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Complex Cardinal Numerals and the Strong Minimalist Thesis

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Abstract: Different analyses of complex cardinal numerals have been proposed in Generative Grammar. This article provides an analysis of these expressions based on the Strong Minimalist Thesis, according to which the derivations of linguistic expressions are generated by a simple combinatorial operation, applying in accord with principles external to the language faculty. The proposed derivations account for the asymmetrical structure of additive and multiplicative complexes and for the instructions they provide to the external systems for their interpretation. They harmonize with those of coordinate nouns, and thus offer a unified Minimalist account of their core properties. Firstly, the empirical problem addressed is stated. Secondly, the theoretical framework is presented. Thirdly, Minimalist derivations for additive and multiplicative complexes are provided. Fourthly, the proposed derivations are contrasted with derivations not relying on the Strong Minimalist Thesis. Lastly, consequences for linguistic theory are identified as well as questions open to further inquiry.

Keywords: minimalism; Merge; principles external to the language faculty; complex cardinal numerals; derivations; functional heads; extended projections; unpronounced interpreted heads; language and mathematics

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Citation: Di Sciullo, A.M. Complex Cardinal Numerals and the Strong Minimalist Thesis. *Philosophies* **2022**, 7, 81. https://doi.org/10.3390/philosophies7040081

Academic Editor: Peter Kosta

Received: 27 May 2022
Accepted: 28 June 2022
Published: 16 July 2022

1. Purpose

This article targets the derivation of complex cardinal numerals used for counting and measuring. It builds on previous work on their asymmetric structure involving silent elements, and the domain specificity of language with respect to mathematics. It highlights the relevance of the Strong Minimalist Thesis (SMT) (Chomsky 1995 [1], 1998 [2], 2005 [3], et seq.) for the development of an explanatory theory of the language internal to the mind (I-language) able to derive the syntactic properties of linguistic expressions and the instructions they provide to the external systems for their interpretation.

Cardinal numerals pose an interesting puzzle for linguistic theory. In English additive complexes, (1), a functional category, namely the coordinate conjunction, may intervene between the numeral constituents, (1a, b). In multiplicative complexes with precise interpretation, no functional category is pronounced between the numeral constituents, (2); whereas this is the case in expressions of large numerosity with vague interpretation, where the preposition *of* is pronounced, (3). Similar facts are observed in other languages, including the Romance languages, as we illustrate in (4) with Italian (I).

(1) a. hundred two
b. hundred and two
(2) a. two hundred
b. two hundred thousand
(3) a. hundreds of thousands of millions
b. hundreds of thousands of millions of trillions
(4) a. centodue, cento e due (I)
"hundred two", "hundred and two"
b. duecento, duecentomila "two hundred", "two hundred thousand"
c. centinaia di migliaia di milioni
"hundreds of thousands of millions"

Considering the linear order of the constituents in English complex numerals, we observe that the base precedes the digit in additive complexes, whereas the inverse order holds in multiplicative complexes. The position of the digit with respect to the base could be determinant in the derivation of additive vs. multiplicative complexes. However, the order of the digit with respect to the base is subject to variation. There are languages such as in Latin (L), where the order of the digit with respect to the base is inverted near the tenth, (5), and the preposition *de* intervenes between the numeral constituents. Similar facts are observed in Italian, (6), where in this case the base precedes the digit near the tenth. Assuming that complex cardinal numerals are derived by the computations of I-language, we expect structure dependencies to be observed in the form of these expressions, as it is the case more generally for linguistic expressions.

(5) duo de viginti, un de viginti (L)
two DE twenty, one DE twenty
"eighteen", "nineteen"

(6) diciasette, diciotto, dicianove (I)
ten seven, ten eight, ten nine
"seventeen", eighteen", "nineteen"

The following questions arise. Is there a structural difference between (1a) and (1b) related to the presence or absence of the coordinate conjunction? Is a functional category generated in the derivation of (2), even if, in the languages under consideration it is not externalized? Why is a functional element, namely the preposition *of*, pronounced with expressions of large numerosity with vague interpretation, such as (3)? We consider these questions from a Minimalist Perspective.

This article provides syntactic derivations for the internal structure of additive and multiplicative complexes, including instructions to the external systems, Conceptual Intentional (CI) and sensorimotor (SM), for their interpretation. It does so with the simplest theory of I-language, i.e., the language internal to the mind, which meets descriptive adequacy. That is, with a theory that accounts for the facts with a minimum of technical apparatus. In this framework, I-language is general enough to account for all languages, and there is no space for construction-specific rules. Furthermore, syntactic operations are determinate, and there is no space for optionality.

The organization of this article is the following. Firstly, the theoretical framework is presented, as well as some differences between the theory of I-language and the theory of natural numbers. Secondly, derivations for complex cardinal numerals are provided. Thirdly, alternative derivations are discussed. Lastly, consequences for linguistic theory, and problems left open for further inquiries are identified.

2. Minimalist Program and Strong Minimalist Thesis

The Minimalist Program is a research space dedicated to the understanding of the human ability for language and thinking. This ability is a species-specific trait that emerged recently in evolutionary times, given archeological records. I-language is a computational system relating thoughts to SM expressions. This system generates infinite arrays of hierarchical structures based on finite means. The externalization of linguistic expressions (E-language) however, is not a direct reflection of the computations of I-language. While we are not conscient of the computation of I-language, we can formulate theories that may provide deeper understanding of human-specific cognitive computations for language. In this perspective, research in the Minimalist Program aims to formulate an explanatory theory of I-language.

According to SMT, I-language is reduced to the simplest computational operation, which applies in accord with principles external to I-language, called third factors principles of computational efficiency in Chomsky (2005), the first principle is genetic endowment and the second is experience. These three factors are part of language design and contribute to the growth of language in the individual.

Simplicity is a fundamental methodological principle of Minimalist inquiry, as it is in science. It is also a basic property of biological organisms. A truly explanatory theory of I-language also meets the criteria of evolvability, the rapid emergence of language, and naturalistic acquisition of language by children (Lenneberg 1969 [4], Chomsky 2011 [5], 2020 [6], Yang and Roeper (2011) [7], Goodluck, and Kasanina (2020) [8], Lightfoot 2020 [9]). An explanatory theory of I-language is the simplest theory, in terms of levels of representation, number and complexity of syntactic operations, as well as theory internal conditions on these operations meeting descriptive adequacy.[1]

In this framework, I-language consists of the binary set formation operation Merge, (7). This operation is the simplest combinatorial structure building operation. Merge is unbounded and derives the discrete infinity of language based on finite means. Formatives are taken from the Lexicon (Lex) and placed into the workspace (WS), where derivations take place. External Merge (eM) combines two elements, α and β, that have not been combined previously in the derivation. Internal Merge (iM) combines an element which has been subject to Merge at a previous step of the derivation, such as α in (7).[2] In Chomsky, Gallego and Ott (2019) [10], Merge applies freely. Previously proposed syntactic features such as the Extended Projection Principle (EPP) and the edge feature ([EF]) no longer drive IM. This contributes to simplifying the central combinatorial operation of I-language. See also Chomsky (2019) [11], and (2021) [12] for discussion.

(7) eM: (α, β): {α, β}
iM: (α, {{$\cancel{\alpha}$, β}): {α{$\cancel{\alpha}$, β}}

The set-theoretical definition of Merge does not undermine natural language first principle of structure dependency. Agreement, displacements of syntactic constituents, and other syntactic dependencies, rely on asymmetrical c-command or sister-containment relation (see Chomsky 1995, and seq.). While eM does not impose any ordering relation between the merged constituent, which are linearized at SM, iM derives dominance relations between constituents. Thus, the derivations generated by set-theoretical Merge, can be represented in terms of hierarchical structures.

Given SMT, Merge applies in accord with third factor principles of computational efficiency. The latter are external to I-language, akin to natural laws, and subject to intensive research in Minimalist syntax related to the restricted recourses of the human brain for processing language, as opposed to vision and audition for example. It might be the case that these principles contribute to minimizing the role of symmetrical relations between constituents in syntactic derivations, as well as their externalization, as discussed in Di Sciullo (2015) [13]. According to this hypothesis, principles minimizing symmetrical relations subsume Agree (Chomsky 2000 [14], Pesetsky and Torrego 2007 [15]) and the Phase Impenetrability Condition (PIC) (Uriagereka (1999) [16], Chomsky (2001) [17], 2008 [18]); principles minimizing externalization subsume Pronounce the Minimum (Chomsky 2001) [19] and Condition on Spell-Out (Collins 2007) [20].

While Merge combines syntactic constituents, feature sharing between constituents is subject to the operation AGREE,[3] according to which feature agreement between syntactic constituents relies on asymmetric c-command, would fall into the principles minimizing symmetry. According to PIC, displacement of a constituent out of a phase is only possible if the constituent has first moved to the left edge of the phase,[4] that is to a hierarchically prominent position. This is the case for α, the Specifier position, but not β, the Complement position in the [Specifier [Head Complement]] configuration in (8), where α and β are syntactic constituents, and F is a functional Head. The PIC is sensitive to asymmetric c-command relation, as the Specifier asymmetrically c-commands the Complement.

(8) a. $[_{Ph2}\ \alpha\ [\ F\ \ [_{Ph1}\ \cancel{\alpha}\ [\ F\ \ \beta\ ]]]]$

b. * $[_{Ph2}\ \beta\ [\ F\ \ [_{Ph1}\ \ \alpha\ [\ F\ \ \cancel{\beta}\ ]]]]$

According to our proposal, principles minimizing externalization would include Pronounce the Minimum (hereafter PM), according to which the copies of a displaced constituents are generally not pronounced (Chomsky 2001). They would also include the Condition on Spell-Out, Collins (2007) (hereafter COS), according to which either the Specifier or the Head of a phase is pronounced, but not both, (9).

(9) Condition on Spell-Out
 a. Edge(x) must be phonetically overt.
 b. The condition in (a) applies in a minimal way so that either the Head or the Specifier, but not both, are spelled out overtly.

(Collins 2007: 3)

According to current assumptions in the Minimalist Program, Merge applies freely Chomsky, Gallego and Ott (2019). It might also be the case for principles external to the language faculty, such as principles minimizing externalization, as illustrated below. The structures in (10) consist of two phases, Ph1 and Ph2. A syntactic constituent α is displaced (iM) from the Specifier of Ph1 to the Specifier of higher phase Ph2. Given PM, the lower copy of α is not pronounced ($\cancel{\alpha}$). However, we proposed that (10a) and (10b) may differ with respect to the timing of COS. The latter applies to the lower phase in (10a), and consequently only the lower F head is pronounced; whereas, in (10b) COS applies also at Ph1, and consequently both F heads are silent (<F>).

(10) a. $[_{Ph2}\ \alpha\ [\ \langle F\rangle\ \ [_{Ph1}\ \cancel{\alpha}\ \ [\ F\ \ \beta\ ]]]]$

b. $[_{Ph2}\ \alpha\ \ [\ \langle F\rangle\ \ [_{Ph1}\ \cancel{\alpha}\ \ [\ \langle F\rangle\ \beta\ ]]]]$

Given these assumptions, we provide derivations for complex cardinal numerals mainly focusing on English. The properties of additive and multiplicative complexes, including differences in the externalization of the conjunction, follow from an explanatory linguistic relying on SMT.

3. A Minimalist Approach

In this section, we provide further support for the derivation of complex cardinal numerals by Merge in accord with principles external to the language faculty. Before doing so, we discuss differences between language and arithmetic in this perspective, pointing to differences between Merge, the successor function, and the operations of Boolean algebra.

3.1. Language and Arithmetic

3.1.1. Merge/Successor Function

Complex cardinal numerals share core properties with natural numbers used in arithmetic, including the fact that they are infinite sets; but differ from them in their internal structure and variation, which suggests that they are constructed by the operations of I-language, rather than by mathematical operations, such as the successor function.

In arithmetic, the infinity of natural numbers can be represented as the infinite sequence: < ————-0-1-2-3————>. It is not possible to identify the greatest natural number, as it is not possible to identify the longest sentence. Neither can be described by enumeration. A recursive function can be used to generate them. In the theory of natural numbers, this is the case for the successor function, see (11), where 0 is a natural number, and each natural number has a successor, which is also a natural number. By this base case and recursive rule, it is possible to construct the set of all natural numbers.[5]

(11) a. 0 is in N.
b. If n is in N, then n+1 is in N.

Chomsky (2008: 139) hypothesizes that Merge can give rise to the successor function and defines counting in terms of Merge as follows. "Suppose that a language has the simplest possible lexicon: just one LI, call it "one". The application of Merge to the LI yields {one}, call it 'two'. Application of Merge to {one} yields {{one}}, call it "three". Etc." (12). Merge applied in this fashion yields the successor function, whereby every numerosity N has a unique successor.

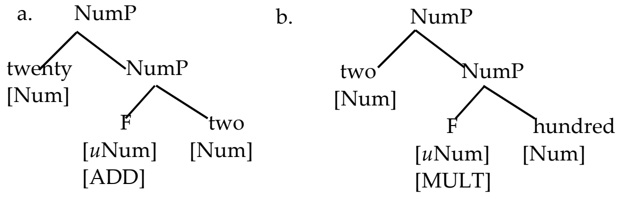
(12) Counting and the successor function
1= {LI}, 2= {LI, {LI}}, 3 = {LI, {LI, {LI}}}, . . .

A closer look at the properties of numerals in natural languages however reveals no trace of the successor function (Hiraiwa 2017) [21]. The hypothesis that unbounded Merge yields arithmetic is abandoned in Chomsky (2018). An evolutionary argument to this effect is discussed in Casares (2019) [22], and Guerrero et al (2020) [23] brings experimental support from language acquisition. See also Roeper (2022) [24] for discussion. In recent Minimalist work (Chomsky 2019, and 2021), the recursion of a syntactic constituent follows from its being placed in the WS. In this view, no operation or construction-specific stipulation is needed to derive the discrete infinity of language.

3.1.2. Merge/Boolean Algebra

According to Di Sciullo (2015), the internal minimal structure of complex cardinal numerals is derived by simplest Merge, applying in accord with factors external to the language faculty. In the structures in (13), from Di Sciullo (2015), NumP is headed by a functional category (F).[6] F does not contribute/project its label in additive complexes, see (13a), as it is the case, more generally in coordinate structures, (Chomsky 2013) [25]. This is also the case, according to our hypothesis, for the functional F head in multiplicative complexes, see (13b). In the structures in (13), F has valued and unvalued (*u*) features. The unvalued numeral feature ([*u*Num]) are valued in the derivation by a matching valued feature, ([Num]). The valued features include addition ([ADD]), multiplication ([MULT]), as well as division [DIV] and subtraction [SUB]. These features are also part of time counting expressions such as *ten thirty* [ADD] and [MULT], *a quarter to two* [DIV], *ten to five* [SUB], as discussed in Di Sciullo (2016) [26], 2017 [27], and Di Sciullo and Español-Echevarría (2017) [28], drawing evidence from several Romance languages and dialects thereof. The hypothesis that the finite set of arithmetic operations are reduced to a finite set of features in syntactic derivations is motivated on empirical grounds. This hypothesis is also independently motivated, as these features are part of other categories than complex cardinal numerals, as evidenced below. Furthermore, this hypothesis is attuned with Minimalist assumptions according to which valued features provide instructions to the external CI system for semantic interpretation, a point to which we come back below.

(13)



Our analysis, according to which complex cardinal numerals are headed by feature bearing functional category, is compatible with other analysis of complex cardinal numerals proposed within the Minimalist Program. For example, it is compatible with Stravou and Terzi's (2008) [29] analysis, according to which phi-feature [±PL] is in head position and enters into agreement with the NP it modifies. It is also compatible with Watanabe's (2010) [30] analysis, according to which numerals are licensed by the features [±singular] and [±augmented]. This proposal accommodates the dissociation of numerals from the plural morphology. Our analysis is also compatible with Scontras's (2022) [31] analysis, where the feature [CARD] (cardinal) heads NumP in cardinal numeral structures, and the feature [ORD] (ordinal) heads NumP in ordinal numeral structures. According to our analysis, however, both multiplicative and additive complexes are headed by a functional category with valued and unvalued features. The unvalued features are valued in the derivation by matching valued heads. Only valued features, including [Num], [ADD] and [MULT], are legible at the semantic interface, and provide instructions to the CI system for semantic interpretation. The hypothesis that semantic features are instructions to the external systems is in line with the feature-based interpretation of linguistic expressions in Generative Grammar, going back to Chomsky (1965). This hypothesis is also compatible with the view that the semantics of linguistic expressions is not truth-conditional, as in Pietroski (2018).

As mentioned previously, [ADD] and [MULT] are needed independently. They are part of the features of complex ordinal numerals, such as *twenty first* and *two hundredth*, as well other categories, including nouns and verbs, such as *to add*, *the addition*, *the sum*; *to multiply*, *the multiplication*, *the product*, whether they are part of expressions describing arithmetic operations, or not, as we illustrate in (14) and (15). The fact that [ADD] and [MULT] are part of the constitutive features of different categories provides additional support to the hypothesis that complex cardinal numerals are generated by I-language, and not by other cognitive systems, such as the ones involved for mathematics.

(14) a. Twenty plus one if twenty-one. / Add one to twenty. / The addition/sum of twenty and one is twenty-one.
b. Two times hundred is two hundred. / Multiply one and twenty. / The addition /sum of twenty and one is twenty-one.

(15) a. This table plus this bookshelf is a great match. / Add a conclusion to the paper. /The addition/sum of these arguments strongly supports this hypothesis.
b. This recipe needs to be read twice. / In the circumstances, the possibilities mul tiply by two. /They observed an uncontrolled multiplication of cells.

If complex cardinal numerals are subject to the computations of I-language, and not to mathematics, it comes as no surprise that the combination of simplex numerals does not follow from Boolean algebra. For example, according to the Commutative law, two numbers can be added or multiplied with the same result irrespective of their order, see (16). In order words, the interchanging of the order of the operands in a Boolean equation does not change its result. The examples in (17) with X = 2 and Y = 100, illustrate the commutative law for addition in (17a), and multiplication in (17b). The examples in (18), where X = *two* and Y = *hundred*, show that this is not the case for complex cardinal numerals.

(16) Commutative law
a. Addition: $X + Y = Y + X$
b. Multiplication: $X \bullet Y = Y \bullet X$

(17) a. $2 + 100 = 100 + 2 = 102$
b. $2 \bullet 100 = 100 \bullet 2 = 200$

(18) a. hundred two vs. two hundred (102 ≠ 200)
b. two hundred vs. hundred two (200 ≠ 102)

The fact that the combinatorial properties of cardinal numerals are not subject to Boolean algebra provides additional support for the domain specificity of language with respect to mathematics, including the generation of complex cardinal numerals by the computational procedure of I-language.

The semantic operator features we proposed, including [ADD] and [MULT] are not direct counterparts of the Boolean operators + and •. Evidence to this effect comes from the fact that algebraic operators must be overt in algebraic equations, whereas this is not necessarily the case for natural language operators. In additive complexes, the functional head associated with [ADD] can be silent in several languages, including English, French, Hausa, Igbo, Welsh and Arabic, reported in Hurford (2001) [32], Zabbal (2005) [33], Al-Bataineh and Branigan (2020) [34]. With multiplicative complexes, the functional Head is assumed to be silent in the language of the world. However, adopting our analysis, Al-Bataineh and Branigan (2020) provide evidence that [MULT] is externalized in Arabic multiplicative complexes. The derivation of additive and multiplicative complexes gives rise to interface asymmetries, whereby interpreted but unpronounced constituents are generated by Merge. This brings additional support for their generation by Merge applying in accord with principles of efficient computation.

3.1.3. Language Variation

In the Minimalist framework, I-language is an organic system that generates thoughts. Their externalization in E-language is subject to variation. There is a discrepancy between I-language and E-language, the latter predating the former in evolutionary terms. In this framework, there is no one-to-one relation between thoughts and their externalizations. Given this discrepancy, it is expected that parts of linguistic expressions are semantically interpreted without being externalized.

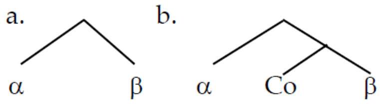
Complex cardinal numerals are not different from other expressions generated by I-language. They include categories that are interpreted without being pronounced. This is the case for the functional head in complex cardinal numerals. Several works provide empirical evidence that complex cardinal numerals in the world languages have grammatical properties, including phi-features, agreement, case, and other functional elements. For example, in Corsican, a dialect spoken in southern Italy, feminine gender [e] is part of multiplicative complexes, e.g., *dui-e centu* (two-FEM. hundred), *dui-e mila* (two-FEM. thousand). In Arabic, Case is part of multiplicative complexes. Complements are marked with genitive Case (*-i*), and the Case of the first constituent depends on the position of the numeral in the sentence: NOM (*-u*) in subject position, ACC (*-a*) in complement position, e.g., *I saw 400 men* (*arba-a* ACC), *the city of the 400 men* (*arba-i* GEN).[7]

In the following sections, we bring further support for the asymmetrical structures in (13). The functional F is replaced by Co, which we will take to be a conjunction encompassing both the coordinate conjunction and the preposition *of*. Evidence will be provided in support of an extended projection for Co. Sep-by-step derivations of additive and multiplicative complexes will be presented, and independently motivated.

3.2. Minimal Structures

Given SMT, Merge applies in accord with third factors of computational efficiency. Principles. According to our view, these principles can be subsumed under principles minimizing symmetry on the one hand, and principles minimizing externalization on the other. In this context, the minimal structure of complex cardinal numerals cannot be reduced to the symmetrical structure in (19a), where α and β, two numerals, are sisters. The asymmetrical structure in (19b), where α asymmetrically c-commands β, is available as early as possible in the derivation. We will take the functional category in (19b) to be a coordinator (Co) in additive and multiplicative complexes. Co is semantically interpreted but not externalized in some cases. Co is externalized as a coordinate conjunction in additive complexes and as the grammatical formative *of* in multiplicative complexes.

(19) a. b.



The position and the function of the coordinate conjunction and the preposition is the same in complex cardinal numerals. Both *and* and *of* share properties, as evidenced in English by the fact that *and* is also found in multiplicative complexes such as *hundreds and thousands* to express emphatically large but vague numerosity. The fact that they occur between the numerals in both additive and multiplicative complexes and do not precede or follow the numerals, see (20), (21), brings independent support to (19b).[8]

(20) a. hundred and two
b.*and hundred two
c *hundred two and

(21) a. thousands of millions
b.*of thousands of millions
c. *thousands millions of

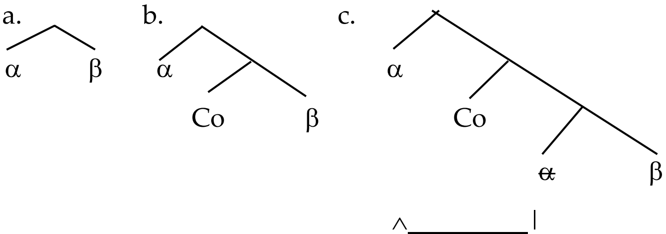
This regularity is also observed with simple coordinate NP/DPs. Namely, Co does not precede or follow both NP/DPs, but it appears between the two NP/DPs. According to Zwart (2000) [35], cross-linguistic generalizations emerge in the externalization of the coordinator's counterparts of English *and* in NP/DP-coordination in structures where Co appears only once. The generalization is that the coordinate conjunction invariably occurs between the two conjuncts. This indicates, according to Kayne (2020) [36], that the structure of coordination is asymmetrical. This is also the case for complex additive cardinal numerals, (22).

(22) a. NumP Co NumP
b. *Co NumP NumP
c. *NumP NumP Co

The question arises whether the derivation of complex cardinal numerals starts by externally merging α and β, where α and β are distinct NumPs, (24a), and then externally merging Co to the previously derived constituent, (23b), and finally internally merging Co to the previously derived constituent, (23c), see also (24). Another possibility would be to first Merge Co with one of the conjuncts, (25a), and externally merge the resulting structure with the other conjunct, (25b), see also (26). Assuming that Head Movement is not available with strictly Markovian derivations, the derivation in (23)/ (24) prevails over the one in (25)/(26).

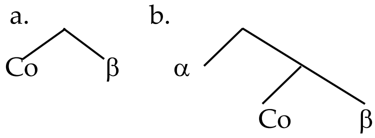
(23) a. eM (α, β): {α, β}
b. eM (Co, {α, β}): {Co, {α, β}}
c. iM (α, {Co, {α, β}}): {α, {Co, {α, β}}}

(24) a. b. c.



(25) a. eM (Co, β): {Co, β}
b. eM (α, {Co, β}): {α, {Co, β}}

(26) a. b.

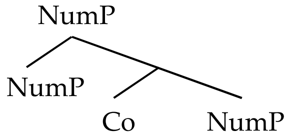


Furthermore, Co is externalized by the coordinate conjunction in additive cardinal numerals, (27), and by the preposition *of* in multiplicative structures in different languages, as we illustrated in (28) with French, Romanian and Italian. This also supports the hypothesis that a functional Co head intervenes between the two numerals in the minimal structure of complex cardinal numerals, as in (29).

(27) a. vingt-et-un (F)
"twenty and one"
b. douazeci si un(u) (R)
two ten and one
"twenty-one"

(28) a. centaines de milliers (F)
"hundreds of thousands"
b. centinaia di migliaia di milioni (I)
"hundreds of thousands of millions"

(29)



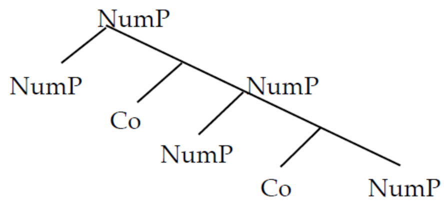
Summarizing, the asymmetrical structure in (29) is the minimal structure, which is part of the derivation of complex cardinal numerals, given SMT. They are also part of the derivations of morphological structures, as argued on independent grounds in Di Sciullo (2005) [37], (2014) [38], (2017), where morphological complexes are derived by the merger of minimal trees, where feature valuation takes place under Agree, and where valued features provide instructions to the CI system for semantic interpretation. The view that certain functional categories do not project their categorical features is echoed in Chomsky's (2013) for the coordinate conjunction. Languages vary with respect to the externalization of Co, which remains silent in some cases. In the next section, we argue that Co has an extended projection.

3.3. Extended Projection

The hypothesis that Co has an extended projection is substantiated on both theoretical and empirical grounds. Extended functional projections have been proposed in the nominal as well as in the verbal domain, as well as for functional categories, including prepositions and complementizer, as discussed in Rizzi and Cinque (2017). Like other functional categories, Co is a closed class category and is associated with different conceptual features. Empirical evidence that more than one Co head is externalized in multiplicative cardinal numerals comes from multiplicative complexes, such as (30) and (31). With composite cardinal numerals, including additive and multiplicative structure, such as (32), more than one overt and covert additive Co is part of the complex cardinal numeral. This suggests that Co has an extended projection, on a par with other functional categories, where Co does not provide/project its label, (33).

(30) a. hundreds of thousands of millions
b. hundreds of thousands of millions of trillions
(31) a. dizaines de centaines de milliers de millions (F)
b. dizaines de centaines de milliers de millions de trillions
(32) a. two million two hundred and thirty-two
b. two trillion two million two hundred and thirty-two

(33) ...



Co to have its own extended projection in complex cardinal numerals. This hypothesis is independently motivated for coordinate DPs/NPs in Di Sciullo (2020) [39], 2021 [40] based on the distribution and interpretation of the coordinate conjunction in Latin, Modern Greek, and English coordinate DPs/NPs, as summarized below.

Empirical evidence that conjunctions are associated to extended functional projections come from several languages, where a conjunction can be attached to each conjunct, as illustrated (34a) with Latin and in (34b) with Modern Greek.

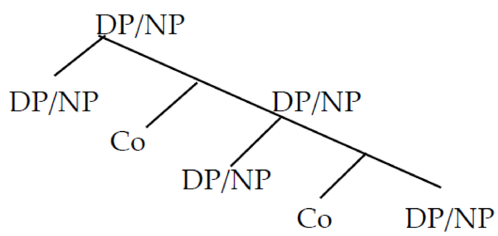
(34) a. et ego et Cicero meus flagitabit (L)
and I and Cicero my will-demand.1PL.
b. ke egho ke o Petros to thelume (MG)
and I and the Petros it want.1PL.
"both Peter and I want it" (Johannessen 1998: 91) [41]

English phrasal coordination may also include more than one coordinated conjunction, as in (35). In (35a) with emphatic stress, the coordinate conjunction can be interpreted as the distributive operator *both*, on a par with (35b). In (35c) the complex coordination structure is in the scope of the distributive operator, which indicates that the coordinate conjunction has an extended projection, where each occurrence of the conjunction occupies a different position. In (35a), the distributive interpretation of the conjunction is provided by a null distributive conjunction operator feature that asymmetrically c-commands it. In (35b), *both* spells out the distributive operator feature.

(35) a. John met and John and Mary.
b. John met both John and Mary.
c. John met both and John and Mary.

We hypothesize that Co is a functional head associated with an extended projection on a par with other functional categories. Each projection includes a position, which can host externally or internally merged constituents, and each extended conjunction projection head bares one conceptual operator feature, as schematized in (36), where Co does not provide/projects its label, see (36).

(36) ...



The extended projection of Co is needed in the derivations of complex NumPs and DPs, and other syntactic constituents to the interfaces. Derivational differences in the externalization of Co or lack thereof are discussed, as well as differences in semantic interpretation in the next sections.

3.4. Instructions to SM

Complex cardinal numerals include functional heads (Co) that can be pronounced or remain silent. This is the case in additive complexes, such as *hundred (and) one*. In multiplicative complexes, the functional head is not pronounced in definite cardinal numerals, such as *two hundred*, but can be pronounced in imprecise numerals, such as *hundreds (of) thousands*. In the following paragraphs, differences are derived in accord with SMT.

Additive cardinal numerals share syntactic properties with coordinate structures. They are islands with respect to extraction (Ross 1967) [42]. Their internal structure is asymmetrical (Munn 1987 [43], 1993 [44]; Kayne 1994 [45]), and they are derived by phases (Thiersch 1993 [46]). A derivation of complex cardinal in terms of Merge and feature valuation is proposed in Di Sciullo (2015). According to this analysis, Numerals (Num) merge with a functional head with valued as well as unvalued features (*u*Num) to be valued in the derivation. In additive and multiplicative numerals, Co project's higher functional structure. The latter is the locus for feature valuation giving rise to displacement of phrasal constituents in higher Specifier positions of Co's extended projection.

A convergent derivation for numerals such as *twenty-one*, where the coordinate conjunction (Co) is silent, proceeds as follows, see (37). At the first step of the derivation, a numeral and Co are e-merged, and the [*u*Num] feature of the conjunction is valued by the matching [Num] feature of the numeral, (37a). At the second step of the derivation, another numeral taken from Lex is externally merged to the previously derived constituent. The Condition on Spell-Out (COS) applies and Co is not pronounced $<\text{Co}_{[\cancel{u\text{Num}}]}>$, (37b). At the third step, $\text{Co}_{\{[u\text{Num}],\,[\text{ADD}]\}}$ taken from the lexicon is e-merged to the precious constituent, (37c). At the fourth step, $\text{NumP}_{[\text{Num}]}$ is internally merged to the previously derived constituent and values the [*u*Num] feature of $\text{Co}_{\{[\cancel{u\text{Num}}],[\text{ADD}]\}}$. Given PM, the copy of the displaced NumP is not pronounced. COS applies, and given that the Specifier is pronounced, the higher Co head is not pronounced $<\text{Co}_{\{[\cancel{u\text{Num}}],\,[\text{ADD}]\}}>$, see (37d). When Co is pronounced, as in *twenty and one*, PM and COS apply after the displacement of Num, and the lower Co is pronounced, see (38d).

(37) a. $[\text{Co}_{[\cancel{u\text{Num}}]}\ \text{NumP}_{[\text{Num}]}]$
b. $[\text{NumP}_{[\text{Num}]}\ [<\text{Co}_{\cancel{[u\text{Num}]}}>\ \text{NumP}_{[\text{Num}]}]]$ COS
c. $[\text{Co}_{\{[u\text{Num}],\,[\text{ADD}]\}}\ [\text{NumP}_{[\text{Num}]\}}\ [<\text{Co}_{[\cancel{u\text{Num}}]}>\ \text{NumP}_{[\text{Num}]}]]]$
d. $[\text{NumP}_{[\text{Num}]}\ [<\text{Co}_{\{[\cancel{u\text{Num}}],\,[\text{ADD}]\}}>\ [\cancel{\text{NumP}_{[\text{Num}]}}\ [<\text{Co}_{[\cancel{u\text{Num}}]}>\ \text{NumP}_{[\text{Num}]}]]]]$ PM, COS
(*twenty one*)

(38) a. $[\text{Co}_{[\cancel{u\text{Num}}]}\ \text{NumP}_{[\text{Num}]}]$
b. $[\text{NumP}_{[\text{Num}]}\ [\text{Co}_{\cancel{[u\text{Num}]}}\ \text{NumP}_{[\text{Num}]}]]$
c. $[\text{Co}_{\{\cancel{[u\text{Num}],}\,[\text{ADD}]\}}\ [\text{NumP}_{[\text{Num}]\}}\ [\text{Co}_{[\cancel{u\text{Num}}]}\ \text{NumP}_{[\text{Num}]}]]]$
d. $[\text{NumP}_{[\text{Num}]}\ [<\text{Co}_{\{[\cancel{u\text{Num}}],\,[\text{ADD}]\}}>\ [\cancel{\text{NumP}_{[\text{Num}]}}\ [\text{Co}_{[\cancel{u\text{Num}}]}\ \text{NumP}_{[\text{Num}]}]]]]$ PM, COS
and (*twenty and one*)

[MULT] can be externalized in multiplicative complexes in certain languages. In English, [MULT] is externalized as the grammatical formative *of* in imprecise numerals, such as *hundreds of thousands*, which is at least *two hundred thousand*. The difference in the externalization of Co in the case of multiplicative complexes such as *hundred thousand* and *hundreds of thousands* proceeds along the same lines as (37), (38) with respect to SMT, see (39), (40).

(39) $[\text{NumP}_{[\text{Num}]}\ [<\text{Co}_{\{[\cancel{u\text{Num}}],\,[\text{MULT}]\}}>\ [\cancel{\text{NumP}_{[\text{Num}]}}\ [<\text{Co}_{[\cancel{u\text{Num}}]}>\ \text{NumP}_{[\text{Num}]}]]]]$
[PM, COS [COS (*hundred thousand*)

(40) $[\text{NumP}_{[\text{Num}]}\ [<\text{Co}_{\{[\cancel{u\text{Num}}],\,[\text{MULT}]\}}>\ [\cancel{\text{NumP}_{[\text{Num}]}}\ [\text{Co}_{[\cancel{u\text{Num}}]}\ \text{NumP}_{[\text{Num}]}]]]]$
[PM, COS [COS of (*hundreds of thousands*)

The derivations in (37)–(38) and (39)–(40) rely on simplest Merge, applying in accord with principles of computational efficiency, minimizing symmetry and minimizing externalization. A difference in the timing of principles minimizing externalization, namely COS, derives the pronunciation or silence of Co.

The derivations above for multiplicative NumPs are consistent with those previously proposed for coordinate DPs and NPs (Di Sciullo 2014, 2021). Simple DP coordination, such as *Paul and Mary*, and multiple DPs coordination, such as *Paul and Mary*, *John, Paul and Mary* are derived by phases (Te Velde 2006 [47]). The pronunciation/silence of the Co-head follows from the displacement of the DP occupying the Specifier of the lower phase to the Specifier of the next higher phase, in conjunction with principles reducing complexity, minimizing symmetry and externalization, see (41). Similarly, with simple coordinate NPs, Co can be externalized, e.g., *syntax and semantics*, or remain covert, e.g., *syntax semantics*, see (42) and (43).

(41) [John$_{[D]}$ [$_{Co}$ <~~and$_{[uD]}$~~> [Paul$_{[D]}$ [$_{Co}$ <~~and$_{[uD]}$~~> [~~Paul$_{[D]}$~~ [$_{Co}$ and$_{[\cancel{uD}]}$ [Mary$_{[D]}$]]]]]]

(*John, Paul and Mary*)

(42) [syntax$_{[N]}$ [$_{Co}$ <~~and$_{[uN]}$~~> [~~syntax$_{[N]}$~~ [$_{Co}$ and$_{[\cancel{uN}]}$ [semantics$_{[N]}$]]]]]

(*syntax and semantics*)

(43) [syntax$_{[N]}$ [$_{Co}$ <~~and$_{[uN]}$~~> [~~syntax$_{[N]}$~~ [$_{Co}$ <and$_{[\cancel{uN}]}$> [semantics$_{[N]}$]]]]]

(*syntax semantics*)

The derivation of complex cardinal is independently motivated and provides a simple unified analysis for these structures, where the conjunction can be pronounced or remain silent. This follows from a difference in the timing of principles external to the language faculty minimizing externalization. Thus, the unified account provided by our analysis rests on the assumption that, as it is the case for Merge, principles of efficient computation also apply freely. The next section explores differences in the semantic features of Co in complex cardinal numerals.

3.5. Instructions to CI

I-language generates feature-bearing hierarchical structures. The valued features are instructions provided to the CI system to compute semantic interpretation compositionally, based on the semantic features of their parts and their syntactic organization. This is the case for the categorial feature [Num], as well as for the semantic operator features including [ADD] and [MULT]. In the case of additive numerals, such as *twenty-two*, the instruction is to interpret the complex numeral as the result of adding one numeral to the other. In the case of multiplicative numerals, such as *two hundred*, the instruction is to interpret the complex numeral as the result of multiplying one numeral with the other. Thus, the features [ADD] and [MULT] provide instructions to CI system for the compositional semantic interpretation of complex numerals based on the interpretation of their parts and their syntactic organization.

Differences in semantic interpretation are observed when the coordinator is silent as opposed to when it is pronounced. Thus is the case in English and other languages, including the Romance languages, where the coordinator can be pronounced or remain silent in additive complexes, as in *twenty (and) one*. The examples (44) and (45) illustrate differences with respect to semantic agreement, and the examples (46) and (47) illustrate differences with respect to scope of distributive vs. selective operators. Differences in interpretation can also be observed with multiplicative complex cardinal numerals, such as *hundred thousand* and *hundreds of thousands*.[9]

(44) a. Twenty-one is a natural number.
     b. *Twenty and one is a natural number.
(45) a. *Twenty-one are natural numbers.
     b. Twenty and one are natural numbers.
(46) a. Both twenty and one are natural numbers.

b. *Both twenty-one are natural numbers.
(47) a. *Only twenty and one is a natural number.
b. Only twenty-one is a natural number.

Similar differences in agreement and interpretation are observed in Di Sciullo (2021) with binominals, such as *lion leopard* as opposed to co-compounds, such as *mother and child*. *Lion leopard* is interpreted as one complex entity, which shares part of the semantics of each of its nominal constituents. *Mother and child* is interpreted as a group consisting of a mother and a child. The examples in (48) and (49) illustrate differences with respect to semantic agreement, and the examples in (50) and (51) illustrate differences with respect to scope of distributive vs. selective operators.

(48) a. The mother and child are/*is in the garden.
b. The lion-leopard *are/is in the garden.
(49) a. This mother and child are often together.
b. ≈This lion-leopard are often together.
(50) a. Both mother and child are in the garden.
b. ≠Both lion-leopard are in the garden.
(51) a. ≠Only the mother and child is in the garden.
b. Only the lion-leopard is in the garden.

The semantic features of Co in complex cardinal numerals and in coordinate NPs are not identical. For example, *twenty-two* is the sum of twenty and two, and *two hundred* is a multiple of *one hundred*. However, the interpretation of *mother and child* is not the sum of a mother and a child, it is a group consisting of a mother and a child. Likewise, *lion leopard* is not a multiple of leopard, it is an intersection of lion and leopard. [MULT] and [ADD] are part of the feature structure of cardinal numerals, given the properties of numerals. These features share properties but are not identical with the intersection [I] and the group [G] features, which are part of the feature structure of conjoined NPs. As proposed in Di Sciullo (2021), the operator features associated with the coordinate conjunction are part of the extended projection of the conjunction, providing differences in the interpretation of coordinate NP structures, where the list feature [L] occupies a lower position than the intersection feature [I], which itself occupies a lower feature than the Group [G] feature, which itself occupies a lower position than the distributive [D] feature in the extended projection of Co: [$\text{Co}_{[D]}$ [$\text{Co}_{[G]}$ [$\text{Co}_{[I]}$ [$\text{Co}_{[L]}$ [Co. It might be the case that operator features, such as [ADD] and [MULT], occupy higher regions in the extended projection or sub-projections of Co and emerge at later stage in language acquisition. These questions will be left to further research.

3.6. Section Summary

Given SMT, complex cardinal numerals are derived by Merge, which applies in accord with principles of efficient computation. Their minimal asymmetrical structure is derived by eM applying in accord with principles minimizing symmetry: a numeral phase is first merged with a functional head Co before a second numeral is eM to the previously derived constituent. Co has an extended projection, and the numeral constituents are iM to higher positions in that projection for feature valuation. Differences in the externalization of Co at SM follow from the timing of principles minimizing externalization. The extended projection of Co is the locus of operator features, [ADD] and [MULT], which provide instructions to the CI system for semantic interpretation of complex cardinal numerals.

4. Previous Analyses

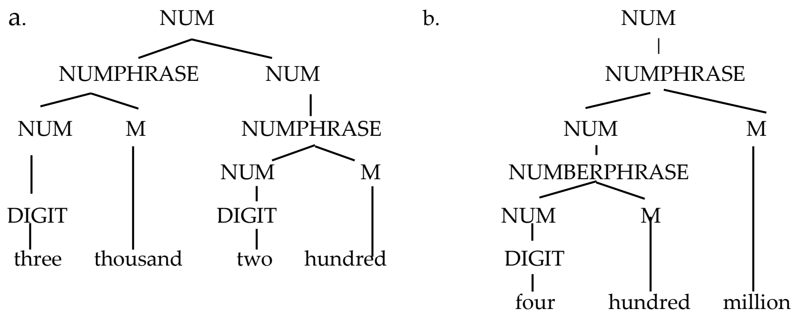
Different derivations of complex cardinal numerals have been proposed through the development of Generative Grammar, which differ from the ones detailed in this article. In this section, we briefly discuss Hurford (1975, *et seq.*) [48] and Ionin and Matushansky (2006 et seq.) [49] [50] analysis.

4.1. Phrase Structure Rules

Hurford's analysis relies on the Standard Theory (Chomsky 1965). [51] Complex cardinal numerals are derived by phrase structure rules, which are part of the rewriting rules of the Base component of the grammar. In phrase structure rules, a syntactic category occurring on the left side of the rewrite symbol (→) is rewritten as one or more categorial symbols to the right of the rewrite symbol. According to Hurford (2001):19786 [52],“ most syntactic numerals in most languages can be represented from a universal schema of just two simple phrase structure rules”, see (52).

(52) a. NUM → {DIGIT
NUMPHRASE (NUM)}
b. NUMPHRASE → NUM M

In (52), NUM is the syntactic category of numerals, which is the set of possible numeral expressions in a language. DIGIT is the category of any single numeral up to the value of the base number (e.g., English *one*, *two*, . . . , *nine*). M is the category of noun-like numeral forms used as a multiplicative base (e.g., English *-ly*, *thousand*, and *billion*). These rules generate the structures of structures such as the ones in (53).

(53) a.


According to this analysis, additive and multiplicative complexes are generated by different category-specific rules. The latter describes the constituent structure and the linear order of their overt constituents. However, these do not account for unpronounced and semantically interpreted constituents. Instead, two semantic interpretative rules are postulated: “If a numeral has two immediate constituents (i.e., is not just a single word) the value of the whole is calculated by adding the values of the constituents; thus, *sixty-four* means 60 + 4. If a numeral phrase (as distinct from a numeral) has two immediate constituents, then the value of the whole is calculated by multiplying the values of the constituents; thus, *two hundred* means 2 × 100.” (Hurford 1975: 10758). However, as discussed below, the features associated with unpronounced and interpreted heads are syntactically active in the derivation of complex cardinal numerals. Furthermore, the rule in (52a) includes choice points, in the form of disjunction ({ }) and optionality (), which add complexity to the derivations, (54).

(54) a. NUM → DIGIT
b. NUM → NUMPHRASE
c. NUM → NUMPHRASE NUM
d. NUMPHRASE → NUM M

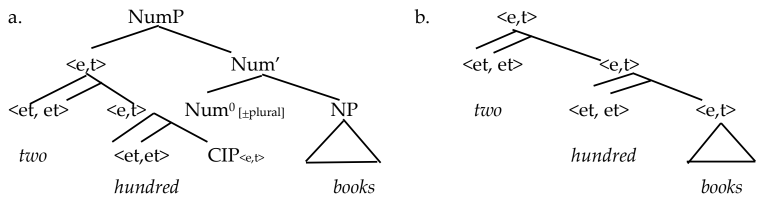
Hurtford's analysis is an important step in the cross-linguistic description of complex cardinal numerals in the framework of the Standard Theory. While valuable with respect to its descriptive capacity, it requires several construction-specific rules to generate the structure of complex cardinal numerals, which reduces its explanatory capacity. Phrases-structure rule were subsumed under a universal phrase structure rule schemata in Government and Binding Theory and further subsumed in the unique operation in the Minimalist Program.

4.2. Regular Syntactic Rules and Type Theoretic Semantics

Ionin and Matushansky's (2006) analysis focuses on the derivation and interpretation of cardinal numerals contained in nominal expressions, such as *two hundred books* and *two hundred and two books*.

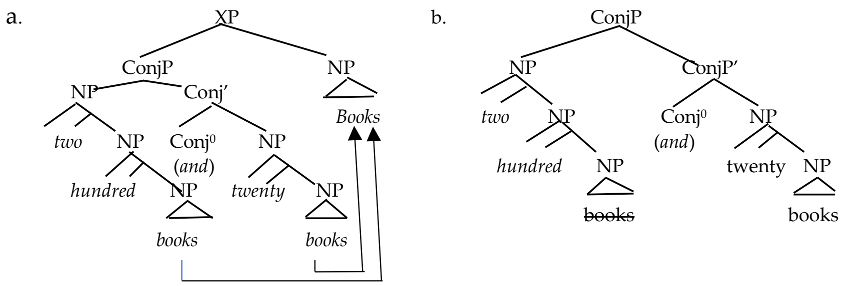
Complex cardinals are constructed by regular syntactic rules, such as complementation and coordination according to the X-bar phrase structure rule schemata (Chomsky 1981). According to X-bar Theory, a syntactic projection of a first syntactic layer including a Head and its Complement, and a second layer consisting of a specifier and the previously derived Head-Complement structure. As we mention in footnote 1, while X-bar Theory contributed to simplify the Standard Theory, it has been eliminated in the Minimalist Program.

The proposed semantic interpretation of complex cardinal numerals relies on type-theoretical semantics. Simplex cardinals to be associated with the semantic type of modifiers <<e,t>, <e,t>>. that is, elements of the set of functions from entities to truth-values. They combine with <e,t> categories and yield <e,t> categories, see (55). Set theoretic semantics is used to derive the semantics of complex cardinals from the semantic type of their parts and their syntactic organization (Heim, and Kratzer (1998) [53]. See Bartsch (1973) [54], Chierchia (1985) [55], Landman (2003) [56], Rothstein (2017) [57], for the semantics of complex cardinal numerals.

(55)


Considering the syntactic derivation of multiplicative complexes, Ionin and Matushansky suggest that different languages may use different means to construct complex cardinal numerals. For the structure in (55a) is proposed for languages such as English, which do not have overt classifiers (CLP). The structure in (55b), is proposed for languages such as Mandarin Chinese with a classifier system, where nouns must generally appear with a classifier to be modified by a numeral. The structure in (55b) is proposed for languages where number morphology is present in cardinals contained in NPs. The Num^0 head is associated with phi-features, [±Plur], which play a role in the agreement relation between the numeral complex in Specifier position of NumP and the nominal constituent in the Complement position. See Kayne (2016) [58] and Cinque (2022) [59] for the syntactic projection of numeral classifiers. See Krifka (1995) [60] and Chierchia (1998) [61] for different approaches to numeral classifiers.

For additive complexes contained in nominal expressions, two derivations are proposed. According to Ionin and Matushansky (2006: 340-341) there are two ways in which *two hundred books and twenty books* could be converted into *two hundred and twenty books*: (i) right-node raising of the lexical NP *books*, (56a), and (ii) PF-deletion of that NP in the first conjunct, (56b). Languages differ whether one or the other strategy is preferred, and some languages use both.

(56)


The pronunciation or silence of the coordinate conjunction does not follow from the derivations. Instead, Ionin and Matushansky (2006: 339) appeal to the notion of *asynthetic* *coordination*, which is taken to be a type of coordination where no coordinator is present, as in *twenty-two*. They point to the fact that synthetic and asynthetic coordination are truth conditionally equivalent, and they illustrate this with the examples in (56) and (57) from the domain of measurement, also discussed in Gawron (2002) [62]. Similar examples come from time counting expressions, as we illustrated in (58). However, from a naturalistic perspective, the semantic interpretation of linguistic expressions is not truth conditional (Pietroski 2005 [63], 2018 [64]; Chomsky, 2019, 2021).

(56) a. six feet six inches of finest silk
b. six feet and six inches of finest silk
(57) a. two dollars (and) seventy-five cents
b. two dollars (and) seventy-five
(58) a. une heure (et) trente (F)
one hour (and) thirty
"one thirty"
b. une heure (et) trente minutes
one hour (and) thirty minutes
"one thirty"

Ionin and Matushansky's analysis is simpler, thus more explanatory, than Hurford's since no phrase-structure specific rules are used in the derivations. Nonetheless, X-bar Theory, and rightward displacement rules, such as right node raising, postulated in previous stage of syntactic theory, are no longer part of Minimalist syntax. Furthermore, different structures are postulated for additive complexes and modification complexes, and the externalization of the conjunction in additive complexes does not follow from the derivations. Putting aside ongoing research on no truth-conditional semantics (Pietrosky 2005, 2018), Ionin and Matushansky's derivations contrasts from those proposed in this article, whereby only one operation, Merge, derives both additive and multiplicative complexes, and where the pronunciation or silence of the coordinate conjunction in additive complexes follows from the timing of principles minimizing externalization external to I-Language. The derivations provide instructions to the CI system for semantic interpretation are independently motivated for coordinate NPs and DPs.

4.3. Section Summary

Both Hurtford's and Ionin and Matushansky's analyses provide important insights on the properties of complex cardinal numerals. However, in the case of multiplicative complexes, no functional head intervene between the numeral constituents, which does not account for the presence of a preposition in large numerals with vague interpretation in English and in the Romance languages, as well as the presence of multiplicative morphology in numerals with precise interpretation in languages such as Arabic, in Al-Bataineh and Branigan (2020). Furthermore, in additive complexes, the presence or absence of the coordinate conjunction does not follow form the syntactic derivation. Moreover, both analyses rely on operations that have been questioned and eliminated through the development of Generative Grammar.

By reducing the computational procedure of I-language to its minimum and appealing to principles external to the language faculty, new explanatory insight may emerge on the derivation on complex cardinal numerals and the instruction they provide for their interpretation.

5. Conclusions

The empirical problem we aimed to solve is to derive the presence or absence of a functional category, between the numerical constituents in additive and multiplicative complexes. An explanatory solution to this problem would be the one which is both descriptively adequate and does neither involve the notion of optionality or construction specific rules. The proposed derivations are not motivated by the position of the constituents of complex cardinal numerals, traditionally referred to as the digit and the base, but on their structural asymmetry. According to our proposal, numeral constituents are externally merged to a functional category, and internally merged in higher position of its functional extended projection. Given third factor principles minimizing externalization, the externalization of the functional head or lack thereof, is derived. Given third factor principles minimizing symmetry, the unvalued feature of the functional head is valued in a higher position of its extended projection, which also provides featural instructions to the CI system for their interpretation.

According to the proposed analysis, complex cardinal numerals are generated by Merge, applying in accord with principles of efficient computation. Their derivations give rise to extended projections hosting formal and semantic operator features. Their asymmetrical structure and the interpretability of their parts is also derived in accord with principles minimizing symmetry and externalization. Because of the simplicity of the computational procedure of I-language, unified derivations can be provided for NumPs, DP and NPs. This brings support to the facilitating role of the SMT in the formulation of explanatory theories of I-language.

Merge generates the fine asymmetrical structure of complex cardinal numerals given its genetically determined properties. This is not the case for mathematical computation, where copies or displaced constituents and other unexpressed but interpreted constituents are not available. It thus comes as no surprise that the laws of Boolean logic do not apply in the derivation of complex cardinal numerals. Furthermore, the instructions provided by Merge to the external systems for their interpretation rely on independently needed formal features, such as [*u*Num], and substantive features, including [MULT] and [ADD]. The position of the latter in the extended projection of Co, and their emergence in the growth of language in the child is open to further inquiry.

The SMT enables simple working hypotheses to be formulated and validated on experimental grounds, both in corpus-based language acquisition studies and in neurosciences. For example, Amalric and Dehaene (2019) [65] reports findings indicating that the activation of language areas correlates with syntactic complexity, not with mathematical content. See also Friederici (2020) [66] for neuroimaging results suggesting an invariant neurobiological basis for Merge. Such results further support the domain specificity of language with respect to mathematics. Current works in language acquisition indicate that complex functional sequences, such as the one postulated in Syntactic Cartography develop gradually in the child. For example, based on a corpus of natural productions, current research in this area indicates that the lower parts of syntactic tree are acquired first (Friedmann et al. 2021) [67]. This suggests that the language acquisition path is sensitive to the geometry of syntactic trees as well as to the gradual installment of principles of computational efficiency external to the language faculty. These results bring additional support to the theory of I-language and the facilitating role of SMT explored in the Minimalist Program.

Funding: This work has been supported in part by funding from the Social Sciences and Humanities Research Council of Canada to the Major Collaborative Research on Interface Asymmetries, grant number 214-2003-1003, and by funding from FQRSC for the structuring research on Dynamic Interfaces, grant number 179523.

Conflicts of Interest: The authors declare no conflict of interest.

Notes

1 In the Minimalist Program (Chomsky 1995 *et seq.*), category-specific rules, postulated in previous stages of the theory of Generative Grammar are subsumed under Merge. This includes phrase structure rules (Chomsky 1965), and phrase structure rule schemata (X-bar Theory), as well as transformations, such as Move NP, which applies for example in the derivations of passive structures, and Move-wh, which applies for example in the derivation of questions in the Government and Binding model (Chomsky 1981), as well as Head Movement, which displaces a syntactic Head to a higher Head position in a syntactic derivation. Specific constraints on the application of these rules, such as the Head Movement Constraint, the Strict Cycle, the Extension Condition, now follow from the strictly Markovian nature of syntactic derivations. See Chomsky (2019) for discussion.

2 A further distinction between Set-Merge and Pair-Merge was introduced in Chomsky (2000), to capture argument-adjunct asymmetries. Set-Merge derives unordered sets, {x,y} and yields the symmetrical property of complementation (arguments),

while pair-Merge derives ordered sets, <x,y> and yields the asymmetric property of adjunction (modifiers). The distinction between Set-Merge and Pair-Merge gives rise to issues, which are beyond the scope of this article. See for example, Omune, J. 2015. Immediate-local Merge as, Pair-Merge. *Coyote Papers*. University of Arizona Circle 22:12–21. Oseki, Y. 2015. Eliminating Pair-Merge. Proceedings of the 32nd West Coast Conference on Formal Linguistics: 303–312. Somerville, MA: Cascadilla Proceedings Project. Somerville, MA, USA. Kitahara, H., M. Nomura, M. Oishi. J. Omune. 2019. Pair-Merge Under Merge. (Chomsky 2019, 2021).

[3] AGREE is the operation responsible for moving feature values from one element to another, and thus provides a tool to model dependencies between syntactic constituents in the structure they are part of. Syntactic constituents are associated with valued ([F]) and unvalued ([*u*F]) features. Unvalued features must be valued before the interfaces to satisfy Full Interpretation. Feature valuation between a probe and a goal is done via minimal search, where the probe sister-contains its goal, and according to our view, also falls into third factor principles external to the language faculty.

[4] According to Derivation by Phases, a phase is a unit of syntactic computation that exhibits independence at the interfaces. Phases are transferred to the interfaces when the next higher phase is completed. Phase Theory subsumes several constraints on syntactic derivations proposed in earlier stages of Generative Grammar, and according to our view, also falls into principles external to the language faculty.

[5] See B. H. Partee, Alice ter Meulen and R.E. Wall. 1993 *Mathematical Methods in Linguistics*. Studies in Linguistics and Philosophy. Dordrecht, Kluwer, for discussion on proof theoretical as well as declarative constructions of the set of natural numbers. I thank an anonymous reviewer for this reminder.

[6] NumP is a syntactic projection standing for numeral phrase. NumP is located between DP and NP in the extended nominal projection: as discussed in Cinque (2005), Rizzi and Cinque (2016), and Cinque (2022) among other works.

[7] Many thanks to Mohamed Guerssel for providing examples of multiplicative complexes in Arabic.

[8] As mentioned by an anonymous reviewer, the fact that the position and function of conjunctions, such as *and*, and prepositions, such as *of*, is the same in numeral structures is not to be unexpected. From our perspective, these elements are both functional heads, thus they occupy the same structural position. The coordinate conjunction does not project its label, this can also be the case for the grammatical formative *of*. Like prepositions, and other functional categories, conjunctions have extended projections, see Section 3.3, as well as Di Sciullo (2020) and (2021) for discussion.

[9] For example, one hundred thousand is/*are a natural number vs. hundreds of thousands *is/are (a) natural numbers.

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