

Refractive Index and Dispersion Measurement Principle with Polarization Change in Total Internal Reflection

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Abstract: Refractive index measurements have been an important task for a long time because that index plays an essential role in describing the optical properties of a material. Many methods have been developed to perform that task. Some of them use interferometry to achieve high precision. However, these configurations are complicated. Some measure the critical angle using simple structures, but their accuracy is unsatisfactory because it is difficult to judge the exact critical angle with intensity variations. Here, we propose several new schemes based on measuring the polarization change in the total internal reflection. The proposed method has the merits of simple structure and easy incident angle determination that gives the maximum phase change. Additionally, it is possible to find the material dispersion by measuring the wavelength dependence of the polarization ellipticity. Some useful formulas relating the refractive index to the maximum phase change are obtained. This work can provide valuable alternatives for refractive index measurement.

Keywords: refractive index measurement; total internal reflection; polarization change; material dispersion



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1. Introduction

Refractive indexing is one of the most important characteristic parameters of optical materials, especially transparent ones. It is defined as the ratio of the velocity of light in a vacuum and in the material. Many methods based on various principles [1] have been proposed to measure the refractive index of a material or liquid, such as prism coupling [2], interferometry [3], ellipsometry [4], Brewster angle [5], photoacoustic microscopy [6,7], and holography [8]. One method uses the total internal reflection (TIR) to achieve that measurement goal. It is noted that image reconstruction can be achieved by detecting the optical evanescent wave of TIR [7]. Usually, there are two categories utilizing TIR. The first measures the critical angle θ_c of a material by detecting the reflection or refraction intensity variation across the critical angle [9–11]. The refractive index can then be calculated via the relationship between the critical angle and the refractive index. Some useful refractometers, such as the Pulfrich type, utilize this principle [12]. The advantages of the θ_c measurement method are the following: simple structure, low cost, and easy handling. However, its disadvantages lie mainly in its time consumption and lower precision because it takes time to scan the incident angle. The reflected light intensity variation across the critical angles is difficult to judge precisely [10]. The second method measures the phase shift Φ carried by the reflected beam after TIR, and for that purpose, a phase meter [13] or an interferometer [14] is needed. Contrary to the θ_c -measurement method, the advantages of the Φ -measurement method are high-precision and time-saving because angle scanning is not needed. Usually, these configurations are more complicated or require sophisticated skills to perform the task. Of course, the cost is generally higher.

In this work, we propose several different approaches to measure the refractive index based on measuring the TIR polarization changes. The first method derives the analytical

equation between the refractive index and the incident angle to give the maximum phase change. Once that angle is measured by varying the incident angle experimentally, the refractive index can be calculated. The second method measures the polarization ellipticity for any incident angle. The refractive index can then be obtained using the relationship derived between them. Finally, a new way to measure material dispersion is also suggested. The details and merits of these schemes will be given in the next section.

The structure of the work is as follows: Section 1 is the introduction. Section 2 includes four subsections. The first reviews a conventional scheme that is a good contrast to the second method provided by us. The third and fourth subsections suggest two other configurations with different merits. The conclusions and discussion are presented in Section 3.

2. Theory and Numerical Results

As mentioned in the introduction, there are many different ways to determine the refraction index of a transparent material. Here, we review a famous historical method [15] because it is a good contrast with our method.

2.1. Method of Measuring Minimum Angular Deviation of a Prism

Figure 1a shows a typical dispersing prism with an apex angle α and refractive index n_1 (at a specified wavelength of the incident light). A parallel light beam is incident from the left with an incident angle θ_{i1} at the first interface of the prism. It leaves the second interface with an angle θ_{t2} , which is deviated from its original incident direction with an angular deviation δ . The symbols θ_{i1} and θ_{t2} indicate the refraction angle and the incident angle for the first and second refraction of the two interfaces, respectively. From the geometry of the prism triangle, we have

$$\alpha = \theta_{i1} + \theta_{t2}, \text{ and } \delta = \theta_{i1} + \theta_{t2} - \alpha. \quad (1)$$

Using some algebra and manipulation, we find that the angular deviation δ can be written as a function of θ_{i1} and α only. When δ reaches its minimum δ_m , the ray traverses the prism symmetrically, that is, parallel to its base ($\theta_{i1} = \theta_{t2}$). Under that situation, we have

$$\theta_{i1(m)} = (\delta_m + \alpha)/2 \text{ and } \theta_{t1(m)} = \alpha/2, \quad (2)$$

where m in the subscript indicates the minimum value for minimum δ_m , as shown in Figure 1b.

Applying Snell's law to the first refraction and assuming the medium around the prism is air or vacuum with a refractive index $n_2 = 1.0$, the prism's refractive index n_1 is

$$n_1 = \frac{\sin(\theta_{i1(m)})}{\sin(\theta_{t1(m)})} = \frac{\sin((\delta_m + \alpha)/2)}{\sin(\alpha/2)} \quad (3)$$

Thus, by varying θ_{i1} to find δ_m and knowing α , the refractive index of the prism can be determined. Figure 1b shows the angular deviation versus the incident for the case of $\alpha = 60^\circ$ and $n = 1.5$; the minimum δ_m is 37.2° at the incident angle $\theta_{i1} = 48.6^\circ$.

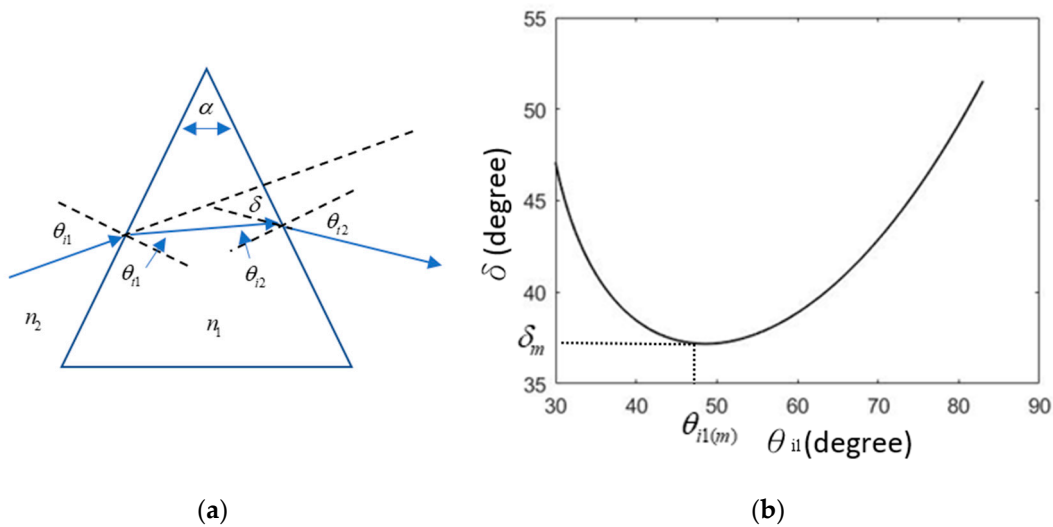


Figure 1. (a) Schematic and notations for a prism scheme. (b) Angular deviation versus the incident angle for the case of $\alpha = 60^\circ$ and $n_1 = 1.5$.

2.2. Method Measuring the Maximum Phase Change in TIR

Now, we consider another scheme proposed by us to measure the refractive index. Figure 2 illustrates a situation in which a light beam is incident into a transparent semi-circle cylinder (or a semi-sphere) with refractive index n_1 , and that of the surrounding medium is n_2 with $n_1 > n_2$. It is well known that when the incident angle is larger than the critical angle $\theta_c = \sin^{-1}(n_2/n_1)$, the total internal reflection (TIR) occurs. As TIR exists, the light beam reflects into the same material without refracting into the medium. Here, the semi-circle shape is chosen for convenience of discussion. The TIR is on the top interface without considering the refraction at the boundaries of the material (i.e., when the light wave enters and leaves the material). It is further assumed that some anti-reflection coating is applied to the boundaries (except the top surface); thus, all of the incident power aimed at the center of the semi-circle with angle θ_{in} is reflected and leaves the material at the same angle without taking the Fresnel loss at the boundaries into account. We also know that under TIR, the reflected light wave will gain different phase changes for different polarizations as follows [12]:

$$\tan\left(\frac{\Phi_p}{2}\right) = \frac{(\sin^2 \theta_{in} - n^2)^{1/2}}{n^2 \cos \theta_{in}}, \quad \tan\left(\frac{\Phi_s}{2}\right) = \frac{(\sin^2 \theta_{in} - n^2)^{1/2}}{\cos \theta_{in}}, \quad (4)$$

where Φ_p and Φ_s are the phase changes for p and s polarizations, respectively, and $n \equiv n_2/n_1$ is the refractive index ratio of n_2 and n_1 , which is less than 1. Figure 2b shows Φ_p (blue line) and Φ_s (red line) versus θ_{in} for the case of $n_1 = 1.5$ and $n_2 = 1.0$ ($n = 2/3$) with the critical angle $\theta_c = 41.8^\circ$. The phase changes start from zero at θ_c and increase monotonically to 180° when θ_{in} reaches 90° , and Φ_p is larger than Φ_s in the whole interval. Because the phase differences exist for the two polarizations, there is a polarization change if the incident wave polarization is not parallel with one of the polarizations. For example, as shown in Figure 3a, if an incident wave is linearly polarized 45° with respect to the x axis, its polarization state will be an elliptical polarization after the TIR. The ellipticity depends on the incident angle and refractive index ratio n . This feature is used as a quarter wave plate that transforms a linear polarization into a circular polarization, called a Fresnel rhomb [12]. Its advantage over the traditional birefringent wave plate is that it is much less wavelength-sensitive. The phase difference is defined as $\Delta\Phi = \Phi_p - \Phi_s$, and it can be written as

$$\tan\left(\frac{\Delta\Phi}{2}\right) = \frac{\cos \theta_{in} (\sin^2 \theta_{in} - n^2)^{1/2}}{\sin^2 \theta_{in}} \quad (5)$$

Figure 3b shows $\Delta\Phi$ for the case of $n = 2/3$ ($n_1 = 1.5$ and $n_2 = 1.0$). Note that at the two incident angles, 50.23° or 53.26° , $\Delta\Phi$ reaches 45° ; thus, after two TAR reflections, the total phase change accumulated is 90° (i.e., quarter wave), which is the Fresnel rhomb principle. It is also interesting to find that there is a maximum for the $\Delta\Phi$; in this case, it is $\Delta\Phi_{(M)} = 45.3^\circ$ for $\theta_{in(M)} = 51.67^\circ$, where the (M) in the subscript indicates the maximum. By differentiating the right side of Equation (5) with respect to θ_{in} and setting it to zero to find $\theta_{in(M)}$, we can have

$$\Delta\Phi_{(M)} = 2 \cdot \tan^{-1}\left(\frac{1-n^2}{2n}\right) \text{ at } \theta_{in(M)} = \sin^{-1}\left[\left(\frac{2n^2}{1+n^2}\right)^{1/2}\right]. \quad (6)$$

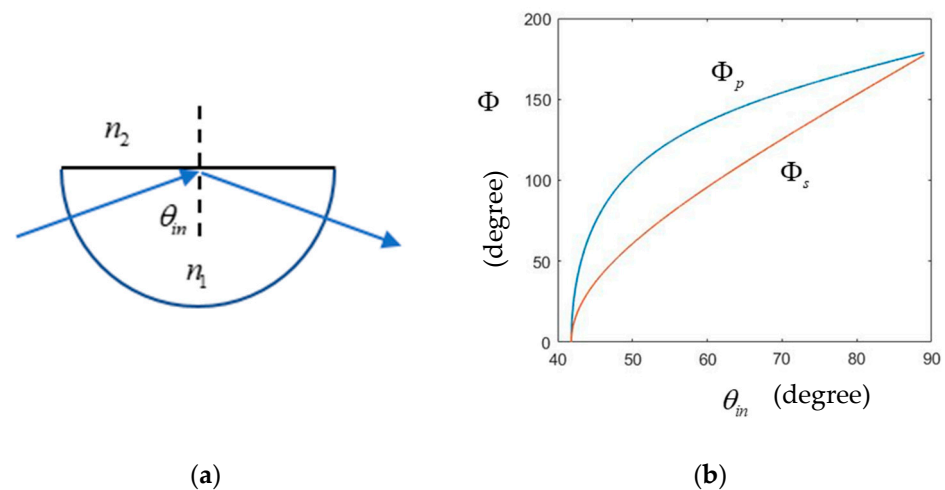


Figure 2. (a) Schematic and notations for a transparent semi-circle cylinder with TIR. (b) Plot for phase vs. incident angle for the two polarizations under TIR.

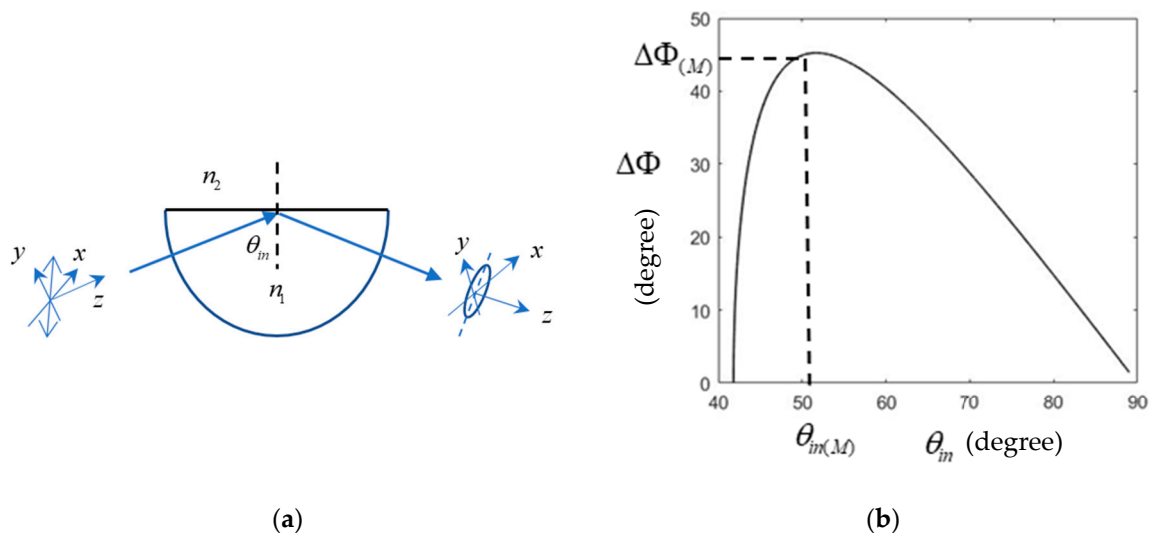


Figure 3. (a) Schematic for the polarization state change, from a linear one to an elliptical one, after TIR. (b) Plot for phase difference vs. incident angle under TIR. The wave is propagating along the z direction, and y and x are the parallel (p-polarization) and vertical (s-polarization) directions, respectively.

Equation (6) is of value because it gives analytical results for $\theta_{in(M)}$ and $\Delta\Phi_{(M)}$. It is interesting to compare Figure 3b with Figure 1b, where both have extreme incident angle values for giving minimum deviation angle or maximum phase change. It is natural to

expect this property can be used to measure a unknown refractive index as in the prism case. Since the phase change leads to polarization state change, we need to modify the configuration properly to detect it. Considering Figure 4, without plotting the hemi-circle boundary, there are two polarizers added outside the material, and the Jones matrices [16] are employed to analyze the polarization change. As shown in the figure, the light wave is incident along the z direction. There are two polarizers, P_1 and P_2 , which make the angle θ_1 and θ_2 respect their local x coordinate, respectively. The Jones matrices indicated at the bottom of the figure for P_1 , TIR, and P_2 are $P_1(\theta_1)$, $TIR(\Delta\Phi)$, and $P_2(\theta_2)$, respectively. Since we need to polarize the light beam at $\theta_1 = 45^\circ$ to gain $\Delta\Phi$, we can assume, without losing generality, that the Jones vector of the incident light field is $\vec{E}_{in} = (1/\sqrt{2}) \cdot [1 \ 1]^T$, a unit vector also polarized at $\theta_1 = 45^\circ$ and the T in the superscript is a transport operation. Using the cascade matrices manipulation, the output field is

$$\vec{E}_{out} = P_2(\theta_2) \cdot TIR(\Delta\Phi) \cdot P_1(\theta_1 = 45^\circ) \cdot \vec{E}_{in} = \frac{1}{\sqrt{2}} \begin{bmatrix} \cos^2 \theta_2 + \sin \theta_2 \cos \theta_2 \\ \sin \theta_2 \cos \theta_2 + e^{j\Delta\Phi} \sin^2 \theta_2 \end{bmatrix} \quad (7)$$

Finally, we can obtain the output intensity as follows:

$$I_{out}(\theta_2, \Delta\Phi) = \vec{E}_{out} \cdot \vec{E}_{out}^* = \frac{1}{2} [1 + \sin(2\theta_2) \cdot \cos(\Delta\Phi)] \quad (8)$$

It is noted that when θ_2 of P_2 is at the angle of 45° or 135° , the output intensities are denoted as I_{45} or I_{135} , respectively.

$$\begin{aligned} I_{out}(\theta_2 = 45^\circ, \Delta\Phi) &\equiv I_{45} = \frac{1}{2} [1 + \cos(\Delta\Phi)] = \cos^2\left(\frac{\Delta\Phi}{2}\right) \\ I_{out}(\theta_2 = 135^\circ, \Delta\Phi) &\equiv I_{135} = \frac{1}{2} [1 - \cos(\Delta\Phi)] = \sin^2\left(\frac{\Delta\Phi}{2}\right) \end{aligned} \quad (9)$$

Taking I_{135} measurement as an example, if we keep increasing the incident angle θ_{in} from θ_c and detect the intensity I_{135} , it reaches its maximum when $\theta_{in} = \theta_{in(M)}$, as can be seen from Figure 3b and Equation (9). This behavior is shown in Figure 5.

Consequently, by varying the incident angle, the maximum of I_{135} can be reached to determine $\theta_{in(M)}$, and the right half of Equation (6) can be used to calculate n as

$$n = \left(\frac{\sin^2 \theta_{in(M)}}{2 - \sin^2 \theta_{in(M)}} \right)^{1/2} \quad (10)$$

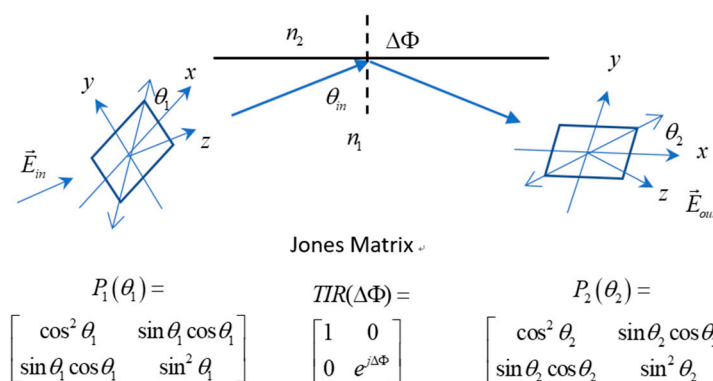


Figure 4. Configuration, coordinates, and Jones matrices of each element.

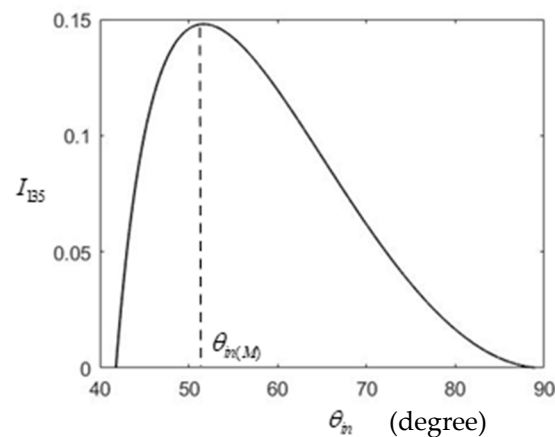


Figure 5. Plot of output intensity I_{135} versus θ_{in} for the case of $n = 2/3$.

Since $n = n_2/n_1$ is the refractive index ratio, we need to know the n_2 of the surrounding medium. If we take $n_2 = 1.0$ for air, the material's refractive index is $n_1 = 1/n$. This is compared with the scheme that determines the refractive index by finding the critical angle, which is not easy to exactly determine. This method works well by finding $\theta_{in(M)}$, which should be easier to determine more precisely because this angle is usually not close to the critical angle. It is noted that a previous work [17] calculated the refractive index by measuring the maximum phase change. They used a polarization camera to detect it. In our scheme, the refractive index is calculated from the incident angle that gives maximum phase change. Their method utilized a more complicated setup, but it can reach high accuracy. Our measurement configuration is simpler, and the precision depends on the resolution of the rotation stage and the intensity detector.

2.3. Method for Measuring Ellipticity of TIR (without Scanning the Incident Angle)

In the last section, we successfully find the refractive index by looking for the incident angle that gives maximum phase change in TIR. This is similar to the method in Section 2.1, which looks for the incident angle that gives the minimum deviation angle. In this section we would like to introduce another possibility in which no incident angle scanning is necessary. This method may be considered an advanced or improved version. This scheme can be seen by examining Equation (5), giving that phase change $\Delta\Phi$ only depends on the incident angle θ_{in} and the index ratio n . Thus, it is possible to find n if $\Delta\Phi$ can be found for a given θ_{in} without using a phase meter or an interferometer. Let us show how it proceeds.

As shown in Figure 4 and Equation (9), this configuration can be used to measure I_{45} and I_{135} by simply rotating the θ_2 of P2 to 45 and 135 degrees, respectively. Note that by taking the square root of the ratio I_{135}/I_{45} and using Equation (5), we have

$$\varepsilon \equiv \frac{|\vec{E}_{135}|}{|\vec{E}_{45}|} = \sqrt{\frac{I_{135}}{I_{45}}} = \frac{\sin(\Delta\Phi/2)}{\cos(\Delta\Phi/2)} = \tan\left(\frac{\Delta\Phi}{2}\right) = \frac{\cos\theta_{in}(\sin^2\theta_{in} - n^2)^{1/2}}{\sin^2\theta_{in}} \quad (11)$$

where ε is the ellipticity of the polarization ellipse shown in Figure 3a. The ellipticity is defined as the ratio of the short axis to the long axis of the polarization ellipse of the field, which is the ratio of field amplitude taken at 135° and 45° , respectively. After finding ε at any selected incident angle θ_{in} , the refractive index ratio, from Equation (11), is obtained as

$$n = \left[\sin^2\theta_{in} - \left(\frac{\varepsilon \sin^2\theta_{in}}{\cos\theta_{in}} \right)^2 \right]^{1/2} \quad (12)$$

Thus, the material refractive index can be determined. The advantage of this method is that we do not need to vary the incident angle to find n . However, it is suggested to select the incident angle carefully to have a larger ε value (i.e., bigger I_{135}), in order to increase the signal intensity or S/N ratio.

2.4. Dispersion Measurement with Polychromatic Light

In the last section, we present a scheme for obtaining the refractive index by finding the ellipticity of the polarization. Here, we would like to show how to use this feature to obtain the dispersion of the material by employing a polychromatic source [18]. It is known that material dispersion is a property such that the refractive index depends on the wavelength of the incident light wave, i.e., the refractive index n is actually a function of wavelength λ , written as $n(\lambda)$ from now on. Considering that we can prepare an incident flat-top spectrum $S_{in}(\lambda)$ in the dispersion interval of interest from λ_1 to λ_2 , as shown in Figure 6. As illustrated in Figure 7, it is incident into a material with $n_1(\lambda)$ and with $n_2 = 1.0$; thus, the index ratio is $n(\lambda) = 1/n_1(\lambda)$. Initially, this spectrum is polarized at 45° as in the last section; however, because of the material dispersion, each wavelength component experiences a different amount of the polarization change $\Delta\Phi(\lambda)$. Consequently, the output spectrum $S_{out}(\lambda)$ has different ellipticity of polarization ellipse for different wavelengths, as shown in the right of the figure. Similarly, as conducted in the last section, this time, a spectrometer is utilized to detect $S_{out}(\lambda)$ by setting the polarization angle of P2 to 45 and 135 degrees, and they are denoted as $S_{45}(\lambda)$ and $S_{135}(\lambda)$, respectively. The ellipticity dependence of λ , as seen in Equation (11), is the square root of the ratio of $S_{135}(\lambda)/S_{45}(\lambda)$, that is

$$\varepsilon(\lambda) = \sqrt{\frac{S_{135}(\lambda)}{S_{45}(\lambda)}} = \frac{\sin(\Delta\Phi(\lambda)/2)}{\cos(\Delta\Phi(\lambda)/2)} = \tan\left(\frac{\Delta\Phi(\lambda)}{2}\right) = \frac{\cos\theta_{in}(\sin^2\theta_{in} - n(\lambda)^2)^{1/2}}{\sin^2\theta_{in}} \quad (13)$$

Consequently, as in Equation (12), the dispersion of the material $n_1(\lambda) = 1/n(\lambda)$ is

$$n_1(\lambda) = [n(\lambda)]^{-1} = \left[\sin^2\theta_{in} - \left(\frac{\varepsilon(\lambda) \sin^2\theta_{in}}{\cos\theta_{in}} \right)^2 \right]^{-1/2} \quad (14)$$

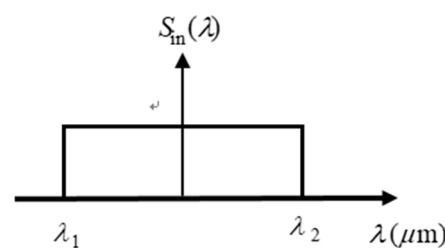


Figure 6. An incident spectrum with a flat-top distribution.

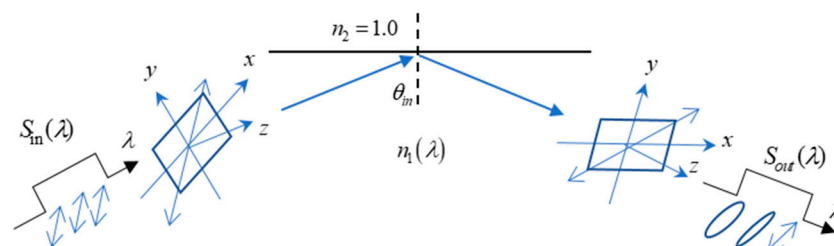


Figure 7. Schematic for the illustration of polarization ellipse change for different spectrum wavelength components.

3. Conclusions and Discussion

Several original schemes were proposed to measure the refractive index n of a material based on the TIR. After reviewing the traditional prism method by looking for the minimum deviation incident angle, our first scheme illustrated how n could be determined by measuring the maximum phase change angle. This can be achieved because a valuable relationship between the incident angle and n is established when the maximum phase change occurs. Compared to the prism method, the advantage of the proposed method is that the apex angle is not needed. The second method we suggested determines the refractive index by measuring the TIR polarization ellipticity at an arbitrary incident angle. The merit of this method is that it can save time because the incident angle does not need to be scanned. Another advantage is that the ellipticity comes from the intensity ratio at two different output polarization angles. We do not need to worry about the absolute value of the intensity, which may be affected by some factors such as the partial reflection at the material boundaries or the absorption of the polarizers. The third method determines the material dispersion with a polychromatic light by measuring the polarization ellipticity wavelength dependence. This is a simple way to obtain material dispersion information. Although incident angle scanning is not required, it is still suggested to select it close to the $\theta_{in(M)}$ in order to have a larger ellipticity value. It is noted that the above schemes can work only if the output intensities can be detected well (i.e., signal-to-noise ratio is high enough). Since there is no loss in the TIR and the Fresnel losses at the boundaries are ignored, material absorption is the only factor that affects the output intensities. Thus, a highly transparent material with lower absorption and smaller thickness is more favorable to utilizing these methods.

Finally, we may say something about the accuracy or the precision of above-mentioned schemes. Our methods depend on how accurately or precisely the angle $\theta_{in(M)}$ or a specified angle can be measured; thus, the uncertainty comes mainly from the precision of the rotation stage varying or selecting the incident angle. It is helpful to estimate this effect based on some available information from existing products or from published works. First, we need to know how the uncertainty of the angle measurement propagates and affects that of the refractive index calculation. From Equation (11), we have

$$\Delta n_1 \approx \frac{2 \sin \theta_{in(M)} \cdot \cos \theta_{in(M)}}{\left(2 - \sin^2 \theta_{in(M)}\right)^{1/2} \cdot \sin^3 \theta_{in(M)}} \Delta \theta_{in(M)} \quad (15)$$

where $n_2 = 1.0$ is assumed in deriving Equation (15). Now, considering the usual transparent glass with a refractive index in the range of 1.4–1.7, the $\theta_{in(M)}$ falls within the interval of 45–55 degrees, which makes the term in front of $\Delta \theta_{in(M)}$ in Equation (15) have a value of about 1.6–2.4. This means that the uncertainty in the refractive index of the glass is in about the same order as the uncertainty in the incident angle measurement (in radians). If we take a rotation stage (BOCIC MRS-101, used in reference [5]) with precision about 5×10^{-3} degree ($\sim 10^{-4}$ radian) as an example, the refractive index uncertainty is also about 10^{-4} , which is comparable to the result in ref citation [5]. Of course, for real experiments, the uncertainty in the polarizer angle and repeatability should also be considered.

The second uncertainty may come from the precision of the detector, or the sensor used that judges if the maximum of I_{135} is reached in the first method. In the second method, it is the accuracy of the polarization ellipticity (i.e., the intensity values of I_{135} and I_{45}) affecting that of the index measurement. In the third scheme, a spectrometer is utilized to measure the spectrum to infer the dispersion message; thus, the accuracy of the spectrometer should be taken into account. To sum up, this work introduced some advantageous schemes to measure the refractive index or material dispersion, which should be of interest to the related fields.

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