristics of the first four modes of the U_x . The first mode has an energy share of 86.8%, which is slightly higher than that of the first axial velocity mode, indicating more concentrated energy. Similarly to the first FDPOD mode of the U_z , the spatial structure of the first mode of the U_x remains consistent with the RMS distribution shown in Figure 17b. The second and third FDPOD modes represent the local spatial structure of the energy distribution, with a sharp decrease in energy contribution. The fourth mode represents the small-scale energy distribution, where the spatial distribution becomes more dispersed.

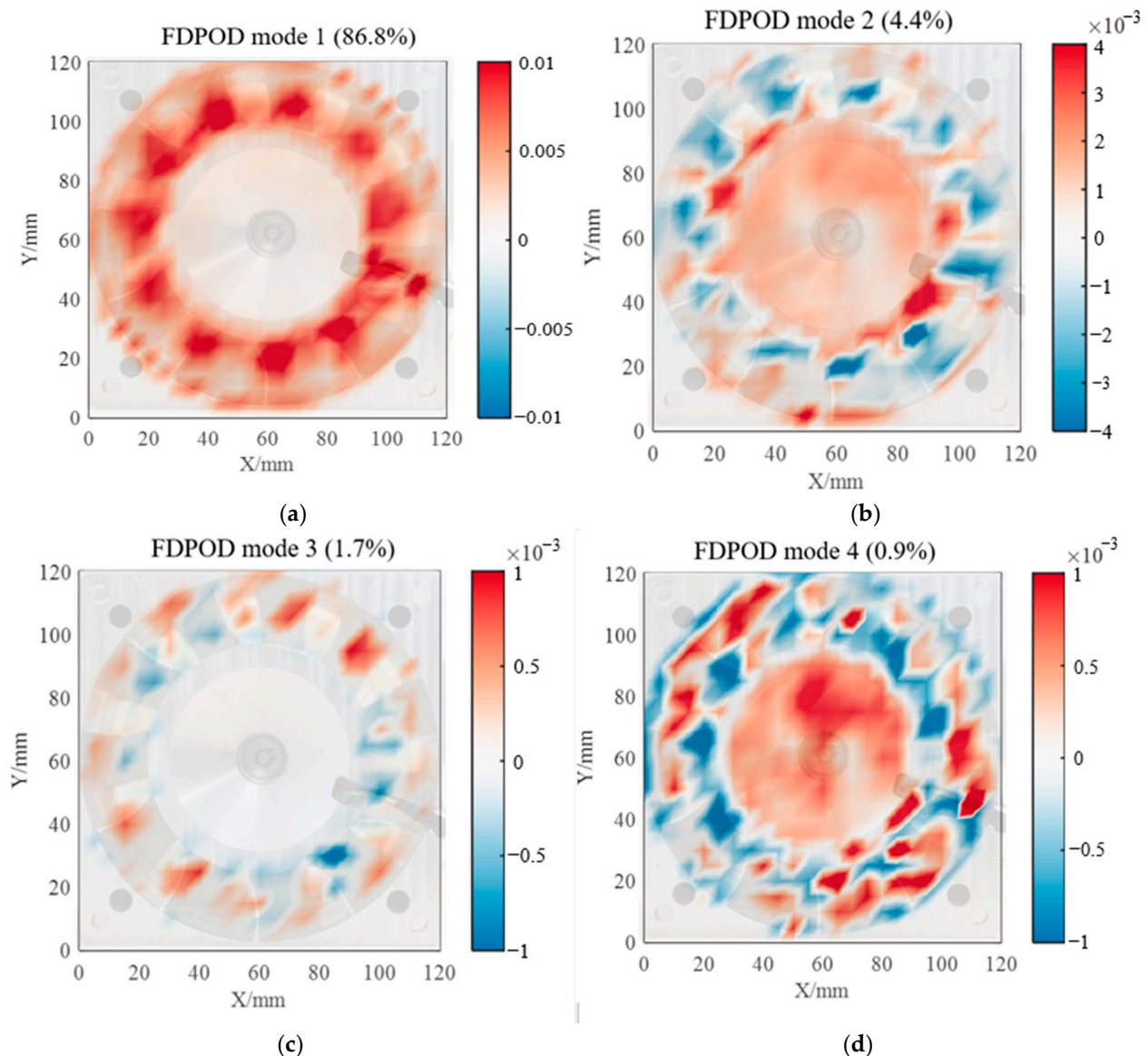


Figure 17. First four modes of U_x distribution: (a) mode 1; (b) mode 2; (c) mode 3; (d) mode 4.

Overall, the energy distribution of the two velocity components shows a high similarity, with the energy mainly concentrated near the blade root and hub region, which is consistent with the conclusion drawn from the previous numerical studies [28]. The comparison of the FDPOD modes shows that the local energy distribution characteristics in both velocity components are similar, indicating that the energy distribution of the turbulence at the fan outlet approximates isotropic characteristics.

5. Conclusions

This study introduces a frequency-domain proper orthogonal decomposition (FD-POD) method to address the limitations of conventional snapshot POD in processing asynchronously sampled data. The method's effectiveness is demonstrated through numerical simulations of a subsonic jet and experimental hot-wire measurements of a mixed-flow fan. The following key findings are derived from the analyses:

- (1) By integrating frequency-domain transformation to decouple phase discrepancies in asynchronous sampling and segmented ensemble averaging of power spectral density (PSD) to suppress spectral leakage errors, FDPOD achieves pioneering modal extraction and flow field reconstruction from asynchronous datasets. In subsonic jet simulations and mixed-flow fan experiments, the cumulative energy error for the first 50 modes remains as low as 0.3% with flow reconstruction accuracy reaching 99.5%, significantly outperforming the dependency of conventional snapshot POD on synchronous sampling constraints.
- (2) FDPOD demonstrates superior efficiency by requiring 80% fewer modes than conventional snapshot POD while maintaining dominant coherent structures, i.e., 99.7% consistent with synchronous POD modes. This advantage proves particularly valuable for multi-scale flow analysis, offering a robust reduced-order modeling framework for large-scale distributed measurement systems.
- (3) The method exhibits exceptional tolerance to temporal disturbances, with hot-wire anemometry measurement errors consistently below 0.03% for both asynchronous and temporally shuffled datasets. This breakthrough resolves long-standing technical barriers in modal decomposition for asynchronous measurement scenarios, establishing a reliable framework for complex unsteady flow analysis.
- (4) By establishing mappings between frequency-domain energy ranking and spatial modes, FDPOD overcomes the synchronous sampling limitations of conventional snapshot POD and provides theoretical foundations for asynchronous monitoring of unsteady phenomena. Its unique advantages in multi-sensor collaborative measurement scenarios highlight broad industrial applicability.

These results establish FDPOD as a practical tool for unsteady flow analysis, particularly in scenarios where synchronous data acquisition is challenging. It should be emphasized that asynchronous sampling in the present study does not refer to randomly varying sampling frequencies or irregular sampling intervals. In both the numerical construction and the hot-wire experiment, the sampling frequency and sampling interval remained constant. The asynchronous characteristic resulted from different acquisition starting times among spatial positions, which prevented the measured signals from being assembled into synchronized instantaneous full-field snapshots. Therefore, the conclusions of this study should be interpreted in the context of deterministic time offsets and sequential experimental measurements. Future work will further explore the applicability of FDPOD to randomly triggered asynchronous measurements, explore extensions to multi-physics systems and the integration of machine learning for adaptive modal extraction from non-uniformly sampled data, aiming to further enhance flow analysis in complex engineering contexts.

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