Design and Dispersion Calibration of Direct-Vision Push-Broom Compressive Double-Amici-Prism Hyperspectral Imager

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Abstract: The design and calibration of the dispersive device in a hyperspectral imager significantly affect the performance of hyperspectral imaging, especially the spectral accuracy. To achieve high-accuracy hyperspectral imaging over the visible band, firstly, the geometric and dispersive parameters of the double Amici prism (DAP) that serves as a dispersive device in the direct-vision push-broom compressive hyperspectral imager (PBCHI) are designed and optimized; secondly, a calibration method based on the numerical calculation of the DAP model is put forward, which can turn the conventional pixel-wise dispersive shift calibration by a monochromator into a group of numerical calculations; lastly, a PBCHI prototype is built to test the performances of the designed and calibrated DAP and the hyperspectral imager. The calibration experiments demonstrate that the mean squared error (MSE) of the dispersive pixel shifts calibrated by the proposed numerical method is 0.1774, which indicates the calibration result of the proposed method is consistent with the directly calibrated result. Furthermore, after this numerical calculation, the spectral signatures of the reconstructed cubes of the DAP-based PBCHI system show consistency with the ground truth. This work will benefit the design and calibration of the DAP-based hyperspectral imager.

Keywords: double Amici prism; dispersion calibration; hyperspectral imaging

1. Introduction

With the superior characteristics of high spectral and spatial resolution, the traditional slit-based hyperspectral imagers have become significant instruments for many applications, including ecology [1–4], geology [5,6], agriculture [7–10], military [11,12], and so on. However, the widespread usages of the traditional slit-based hyperspectral imager are still limited by the well-known annoying issues, such as the enormous quantity of the captured data, which makes transmission and storage overloaded, and the contradiction between the system resolution and the signal-to-noise ratio. Fortunately, the compressive hyperspectral imagers (CHIs) [13–21] were developed. In a typical CHI system, a three-dimensional hyperspectral scene, i.e., hyperspectral data cube, can be modulated by an encoding mask and then projected onto a two-dimensional detector array to generate a spatio-spectral multiplexed measurement [22,23], which can significantly reduce the data volume and enlarge the light flux; after that, the hyperspectral data cube can be reconstructed via various reconstruction algorithms [24–26].

Gratings and prisms are the most commonly used dispersive elements in a CHI system. Generally, the gratings are distinguished with high performance in linear dispersion; however, they suffer from low energy efficiency, complicated fabrication processes, high cost, and high-order harmonic distortion within broadband [27]. On the contrary, the prisms exhibit high optical efficiency, low fabrication complexity, low cost, and large bandwidth but
large nonlinear dispersion [28,29] that leads to extra complexity and incompact layout. An abundance of efforts have been exploited to figure out the nonlinearity and deflection issues of prisms, among which the double Amici prisms (DAP) [30–32], with properties of nearly linear dispersion and direct-vision, have been widely recognized as a versatile solution for the prism-based spectral imaging systems. In the process of hyperspectral imaging, a rigorous calibration strategy for DAP is necessary to improve the quality of spectral image estimates. The procedure was performed to capture multiframe monochromatic images of the encoding mask at different wavelengths in 1 nm increments within the bandpass of the system [20]. Obviously, if the bandpass or the focal length of the lens of the push-broom compressive hyperspectral imager (PBCHI) system is changed, the calibration will need to be repeated, which will complicate the calibration of the DAP.

Herein, to satisfy the requirements of hyperspectral imaging in the visible bands, with respect to the direct-vision PBCHI, the geometric and dispersive parameters of DAP are, firstly, designed and optimized; secondly, a calibration method, which is based on the numerical calculation of the DAP model rather than the pixel-wise direct calibration by a monochromator, is put forward to simplify the calibration process and ensure the calibration accuracy; lastly, a DAP-based PBCHI prototype is built, and a series of experiments are conducted to test the proposed calibration method and the performances of the designed and calibrated DAP.

2. Design of Double Amici Prism and Push-Broom Compressive Hyperspectral Imager

For the common single prisms, a certain input collimated light beam will be dispersed with different spectral resolutions over the visible (VIS) region (400 nm–700 nm); the difference of spectral resolutions between long-wave and short-wave is usually more than one order of magnitude. Therefore, the single prisms are not actually suitable for the push-broom compressive hyperspectral imager (PBCHI) system in which the hyperspectral cube will be very difficult to be resolved accurately. Nevertheless, the dispersion of the double Amici prism (DAP) is more linear, which makes them considered as suitable dispersive devices for PBCHI systems.

A DAP and a corresponding PBCHI prototype were designed and fabricated as shown in Figure 1. Figure 1a represents a ray tracing through a DAP, Figure 1b shows our fabricated DAP, and Figure 1c displays the direct-vision PBCHI system. In our PBCHI system, the slit in the conventional slit-based hyperspectral imager was replaced by a 2D random encoding mask. The PBCHI prototype was mainly composed of six parts including the imaging lens (L1), an encoding mask, the relay lens (L2) (1:1 imaging), the dispersion device (our fabricated DAP), the relay lens (L3) (1:1 imaging), and the imaging sensor. In more detail, the encoding mask was set at the primary imaging plane of L1 and the front focal plane of L2, our designed DAP as the dispersive device was located between L2 and L3, the imaging sensor was set at the back focal plane of L3, and the encoding mask was imaging 1:1 on the image sensor. A hyperspectral target scene was, firstly, modulated by a binary random matrix encoding mask; then, the modulated data cube was dispersed by our designed DAP and projected onto a detector array to generate spatio-spectral multiplexed measurements.

For a DAP, the angles follow the modern sign convention [33] as shown in Figure 1a. Based on the ray tracing, the refraction of a ray with a wavelength of λ through a DAP is given by applying Snell’s law at each interface:

\[
\begin{align*}
  i_1(\lambda) &= i_0(\lambda) - \beta_1, \\
  i_2(\lambda) &= r_1(\lambda) - \alpha_1, \\
  i_3(\lambda) &= r_2(\lambda) - \alpha_2, \\
  i_4(\lambda) &= r_3(\lambda) - \alpha_1, \\
  i_5(\lambda) &= r_4(\lambda) - \beta_1
\end{align*}
\]

\[
\begin{align*}
  r_1(\lambda) &= \arcsin\left(\frac{n_1 \times \sin(i_1(\lambda))}{n_2}\right), \\
  r_2(\lambda) &= \arcsin\left(\frac{n_2 \times \sin(i_2(\lambda))}{n_3}\right), \\
  r_3(\lambda) &= \arcsin\left(\frac{n_3 \times \sin(i_3(\lambda))}{n_4}\right), \\
  r_4(\lambda) &= \arcsin\left(\frac{n_4 \times \sin(i_4(\lambda))}{n_5}\right)
\end{align*}
\]
Here we assume that the optical axis is parallel to the prism base shown as the red dashed line in Figure 1a, $\alpha_1$ is the apex angle of Glass1 and Glass3, $\alpha_2$ is the apex angle of Glass2, and Glass1 and Glass3 are identical and symmetrical about Glass2. By making a vertical line from the apex angle of Glass1 to divide $\alpha_1$ into $\beta_1$ and $\gamma_1$, where $\beta_1 = -\alpha_1 - \frac{\alpha_2}{2}$, and combining Equation (1), the deflection angle $\delta(\lambda)$ of a light beam with a wavelength of $\lambda$ through the DAP can be calculated by

$$\delta(\lambda) = i_0(\lambda) - i_5(\lambda) = i_0(\lambda) - r_4(\lambda) + \beta_1. \quad (2)$$

The difference in deviation angle between the two extreme wavelengths transmitted by the prism is the dispersion angle $\Delta$:

$$\Delta = \delta(\lambda_{\text{max}}) - \delta(\lambda_{\text{min}}) = r_4(\lambda_{\text{max}}) - r_4(\lambda_{\text{min}}), \quad (3)$$

Here, $\lambda_{\text{min}} = 400$ nm, $\lambda_{\text{max}} = 700$ nm.

Usually, the designed parameters of the DAP depend on the requirements of the spectral band numbers, the imaging sensor specifications, and some other optical elements inside of the systems. The dispersion can also be expressed as:

$$\Delta = \arctan \frac{|d(\lambda_{\text{max}})|}{f} + \arctan \frac{|d(\lambda_{\text{min}})|}{f}, \quad (4)$$

where $|d(\lambda_{\text{max}})|$ denotes the distance between the convergence point of wavelength $\lambda_{\text{max}} = 700$ nm and that of the central wavelength $\lambda_0 = 550$ nm on the imaging sensor, and $|d(\lambda_{\text{min}})|$ denotes the distance between the convergence point of wavelength $\lambda_{\text{min}} = 400$ nm and that of the central wavelength $\lambda_0 = 550$ nm on the imaging sensor. Here, $f = 50$ mm represents the focal length of $L_3$. To meet the spectral resolution requirements for achieving hyperspectral imaging in all VIS light bands, the dispersion is calculated as $\Delta \geq 1.13^\circ$ by combining Equation (4) with the system parameters of imaging sensor pixel size $p = 5.5$ $\mu$m and a spectral band number of 60. Usually, a higher spectral resolution can be achieved at a
larger dispersion $\Delta$; however, as an oversized dispersion, $\Delta$ will cause spatial and spectral distortion, so the dispersion $\Delta$ should not be set too large. Therefore, $\Delta = 1.2^\circ$ is chosen as the dispersion angle of our designed DAP.

The linearity of the dispersion is another key characteristic of the designed parameters of the DAP, which can be characterized by standard deviation ($SD$) and spectral sampling rate ($SSR$) as follows:

\[
SD = \sqrt{\frac{\sum_{i=1}^{n} \left( \left| \frac{d\delta(\lambda)}{d\lambda} \right| - \left| \frac{d\delta(\lambda)}{d\lambda} \right|_{\min} \right)^2}{n}},
\]

(5)

\[
SSR = \max \left\{ \frac{\left| \frac{d\delta(\lambda)}{d\lambda} \right|}{\min \left\{ \left| \frac{d\delta(\lambda)}{d\lambda} \right| \right\}} \right\},
\]

(6)

where the gradient of the deviation angle $\frac{d\delta(\lambda)}{d\lambda}$ gives the spectral dispersion per unit wavelength and $n$ represents the number of bands between 400 nm and 700 nm by step of 0.1 nm, i.e., $n = 3000$ for the design. Generally, smaller indices of $SD$ and $SSR$ imply better linearity of the DAP, which can be minimized during the design process. For the designed goal of $\delta(550 \text{ nm}) = 0$ and $\Delta = 1.2^\circ$, i.e., a direct-vision wavelength of 550 nm and a dispersion angle of $1.2^\circ$, we input the parameters of all glasses listed in a standard optical materials catalog, supplied by CDGM Glass Company and SCHOTT, into the above equations to achieve the optimization of the glass group. As a result, the configuration parameters of the designed DAP are listed in Table 1, which achieve the best performance with the most linear dispersion and the most compact layout.

Table 1. Configuration lists of the designed DAPs for the best performance. ($\delta(550) = 0, \Delta = 1.2^\circ$, $i_0(\lambda) = 0$, $400 \text{ nm} \leq \lambda \leq 700 \text{ nm}$).

<table>
<thead>
<tr>
<th>Configurations</th>
<th>Glass1</th>
<th>Glass2</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\Delta$</th>
<th>SD ($10^5$)</th>
<th>SSR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>H-LAK54</td>
<td>H-FK95N</td>
<td>-27.59</td>
<td>114.23</td>
<td>1.2</td>
<td>2.6</td>
<td>5.86</td>
</tr>
<tr>
<td>2</td>
<td>H-FK95N</td>
<td>H-LAK54</td>
<td>54.76</td>
<td>-62.16</td>
<td>1.2</td>
<td>2.63</td>
<td>5.94</td>
</tr>
<tr>
<td>3</td>
<td>N-BK7</td>
<td>H-LAK54</td>
<td>74.09</td>
<td>-93.65</td>
<td>1.2</td>
<td>2.87</td>
<td>7.17</td>
</tr>
<tr>
<td>4</td>
<td>F2</td>
<td>H-FK95N</td>
<td>-21.4</td>
<td>63.16</td>
<td>1.2</td>
<td>3.14</td>
<td>7.69</td>
</tr>
<tr>
<td>5</td>
<td>H-FK95N</td>
<td>F2</td>
<td>31.76</td>
<td>-43.94</td>
<td>1.2</td>
<td>3.16</td>
<td>7.76</td>
</tr>
<tr>
<td>6</td>
<td>SF10</td>
<td>H-FK95N</td>
<td>-12.8</td>
<td>43.8</td>
<td>1.2</td>
<td>3.51</td>
<td>8.77</td>
</tr>
<tr>
<td>7</td>
<td>H-FK95N</td>
<td>SF10</td>
<td>21.89</td>
<td>-25.98</td>
<td>1.2</td>
<td>3.52</td>
<td>8.82</td>
</tr>
<tr>
<td>8</td>
<td>N-BK7</td>
<td>F2</td>
<td>35.66</td>
<td>-58.26</td>
<td>1.2</td>
<td>3.44</td>
<td>9.24</td>
</tr>
</tbody>
</table>

Here, F2, SF10, and N-BK7 are common materials for dispersive prisms. As listed in Table 1, comparing the 1st row configuration parameters (H-LAK54 and H-FK95N) with the 2nd configuration (H-FK95N and H-LAK54), the SD and SSR values of these two DAPs are approximately equal, while the apex angle of the 2nd configuration is less than $65^\circ$ to allow sufficient design flexibility; moreover, comparing the 2nd configuration (H-FK95N and H-LAK54) with the 7th configuration (H-FK95N and SF10), the SD and SSR values of the 2nd configuration are smaller than the 7th, which indicates that the linearity of the DAP with the 2nd configuration is better.

According to the Schott formula in ZEMAX, six constant coefficients, i.e., $a_i (0 \leq i \leq 5)$, are needed to calculate the refractive index of arbitrary wavelength:

\[(n(\lambda))^2 = a_0 + a_1 \lambda^2 + a_2 \lambda^{-2} + a_3 \lambda^{-4} + a_4 \lambda^{-6} + a_5 \lambda^{-8}. \]

(7)

In line with the six constant coefficients of the five materials listed in Table 1, the variations of refractive index with wavelength are shown in Figure 2. In fact, the refractive index of H-LAK54 and SF10 are similar (see Figure 2), while the linearity of H-LAK54 (green curve) is better than that of SF10 (black curve), which implies that H-LAK54 can realize the better dispersion linearity of the DAP. As a result, we choose the 2nd configuration for our designed DAP, i.e., materials of H-FK95N and H-LAK54 and apex angles of $54.76^\circ$ and
−62.16°. In addition, the materials H-FK95N and H-LAK54 have moisture resistance and water resistance which are suitable for processing the DAP.

Figure 2. The refractive index of the glass materials varies with wavelength.

The deflection angles $\delta(\lambda)$ after passing through the DAP vary with the incident wavelengths $\lambda$, as shown in Figure 3a; here, the relationship between the deflection angle of incident wavelength $\lambda$ and the incident angle $i_0(\lambda)$ is discussed. As the incident angle $i_0$ changes, the variation of the deflection angle for different wavelengths is shown in Figure 3. As can be seen from Figure 3a,c, for the case of $\lambda < 550$ nm (such as $\lambda = 400$ nm, $\lambda = 450$ nm, and $\lambda = 500$ nm), the deflection angle increases with the incident angle from $-20^\circ$ to $0^\circ$, while the deflection angle decreases with the increasing incident angle from $0^\circ$ to $20^\circ$, and there exists a maximum deflection angle at $i_0 = 0$. As shown in Figure 3a,b,d, when $\lambda \geq 550$ nm (such as $\lambda = 550$ nm, $\lambda = 600$ nm, $\lambda = 650$ nm, and $\lambda = 700$ nm), there exists a minimum deflection angle at $i_0 = 0$; in addition, the minimum deflection angle at the central wavelength of 550 nm is $0^\circ$.

Figure 3. (a) The deflection angle of different wavelengths varies with the incident angle from $-20^\circ$ to $20^\circ$. (b–d) Details for $\lambda = 550$ nm, $\lambda = 400$ nm, and $\lambda = 700$ nm, respectively.

3. Dispersion Calibration

We propose a novel calibration method based on the numerical calculation of the DAP model. For a DAP-based PBCHI system, the calibration is performed by tracking a distinguishable point of the encoding mask in the direction of dispersion. As shown in
Figure 4, the calibration process consists of the following steps: Step1, the pixel coordinate of the tracked point in the PBCHI system without DAP is first recorded as the reference position $x_{re}$ on the imaging sensor. Step2, the fabricated DAP is accurately fixed in the PBCHI system to achieve a system optical axis parallel to the prism base by searching for the minimum (or maximum) deflection angle. The actual dispersion angle can be calibrated according to Equation (4) by illuminating with the wavelength of $\lambda_{\text{min}}$ and $\lambda_{\text{max}}$, respectively. The actual direct-vision wavelength can be found by adjusting the incident wavelength from $\frac{\lambda_{\text{min}}+\lambda_{\text{max}}}{2} - \Delta \lambda$ to $\frac{\lambda_{\text{min}}+\lambda_{\text{max}}}{2} + \Delta \lambda$ in a step of 1nm to find the wavelength whose dispersion shift is 0 relative to the reference position $x_{re}$, where $\Delta \lambda$ represents the wavelength scan range and is usually set to 20 nm. Step3, the actual apex angles $\alpha_1$ and $\alpha_2$ of the fabricated DAP can be calculated from the above-calibrated dispersion angle and direct-vision wavelength together with the parameters of the DAP glass materials. Step4, combining Equations (1) and (7), the dispersion shift $N_C$ of the DAP at different wavelengths $\lambda$ can be calculated by the DAP apex angles $\alpha_1$ and $\alpha_2$, the focal length $f$ of the lens $L_3$, and the pixel size $p$ of the imaging sensor. Through the above calibration method of numerical calculations combined with simple experimental measurements, the dispersion shift of the fabricated DAP can be calculated, which shows accuracy compared with the direct-measured result. Compared with the traditional calibration method for the dispersion shift at different wavelengths, our calibration method can simplify the calculation process by avoiding a large number of direct measurements, especially in the case that the components in the PBCHI system have been moved or replaced, and realize the calibration of the DAP dispersive parameters.

Figure 4. The flow chart of the calibration method based on the experimental test and numerical calculations of the fabricated DAP.

Due to the fabrication errors in the practical manufacturing processes and the practical material property deviations, the fabricated DAP for practical spectral imagers requires further dispersion calibrations. Based on the PBCHI system as shown in Figure 1c, we performed the dispersion calibration for the DAP. In the calibration, a monochromator working together with a Xenon lamp provided quasi monochromatic light. We first calibrated the apex angles $\alpha_1$ and $\alpha_2$ of the DAP by tracking the position of the encoding mask at different wavelengths, i.e., a sign “+” on the encoding mask was selected as the tracked target, as shown in Figure 5a, and then we compared the measured values with the designed values.
It should be pointed out that the position (here, select a certain pixel point) of the tracked target in the PBCHI system without the DAP is considered as the original reference point, whose pixel coordinated along the dispersion direction is \( x_{re} = 965 \), as shown in Figure 5b. Then, the DAP is placed in the PBCHI system and the optical axis of the DAP is coincident with the axis of the PBCHI system. In this process, by adjusting the wavelength of the monochromator to 550 nm and carefully rotating the DAP horizontally to change the incident angle of the light beam, the movement of the tracked point in the dispersion direction on the imaging sensor will help us to calibrate the optical axis of the DAP. When the incident angle changes to a certain value by rotating the DAP horizontally, the tracked point will no longer move toward the same direction; instead, it will move toward the opposite direction if we continue to rotate the DAP in the same direction; at this time, the incident angle implies the maximum (or minimum) deflection angle and \( i_0(\lambda) = 0 \), i.e., the optical axis of the DAP is coincident with the axis of the PBCHI system. After that, the DAP can be fixed.

![Figure 5](image-url)

**Figure 5.** (a) The schematic diagram of the encoding mask with sign “+” (near the upper edge). (b) The coordinate of the observation point without DAP. (c) The coordinate of the observation point with DAP while \( \lambda = 550 \) nm. (d) The coordinate of the observation point with DAP while \( \lambda = 540 \) nm.

Then, we try to calibrate the actual direct-vision wavelength of the manufactured DAP. As shown in Figure 5c, when the incident wavelength is set to 550 nm, the pixel coordinate of the tracked position along the dispersion direction is \( x_{550\text{nm}} = 971 \), which shifts six pixels from the original reference coordinate \( x_{re} = 965 \), as shown in Figure 5b. By carefully adjusting the wavelength of the monochromator from 530 nm to 570 nm by a step of 1 nm, the position of the observation point was recorded to find the direct-vision wavelength. When the wavelength is adjusted to 540 nm, the pixel coordinate of the tracked point in the dispersion direction is \( x_{540\text{nm}} = 965 \), as shown in Figure 5d, which is identical to the reference coordinate and indicates that \( \lambda = 540 \) nm is the actual direct-vision wavelength of the fabricated DAP used in our PBCHI system.

To ensure the spectral accuracy of the reconstructed hyperspectral cube, it is necessary to perform the dispersion angle calibration for the PBCHI. Therefore, we change finely the wavelength from 400 nm to 700 nm by a step of 5 nm to measure the actual dispersion
positions of the tracked point for various specific bands of the imager, and the pixel number $N$ shifted from the original reference point ($x_r = 965$) is measured and listed in Figure 6. Then, the calibrated dispersion angle $\Delta C$ is calculated by Equation (4), which is approximately 0.04° larger than the designed value, i.e., $\Delta C = 1.24°$. According to the calibrated direct-vision wavelength and dispersion angle, combined with Snell’s law, the apex angle $\alpha_1$ of the DAP in the imager is calculated to be approximately 1.4° larger than the designed value, i.e., $\alpha_1 = 56.16°$, and $\alpha_2$ is calculated to be approximately 1.67° smaller than the designed one, i.e., $\alpha_2 = -63.83°$. The pixel number $N$ can be expressed

$$N = \frac{f \cdot \tan(i_5(\lambda)) - f \cdot \tan(i_0(\lambda))}{p}. \quad (8)$$

Here, $i_5(\lambda)$ is the beam exit angle which is calculated by Equations (1) and (7), $i_5(\lambda)$ is related to the apex angle, and $p$ is the pixel size of the imaging sensor.

![Figure 6](image-url)  
**Figure 6.** The pixel shift for various wavelengths. $N_C$ represents the calculated value from the calibrated parameters of the DAP. $N_E$ represents experimental measured values. $N_T$ represents the theoretical designed value.

As shown in Figure 6, the blue solid line represents the theoretical designed value $N_T$, which is calculated by Equation (8) combined with the designed apex angle and dispersion angle before calibration; the red hollow-dotted line represents the experimental measured value $N_E$, and the black solid-dotted line represents the calibrated value $N_C$ from the calibrated apex angle and dispersion angle of the DAP according to Equation (8), which is approximately the same as the $N_E$. Moreover, the mean squared error (MSE) was employed to give an evaluation of the difference between the calculated dispersion shifts $N_C$ through our method with the direct-measured values $N_T$ of different wavelengths. A lower MSE index implies lower difference and higher accuracy. The MSE index between $N_C$ and $N_E$ is 0.1774, far less than the index between $N_T$ and $N_E$ that is 24.2134, which demonstrates that our calibration can achieve accuracy.

The pixel shift differences before and after the calibration can be respectively expressed as

$$\begin{align*}
\Delta N_A &= N_E - N_T \\
\Delta N_B &= N_E - N_C.
\end{align*} \quad (9)$$

Before calibration, the pixel shift difference $\Delta N_A$ is shown by the black dotted line in Figure 7, and the red solid line is the linear fitting line of the $\Delta N_A$. It can be seen that $\Delta N_A$ increases with the wavelength. $\Delta N_B$ represents the pixel shift difference after calibration as marked by the blue dotted line in Figure 7, and the yellow solid line is the linear fitting line. Obviously, the slope of the linear fitting line of $\Delta N_B$ is close to 0, and $\Delta N_B$ can be considered
basically unchanged with the increase of the wavelength; that is, the calculated value from calibrated parameters of the DAP is basically consistent with the measured value.

Figure 7. The difference of pixel shift for various wavelengths.

4. Experimental Results

To test the performance of our PBCHI system based on the calibrated DAP as shown in Figure 1c, we constructed a series of push-broom hyperspectral imaging experiments as shown in Figure 8a. The scenes, located about 7 m away from $L_1$ of the prototype, were illuminated by LED light. The encoding mask in the PBCHI prototype was generated from a Gaussian random matrix with a size of $96 \times 48$, in which the elements greater than 0 were set to 1 and the elements less than or equal to 0 were all set to 0, and the smallest encoding mask element is four times the size of a single pixel of the imaging sensor. The spectral filtering was performed by a 450–650 nm bandpass filter. We pushed the prototype by a stepper motor and, meanwhile, collected the multi-frame spatio-spectral multiplexed measurements of the scene to achieve a large field of view (FOV), and one frame of measurements by the random-encoding-mask-based PBCHI is shown in Figure 8c.

Figure 8. (a) Photograph of the experimental system. (b) One frame of measurements by slit-based push-broom hyperspectral imager. (c) One frame of measurements by our encoding-mask-based PBCHI.

For comparison, one frame of measurements by the slit-based push-broom hyperspectral imager is shown in Figure 8b, which can be captured by replacing the encoding mask in the PBCHI prototype with a slit. The width of the slit is equal to the single pixel size.
As shown in Figure 9, the ground truth with 139 channels of two hyperspectral scenes can be acquired from the measurements by the slit-based push-broom hyperspectral imager. Figures 10 and 11 show the raw cubes with 31 channels reconstructed by compressive algorithm regularized least squares (RLS) [34,35] for the first scene and the second one, respectively. We push the whole PBCHI prototype to acquire a sequence of multiplexed measurements; in detail, the first scene (Figure 10) is reconstructed from 85 frames of multiplexed measurements, while the second scene (Figure 11) is reconstructed from 50 frames of measurements. The compression ratio is about 4.

It is well-known that the PBCHI is rather sensitive to various noises, such as photon shot noise, dark current noise, background noise, and so on. At the same time, the encoding mask processing errors, the residual calibration errors, the push-broom sampling instability, and the sparse reconstruction process will also introduce significant artifacts, which usually turn out to be severe and appear as defects such as stripes and dead pixels. As a result, compared with the ground truths in Figure 9, there are obvious noises and stripes in the raw reconstructed cubes as shown in Figures 10 and 11.

![Figure 9. The ground truth captured by the slit-based push-broom hyperspectral imager. (a) The first scene. (b) The second scene.](image)

![Figure 10. The raw 31-channel cube of the first scene reconstructed from the PBCHI measurements.](image)
To test the spectral accuracy of the raw reconstructed cubes of the PBCHI prototype, the spectral intensities for different spatial locations of the two scenes are shown in Figure 12. The spectral intensities of the regions marked by the red box in Figure 9a and the corresponding regions in Figure 10 are shown in Figure 12a, and the spectral intensities of the regions marked by the white box in Figure 9b and the corresponding regions in Figure 11 are shown in Figure 12b. Obviously, the spectral signatures of the reconstructed cubes of the PBCHI system show consistency with the ground truth captured by the slit-based push-broom hyperspectral imager, which indicates the spectral accuracy of the DAP-based PBCHI.

5. Conclusions and Discussion

In this paper, firstly, based on the direct-vision push-broom compressive hyperspectral imager (PBCHI), a geometric and dispersive parameter of the double Amici prism (DAP) is optimized to meet the requirements of hyperspectral imaging in the visible bands.

Secondly, a novel calibration method for the dispersion device DAP model is put forward where the apex angles of the fabricated DAP are calibrated; as a result, the dispersion shift of the PBCHI system at different wavelengths can be calculated if the focal...
length \( f \) of the lens \( L_3 \) and the pixel size \( p \) of the image sensor used in the PBCHI system are known.

Lastly, a PBCHI prototype is built to test the performances of the designed and calibrated DAP and the hyperspectral imager, and it is found that the dispersion performance after calibration is more consistent with the experimental test value than before calibration through the comparison of evaluation index MSE and the differences of the pixel shift. Moreover, the reconstructions cube and the spectral intensity demonstrate that the dispersion calibration achieves the hyperspectral cube accuracy reconstruction especially in spectrum, which indicates the spectral accuracy of the PBCHI system.

This work can simplify the calibrated process and turn the conventional pixel-wise dispersive shift calibration by a monochromator into a group of numerical calculations for the DAP-based hyperspectral imager, and the research will be helpful for promoting PBCHI technologies for achieving further widely used applications.

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**References**


