

Article

Enhanced Strategy for Optimizing Net Energy Consumption of Railway Systems Using Speed Profile and Variable Headway

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Abstract

Energy-efficient operation of railway systems is of great importance for both environmental and economic reasons. Minimizing net energy consumption helps to achieve such energy-efficient operation. In this paper, the train's speed profile and headway between trains are controlled to achieve lower traction energy consumption and higher train synchronization for better regenerative braking energy utilization. Eventually, the net energy consumption, defined as the difference between the traction energy consumption and the utilization of regenerative braking energy, is minimized. Two optimization problems are defined to solve the problem efficiently. The first main problem is to find the optimal speeds at each segment of the railway track. The second sub-problem's objective is to find the optimal values of travel time, dwell time, and headway for every suggested solution to the main problem. Both problems are solved using the genetic algorithm. Numerical results are based on the actual operation data of the Beijing Metro Yizhuang Line in China. In the numerical results, the proposed strategy of dividing the problem into two problems and the use of variable headway shows an enhancement in reducing net energy consumption by 7.5% compared to other strategies in the literature.

Keywords: crossover; energy efficient; genetic algorithm; headway; mutation; net energy consumption; railway; speed profile



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1. Introduction

Using a railway system helps passengers avoid traffic congestion. Moreover, optimizing railway operations helps save energy, which is reflected by both the environment and operational cost [1,2].

Railway operation is characterized by a train's timetable, speed profile, and the headway between trains [3]. The timetable includes the arrival/dwell times of the trains at the station [4,5]. The speed profile is much more general than the timetable [6,7]. It includes the entering time and speed of the train for each segment of the track. Moreover, speed–distance curves are generated. Given the speed profile of a train, the timetable can be derived after accounting for dwell times [8,9].

In order to increase the energy efficiency of railway operation, regenerative braking energy can be utilized to compensate for all/part of the traction energy required by ac-

celerating trains [2,10]. For better use of regenerative braking energy, both braking and accelerating trains need to be in close proximity [11]. This requires efficient control of the railway parameters of the trains' speeds and headway.

To handle the optimization problem of energy saving, a heuristic optimization algorithm is usually used [12–16]. The genetic algorithm (GA), particle swarm optimization (PSO), and variable neighborhood search (VNS) are examples of meta-heuristic algorithms used in the literature to solve the energy saving problem of a railway system. Moreover, the GA serves as a common choice for this problem [1].

1.1. Related Work

Energy savings in railway systems are improved through many forms of objective functions. In [12,13,16–18], net energy consumption is minimized. Net energy consumption is defined as the difference between the required traction energy and the utilized regenerated braking energy. In [15], the utilized braking energy is maximized. This could be considered equivalent to minimizing net energy consumption. However, in [14], only traction energy consumption is minimized without taking regenerated braking energy into consideration.

As stated above, the speed profile is a more general concept than the timetable. It enables full control of the train's movement to achieve high energy efficiency. Despite the importance of optimizing the speed profile, only limited work has considered controlling it in detail [12,17,19]. In [12], the railway track is partitioned into segments that include stations and turnaround stations. The entering speed and entering time of each segment are controlled. Moreover, the speed–distance curve is controlled using three main train action strategies, namely, traction, coasting, and braking. In [17], Ran et al. developed a joint optimization framework for train speed profiles and timetables on the Beijing Metro Yizhuang Line. With a fixed headway, their work focuses on adjusting running, dwell, and turnaround times under a detailed physics-based resistance model (including gradient, curve, and tunnel effects). In [19], traction energy consumption is minimized. The decision variables are the detailed speed trajectories generated by slicing and recombining segments of other feasible speed profiles through a curve-splicing technique. Nevertheless, this model does not consider regenerative braking energy, and the headway is considered fixed. Other works in the literature have oversimplified the problem of finding the optimal speed profile by selecting only the best profile from a finite set of profiles [13,14,16,20]. In [15], a timetable is controlled, which is composed of the headway between trains, the dwell times at the stations, running times, and turnaround times.

The headway is considered fixed in most related work [12–15,17–19]. However, by dealing with the headway between trains as a variable, this can play an important role in the synchronization of trains to maximize the amount of regenerated braking energy [16]. Moreover, it can help to satisfy the demand for passengers during peak hours [20]. In [16], the headway is fine-tuned within a specific range, with the aim of increasing the synchronization of the train. Consequently, regenerated braking energy is increased. In [20], a multi-objective optimization problem is formulated to minimize headway deviations and variations, rolling stock usage, and net energy consumption. The decision variables include the selection of running levels (from a predefined set of speed profiles), dwell levels, arrival/departure times, and rolling stock assignments. For a comprehensive overview of the various energy-saving strategies in urban rail systems, including speed profile optimization, timetabling, and regenerative energy utilization, the readers are referred to the survey in [21].

Table 1 summarizes the literature review.

Table 1. Literature review summary.

Paper No.	Objective Function	Decision Variables	Speed Profile	Regen. Energy	Optim. Tool
[12]	Net energy consumption	Speed profile	Detailed	✓	GA
[17]	Net energy consumption	Speed profile	Detailed	✓	PSO-GA
[19]	Energy consumption	Speed profile	Detailed	×	Simulated annealing
[13]	Net energy consumption + operating cost of rolling stocks	Timetable + speed profile strategy + circulation plan	From a set of strategies	✓	PSO
[14]	Energy consumption + passenger travel time + operating cost of rolling stocks	Timetable + speed profile strategy + circulation plan	From a set of strategies	×	VNS + CPLEX
[20]	Net energy consumption + passenger demand	Timetable + speed profile strategy + circulation plan + headway	From a set of strategies	✓	CPLEX
[15]	Passenger travel time + utilized energy + installed number of ESDs	Timetable	×	✓	NSGA-II
[18]	Net energy consumption	Timetable	×	✓	LP
[16]	Net energy consumption	Timetable + speed profile strategy + headway	From a set of strategies	✓	GA
Proposed work	Net energy consumption	Speed profile + headway	Detailed	✓	GA

1.2. Work Motivation and Contribution

Based on the literature review, some features of railway operation can be further investigated to improve energy efficiency and better use of regenerated braking energy. The speed profiles, headways, and how to handle the optimization problem can be improved. The speed profile is considered in detail, similarly to [12], and the headway is considered a variable, similarly to [16].

Considering both the speed profile and headway as decision variables to minimize net energy consumption in one optimization problem is challenging. Similar to most work in the literature, a GA is used to solve the proposed optimization problem. Compared to [12], which deals with the large search space of the entering speeds and the entering times in one optimization problem, two optimization problems can instead be considered. The first addresses the entering speeds of each track segment (continuous optimization problem). The generation of possible feasible solutions for the first optimization problem is significantly improved compared to the work in [12] in terms of computational time. While [12] successfully incorporates railway boundary conditions, optimizing mechanical and timetable parameters simultaneously creates a coupled search space where a high percentage of mutated chromosomes represent physically impossible trajectories. Our novelty lies in structurally decoupling this space. By implementing a segment-by-segment constraint-handling mechanism governed by analytical kinematic bounds, the continuous search domain is restricted a priori to physically viable solutions. This deterministic filtering fundamentally lowers the algorithm's computational complexity by eliminating the heavy overhead usually required for post-genetic constraint-checking or chromosome repair loops. For each candidate speed, another optimization problem for timing (entering times of each track segment, travel time, and headway time) is considered. The second optimization problem is discrete and bounded. It can be efficiently solved with less computational complexity, which retains the bulk of the available computational power for the first optimization problem. Moreover, the strategies for crossover and mutation are

modified for better exploration and exploitation of the candidate solutions. Compared to [17], which optimizes trajectories under a rigid fixed-headway regime, the present work departs significantly by introducing a two-level optimization strategy: a main GA for segment-wise speed profiles and a sub-GA for variable dwell times and variable headways within practical bounds, thereby capturing more operational flexibility. Moreover, instead of the computationally heavy hybrid PSO-GA used by [17], the proposed method leverages an efficient split GA design that improves convergence while still minimizing net energy consumption, effectively isolating the mechanical energy-saving impacts of driving modifications in the master problem from the network synchronization benefits of variable headways managed in the slave sub-problem. Similarly, while [16] incorporates variable headways, it simplifies traction optimization by selecting speed profiles from a static, pre-generated discrete set based on station running times. In contrast, our framework employs a bidirectional feedback loop where the continuous trajectory profiles are dynamically adjusted in the master layer, generating custom, optimized driving strategies that actively shape and support the scheduling updates in the slave sub-problem.

The contributions of this paper are as follows:

- Faster strategy for generating feasible entering speed for each segment with tighter travel time bounds.
- An algorithm for generating the entering time and variable headway within a cycle time constraint. “Note: There is a violation of the cycle time constraint in [12]”.
- Contributions related to the optimization problem handling:
 - New, effective and faster strategy for the crossover operation.
 - New, effective and faster strategy for the mutation process.
 - Structural reduction in search space dimensionality, leading to a smaller number of function evaluations per iteration.
 - Dividing the optimization problem into two problems, as follows:
 - * The main problem for the continuous variable of the entering speed of each segment.
 - * The sub-problem for the discrete entering time, travel time, and variable headway, where the main problem provides bounds on the travel time of each segment. Consequently, the sub-problem is faster and does not require high computational complexity.
- Using variable headway to enhance train synchronization to minimize net energy consumption.

1.3. Paper Organization

The rest of this paper is organized into four sections. The system model and assumptions are presented in Section 2. Analysis of the proposed optimization problem is presented in Section 3. The numerical results are shown in Section 5. Finally, concluding remarks and future work are given in Section 6.

2. System Model and Assumptions

In this section, the model under study is described and the assumptions considered are presented. For clarity, the notation used in this paper is introduced in Table 2.

Table 2. Summary of notation used in the model.

Symbol	Description	Symbol	Description
α	Train's acceleration rate.	R	The running resistance.
$\alpha_{\max}$	Maximum traction force per unit mass.	S_{stations}	Set containing segment indices of stations.
β	Coasting deceleration rate.	$S_{\text{sub}_i}^{\text{Segment}}$	Set containing segment indices of segments belonging to substation i .
b_n	Switching time from coasting to braking at segment n .	$S_{\text{sub}_i}^{\text{Station}}$	Set of stations belonging to substation i .
C	Train's maximum passenger capacity.	Sub_i	Substation of the i th train.
c_n	Switching time from acceleration to coasting at segment n .	$S_{\text{turnaround}}$	Set containing segment indices of turnaround stations.
d_{e_n}	Segment n end distance.	T_{cycle}	The cycle time.
d_{s_n}	Segment n start distance.	T_D	Dwell time vector.
e_n	Entering time of segment n .	$T_D^{\max}$	Maximum dwell time vector.
e_n^{long}	Entering time of segment n corresponding to the longest travel time.	$T_D^{\min}$	Minimum dwell time vector.
e_n^{short}	Entering time of segment n corresponding to the shortest travel time.	$t_{\text{end}}(\text{cyc})$	Cycle cyc end time.
E_{Net}	Overall net energy consumption.	T_{fixed}	Fixed total time of generated random integers of Algorithm 2.
$E_{\text{Net},\text{cyc}}$	Net energy consumption for cycle cyc .	T_H	Headway time vector.
$E_{\text{Reg}}(t, Sub_i, i)$	Regenerative braking energy of train i at time t and substation Sub_i .	T_{H_i}	Headway time between train i and train $i + 1$.
$E_T(t, Sub_i, i)$	Traction energy of train i at time t and substation Sub_i .	$T_H^{\max}$	Maximum headway time vector.
$E_{\text{Trac}}(t, s)$	Traction energy at time t and substation s .	$T_H^{\min}$	Minimum headway time vector.
γ	Train's braking rate.	θ_1	Conversion factor from electricity to kinetic energy.
$\gamma_{\max}$	Maximum braking force per unit mass.	θ_2	Conversion factor from kinetic energy to regenerative braking energy.
K	Number of actual stations.	θ_3	Transmission loss factor of regenerative braking energy.
M	Train's mass.	T_{op}	Operation time.
m_n	Maximum speed at segment n .	$Tr^{\text{long}}(n)$	Longest travel time of segment n .
m_n^{long}	Maximum speed at segment n for longest travel time.	$Tr^{\text{short}}(n)$	Shortest travel time of segment n .
N_{cycles}	Number of cycles.	$t_{\text{start}}(\text{cyc})$	Cycle cyc start time.
N_{segments}	Number of segments.	T_{turn}	Turnaround time.
N_{stations}	Number of stations with up and down direction.	u_n	Upper speed limit of segment n .
N_{sub}	Number of substations.	$U_{\text{Reg}}(t, s)$	Utilization of regenerative braking energy at time t and substation s .
N_{trains}	Number of trains.	$v(t)$	Train velocity at time t .
N_{vars}	Number of decision variables.	$v_{\text{state}}(t_i)$	Speed state at train's i time t_i .
P_{cross}	Crossover probability.	z_n	Braking speed at segment n .
P_{mutation}	Mutation probability.	z_n^{long}	Braking speed at segment n for longest travel time.
q_n	Entering speed of segment n .	z_n^{short}	Braking speed at segment n for shortest travel time.

2.1. The Infrastructure

A two-track railway line with up/down directions is assumed. Moreover, each adjacent group of stations is connected to the same substation. Let $S_{sub_i}^{Station}$ be the set of stations belonging to substation i , the number of substations is N_{sub} , and the number of stations is $N_{stations}$ (K stations in the up direction and the same K stations in the down direction). The railway line has two endpoint stations (station 1 and station K , where $K = N_{stations}/2$). Additionally, two turnarounds are assumed at the endpoint stations with turnaround time assumed to be fixed (T_{turn}) and a depot at station 1. Figure 1 illustrates the infrastructure under study.

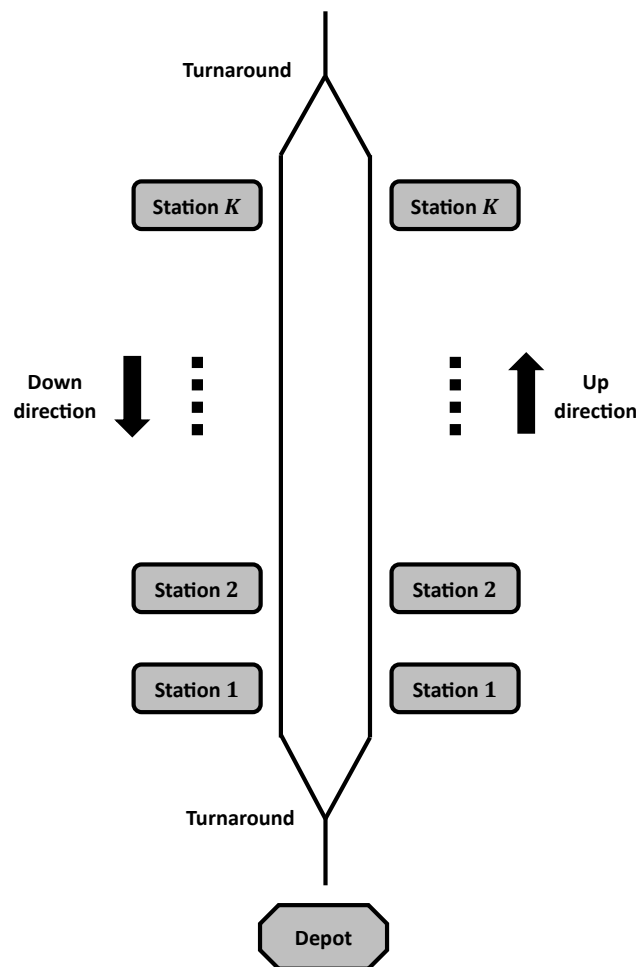


Figure 1. The considered railway line.

It is a common assumption to divide the total route into segments where each segment has its speed limit (u_n). This can be shown for a general segment n , as in Figure 2. Moreover, each segment is defined by its start distance (d_{s_n}) and end distance (d_{e_n}). The partition into segments and the different speed limits for different segments are the result of curvature, track conditions, etc. A total of $N_{segments}$ is defined for the assumed railway. $S_{stations}$ and $S_{turnaround}$ are the sets containing the segment indices of stations and turnaround stations, respectively. Moreover, let $S_{sub_i}^{Segment}$ be the set of segments belonging to substation i .

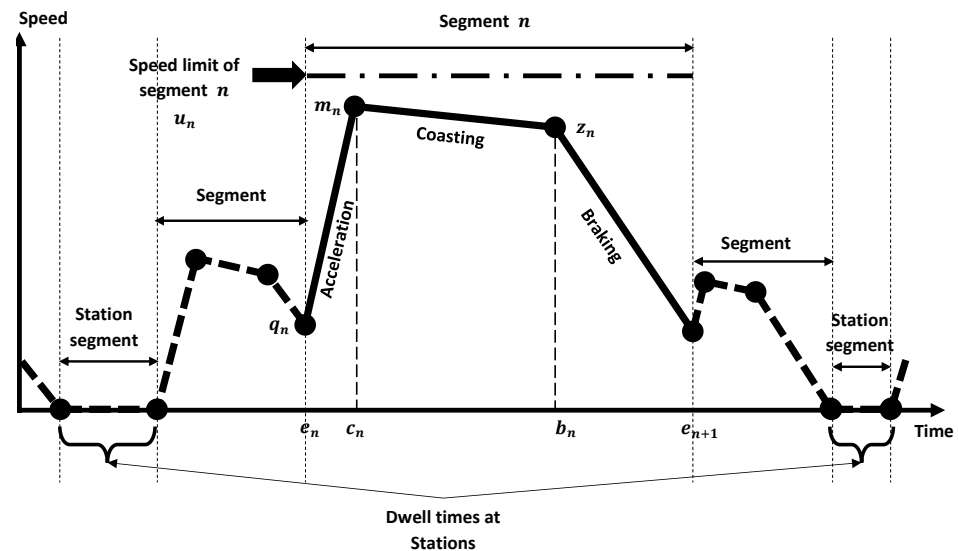


Figure 2. Speed profile and track segments.

2.2. The Trains

N_{trains} identical trains are assumed, where each train has a mass M in kg and a maximum capacity for passengers C . Figure 2 shows the considered phases for a train's speed, namely, acceleration, coasting, and braking. In the acceleration phase, traction force is applied, and the train accelerates with a rate of α m/s. α is given by

$$\alpha = \alpha_{\max} - R, \quad (1)$$

where $\alpha_{\max}$ is the maximum traction force per unit mass and R is the running resistance. It is noted that $\alpha_{\max}$ and R are assumed constant for simplicity.

In the coasting phase, no traction force is applied, and the train moves with natural deceleration at a rate of β m/s. β is given by

$$\beta = R. \quad (2)$$

In the braking phase, no traction force is applied and braking force is applied. The deceleration rate in this case is γ m/s. γ is given by

$$\gamma = \gamma_{\max} + R, \quad (3)$$

where $\gamma_{\max}$ is the maximum braking force per unit mass, which is assumed to be constant.

2.3. The Speed Profile

The speed–time graph in Figure 2 is called the speed profile. For each segment n , the speed profile is characterized by the entering speed q_n , the maximum reached speed m_n , and the braking speed z_n . Moreover, the speed profile is characterized by the entering time e_n , the switching time from acceleration to coasting c_n , and the switching time from coasting to braking b_n . Additionally, the speed profile is defined for a single cycle time. Moreover, it is assumed that the profile is identical for all cycles and for all trains.

From the continuity of the train's speed, it can be observed that the acceleration straight line should intersect the coasting straight line at c_n . Moreover, the coasting straight line should intersect with the braking straight line at b_n . Hence, c_n and b_n could be calculated by solving the two equations

$$q_n + \alpha(c_n - e_n) = m_n \quad (4)$$

and

$$z_n = q_{n+1} + \gamma(e_{n+1} - b_n), \quad (5)$$

where the braking speed z_n can be calculated using the equation

$$z_n = m_n - \beta(b_n - c_n). \quad (6)$$

Using Equations (4)–(6), the switching times c_n and b_n can be calculated as

$$c_n = e_n + \frac{m_n - q_n}{\alpha} \quad (7)$$

and

$$b_n = (\gamma e_{n+1} - m_n - \beta c_n + q_{n+1}) / (\gamma - \beta), \quad (8)$$

respectively.

Based on the aforementioned analysis, the speed profile can be completely defined using only the speeds q_n and m_n and the entering time e_n for each segment. The rest of the parameters can be calculated using these parameters.

2.4. Regenerative Braking Energy

The technology of regenerative braking energy is assumed to be available. It is assumed that if two trains are in proximity, where one train is in the accelerating phase and the other one is in the braking phase, then the energy resulting from braking can be utilized to compensate for part/all of the traction energy required for acceleration. The proximity of trains is assumed similar to the work in [12], where the trains within the same substation are assumed to be in close proximity. The conversion factor from electricity to kinetic energy is θ_1 . The conversion factor from kinetic to regenerative energy is θ_2 . The transmission loss factor of regenerative energy is θ_3 .

The assumptions of constant running resistance, maximum acceleration/braking rates, and fixed conversion efficiencies are adopted to preserve the closed-form tractability of the optimization framework similar to [12]. By using these constant parameters, we effectively decouple the optimization algorithm's performance from highly non-linear environmental variables, allowing for a rigorous validation of the structural efficiency of our split master-slave genetic algorithm strategy. We acknowledge, however, that actual railway systems operate within a complex, heterogeneous environment where parameters—such as track gradients, curvature, and fluctuating passenger loads—introduce significant stochasticity. Furthermore, real-world traction power networks are subject to non-linear electrical constraints, including line impedance and voltage fluctuations, which are omitted here to establish a clear baseline for evaluation. Although this structural simplification is necessary for the present analysis, incorporating such dynamic operational complexities remains an important direction toward future industrial-grade applications.

2.5. Passenger Demand

The passenger demand can be satisfied by many parameters, e.g., the train's capacity C , the dwell time at stations T_D , the number of cycles N_{cycles} , and the headway time between trains T_H .

The train's capacity is assumed to be fixed and identical for all trains. Lower and upper bounds can be defined for the dwell time at stations ($T_D^{\min}$ and $T_D^{\max}$, respectively). Similarly, lower and upper bounds can be defined for the headway time between trains ($T_H^{\min}$ and $T_H^{\max}$, respectively).

The cycle time can be defined in terms of the headway time as follows

$$T_{\text{cycle}} = \sum_{i=1}^{N_{\text{trains}}-1} T_{H_i}, \quad (9)$$

where T_{H_i} is the headway time between train i and train $i + 1$.

The operation time T_{op} can be defined in terms of the cycle time and the number of cycles as

$$T_{\text{op}} = T_{\text{cycle}} \times N_{\text{cycles}}, \quad (10)$$

where N_{cycles} is assumed to be fixed.

3. Analysis of the Proposed Optimization Problem

In this section, the optimization problem is described.

3.1. The Objective Function

The objective function is to minimize net energy consumption E_{Net} . Net energy consumption is defined as the difference between the required traction energy for all trains during operation and the utilization of regenerative braking energy during this time. Given the speed profile, the headway vector, and the number of operation cycles, net energy consumption can be evaluated using Algorithm 1.

First, three distinctive cycles are identified, namely, the first cycle, intermediate cycle, and last cycle. In the first cycle, trains leave the depot one by one, separated by the headway time. Consequently, the net energy consumption in the first cycle needs to be calculated separately. The intermediate cycle (labeled cycle 2 in the algorithm) is identical for all cycles, except for the first and last cycles. In this cycle, all trains are already out of the depot and in operation. It is enough to calculate the net energy consumption for any of these cycles and multiply by the number of intermediate cycles. The last cycle also needs special treatment when calculating net energy consumption, as the trains are ending their operation and going back to the depot one by one. The start and end times of each of these special cycles are identified in Algorithm 1 in lines 3 through 11.

The speed profile is considered for one cycle, and is assumed to be identical for any other cycle and for all trains. Consequently, to determine the train's segment and the speed phase of the train (stopped, acceleration, coasting, or braking), the train's time referenced to the first cycle needs to be calculated. At any operation time t , the i th train time referenced to the first cycle (t_i) is given as follows

$$t_i = \left(t - \sum_{k=1}^{i-1} T_{H_k} \right) - \left\lfloor \frac{\left(t - \sum_{k=1}^{i-1} T_{H_k} \right)}{T_{\text{cycle}}} \right\rfloor \times T_{\text{cycle}}. \quad (11)$$

Comparing t_i to the entering time of each segment from the speed profile, the segment number of the i th train can be determined as follows

$$n_i = \arg \min_n (t_i - e_n); t_i > e_n. \quad (12)$$

After calculating the train's segment (n_i), if the segment is not a station or a turnaround segment, the speed state can be determined as

$$v_{\text{state}}(t_i) = \begin{cases} \text{Acceleration} & , e_{n_i} \leq t_i \leq c_{n_i} \\ \text{Braking} & , b_{n_i} \leq t_i < e_{n_i+1} \\ \text{Otherwise} & , c_{n_i} < t_i < b_{n_i} \text{ or } v(t_i) = 0, \end{cases} \quad (13)$$

where $v(t_i)$ is the train's velocity at time t_i ; moreover, $v(t_i) = 0$ at stations and turnarounds.

Algorithm 1 Simulation algorithm to evaluate net energy consumption.

Input: Speed profile, headway vector, operation time, simulation time.

Output: Net energy consumption.

```

1: for  $i = 1$  to 3 do    ▷ Determine start and end times for each cycle (cycles 1, 2, and last
   cycle).
2:   switch  $cyc$                                 ▷ First cycle.
3:     case 1:
4:        $t_{start} = 1$ .
5:        $t_{end} = T_{cycle} - 1$ .
6:     end case
7:     case 2:                                ▷ Second cycle (repeated identically).
8:        $t_{start} = T_{cycle}$ .
9:        $t_{end} = 2 \times T_{cycle} - 1$ .
10:    end case
11:    case 3:                                ▷ Last cycle.
12:       $t_{start} = T_{op}$ .
13:       $t_{end} = T_{op} + T_{cycle}$ .
14:    end case
15:  end switch
16:  for  $t = t_{start}$  to  $t_{end}$  do
17:    for  $i = 1$  to  $N_{trains}$  do
18:      Calculate train time according to (11).    ▷ Train time referenced to the cycle
time.
19:      if Train time < 0 then                                ▷ Train is still at depot.
20:        Go to next train.
21:      else                                ▷ Train is in operation.
22:        Determine train's segment ( $n_i$ ) using (12).
23:        Determine if the train is in acceleration, braking, or other states ( $v_{state}$ )
using (13).
24:        if  $v_{state} = \text{acceleration or braking}$  then
25:          Determine substation of the train  $n_i$  using (14).
26:          if  $v_{state} = \text{acceleration}$  then
27:             $E_T(t, Sub_i) = E_T(t, Sub_i) + E_T(t, Sub_i, i)$     ▷ Update substation
traction energy at time  $t$  using (15).
28:          end if
29:          if  $v_{state} = \text{Braking}$  then
30:             $E_{Reg}(t, Sub_i) = E_{Reg}(t, Sub_i) + E_{Reg}(t, Sub_i, i)$     ▷ Update
substation regenerative energy at time  $t$  using (16).
31:          end if
32:        end if
33:      end if
34:    end for
35:    Calculate utilization of regenerative braking energy for each substation  $j$  at time
 $t$  using (17).
36:  end for
37:  Calculate net energy consumption of cycle  $cyc$  using (18)
38: end for
39: Calculate overall net energy consumption using (19).

```

The i th train substation can be determined as follows

$$Sub_i = \{i; n_i \in S_{sub_i}^{Segment}\}. \quad (14)$$

The traction energy required by the accelerating train i at operation time t and substation Sub_i is given by (15)

$$E_T(t, Sub_i, i) = M\alpha_{\max}v(t_i)/\theta_1; e_{n_i} \leq t_i \leq c_{n_i}, \quad (15)$$

where M is the train's mass and $v(t_i)$ is the train's speed at time t , which can be easily calculated using the given speed profile.

The regenerative braking energy by the braking train i at operation time t and substation Sub_i is given by

$$E_{Reg}(t, Sub_i, i) = M\gamma_{\max}v(t_i)\theta_2(1 - \theta_3); b_{n_i} \leq t_i \leq e_{n_i+1}. \quad (16)$$

The utilization of regenerative braking energy at operation time t and substation s ($U_{Reg}(t, s)$) can now be evaluated as follows

$$U_{Reg}(t, s) = \min\left(\sum_{i=1}^{N_{trains}} E_T(t, s, i), \sum_{i=1}^{N_{trains}} E_{Reg}(t, s, i)\right). \quad (17)$$

The net energy consumption (E_{Net}^{cyc}) for cycle cyc is defined as

$$E_{Net, cyc} = \sum_{t=t_{start}(cyc)}^{t_{end}(cyc)} \sum_{s=1}^{N_{sub}} (E_{Trac}(t, s) - U_{Reg}(t, s)), \quad (18)$$

where $t_{start}(cyc)$ and $t_{end}(cyc)$ are the cycle cyc start and end times, respectively. $E_{Trac}(t, s)$ is the traction energy at operation time t and substation s .

The total net energy consumption during the operation time is then given by

$$E_{Net} = E_{Net,1} + (N_{cycles} - 2) \times E_{Net,2} + E_{Net,3}. \quad (19)$$

It is worth noting that, unlike conventional multi-objective formulations that balance travel time and energy savings using subjective, arbitrary weight factors, the proposed framework treats travel time as a rigid operational boundary condition. The total round-trip line cycle time is enforced as a strict equality constraint ($T_{cycle} = 4950$ s) based on real-world transit scheduling requirements. Consequently, the optimization tool explores the distribution of segment run times and variable headways strictly within this fixed time limit, eliminating the need for empirical weight adjustments while aligning directly with practical scheduling limitations.

3.2. The Decision Variables

The speed profile parameters and the headway vector represent the decision variables for minimizing net energy consumption. It is noted that the speed profile is designed for a single cycle for a tagged train. It is assumed that the same profile will be used by all trains for all cycles. Moreover, the headway vector is assumed to be the same for all cycles.

The decision variables can be listed as follows:

- e_n : Entering time of each segment.
- q_n : Entering speed of each segment.
- m_n : Maximum reachable speed at each segment.
- T_{H_i} : The headway between train i and train $i + 1$.

The total number of decision variables is

$$N_{vars} = 3 \times N_{segments} + N_{trains} - 1. \quad (20)$$

3.3. The Constraints

Multiple constraints control the movement of trains starting from stations/turnarounds. At each station i , the dwell time ($T_D(i)$) is bounded as follows

$$T_D^{\min}(i) \leq T_D(i) \leq T_D^{\max}(i), \quad (21)$$

where $T_D^{\min}(i)$ and $T_D^{\max}(i)$ are the lower and upper bounds for the dwell time of station i , respectively. Moreover, the turnaround time (T_{turn}) is fixed. Additionally, for any station/turnaround segment, the train's speed is assumed to be zero.

The total operation time T_{op} is fixed. Given the number of cycles N_{cycles} and the operation time T_{op} , the cycle time can be calculated using (10). The headway vector should satisfy the cycle time as in (9). Moreover, the entering times of the first and last segments should satisfy the cycle time as follows

$$e_{N_{\text{segments}}} - e_1 + T_{\text{turn}} = T_{\text{cycle}}. \quad (22)$$

For each segment n , a maximum allowable speed (u_n) is defined where

$$0 \leq q_n, m_n, z_n \leq u_n. \quad (23)$$

The speed profile variables should satisfy the travel time/distance constraints given as

$$\frac{m_n - q_n}{\alpha} + \frac{m_n - z_n}{\beta} + \frac{z_n - q_{n+1}}{\gamma} = e_{n+1} - e_n, \quad (24)$$

and

$$\frac{m_n^2 - q_n^2}{2\alpha} + \frac{m_n^2 - z_n^2}{2\beta} + \frac{z_n^2 - q_{n+1}^2}{2\gamma} = d_{e_n} - d_{s_n}, \quad (25)$$

respectively, where (24) states that the sum of acceleration, coasting, and braking times should satisfy the segment travel time. Moreover, Equation (25) states that the sum of acceleration, coasting, and braking distances should satisfy the segment distance.

3.3.1. Train Control Feasibility

For a given entering speed q_n and a departure speed q_{n+1} of segment n , to check if the train can be controlled to travel the segment distance $d_{e_n} - d_{s_n}$, the following conditions should be satisfied

$$\frac{u_n^2 - q_n^2}{2\alpha} + \frac{u_n^2 - q_{n+1}^2}{2\gamma} \leq d_{e_n} - d_{s_n} \leq \frac{u_n^2 - q_n^2}{2\alpha} + \frac{u_n^2 - q_{n+1}^2}{2\beta}. \quad (26)$$

Using the maximum speed u_n , the left inequality means that the shortest travel distance, assuming the acceleration from q_n to the maximum possible speed u_n , then braking to q_{n+1} without coasting, is shorter than the segment length. The right inequality means that the longest travel distance, assuming the acceleration from q_n to u_n , then coasting to q_{n+1} without braking, is longer than the segment length. This is illustrated in Figure 3a. If the left inequality in (26) is not satisfied, then the train can not be controlled, where the shortest travel distance is larger than the segment length, as shown in Figure 3b. This could happen for low entering and departure speeds and small segments. Moreover, if the right inequality in (26) is not satisfied, then the train can not be controlled, where the longest travel distance is smaller than the segment length, as shown in Figure 3c. This could happen for large entering and departure speeds and large segments.

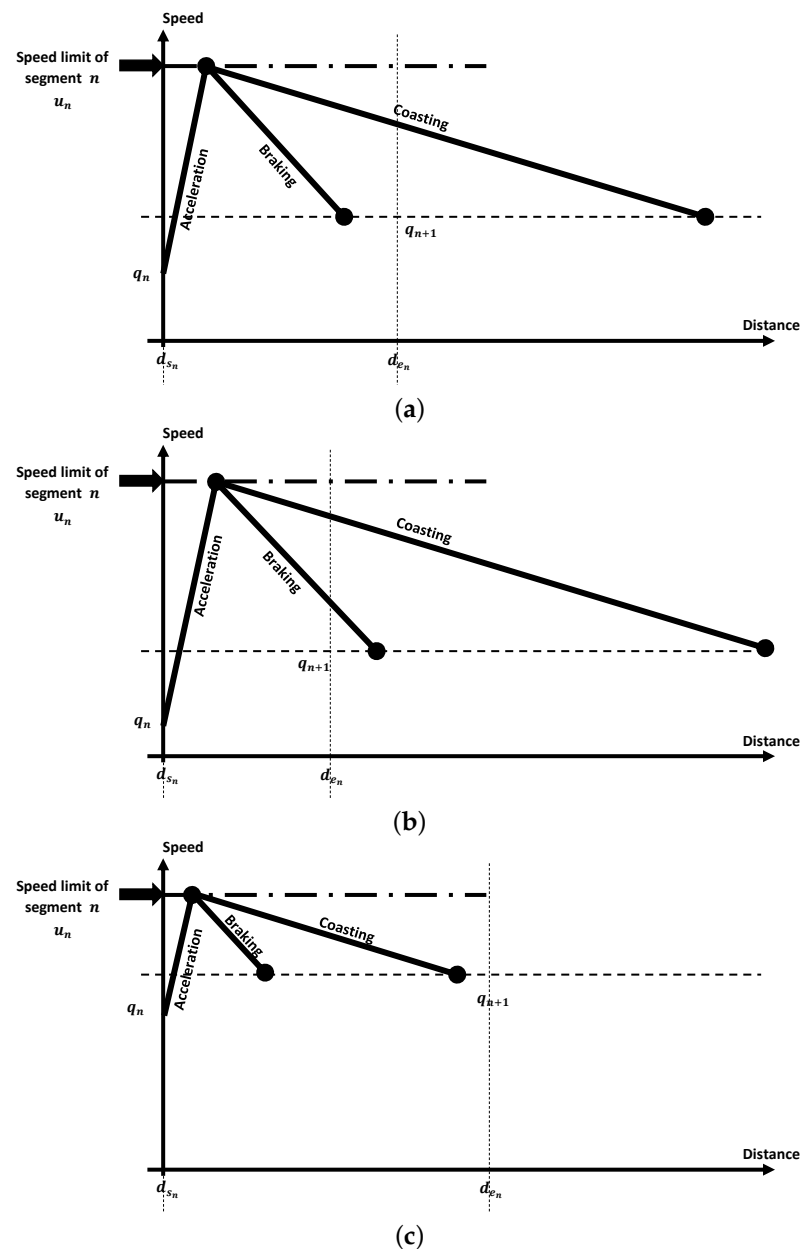


Figure 3. Check feasibility of the entering speed. (a) Feasible case. (b) Infeasible case I. (c) Infeasible case II.

3.3.2. Minimum and Maximum Travel Time Bounds

To get the minimum and maximum boundaries for the segment travel time, multiple strategies are assumed for the fastest and slowest travel times.

The fastest strategy to get the lower bound on the travel time of segment n is to accelerate from q_n to the maximum possible speed u_n , then coast to some speed z_n^{short} that allows the braking to q_{n+1} within the segment length constraint. Using (25) and substituting $m_n = u_n$, the speed z_n^{short} corresponding to the shortest travel time can be evaluated as

$$z_n^{\text{short}} = \sqrt{\frac{\beta\gamma}{\beta - \gamma} \left[2(d_{e_n} - d_{s_n}) - \frac{u_n^2 - q_n^2}{\alpha} - \frac{u_n^2}{\beta} + \frac{q_{n+1}^2}{\gamma} \right]}. \quad (27)$$

Substituting with z_n^{short} in (24), we get

$$e_{n+1}^{\text{short}}(n) = e_n + \left[\frac{u_n - q_n}{\alpha} + \frac{u_n - z_n^{\text{short}}}{\beta} + \frac{z_n^{\text{short}} - q_{n+1}}{\gamma} \right]. \quad (28)$$

Then, the lower bound of the travel time of segment n could be given by

$$Tr^{\text{short}} = e_{n+1}^{\text{short}} - e_n. \quad (29)$$

The slowest strategy to get the upper bound on the travel time of segment n depends on using the coasting speed most of the time. Two slow strategies can be introduced depending on the relation between q_n and q_{n+1} .

If $q_n > q_{n+1}$, then the slowest strategy is as follows, starting from q_n coasting to some speed that allows braking to q_{n+1} . The speed z_n^{long} that separates coasting from braking for this strategy is given by (25) assuming coasting and braking only.

$$z_n^{\text{long}} = \sqrt{\frac{\beta\gamma}{\beta - \gamma} \left[2(d_{e_n} - d_{s_n}) - \frac{q_n^2}{\beta} + \frac{q_{n+1}^2}{\gamma} \right]}. \quad (30)$$

Substituting with z_n^{long} in (24), we get

$$e_{n+1}^{\text{long}}|_{\text{Strategy\#1}} = e_n + \left[\frac{q_n - z_n^{\text{long}}}{\beta} + \frac{z_n^{\text{long}} - q_{n+1}}{\gamma} \right]. \quad (31)$$

Another slow strategy is identified when $q_n \leq q_{n+1}$. In this case, starting with coasting from q_n , a train will not reach q_{n+1} as $q_n \leq q_{n+1}$. In this case, a push is needed for q_n by acceleration to reach some speed that allows applying coasting directly to q_{n+1} without the need for braking. For this strategy, the speed m_n^{long} that separates traction from coasting is given using (25) and assuming traction and coasting only.

$$m_n^{\text{long}} = \sqrt{\frac{\alpha\beta}{\alpha - \beta} \left[2(d_{e_n} - d_{s_n}) + \frac{q_n^2}{\alpha} + \frac{q_{n+1}^2}{\beta} \right]}. \quad (32)$$

Substituting with m_n^{long} in (24), we get

$$e_{n+1}^{\text{long}}|_{\text{Strategy\#2}} = e_n + \left[\frac{m_n^{\text{long}} - q_n}{\alpha} + \frac{m_n^{\text{long}} - q_{n+1}}{\beta} \right]. \quad (33)$$

It is noted that in [12], only the aforementioned slow strategy is assumed, which leads to a larger search space for the optimization problem.

Based on the aforementioned discussion, the upper bound of the travel time of segment n could be given as follows

$$Tr^{\text{long}}(n) = \begin{cases} e_{n+1}^{\text{long}}|_{\text{Strategy\#1}} - e_n & , q_n > q_{n+1} \\ e_{n+1}^{\text{long}}|_{\text{Strategy\#2}} - e_n & , q_n \leq q_{n+1}. \end{cases} \quad (34)$$

Algorithm 2 is used to generate a random sequence of integers within each integer bound with a fixed total summation of these integers. N_{int} is the number of generated integers. LB is the lower bound vector for the generated integers. UB is the upper bound vector for the generated integers. T_{fixed} is the fixed total sum of all generated integers. $Rand_{\text{int}}$ is the generated random integers.

The algorithm is used to generate a random headway vector, where LB and UB are given as $T_H^{\min}$ and $T_H^{\max}$, respectively. Moreover, T_{fixed} is defined as the cycle time in (9). Additionally, the algorithm is used to generate the travel time of each segment, where LB is given as Tr^{short} and $T_D^{\min}$ for the segment travel times and the station segment dwell times, respectively. UB is given as Tr^{long} and $T_D^{\max}$ for the segment travel times and the station segment dwell times, respectively. Moreover, T_{fixed} is defined as the cycle time in (22).

Algorithm 2 Generate random integers within bounds with fixed total summation.

Input: $N_{\text{int}}, LB, UB, T_{\text{fixed}}$.

Output: $Rand_{\text{int}}$.

```

1: if  $\sum_{\forall i} LB(i) \leq T_{\text{fixed}}$  and  $\sum_{\forall i} UB(i) \geq T_{\text{fixed}}$  then      ▷ Check if there exists a feasible
   solution.
2:    $Ranges = UB - LB$ .                                          ▷ Calculate range of each integer.
3:    $x_{\text{Rand}} = \text{round}(LB + \text{rand}(N_{\text{int}}) * ranges)$ .           ▷ Generate initial solution.
4:    $RandSum = \sum x_{\text{Rand}}$ .                                       ▷ Sum of entries of the initial solution.
5:   if  $RandSum > T_{\text{fixed}}$  then
6:      $TotalOvershoot = RandSum - T_{\text{fixed}}$       ▷ Calculate overshoot from  $T_{\text{fixed}}$  to be
       decreased.
7:      $DecreaseAvailable = x_{\text{Rand}} - LB$  ▷ Find the positions/amounts available for the
       decrease required.
8:     for  $i = 1$  to  $TotalOvershoot$  do
9:        $rand_{\text{idx}} = \text{randPos}(DecreaseAvailable)$ . ▷ Select a random position from the
       available positions for decrease.
10:       $x_{\text{Rand}}(rand_{\text{idx}}) = x_{\text{Rand}}(rand_{\text{idx}}) - 1$ . ▷ Decrease the selected position by 1.
11:    end for
12:  else
13:     $TotalUndershoot = T_{\text{fixed}} - RandSum$  ▷ Calculate undershoot from  $T_{\text{fixed}}$  to be
       increased.
14:     $IncreaseAvailable = UB - x_{\text{Rand}}$  ▷ Find the positions/amounts available for the
       increase required.
15:    for  $i = 1$  to  $TotalUndershoot$  do
16:       $rand_{\text{idx}} = \text{randPos}(IncreaseAvailable)$ . ▷ Select a random position from the
       available positions for increase.
17:       $x_{\text{Rand}}(rand_{\text{idx}}) = x_{\text{Rand}}(rand_{\text{idx}}) + 1$ . ▷ Increase the selected position by 1.
18:    end for
19:  end if
20: else
21:   Return  $-1$ , indicating no feasible solution.
22: end if

```

4. The Optimization Tool

The described optimization problem is hard enough that heuristic optimization techniques are commonly used in the literature to solve such problems (see Table 1). Moreover, the GA is used in multiple related works [12,15,16].

The genetic algorithm is used in our work as the main solution algorithm. Compared to [12], some enhanced strategies for creating the initial population, crossover operation, and mutation are introduced.

Figure 4 shows the main procedure of the solution algorithm. It is noted that compared to the single optimization problem in [12], our optimization problem is divided into two problems. The hierarchical optimization tool operates via a master–slave framework. The main GA acts as the outer layer, optimizing the continuous entering speed vector. For every single feasible speed chromosome generated by the main GA, its exact segment travel bounds are mathematically derived using Equations (29) and (34). These bounds are passed downward to seed the sub-GA. The sub-GA then runs as an inner loop for a fixed number

of generations to find the optimal discrete travel times, dwell times, and variable headway values. The minimized net energy consumption obtained by the sub-GA is passed back to the main GA as the definitive fitness score for that speed profile. The total computational execution time for this nested workflow is detailed in Section 5.2.

After evaluating each candidate's speed vector, the stopping criterion is checked. The stopping criterion for both the main and sub-optimization problems is to reach a maximum number of stalled generations with no improvement or to reach the maximum number of generations.

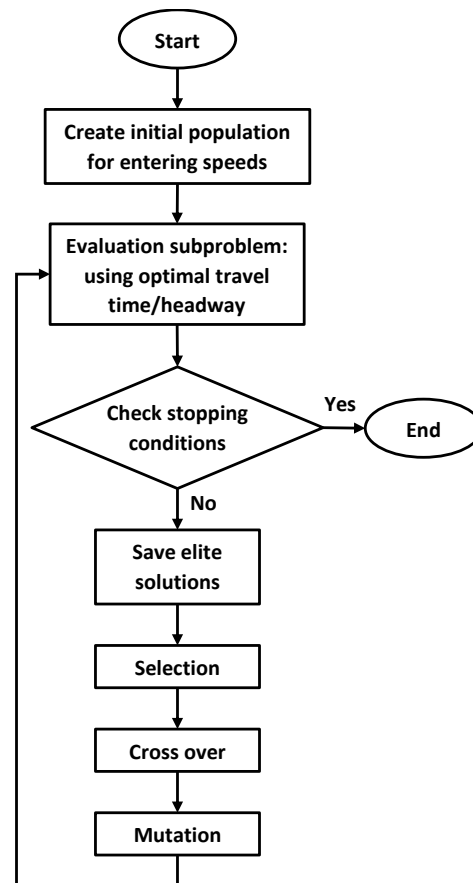


Figure 4. Main procedure of the proposed solution algorithm.

If the stopping criteria are not satisfied, steps to produce the next generation of solutions are initiated. First a percentage of the fittest solutions (elite solutions) are saved from the previous generation to the next one in order not to lose the best solutions found and keep enhancing them. This step is not used in [12]. To produce the next generation of solutions, some solutions are selected to enter the crossover and mutation processes. The roulette wheel selection technique is used based on the fitness of the solutions. The selected solutions are called parents, and the produced new solutions are called children. With some probability of crossover p_{cross} , the crossover process is applied to each consecutive pair of selected parents. The solutions that are selected for reproduction and miss the crossover process survive as they are until the next stage of processing. The solutions then enter the mutation process with probability p_{mutation} . An evaluation is then applied to the new generation, and then, the process repeats itself until one of the stopping criteria are met.

It is noted that in [12], the evaluation process is done in two places each iteration. One time after the generation of the population and another time after the crossover process. However, in our work, the solutions are evaluated only once in each iteration after creating the population. In [12], an additional evaluation step after the crossover process is used to

evaluate the children and compare the performance of both parents and children with a total of four solutions. Then the best two solutions out of the four are selected. This step is not necessary for our algorithm for multiple reasons. It limits the exploratory advantage of the crossover process by keeping only the best solutions among the parents and children. Also, the best solutions then undergo a mutation process, which can alter these solutions to generate new ones that may not be as good as before.

Next, details of some important processes of the GA for the main and sub-optimization problems are described.

4.1. The Main Optimization Problem

4.1.1. Creating the Initial Population

Figure 5 shows the introduced efficient algorithm for creating the initial population of entering speeds of each segment. Instead of completely generating the entering speeds for all segments all at once and then checking the feasibility of the whole solution, the speeds are created and checked segment by segment. The first strategy could take a very long time to create even a single feasible solution. Infeasibility of the speeds of a single segment leads to rejection of the whole solution.

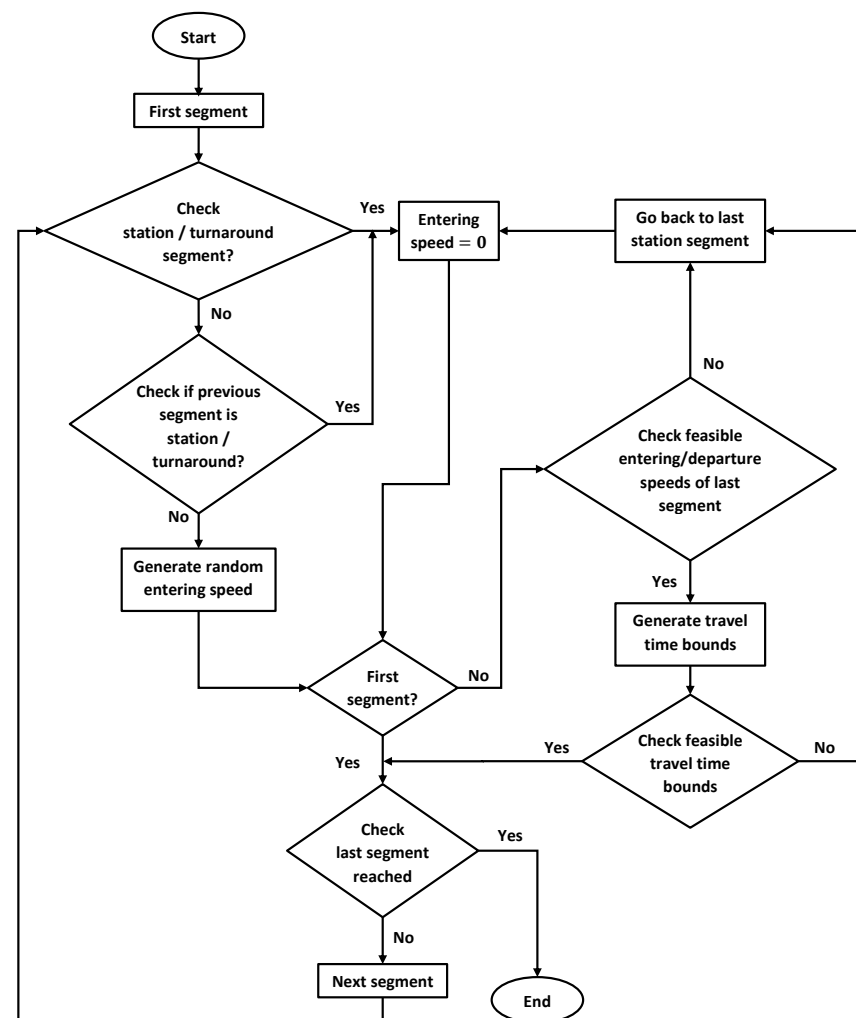


Figure 5. Creating feasible entering speeds of each segment and travel time bounds.

The applied segment-by-segment strategy is more efficient, as it relies on using the station segments as pivot points. If the speeds of any segment are found to be infeasible, then generating new speeds for the segment could lead to the infeasibility of the speeds of

the previous segment. This error propagation can be stopped at the last previous station's segment. The station/turnaround segments are characterized by zero entering and leaving speeds. This way, instead of rejecting the whole solution, the speeds are regenerated starting from the last pivot point (last station/turnaround segment before the infeasible segment). Furthermore, the creation of a feasible speed vector is faster and more efficient.

As shown in Figure 5, after ensuring the feasibility of the entering and leaving speeds of any segment, the travel time bounds are calculated using (29) and (34) for the lower and upper bounds, respectively. The feasibility of the travel times is checked using the following equation

$$\lfloor Tr^{\text{long}} \rfloor > \lceil Tr^{\text{short}} \rceil. \quad (35)$$

This is based on the fact that time is considered discrete. This process is repeated until reaching the last segment of the line.

4.1.2. The Crossover Process for the Main Optimization Problem

Figure 6 shows the crossover operation for the speed vector of the main optimization problem. The segment-by-segment strategy separates the segments into independent sets in terms of entering speeds, where the station segments act as separation segments between these sets. These independent sets of segments are exchanged between each pair of parents to create new children. Using this technique avoids the requirement to check the feasibility of the speeds for the new offspring.

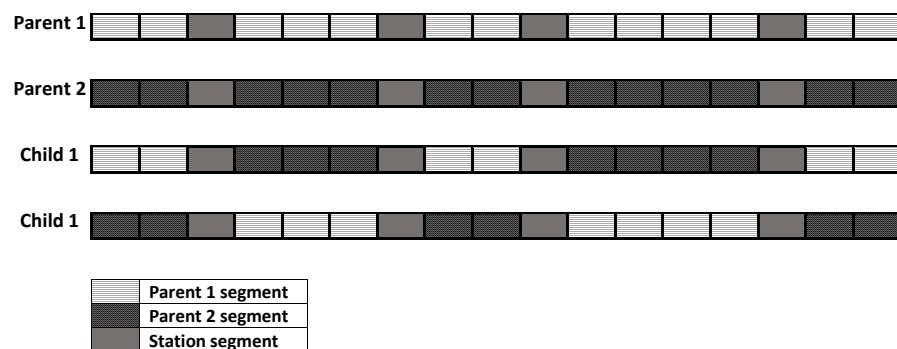


Figure 6. Crossover of the main optimization problem for the entering speed of each segment.

It is noted that in [12], the crossover operation is much more computationally expensive. The children's speeds are generated as a linear combination of the parents' speeds. These new speeds have many disadvantages and could be infeasible. Moreover, if they are feasible, the travel times and maximum speeds of each segment need to be recalculated.

4.1.3. The Mutation Process for the Main Optimization Problem

Figure 7 shows how the mutation process is executed in the main optimization problem. First a random segment other than station or turnaround segments is selected. Next, the set of segments between the stations just before and after this randomly selected segment is selected for the mutation process. The speeds of these selected segments are regenerated randomly using the same segment-by-segment procedure without affecting the old speeds of any other segment outside the set of selected segments. After generating random entering speeds for the selected segments, a random travel time is generated for these segments using the new travel time bounds associated with the new entering speeds, satisfying the condition in (22).

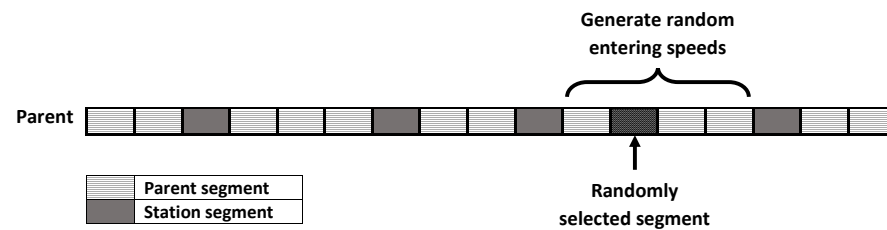


Figure 7. Mutation of the main optimization problem for the entering speeds of a randomly selected segment between stations.

In [12], the mutation process is performed as follows. First a random segment other than the turnaround segments is selected. If this segment is a station segment, a random dwell time is generated, and the entering and dwell times of all segments after this station segment are adjusted by the difference between the old and new dwell times. However, if the selected segment is another segment with non-zero speeds. Then, the entering speeds for the selected segment are randomly generated within the constraints. Next, the entering and dwell times of all segments after this segment are adjusted by the difference between the old and new travel times. This mutation technique has multiple drawbacks. First, the time adjustment due to the dwell time difference or the travel time difference makes the total cycle time different, and the condition in (22) fails. Moreover, applying the mutation multiple times to the same solution from one generation to another accumulates the constraint violation of (22) and makes it larger. Second, for the mutation of non-station segments and the regeneration of the entering speeds of only this segment, even if the speed constraints are satisfied for this segment, the segments just before and after this segment now have one of their entering speeds changed. These new speeds could be infeasible.

4.2. The Sub-Optimization Problem

Each feasible solution of the main optimization problem has feasible entering speeds and travel time bounds for each segment. For each of these solutions, in the sub-optimization problem, the optimal travel time of each segment as well as the optimal headway vector are found using the GA. First, an initial population is generated using the travel time bounds and the lower and upper bounds of the headway time using Algorithm 2. Next, each of these solutions is evaluated using Algorithm 1 to calculate the net energy consumption. Some solutions are selected for the reproduction process of crossover and mutation using the roulette wheel selection technique based on the fitness of the solutions.

Next, details of the crossover and mutation operations are explained.

4.2.1. The Crossover Process for the Sub-Optimization Problem

For each pair of selected solutions, “called parents”, new offspring solutions, “called children”, are created using the crossover operation. It is noted that the same technique for crossover is applied to both the travel time and the headway time. Figure 8 shows the crossover process for the sub-optimization problem of travel time and headway.

For the travel time vector, first, three randomly selected points along the segments are selected. These points divide the segments into four regions. The crossover is performed as follows: For child 1, the travel times of the segments of the first region are the same as those of parent 1. Moreover, the travel times of the segments of the third region are the same as those of parent 2. The travel times of the segments of the even regions are generated randomly using Algorithm 2 to ensure that the condition in (22) is satisfied. The same procedure is repeated for child 2 while reversing the order of the parents. Parent 2 travel times are set in region one, while parent 1 travel times are set in region three. The same

technique is used for the crossover of the headway vector, and the randomly generated regions should satisfy the condition in (9).

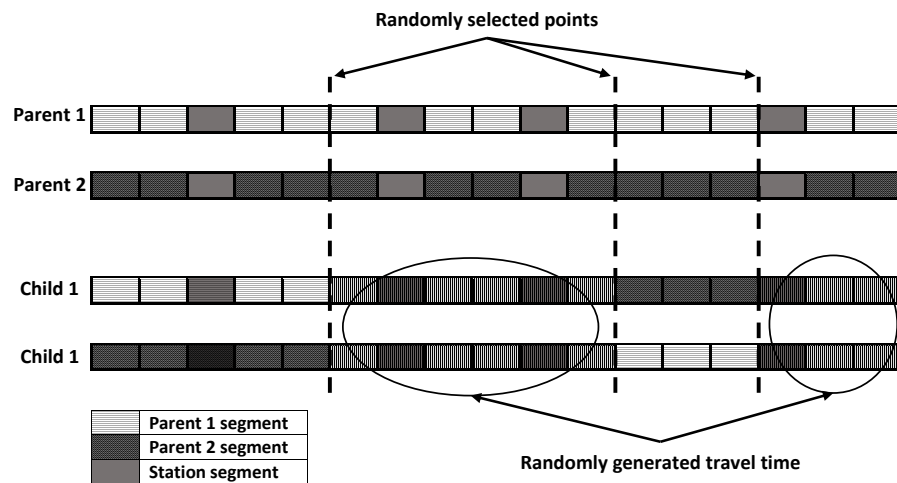


Figure 8. Crossover of the sub-optimization problem for the headway time and the travel time of each segment.

4.2.2. The Mutation Process for the Sub-Optimization Problem

Figure 9 shows the mutation process used for mutation of the solutions of travel/-headway times. For each parent solution, the solution is divided into two halves. Next, a randomly selected half is kept as is. The other half is generated randomly within the allowable bounds, satisfying (22) for the travel times or (9) for the headway vector.

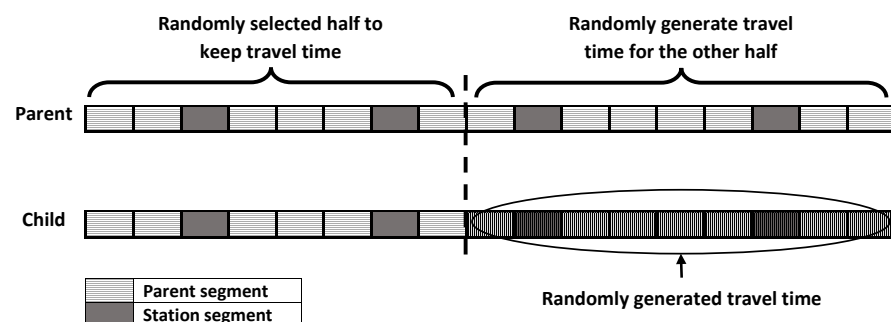


Figure 9. Mutation of the sub-optimization problem for the headway time and the travel time of half of the segments.

5. Numerical Results

In this section, some numerical results are presented to (1) compare the proposed solution mechanisms to the techniques proposed in [12], (2) prove the convergence of the solution algorithm, (3) show the optimal timetable and speed profile, and (4) show the maximum number of trains per station/turnaround.

The actual data from the Beijing Metro Yizhuang Line in China is considered a case study to test the proposed energy-efficient strategies and compare them with related models in the literature. Speed limits and segment intervals of the Beijing Metro Yizhuang Line in China are introduced in Table 3 [22].

Based on Table 3, there exist three segments between each pair of stations. Consequently, the indices of the station segments are set as $S_{\text{stations}} = \{1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53\}$ for the upward direction and $S_{\text{stations}} = \{55, 59, 63, 67, 71, 75, 79, 83, 87, 91, 95, 99, 103, 107\}$ for the downward direction. The indices of the turnaround segments are set as $S_{\text{turnaround}} = \{54, 108\}$.

Table 3. Speed limits and segment intervals of the Beijing Metro Yizhuang Line.

Section	Speed Limit (m/s)	Segment (m)	Speed Limit (m/s)	Segment (m)	Speed Limit (m/s)	Segment (m)
SJZ-XC	10	0–128	20	128–1147	14	1147–1332
XC-XHM	13	0–130	20	130–1085	13	1085–1286
XHM-JG	13	0–129	20	129–1897	11	1897–2086
JG-YZQ	13	0–128	20	128–2073	13	2073–2265
YZQ-YZWHY	13	0–120	20	120–2143	12	2143–2331
YZWHY-WYJ	14	0–130	20	130–1168	13	1168–1354
WYJ-RJDJ	14	0–130	20	130–1087	13	1087–1280
RJDJ-RCDJ	14	0–130	20	130–1350	13	1350–1544
RCDJ-TJNL	13	0–130	20	130–796	13	796–992
TJNL-JHL	14	0–123	20	123–1781	13	1781–1975
JHL-CQN	14	0–128	20	128–2165	13	2165–2369
CQN-CQ	14	0–130	20	130–1151	13	1151–1349
CQ-YZHCZ	13	0–133	20	133–2300	10	2300–2610

The dwell time limits are set based on the original timetable given in [12], where lower and upper bounds are set around the original dwell time in Table 4. This technique is used to set the limits of the dwell time in order not to deviate by a large value from the original dwell time value, which is related to passenger demand.

Table 4. Upper and lower bounds of dwell time for every station (s).

Station number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	T
$T_D^{\min}$	25	28	25	25	30	26	25	28	24	25	25	32	43	35	180
$T_D^{\max}$	32	32	32	32	37	32	32	35	32	32	32	37	51	42	180
Station number	15	16	17	18	19	20	21	22	23	24	25	26	27	28	T
$T_D^{\min}$	35	42	29	28	25	26	26	28	27	30	28	25	27	26	180
$T_D^{\max}$	42	47	37	35	32	32	32	35	32	37	34	32	32	32	180

The system parameters used in the simulation are set in Table 5. The optimization algorithm settings are introduced in Table 6. The system parameters and genetic operator settings listed in Tables 5 and 6 are adopted from proven baselines in the related railway optimization literature [12]. Utilizing these well-established values ensures that the physical constraints of the transit system are accurately represented, while the genetic mechanisms maintain an optimal balance between solution exploration and exploitation to avoid premature convergence.

Table 5. System parameters.

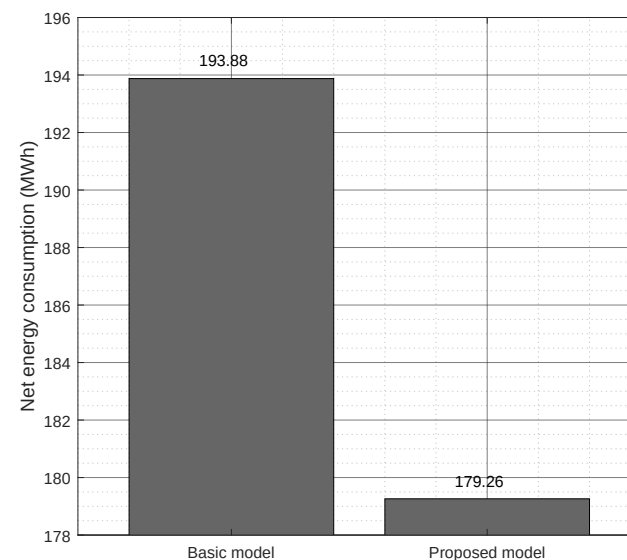
Parameter	Value	Parameter	Value
α	0.8	θ_1	0.9
β	0.02	θ_2	0.6
γ	0.4	θ_3	0.1
M	287,080 kg	$T_H^{\max}$	100 s
N_{cycles}	13	$T_H^{\min}$	80 s
N_{segments}	108	T_{cycle}	4950 s
N_{stations}	28	T_{turn}	180 s
N_{trains}	55	-	-

Table 6. Parameters of the solution algorithm.

Parameter	Value	Parameter	Value
Population size	200/30	Reproduction prob.	0.2
Maximum generations	100/20	P_{cross}	0.8
Max. stall generations	20/5	P_{mutation}	0.5
Elite count	5% of Population size	-	-

5.1. Comparison of Solution Algorithms

Figure 10 compares the proposed model to the basic model in [12]. The enhancement in net energy consumption is about 7.5%. Despite the level of enhancement, the proposed model includes features that enable better control of the system when other features are considered in future work. For example, to consider passenger demand, the variable headway would allow headway times to be reduced or increased according to passenger demand at any time during a cycle.

**Figure 10.** Comparison between the proposed model and the basic model in [12] in terms of net energy consumption.

5.2. Convergence of Solution

Figure 11 illustrates the convergence of net energy consumption with the number of iterations. It shows that performance did not improve after 44 iterations up to iteration 64, i.e., for 20 consecutive iterations, “stall time”. This proves that, given the optimization algorithm setting, this is the best solution reached.

Figure 12 also proves the convergence of the proposed algorithm to the near-optimal solution when increasing the population size (computational complexity) to better explore the solution space. The population sizes for the main/sub-problems are first set to 200/30, respectively. Next, the population size is increased to 800/30 and finally to 2000/100. The enhancement from the 200/30 setting to the 800/30 setting is about 2.01%, and the enhancement from the 800/30 setting to the 2000/100 setting is about 2.1%. The small increase in enhancement while increasing the population size proves the convergence to a near-optimal solution.

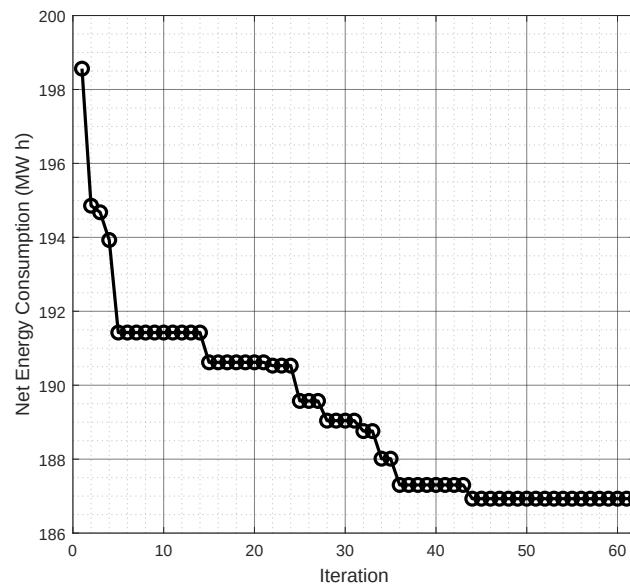


Figure 11. Solution convergence with the number of iterations at 200/30 population.

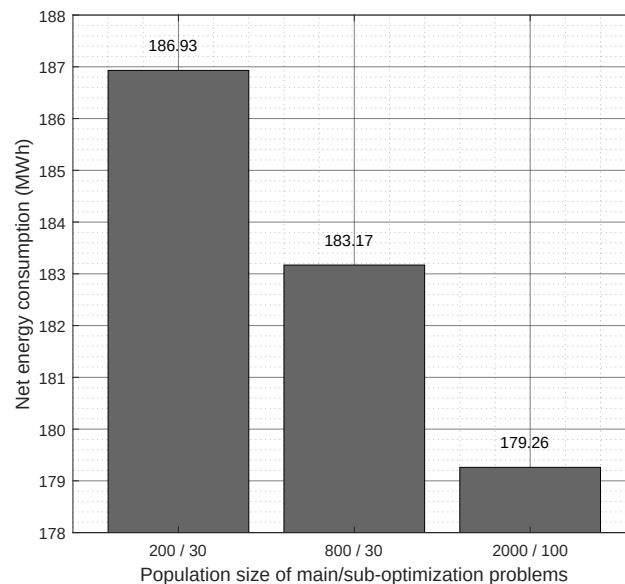


Figure 12. Solution convergence with population size.

This analysis effectively serves as a parameter sensitivity study for the algorithm's population size. The fact that scaling up the population yields only a small performance enhancement confirms that the framework delivers highly consistent results and is not overly sensitive to parameter modifications. This baseline stability highlights a core benefit of the structural design of the optimization problem, which restricts the search space to mathematically feasible domains and ensures robust, reliable convergence without requiring intensive parameter tuning loops.

It is noted that the proposed optimization framework is structured primarily for off-line timetable design, where the total computational workload follows an algorithmic order of complexity defined by $\mathcal{O}(G_{\text{main}} \cdot N_{\text{main}} \cdot G_{\text{sub}} \cdot N_{\text{sub}})$, where G_{main} and G_{sub} are the number of generations of the master and slave optimization problems, respectively. N_{main} and N_{sub} are the population sizes of the master and slave optimization problems, respectively. Within each joint generation step, the number of objective function evaluations scales strictly as a product of the population dimensions ($N_{\text{main}} \times N_{\text{sub}}$), resulting in exactly 6000 evaluations for the baseline setup (200×30) and up to 200,000 evaluations for the

massive setup (2000×100). Because a 33.3-fold expansion of the search landscape yields a marginal net energy improvement of only 2.1%, the baseline setup is mathematically justified as an optimized and highly stable configuration for offline deployment. Furthermore, this decoupled master–slave architecture offers high flexibility for online, real-time traffic rescheduling. By fixing the continuous outer-loop speed profile variables (G_{main} and N_{main}) to pre-computed baseline values during active transit operations, the active execution complexity collapses strictly to the inner sub-problem level, $\mathcal{O}(G_{\text{sub}} \cdot N_{\text{sub}})$. This constraint narrows the live optimization process down to the discrete timetable adjustments. Consequently, the algorithm executes rapidly, making it highly effective for real-time traffic rescheduling during disruptions.

5.3. Optimal Timetable, Speed Profile, and Headway Vector

The results of solving the optimization problem are shown in Figures 13 and 14, and Table 7 for the 200/30 population size setting.

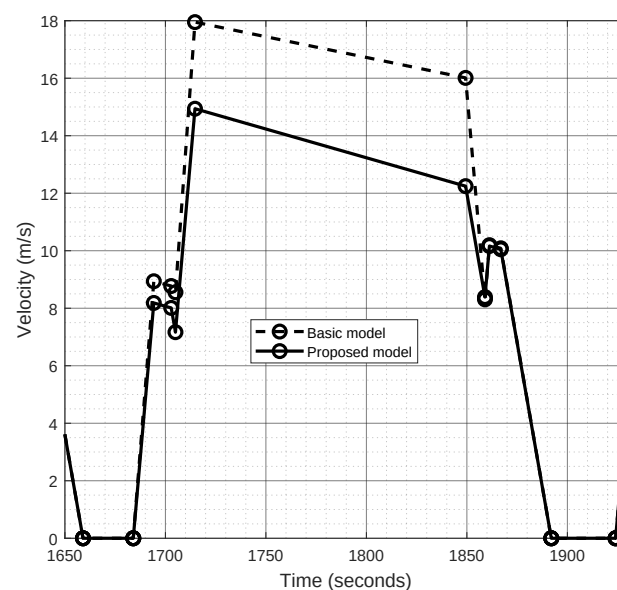


Figure 13. Part of the optimal speed profile.

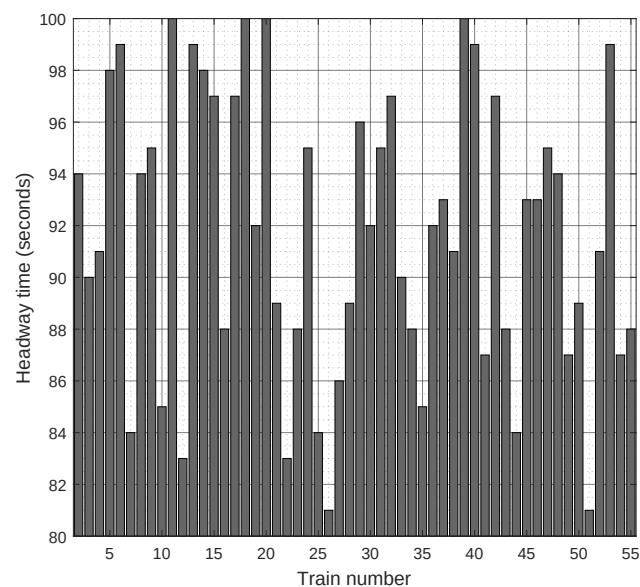
Figure 13 shows a section of the optimal speed profile. As stated before, the speed profile gives full information on the detailed movement of the train. It introduces the speed at every time instant as well as the entering and dwell times for each station. Figure 13 shows the speed profile between two stations, where the stations can be identified by the time intervals of zero speeds. There exist three segments between the two stations: two short segments right after the first station and right before the second station, and another long segment in between. Each of the three segments is formed for the main stages of acceleration, followed by coasting, and ending with braking. By comparing the two curves in Figure 13, a significant difference can be observed in the peak velocity reached during the long central segment. The basic model accelerates to a much higher maximum velocity of 18 m/s, whereas the proposed model caps its peak velocity at 15 m/s. Despite this lower top speed, both models overlap perfectly during the zero-speed dwell periods, meaning they maintain the exact same departure and arrival times at the stations. Because running at lower speeds consumes less power, the proposed model's strategy of avoiding high peak velocities yields substantial energy savings while fully satisfying the operational schedule constraints.

Table 7. Optimal timetable for the proposed model.

Station number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	T
Entering time (s)	0	150	294	489	684	879	1039	1193	1353	1482	1659	1892	2048	2281	2316
Dwell time (s)	25	28	25	25	30	26	25	28	24	25	25	32	43	35	180
Station number	15	16	17	18	19	20	21	22	23	24	25	26	27	28	T
Entering time (s)	2496	2716	2880	3089	3268	3402	3571	3725	3872	4072	4270	4454	4594	4744	4770
Dwell time (s)	35	42	29	28	25	26	26	28	27	30	28	25	27	26	180

By extracting the entering and dwell times of each station from the optimal speed profile, the optimal timetable is constructed in Table 7.

The optimal headway vector presented in Figure 14 shows variable headway values. This proves that variable headway can achieve better net energy consumption than the fixed-headway case.

**Figure 14.** Optimal headway vector.

5.4. Maximum Number of Trains per Station/Turnaround

The maximum number of trains that can simultaneously occupy stations and turnarounds represents an important operational parameter for infrastructure planning. Unlike previous studies, this aspect is explicitly considered in the present work, as it directly influences the required capacity of stations and turnaround facilities. The results indicate that a maximum of two trains can be present simultaneously at any station, whereas up to four trains can occupy a turnaround. This difference is primarily determined by the relationship between the headway and the dwell or turnaround times. Since the turnaround time is much larger than the dwell time at any station, there is a larger number of trains that could exist at the same time at turnarounds than at stations.

6. Conclusions

This work introduces a novel solution algorithm for energy-efficient operation of railway systems. In this work, net energy consumption is minimized by controlling the speed profile of the train and the headway times between trains. By controlling both the speed profile and headway time, it was possible to synchronize the trains to better utilize regenerative braking energy. The proposed algorithm for solving the optimization problem divides the problem based on the nature of the decision variables. The main optimization problem is focused on the continuous decision variables of the entering speeds of each

segment of the railway. The sub-optimization problem is concerned with the discrete decision variables of travel time, dwell time, and headway time. The genetic algorithm with efficient novel crossover and mutation operations is used to solve both the main and sub-problems. The proposed algorithm showed an enhancement compared to previous algorithms in the literature. Moreover, the convergence of the proposed algorithm is confirmed. The optimal speed profile and timetable are introduced, and the maximum number of trains at each possible stop along the railway line at any time is calculated for the first time. This allows the proper design of station and turnaround capacities.

This work can be further extended by considering passenger demand at each station. This would change the perspective on the objective function when considering passenger demand as a high priority. Moreover, a more general infrastructure could be considered instead of the two-track railway line addressed in this work. Building upon the computational efficiency provided by our hierarchical split-GA structure, future work will also focus on integrating non-linear train dynamics—such as gradient-dependent resistance and time-varying passenger mass—as well as complex electrical network constraints, to improve real-world deployment accuracy and robustness under operational uncertainty.

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