

Article

Voltage Stability Analysis in HVDC Systems Using Jacobian Singularity and Saddle-Node Bifurcations

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Abstract

This paper introduces a methodology for evaluating the voltage stability margin in high-voltage direct-current (HVDC) systems, which analyzes the singularity of the power flow Jacobian matrix—computed via the Newton–Raphson method—and identifies saddle-node bifurcations. The continuation power flow method is employed to model progressive load increases, enabling the continuous tracking of power flow solutions and the determination of voltage collapse points. Within this framework, the system’s behavior is analyzed under contingency conditions, particularly transmission line outages, assessing its capability to maintain secure operating conditions under increasing demand scenarios. The main objective is to identify the most critical line in the system, defined as that which leads to the greatest reduction in loadability when unavailable, prior to voltage collapse. This approach allows for the early identification of structural vulnerabilities, supporting decision-making processes aimed at risk mitigation and operating cost optimization. The proposed methodology is validated using two systems: the six-terminal CIGRE-B4 HVDC system and an 11-node HVDC test feeder.

Keywords: voltage stability margin; saddle-node bifurcation; transmission contingencies; loadability factor; Jacobian matrix singularity; Newton–Raphson

1. Introduction

The large-scale integration of renewable energy sources and the widespread deployment of power electronic devices have considerably increased the complexity and uncertainty of modern power systems, particularly affecting voltage stability and congestion management. The inherent variability of renewable generation, when combined with network contingencies, can induce abrupt power flow redistributions and elevate the risk of voltage collapse [1]. Conventional approaches such as security-constrained optimal power flow (SCOPF) and sensitivity-based methods exhibit significant limitations in this context, including high computational requirements and a reduced capacity to capture the nonlinearities that characterize operation under stress conditions [2].

In response to these challenges, high-voltage direct-current (HVDC) transmission has emerged as a strategic solution. Its ability to provide precise and fast power flow control enhances the secure and flexible operation of power systems with a high penetration of renewables [3]. Compared to conventional high-voltage alternating-current (HVAC) systems, HVDC offers well-documented advantages for long-distance and submarine



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transmission, including lower power losses, the absence of electromagnetic interference, and superior compatibility with offshore renewable generation [4].

Nevertheless, these technical advances have also increased the complexity of operating and analyzing modern power grids, making voltage stability assessment a fundamental concern [3]. As systems are pushed closer to their physical limits, accurately determining the proximity to voltage collapse becomes critical. In response to this issue, several static methods have been developed, including eigenvalue and singular value analysis, modal analysis, and sensitivity-based techniques, all aimed at capturing system behavior under evolving operating conditions [5–9].

Within this context, the Jacobian matrix plays a central role in voltage stability analysis, as its properties reflect the sensitivity of the system to variations in the state variables [10]. As the system approaches its stability limit, the Jacobian tends towards singularity, indicating a divergence in the solution of the power flow equations—a condition that is closely associated with a saddle-node bifurcation (SNB) [11,12]. At such a bifurcation, two equilibrium solutions converge to a critical point and subsequently disappear [13].

This critical point marks the onset of voltage collapse, a progressive degradation phenomenon characterized by a cascade of voltage reductions that can ultimately culminate in large-scale blackouts [14]. Collapse may be triggered either by gradual operational changes or by abrupt disturbances, e.g., the disconnection of generation units or the outage of heavily loaded transmission lines, both of which impose increased reactive power demands and compromise voltage stability [15]. Notably, automatic correction mechanisms such as excitation systems and on-load tap changers can exacerbate collapse by overloading generators and increasing transmission losses, thereby accelerating cascading disconnections and system-wide failure [2]. This complexity is further amplified in modern networks by operational constraints and mode-switching behavior at PV/PQ nodes, generators, and converters. Conventional switching logic may yield non-physical power flow solutions and erroneous load margin calculations [15,16], particularly in HVDC systems, where power electronic converters introduce additional dynamics and control modes that must be explicitly accounted for in stability analysis [1].

Consistent with this framework, load margin studies have shown that, as the system approaches the SNB point, the Jacobian matrix becomes nearly singular, causing convergence failures in conventional power flow algorithms [17,18]. This observation confirms that the challenge is not only physical but also computational: it necessitates methods capable of tracing the system's operating trajectory near critical conditions. Jacobian singularity analysis and saddle-node bifurcation (SNB) detection emerge as particularly effective approaches in this regard.

In light of the above, this work seeks to determine the critical point regarding voltage stability in HVDC systems by analyzing the singularity of the power flow Jacobian matrix, identifying the corresponding SNB from the parametric variation in the load. To this effect, the proposed methodology follows a cohesive sequential framework. First, the HVDC system is modeled in MATLAB 2025b using Newton–Raphson continuation power flow (CPF). Second, the system is progressively stressed through a load sweep to estimate the stability margin and identify the critical voltage collapse point from the Jacobian matrix. Third, contingency scenarios are simulated to assess system robustness under disturbed operating conditions. To validate this approach, our work considers standard CIGRE test systems that are widely used in academic power system studies. Specifically, the six-terminal CIGRE-B4 HVDC system and the 11-node MT-HVDC test feeder proposed by [19] are employed to assess the performance of the method across networks with different levels of topological complexity.

Despite the maturity of CPF-based voltage stability methods for AC systems, their systematic application to pure-DC resistive HVDC networks is still scarce in the literature. Existing studies on the CIGRE B4 and MT-HVDC benchmarks have focused predominantly on dynamic stabilization [20], optimal power flow and economic dispatch [19], and droop control design [21,22], rather than on the static loadability boundary and its sensitivity to transmission line outages. This work fills this gap by providing, for the first time, a complete CPF-based characterization of the voltage collapse boundary for these DC benchmarks, including a systematic N-1 contingency analysis grounded in Jacobian singularity theory, wherein the minimum singular value $\sigma_{\min}(J)$ and the condition number $\text{cond}(J)$ are jointly monitored along the solution trajectory as complementary early-warning indicators of imminent voltage collapse.

The remainder of this paper is organized as follows: Section 2 presents the mathematical formulation of the voltage stability margin estimation problem under HVDC transmission line outages. Section 3 describes the proposed methodology, focusing on Jacobian matrix singularities and the identification of the SNB from parametric load variations. Afterwards, Section 4 outlines the test systems used in the study, and Section 5 presents and discusses the results obtained for the evaluated scenarios. Finally, Section 6 provides the main conclusions of this work and suggests potential directions for future research.

2. Mathematical Formulation

Voltage stability constitutes a fundamental challenge in modern power systems, particularly in the context of HVDC transmission. In such systems, the ability to sustain stable voltage profiles under varying load conditions is essential for a reliable and secure operation. As the rising electricity demand and the large-scale integration of renewable energy sources increasingly push power systems towards their operational boundaries, the susceptibility to voltage collapse emerges as a critical concern.

Definition 1 (Voltage stability). *Voltage stability denotes a power system's ability to maintain acceptable voltage magnitudes at all buses under normal operating conditions, as well as after the occurrence of a disturbance [14].*

Within this framework, for security assessment and operational planning, it is essential to determine the maximum load that a system can withstand under transmission line outage scenarios.

2.1. DC Power Flow Formulation and Loadability Factor

In this work, the analysis focuses on HVDC systems modeled as resistive DC-equivalent networks. The formulation differs from that of the conventional AC power flow in that voltage angles, sinusoidal components, and reactive power are not considered, since the electrical behavior is governed solely by voltage magnitudes and conductances [2,23]. The static resistive DC network model adopted herein is consistent with prior work on MT-HVDC voltage stability assessment [19,20], and it is appropriate for the purpose of steady-state loadability analysis; the dynamic converter behavior, droop control, and operational constraints are relevant to transient and small-signal stability studies and are identified as extensions of this framework in Section 6.

Under this framework, the power flow is based on the active power balance at each network node. For a given node k , the power equation is expressed as follows:

$$P_k = P_k^g - \lambda P_k^d = V_k \sum_{m \in \mathcal{N}} G_{km} V_m, \quad \forall k \in \mathcal{N} \quad (1)$$

where $\mathcal{N}$ denotes the set of all nodes in the system; P_k is the net active power injected at node k ; P_k^g represents the active power generation at node k ; P_k^d is the base case's active power demand at node k ; λ is the loadability factor (scalar parameter); V_k and V_m are the nodal voltage magnitudes at nodes k and m , respectively; and G_{km} corresponds to the elements of the system conductance matrix.

The loadability factor λ quantifies the loading level of the power system relative to its base-case demand. As λ progressively increases, the demand at each load node scales uniformly. This allows tracing of the system's operating point along the P-V curve. The static voltage stability limit is reached when the minimum eigenvalue of the Jacobian matrix approaches zero, indicating that the system has reached its maximum loadability [5]. This parametric formulation enables the systematic identification of the SNB point, beyond which no feasible power flow solution exists.

2.2. Continuation Power Flow Method

Given that voltage stability assessment requires tracking the evolution of the system's operating point under progressively increasing load demand, the continuation power flow (CPF) method is adopted as the main analytical tool in this study, as it enables a systematic characterization of the system's trajectory up to the stability limit [24]. Within this framework, the conventional power flow problem is reformulated as a parametric system in which the equilibrium equations are expressed as a function of a scalar loading parameter (λ).

$$F(V, \lambda) = 0 \quad (2)$$

The method consists of two main stages that are sequentially applied to trace the solution trajectory. First, in the predictor step, a new operating point is estimated from the current state, using the tangent vector to the solution path, which is obtained by differentiating the augmented system of equations. This is given by

$$\frac{\partial F}{\partial V} \cdot t_V + \frac{\partial F}{\partial \lambda} \cdot t_\lambda = 0 \quad (3)$$

In this expression, $\frac{\partial F}{\partial V} \equiv \mathbf{J}$ denotes the Jacobian matrix with respect to the nodal voltages, while t_V and t_λ correspond to the components of the tangent vector. As a result, this stage provides an initial approximation to the system's evolution in response to an incremental variation in the loading parameter. Subsequently, this estimate is improved during the correction step, where the predicted point is iteratively refined by solving the augmented nonlinear system via a Newton–Raphson procedure [17].

Remark 1. In addition, an arc-length constraint is incorporated to ensure numerical robustness. This allows tracking of the solution trajectory accurately, even near critical operating conditions, where the Jacobian becomes singular [24].

3. Methodology

To solve the Equation (3), the tangent vector to the solution path is obtained by exploiting the rank-deficiency structure of the power flow Jacobian matrix [5]. Under regular operating conditions, the Jacobian is non-singular, and the tangent vector is computed by solving the differentiated form of the parametric power flow system, which expresses the rate of change in the nodal voltages with respect to the loading parameter.

However, as the loading parameter approaches its critical value, the Jacobian progressively loses conditioning, and its smallest singular value decreases toward zero. In the limit, the matrix becomes rank-deficient, and its null vector—associated with the zero

eigenvalue—defines the only direction along which the power flow equations can still be satisfied, thereby providing the natural tangent direction for the predictor step without requiring an explicit inversion of the Jacobian.

This behavior has a direct physical interpretation: the null vector of the Jacobian at the critical point represents the pattern of voltage deviations that the system would experience under an infinitesimal load increment beyond the stability boundary, a direction along which no corrective voltage response is available. From a numerical standpoint, the progressive rank deficiency of the Jacobian along the solution trajectory serves as a reliable early-warning indicator of proximity to voltage collapse [25], as it can be monitored continuously through the minimum singular value at each correction step without the need for computational overhead.

The same property that enables the predictor step also underlies the identification of the SNB point, which corresponds to the operating condition at which the Jacobian becomes exactly singular. At this point, the two equilibrium solutions of the power flow equations—one stable and one unstable—coalesce into a single degenerate solution, beyond which there is no feasible operating point for higher loading levels. The loading parameter value at which this coalescence occurs defines the maximum loadability of the HVDC system and establishes the static voltage stability limit. Therefore, detecting this point through the singularity of the Jacobian provides both a mathematically rigorous criterion for voltage collapse and a computationally tractable indicator that can be extracted directly from the information already generated by the CPF algorithm.

The critical loading parameter is the point at which the minimum singular value of the Jacobian reaches its lowest value along the solution trajectory computed by Algorithm 1. This procedure is then repeated for each transmission line outage, and the difference between the base-case and contingency critical parameters is used to quantify the impact of each line outage on the static voltage stability of the HVDC system.

The proposed methodology offers a computationally efficient and mathematically rigorous approach to voltage stability assessment in HVDC systems. By combining the CPF algorithm with the systematic monitoring of the minimum singular value of the Jacobian matrix, the method avoids the convergence failures that affect conventional Newton–Raphson-based power flow solvers near the stability boundary while providing a continuous and quantitative indicator of proximity to voltage collapse.

The arc-length parameterization employed in the correction step ensures that the solution trajectory can be traced reliably up to and beyond the nose point of the P-V curve without requiring any reformulation of the underlying power flow equations. Furthermore, the identification of the SNB point through Jacobian singularity establishes a direct link between the numerical behavior of the algorithm and the physical phenomenon of voltage collapse, making the results both interpretable and consistent with the theoretical framework of nonlinear dynamical systems.

Remark 2. *The extension to the transmission line outage analysis preserves the same computational structure, requiring only a reconstruction of the nodal conductance matrix for each modified topology. This makes the overall framework scalable and suitable for the systematic security assessment of HVDC networks under credible outage scenarios.*

Algorithm 1 Voltage stability analysis of HVDC systems via CPF and Jacobian singularity

Require: Network data $\mathcal{L}$, slack bus s , V_s , base loading P^0 , step size Δs , tolerances $\varepsilon_F, \varepsilon_V$
Ensure: P–V curve, λ^* , $\sigma_{\min}(J)$ trajectory, contingency load margins $\{\lambda_c\}$

- 1: Construct nodal conductance matrix G from $\mathcal{L}$
- 2: Compute reduced system: $G_{\text{red}}, G_{dg}, P_{\text{PQ}}^0$
- **Base Case** —
- 3: Initialize $V^{(0)} \leftarrow \mathbf{1}, \lambda_0 \leftarrow 0$
- 4: Solve $F(V, \lambda_0) = 0$ via the Newton-Raphson method to obtain base-case voltages
- 5: Set $\lambda_1 \leftarrow \lambda_0 + \delta\lambda_{\text{ini}}$, solve for V_1
- 6: **for** step = 1, 2, ..., N_{max} **do**
- 7: **Predictor:** compute tangent vector (t_V, t_λ) by solving

$$\frac{\partial F}{\partial V} t_V + \frac{\partial F}{\partial \lambda} t_\lambda = 0, \quad \|(t_V, t_\lambda)\| = 1$$

- 8: Predict next point: $V^{\text{pred}} \leftarrow V_k + \Delta s t_V, \lambda^{\text{pred}} \leftarrow \lambda_k + \Delta s t_\lambda$
- 9: **Corrector:** solve augmented system via the Newton-Raphson method

$$\begin{bmatrix} J & \partial F / \partial \lambda \\ t_V^\top & t_\lambda \end{bmatrix} \begin{bmatrix} \Delta V \\ \Delta \lambda \end{bmatrix} = - \begin{bmatrix} F \\ g \end{bmatrix}$$

$$\text{where } g = (V - V_k)^\top t_V + (\lambda - \lambda_k) t_\lambda - \Delta s^2$$

Note that, in the implementation, the tangent vector (t_V, t_λ) is not explicitly normalized prior to its use in the arc-length constraint. Since the predictor advances Δs along the unit tangent direction, the unnormalized difference vector satisfies $\|(t_V, t_\lambda)\| \approx \Delta s$, and the constraint $g = (V - V_k)^\top t_V + (\lambda - \lambda_k) t_\lambda - \Delta s^2 = 0$ is algebraically equivalent to the standard form $[\dots] - \Delta s = 0$ as written in terms of the normalized tangent vector.

- 10: **if** corrector converges **then**
- 11: Update $(V_{k+1}, \lambda_{k+1}) \leftarrow (V^{\text{pred}} + \Delta V, \lambda^{\text{pred}} + \Delta \lambda)$
- 12: Compute $\sigma_{\min}(J_{k+1}), \text{cond}(J_{k+1}), \det(J_{k+1})$
- 13: Record $\lambda_{k+1}, V_{\min}^{k+1}, \sigma_{\min}^{k+1}$
- 14: **else**
- 15: **break**
- 16: **end if**
- 17: **if** $\lambda_{k+1} > \lambda_{\text{max}}$ **or** $\lambda_{k+1} < -0.1$ **then**
- 18: **break**
- 19: **end if**
- 20: **end for**
- 21: $\lambda^* \leftarrow \lambda_k$ such that $\sigma_{\min}(J_k) = \min_k \sigma_{\min}(J_k)$
- **Transmission line outage analysis** —
- 22: **for** each line $c \in \mathcal{L}$ **do**
- 23: Remove line c , recompute $G^{(c)}, G_{\text{red}}^{(c)}$
- 24: Repeat steps 3–23 on the modified system
- 25: Record $\lambda_c^*, \sigma_{\min,c}, \Delta \lambda_c \leftarrow \lambda^* - \lambda_c^*$
- 26: **end for**
- 27: **return** P–V curves, $\lambda^*, \{\lambda_c^*\}, \sigma_{\min}$ trajectories

4. Test Systems

The proposed methodology was evaluated using two benchmark test systems operated under a HVDC configuration: the six-terminal CIGRE-B4 HVDC system and the 11-node MT-HVDC test feeder proposed by [19]. The CIGRE-B4 system was adopted as the reference benchmark for meshed multi-terminal HVDC grids, while the 11-node MT-HVDC feeder served as a medium-scale DC benchmark with renewable integration. This combination

ensured validation across different scales and topologies, enabling direct a comparison with previously published results.

4.1. The Cigre 6-Bus System Data

The proposed methodology was validated on the six-terminal HVDC benchmark system developed by the CIGRE-B4 working group, which is depicted in Figure 1. This system has been widely adopted as a benchmark for the analysis of multi-terminal DC (MTDC) grids. The network comprises six DC nodes interconnected through seven transmission lines arranged in a meshed topology, with three nodes (1, 2, and 3) coupled to AC systems via voltage source converters (VSCs), represented as DC/DC blocks in the diagram. The remaining nodes (4, 5, and 6) operate as load terminals that aggregate the equivalent demand of the connected AC subsystems [20].

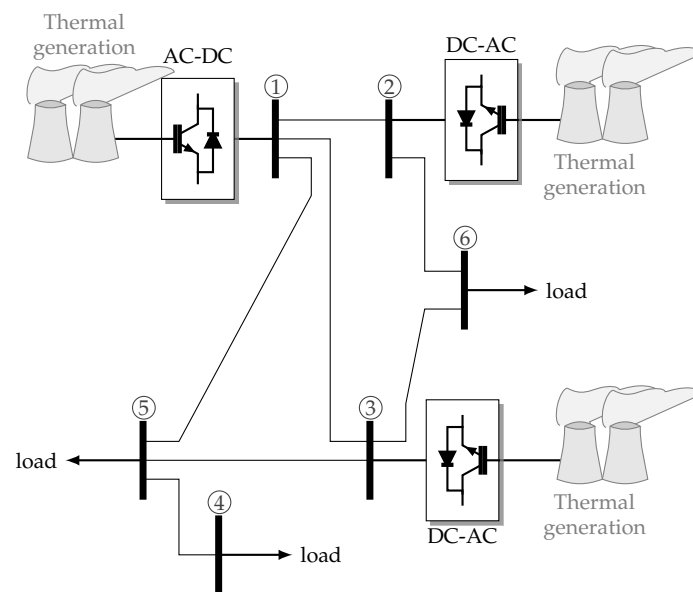


Figure 1. Six-terminal HVDC benchmark system proposed by the CIGRE-B4 working group.

The line resistances and nodal load consumptions are summarized in Table 1. The former range from $1.71\ \Omega$ to $5.70\ \Omega$, and the total system demand amounts to 6300 MW, distributed across nodes 4, 5, and 6 with 1550 MW, 2500 MW, and 2250 MW, respectively.

Table 1. Parameters of the six-terminal HVDC system proposed by the CIGRE-B4 working group.

Line Parameters					
From	To	$R_{ij}\ [\Omega]$	From	To	$R_{ij}\ [\Omega]$
1	2	5.70	3	6	4.75
2	3	2.28	1	5	1.90
2	4	1.71	5	6	1.90
1	3	2.28	–	–	–
Load Consumptions					
Node	$P\ [\text{MW}]$	Node	$P\ [\text{MW}]$	Node	$P\ [\text{MW}]$
4	1550	5	2500	6	2250

4.2. 11-Node Mt-Hvdc Test System Data

Figure 2 depicts the 11-node MT-HVDC test system proposed by [19], which is composed of 11 DC nodes interconnected through 17 transmission lines. The system integrates three thermal generation plants connected via AC/DC and DC/AC converter

stations at nodes 1, 2, and 3; two photovoltaic (PV) plants interfaced through DC/DC converters at nodes 4 and 5; and six load nodes (6, 7, 8, 9, 10, and 11), with a peak demand of 4750 MW.

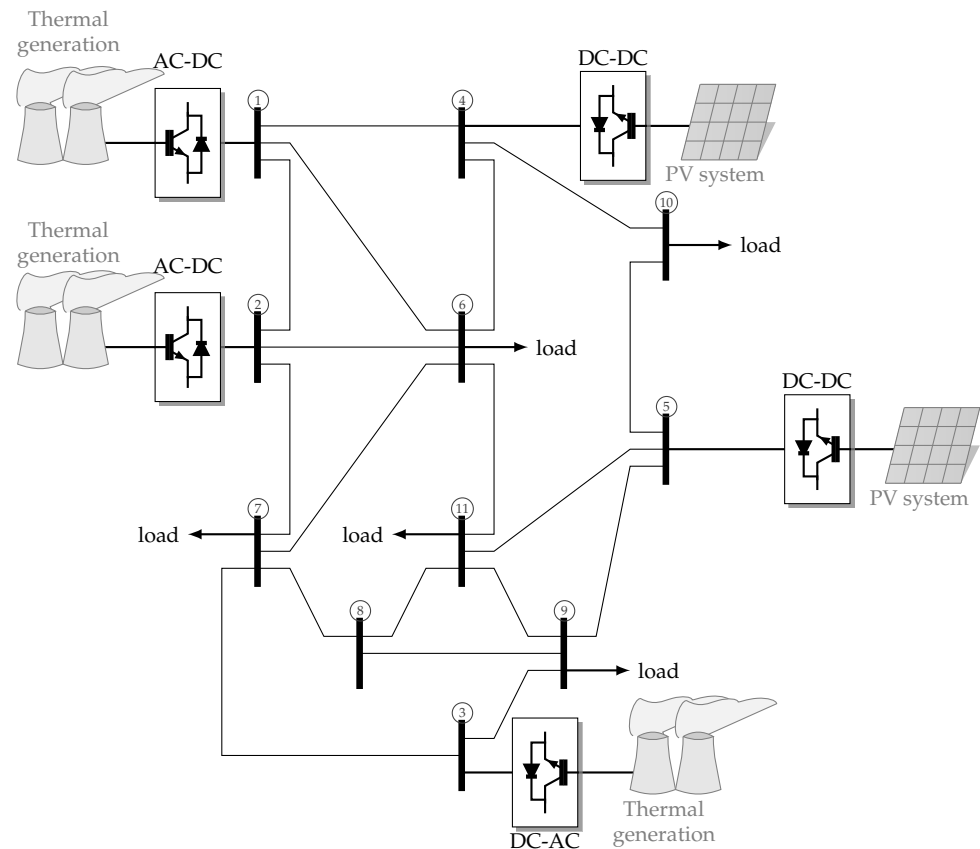


Figure 2. The 11-terminal HVDC benchmark system proposed by [19].

The branch resistances and peak load consumptions are reported in Table 2. The former range from $2.37 \, \Omega$ to $5.25 \, \Omega$, and the maximum thermal currents span between 1.50 and 3.60 kA.

Table 2. Parametric information regarding branch resistances and peak load consumptions for the 11-node MT-HVDC system.

Line Parameters							
Line #	From	To	$R_{ij} [\Omega] - I_{ij}^{\max} [\text{kA}]$	Line #	From	To	$R_{ij} [\Omega] - I_{ij}^{\max} [\text{kA}]$
1	1	2	3.85 (3.50)	10	5	9	3.34 (3.00)
2	1	4	4.22 (3.20)	11	5	10	4.12 (2.80)
3	1	6	4.85 (3.10)	12	5	11	3.78 (2.00)
4	2	6	2.37 (3.50)	13	6	7	4.65 (3.00)
5	2	7	3.25 (3.10)	14	6	11	5.25 (2.60)
6	3	7	2.95 (2.90)	15	7	8	3.14 (3.00)
7	3	9	4.36 (3.20)	16	8	9	4.55 (1.50)
8	4	6	4.02 (3.50)	17	8	11	5.14 (1.60)
9	4	10	3.87 (3.60)	—	—	—	—
Load Consumptions							
Node	$P [\text{MW}]$		Node	$P [\text{MW}]$			
6	850		7	750			
8	950		9	800			
10	650		11	700			

The economic, environmental, and operational coefficients of the thermal and PV plants are summarized in Table 3, including the cost coefficients (a , b , c), the emission parameters (α , β , γ), and the minimum and maximum active power generation limits.

Table 3. Thermal and PV plant coefficients for the 11-node MT-HVDC system.

Gen.	c (\$)	b (\$/MWh)	a (\$/MW ² h)	γ (kg)
p_1^{cg}	150	14	0.10	3.002
p_2^{cg}	125	18	0.07	4.903
p_3^{cg}	180	22	0.05	5.236
p_4^{rg}	0	40	0	0
p_5^{rg}	0	42	0	0
Gen.	β (kg/MWh)	α (kg/MW ² h)	$p^{cg,min}$ (MW)	$p^{cg,max}$ (MW)
p_1^{cg}	−4.268	0.075	150	1350
p_2^{cg}	−5.324	0.087	300	1800
p_3^{cg}	−6.576	0.060	250	2400
p_4^{rg}	32	0	0	2500
p_5^{rg}	29	0	0	2000

5. Numerical Validations and Result Analysis

The following sections present the results obtained for the two case studies considered in this work, including the exact model, the assessment of the Jacobian singularity, and the identification of the SNB point. These results are analyzed for each transmission line contingency in the evaluated systems, enabling a comprehensive comparison of the proposed methodologies' performance.

All simulations were carried out in MATLAB R2024b, on a personal computer equipped with an AMD Ryzen processor (2.00 GHz), 16 GB of RAM, and running a 64-bit version of Microsoft Windows 11.

5.1. The Cigre 6-Bus System Results

Figure 3 presents the DC PV curve of the CIGRE 6-bus HVDC system, depicting the evolution of V_{min} as a function of the loading factor λ . The curve exhibits the typical nonlinear behavior reported in the literature on voltage stability, with a pronounced inflection near $\lambda \approx 7.5$ that corresponds to an SNB, where the Jacobian matrix becomes singular and beyond which no physically feasible power-flow solutions exist.

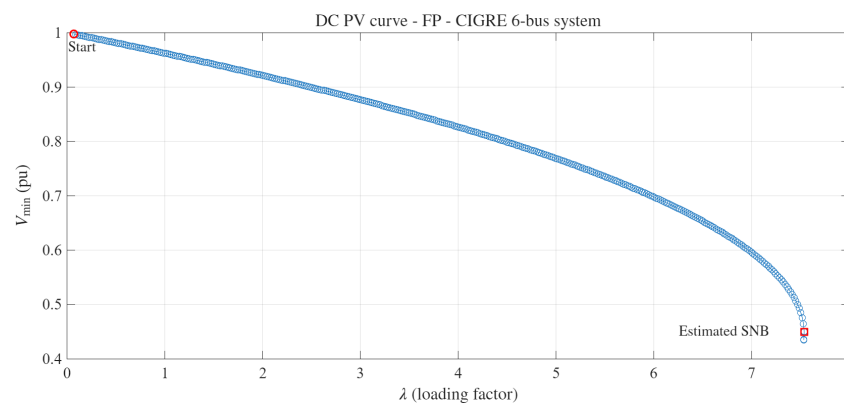


Figure 3. DC P-V curve of the CIGRE 6-bus system.

The CPF was first applied under base operating conditions, reaching a maximum loadability factor of $\lambda_{max} = 7.5370$, with an associated minimum bus voltage of

$V_{\min} = 0.4501$ p.u. The minimum singular value of the Jacobian at this point, $\sigma_{\min}(J) = 5.676 \times 10^{-1}$, corresponds to the last converged point of the CPF trajectory, not the exact SNB. The diagnostic value of $\sigma_{\min}(J)$ lies in its monotonic decrease along the solution trajectory, as illustrated in Figure 4, rather than in its absolute magnitude, which depends on the numerical scaling of the system's conductance matrix. Therefore, the bifurcation point is identified through this monotonic trend towards zero, not through a fixed absolute threshold.

A comprehensive contingency analysis was subsequently conducted by sequentially removing each of the seven transmission lines and re-evaluating the maximum loadability. The results, summarized in Table 4, reveal a significant heterogeneity in the impact of individual line outages on the voltage stability margin.

Table 4. Summary of transmission line outages sorted by severity for the CIGRE 6-bus HVDC system.

ID	From	To	$\lambda_{\max}^{\text{cont}}$	$V_{\min}$ (pu)	$\Delta\lambda_{\max}$	Reduction (%)	$\sigma_{\min}(J)$	Status
3	2	4	—	—	—	—	—	No convergence
6	1	5	3.9251	0.4775	3.6119	47.92	3.98×10^{-1}	Loadability reduced
7	5	6	3.9256	0.4830	3.6115	47.92	9.15×10^{-5}	Loadability reduced
4	1	3	5.1981	0.4483	2.3390	31.03	2.92×10^{-1}	Loadability reduced
2	2	3	5.4416	0.5001	2.0954	27.80	4.78×10^{-3}	Loadability reduced
1	1	2	5.7168	0.4536	1.8202	24.15	5.96×10^{-1}	Loadability reduced
5	3	6	8.0367	0.6964	−0.4997	−6.63	2.19×10^1	Loadability increased

The most severe outages occur in lines 1–5 and 5–6, which produce nearly identical reductions of $\Delta\lambda_{\max} \approx 3.61$ (~47.9%). Both lines connect bus 5 (a load terminal with a 2500 MW demand) to the rest of the system, so their removal effectively isolates a major portion of the system load. The outage of line 1–3 also represents a critical scenario (31.0% reduction), while lines 2–3 and 1–2 lead to moderate reductions of 27.8% and 24.2%, respectively. In contrast, the outage of line 2–4 results in the immediate non-convergence of the CPF algorithm, indicating a structurally infeasible operating condition: this line is the only path supplying the 1550 MW load at bus 4, and its disconnection isolates the load from the rest of the network.

A particularly noteworthy result was observed for line 3–6, whose outage yields an unexpected increase in the maximum loadability up to $\lambda_{\max} = 8.0367$ (~6.6% above the base case). This non-monotonic behavior is attributed to a favorable redistribution of power flows that relaxes the operating constraints imposed by this line under the original topology, and it underscores the importance of full nonlinear analyses, since linearized sensitivity indices may fail to capture such phenomena. Based on these results, the transmission lines can be ranked by criticality: 2–4 (structural) > 1–5 $\approx$ 5–6 > 1–3 > 2–3 > 1–2 > 3–6. This provides valuable insights for the prioritization of protection schemes and preventive maintenance strategies.

The evolution of the minimum singular value of the Jacobian, $\sigma_{\min}(J)$, shown in Figure 4, confirms the previous analysis: as the system approaches the critical point, $\sigma_{\min}(J)$ decreases progressively and tends to zero in the vicinity of the SNB, in agreement with voltage stability theory.

This trend is complemented by the Jacobian condition number, $\text{cond}(J)$, depicted in Figure 5, which exhibits a steep growth as the bifurcation point is approached, reflecting a severe deterioration in numerical conditioning and the increasing sensitivity of nodal voltages to small perturbations. Together, both indicators provide a reliable early-warning mechanism for imminent voltage collapse without additional computational overhead.

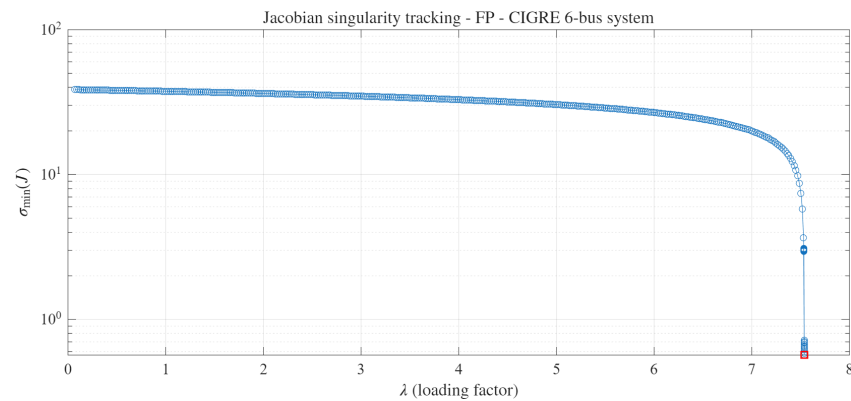


Figure 4. Tracking the Jacobian singularity for the CIGRE 6-bus HVDC system.

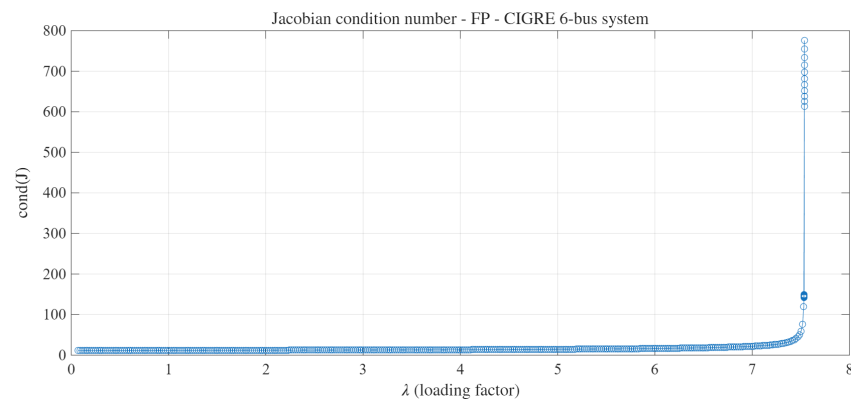


Figure 5. Jacobian condition number $\text{cond}(J)$ for the CIGRE 6-bus HVDC system.

The bus voltage profiles, presented in Figure 6, allow for identifying each node's contribution to the instability phenomenon. All buses exhibit a progressive voltage decrease with increasing load, albeit with markedly different sensitivities: certain buses show more pronounced drops, reaching values close to 0.45 p.u. near the collapse point. These buses can be regarded as the weak nodes of the system, since they are more susceptible to load variations and play a decisive role in the overall voltage stability.

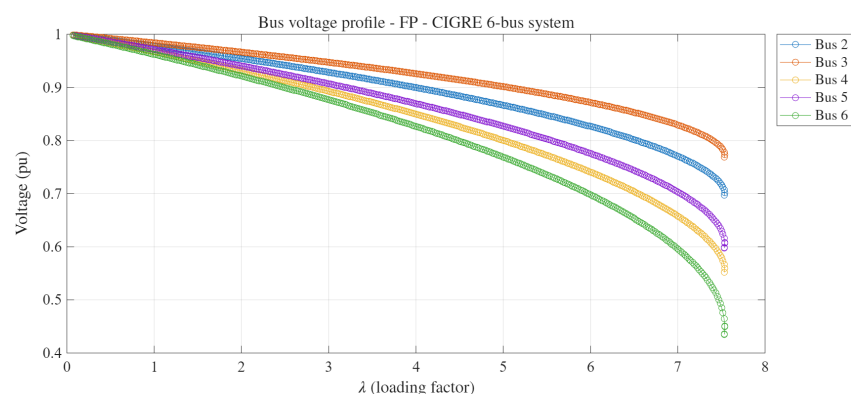


Figure 6. Bus voltage profiles vs. loading factor λ for the CIGRE 6-bus HVDC system.

The impact of each contingency on the stability margin is graphically summarized in Figures 7 and 8, which depict the absolute values of $\lambda_{\max}$ and the corresponding deviations $\Delta\lambda_{\max}$ with respect to the base case. Both visualizations consistently identify lines 1–5 and 5–6 as the most critical and confirm the anomalous behavior of line 3–6. This non-monotonic response highlights the complexity of HVDC networks, where line outages do not necessarily translate into a deterioration in the stability margin and where structural and operational aspects must be jointly considered in any voltage stability assessment.

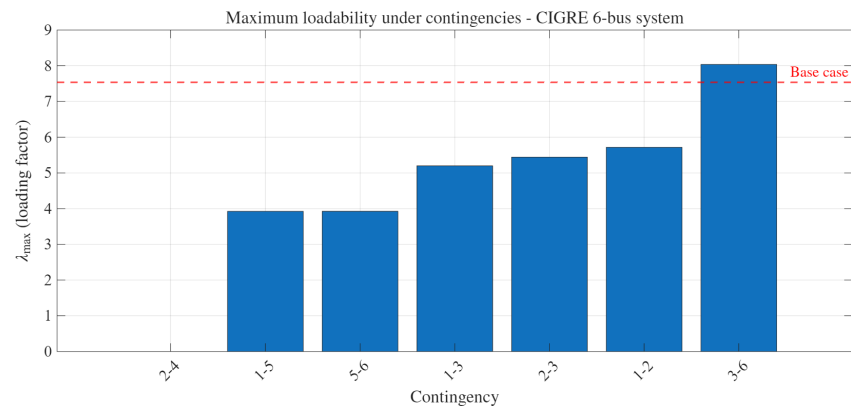


Figure 7. Maximum loadability factor $\lambda_{\max}$ under transmission line outages for the CIGRE 6-bus HVDC system.

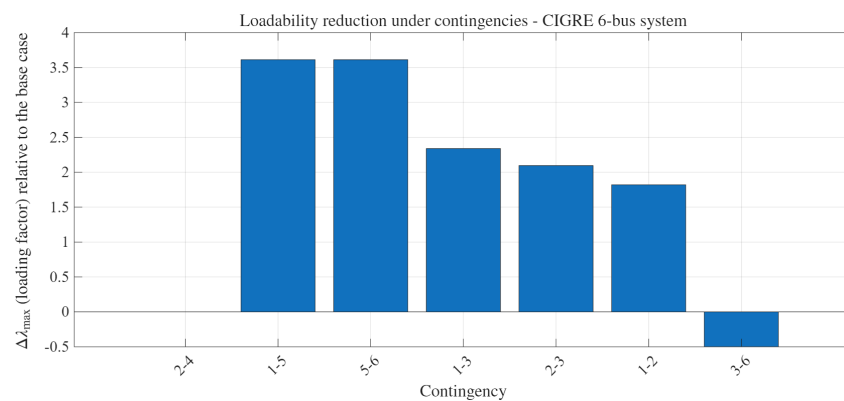


Figure 8. Loadability reduction $\Delta\lambda_{\max}$ relative to the base case under transmission line outages for the CIGRE 6-bus HVDC system.

5.2. The 11-Node Mt-Hvdc System Results

The DC PV curve of the 11-node system, shown in Figure 9, exhibits nonlinear behavior that is typically observed in electrical systems operating close to voltage instability conditions. As the loading factor λ increases from nominal conditions ($\lambda \approx 1$), the minimum system voltage decreases progressively. A critical region is observed around $\lambda \approx 5.2$, where the slope of the curve steepens markedly, indicating an accelerated voltage drop. The point labeled as Estimated SNB corresponds to the maximum loadability limit of the system at which an SNB occurs and beyond which no stable power-flow solution exists.

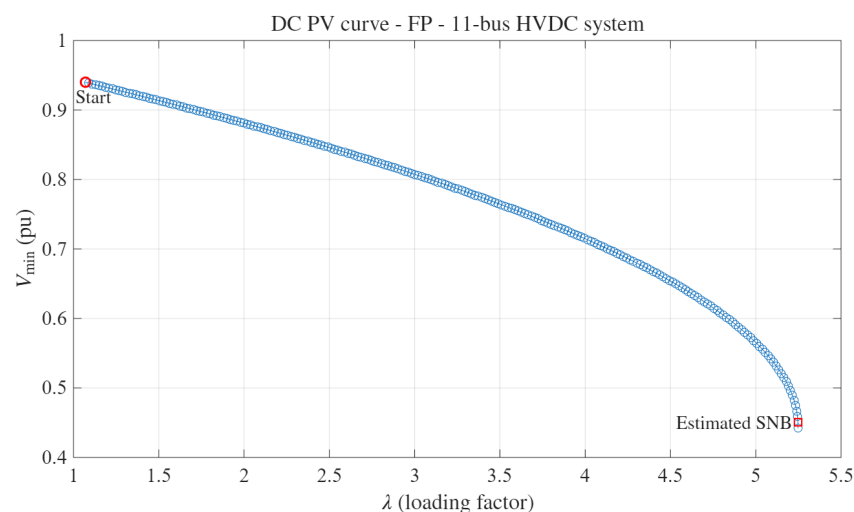


Figure 9. DC P-V curve of the 11-node MT-HVDC system.

In comparison with its 6-bus counterpart, the 11-bus system exhibits a smaller loadability margin, which suggests a higher structural sensitivity to load increments.

The results of the contingency analysis, summarized in Table 5, reveal a wide range of impacts on the voltage stability margin of the 11-bus HVDC system. With the base case ($\lambda_{\max} \approx 5.25$) as reference, most contingencies reduce this margin, highlighting the strong influence of the network topology on voltage stability. The most severe outages correspond to lines 1-2 and 1-4, with reductions of 29.85% and 26.12%, respectively. Both lines emerge from the slack bus and constitute primary supply paths for a substantial portion of the system load; their disconnection forces a redistribution of currents through longer and more resistive routes.

Table 5. Summary of transmission line outages sorted by severity for the 11-bus HVDC system.

ID	From	To	$\lambda_{\max}^{\text{cont}}$	$V_{\min}$ (pu)	$\Delta\lambda_{\max}$	Reduction (%)	$\sigma_{\min}(J)$	Status
1	1	2	3.6806	0.4547	1.5663	29.85	1.45×10^{-2}	Loadability reduced
2	1	4	3.8766	0.4537	1.3703	26.12	5.63×10^{-2}	Loadability reduced
14	6	11	4.3433	0.4319	0.9036	17.22	5.34×10^{-2}	Loadability reduced
3	1	6	4.4352	0.4619	0.8117	15.47	9.61×10^{-2}	Loadability reduced
11	5	10	4.7455	0.4434	0.5014	9.56	7.93×10^{-2}	Loadability reduced
13	6	7	4.8483	0.4503	0.3986	7.60	3.13×10^{-2}	Loadability reduced
9	4	10	4.8866	0.4483	0.3603	6.87	6.33×10^{-2}	Loadability reduced
10	5	9	4.8903	0.4171	0.3566	6.80	3.23×10^{-2}	Loadability reduced
6	3	7	4.9937	0.4246	0.2531	4.82	1.10×10^{-1}	Loadability reduced
5	2	7	5.0043	0.4558	0.2426	4.62	2.35×10^{-2}	Loadability reduced
17	8	11	5.1498	0.4401	0.0971	1.85	2.57×10^{-2}	Loadability reduced
8	4	6	5.1505	0.4469	0.0964	1.84	1.19×10^{-1}	Loadability reduced
4	2	6	5.1831	0.4512	0.0638	1.22	1.76×10^{-1}	Loadability reduced
16	8	9	5.2425	0.4417	0.0044	0.08	9.89×10^{-2}	Loadability reduced
12	5	11	6.0994	0.4401	−0.8525	−16.25	1.08×10^{-1}	Loadability increased
15	7	8	6.6267	0.4313	−1.3798	−26.30	1.65×10^{-1}	Loadability increased
7	3	9	6.6306	0.4349	−1.3838	−26.37	1.20×10^{-1}	Loadability increased

A second tier of critical contingencies is observed for lines 6–11, 1–6, and 5–10, with reductions between 9.5% and 17.2%, while the remaining outages produce only marginal effects. Conversely, the disconnection of lines 5–11, 7–8, and 3–9 yields an unexpected increase of up to 26.4% in the maximum loadability, indicating that, under the original topology, these lines impose unfavorable flow distributions whose removal enables a more efficient power redistribution. This non-monotonic response reaffirms the importance of full nonlinear analyses for HVDC voltage stability assessment, particularly in structurally complex networks such as the 11-bus feeder, which exhibits a higher proportion of such behaviors than its 6-bus counterpart.

The behavior of the minimum singular value of the Jacobian, $\sigma_{\min}(J)$, shown in Figure 10, confirms the trend observed in the PV curve: as λ increases, $\sigma_{\min}(J)$ decreases progressively and approaches zero in the vicinity of the critical point, indicating that the system tends towards singularity conditions associated with the loss of a feasible power flow solution.

This is complemented by the Jacobian condition number, $\text{cond}(J)$, depicted in Figure 11, which grows steadily with λ and reaches values exceeding 2000 near the bifurcation point, reflecting a severe deterioration in numerical conditioning, whereby small load perturbations may induce large variations in the nodal voltages. Together, the abrupt decline of $\sigma_{\min}(J)$ and the steep growth of $\text{cond}(J)$ constitute robust and complementary indicators of imminent voltage collapse.

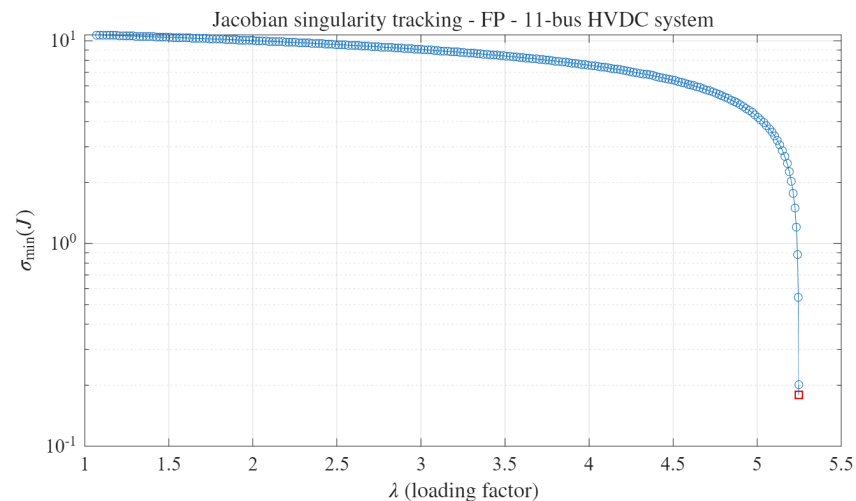


Figure 10. Tracking the Jacobian singularity for the 11-node MT-HVDC system.

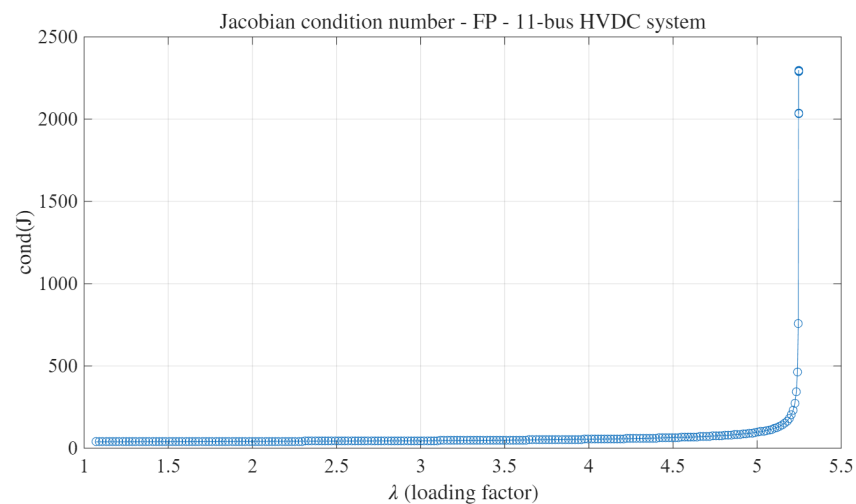


Figure 11. Jacobian condition number $\text{cond}(J)$ for the 11-node MT-HVDC system.

The bus voltage profiles for the 11-bus system, presented in Figure 12, reveal that all buses experience a progressive voltage decrease as the load increases, although significant differences in nodal sensitivity are observed. Some buses exhibit more pronounced drops, reaching values close to 0.45 p.u. under critical conditions. These can be regarded as the weak nodes of the system, since they are more vulnerable to demand variations and contribute decisively to the voltage collapse phenomenon. The greater dispersion of the voltage curves in comparison with the 6-bus system reflects a higher structural complexity in which inter-nodal interactions exert a more pronounced influence on the overall stability.

A complementary view of the contingency impact is provided in Figures 13 and 14, which depict the maximum loadability factor and its variation $\Delta\lambda_{\max}$ across all simulated outages. The figures confirm that lines 1–2 and 1–4 produce reductions exceeding 1.3 units in the loading factor, which evidences their essential role in the voltage support of the system. Meanwhile, the negative values of $\Delta\lambda_{\max}$, associated with lines 3–9, 7–8, and 5–11, illustrate the non-monotonic behavior discussed earlier in this document.

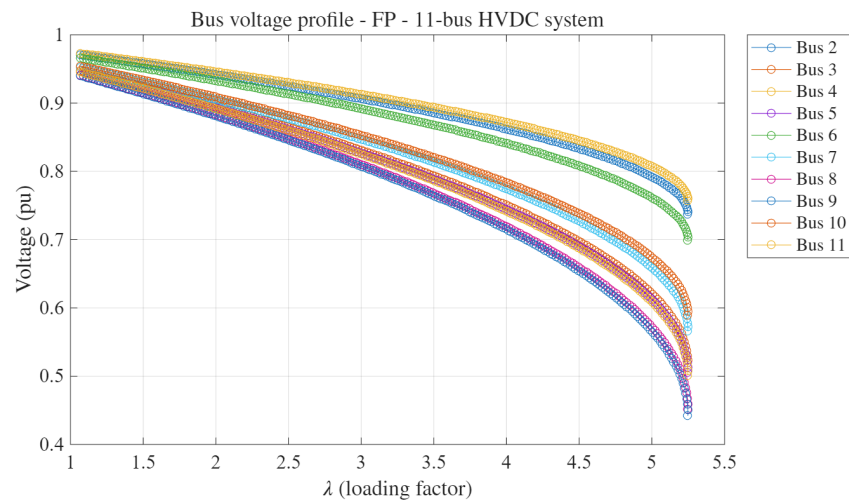


Figure 12. Bus voltage profiles *vs.* the loading factor λ for the 11-node MT-HVDC system.

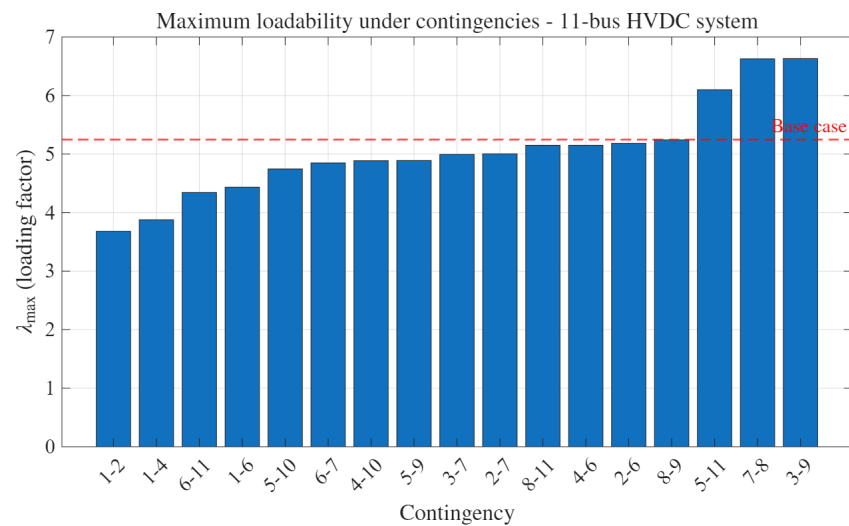


Figure 13. Maximum loadability factor $\lambda_{\max}$ under transmission line outages for the 11-node MT-HVDC system.

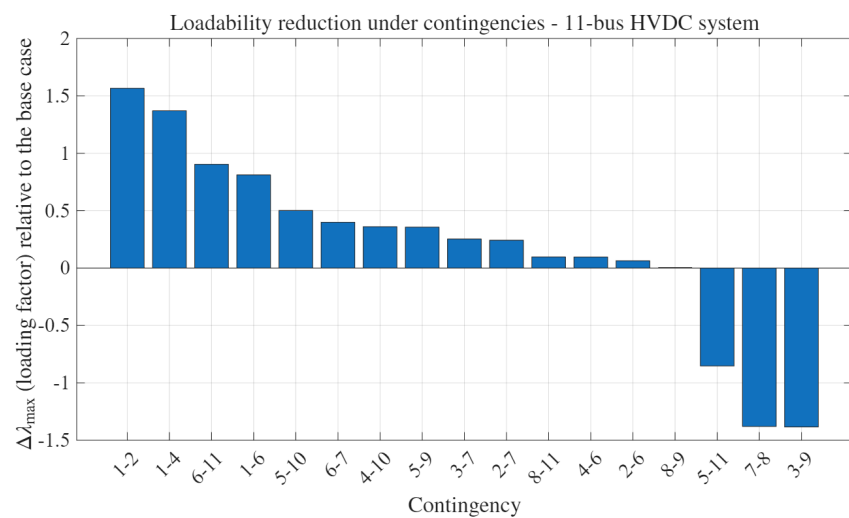


Figure 14. Loadability reduction $\Delta\lambda_{\max}$ relative to the base case under transmission line outages for the 11-node MT-HVDC system.

5.3. Effect of Non-Uniform Loading Patterns

To assess the robustness of the proposed methodology with respect to the load distribution assumption, an additional test was conducted on both benchmark systems: the loadability factor was applied non-uniformly across load nodes. Specifically, node-specific participation factors α_k were introduced, such that the demand at each load node was expressed as $P_k(\lambda) = \lambda \alpha_k P_k^0$, wherein heavier-loaded nodes received proportionally larger weights. The factors were normalized to maintain the total load growth rate approximately comparable to that of the uniform case, ensuring a meaningful comparison between both scenarios.

For the CIGRE 6-bus system, the DC P-V curves for the uniform and non-uniform cases closely followed the same trajectory up to $\lambda \approx 4$, after which a slight divergence was observed, with the non-uniform case reaching voltage collapse at a marginally lower loading level ($\lambda_{\text{non-uniform}}^* \approx 7.48$ vs. $\lambda_{\text{uniform}}^* \approx 7.55$, a reduction of approximately 0.9%). The $\sigma_{\min}(J)$ trajectories are virtually identical throughout the loading range and diverge only near the critical point, where the non-uniform case reaches singularity slightly earlier, as shown in Figures 15–17.

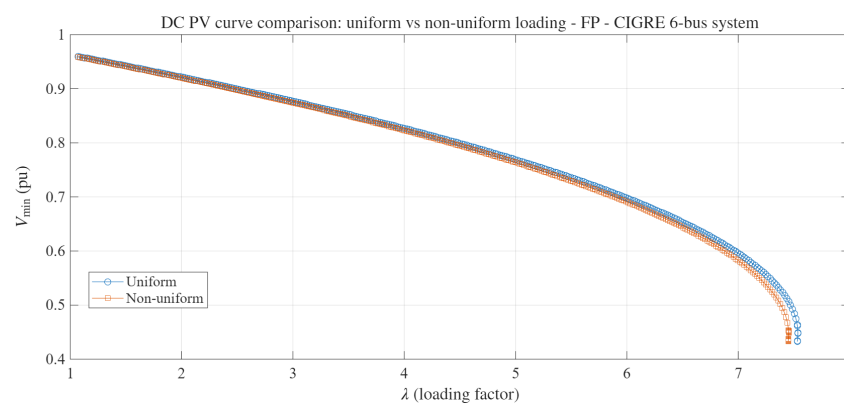


Figure 15. DC P-V curve comparison under uniform and non-uniform loading patterns for the CIGRE 6-bus HVDC system.

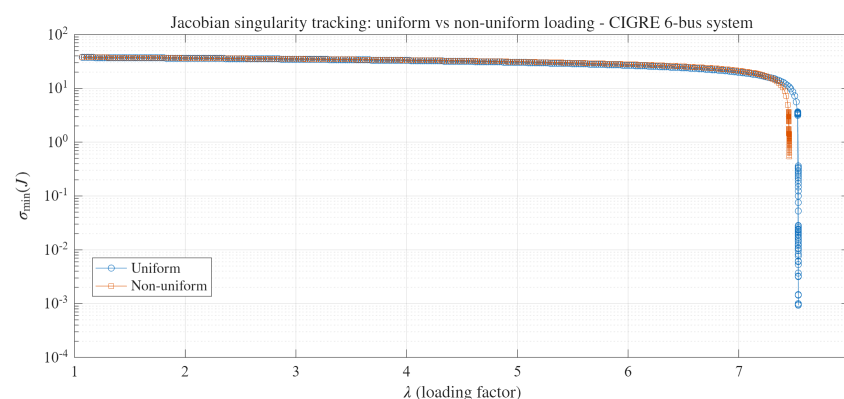


Figure 16. Jacobian singularity: tracking $\sigma_{\min}(J)$ under uniform and non-uniform loading patterns for the CIGRE 6-bus HVDC system.

For the 11-node MT-HVDC system, the two loading patterns produced DC P-V curves that were practically indistinguishable across the entire trajectory. The estimated critical loading factors are $\lambda_{\text{uniform}}^* \approx 5.25$ and $\lambda_{\text{non-uniform}}^* \approx 5.22$, corresponding to a difference of less than 0.6%. Likewise, the $\sigma_{\min}(J)$ trajectories were nearly identical, with the non-uniform case reaching marginally lower singular values near the bifurcation point, as illustrated in Figures 18–20.

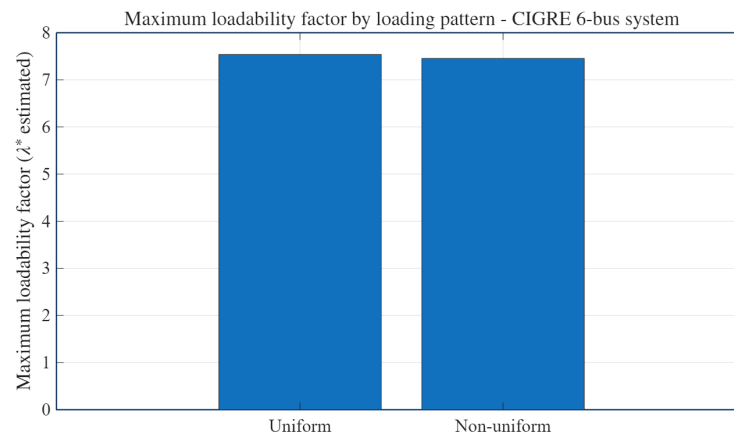


Figure 17. Maximum loadability factor λ^* under uniform and non-uniform loading patterns for the CIGRE 6-bus HVDC system.

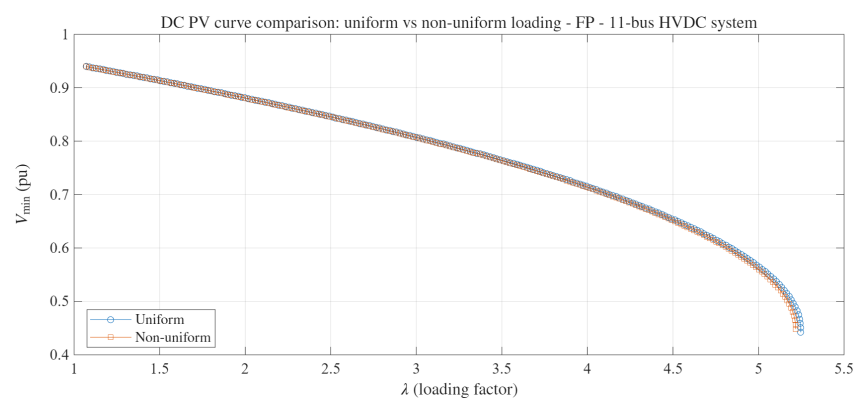


Figure 18. DC P-V curve comparison under uniform and non-uniform loading patterns for the 11-node MT-HVDC system.

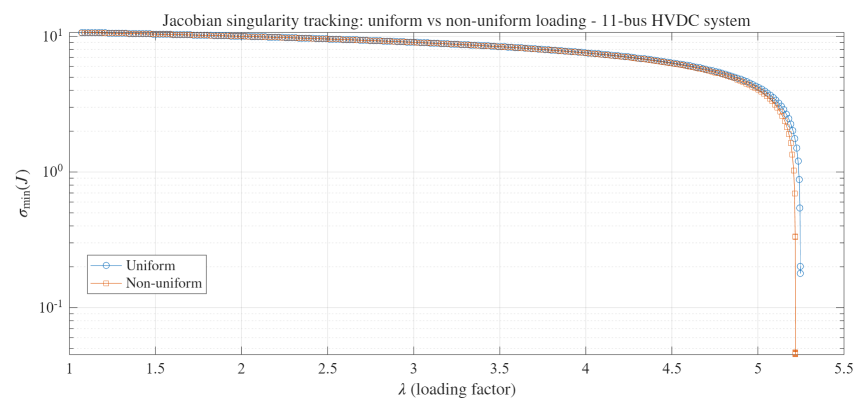


Figure 19. Jacobian singularity: tracking $\sigma_{min}(J)$ under uniform and non-uniform loading patterns for the 11-node MT-HVDC system.

In both systems, the quantitative differences in λ^* between the two scenarios remained below 1%, confirming that the uniform scaling assumption adopted in this work yields results that are representative of the system's static voltage stability characteristics and does not introduce significant bias in the critical loading factor estimation. This finding also demonstrates that the monotonic decrease in $\sigma_{min}(J)$ towards zero as a reliable early-warning indicator of imminent voltage collapse holds regardless of the load distribution pattern, thus supporting the generality of the proposed methodology.

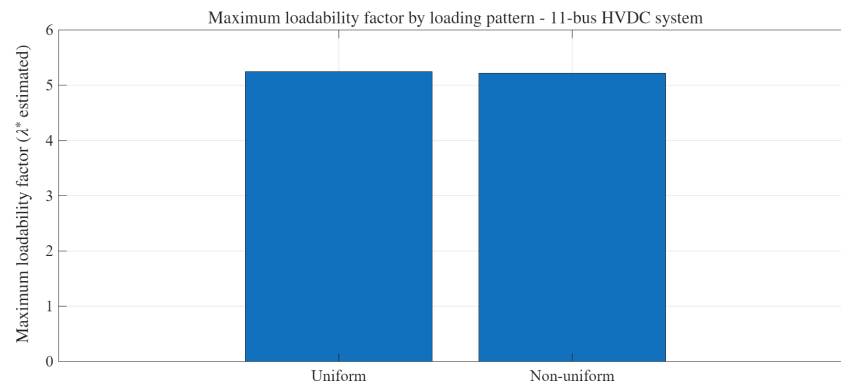


Figure 20. Maximum loadability factor λ^* under uniform and non-uniform loading patterns for the 11-node MT-HVDC system.

6. Conclusions and Future Work

This work presented a voltage stability assessment framework for HVDC transmission systems, integrating the CPF method with Jacobian-based singularity indicators and systematic transmission line outage analysis. This approach was validated on the CIGRE 6-bus HVDC system and the 11-node MT-HVDC system, yielding the following conclusions.

First, the CPF algorithm was successfully employed to trace the complete DC P-V curves of both systems, accurately identifying the SNB point as the ultimate boundary of static voltage stability. For the CIGRE 6-bus system, a maximum loadability factor of $\lambda_{\max} = 7.5370$ was obtained under base operating conditions, whereas the 11-node system exhibited a substantially smaller margin of $\lambda_{\max} \approx 5.25$. This contrast highlights that loadability is not solely governed by the system size, but also by structural attributes such as topological redundancy, the spatial distribution of loads, and the electrical distance between generation and demand nodes.

Second, the joint monitoring of the minimum singular value $\sigma_{\min}(J)$ and the condition number $\text{cond}(J)$ of the power flow Jacobian proved to be a robust and computationally efficient mechanism for the early detection of voltage collapse. In both case studies, $\sigma_{\min}(J)$ decreased monotonically towards zero, while $\text{cond}(J)$ exhibited a steep growth as the loading factor approached the SNB point. These complementary indicators provide a reliable quantitative mechanism for warning of imminent instability without incurring additional computational burden, which can be readily integrated into real-time monitoring schemes.

Third, the systematic contingency analysis revealed a markedly heterogeneous impact of individual line outages on the voltage stability margin. In the CIGRE 6-bus system, the outage of line 2–4 led to immediate non-convergence of the CPF, denoting structurally infeasible operating conditions, while lines 1–5 and 5–6 induced loadability reductions of approximately 47.9%. Analogously, in the 11-node system, lines 1–2 and 1–4 emerged as the most critical, with reductions of 29.85% and 26.12%, respectively. These findings confirm that lines directly connected to the slack bus or constituting the sole supply path to high-demand nodes deserve priority in protection coordination and preventive maintenance planning.

Finally, a particularly relevant phenomenon was identified in both systems: the disconnection of certain lines (3–6 in the 6-bus system, and 3–9, 7–8, and 5–11 in the 11-node system) resulted in an unexpected increase in the maximum loadability. This non-monotonic behavior, with loadability gains of up to 26.4%, evidences that, in the original topology, these lines impose unfavorable flow distributions whose removal enables a more efficient power redistribution. This result has two important implications. On one hand, it demonstrates the limitations of linearized sensitivity-based indices, which,

by construction, cannot capture such effects. On the other hand, it opens the possibility of conceiving topology reconfiguration strategies as a complementary tool for enhancing voltage stability margins in HVDC networks.

Despite its demonstrated effectiveness, the proposed methodology has several limitations that should be acknowledged. First, the validation was conducted exclusively on small-scale benchmark systems (6 and 11 nodes), and the computational scalability of their implementation in large-scale meshed MT-HVDC networks has not been demonstrated. Second, the framework relies solely on static power flow analysis and does not account for the dynamic behavior of voltage source converters (VSCs) and their control strategies, which may influence stability under transient conditions. Third, the formulation is deterministic and assumes fixed generation and demand profiles, which may be insufficient in systems with a high penetration of renewable energy, which may exhibit significant uncertainty in generation and load. Finally, the proposed approach has only been validated through numerical simulations in MATLAB, and its applicability in real-time energy management systems is yet to be verified through hardware-in-the-loop experimental platforms.

Several research directions emerge from this study. For instance, the proposed methodology can be extended to large-scale meshed MT-HVDC systems and hybrid AC/DC grids, which would require the adoption of sparse solvers and parallelization schemes to ensure computational tractability. The incorporation of dynamic VSC models, together with their associated control strategies, would enable a joint assessment of static and dynamic stability phenomena [26–28]. Likewise, the formulation of probabilistic CPF schemes is particularly relevant in the context of high renewable energy penetration, where generation and demand both exhibit significant uncertainty. Other promising directions include the development of optimization- and machine-learning-based methods for the efficient screening of critical contingencies, the exploitation of the observed non-monotonic behavior through topology reconfiguration strategies within a security-constrained optimal power flow (SC-OPF) framework, the integration of Jacobian-based indicators into adaptive protection and special protection schemes (SPS), and the experimental validation of the proposed approach on hardware-in-the-loop platforms aimed at deployment in real-time energy management systems.

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References

- Schipper, J.; Bathini, V.; Mukhedkar, R.; Watson, N.; Pirnia, M.; Nair, N.K.; Watson, J. Steady-state power system analysis revisited for hybrid AC–DC grids. *Renew. Sustain. Energy Rev.* **2025**, *215*, 115564. <https://doi.org/10.1016/j.rser.2025.115564>.
- Zhou, Y.; Liu, Y.; Huang, S.; Guo, C.; Cao, Y.; Li, Y. A review of HVDC-based transmission congestion alleviation strategies for modern power systems. *Electr. Power Syst. Res.* **2026**, *255*, 112748. <https://doi.org/10.1016/j.epsr.2026.112748>.
- Hou, J.; Deng, H.; Vorobev, P.; Peng, J.C.H. Stability of two-terminal high-voltage direct-current power transmission systems: Nexus between parameters and controls. *Int. J. Electr. Power Energy Syst.* **2025**, *172*, 111154. <https://doi.org/10.1016/j.ijepes.2025.111154>.
- Abo-Khalil, A.G.; Hassan, A.; Sayed, K. HVDC Systems and Renewable Energy in a Comparative Study of Technologies and Applications. *e-Prime—Nexus Electr. Electron. Intell. Eng.* **2026**, *17*, 201167. <https://doi.org/10.1016/j.eprime.2026.201167>.
- Montoya Giraldo, O.D.; Ramírez-Vanegas, C.A.; Mesa, F. Generalized Power Flow Algorithms for Monopolar DC Networks Using Broyden's Method. *Ingeniería* **2025**, *30*, e24009. <https://doi.org/10.14483/23448393.24009>.
- Ibrahim, M.; Ang, S.; Dani, M.; Petra, R.; Salam, M. Voltage stability assessment for solar photovoltaic penetration using Reactive Power–Voltage and Active Power–Voltage modal analysis. *Energy Rep.* **2023**, *9*, 486–493. <https://doi.org/10.1016/j.egyr.2023.05.124>.
- Chen, H.; Jiang, T.; Yuan, H.; Jia, H.; Bai, L.; Li, F. Wide-area measurement-based voltage stability sensitivity and its application in voltage control. *Int. J. Electr. Power Energy Syst.* **2017**, *88*, 87–98. <https://doi.org/10.1016/j.ijepes.2016.12.011>.
- Li, Z.; Xue, Y.; Chen, M.; Swanson, A.; Yang, C.; Li, J. Analysis of the Impact of HVDC Frequency Control Parameters on HVDC Power Oscillation and Interarea Oscillation. In Proceedings of the 2024 International Conference on HVDC (HVDC), Urumqi, China, 8–9 August 2024; IEEE: New York, NY, USA, 2024; pp. 355–360. <https://doi.org/10.1109/hvdc62448.2024.10723005>.
- Maria, C.; Ajeesh, A.; Shereef, R.; Samuel, N.; Jayan, P.P. Real-time Digital Simulation of CIGRE HVDC System using Full Spectrum Simulator. In Proceedings of the 2020 IEEE International Conference on Power Electronics, Smart Grid and Renewable Energy (PESGRE2020), Cochin, India, 2–4 January 2020; IEEE: New York, NY, USA, 2020; pp. 1–6. <https://doi.org/10.1109/pesgre45664.2020.9070761>.
- Lou, Y.; Ou, Z.; Tong, Z.; Tang, W.; Li, Z.; Yang, K. Static Voltage Stability Evaluation on the Urban Power System by Continuation Power Flow. In Proceedings of the 2022 5th International Conference on Energy, Electrical and Power Engineering (CEEPE), Chongqing, China, 22–24 April 2022; IEEE: New York, NY, USA, 2022; pp. 833–838. <https://doi.org/10.1109/ceepe55110.2022.9783364>.
- Rao, S.D.; Tylavsky, D.J.; Feng, Y. Estimating the saddle-node bifurcation point of static power systems using the holomorphic embedding method. *Int. J. Electr. Power Energy Syst.* **2017**, *84*, 1–12. <https://doi.org/10.1016/j.ijepes.2016.04.045>.
- Marcial, A.; Perninge, M. Finding a closest saddle–node bifurcation in power systems: An approach by unsupervised deep learning. *Electr. Power Syst. Res.* **2024**, *235*, 110632. <https://doi.org/10.1016/j.epsr.2024.110632>.
- Li, X.; Cao, L.; Liu, G.; Liu, J.; Chen, Y. Spike-adding in neuron systems induced by bifurcation or non-bifurcation: A comparative study via solution continuation. *Chaos Solitons Fractals* **2026**, *203*, 117633. <https://doi.org/10.1016/j.chaos.2025.117633>.
- Prabhakar, P.; Kumar, A. Voltage stability boundary and margin enhancement with FACTS and HVDC. *Int. J. Electr. Power Energy Syst.* **2016**, *82*, 429–438. <https://doi.org/10.1016/j.ijepes.2016.03.038>.
- Campaña, M.; Masache, P.; Inga, E.; Carrión, D. Voltage stability and electronic compensation in electrical power systems using simulation models (In Spanish). *Ingenius* **2023**, *9*–23. <https://doi.org/10.17163/ings.n29.2023.01>.
- Zeng, L.; Chiang, H.D.; Neves, L.S.; Alberto, L.F.C. On the accuracy of power flow and load margin calculation caused by incorrect logical PV/PQ switching: Analytics and improved methods. *Int. J. Electr. Power Energy Syst.* **2023**, *147*, 108905. <https://doi.org/10.1016/j.ijepes.2022.108905>.
- Wu, Y.; Dong, X.; Lu, T.; Yu, S.; Wang, C.; Li, Z. A load margin calculation method considering optimized reactive power support. *Int. J. Electr. Power Energy Syst.* **2025**, *168*, 110706. <https://doi.org/10.1016/j.ijepes.2025.110706>.
- Ndjankeu, N.M.; Moulum, P.A.; Kenfack, P.; Naoussi, S.R.D. Modern sensitivity index-based optimal UPFC location for power quality enhancement in the Southern Cameroon interconnected grid considering the feasibility of the load flow under N-k contingency scenarios. *Energy Rep.* **2025**, *13*, 5160–5186. <https://doi.org/10.1016/j.egyr.2025.04.049>.
- Contreras-Sepúlveda, W.; Montoya, O.D.; Gil-González, W. An economic-environmental energy management system design for MT-HVDC networks via a semi-definite programming approximation with robust analysis. *Ain Shams Eng. J.* **2024**, *15*, 102968. <https://doi.org/10.1016/j.asej.2024.102968>.
- Montoya, O.D.; Gil-González, W.; Garces, A. Numerical methods for power flow analysis in DC networks: State of the art, methods and challenges. *Int. J. Electr. Power Energy Syst.* **2020**, *123*, 106299. <https://doi.org/10.1016/j.ijepes.2020.106299>.
- Li, C.; Zhang, X.; He, P.; Zhen, Z.; Zhao, K. Adaptive Droop Control of VSC-MTDC System Based on Virtual Inertia. *Electronics* **2023**, *12*, 2324. <https://doi.org/10.3390/electronics12102324>.
- Li, C.; Zhang, X.; He, P.; Zhao, K.; Liu, L. Improved coordinated control strategy for VSC-MTDC system with DC voltage secondary regulation. *Front. Energy Res.* **2024**, *12*, 1363267. <https://doi.org/10.3389/fenrg.2024.1363267>.

23. Glover, J.D.; Sarma, M.S. *Power System Analysis and Design*; Brooks/Cole: St. Paul, MN, USA, 2002.
24. Wang, T.; Wang, S.; Ma, S.; Guo, J.; Zhou, X. An Extended Continuation Power Flow Method for Static Voltage Stability Assessment of Renewable Power Generation-Penetrated Power Systems. *IEEE Trans. Circuits Syst. II Express Briefs* **2024**, *71*, 892–896. <https://doi.org/10.1109/tcsii.2022.3209335>.
25. Wang, Y.; Xin, H.; Wang, G. Iterative Searching Method for Closest Saddle-Node Bifurcation Point in Multi-Infeed HVDC Systems. *IEEE Trans. Power Syst.* **2024**, *39*, 6099–6102. <https://doi.org/10.1109/tpwrs.2024.3390410>.
26. Haro-Larrode, M.; Santos-Mugica, M.; Etxegarai, A.; Eguia, P. Methodology for Tuning MTDC Supervisory and Frequency-Response Control Systems at Terminal Level under Over-Frequency Events. *Energies* **2020**, *13*, 2807. <https://doi.org/10.3390/en13112807>.
27. Lu, Y.; Shi, G.; Chen, Q.; Qiu, P.; Zhou, J.; Yang, R.; Zhang, J. Stability analysis and stabilization control of a grid-forming VSC-HVDC system. *Front. Energy Res.* **2024**, *12*, 1437287. <https://doi.org/10.3389/fenrg.2024.1437287>.
28. Bin, Z.; Kong, X.; Zhao, K.; Wu, X.; Yuan, Y.; Ren, X. Stability Analysis of a Receiving-End VSC-HVDC System with Parallel-Connected VSCs. *Electronics* **2025**, *14*, 2178. <https://doi.org/10.3390/electronics14112178>.

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