Scales and Hierarchies: Planckian Signature in Standard Model

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Abstract: A model of a physical vacuum defined by a Gross–Pitaevskij equation and characterized by dissipative features close to the Planck scale is proposed, which provides an emergent explanation of scales, hierarchies and Higgs mass generation of the Standard Model. A fundamental nonlocal and nonlinear texture of the vacuum is introduced in terms of planckeons, sub-Planckian objects defined by a generalized Compton wavelength, which lead to find Planckian signatures of the Standard Model.

Keywords: planck scale; dissipative vacuum; planckeons; gross–pitaevskij equation; standard model; higgs mechanism

1. Introduction

The Standard Model of particle physics is the fundamental theoretical scheme which provides a detailed description of the elementary particles of matter and their interactions up to at least a few TeV as revealed in experiments at the LHC and in low-energy precision experiments. In this theory, the spontaneous symmetry breaking mechanism generates the masses of the weak gauge bosons, gives rise to the appearance of the Higgs boson, as well as of the fermion masses and mixings. Despite its extraordinary predictive power and the recent discovery of the Higgs boson, which seems to complete the particle spectrum of the theory, the Standard Model is affected by several flaws which indicate the necessity to develop an extension of this theory. In particular, there is the problem of Higgs boson’s couplings, which still requires a satisfactory solution, and there is a quadratically divergent counterterm in the mass of the Higgs boson which would push its value towards the Planck scale. On the other hand, some extra new physics is still needed in order to explain other questions, such as the origin of tiny neutrino masses, the matter–antimatter asymmetry in the Universe, the strong CP puzzle, the origin of dark matter, and the vacuum energies associated with the cosmological constant and initial inflation. The origin of this new physics and how it interacts with the Standard Model is yet unknown [1].

In order to develop a beyond Standard Model physics, namely, to treat processes beyond the TeV scale, the Higgs sector seems to play a key role in the sense that it provides a portal between the visible and dark sector [1–4], is the ultimate source of the interactions with right-handed neutrinos in the leptogenesis framework which can reproduce the baryon asymmetry [5], and leads to a possible explanation of the origin of the electroweak symmetry breaking if the Higgs field is associated to additional scalar particles [6–9]. However, in the light of the interactions of the Higgs boson with any heavier state of new physics [10–14], quantum corrections to the Higgs mass are involved that are quadratic in the mass of the heavy particle. This implies that the Standard Model turns out to be affected by a hierarchy problem, namely, in light of the absence of new physics signatures at the TeV scale and beyond, the observed Higgs mass appears rather “unnatural” and there is the problem to explain the smallness of the electroweak scale [15]. On the other hand, the hierarchy problem of the Standard Model can also be considered as a fine-tuning problem, regarding the careful choice of the parameters of new high-scale physics in order
to reproduce the observed low-energy parameters. As a consequence, there is the problem to explain the extreme smallness of the cosmological constant of vacuum energy density, which drives the accelerating expansion of the universe with respect to the Higgs mass, QCD scale, and Planck scale.

As regards the treatment of the hierarchy problem of the Standard Model at the TeV scale, if several proposals exist in the literature, such as supersymmetric extensions [16–19], strong dynamics or technicolor [20,21], extra dimensions [25–27], and the recent decoupling methods [28], it must be emphasized that all these models imply a fine-tuning process in order to stay compatible with experiments [15,29–32].

An important aspect of the fine-tuning problem connected to the hierarchy problem of the Standard Model lies in the explanation of the origin of the electroweak scale, so that one needs a scale-generation mechanism, namely a scalegenesis. In this regard, if the Coleman–Weinberg mechanism and the dimensional transmutation associated with QCD historically seemed two possible convenient ways [33], both these mechanisms, however, turn out to be not compatible with the observed masses of Standard Model particles in order to generate the electroweak scale. An electroweak scalegenesis could be obtained by developing a scale invariant extension of the Standard Model. In this regard, by considering a scalar field coupled to the Higgs field, if the TeV scale is generated by the dynamics in the new (hidden) sector, the electroweak symmetry breaking is determined by the Higgs–portal coupling. This programme has led to the development of many possible scale invariant extensions as a hidden sector, associated with other issues such as dark matter, neutrino masses, and baryogenesis.

Among the different proposals of treating the hierarchy problem of the Standard Model, an important place is today occupied by the emergent models. In this kind of approach, the key idea is that if one considers a critical statistical system close to the Planck scale, the only long-range correlations—light mass particles—that might exist in the infrared self-organise into multiplets just as they do in the Standard Model; in other words, a many-body system exhibits collective behaviour in the infrared that is qualitatively different from that of its more primordial constituents as probed in the ultraviolet [3,4]. Recently, an interesting emergent approach of the scale hierarchies of the Standard Model considers the interplay of Poincaré invariance, mass generation, and renormalization group invariance, leading to the fact that the measured cosmological constant scale is associated with higher dimensional terms in the action, suppressed by power of the large emergence scale, thus implying that the cosmological constant scale and neutrino masses should be of similar size [34].

In this paper, in the spirit of an emergent explanation of the hierarchies of the Standard Model, we provide an alternative explanation of the hierarchy scales of the Standard Model by invoking a model of a physical vacuum which is characterized by dissipative features close to the Planck scale, expressed by an opportune dispersion relation. The structure of this paper is the following. In Section 2 we introduce the mathematical formalism of the vacuum with dissipative features. In Section 3 we explore the perspectives introduced by our model as regards the explanation of the scale hierarchies and the Higgs mass mechanism. In Section 4 we analyse in what sense, in our approach of dissipative vacuum, it is possible to obtain the Standard Model particles as emergent facts and to find Planck signatures in the Standard Model. In Section 5 we explore in what sense one can provide in this model an emergent treatment of the gauge symmetry associated with a tiny nonzero value of the cosmological constant. Finally, in Section 6 we summarize the main results of the paper.

2. A Vacuum with Dissipative Features

Several current researchers consider the possibility that spacetime is an emergent fact from a physical vacuum characterized by dissipative features. For example, in Ref. [35], Liberati and Maccione analysed the dynamics of the matter propagating on an emergent spacetime as collective excitation in hydrodynamics, showing that the energy exchange
between collective excitations of the deep level and the spacetime fundamental degrees of freedom may be treated in terms of dissipative hydrodynamics. The Liberati and Maccione framework exhibits an epistemological affinity with the adoption of a nonlinear Schrödinger equation. In analogous way, Zloshcastiev explored the possibility that collective degrees of freedom which emerge in the condensate-like Bose systems, due to quantum interactions between original constituent particles, are described through a nonlinear logarithmic Schrödinger-like equation. There, the nonlinear coupling is related to the temperature, showing in this way that in the case of a varying nonlinear coupling an additional force occurs, which is parallel to a gradient of the coupling [36]. On the other hand, Volovik and Jannes [37] suggested that spacetime emerges as a collective excitation from an underlying microscopic Bose–Einstein condensate in the long-wavelength limit while, for shorter wavelengths, the spectrum of excitations of the vacuum becomes nonlinear, the emergent Lorentz invariance is violated and one has a phononic dispersion relation in the Bose–Einstein condensate, which can be expressed in the Bogoliubov form:

\[ \omega^2 = c^2 k^2 - \frac{1}{4} \frac{\hbar^2}{m^2} k^4 \]  

(1)

where \( c \) is the speed of light, \( k \) is the wave number, \( \hbar \) is Planck’s reduced constant, and \( m \) is the mass of the particles of the condensate.

In line with the research of Liberati and Maccione, Zloshcastiev and Volovik and Jannes, our purpose is to find the real particles of ordinary quantum mechanics as emergent structures from the virtual particle–antiparticle pairs of a Bose–Einstein condensate, where the energy fluctuations are characterized by dissipative features, and the collective degrees of freedom are described through a nonlinear Schrödinger-like equation.

On the basis of the results obtained by one of the authors (DF) in the context of the dynamic quantum vacuum model [38], we consider a physical vacuum given by a Bose–Einstein condensate of virtual particles which can be associated with elementary reduction-state processes and corresponding to opportune changes of the quantum vacuum energy density. In order to characterize the fluctuations of the quantum vacuum energy density, we invoke an idea originally suggested by Staniukovich, Melnikov, and Bronnikov, of a gravitational vacuum intended as a set of virtual, radiating planckeons with Planck size and mass

\[ M_{\text{dim}} = \frac{\hbar}{G l_p} \]  

(2)

where \( l_p = \sqrt{\frac{\hbar G}{c^3}} \) is the Planck length, \( G \) being Newton’s universal gravitation constant, which leads to obtain that the minimum universe scale factor corresponds to the chronon scale [39]. Moreover, we underline that the idea of planckeons has been applied in the context of the evaporation of primordial black holes, in order to explore the possibility that cosmological dark matter can be generated by black hole relics. In this regard, it has been proposed that black holes cease to evaporate and, in the last stage of evaporation, become stable relics, called planckeons, which would have mass of the order of Planck mass and a lifetime longer than the age of the universe [40]. More recently, it has been shown how, with a reheating temperature close to the Planck scale, high-energy collisions can produce the correct relic abundance of planckeons to account for all of the observed dark matter, and analytic expressions for the density of planckeons and its evolution have been obtained [41–43].

In light of these results, we assume that the most elementary constituents, at a fundamental level, which generate the variable energy density of the physical vacuum, are planckeons of dimensions and mass lying in the Planckian range. In this picture, therefore, both microscopic systems and macroscopic objects can be seen as emergent physical structures which derive from the planckeons; more precisely, they can be seen as the collective behaviour of a more fundamental variable quantum vacuum energy density, which in turn is associated with opportune aggregates of planckeons.
In order to throw new light towards a unifying treatment of microscopic systems and macroscopic regime, some authors have recently demonstrated the existence of a unified expression for the radii of black holes and fundamental particles, and, for this reason, also referred to as the Compton–Schwarzschild correspondence [34–38]. Trying inspiration from this kind of research, in Ref. [44], one of the authors [DF] has shown that, at a fundamental level, the scale where Compton wavelength and Schwarzschild radius are unified can be expressed by the following expression

\[ R'_{C} = R'_{S} = \sqrt{\left( \frac{\beta \hbar c}{\Delta \rho_{qE} V} \right)^2 + \left( \frac{\beta l^2 p}{\hbar} \frac{\Delta \rho_{qE} V^2}{\hbar c^2} \right)^2} \]  

Equation (3) can be called the “generalized Compton wavelength”. In this equation, \( \Delta \rho_{qE} \) is the variable energy density of the vacuum in a given volume \( V \), and the parameter \( \beta \) is a fluctuating quantity which can be associated with the Planck scale, in analogy with what happens in quantum foam scenarios such as loop quantum gravity, as well as cellular automaton interpretation of quantum mechanics [45–52]. This parameter has a fundamental importance: not only is it present in all theories on the Planck scale, but it is linked to a general phenomenon of complex systems, i.e., the loss of the degrees of freedom that occurs in the transition from one scale to another, making one class of phenomena “invisible” and others emerging. This parameter introduces essential ingredients of nonlinearity and nonlocality, which remain as subtle “signatures” in the various levels of the physical world [53]. We underline that the quantity (3) has been obtained by starting from the following generalized uncertainty relation, which are valid at the Planck scale:

\[ \Delta x \Delta p \geq \frac{\hbar}{2} \left( 1 + \beta l^2 p \frac{\Delta \rho_{qE}^2 V^2}{\hbar^2 c^2} \right) \]  

which expresses how the variable energy density of the vacuum can be associated to a deformation of the geometry of the background.

In summary, in our approach, we assume that the fundamental energy fluctuations of the physical vacuum are the physical manifestations—or, better, forms of collective organization—of these planckeons of mass and dimensions lying in the Planck range, intended simply as particles that live at the smallest scale on which one can construct a metric, i.e., at the scale (3) at which Compton wavelength and Schwarzschild radius are comparable. In other words, the planckeons are just the ultimate particles which can be defined at this scale, namely, express the activity of the vacuum associated with the Planck scale. In this sense, the planckeons may also be defined as elementary objects, of the Planck scale, which simultaneously have the properties of elementary particles and of black holes. In this picture, moreover, we can say that, below the generalized Compton wavelength, the planckeons are simply nonlocal entities, are everywhere, while above that scale, opportune aggregates of the planckeons lead to the localizations of actual, real particles with the corresponding (Fermi or Bose–Einstein) statistics.

In our model, our aim is to find the ordinary particles as the result of processes of emergence from opportune condensates of planckeons. This implies that one must assume that the energy of the planckeons is variable in the sense the planckeons can have energy that is not always the one corresponding to its maximum value. In order to mathematically define the mass of the planckeons, taking into account the essential role of gravity in the formation of an elementary particle (see for example [54]), we introduce inside Equation (2) the parameter

\[ \gamma = \left( \frac{m \sqrt{4 \pi \varepsilon_0 G}}{e} \right)^2 \]  

which corresponds to the ratio of gravitational and electromagnetic interactions for an elementary particle of mass \( m \) and charge \( e \), where \( \varepsilon_0 \) is the dielectric permittivity of the
vacuum and $G$ is the gravitational constant. In this way, we define the dimensional mass of the planckeons on the basis of relation

$$M_P = \frac{\gamma \hbar}{c \ell_p}$$  \hspace{1cm} (6)

where $\gamma$ is given by (5) and the mass $m$ appearing in (5) must be interpreted as the mass of the virtual sub-particles of the medium. Because of the variability of the parameter $\gamma$, the planckeons can have different masses. Each planckeon can be associated to a corresponding (variable) elementary energy density of the vacuum given by relation

$$\Delta \rho_{qE} = \frac{M_P c^2}{V}$$  \hspace{1cm} (7)

which implies that the total energy density in a certain region can be seen as a form of collective organization of the aggregates planckeons of that region. In the bath constituted by these elementary quantum vacuum energy density fluctuations, the virtual particles of the medium continuously appear and disappear and give rise to a total zero spin, thus constituting an organized Bose ensemble. As a consequence of these processes of the vacuum where virtual particles generating a Bose–Einstein condensate appear and annihilate, space-time can be seen as a collective excitation emerging from the elementary modes of the vacuum defined by frequency

$$\omega_i = \frac{2\Delta \rho_{qE} V}{\hbar n}$$  \hspace{1cm} (8)

associated with the Bose–Einstein condensate of the virtual sub-particles, where these modes are characterized by dissipative features close to the Planck scale. In Equation (8), $\Delta \rho_{qE}$ are the energy density fluctuations of the vacuum in the volume $V$, $n$ is the number of the virtual particles in the volume $V$. By substituting Equation (7) into Equation (8), one obtains the link between the frequency of the elementary modes of the vacuum and the variable mass of its virtual planckeons

$$\omega_i = \frac{M_P c^2}{\hbar n}$$  \hspace{1cm} (9)

The dissipative features characterizing the Bose–Einstein condensate of the planckeons can be described by invoking a dispersion relation, which can be seen as a generalization of the Jannes–Volovik Equation (1). In our model, the phononic dispersion relation (1) of Volovik–Jannes becomes a dispersion relation which describes the dissipative features near the Planck scale, and reads

$$\omega^2 = c^2 k^2 - \frac{1}{4} \frac{\hbar^2}{M_P^2} k^4$$  \hspace{1cm} (10)

The physical meaning of this relation, at the most fundamental level, can be analysed by considering the unified scale (3), namely the generalized Compton wavelength, which describes the microscopic geometry of the vacuum associated with the planckeons, in other words, which expresses the activity of the vacuum associated with the Planck scale. By substituting (6) and (7) into Equation (3), the generalized Compton wavelength can be expressed as

$$R'_C = R'_S = \sqrt{\left(\frac{\beta \ell_p}{\gamma}\right)^2 + (\beta \gamma \ell_p)^2}$$  \hspace{1cm} (11)
and, in light of the existence of this underlying scale (11), we can assume that the wave number appearing in the dispersion relation (10) may be assimilated to the generalized Compton wavelength (11), namely:

\[
k = \frac{1}{\sqrt{\left(\frac{\beta}{T}\right)^2 + (\beta \gamma l_p)^2}}
\]

(12)

Therefore, by substituting (12) into Equation (10), one obtains the final expression of the dispersion relation in our model of dissipative vacuum:

\[
\omega^2 = c^2 \left(\frac{\beta}{T}\right)^2 \frac{1}{(\beta \gamma l_p)^2} - \frac{1}{4} \frac{\hbar^2}{M_p^2} \left(\left(\frac{\beta}{T}\right)^2 + (\beta \gamma l_p)^2\right)^2
\]

(13)

namely, taking account of (7),

\[
\omega^2 = c^2 \left(\frac{\beta}{T}\right)^2 \frac{1}{(\beta \gamma l_p)^2} - \frac{1}{4} \frac{c^2 \gamma^2 l_p^2}{\gamma^2} \left(\left(\frac{\beta}{T}\right)^2 + (\beta \gamma l_p)^2\right)^2
\]

(14)

In the light of Equation (14), one can say that, in this approach, the fluctuations of the quantum vacuum energy density determined by the planckeons are the fundamental entities which generate the dissipative features of the vacuum, and here the second term \[\frac{1}{4} \frac{c^2 \gamma^2 l_p^2}{\gamma^2} \left(\left(\frac{\beta}{T}\right)^2 + (\beta \gamma l_p)^2\right)^2\] is responsible of the magnitude of the Lorentz violation. Moreover, by virtue of the role of the generalized Compton wavelength (11) in generating a scale where elementary particles and black holes are unified, the planckeons can be also defined as microblack holes associated with the dissipative vacuum.

Now, in our approach, the behaviour and evolution of the physical vacuum are ruled by the following nonlinear Schrödinger equation

\[
\frac{i}{\hbar} \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + v m |\psi|^2 \psi + U \psi
\]

(15)

where \(m\) is the mass of each virtual particle of the physical vacuum, \(U\) is the potential energy relating to the single virtual particle, and \(v\) is a viscosity coefficient having the dimensions \[\text{length}^2 \text{time}^{-1}\] which can be expressed as \[v = \frac{a^2 k_0}{2 \hbar \omega}\] where \(a\) is the scattering length between the virtual particles, \(\omega\) is the frequency of the elementary modes given by the dispersion relation (14), and \(k\) is an adimensional parameter corresponding to the size of the condensate of planckeons in the region of consideration (namely, represents a sort of effective parameter of density of the planckeons). The parameter \(k\) plays a fundamental role in the model, as we will see in Section 3. Equation (15) can be considered as a new peculiar version of the Gross–Pitaevskij equation with the presence of a new fundamental term, depending of \[|\psi|^2\] and the viscosity coefficient \(v\) linked with the scattering length of the virtual sub-particles of the vacuum, the generalized Compton wavelength, and the parameter measuring the effective density of the planckeons. We must emphasize here that the consideration of the nonlinear Schrödinger Equation (15) as a basis to describe the behaviour of the dissipative vacuum finds its motivation in (and is in affinity with) some important recent research which explores the mechanism of spontaneous symmetry breaking associated with Goldstone and Higgs fields (as well as the elementary excitations for a weakly-interacting Bose gas at a finite temperature) in the context of Gross–Pitaevskij and Klein–Gordon nonlinear equations [55].

In our model, in light of the dispersion relation (14), Equation (15) may be conveniently expressed as:
\[ i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + \frac{a^2 k}{2\pi} \left( c^2 \left( \frac{\partial \psi}{\partial x} \right)^2 + (\beta \gamma) \frac{\partial \psi}{\partial \gamma} \right) \left( \frac{1}{4\gamma^2} \left( \frac{\partial \psi}{\partial \gamma} \right)^2 + (\beta \gamma) \frac{\partial \psi}{\partial \gamma} \right) - \frac{1}{4} \frac{c^2 t_p^2}{\gamma^2} \left( \frac{\partial \psi}{\partial \gamma} \right)^2 \left( \frac{\partial \psi}{\partial \gamma} \right)^2 \right) m|\psi|^2 \psi + U\psi \]  

(16)

The nonlinearity of Equation (16) emerges as a result of the interactions of the fundamental planckeons of the vacuum, which give rise to states of collective organization. In order to explore the consequences of the nonlinear Schrödinger Equation (16), we recast this equation by the Madelung transformation, writing the wave function as

\[ \psi = \text{Re} e^{iS/h} \]  

(17)

By substituting (17) into Equation (16) and separating real and imaginary parts, one obtains a continuity equation

\[ \frac{\partial \rho}{\partial t} + \frac{\hbar}{M} \nabla (\rho \nabla S) = 0 \]  

(18)

and the Hamilton–Jacobi equation

\[ \frac{\partial S}{\partial t} + \frac{1}{2M} (\nabla S)^2 + U - \frac{a^2 k}{2\pi} \left( c^2 \left( \frac{\partial \psi}{\partial x} \right)^2 + (\beta \gamma) \frac{\partial \psi}{\partial \gamma} \right) \left( \frac{1}{4\gamma^2} \left( \frac{\partial \psi}{\partial \gamma} \right)^2 + (\beta \gamma) \frac{\partial \psi}{\partial \gamma} \right) \nabla^2 S = 0 \]  

(19)

where

\[ Q = \frac{p_1 + p_2}{\rho} = \frac{\hbar^2}{8m} \left( \frac{\nabla \rho}{\rho} \right)^2 - \frac{\hbar^2}{4m} \nabla^2 \rho \]  

(20)

is the quantum potential. In Equation (20), \( \rho = n/V \), where \( n \) is the number of the virtual particles in the volume \( V \) and \( p_1 + p_2 \) is the pressure generated by the collisions of the virtual planckeons of the medium, where

\[ p_1 = -\frac{D^2}{c^2} \left[ \nabla^2 \Delta \rho_{qV} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \Delta \rho_{qV} \right] \]  

(21)

and

\[ p_2 = \frac{D^2}{2\Delta \rho_{qV} c^2} \left[ \left( \nabla \Delta \rho_{qV} \right)^2 - \frac{1}{c^2} \left( \frac{\partial}{\partial t} \Delta \rho_{qV} \right)^2 \right] \]  

(22)

where

\[ D = \frac{\hbar^2 n}{2\Delta \rho_{qV} V} \]  

(23)

On the basis of relation (20), in the case of bound systems, the points where the quantum potential tends to zero indicate the boundary of the region where the virtual planckeons of the vacuum are delocalized [56].

Since, on the basis of Equation (20), the quantum potential cannot be ever null because it represents the internal pressure divided by the density distribution of the particle–antiparticle pairs (and in fact the internal pressure cannot ever be null), in the limit of \( Q \to \infty \), one obtains the configuration of the maximum degree of nonlocality. In virtue of these features of the quantum potential (20), one can say that the texture of the planckeons is characterized by an inner nonlocality, which implies that even what appears localized (real) is woven into the nonlocality. In other words, one deals with a Bose–Einstein condensate of virtual particles of the vacuum, namely the planckeons, which constitutes a fundamental nonlocal texture.

Because of the fundamental nonlocality of the dissipative vacuum associated with the quantum potential (20), the planckeons, intended as pre-local objects which obey the
Gross–Pitaevskij Equation (16), can be seen as extreme bridges of nonlocality. Here, a crucial point lies in the fact that the dispersion relation (14), determined by the generalized Compton wavelength (11), and ultimately by the generalized uncertainty relations (4), fixes the statistics of the planckeons. In this regard, if one considers the field $\psi$ of the vacuum as an infinite set of decoupled harmonic oscillators, by following Refs. [57, 58], one can formulate the following commutation relation between the creation and annihilation operators, which, respectively, create and annihilate virtual particles from the vacuum:

$$\hat{a}_{M}^{\dagger} \hat{a}_{M} - q \hat{a}_{M}^{\dagger} \hat{a}_{M} = \frac{1}{1 - \alpha} \left[1 - \tilde{\alpha} (a_{M}^{+} a_{M} + a_{M} a_{M}^{+} - 2a_{M}^{+} a_{M})\right] \delta_{MM}$$

where $\tilde{\alpha} = 2\beta l_{p}^{2} \left[c^2 \left(\frac{\theta_{p}}{\gamma}\right)^2 + (\theta_{l_{p}})^2\right] - \frac{1}{4} \beta l_{p} \gamma \left[(\theta_{p})^2 + (\theta_{l_{p}})^2\right]$.

Relation (24) can be considered as a sort of generalization of the so-called infinite statistics introduced by Ng in [59–63], represented by the $q$ deformation of the commutation relations of the oscillators, expressed by the following relation

$$\hat{a}_{M}^{\dagger} \hat{a}_{M} - q \hat{a}_{M}^{\dagger} \hat{a}_{M} = \delta_{MM}$$

where the special cases $q = \pm 1$ correspond to bosons and fermions. The infinite statistics characterizing the virtual particles of the background implies that each opportune set of planckeons can give rise to a boson or a fermion depending of the value of the deformation parameter $q$ appearing in the commutation relations of the oscillators (25). On the basis of Equation (24), one can provide an interpretation of the infinite statistics described by the commutation relations of the oscillators, expressed by the following relation

$$\hat{a}_{M}^{\dagger} \hat{a}_{M} - q \hat{a}_{M}^{\dagger} \hat{a}_{M} = \delta_{MM}$$

3. Scales, Hierarchies, and Higgs Mass in the Dissipative Vacuum

The model, here developed, of the physical vacuum—characterized by dissipative features originating from the planckeons—allows us to shed new light on the explanation of the formation of the mass of elementary particles, introducing new perspectives in the interpretation of the Higgs boson, and explaining in what sense not all the values of energy coming from the vacuum can give origin to a minimal substitution. In this picture, a key point is that the large hierarchy problem between electroweak scale and Planck scale is faced from the perspective of new concepts at the Planck scale, namely, the dissipative features of the vacuum associated with the planckeons. This scenario implies that the Higgs mechanism can be associated to the Planck scale and, thus, on new procedures which have to integrate conventional quantum field theory. In particular, our insight is that the Higgs mass would be fixed by the dissipative features of the vacuum at the unified scale (11), namely, the generalized Compton wavelength describing the ultimate geometry of the planckeons, which expresses the activity of the vacuum associated with the Planck scale. As a consequence, in this programme, we forbid any kind of new intermediate energy scale between weak and Planck scale in order to avoid the large hierarchy between Higgs and heavy intermediate particle’s mass.

3.1. Higgs Field and the Renormalization Equation

The Standard Model predicts that the masses of the W and Z gauge bosons and charged fermions are emergent facts from the Higgs boson with a finite Higgs vacuum expectation value (vev). In the theoretical plant of this theory, the renormalized Higgs mass
squared $m^2_{h\, ren}$ is characterized by a divergent counterterm, in the sense that it is connected to the bare Higgs mass $m^2_{h\, bare}$ by relation
\[ m^2_{h\, bare} = m^2_{h\, ren} + \delta m^2_h \]  
where $\delta m^2_h$, by neglecting the contributions from lighter mass quarks, is
\[ \delta m^2_h = \frac{K^2}{16\pi^2} \frac{6}{\nu^2} \left( m^2_t + m^2_Z + 2m^2_W - 4m^2_t \right) \]  
where $K$ is an ultraviolet scale identifying the limit of the Standard Model, $\nu$ is the Higgs vev, $m_h$ is the Higgs mass, $m_Z$ is the mass of the $Z$ boson, $m_W$ is the mass of the $W$ boson, $m_t$ is the mass of the top quark. A crucial result of this theory lies in the fact that the quadratic divergence in the Higgs mass self energy would dissolve, and the corresponding Higgs mass hierarchy problem would be resolved, when the coefficient of $K^2$ vanishes. This occurs if the Veltman condition [15,64] given by
\[ m^2_t + m^2_Z + 2m^2_W = 4m^2_t \]  
is satisfied, leading to an equality of renormalized and bare masses with no hierarchy problem. However, the pole masses of $W$, $Z$ bosons and top quark involved in the Veltman constraint imply a Higgs mass of 314 GeV, much above the measured value of 125 GeV [34].

In our approach of Gross–Pitaevskij dissipative vacuum, our starting point is to consider a quantum vacuum energy linked with the fluctuations of the dissipative vacuum, given by relation
\[ \rho_{qE} = \frac{1}{2} \frac{t_p c^2 k}{\hbar m l_p} \sum g_i \int_0^{M_{planck}^{max}} \frac{d^3 m}{(2\pi)^3} \left[ \frac{1}{c^2 p^2 + (\beta \gamma l_p)^2} + m^2 \right] \]  
where $m$ is the mass of the particle, $t_p$ is Planck time, $g_i = (-1)^j (2j + 1)$ is the degeneracy factor for a particle $i$ of spin $j$, and $g_i > 0$ for bosons and $g_i < 0$ for fermions, and $M_{planck}^{max}$ is the maximum value of the mass of the planckeons. The factor $f$ is 1 for bosons, 2 for each charged lepton, and 6 for each flavour of quark (2 charge factors for the quark and antiquark, each with 3 colours). Here, by invoking a Lorentz covariant regularization and using some results obtained in Ref. [34], one obtains
\[ \frac{1}{2} \frac{t_p c^2 k}{\hbar m l_p} \sum g_i \int_0^{M_{planck}^{max}} \frac{d^3 m}{(2\pi)^3} \left[ \frac{1}{c^2 p^2 + (\beta \gamma l_p)^2} + m^2 \right] = -\hbar g_i \frac{m^4}{64\pi^2} \left[ \frac{2}{\epsilon} - \frac{3}{2} - \theta - \ln \left( \frac{m^2}{4\pi \mu^2} \right) \right] + \ldots \]  
where $D = 4 - \epsilon$ is the number of dimensions, $\mu$ is the renormalization scale, and $\theta$ is Euler’s constant. On the basis of Equation (30), the renormalization, which leads to a vanished zero-point energy for photons and for which Standard Model particles are induced by the Higgs field, can be ultimately associated with opportune networks of the elementary virtual planckeons of the vacuum, which express the activity of the vacuum at Planck scale linked with the generalized uncertainty relations. In other words, in light of the renormalization Equation (30) of the Gross–Pitaevskij vacuum, the ultimate source of the action of the Higgs field is represented by the virtual planckeons providing the dissipative features of the vacuum. Equation (30) and the Pauli constraint that cancels the zero-point energy lead to specific fundamental constraints on the collective excitations of the planckeons of the vacuum, expressed by relations
\[ \sum_i g_i \left( \frac{\gamma_i h}{c l_p} \right)^4 = 0 \]
\[ \sum_i s_i \left( \frac{\gamma_i \hbar}{c p} \right)^4 \ln \left[ \frac{\gamma_i \hbar}{c p} \right]^2 = 0 \]  

(32)

which directly justify the requirement of the mass of Higgs boson of the value 125 GeV.

Moreover, if in the current extensions of the Standard Model the appearance of Standard Model fermions when the zero-point energy is negative is assured by invoking some extra bosons such as 2 Higgs Doublet Models [65], now in our model of Gross–Pitaevskij vacuum we can provide a deeper characterization of the origin of these extra bosons, as emergent entities from more fundamental networks the virtual planckeons of the vacuum.

3.2. Higgs Boson, Spontaneous Symmetry Breaking, Vector Bosons, and Photon as Emergent Facts

If, in light of the experimental results obtained at LHC, the Standard Model works as a consistent theory up to the Planck scale, the electroweak vacuum sits very close to the border of stable and metastable, suggesting possible new critical phenomena in the ultraviolet [66]. In current models, which face the hierarchy problem of the Standard Model, the question of vacuum stability depends on whether the Higgs self-coupling crosses zero or not deep in the ultraviolet and involves a delicate balance of Standard Model parameters. Instead, in our model of physical vacuum with dissipative features in the form of energy fluctuations associated to the collective organization of the virtual planckeons and described by the Gross–Pitaevskij Equation (21), the hierarchy problem can receive a more satisfactory re-reading in the sense that the actions of the Higgs field and other particle masses have their fundamental origin in the physics close to the Planck scale in a causal way inside an emergent picture.

Let us consider the Gross–Pitaevskij Equation (16) of the dissipative vacuum. Here, if in some representation the function

\[ H = -\hbar^2 \frac{\nabla^2}{2m} + \frac{a^2}{2\pi} k \left( \frac{1}{(\beta l_p)^2 + (\beta \gamma l_p)^2} - \frac{1}{4} \frac{c^2 l_p^2}{\gamma^2 \left( (\beta l_p)^2 + (\beta \gamma l_p)^2 \right)^2} \right) m|\psi|^2 + U \]  

appearing in the Gross–Pitaevskij Equation (16), can be written as a second-order differential function with respect to some variable \( X \) or \( iX \), then the Gross–Pitaevskij Equation (16) becomes the equation of motion of a virtual particle moving in the following rotational-invariant effective potential, which turns out to be characterized by a Mexican-hat shape:

\[ V(\psi) = \left\{ \frac{a^2}{2\pi} k \left( \frac{1}{(\beta l_p)^2 + (\beta \gamma l_p)^2} - \frac{1}{4} \frac{c^2 l_p^2}{\gamma^2 \left( (\beta l_p)^2 + (\beta \gamma l_p)^2 \right)^2} \right) m|\psi|^2 + U \right\} + V_0 \]  

(34)

where \( V_0 = V = \frac{1}{(\beta l_p)^2 + (\beta \gamma l_p)^2)^{3/2}} \), with the role of time coordinate being assigned to \( X \) or \( iX \). The Mexican-hat shape of the potential (34) lies in the fact that its local maximum is located at \( |\psi| = 0 \), whereas the degenerate minima lie on the circle \( |\psi| = 1 \) where the energy of the “particle” reaches its minimum. The effective potential (34), by virtue of its features, can be considered a plausible candidate to express the Higgs potential. Therefore, the effective potential (34) represents our specific hypotheses related to the Higgs field.

By starting from the effective potential (34), the Higgs mass can be derived as an emergent fact from the fundamental properties of the dissipative vacuum, as follows. In
our approach, by applying the Unitary gauge in order to remove the nonphysical degrees of freedom, the Higgs doublet can be expressed as

\[ \phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \text{h} \end{pmatrix} \]

where \(\text{h}\) indicates the excitations of the dissipative vacuum from the vacuum expectation value \(\varepsilon\) which is related to the rest mass of the \(W\) boson as

\[ \varepsilon = \frac{\sqrt{2}c^2}{g} M_W \]

where \(g\) represents the electroweak coupling constant. Then, taking account of the results obtained in Ref. [67], by applying the Unitary gauge constraint, the effective potential (34) may be expressed as

\[ V(\psi) = \frac{1}{4} \lambda \left( \left( \frac{\beta_l}{\gamma} \right)^2 + (\beta \gamma l_p)^2 \right)^2 + \lambda \left( \left( \frac{\beta_l}{\gamma} \right)^2 + (\beta \gamma l_p)^2 \right)^3 h^2 + \lambda \left( \left( \frac{\beta_l}{\gamma} \right)^2 + (\beta \gamma l_p)^2 \right)^{3/2} h^3 + \frac{1}{4} \lambda h^4 \]

where \(\lambda\) is a positive constant which satisfies relation

\[ \left( \frac{\beta_l}{\gamma} \right)^2 + (\beta \gamma l_p)^2 \]

\[ = \frac{1}{\lambda} \left( \left( \frac{\beta_l}{\gamma} \right)^2 + (\beta \gamma l_p)^2 \right)^{3/2} \]

namely

\[ \lambda = \left[ \frac{a^2}{2\pi k} \left( \left( \frac{\beta_l}{\gamma} \right)^2 + (\beta \gamma l_p)^2 \right) - \frac{1}{4} \frac{\beta_l^2}{\gamma^2} \left( \left( \frac{\beta_l}{\gamma} \right)^2 + (\beta \gamma l_p)^2 \right)^2 \right]^{1/2} m|\psi|^2 \left( \left( \frac{\beta_l}{\gamma} \right)^2 + (\beta \gamma l_p)^2 \right)^{3/2} \]

Now, the second term in the potential (37) (quadratic in \(h\)) allows us to obtain the following expression for the Higgs mass

\[ m_H^2 = \frac{2}{\lambda^2} \left[ \left( \frac{\beta l}{\gamma} \right)^2 + (\beta \gamma l_p)^2 \right] m|\psi|^2 \left( \left( \frac{\beta l}{\gamma} \right)^2 + (\beta \gamma l_p)^2 \right)^{3/2} \]

Equation (40) establishes the condition which must be satisfied by the planckons in order to generate the Higgs mass. On the basis of Equation (40), the Higgs mass turns out to be directly determined by fundamental properties of the dissipative vacuum and, in particular, can be seen as an emergent fact from the generalized Compton wavelength, which is the crucial entity generating the dissipative activity of the vacuum associated with the Planck scale.

Let us see now how, by starting from the effective potential (34) of the dissipative vacuum, one can develop a mathematical treatment of the issues related to the spontaneous symmetry breaking invoked in the Standard Model. In this regard, before all, the potential (34) leads to defining the potential energy density of the dissipative vacuum as follows:
\[ V(\psi) \equiv \frac{V(\psi)}{k^3\left(\frac{\beta\psi}{\gamma} + (\beta\gamma l_p)^2\right)^{3/2}} = \frac{1}{k^3\left(\frac{\beta\psi}{\gamma} + (\beta\gamma l_p)^2\right)^{3/2}} \left\{ \frac{\beta^2 m}{\pi^2 k} \left[ \frac{c^2}{(\beta\psi)^2 + (\beta\gamma l_p)^2} - \frac{1}{4} \frac{c^2}{(\beta\psi)^2 + (\beta\gamma l_p)^2} \right] |\psi|^2 + U \right\} + V_0 \quad (41) \]

where \( k \) is a parameter corresponding to the size of the condensate of planckeons in the region of consideration. From the potential energy density (41), we can consider the following effective action of the dissipative vacuum:

\[ S = \tilde{S}(\psi_i, \psi) - \int V(\psi) \quad (42) \]

Here, although the exact form of the effective action \( \tilde{S}(\psi_i, \psi) \) depending on the wave function \( \psi \) as well as on all the other fields \( \psi_i \), is unknown, some plausible conjectures can be made in order to include vacuum effects which account for the small fluctuations of the Bose–Einstein condensate of the virtual particles of the vacuum. Here, since the quantum wave amplitude of the planckeons turns out to be much smaller than the background value of the condensate wave function amplitude, the field-theoretical models can be constructed in a covariant manner. In line of principle, since we are dealing with low-energy effective models, we are free to use any form of the covariant action for the psi-field—as long as it is physically transparent, self-consistent, mathematically manageable, and the corresponding field equation contains the requested nonlinearity. Moreover, it is likely that \( \tilde{S} \) will contain couplings of the psi-particle to other fields.

By following Ref. [68], a simple toy-model in order to mathematically describe the effective action \( \tilde{S} \) of the dissipative vacuum is the self-interacting vacuum involving only the complex psi-field. In this simple model, in D-dimensional spacetime, the Lagrangian can be expressed in the covariant form

\[ \mathcal{L} = k \sqrt{\left(\frac{\beta l_p}{\gamma}\right)^2 + (\beta\gamma l_p)^2} \partial_\mu \psi \partial^\mu \psi^* - V(\psi) \quad (43) \]

where the potential energy density is given by Equation (41) and \( k \) is a parameter corresponding to the size of the condensate of planckeons. This model is invariant under a global change of phase of \( \psi \), but in the vacuum state, the value of \( \psi \) must be nonzero, with a magnitude close to \( 1/\sqrt{k^3\left(\left(\frac{\beta l_p}{\gamma}\right)^2 + (\beta\gamma l_p)^2\right)^{3/2}} \) and arbitrary phase. This means that the model implies the existence of a degenerate family of vacuum states, which, together with the Goldstone theorem, would suggest the presence of the Nambu–Goldstone bosons in the theory. This can be obtained by introducing the shifted real-valued fields

\[ \psi = k^{-3} \left(\frac{\beta l_p}{\gamma}\right)^2 + (\beta\gamma l_p)^2 \right)^{-3/2} + \frac{1}{2k\sqrt{\left(\frac{\beta l_p}{\gamma}\right)^2 + (\beta\gamma l_p)^2}} (\varphi_1 + i\varphi_2) \quad (44) \]

and expanding the potential near the minimum as follows

\[ \mathcal{L} = \frac{1}{2} \left[ (\partial_\mu \varphi_1)^2 + (\partial_\mu \varphi_2)^2 \right] - \frac{1}{4} m_\varphi^2 \varphi_1^2 - \sqrt{2k} \frac{\beta l_p}{\gamma} \left[ \left(\frac{\beta l_p}{\gamma}\right)^2 + (\beta\gamma l_p)^2 \right]^{1/2} \varphi_1 \varphi_2 (\varphi_1^2 + \varphi_2^2) \]

\[ - \frac{1}{4} k^{D-3} \left(\left(\frac{\beta l_p}{\gamma}\right)^2 + (\beta\gamma l_p)^2 \right)^{D-3} \varphi_1^2 \varphi_2^2 + \mathcal{O}(\varphi^5) \quad (45) \]
In Equation (45) we have defined the quantity

\[ m_\phi = \frac{2\sqrt{T_p}}{k_1^{1/2} \beta_1^{1/2} \left[ \left( \frac{\beta_1}{T_p} \right)^2 + (\beta \gamma l_p)^2 \right]^{1/2} \left( \frac{\gamma h}{c l_p} \right)^4} \]  

(46)

as the effective mass of the fluctuation of the condensate of the planckeons of the dissipative vacuum (and therefore must not be confused with the mass \( M_{dim} \) of the planckeons themselves). As regards the behaviour of the parameter \( \beta \), we apply now the results of Refs. [68,69], namely

\[ \beta \sim (E_0 - E)^{-1} \]  

(47)

where \( E_0 \) is the energy of the “ground state” of the dissipative vacuum associated with the generalized Compton wavelength, defined as

\[ E_0 = \frac{hc}{\sqrt{\left( \frac{\beta_1}{T_p} \right)^2 + (\beta \gamma l_p)^2}} \]  

(48)

In this way, we obtain

\[ m_\phi k_1^{1/2} \left[ \left( \frac{\beta_1}{T_p} \right)^2 + (\beta \gamma l_p)^2 \right]^{1/2} \sim \sqrt{E_0 - E} \]  

(49)

which means that the mass of the fluctuation of the condensate of the planckeons of the dissipative vacuum is not determined solely by the Planck scale, in the sense that for energy very small compared to \( E_0 \), it tends to the constant value

\[ m_\phi^{(0)} \equiv m_\phi(E = 0) \sim \sqrt{E_0 / k_1^{1/2} \sqrt{\left( \frac{\beta_1}{T_p} \right)^2 + (\beta \gamma l_p)^2}} \]  

(50)

namely, by inserting (48):

\[ m_\phi^{(0)} \sim \sqrt{hc / k_1^{1/2} \left[ \left( \frac{\beta_1}{T_p} \right)^2 + (\beta \gamma l_p)^2 \right]} \]  

(51)

Instead, at high energies, this mass turns out to be subjected to changes as a consequence of the dynamical nature of the dissipative vacuum. Thus, one obtains that, in the broken symmetry regime, this model describes two kinds of particles, one massive and one massless, where the latter are the Nambu–Goldstone bosons which describe the spatial variations of the vacuum’s phase.

In this toy-model based on the Lagrangian (43), the effective mass of the fluctuation of the condensate of the planckeons can be therefore considered the starting-point in order to define the mass of the vector bosons \( W \) and \( Z \), finding suggestive consequences as regards the relation between the gauge couplings and the fundamental parameters of the dissipative vacuum. In fact, on the basis of the general Equation (46) and some mathematical formalism developed in Ref. [67], if \( k_W \) is the constant associated to the aggregate of planckeons corresponding to the appearance of a \( W \) boson, we can define the mass of \( W \) bosons as

\[ M_W = \frac{2\sqrt{T_p}}{k_W^{1/2} \beta_1^{1/2} \left[ \left( \frac{\beta_1}{T_p} \right)^2 + (\beta \gamma l_p)^2 \right]^{1/2} \left( \frac{\gamma h}{c l_p} \right)} \]  

(52)
In an analogous way, if $k_Z$ is the constant associated to the aggregate of planckeons corresponding to the appearance of a $Z$ boson, we can define the mass of $Z$ bosons as

$$M_Z = \frac{2\gamma h}{k_Z^{1/2}\beta^{1/2}c\sqrt{\tau_p}\left[\left(\frac{\beta p}{\gamma}\right)^2 + (\beta \gamma l_p)^2\right]^{1/2}}$$

(53)

In this way, in light of relation (52), as regards the $W$ bosons, we find:

$$\frac{2\gamma h\sqrt{\tau_p}}{k_W^{1/2}\beta^{1/2}c\sqrt{\tau_p}\left[\left(\frac{\beta p}{\gamma}\right)^2 + (\beta \gamma l_p)^2\right]^{1/2}} = \frac{g}{2\left(\left(\frac{\beta p}{\gamma}\right)^2 + (\beta \gamma l_p)^2\right)^{3/2}}$$

(54)

namely

$$g = \frac{4\gamma h\left(\left(\frac{\beta p}{\gamma}\right)^2 + (\beta \gamma l_p)^2\right)^{5/4}}{k_W^{1/2}\beta^{1/2}c\sqrt{\tau_p}}$$

(55)

Instead, in light of Equation (53), as regards the $Z$ boson, we find:

$$\frac{2\gamma h}{k_Z^{1/2}\beta^{1/2}c\sqrt{\tau_p}\left[\left(\frac{\beta p}{\gamma}\right)^2 + (\beta \gamma l_p)^2\right]^{1/2}} = \frac{1}{2\left(\left(\frac{\beta p}{\gamma}\right)^2 + (\beta \gamma l_p)^2\right)^{3/2}} \sqrt{g^2 + g'^2}$$

(56)

namely

$$g^2 + g'^2 = \frac{16\gamma^2 h^2\left(\left(\frac{\beta p}{\gamma}\right)^2 + (\beta \gamma l_p)^2\right)^{5/2}}{k_Z\beta l_p c^2}$$

(57)

and, thus, substituting the expression (55) of $g$, we obtain the final expression for $g'$:

$$g'^2 = \frac{16\gamma^2 h^2\left(\left(\frac{\beta p}{\gamma}\right)^2 + (\beta \gamma l_p)^2\right)^{5/2}}{k_Z\beta l_p c^2} - \frac{16\gamma^2 h^2\left(\left(\frac{\beta p}{\gamma}\right)^2 + (\beta \gamma l_p)^2\right)^{5/2}}{k_Z\beta l_p c^2}$$

(58)

namely

$$g'^2 = \frac{16}{\gamma l_p}\left(\left(\frac{\beta p}{\gamma}\right)^2 + (\beta \gamma l_p)^2\right)^{5/2} \frac{\gamma^2 h^2}{c^2\beta} \left[\frac{1}{k_Z} - \frac{1}{k_W}\right]$$

(59)

On the basis of relations (55) and (59), we have thus obtained that the gauge couplings of the vector bosons of the Standard Model can also receive a re-reading in terms of quantities characteristic of the dissipative vacuum and, in particular, depend on the size of the opportune aggregate of planckeons, the mass of the planckeons, as well as the generalized Compton wavelength describing the ultimate geometry of the vacuum. In other words, the couplings (55) and (59) express in what sense Planck-scale signatures can be found in the Standard Model. Moreover, since in the Standard Model the elementary charge is expressed in terms of the gauge couplings $g$ and $g'$ as follows

$$e = gg'\sqrt{g^2 + g'^2}$$

(60)
now, by substituting (55), (57) and (59) inside (60), we obtain

\[
e = 64 \left[ \left( \frac{\beta \beta}{\tau} \right)^2 + \left( \beta \gamma l_p \right)^2 \right]^{15/4} \frac{\gamma^3 h^3}{k_w^{1/2} \beta^{3/2} k_Z^{1/2}} \sqrt{\frac{1}{k_Z} \frac{1}{k_W}}
\]

(61)

According to Equation (61), the elementary charge, in the approach of the dissipative vacuum, can also be seen as an emergent fact, a collective phenomenon from opportune condensates of the planckeons. Finally, the Fermi constant can also receive a new re-reading, as follows:

\[
G_F = \sqrt{\frac{g^2}{8M_W^2}} = \sqrt{\frac{k_Z \left( \left( \frac{\beta \beta}{\tau} \right)^2 + \left( \beta \gamma l_p \right)^2 \right)^3}{2k_W}}
\]

(62)

namely, it is a collective result of the dissipative vacuum, being linked with the parameters associated with the size of the aggregates of planckeons corresponding to the appearance of W and Z bosons, as well as with the generalized Compton wavelength. Here, since the quantity

\[
v = \sqrt{\frac{k_Z \left( \left( \frac{\beta \beta}{\tau} \right)^2 + \left( \beta \gamma l_p \right)^2 \right)^3}{k_W}}
\]

(63)

represents the electroweak scale, which is equal to about 246 GeV, one can also obtain a relation between the parameters \( k_Z \) and \( k_W \) regarding the condensates of planckeons associated with Z bosons and W bosons, respectively:

\[
\sqrt{\frac{k_Z \left( \left( \frac{\beta \beta}{\tau} \right)^2 + \left( \beta \gamma l_p \right)^2 \right)^3}{k_W}} = 246 \text{ GeV}
\]

(64)

From Equation (64), it follows that the Higgs mass of 125 GeV can be expressed in terms of the parameters \( k_Z \) and \( k_W \) regarding the condensates of planckeons associated with Z bosons and W bosons as

\[
m_H = \frac{125}{246} \sqrt{\frac{k_Z \left( \left( \frac{\beta \beta}{\tau} \right)^2 + \left( \beta \gamma l_p \right)^2 \right)^3}{k_W}}
\]

(65)

Then, by comparing Equation (65) with Equation (40), one obtains

\[
\frac{15,625}{60,516} k_Z \left( \left( \frac{\beta \beta}{\tau} \right)^2 + \left( \beta \gamma l_p \right)^2 \right)^3 = 2 \left[ \frac{\text{c}_{2\pi}}{\sqrt{1 + \frac{1}{\text{c}_{2\pi}} \left( \left( \frac{\beta \beta}{\tau} \right)^2 + \left( \beta \gamma l_p \right)^2 \right)^2}} \right]^{1/2} \frac{m_{\psi}^2 \left( \left( \frac{\beta \beta}{\tau} \right)^2 + \left( \beta \gamma l_p \right)^2 \right)^{3/2}}{}
\]

(66)

namely

\[
\frac{15,625}{60,516} k_Z = 2 \left[ \frac{\text{c}_{2\pi}}{\sqrt{1 + \frac{1}{\text{c}_{2\pi}} \left( \left( \frac{\beta \beta}{\tau} \right)^2 + \left( \beta \gamma l_p \right)^2 \right)^2}} \right]^{1/2} \frac{m_{\psi}^2 \left( \left( \frac{\beta \beta}{\tau} \right)^2 + \left( \beta \gamma l_p \right)^2 \right)^{9/2}}{}
\]

(67)
Equation (67) leads directly to an expression of the effective parameter of the density of planckeons associated with the action of the Higgs field:

\[
[k]^{1/2} = \frac{15,625}{121,032} k_Z \left( \frac{\beta l_p}{\gamma} \right)^2 + (\beta \gamma l_p)^2 \right)^{9/2} 
\]

\[
f_k \left[ \frac{2\pi}{\alpha} \left\{ \frac{c^2}{(\beta l_p)^2} + (\beta \gamma l_p)^2 \right\}^{-1} \right]^{1/2} m |\psi|^2 \right]^{1/2} 
\]

namely:

\[
k = \frac{0.0167}{k_Z^2} \left( \frac{\beta l_p}{\gamma} \right)^2 + (\beta \gamma l_p)^2 \right)^9 
\]

\[
f_k \left[ \frac{2\pi}{\alpha} \left\{ \frac{c^2}{(\beta l_p)^2} + (\beta \gamma l_p)^2 \right\}^{-1} \right]^{1/2} m |\psi|^2 \right]^{1/2} 
\]

In summary, we can say that the approach based on Equations (33)–(69) shows how the condensates of planckeons (and fundamental properties of the dissipative vacuum such as the generalized Compton wavelength) have a selective function in generating the appearance of Higgs mass, of the mass of W and Z bosons, as well as the electroweak scale, the gauge couplings of vector bosons, and the elementary electric charge.

Another toy-model in order to mathematically describe the effective action \( S \) of the dissipative vacuum, which can lead to other suggestive consequences, lies in coupling the condensate of planckeons to the Abelian gauge field, by starting from the following Lagrangian in D-dimensional spacetime:

\[
L = k \left( \frac{\beta l_p}{\gamma} \right)^2 + (\beta \gamma l_p)^2 D_\mu \psi^* D^\mu \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \mathcal{V}(\psi) 
\]

where \( k \) is the usual parameter quantifying the size of the condensate of the planckeons, \( D_\mu = \partial_\mu + i e k \frac{\alpha}{4} \left( \frac{\beta l_p}{\gamma} \right)^2 + (\beta \gamma l_p)^2 \right\}^{D-4} A_\mu \) and \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \). The Lagrangian (70) is invariant under the \( U(1) \) local transformation and describes psi-particles and antiparticles interacting with massless photons. To see what happens in the regime of spontaneously broken symmetry, we apply again the constraint (44), thus obtaining

\[
L = \frac{1}{2} (\partial \varphi_1)^2 - \frac{1}{2} m_\varphi \varphi_1^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_\gamma B_\mu B^\mu + \ldots 
\]

where the quantity

\[
B_\mu = A_\mu + \frac{k}{e \sqrt{2}} \left( \frac{\beta l_p}{\gamma} \right)^2 + (\beta \gamma l_p)^2 \partial_\mu \varphi_2 
\]

refers to the new gauge field having mass

\[
m_\gamma = \frac{e \sqrt{2}}{k \left( \frac{\beta l_p}{\gamma} \right)^2 + (\beta \gamma l_p)^2} 
\]

Relation (73) physically means that the photon acquires mass \( m_\gamma \) while no massless Goldstone bosons appear. This model seems to be compatible with the Coleman–Weinberg idea regarding spontaneous symmetry breaking as an effect induced by the vacuum [38]. Above all, it shows that the possible effect of the dissipative vacuum, and in particular
of the condensate of the planckeons of the vacuum, lies in generating a mass of photons, where this mass turns out to be strictly related to the generalized Compton wavelength. In fact, here one can opportunistically choose the value of the parameter $k$ describing the size of the condensate of planckeons in order to assure the smallness of the mass (73) of photons. In this regard, by virtue of the large value of the quantity $k \sqrt{\left( \frac{\beta \mu}{\gamma} \right)^2 + \left( \beta \gamma l_\gamma \right)^2}$, it is tempting to conjecture the cosmological-scale value for the volume $k^3 \left( \frac{\beta \mu}{\gamma} \right)^2 + \left( \beta \gamma l_\gamma \right)^2 \right)^{3/2}$, in other words, to claim that it represents the volume of the (observable part of the) Universe. Then, if one inserts the current value of volume of the (observable part of the) Universe given by about ten billion light years, the above-mentioned characteristic masses (51) and (73) can be respectively estimated as

$$m^{(0)}_\phi c^2 \sim 10^{-3} \div 10^{-2} \text{eV}, \quad m_\gamma c^2 \sim 10^{-35} \text{eV}$$

(74)

where for the former mass we imposed $E_0$ to be the Planck one (which is valid if the external fields are weak enough as not to change the vacuum energy significantly). These small yet nonvanishing masses indicate that their gravitational effect and contributions to the density of matter in the Universe can be quite substantial. Moreover, by virtue of the presence of the elementary charge $e$ in the Formula (73), one can explain why it is the photon which mediates the long-range interactions between the electrically charged elementary particles: in analogy with superconductivity, the photons in this model can be interpreted as the pairs of virtual particles and antiparticles of the vacuum associated with the condensate of planckeons.

In summary, in light of the two models we have here considered for describing the effective action of the dissipative vacuum, and which are based on Equations (34)–(74), we can say that we deal here with two complementary descriptions as regards the issue of estimating the values of the generated masses of the otherwise massless particles such as the photon: the fundamental one of the planckeons and the emergent one given by an opportune vacuum condensate of planckeons. However, whereas this condensate of planckeons is opportunistically formed, it can be regarded as the most fundamental object (due to its ground state being described by a single wave function only) while the particles and interactions observed by a physical observer are represented by its different modes, collective ones, and excitations. In this approach, the generation of opportune condensates of planckeons is the fundamental physical property which gives, in turn, rise to massless and massive particles, thus causing in principle the spontaneous symmetry breaking. Each elementary particle can be seen as an emergent fact from the fundamental dissipative vacuum in the sense that its appearance can be seen as a collective mode associated with opportune condensate of the planckeons. The mass generation mechanism based on vacuum fluctuations is universal in a sense that it may supplement the electroweak one (by generating the masses of the photon and Higgs boson, for instance), but also it can enhance or even replace the latter, under certain physical circumstances. The role of the Bose–Einstein condensate of planckeons seems to be natural here because the mass generation by such a highly nonclassical object naturally serves as a physical realization of Mach’s principle. The straightforward computation we have made in the picture of the two toy-models shows that the photon mass, gained due to its interaction with the quantum-gravitational vacuum represented by the condensate of planckeons of the dissipative vacuum, can be expressed as a ratio of the elementary electrical charge and the length related to one of the parameters of nonlinearity.

On the other hand, it is worth mentioning that this description can also be reconciled with the current cosmological paradigm. In fact, on the basis of the results obtained in Ref. [68], the perspective is opened that the curved-spacetime description of the Universe’s large-scale evolution is valid only in the long-wavelength approximation, and it is not the only possible or most convenient: one can also describe it in (hydro-) dynamical terms
by invoking a Bose liquid which flows in a certain way when viewed as an embedding manifold into the Euclidean space. Such description has the potential to watch in a different manner at some long-standing problems of both the standard and inflationary cosmologies, also hinting at the possible ways of facing quantum gravity. In this regard, further investigations are obviously required.

3.3. About TeV Scale of Strong Interactions and the “Negative Mass Square Problem”

As regards the treatment of a dynamical spontaneous symmetry breaking in the hidden sector generating the TeV scale of strong interactions, the model of the dissipative vacuum leads to appealing results in terms of opportune energy fluctuations linked with the condensates of virtual planckeons too. In this regard, taking account of the results of Yamada [70], a spontaneous dynamical scale symmetry breaking can be obtained by considering the mean-field approximated effective Lagrangian of the form

\[ \mathcal{L}_{MFA} = \left( [\partial^\mu s_R]^+ \partial_\mu s_R \right) - e^2 (s_R^+ s_R) - \lambda_C (H^+ H)^2 + N_f \left( N_f \lambda_S + \lambda_S^f \right) f^2 + \frac{\lambda_S^f}{2} (\phi^a)^2 - 2 \lambda_S^f \phi^a \left( s_R^+ t_{ij} s_R \right) \] (75)

In Equation (72) \( s_R \) is a scalar field depending on the energy fluctuations of the vacuum, \( f = (s_R^+ s_R) / N_f \), \( H \) is the Higgs doublet field which ultimately emerges from the renormalization constraint (30), and therefore from the fundamental network of planckeons of the vacuum, \( \phi^a = 2 \left( s_R^+ t_{ij} s_R \right) \) are auxiliary fields with \( t_{ij} \) generators of the flavour \( SU(N_f) \) transformation, and \( \varepsilon = \frac{kM_{Planck}^2}{\Lambda} \) is a “constituent” scalar variable quantum vacuum energy of the hidden sector of the strong interactions—which is determined by opportune collective excitations of the virtual planckeons of the dissipative vacuum—given by relation

\[ \varepsilon^2 = 2 \left( N_f \lambda_S + \lambda_S^f \right) f - \lambda_{SC} (H^+ H) \] (76)

Here, if one sets \( \phi^a = 0 \) in the mean-field approximation lagrangian, one can obtain the effective potential

\[ V_{MFA} = k^2 \frac{M_{Planck}^4}{\Lambda^2} (\bar{s}_R^+ \pi_R) + \lambda_C (H^+ H)^2 - N_f \left( N_f \lambda_S + \lambda_S^f \right) f^2 + \frac{N_f}{2} \frac{\lambda_{SC}^f}{\Lambda} \ln \frac{M_{Planck}^2}{\Lambda} \] (77)

where \( \bar{s}_R \) is the background field of the scalar field \( s_R \). The effective potential (77), by applying the dimensional regularization and the \( \overline{MS} \) scheme to eliminate ultraviolet divergence, can be considered as the real mediating entity that is responsible of the spontaneous symmetry breaking occurring at the TeV scale, by leading directly to the following expressions for the changes of the vacuum energy associated to the collective excitations of the virtual planckeons, and the Higgs mass:

\[ \left\langle \frac{1}{2} M^2_P c^4 \right\rangle = \frac{4N_f \lambda_C \lambda_S - N_f \lambda_{SC}^2 + 4 \lambda_C \lambda_S^f}{2\lambda_C} \left\langle (s_R^+ s_R) / N_f \right\rangle \] (78)

\[ M^2_H \equiv 2N_f \lambda_{SC} \left\langle (s_R^+ s_R) / N_f \right\rangle \] (79)

where here a small \( \lambda_{SC} \) is assumed. On the basis of Equations (78) and (79), we can say that a scale-generation mechanism in the interactions predicted by the Standard Model emerges naturally from opportune collective excitations of the virtual planckeons of the dissipative vacuum expressing the activity of the vacuum at the Planck scale, which is associated with the generalized uncertainty relations. Here, the variable quantum vacuum energy, the scalar field \( s_R \) depending on the energy fluctuations of the dissipative vacuum, as well as the scalar couplings \( \lambda_{SC}, \lambda_C, \lambda_S \) associated with the vacuum and with the singlet field depending on the quantum vacuum energy density fluctuations, can be considered the ultimate parameters that are responsible of the generation of the action of the Higgs boson in the high-energy regime. In other words, in the approach based on Equations (75) and (79),
the action of the Higgs boson can be considered as a “mechanism”, as an emerging process. It is the interplay of opportune fluctuations of the energy of the dissipative quantum vacuum associated with the condensates of the planckeons, which indeed determines the action of the Higgs boson generating a spontaneous symmetry breaking at the $\text{TeV}$ scale.

Moreover, by exploring quantum gravity effects on the effective potential (74), one obtains the following renormalization group equations for the scalar mass

$$\frac{\partial}{\partial t} m^2 = (-2 + \gamma_m^g) m^2$$

(80)

where

$$\gamma_m^g = \frac{g N}{6\pi} \left[ \frac{20}{(1 - v_0)^2} + \frac{1}{(1 - \frac{v_0}{4})^2} \right]$$

(81)

is the graviton anomalous dimension, where $v_0 = 16\pi g N U_0$ with $U_0 = \frac{\Lambda_{CC}}{L}$ being the dimensionless cosmological constant. The graviton anomalous dimension $\gamma_m^g$ given by relation (81) implies a radical change of the energy scaling of coupling constants above the Planck scale. If $\gamma_m^g$ is larger than 2, the scalar mass parameter becomes irrelevant. In this situation, one gets the following potential solutions to the gauge hierarchy problem.

On one hand, one deals with the resurgence mechanism in which the small Higgs mass parameter $m^2 \approx 10^{-36}$ is self-organized by quantum gravity effects. In other words, the scalar mass parameter shrinks towards zero above the Planck scale and then increases such that $m^2 \approx 0, 2$ at the electroweak scale, as a consequence of the decoupling of quantum gravity effects below the Planck scale. On the other hand, one has the perspective of classical scale invariance, namely, the scale invariance at the Planck scale could be naturally realized as a consequence of the irrelevance of the scalar mass parameter above the Planck scale. Inside our approach, one can say that a gauge hierarchy problem is originated when there is a large intermediate scale between the Planck scale and the electroweak scale, for example, the grand unification scale, $\Lambda_{\text{GUT}}$ [70].

Moreover, by taking into account the results of Ref. [71] as regards a fundamental intrinsic relation between mass, gravity, space-time symmetry, and the Higgs mechanism, which emerges by involvement of the de Sitter (false) vacuum as its basic ingredient, our model of physical vacuum with dissipative features in the form of energy fluctuations associated to the virtual planckeons suggests a natural explanation for the anomalous results known as “negative mass square problem”. In light of the data on the solar and atmospheric neutrino [72], the mass-squared difference for the neutrino oscillation is given, in two-flavour mixing approximation, by the following values

$$\Delta m^2_{\text{atm}} = 2.5 \cdot 10^{-3} \text{eV}^2; \quad \Delta m^2_{\text{sol}} = 6.9 \cdot 10^{-5} \text{eV}^2$$

(82)

which lead to a relation between the gravity–electroweak unification scale and observational data. In our approach, in the interaction vertex, a particle can be described by an eigenstate of the de Sitter Casimir invariants, given by relation

$$I'_0 = k^2 \left[ \mu^2 \epsilon^2 \pm \frac{\hbar^2}{2r^2_0} \right]$$

(83)

where $r_0$ is the de Sitter radius, which is ultimately associated and derived from the more fundamental generalized Compton wavelength (11), namely:

$$r^2_0 = \left( \frac{\beta l_p}{\gamma} \right)^2 + (\beta \gamma l_p)^2$$

(84)
By taking into account Refs. [66,68], the state (83), by following an evolution in the Minkowski space, assumes the form of a linear superposition of two different mass eigenstates related to the condensate of the planckeons

\[
m_1^2 = k^2 \left\{ \mu^2 + \frac{\hbar^2}{2c^2 \left[ \left( \frac{\beta \mu}{T} \right)^2 + (\beta \gamma l_p)^2 \right]} \right\}, \quad m_2^2 = k^2 \left\{ \mu^2 - \frac{\hbar^2}{2c^2 \left[ \left( \frac{\beta \mu}{T} \right)^2 + (\beta \gamma l_p)^2 \right]} \right\}
\]

(85)

with equal weights. Moreover, always following Ref. [73], the mass-squared difference (82) can be connected directly with the unification scale \( M_{\text{unif}} \) of gravity and electroweak scale as follows

\[
\Delta m^2 = \frac{8\pi}{3} k^2 \left( \frac{M_{\text{unif}}}{m_{\text{pl}}} \right)^4 \frac{m_{\text{pl}}^4}{m_{\text{pl}}^4}
\]

(86)

The resulting symmetry induced by the vacuum energy density of the Gross–Pitaevskij vacuum in the gravito–electroweak vertex generates an exact bi-maximal mixing for neutrinos, which leads to the following mass-squared difference between atmospheric and solar neutrinos linked with the condensate of the planckeons:

\[
\Delta m^2 = \frac{\hbar^2 k^2}{2c^2 \left[ \left( \frac{\beta \mu}{T} \right)^2 + (\beta \gamma l_p)^2 \right]}
\]

(87)

for both the right- and left-handed fields. On the basis of Equation (87), the physical meaning of the mass-squared difference between atmospheric and solar neutrinos lies in the collective excitations of the virtual planckeons, namely, it can be seen as an emergent effect from the dissipative features of the Gross–Pitaevskij vacuum close to the Planck scale.

Now, by equating (86) and (87), one can obtain an expression for the gravito–electroweak scale \( M_{\text{unif}} \) in the dissipative vacuum as follows

\[
8\pi \frac{k^2}{3} \left( \frac{M_{\text{unif}}}{m_{\text{pl}}} \right)^4 \frac{m_{\text{pl}}^4}{m_{\text{pl}}^4} = \frac{\hbar^2 k^2}{2c^2 \left[ \left( \frac{\beta \mu}{T} \right)^2 + (\beta \gamma l_p)^2 \right]}
\]

(88)

thus leading to the following expression of the unification scale

\[
\left( \frac{M_{\text{unif}}}{m_{\text{pl}}^4} \right)^4 = \frac{3 \hbar^2 m_{\text{pl}}^4}{16\pi c^2 \left[ \left( \frac{\beta \mu}{T} \right)^2 + (\beta \gamma l_p)^2 \right]}
\]

(89)

namely

\[
M_{\text{unif}} = \frac{1}{2} \left( \frac{3 \hbar^2 m_{\text{pl}}^4}{\pi c^2 \left[ \left( \frac{\beta \mu}{T} \right)^2 + (\beta \gamma l_p)^2 \right]} \right)^{1/4}
\]

(90)

On the basis of relation (90), in our model of dissipative vacuum, the gravito–electroweak scale \( M_{\text{unif}} \) turns out to be directly fixed by the generalized Compton wavelength, which measures the activity of the vacuum at the Planck scale, but turns out to be independent of the parameter \( k \) indicating the size of the condensate of the planckeons. In this way, the mass-squared differences of neutrinos determined by fluctuations of the quantum vacuum energy density associated with the virtual planckeons of the dissipative Gross–Pitaevskij vacuum, lead to the following corresponding values of the unification scale for solar and atmospheric neutrinos:

\[
M_{\text{unif(atm)}} \approx 14.5 \text{ TeV}; \quad M_{\text{unif(sol)}} \approx 5.9 \text{ TeV}
\]

(91)
The values (91) turn out to be in good agreement with the results obtained in Ref. [66], as well as in other previous theories of electroweak unification [74–76]. The novelty of this approach, with respect to the previous theories of electroweak unification, lies in the fact that the unification scale for solar and atmospheric neutrinos emerges naturally from the interplay of the virtual planckeons of the Gross–Pitaevskij vacuum close to the Planck scale.

4. The Link with Quantum Jumps: About Emergence of Particles at the Planck Scale in the Standard Model

The approach suggested in this paper allows us to shed new light as regards the emergence of particles at the Planck scale, leading to a direct relation between the planckeons describing the activity of the vacuum at the Planck scale and the Licata–Chiatti quantum jumps theory.

By following Ref. [77], one can assume that a quantum jump, a process of localization of the skeleton of a particle, can be described through an internal (inaccessible) wave function factor $\phi(\tau')$ of an internal time, when the information associated with the elementary cells of a de Sitter–Planck vacuum satisfies relation $A/l_p \approx 10^{-13}\text{cm}$ where $A$ is the area of the de Sitter micro-region encoding the information about that region. This area, which is linked with the information of the ultimate vacuum, can be expressed as $A = I(\delta l)^2$ where, taking account of the results obtained in Refs. [59–63], the quantity $\delta l$ represents the average minimum uncertainty, i.e., the average separation between neighbouring cells, of the background, which is determined by the fluctuations of the quantum vacuum energy density, namely, by the ultimate texture of the planckeons, and is given by the following relation

$$\delta l \geq \left(\frac{2\pi^2}{3}\right)^{1/3} c\theta_0$$

(92)

where $c$ is light speed and $\theta_0$ is the minimum proper time corresponding to the switching of a cell, which generates the “bare” state of a particle (and thus determines, as a derived fact, the time interval between two successive localizations of the same particle).

Now, in the de Sitter geometry of the dissipative vacuum characterized by semi-local cells, the intrinsic positional uncertainty (92) can be associated to a corresponding spatial length, which we denote $l_0$, expressing the minimum spatial length, where the switching of opportune cells can give origin to the appearance of the “bare state” of a particle, such as a fermion of the Standard Model. In order to define this peculiar spatial length characterizing the ultimate texture of Planck scale in a region of the de Sitter universe, by following Refs. [59–63], we assume that the spatial volume occupied on the average by each cell is $l_0^3/l_p^3$, and thus that a spatial region of size $l_0$ contains a number of cells given by

$$N = l^3 / \left(l/l_p\right)^3 = (l/l_p)^2$$

(93)

We can therefore define $l_0$ through the following equation:

$$(\delta l)^3 = l_0^2$$

(94)

which yields

$$l_0 = \frac{2\pi^2}{3} (c\theta_0) / l_p^2$$

(95)

Equation (95) expresses, in the de Sitter background of the dissipative vacuum, the minimum size of each spatial length which is ultimately determined by the intrinsic positional uncertainty (94) characterizing the Planck lattice, giving rise to the appearance of the “bare” state of a particle.

The wave function $\phi(\tau')$ describing a process of localization of the skeleton of a particle physically regards the virtual particles of the dissipative vacuum, while $\tau'$ is a kind of internal time of the background of the planckeons, which acts as when the real
elementary particles are someway “dormant” relative to external time (namely after their annihilation or before their creation). As regards the features and evolution of this wave function, by following the Licata–Chiatti quantum jumps approach, in the de Sitter–Planck background, this function is oscillatory in the internal time and it is just these elementary oscillations which originate the different types of elementary particles that can be created or annihilated. Moreover, the wave function $\phi(\tau')$ has the property to be real and harmonic in the internal time variable $\tau'$, null at the boundary and outside of the interval $\left[-A/l_p^c, A/l_p^c\right]$, where $A/l_p^c \approx 10^{-13}$ cm, and obeys the following general equation:

$$\left\{ \begin{align*}
-\hbar^2 \frac{\partial^2}{\partial (2\pi \tau')^2} \phi(\tau') &= (mc^2)^2 \phi(\tau') & \text{if } \tau' \in \left[-A/l_p^c, A/l_p^c\right] \\
\phi(\tau') &= 0 & \text{otherwise}
\end{align*} \right.$$  \hspace{1cm} (96)

where $m$ is the mass of the virtual particles of the de Sitter–Planck background, which represents the skeleton, namely the “bare” state of the particle mass of the observable world. By defining $\theta_0 = \frac{A}{l_p^c}$, on the basis of Equation (96), in the Licata–Chiatti model of quantum jumps, the virtual particles emerging from the ultimate texture of cells of Planck scale of the de Sitter background generate the usual real elementary particles when their mass satisfies the following relation

$$mc^2 = nt \frac{\hbar}{\theta_0}$$  \hspace{1cm} (97)

where $c\theta_0 \approx 10^{-13}$ cm corresponds to the chronon scale, $nt = 0, 1, 3/2, \ldots$ is an integer for odd solutions, a half-integer for even solutions.

Now, in our model of dissipative vacuum developed in this paper, as a consequence of the crucial role of the condensate of the planckeons at the scale represented by the generalized Compton wavelength (11), it follows that the minimum size of each spatial length (95) which give rise to the appearance of the “bare” state of a particle can be expressed by relation:

$$\frac{2\pi^2}{3} (c\theta_0)^3 / l_p^2 = k \sqrt{\left(\frac{\beta l_p}{\gamma}\right)^2 + \left(\beta \gamma l_p\right)^2}$$  \hspace{1cm} (98)

Equation (98) expresses in what sense the generalized Compton wavelength and the size of the condensate of the planckeons lead in a direct way to the minimum size of each spatial length characterizing the Planck lattice, which gives rise to the appearance of the “bare” state of a particle, i.e., to the chronon scale. In other words, on the basis of Equation (98), we can say that the chronon scale can be considered as a direct manifestation, at an upper level, of the generalized Compton wavelength as well as of the size of the condensate of the planckeons:

$$c\theta_0 = \left(\frac{3l_p^2}{2\pi^2} k \sqrt{\left(\frac{\beta l_p}{\gamma}\right)^2 + \left(\beta \gamma l_p\right)^2}\right)^{1/3}$$  \hspace{1cm} (99)

By substituting Equation (99) into Equation (97), this latest equation becomes

$$mc = nt \frac{\hbar}{\left(\frac{3l_p^2}{2\pi^2} k \sqrt{\left(\frac{\beta l_p}{\gamma}\right)^2 + \left(\beta \gamma l_p\right)^2}\right)^{1/3}}$$  \hspace{1cm} (100)

which expresses the constraint which must be satisfied by the virtual planckeons of the dissipative vacuum described by a Gross–Pitaevskij evolution in order to give origin to the ordinary elementary particles. The physical meaning of Equation (100) is thus the following: the mass of the planckeons of the dissipative vacuum characterized by a Gross–Pitaevskij
evolution law can be considered as the ultimate element which gives rise to the appearance of particles of the Standard Model at the Planck scale and, conversely, can be considered as the Planck signature of Standard Model particles.

5. Gauge Symmetry, Scale of Emergence, and Cosmological Constant

In the Standard Model, gauge symmetries assume a relevant role in fixing the features of the interactions of elementary particles, but there is the problem regarding their origin, whether they unify in the ultraviolet or are emergent in the infrared and disappear above the scale of emergence. Emergence of the Standard Model can be considered a sort of a phase transition, connected with the stability of the Higgs vacuum, which implies that leptons, quarks, gauge, and Higgs bosons can be seen as the stable long-range collective excitations of some critical statistical system that sits close to the Planck scale. In this way, the Standard Model manifests itself as an effective theory with action containing an infinite series of higher-dimensional operators whose contributions are suppressed by powers of the scale of emergence. However, within the emergence scenario, the degrees of freedom above the scale of emergence remain an open issue, and there is the problem of connecting the scale of emergence to the vacuum stability.

In the approach of the dissipative vacuum, if hadrons and their interactions described by fundamental QCD quark and gluon degrees of freedom and the structure of ordinary matter described inside QED, are emergent from the nonlinearity and nonlocality of the fundamental background of the planckeons, intended as activity of the vacuum at Planck scale linked with the generalized uncertainty relations, one expects that at a deeper level, the gauge symmetries would be emergent and would dissolve in the ultraviolet and that these processes are ruled by opportunite behaviour of the collective excitations of the virtual planckeons of the dissipative vacuum too. While in standard unification models the maximum symmetry is at the highest energies of the extreme ultraviolet and a spontaneous symmetry is applied in the infrared, which is associated with the Higgs mechanism [78], instead in our model one can suggest that new critical phenomena in the ultraviolet appear as the long-range consequence of the unification scale \( M_{\text{unif}} \) given by (87) and determined by the virtual planckeons of the vacuum described by the Gross–Pitaevskij Equation (16).

According to the approach suggested in this paper, the Gross–Pitaevskij vacuum and, in particular, the scale \( M_{\text{unif}} \) associated with the virtual planckeons generating the gravity–electroweak unification scale and which is given by Equation (90), is the turning key that allows a resolution of the cosmological constant issue and the hierarchy problem. As a consequence, the gauge symmetries of the Standard Model would be emergent from the collective features of the network of the planckeons of the vacuum characterizing this scale \( M_{\text{unif}} \).

If one takes the Standard Model gauge symmetries as emergent and dynamically generated by the collective behaviour of the elementary particles of the dissipative vacuum ruled by the Gross–Pitaevskij Equation (16), the Standard Model can be interpreted as an effective theory where at low energies, the physics is determined by a relatively small number of operators with mass dimension at most four. For these terms, gauge invariance and renormalizability restrict the number of possible operator contributions and strongly constrain the global symmetries of the system. Instead, the extra symmetry breaking terms associated with higher dimensional operators only become active in the particle dynamics when mass and energy scales close to the large emergence scale are approached.

In our theory, the Gross–Pitaevskij vacuum characterized by fluctuations determined by the network of its fundamental planckeons emerges as a possible candidate to define the ultraviolet scale \( M_\text{u} \) that, according to the experimental constraints on the size of the Pauli term, tiny neutrino masses, axion masses, and proton decay, should be between \( 10^{15} \) GeV and the Planck scale of \( 1.2 \times 10^{19} \) GeV. In other words, the lattice of the virtual planckeons of the Gross–Pitaevskij vacuum is the fundamental background that explains the appearance of the ultraviolet scale in a natural and direct way. So, the specific collective behaviour of the virtual planckeons, as a consequence of the activity of the vacuum at the
Planck scale associated with the generalized Compton wavelength, imply that at the highest
energies, the system becomes increasingly chaotic, characterized by a maximum symmetry
breaking, in contrast to unification models which exhibit the maximum symmetry in the
extreme ultraviolet.

In the context of an emergent picture of the Standard Model gauge symmetries from
the interplay of the virtual planckeons generating dissipative features of the vacuum, some
considerations can be made also as regards hints of new physics and new patterns of global
symmetry violation associated with higher dimensional operators beyond the Standard Model.
In this regard, the lowest-dimension operators that violate lepton and baryon numbers,
respectively, are the Weinberg dimension-five operator regarding neutrino mass and the
four-fermion operator associated with proton decay. The lepton number violation at energies
typical of the very early universe is described by the Weinberg dimension-five operator

\[ O_5 = \frac{(HL)_{ij}^T \lambda_{ij}(H)_j}{M_u} \]  

concerning Majorana neutrino mass, where here the Higgs doublet \( H \) ultimately emerges
as the result of more elementary fluctuations of the Gross–Pitaevskij vacuum determined
by the collective excitations of the virtual planckeons describing its dissipative features, \( L_i \)
denotes the SU(2) left-handed lepton doublets and \( \lambda_{ij} \) is a matrix in flavour space, \( M_u \) is
the ultraviolet scale, about \( 10^{15} \) GeV, itself determined by opportune collective networks
of the planckeons of the vacuum. In an analogous way, the dissipative features of the vacuum
determined by collective excitations of the virtual planckeons, by the activity of the vacuum
at the Planck scale linked with the generalized Compton wavelength, can be considered as
the fundamental origin of the possible violation of the baryon number associated with the
four-fermion operator

\[ O_6 = \frac{1}{M_u^2} QQQL \]  

where \( L \) and \( Q \) are the lepton and quark doublets, respectively, which becomes active at
energies typical of the very early universe and might play an important role in under-
standing the matter–antimatter asymmetry in the Universe. Moreover, we emphasize
here the importance of operators, with dimensions greater than four, that give corrections
to flavour-changing processes that are highly suppressed in the Standard Model, such
as, for example, the one contributing to Kaon mixing or operators involving large extra
dimensions expressed in terms of towers of Kaluza–Klein excitations which could give
rise to a physics beyond the Standard Model that cannot be described by an effective field
theory [79]. Indeed, as regards physics beyond the Standard Model and corresponding
symmetry violations, it has been claimed that two different scenarios are possible: one
where the new physics is dominated by interference terms between dimension-six contribu-
tions and the Standard Model (a scenario that occurs whenever the scale of new physics is
high in comparison with the electroweak scale), the other where one should also consider
interference terms between dimension-six contributions and dimension-eight contributions
(which indicates an underlying structure in the beyond Standard Model background and
can occur if the dimension-six operators are purely CP-odd and the observable is a CP-even
quantity) [80].

On the other hand, in light of cosmology observations, we know that, as regards the
real content of the Universe, just 5% is built from Standard Model particles, in the sense
that 26% involves dark matter (possibly made of new elementary particles) and 69% is
dark energy [81]. Dark energy, which is introduced in order to “explain” the accelerated
evolution of the universe, refers to some mysterious form of diffuse energy presumably
permeating all corners of the universe, and is currently associated to the energy density
of the vacuum [82–84]. Dark matter is invoked in order to “reproduce” the anomalous
dynamics of galaxies and of galaxy clusters. New axion-like particles with masses and
couplings suppressed by powers of the large emergence scale might be a vital ingredient in
understanding dark matter. The model suggested in this paper, by considering an emergent approach from fluctuations describing the dissipative features of the vacuum, associated with collective excitations of virtual planckeons close to the Planck scale, has the potential to introduce interesting perspectives of explanation also of dark matter and dark energy.

Moreover, it must be emphasized that, in models of the emergent Standard Model, the scale of emergence can have a relation with new dynamical scenarios in the sense that, if electroweak symmetry breaking and emergence were to happen at the same scale, then the physics of inflation would involve totally new physics with different unknown degrees of freedom. In this regard, the model of the physical vacuum with dissipative features expressed by fluctuations generated by the virtual planckeons of the vacuum, on the basis of Equation (40) which expresses the origin of the Higgs field and Equation (87) which expresses the gravity-electroweak unification scale, suggests new perspectives of explanation of the issue whether chirality (and neutrinos) might assume a special role in any ultraviolet critical phenomena and a consequent emergent gauge symmetry in the infrared.

Some interesting considerations can also be made as regards the treatment of the cosmological constant in the sense that the contributions appearing in it—and represented by the Higgs and QCD condensates and a renormalized version of the bare gravitational term $\rho_\Lambda$ [82,85]—can be seen as the results of specific collective behaviours of the virtual particles of the physical vacuum with dissipative features. In our approach, the net vacuum energy density may thus be expressed in the following form

$$\rho_{\text{vac}} = \rho_{\text{qeE}} + \rho_{\text{potential}} + \rho_\Lambda$$

where $\rho_{\text{potential}}$ is the potential energy density given by Equation (41). The vacuum energy density (103), which turns out to be renormalization scale invariant and drives the accelerating expansion of the Universe, satisfies relation

$$\frac{d}{d\mu^2}\rho_{\text{vac}} = 0$$

which means that it is independent of the way it is computed. In light of its explicit $\mu^2$ dependence and of the network of the planckeons of the dissipative vacuum, the contributions to the quantum vacuum energy density $\rho_{\text{qeE}}$ in Equation (103) are scale dependent. In line with the results obtained in Ref. [34], one finds the following results: for QCD, the degrees of freedom depend on the resolution, deep in the ultraviolet one has asymptotic freedom, for massless quarks the quantum vacuum energy density vanishes, in the infrared confinement and dynamical chiral symmetry breaking take over. The Higgs potential is renormalization scale-dependent as a consequence of the scale dependence of the Higgs mass and Higgs self-coupling, which ultimately derives from the elementary virtual planckeons of the dissipative vacuum on the basis of Equation (30), and determines the stability of the electroweak vacuum ultimately emerging from the unification scale (90).

In this emergent picture of the Standard Model and of its gauge symmetries from the ultimate Gross–Pitaevskij vacuum intended as a network of virtual planckeons describing its dissipative features at the Planck scale, if we suppose that the vacuum is translational invariant and flat space-time is consistent at dimension four, just as suggested by the success of the Standard Model, then the contribution of the Renormalization Group invariant scales $\Lambda_{\text{QCD}}$ and electroweak $\Lambda_{\text{ew}}$ into the cosmological constant lies in the scale of the leading term suppressed by $\Lambda_{\text{ew}}/M$, where the scale of emergence $M$ depends on opportune fluctuations of the dissipative vacuum determined by the network of the virtual planckeons (which lead to $\rho_{\text{vac}} \sim \left(\frac{\Lambda_{\text{ew}}^2}{M}\right)^4$ with one factor of $\Lambda_{\text{ew}}^2/M$ for each dimension of space-time). This scenario allows an explanation of the reason why the cosmological constant scale 0.002 eV is similar to what we expect for the neutrino masses [86] and Majorana neutrinos [87], as a consequence of the interplay and the collective excitations of fundamental virtual planckeons of the dissipative vacuum close to the Planck scale. In summary, we can conclude that,
according to the approach developed in this paper, we can justify in what sense the scale of emergence should be deep in the ultraviolet, much above the Higgs and other Standard Model particle masses, close to the Planck scale, just as a consequence of the collective behaviour of the virtual planckeons of the Gross–Pitaevskij vacuum.

Finally, we must mention that several issues put the Standard Model at risk. In particular, scenarios towards a new physics could be opened by the so-called g-2 anomaly, regarding the discrepancy between theory and data of the magnetic dipole moment of the muon. To date, the most part of physical explanations of the muon g-2 discrepancy invokes new scalar fields, for example, axions, i.e., light, periodic scalar bosons originating from the breaking of an approximate U(1) symmetry. In Ref. [88], an approach was suggested, based on heavy axion-like particles with couplings to leptons and photons, which provides a tantalizing potential solution to the muon g-2 anomaly. However, this recent approach which invokes axions does not manage to specify the origins of the axion couplings and other relevant degrees of freedom. This could be the clue of the existence of new fundamental particles existing in nature, that could be probed in future searches. In light of the treatment made in this paper, the perspective is opened that these new fundamental particles, which take account of the origin of axions, could be associated with the planckeons describing the activity of the vacuum at the Planck scale linked with the generalized Compton wavelength. In this regard, further research will yield more information.

6. Conclusions

Since their appearance in the early 1900s, Planck’s natural units have posed the question we now call quantum gravity. Much of the difficulty comes from the fact that nobody really knows what these units mean. They appear as a mysterious sphinx placed at the centre of convergence between quantum mechanics and relativity, and only a theory capable of including gravitation in a unified picture with the other forces will be able to reveal their secret. Most physicists have understood natural units as elementary pieces of space-time, beyond which the very description of the physical world loses meaning. The idea of a Rubik space-time has been taken up by many authors [89–96]. These approaches, based on an idea of fixed tessellation, have been progressively replaced by others that interpret the Planck scale in dynamic (or pre-dynamic) terms such as loops and strings (for an introduction, see Refs. [97,98]). Obviously, these are different approaches. In fact, it is possible to say that if the theories with Rubik’s construction tried to find the traces of a reconciliation between quantum field theory and relativity on an extreme microphysical scale of space-time, the new classes of theories, on the contrary, have laid the foundations for an emergent description of space-time and particles.

In this work, we have introduced planckeons, which are not to be understood as ultimate building blocks, but as a special class of particles which obey the constraints of the Planck scale, ingredients which introduce nonlocality and nonlinearity by solving some serious problems of scales and hierarchies which afflict the Standard Model. In other words, we could say that planckeons are like dust introduced into the mechanism of the Standard Model in order to match the theory with the observations, a not too radical intervention which allows us to keep the advantages of the Standard Model. We want to remind here that the equation that guides the vacuum of the planckeons is the Gross–Pitaevskij equation, well known—on other scales—in collective phenomena. In fact, here the Higgs mechanism finds interpretation in terms of organization of opportune condensates of planckeons around a bare charge. In the approach suggested in this paper, by starting from the nonlinear and nonlocal Gross–Pitaevskij Equation (16), one directly arrives at the effective potential (34), expressing the interactions of the fundamental planckeons, which is characterized by a Mexican-hat shape, making it a plausible candidate for the Higgs potential. In this way, the Higgs mechanism and the spontaneous symmetry breaking emerge naturally from the ultimate dynamics of opportune condensates of the planckeons of the dissipative vacuum and, in particular, the Higgs mass turns out to be an emergent fact from the generalized Compton wavelength which is associated with the dissipative features of the vacuum. We
have explored two different toy-models in order to characterize the effective action of the dissipative vacuum, the one which considers the self-interacting vacuum involving only the complex psi-field, the other which lies in coupling the condensate of planckeons to the Abelian gauge field. The first model shows that, in the broken symmetry regime, one can describe two kinds of particles, one massive and one massless, where the latter are the Nambu–Goldstone bosons which describe the spatial variations of the vacuum’s phase. In the other model, the condensate of the planckeons of the vacuum lead to generating a mass of photons, strictly related to the generalized Compton wavelength, and the smallness of the photon’s mass is assured by opportunely choosing the value of the parameter describing the size of the condensate of planckeons. A fascinating result of our approach lies in the possibility to find Planck-scale signatures in the Standard Model, by formulating the gauge couplings of the vector bosons in terms of the size of the opportune aggregate of planckeons, the mass of the planckeons, as well as the generalized Compton wavelength describing the ultimate geometry of the vacuum and by demonstrating that also the elementary charge can be seen as a collective phenomenon from opportune condensates of planckeons.

Moreover, the mathematical framework of our approach shows how a scale-generation mechanism in the interactions predicted by the Standard Model emerges naturally as a consequence of opportune collective excitations of the virtual planckeons of the dissipative vacuum, showing that the interplay of opportune fluctuations of the energy of the vacuum associated with the condensates of the planckeons determines the action of the Higgs boson generating a spontaneous symmetry breaking at the TeV scale. Here, one can also provide a natural explanation for the “negative mass square problem”, in the sense that a unification scale for solar and atmospheric neutrinos, given by Equation (90), emerges directly from the interplay of the virtual planckeons of the Gross–Pitaevskij vacuum close to the Planck scale. An appealing result of this formalism is that, while the mass of solar and atmospheric neutrinos depends on the parameter describing the size of the condensate of planckeons, instead, the unification scale responsible for the mass difference between solar and atmospheric neutrinos turns out to be independent of the value of this parameter and depends only on the generalized Compton wavelength, and thus on the mass of the virtual sub-particles of the vacuum.

On the other hand, the generalized Compton wavelength generating the dissipative features of the vacuum leads in a direct way to the minimum size of each spatial length characterizing the Planck lattice, which gives rise to the appearance of the “bare” state of a particle, i.e., to the chronon scale, thus shedding new light as regards the issue of the emergence of particles at the Planck scale, leading to a direct relation with the Licata–Chiatti quantum jumps theory. In this regard, on the basis of Equation (98), the chronon scale can be considered as a direct manifestation, at an upper level, of the generalized Compton wavelength and the size of the condensate of the planckeons.

Finally, our model leads to the fundamental prediction that the scale of emergence should be deep in the ultraviolet, much above the Higgs and other Standard Model particle masses, close to the Planck scale, just as a consequence of the collective behaviour of the virtual planckeons of the Gross–Pitaevskij vacuum and, therefore, that the cosmological constant and the hierarchy problem could be resolved near to the unification scale (90) associated with the virtual planckeons generating the gravity–electroweak unification scale. This introduces the perspective that the gauge symmetries of the Standard Model would be emergent from the collective features of the network of the planckeons of the vacuum characterizing the unification scale (90).

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