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Moving Bragg Solitons in a Dual-Core System Composed of a Linear Bragg Grating with Dispersive Reflectivity and a Uniform Nonlinear Core

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Abstract: The existence and stability of moving Bragg grating solitons are systematically investigated in a dual-core system, where one core is uniform and has Kerr nonlinearity, and the other is linear with Bragg grating and dispersive reflectivity. It is found that moving soliton solutions exist throughout the upper and lower bandgaps, whereas no soliton solutions exist in the central bandgap. Similar to the quiescent solitons in the system, it is found that when dispersive reflectivity is nonzero, for certain values of parameters, sidelobes appear in the solitons' profiles. The stability of the moving solitons is characterized using systematic numerical stability analysis. Additionally, the impact and interplay of dispersive reflectivity, soliton velocity, and group velocity on the stability border are analyzed.

Keywords: Bragg grating soliton; dual-core; Kerr nonlinearity; dispersive reflectivity



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1. Introduction

Fiber Bragg gratings (FBGs) are optical media in which refractive index variations are periodically or aperiodically written on the optical fiber core. The linear properties of FBGs have been extensively studied in the last few decades for applications such as dispersion compensation, optical filtering, and sensing [1–3]. Additionally, FBGs have attracted significant interest in various nonlinear applications, including pulse compression and optical switching [4–8]. The linear cross-coupling induced by FBGs between forward- and backward-propagating waves leads to a bandgap in the system's linear spectrum, which gives rise to a strong effective dispersion. Experimental findings indicate that the dispersion induced by FBGs can be larger than that of uniform silica fiber by up to six orders of magnitude [9,10]. The substantial dispersion arising from FBGs opens up the possibility of the existence of slow lights in a medium, which has attracted considerable attention due to its numerous potential applications [11–13].

Solitons (or, more generally, solitary waves) are formed when the nonlinearity of the medium is counterbalanced by its dispersion. The characteristics of solitons have been investigated in various optical media, including Kerr nonlinear media, media with second-order nonlinearity, and photonic crystals [14–20]. In the case of FBGs, a balance between Kerr nonlinearity and the dispersion induced by FBGs at high intensity results in the formation of Bragg grating (BG) solitons [10,21,22]. Theoretical findings indicate that BG solitons form a two-parameter family of soliton solutions in uniform FBGs, existing throughout the entire bandgap [21,22]. The first parameter represents the velocity of BG solitons, ranging from zero to the speed of light in the medium, while the other parameters determine the peak, width, and detuning frequency of the solitons. Stability analysis reveals that approximately half of the soliton family is stable [23–25]. Thus far, BG solitons with a velocity as low as 23% of the speed of light in the medium have been

experimentally observed [26]. BG solitons have been extensively studied in other structures, including photonic crystals [27–29], dual-core fibers [30–33], waveguide arrays [34–36], and microcavities with a periodic potential [37]. They have also been investigated in other complex nonlinear media, such as quadratic nonlinearity [38–40], sign-changing Kerr nonlinearity [41], and cubic-quintic nonlinearity [42–45].

Over the past three decades, there has been significant interest in dual-core systems and multicomponent solitons due to their potential applications in optical signal processing, logic gates, and switching [46–52]. In particular, semilinear dual-core systems, where one core is linear, and the other is nonlinear, exhibit rich nonlinear dynamics and switching characteristics [53,54]. Bragg gratings in dual-core systems also find important applications in optical systems. For example, it has been shown that grating-assisted couplers may be employed as optical add/drop elements in WDM optical networks [55–58]. It has been theoretically demonstrated that both quiescent and moving BG solitons exist in the semilinear coupler with Bragg gratings written in the nonlinear core only [30,59]. Thus, nonlinear couplers with Bragg gratings may open a pathway for novel optical applications. For example, Bragg grating solitons in coupled systems can be instrumental in building novel all-optical devices such as optical diodes [60] and logic gates [61].

In the case of complex Bragg gratings, such as nonuniform Bragg gratings, deep Bragg gratings, Bragg superstructures, and gratings written on photonic wires, band gaps may be broad and inhomogeneous [62–65]. In order to analyze the existence and dynamics of BG solitons in these nonstandard Bragg gratings, the standard model must be modified. A phenomenological approach for this modification involves considering the spatial dispersion of Bragg reflectivity, as discussed in Ref. [66]. The presence of dispersive reflectivity significantly affects both the bandgap and the stability region. Notably, dispersive reflectivity leads to the emergence of side lobes in the soliton's profile, which are more pronounced for higher values of the dispersive reflectivity parameter. Such sidelobes do not exist in BG solitons in uniform gratings. Additionally, it has been observed that the stability region expands for both quiescent and moving BG solitons [66,67].

In most of the previously reported nonlinear dual-core systems, Bragg gratings were typically placed in the nonlinear core. However, a distinctive approach was taken in [31], where a semilinear dual-core system was analyzed with the Bragg grating situated in the linear core, and the other core exhibited only Kerr nonlinearity. The model of [31] supports both quiescent and moving BG solitons and its linear spectrum differs significantly from that of previous models. The system presented in [31] was further generalized by introducing dispersive reflectivity, leading to an exploration of the existence, stability, and interactions of quiescent solitons in Ref. [68]. In this paper, we analyze the existence and stability of moving BG solitons in the generalized model of Ref. [68]. The remainder of the paper is organized as follows: in Section 2, we discuss the mathematical model and its characteristics. A detailed discussion of the results is provided in Section 3. In particular, the analysis of the linear spectrum and the bandgap structure is presented in Section 3.1. The features of the soliton solutions are discussed in Section 3.2. The effects of soliton velocity and dispersive reflectivity on the stability of the moving solitons are analyzed in Section 3.3. A summary of the main results is presented in Section 4.

2. The Model

Following the procedure described in the Refs. [30,31,66], the following system of normalized partial differential equations can be derived as the governing equations for

the propagation of light in two linearly coupled cores, where one core has only Kerr nonlinearity, and the other one contains a Bragg grating with dispersive reflectivity:

$$\begin{aligned} iu_t + iu_x + \left(|v|^2 + \frac{1}{2}|u|^2\right)u + \phi &= 0, \\ iv_t - iv_x + \left(|u|^2 + \frac{1}{2}|v|^2\right)v + \psi &= 0, \\ i\phi_t + ic\phi_x + u + \lambda\psi + m\psi_{xx} &= 0, \\ i\psi_t - ic\psi_x + v + \lambda\phi + m\phi_{xx} &= 0. \end{aligned} \quad (1)$$

In Equation (1), the forward- and backward-propagating waves are represented by u and v in the nonlinear core, and ϕ and ψ are their counterparts in the linear core containing the FBG. The coefficient of linear coupling between the cores is normalized to be 1. The Bragg grating-induced linear coupling coefficient between the left- and right-propagating waves is denoted by λ ; λ is real and $\lambda > 0$. The group velocity in the nonlinear core is set to 1, and c represents the relative group velocity in the linear core. The real coefficient $m > 0$ represents the strength of dispersive reflectivity. Our analysis is confined to $0 \leq m \leq 0.5$, as values beyond $m > 0.5$ hold no practical significance [66].

3. Discussion of Results

3.1. Analysis of the Linear Spectrum

In order to analyze the characteristics of moving BG solitons, Equation (1) is transformed into moving co-ordinates using the transformation $\{X, T\} = \{x - \eta t, t\}$, resulting in the following system of nonlinear coupled mode equations:

$$\begin{aligned} iu_T + i(1 - \eta)u_X + \left(|v|^2 + \frac{1}{2}|u|^2\right)u + \phi &= 0, \\ iv_T - i(1 + \eta)v_X + \left(|u|^2 + \frac{1}{2}|v|^2\right)v + \psi &= 0, \\ i\phi_T + i(c - \eta)\phi_X + u + \lambda\psi + m\psi_{XX} &= 0, \\ i\psi_T - i(c + \eta)\psi_X + v + \lambda\phi + m\phi_{XX} &= 0, \end{aligned} \quad (2)$$

where η is the normalized soliton velocity, and $\eta = 1$ corresponds to the speed of light in the medium. In order to obtain the dispersion relation, Equation (2) is first linearized by setting $|u|^2$ and $|v|^2$ in the first two equations of Equation (2) to zero [21]. Upon the substitution of $u, v, \phi, \psi \sim \exp(ikX - i\Omega T)$ into the linearized form of Equation (2) and some straightforward algebraic manipulations, one arrives at the following equation:

$$\begin{aligned} \Omega^4 + 4\eta k\Omega^3 - \left((\lambda - mk^2)^2 + (c^2 - 6\eta^2 + 1)k^2 + 2\right)\Omega^2 - 2\eta k\left((\lambda - mk^2)^2 + (c^2 - 2\eta^2 + 1)k^2 + 2\right)\Omega \\ - \eta^2 k^2\left((\lambda - mk^2)^2 + (c^2 - \eta^2 + 1)k^2 + 2\right) + k^2(\lambda - mk^2)^2 + (ck^2 - 1)^2 = 0. \end{aligned} \quad (3)$$

where Ω is the frequency of the soliton in the moving frame, which is related to the Bragg frequency in the laboratory reference frame by the following relation:

$$\Omega(k) = \omega(k) - \eta k. \quad (4)$$

The dispersion relation results in three disjoint bandgaps, similar to the case of quiescent solitons: a central gap and upper and lower gaps. However, the central bandgap and both upper and lower gaps overlap with one branch of the continuous spectrum; therefore, they are not genuine bandgaps. Similar to other models of BG solitons (e.g., Ref. [69]), when the velocity of solitons is nonzero, the edges of the upper and lower gaps do not occur at $k = 0$. Additionally, similar to the case of quiescent solitons [70], the condition for closing the central gap is $m = c\lambda$. However, at any fixed value of the velocity parameter η , an increase

in c causes the upper and lower gaps to expand (e.g., see the inset of Figure 1a). As for the effect of m , as m increases, a reduction in the size of the upper and lower gaps is observed for fixed values of c and η (see the inset of Figure 1b). It is also worth noting that as c increases, the effect of m and η is diminished.

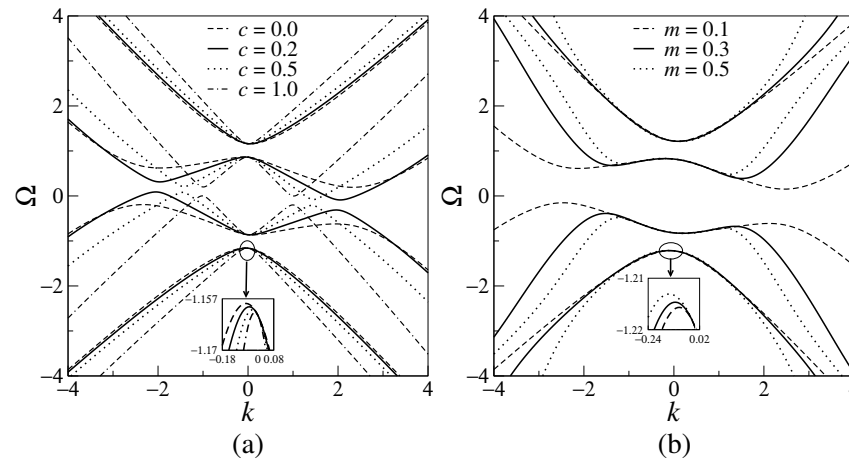


Figure 1. Dispersion diagrams for different parameter values. (a) $m = 0.1$, $\eta = 0.1$, and $\lambda = 0.3$, and (b) $c = 0.0$, $\eta = 0.1$, $\lambda = 0.4$. The insets show that the edges of the lower and upper gaps do not occur at $k = 0$. For clarity, the insets are only shown for one of the branches in the lower bandgap.

The widths of both the upper and lower gaps strongly depend on the velocity parameter η and c for any fixed value of λ . An increase in velocity parameter η results in a decrease in the widths of the upper and lower gaps (see Figure 2 for $\lambda = 0.2$). It is noteworthy that the upper and lower gaps close at a critical value of η . The critical value η_{cr} is influenced by the parameter c , and it follows the empirical relation given by

$$\eta_{cr} = \frac{c + 1}{2}. \quad (5)$$

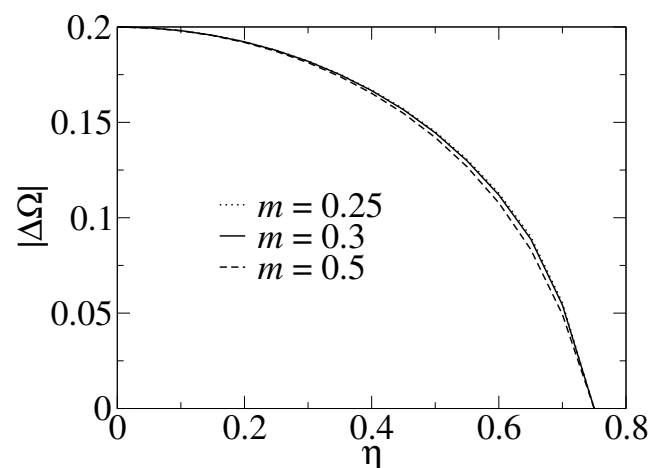


Figure 2. Variation of the width of the upper/lower gap with the velocity parameter η for various values of m for $c = 0.5$ and $\lambda = 0.2$.

The condition obtained for the critical value η_{cr} is similar to the result of Ref. [69]. However, in the standard model, the bandgap closes at $|\eta| = 1$ for moving solitons [71]. Additionally, the upper and lower gaps reduce slightly for fixed values of η and c as m increases. Nevertheless, the upper and lower gaps remain unchanged with m at two specific values of η , namely, $\eta = 0$ and $\eta = \eta_{cr}$.

3.2. BG Soliton Solutions

There are no exact analytical solutions for the moving BG solitons in the system of Equation (2). Therefore, the moving soliton solutions need to be determined numerically. To this end, we assume that the stationary moving soliton solutions are of the form

$$\begin{aligned} \{u(X, T), v(X, T)\} &= \{U(X), V(X)\} \exp(-i\Omega T), \\ \{\phi(X, T), \psi(X, T)\} &= \{\Phi(X), \Psi(X)\} \exp(-i\Omega T). \end{aligned} \quad (6)$$

Substituting Equation (6) into Equation (2), we arrive at the following system of ordinary differential equations:

$$\begin{aligned} \Omega U + i(1 - \eta)U_X + \left[|V|^2 + \frac{1}{2}|U|^2\right]U + \Phi &= 0, \\ \Omega V - i(1 + \eta)V_X + \left[|U|^2 + \frac{1}{2}|V|^2\right]V + \Psi &= 0, \\ \Omega \Phi + i(c - \eta)\Phi_X + U + \lambda\Psi + m\frac{d^2\Psi}{dX^2} &= 0, \\ \Omega \Psi - i(c + \eta)\Psi_X + V + \lambda\Phi + m\frac{d^2\Phi}{dX^2} &= 0. \end{aligned} \quad (7)$$

Equation (7) is solved numerically for various parameter values by using the relaxation algorithm to obtain the moving BG soliton solutions. It is found that moving BG soliton solutions exist throughout the upper and lower band gaps. On the other hand, similar to the quiescent case, there are no soliton solutions in the central band gap. Moreover, similar to other BG models with dispersive reflectivity, when $m \neq 0$, in certain parameter ranges, sidelobes appear in the solitons' profiles. On the other hand, in the absence of dispersive reflectivity, there are no sidelobes in the solitons' profiles. Examples of solitons with and without sidelobes are shown in Figure 3. It should be noted that the amplitudes of the solitons in Figure 3 are shown in a logarithmic scale in order to make the sidelobes visible.

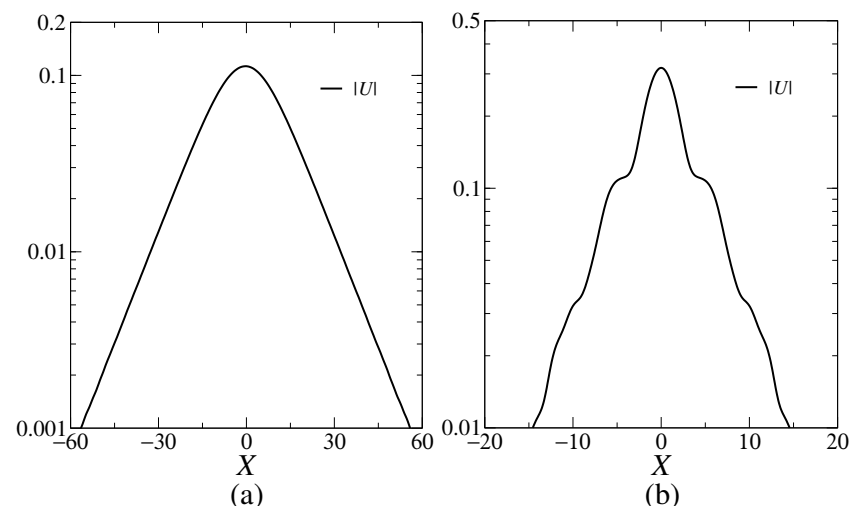


Figure 3. Moving BG soliton solutions in a logarithmic scale: (a) Soliton without sidelobes for $\lambda = 0.200$, $c = 0.500$, $\Omega = 1.090$, $m = 0.00$, and $\eta = 0.3$. (b) Soliton with sidelobes for $\lambda = 1.000$, $c = 0.000$, $\Omega = 1.580$, $m = 0.460$, and $\eta = 0.01$. Only the U component is shown here.

In the other models of BG solitons with dispersive reflectivity, it has been shown that the collisions of moving solitons leads to rich dynamics, particularly in the case of solitons with sidelobes [67]. The analysis of the collisions of moving solitons is beyond the scope of the current paper. It is worth noting that these collisions can be utilized to manipulate light and design novel optical processing elements.

3.3. Stability Analysis

We have conducted a systematic numerical stability analysis in the model described by Equation (2) to analyze the stability of moving solitons. The analysis was performed by solving Equation (2) using the symmetrized split-step Fourier method. The moving BG solitons were propagated up to $t = 2000$ in all simulations, and absorbing boundary conditions were applied to attenuate the radiation that reached the boundaries of the computational window.

Our analysis shows that stable and unstable solitons exist in the upper and lower bandgap (see below). It is found that the evolution of unstable solitons may lead to different outcomes. As is shown in Figure 4a, the evolution of unstable solitons that are located deep in the unstable region results in their complete destruction accompanied by a significant amount of radiation. On the other hand, unstable solitons close to the stability boundary can spontaneously split into two oppositely traveling robust moving solitons for both uniform ($m = 0$) and nonuniform ($m \neq 0$) FBGs (see Figure 4b). Another possibility is that the unstable soliton may evolve into another moving soliton with higher velocity than the original soliton (see Figure 4c).

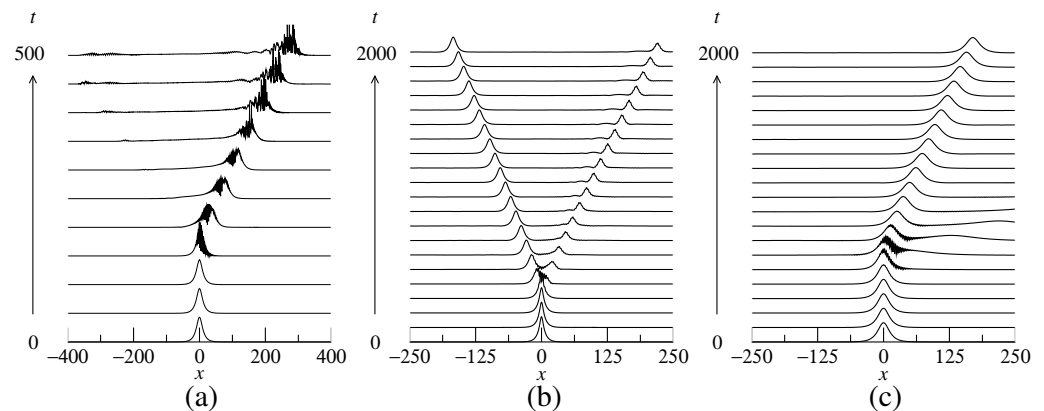


Figure 4. Evolution of unstable moving BG solitons. (a) Complete destruction of soliton for $\lambda = 0.2$, $c = 1.00$, $m = 0.060$, $\Omega = 1.000$, and $\eta = 0.2$; (b) spontaneous splitting of soliton for $\lambda = 0.2$, $c = 0.10$, $m = 0.00$, $\Omega = -1.06$, and $\eta = 0.1$; (c) generation of a soliton with higher velocity than the original soliton for $\lambda = 0.2$, $c = 1.00$, $m = 0.20$, $\Omega = 1.04$, and $\eta = 0.2$. Only the $|u|$ component is shown here.

In order to better understand and visualize the effects of system parameters on the stability of the moving solitons, the results of the stability analysis are presented in a parametric plane. Figure 5 displays the effect of increasing velocity on the stability regions for uniform FBGs ($m = 0$) with $\lambda = 0.2$ in the plane of (c, Ω) . As is shown in this figure, in the upper gap, the stable region is increased slightly as η is increased from 0.1 to 0.2. However, increasing η to 0.3 results in the shrinkage of the stable region. In the lower gap, a general trend is observed, where the size of the stable region decreases with an increase in η .

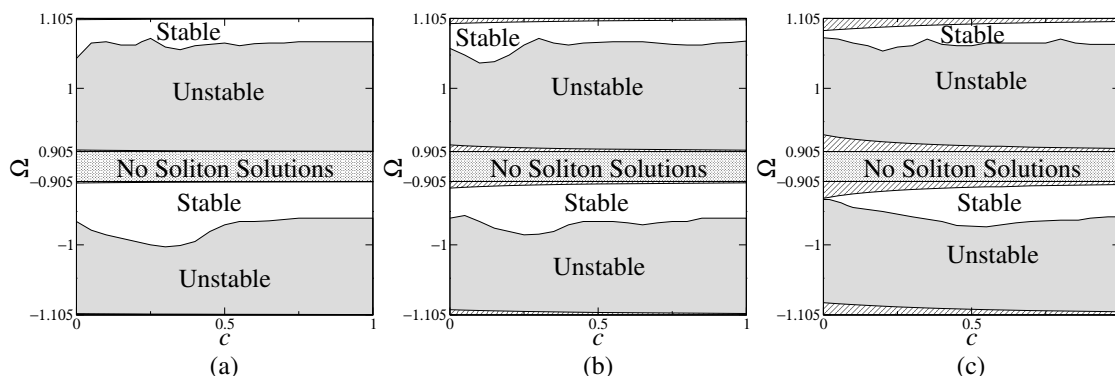


Figure 5. Stability regions in the case of uniform FBGs (i.e., $m = 0$) in the plane of (c, Ω) at $\lambda = 0.20$ for (a) $\eta = 0.10$, (b) $\eta = 0.2$, and (c) $\eta = 0.3$.

In the case of nonuniform FBG ($m \neq 0$), the results of the stability analysis are summarised in Figures 6 and 7 for the upper and lower gaps, respectively, for $\lambda = 0.2$ in the plane of (m, Ω) . As is shown in Figure 6, in the upper gap, for a given c , a general trend is that the stable region decreases when η increases. Another noteworthy feature is that for $\eta = 0.1$ and 0.2 , as c is increased from 0 to 0.5, the stable region decreases; however, increasing c from 0.5 to 1.0 leads to the enlargement of the stable region. Additionally, in the case of $\eta = 0.3$, the stable region is enlarged as c increases from 0 to 1. It should also be noted that the cusp for $c = 0.5$ occurs at $m = c\lambda$. A similar behavior has been observed for quiescent solitons.

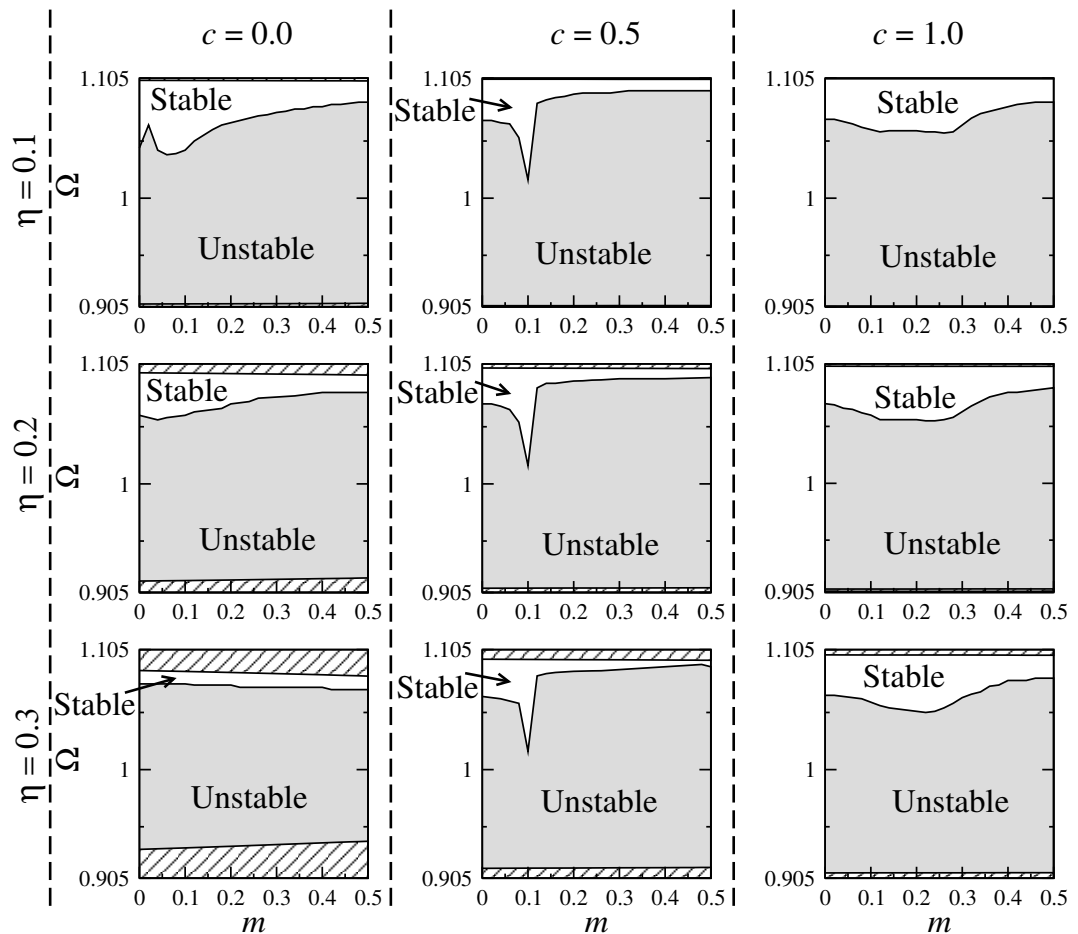


Figure 6. Stability diagram in (m, Ω) plane in the upper gap for $\lambda = 0.2$. The areas shown by the hatched lines are outside the gap and do not contain any soliton solutions.

In the lower gap, for a given $\eta = 0.1$, as c is increased from 0 to 0.5, the stable region decreases, and a further increase in c from 0.5 to 1.0 results in the expansion of the stable region (see Figure 7). However, for $\eta = 0.2$ and 0.3 , increasing c from 0 to 1.0 gives rise to the expansion of the stable region. As for the effect of η , for $c = 0$, the stable region shrinks as η increases from 0.1 to 0.3. However, for $c = 0.5$ and $c = 1.0$, the stable region expands as η is increased from 0.1 to 0.2, and a further increase in η leads to the shrinkage of the stable region.

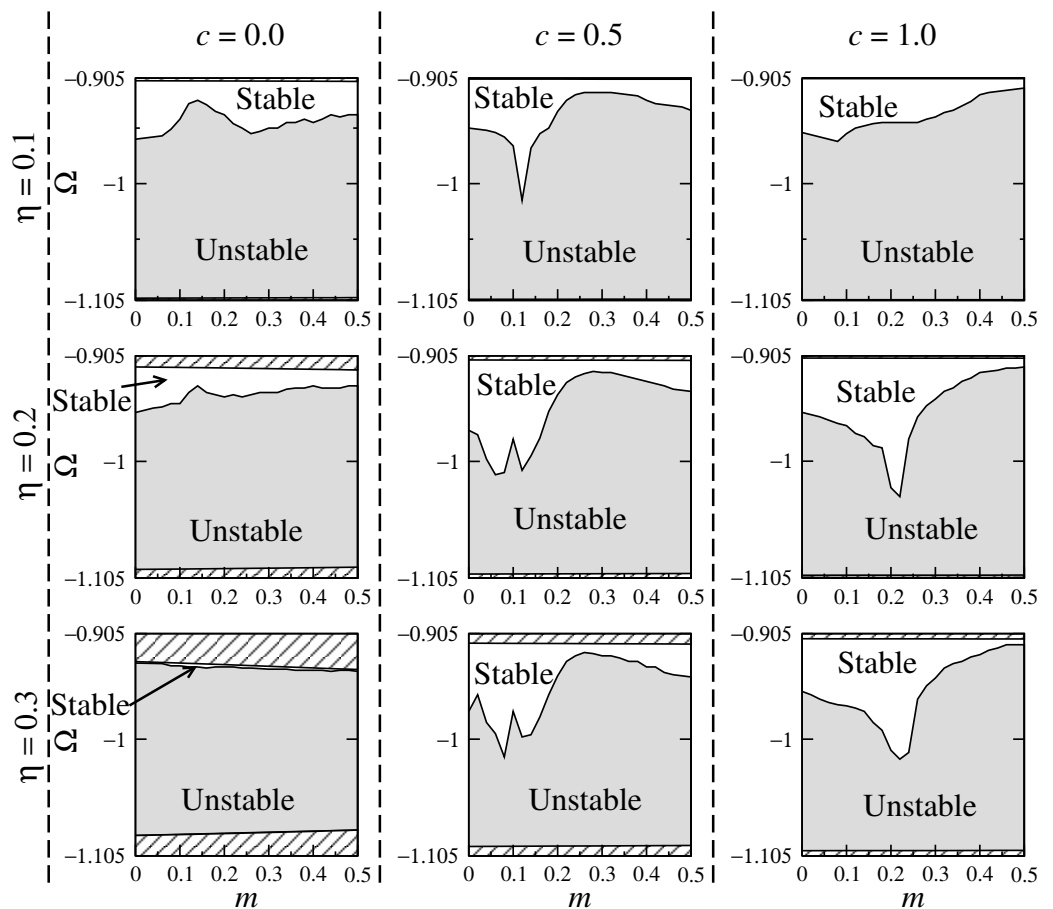


Figure 7. Stability diagram in (m, Ω) plane in the lower gap for $\lambda = 0.2$. The shaded regions show unstable solitons. The areas shown by the hatched lines are outside the gap and do not contain any soliton solutions.

4. Conclusions

In this paper, we have investigated the existence and stability of moving BG solitons in a dual-core system, where one core is linear and contains a Bragg grating with dispersive reflectivity, and the other core is uniform and has Kerr nonlinearity. The system's linear spectrum comprises three distinct bandgaps: the upper, lower, and central bandgaps. As the soliton velocity and dispersive reflectivity parameters increase, both the upper and lower gaps decrease. However, an increase in c tends to counteract the effects of velocity and dispersive reflectivity. We empirically determined a critical value of soliton velocity for which these gaps close, and this critical value depends solely on c . The numerical stability analysis indicates the existence of moving BG solitons throughout the upper and lower gaps, with no solitons found in the central bandgap. We have studied the stability of moving BG solitons by means of systematic numerical stability analysis and identified nontrivial stability borders. It has been observed that for $m = 0$, the stability area in the lower gap in the plane of (c, Ω) decreases as the velocity of solitons increases. In the case of $m \neq 0$, the size of the stable region in the plane of (m, Ω) is affected by c and η . A key finding is that as η is increased, the stability region in the upper gap reduces for a fixed c .

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References

1. Kashyap, R. *Fiber Bragg Gratings*; Academic Press: San Diego, CA, USA; London, UK, 1999.
2. Krug, P.A.; Stephens, T.; Yoffe, G.; Ouellette, F.; Hill, P.; Dhosi, G. Dispersion compensation over 270 km at 10 Gbit/s using an offset-core chirped fibre Bragg grating. *Electron. Lett.* **1995**, *31*, 1091.
3. Loh, W.H.; Laming, R.I.; Robinson, N.; Cavaciuti, A.; Vaninetti, F.; Anderson, C.J.; Zervas, M.N.; Cole, M.J. Dispersion compensation over distances in excess of 500 km for 10-Gb/s systems using chirped fiber gratings. *IEEE Photon. Technol. Lett.* **1996**, *24*, 944–946.
4. Radic, S.; George, N.; Agrawal, G.P. Theory of low-threshold optical switching in nonlinear phase-shifted periodic structures. *J. Opt. Soc. Am. B* **1995**, *12*, 671–680.
5. De Sterke, C.M.; Eggleton, B.J.; Krug, P.A. High-intensity pulse propagation in uniform gratings and grating superstructures. *J. Light. Technol.* **1997**, *15*, 1494–1502.
6. Winful, H.G.; Marburger, J.H.; Garmire, E. Theory of bistability in nonlinear distributed feedback structures. *Appl. Phys. Lett.* **1979**, *75*, 379–381.
7. Cao, H.; Atai, J.; Shu, X.; Chen, G. Direct design of high channel-count fiber Bragg grating filters with low index modulation. *Opt. Express* **2012**, *20*, 12095–12110.
8. Cao, H.; Shu, X.; Atai, J.; Gbadebo, A.; Xiong, B.; Fan, T.; Tang, H.; Yang, W.; Yu, Y. Optimally-designed single fiber Bragg grating filter scheme for RZ-OOK/DPSK/DQPSK to NRZ-OOK/DPSK/DQPSK format conversion. *Opt. Express* **2014**, *22*, 30442–30460.
9. Eggleton, B.J.; de Sterke, C.M.; Slusher, R.E. Nonlinear pulse propagation in Bragg gratings. *J. Opt. Soc. America. B Opt. Phys.* **1997**, *14*, 2980–2993.
10. De Sterke, C.M.; Sipe, J.E. Gap solitons. *Prog. Opt.* **1994**, *33*, 203–260.
11. Marangos, J. Slow light in cool atoms. *Nature* **1999**, *397*, 559–560.
12. McDonald, K.T. Slow light. *Am. J. Phys.* **2000**, *68*, 293–294.
13. Krauss, T.F. Why do we need slow light? *Nat. Photon.* **2008**, *2*, 448–450.
14. Chiao, R.Y.; Garmire, E.; Townes, C.H. Self-Trapping of Optical Beams. *Phys. Rev. Lett.* **1964**, *13*, 479–482.
15. Hasegawa, A.; Tappert, F. Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers. I. Anomalous dispersion. *Appl. Phys. Lett.* **1973**, *23*, 142–144.
16. Hayata, K.; Koshida, M. Multidimensional solitons in quadratic nonlinear media. *Phys. Rev. Lett.* **1993**, *71*, 3275–3278.
17. Torner, L.; Mazilu, D.; Mihalache, D. Walking Solitons in Quadratic Nonlinear Media. *Phys. Rev. Lett.* **1996**, *77*, 2455–2458.
18. Kivshar, Y.S.; Agrawal, G.P. *Optical Solitons: From Fibers to Photonic Crystals*; Academic Press: San Diego, CA, USA, 2003; p. xvi; 540p.
19. AlQahtani, S.A.; Shohib, R.M.A.; Alngar, M.E.M.; Alawwad, A.M. High-stochastic solitons for the eighth-order NLSE through Itô calculus and STD with higher polynomial nonlinearity and multiplicative white noise. *Opt. Quantum Electron.* **2023**, *55*, 1227.
20. AlQahtani, S.A.; Al-Rakhami, M.S.; Shohib, R.M.A.; Alngar, M.E.M.; Pathak, P. Dispersive optical solitons with Schrödinger–Hirota equation using the P^6 -model expansion approach. *Opt. Quantum Electron.* **2023**, *55*, 701.
21. Christodoulides, D.N.; Joseph, R.I. Slow Bragg solitons in nonlinear periodic structures. *Phys. Rev. Lett.* **1989**, *62*, 1746–1749.
22. Aceves, A.B.; Wabnitz, S. Self-induced transparency solitons in nonlinear refractive periodic media. *Phys. Lett. A* **1989**, *141*, 37–42.
23. Malomed, B.A.; Tasgal, R.S. Vibration modes of a gap soliton in a nonlinear optical medium. *Phys. Rev. E* **1994**, *49*, 5787–5796.
24. De Rossi, A.; Conti, C.; Trillo, S. Stability, Multistability, and Wobbling of Optical Gap Solitons. *Phys. Rev. Lett.* **1998**, *81*, 85–88.
25. Barashenkov, I.V.; Pelinovsky, D.E.; Zemlyanaya, E.V. Vibrations and Oscillatory Instabilities of Gap Solitons. *Phys. Rev. Lett.* **1998**, *80*, 5117–5120.
26. Littler, I.C.M.; Mok, J.T.; Eggleton, B.J.; de Sterke, C.M. Dispersionless slow light using gap solitons. *Nat. Phys.* **2006**, *2*, 775–780.
27. Skryabin, D. Coupled core-surface solitons in photonic crystal fibers. *Opt. Express* **2004**, *12*, 4841–4846.
28. Monat, C.; de Sterke, M.; Eggleton, B.J. Slow light enhanced nonlinear optics in periodic structures. *J. Opt.* **2010**, *12*, 104003.
29. Atai, J.; Malomed, B.A.; Merhasin, I.M. Stability and collisions of gap solitons in a model of a hollow optical fiber. *Opt. Commun.* **2006**, *265*, 342–348.
30. Atai, J.; Malomed, B.A. Bragg-grating solitons in a semilinear dual-core system. *Phys. Rev. E* **2000**, *62*, 8713–8718.
31. Atai, J.; Malomed, B.A. Solitary waves in systems with separated Bragg grating and nonlinearity. *Phys. Rev. E* **2001**, *64*, 066617.
32. Mak, W.C.K.; Chu, P.L.; Malomed, B.A. Solitary waves in coupled nonlinear waveguides with Bragg gratings. *J. Opt. Soc. Am. B* **1998**, *15*, 1685–1692.
33. Tsofe, Y.J.; Malomed, B.A. Quasisymmetric and asymmetric gap solitons in linearly coupled Bragg gratings with a phase shift. *Phys. Rev. E* **2007**, *75*, 056603.

34. Mandelik, D.; Morandotti, R.; Aitchison, J.S.; Silberberg, Y. Gap solitons in waveguide arrays. *Phys. Rev. Lett.* **2004**, *92*, 093904.
35. Dong, R.; Rüter, C.E.; Kip, D.; Cuevas, J.; Kevrekidis, P.G.; Song, D.; Xu, J. Dark-bright gap solitons in coupled-mode one-dimensional saturable waveguide arrays. *Phys. Rev. A* **2011**, *83*, 063816.
36. Tan, Y.; Chen, F.; Beliçev, P.P.; Stepić, M.; Maluckov, A.; Rüter, C.E.; Kip, D. Gap solitons in defocusing lithium niobate binary waveguide arrays fabricated by proton implantation and selective light illumination. *Appl. Phys. B* **2009**, *95*, 531–535.
37. Gorbach, A.; Malomed, B.; Skryabin, D. Gap polariton solitons. *Phys. Lett. A* **2009**, *373*, 3024–3027.
38. Conti, C.; Trillo, S.; Assanto, G. Doubly resonant Bragg simultons via second-harmonic generation. *Phys. Rev. Lett.* **1997**, *78*, 2341–2344.
39. He, H.; Drummond, P.D. Ideal soliton environment using parametric band gaps. *Phys. Rev. Lett.* **1997**, *78*, 4311–4315.
40. Mak, W.C.K.; Malomed, B.A.; Chu, P.L. Three-wave gap solitons in waveguides with quadratic nonlinearity. *Phys. Rev. E* **1998**, *58*, 6708–6722.
41. Atai, J.; Malomed, B.A. Spatial solitons in a medium composed of self-focusing and self-defocusing layers. *Phys. Lett. A* **2002**, *298*, 140–148.
42. Atai, J.; Malomed, B.A. Families of Bragg-grating solitons in a cubic-quintic medium. *Phys. Lett. A* **2001**, *284*, 247–252.
43. Atai, J. Interaction of Bragg grating solitons in a cubic-quintic medium. *J. Opt. B Quantum Semiclass.* **2004**, *6*, S177–S181.
44. Dasanayaka, S.; Atai, J. Moving Bragg grating solitons in a cubic-quintic nonlinear medium with dispersive reflectivity. *Phys. Rev. E* **2013**, *88*, 022921.
45. Dasanayaka, S.; Atai, J. Stability and collisions of moving Bragg grating solitons in a cubic-quintic nonlinear medium. *J. Opt. Soc. Am. B* **2013**, *30*, 396–404.
46. Jensen, S.M. The Nonlinear Coherent Coupler. *IEEE Trans. Microw. Theory Techn.* **1982**, *30*, 1568–1571.
47. Maier, A.A. Optical transistors and bistable devices utilizing non-linear transmission of light in systems with unidirectional coupled waves. *Sov. J. Quantum Electron.* **1982**, *12*, 11490–1494.
48. Maier, A.A. Possible practical use of self-switching of radiation in coupled waveguides for amplification of the useful modulation of a signal. *Sov. J. Quantum Electron.* **1987**, *17*, 1013–1017.
49. Yang, C.C. All-optical ultrafast logic gates that use asymmetric nonlinear directional couplers. *Opt. Lett.* **1991**, *16*, 1641–1643.
50. Chen, Y.; Atai, J. Parametric spatial solitary waves. *Opt. Lett.* **1994**, *19*, 1287–1289.
51. Chen, Y.; Atai, J. Stability of fundamental solitons of coupled nonlinear Schrödinger equations. *Opt. Commun.* **1998**, *150*, 381–389.
52. Nistazakis, H.E.; Frantzeskakis, D.J.; Atai, J.; Malomed, B.A.; Efremidis, N.; Hizanidis, K. Multichannel pulse dynamics in a stabilized Ginzburg-Landau system. *Phys. Review. E Stat. Nonlinear Soft Matter Phys.* **2002**, *65*, 036605.
53. Atai, J.; Chen, Y. Nonlinear couplers composed of different nonlinear cores. *J. Appl. Phys.* **1992**, *72*, 24–27.
54. Atai, J.; Chen, Y. Nonlinear mismatches between two cores of saturable nonlinear couplers. *IEEE J. Quant. Electron.* **1993**, *29*, 242–249.
55. Archambault, J.L.; Russell, P.S.J.; Barcelos, S.; Hua, P.; Reekie, L. Grating-frustrated coupler: A novel channel-dropping filter in single-mode optical fiber. *Opt. Lett.* **1994**, *19*, 180–182.
56. Erdogan, T. Optical add-drop multiplexer based on an asymmetric Bragg coupler. *Opt. Commun.* **1998**, *157*, 249–264.
57. Aslund, M.; Poladian, L.; Canning, J.; de Sterke, C. Add-drop multiplexing by dispersion inverted interference coupling. *J. Light. Technol.* **2002**, *20*, 1585–1589.
58. An, H.; Ashton, B.; Fleming, S. Long-period-grating-assisted optical add-drop filter based on mismatched twin-core photosensitive-cladding fiber. *Opt. Lett.* **2004**, *29*, 343–345.
59. Shnaiderman, R.; Tasgal, R.S.; Band, Y.B. Creating very slow optical gap solitons with a grating-assisted coupler. *Opt. Lett.* **2011**, *36*, 2438–2440.
60. Li, H.; Deng, Z.; Huang, J.; Fu, S.; Li, Y. Slow-light all-optical soliton diode based on tailored Bragg-grating structure. *Opt. Lett.* **2015**, *40*, 2572–2575.
61. Shapira, Y.P.; Horowitz, M. Optical AND gate based on soliton interaction in a fiber Bragg grating. *Opt. Lett.* **2007**, *32*, 1211–1213.
62. Raphael Broderick, N.G.; de Sterke, C.M. Theory of grating superstructures. *Phys. Rev. E* **1997**, *55*, 3634–3646.
63. Khurgin, J.B. Light slowing down in Moiré fiber gratings and its implications for nonlinear optics. *Phys. Review. A At. Mol. Opt. Phys.* **2000**, *62*, 013821.
64. Janner, D.; Galzerano, G.; Valle, G.D.; Laporta, P.; Longhi, S.; Belmonte, M. Slow light in periodic superstructure Bragg gratings. *Phys. Review. E Stat. Nonlinear Soft Matter Phys.* **2005**, *72*, 056605.
65. Gnan, M.; Bellanca, G.; Chong, H.M.H.; Bassi, P.; De La Rue, R.M. Modelling of Photonic Wire Bragg Gratings. *Opt. Quantum Electron.* **2006**, *38*, 133–148.
66. Atai, J.; Malomed, B.A. Gap solitons in Bragg gratings with dispersive reflectivity. *Phys. Lett. A* **2005**, *342*, 404–412.
67. Royston Neill, D.; Atai, J.; Malomed, B.A. Dynamics and collisions of moving solitons in Bragg gratings with dispersive reflectivity. *J. Opt. A: Pure Appl. Opt.* **2008**, *10*, 085105.
68. Ahmed, T.; Atai, J. Bragg solitons in systems with separated nonuniform Bragg grating and nonlinearity. *Phys. Rev. E* **2017**, *96*, 032222.
69. Chowdhury, S.A.M.S.; Atai, J. Moving Bragg grating solitons in a semilinear dual-core system with dispersive reflectivity. *Sci. Rep.* **2017**, *7*, 4021.

70. Ahmed, T.; Atai, J. Effect of Dispersive Reflectivity on the Stability of Gap Solitons in Systems with Separated Bragg Grating and Nonlinearity. In Proceedings of the P4th International Conference on Photonics, Optics and Laser Technology, Rome, Italy, 27–29 February 2016, pp. 231–235.
71. Conti, C.; Trillo, S. Bifurcation of gap solitons through catastrophe theory. *Phys. Rev. E* **2001**, *64*, 036617.

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