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High-Performance NOON State from a Quantum Dot Single Photon for Supersensitive Optical Phase Measurement

Zhixuan Rao ^{1,†}, Jiawei Yang ^{1,†}, Luyu Liu ¹ and Ying Yu ^{1,2,*}

¹ State Key Laboratory of Optoelectronic Materials and Technologies, School of Electronics and Information Technology, Sun Yat-sen University, Guangzhou 510006, China; raozhx@mail2.sysu.edu.cn (Z.R.); yangjw8@mail2.sysu.edu.cn (J.Y.); liuly75@mail2.sysu.edu.cn (L.L.)

² Hefei National Laboratory, Hefei 230088, China

* Correspondence: yuying26@mail.sysu.edu.cn

† These authors contributed equally to this work.

Abstract: We investigate the utilization of advanced single photons produced by quantum dots (QDs) in a microcavity for quantum metrology. Through the integration of lateral excitation and the Purcell effect in an Fabry–Perot microcavity, we realized single-photon emission with an extraction efficiency of 46.39%, high purity of 96.91%, and high indistinguishability of 98.32%. Our QD-generated single photons enabled the creation of high-quality NOON states ($N = 2$) for phase measurement, yielding an interference contrast of 79.79% and surpassing the standard quantum limit (SQL) with phase super-sensitivity. Our results underscore the immense potential of QD-derived single photons for propelling quantum metrology forward, facilitating enhanced precision measurements across diverse applications.

Keywords: single photon source; NOON state; quantum metrology; phase measurement

1. Introduction

Quantum metrology [1] primarily involves using quantum resources for precise estimation of physical parameters, particularly the optical phase [2]. By employing quantum sources such as squeezed states [3–5] and entangled states [6–9] generated via high-performance single photons (SPS), it is possible to surpass the standard quantum limit (SQL) dictated by fundamental quantum mechanics principles [10] such as the Heisenberg uncertainty principle, which scales inversely with the square root of the number of resources (N). This kind of uncertainty in measurement can also be understood as photon number fluctuations originating from scattering upon reaching the detector; hence, it is also referred to as the shot-noise limit (SNL). By introducing quantum resources, the measurement precision limit is enhanced to $\sqrt{N}$ times the SQL, also known as the Heisenberg limit (HL). While entangled states produced through nonlinear processes have demonstrated phase super-sensitivity beyond the SQL in various configurations [7,11], challenges such as the loss and dephasing of quantum states [12,13] have limited the realization of a true quantum advantage, often resulting in only achieving phase super-resolution, which shortens the de Broglie wavelength [14–16]. Self-assembled semiconductor quantum dots (QDs) with their atom-like, two-level structure present a promising avenue for creating high-purity, indistinguishable single-photon sources, particularly when employing rapid-pulsed resonant excitation to mitigate dephasing and time jitter [17]. Additionally, their compatibility with semiconductor technology makes them highly attractive for quantum light sources with efficient photon generation. Embedding QDs into microcavities enhances the coupling between the QD dipole and the confined electromagnetic mode, leading to improved photon extraction efficiency and an increased probability of emitting coherent photons with indistinguishability via the Purcell effect [18–20]. Resonant fluorescence emitted by QD, minimally affected by dephasing effects, and exhibiting near-unity indistinguishability



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broadens the applications of QD SPS in multiphoton interference [21] and quantum entanglement [22]. Recent advances include the consecutive generation of two-photon NOON states [8] and three-photon GHZ states [9] based on QD SPS, demonstrating remarkable phase super-sensitivity. Compared to the 18-qubit entangled state generated by a probabilistic single-photon source relying on parametric down-conversion [23], the number of entangled photons achievable with a deterministic single-photon source remains low, which restricts further improvement in measurement accuracy in quantum metrology. To obtain a multiphoton entangled state, achieving QD SPS with high collection efficiency and high indistinguishability becomes crucial [24].

Despite advancements, the collection of resonant fluorescence from QDs still necessitates the use of polarizers [3,25], which limits its brightness. In this study, we propose side excitation of QD SPS within a Purcell-enhanced microcavity [26], which resulted in resonant single photons with a high indistinguishability of 98% and minimal single-photon loss, while extraction efficiency reached 46.39%. By adopting this technique, we were able to prepare high-performance NOON states with $N = 2$. Furthermore, by injecting these NOON states into a phase-tunable Mach–Zehnder interferometer (MZI), we achieved an interference contrast of 79.79%, thereby demonstrating phase super-sensitivity.

2. Principle

The NOON state is an N -photon entangled state with two orthogonal modes, which can be expressed as

$$|\Psi\rangle_{\text{NOON}} = \frac{1}{\sqrt{2}}(|N, 0\rangle_{A,B} + |0, N\rangle_{A,B}) \quad (1)$$

Injecting the path-entangled NOON state into MZI, the $|\Psi\rangle_{\text{NOON}}$ state with the phase evolution can be expressed as

$$|\Psi'\rangle_{\text{NOON}} = \frac{1}{\sqrt{2}}(|N, 0\rangle_{A,B} + e^{iN\varphi}|0, N\rangle_{A,B}) \quad (2)$$

With appropriate observables, such as the coincidence of single-photon detectors at the two output ports of MZI, we can obtain [27]

$$P_{D_1,D_2} = |N, 0\rangle\langle N, 0| + |0, N\rangle\langle 0, N| \propto \frac{1}{2}[1 + \cos(N\varphi)] \quad (3)$$

Consequently, we can obtain the measurement error of the observable as

$$\Delta P = \sqrt{\langle P_{D_1,D_2}^2 \rangle - \langle P_{D_1,D_2} \rangle^2} \propto |\sin(N\varphi)| \quad (4)$$

The error in phase estimation can be expressed as [27]

$$\Delta\varphi = \Delta P / \frac{\partial P}{\partial \varphi} \propto \frac{1}{N} \quad (5)$$

For an ideal NOON state, the limit achievable for the phase measurement is $1/N$ (HL), which represents an improvement of $\sqrt{N}$ times compared to the SQL. In this study, we beat the SQL and achieved phase super-sensitivity by using the high-performance NOON state.

For classical interference systems, a phase estimation can be achieved by observing the variation in interference fringes in CCD imaging as a function of phase shift. Upon introducing quantum resources, to prevent quantum effects from being overwhelmed by noise, it is necessary to use avalanche photodiodes (APDs) to detect coincidence events at two outputs of the interferometer. To establish the relationship between coincidence events

and phase evolution, we quantized the beam splitter (BS) and phase shifter (PS) within the interferometer as follows:

$$\widehat{PS}(\varphi) = \begin{pmatrix} e^{i\varphi} & 0 \\ 0 & 1 \end{pmatrix} \widehat{BS} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \quad (6)$$

Here, φ represents the phase shift introduced by the PS, which is typically implemented by rotating a thin glass plate in the experiment. The phase evolution process through the interferometer can be described as $\widehat{MZI}(\varphi) = \widehat{BS}\widehat{PS}\widehat{BS}$. For two independent single photons incident from the two input ports of the interferometer, which can be denoted as $\widehat{a}^\dagger|0\rangle$ and $\widehat{b}^\dagger|0\rangle$ ($|0\rangle$ represents the vacuum state), the output state after the evolution of $\widehat{MZI}$ can be expressed as [8]

$$|\Psi\rangle = \frac{1}{4} \left[-i(e^{-2i\varphi} - 1)\widehat{e}^\dagger\widehat{e}^\dagger + i(e^{-2i\varphi} - 1)\widehat{f}^\dagger\widehat{f}^\dagger - (e^{-i\varphi} - 1)\widehat{e}^\dagger\widehat{f}^\dagger + (e^{-i\varphi} + 1)\widehat{f}^\dagger\widehat{e}^\dagger \right] |00\rangle \quad (7)$$

where $\widehat{e}(\widehat{f})$ and $\widehat{e}^\dagger(\widehat{f}^\dagger)$ represent the modes of individual single photons exiting from ports $e(f)$, respectively. Equation (7) indicates that non-entangled photon pairs cannot halve the oscillation period of the interference fringes.

For two identical single photons undergoing biphoton interference in the first beam splitter, the output state evolves as [8]

$$|\Psi\rangle_{NOON} = \frac{1}{4} \left[-i(e^{-2i\varphi} - 1)\widehat{e}^\dagger\widehat{e}^\dagger + i(e^{-2i\varphi} - 1)\widehat{f}^\dagger\widehat{f}^\dagger - 2(e^{-2i\varphi} + 1)\widehat{e}^\dagger\widehat{f}^\dagger \right] |00\rangle \quad (8)$$

From the term $\widehat{e}^\dagger\widehat{f}^\dagger$ in Equation (8), we can extract the relationship between coincidence events at the two output ports of the interferometer and the phase shift as follows:

$$P_{D1, D2} = \frac{1}{2}(\cos(2\varphi) + 1) \quad (9)$$

Inputting the NOON state generated by high indistinguishability single photons from quantum dots into the interferometer, the coincidence events exhibit periodic oscillations with a period of π as the phase varies.

In the actual experiments, the HOM interference contrast of indistinguishability needed to be taken into account. Therefore, because the coincidence counting theory needed to contain two components—distinguishable and completely indistinguishable—we modified Equation (9) as follows:

$$P_{D1, D2} = P_{V_{HOM}} + P_{1-V_{HOM}} = \frac{1}{4}[(3 - V_{HOM}) + (1 + V_{HOM})\cos(2\varphi)] \quad (10)$$

The contrast of the interference fringes is defined as $C = \frac{P_{max} - P_{min}}{P_{max} + P_{min}} = \frac{1 - V_{HOM}}{3 - V_{HOM}}$. Therefore, the higher the polarization-dependent interference contrast in the HOM experiment (V_{HOM}), the more perfect the preparation of the NOON state and the higher the precision of phase measurement will be. This means that, whenever possible, we needed to obtain single photons with high indistinguishability and purity.

3. Results

3.1. Preparation of the High-Performance NOON State

To achieve high-performance single photons, we introduced a single QD deterministically coupled to a Purcell-enhanced defect-type Fabry–Perot (FP) microcavity [25]. This microcavity consisted of 7 pairs of SiO₂/TiO₂ top distributed Bragg reflectors (DBRs) and 46 pairs of GaAs/Al_{0.95}Ga_{0.05}As bottom DBRs, with low-density InAs/GaAs QDs embedded at the center. A parabolic-shaped lens defect imposed lateral confinement on the emission of QDs, leading to a significant reduction in the mode volume of the optical

field. The utilization of dielectric layers as the top DBR allowed us to achieve good curvature, even at thicknesses of a few micrometers. The quasi-Gaussian mode field of the 2λ -GaAs film (the transverse light field in Figure 1a) facilitates low-loss long-distance laser transmission within the sample (λ represents the wavelength of the material), while the design of the top and bottom DBRs facilitated directional emission of single photons (the longitudinal light field in Figure 1a). By employing side excitation, where the pump laser and signal propagation directions were spatially orthogonal, we achieved low-loss resonant fluorescence of the QD. By adjusting the geometrical parameters of the defect cavity, we achieved a maximum Q factor of 15614, with the highest extraction efficiency of the single photon source reaching approximately 94.9% under the conditions of $B = 4 \mu\text{m}$, $S = 480 \text{ nm}$, and $H = 350 \text{ nm}$. Further details of the sample can be found in reference [25].

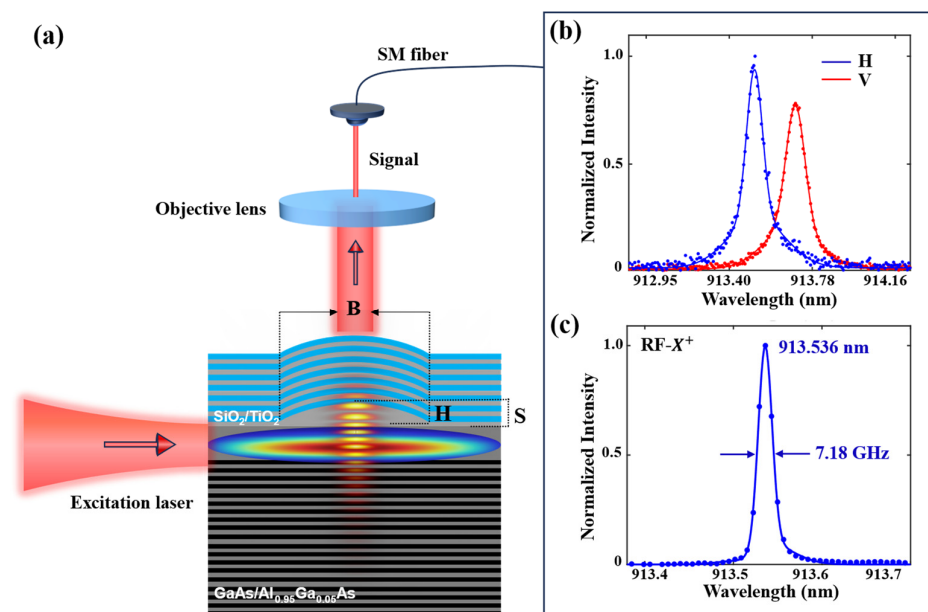


Figure 1. (a) The schematic diagram of sample structure and side excitation principle. The sample primarily consisted of 7 pairs of SiO₂/TiO₂ top DBRs and 46 pairs of GaAs/Al_{0.95}Ga_{0.05}As bottom DBRs. The laterally incident laser was coupled to the GaAs film waveguide in the middle of the cavity and excited the QD. The QD signal emitted from the top DBR and was collected by the objective lens into a single-mode optical fiber. The transverse and longitudinal optical fields in the center of the sample represented the waveguide mode and QD light field, respectively. (b) Characterization of the cavity mode PL spectra. The wavelengths of two orthogonally polarized cavity modes are 913.536 nm and 913.756 nm, with a cavity mode splitting of 79.01 GHz. (c) Resonant fluorescence PL spectra of a charged exciton. The QD wavelength resonated with the cavity mode H; and underside excitation, most of the pump laser is filtered out.

The optical fiber was securely positioned on a three-dimensional position stage with metal tape adjacent to the sample. The laser, coupled with the waveguide mode of the cavity, propagated laterally within the waveguide and excited the QD along its propagation path. Both the sample and the fiber were placed in a liquid-helium-cooled cryostat, and the resonance fluorescence, perpendicular to the direction of the excitation laser, was collected into the single-mode fiber through an objective lens with NA ~ 0.65 . This excitation scheme significantly reduced the loss of single photon during collection, enabling us to achieve a high extraction efficiency of 87% in our previous work [26].

We characterized the photoluminescence (PL) spectra of the cavity mode of the defect cavity (Figure 1b): the asymmetry of the semiconductor material resulted in two polarized-orthogonal non-degenerate modes, a phenomenon also observed in open cavities [28]. At a low temperature of 7.8 K, these modes appeared at wavelengths of 913.536 nm and 913.756 nm, with linewidths of 0.105 nm ($\delta\omega_H = 37.72 \text{ GHz}$) and 0.141 nm ($\delta\omega_V = 50.63 \text{ GHz}$), respectively,

which corresponded to Q values of 8700 and 6481. Notably, the wavelength of the QD charged exciton (CX) aligned with the stronger cavity mode H. With a large cavity mode splitting of 79.01 GHz, we ultimately obtained a highly polarized single photon with a polarization degree of 80.43% (Figure 1c).

As a result of the Purcell effect, the lifetime of the CX was shortened to 97 ps (red curve in Figure 2a), corresponding to a Purcell factor $F_p \sim 9$. Additionally, we achieved a high single-photon count rate of 3.46 MHz (Figure 2b) with the low-loss optical set-up and obtained a high extraction efficiency of 46.39% (the specific characterization process of extraction efficiency can be obtained from reference [26]). As the pump pulse needed to be coupled into the waveguide from the cleaved facet of the sample laterally, a high pulse power of 75 μW was required for the “ π ” pulse. Benefiting from the spatially orthogonal filtering scheme of side excitation, we achieved a value of $g^{(2)}(0) = 0.0387$ for the zero-time delay of the second-order autocorrelation function, corresponding to a single photon purity of 96.13% (Figure 3a). The slight impurity in the single photon originates mainly from scattered light resulting from the lateral propagation of the laser through the Au marker; further discussion can be found in reference [26]. Furthermore, the coherence of the single photons was measured using a Hong–Ou–Mandel (HOM) interferometer. We used a pulsed laser consisting of two picosecond pulses, separated by 2.5 ns, emitted every 12.48 ns. We observed a standard pattern of five peaks in the second-order correlation function, corresponding to possible coincidence events. Only identical single photons separated by 2.5 ns can undergo two-photon interference showing antibunching at zero-time delay (A_3 in Figure 3b,d), while orthogonally polarized photon pairs exhibit bunching. The QD coupled to the defect cavity showed significant HOM interference contrast, demonstrating superior single-photon indistinguishability. By calculating the second-order autocorrelation function at zero-time delay, we obtained the experimentally measured raw indistinguishability as $V_{raw} = 1 - \frac{g_{\parallel}^{(2)}(0)}{g_{\perp}^{(2)}(0)} = 78.97\%$ (Figure 3b). Considering the effects of the HOM interferometer contrast and single-photon purity [29], we corrected the indistinguishability to $V_{HOM} = 88.84\%$. The utilization of the side excitation scheme allowed us to achieve single photons with high efficiency, high purity, and high indistinguishability.

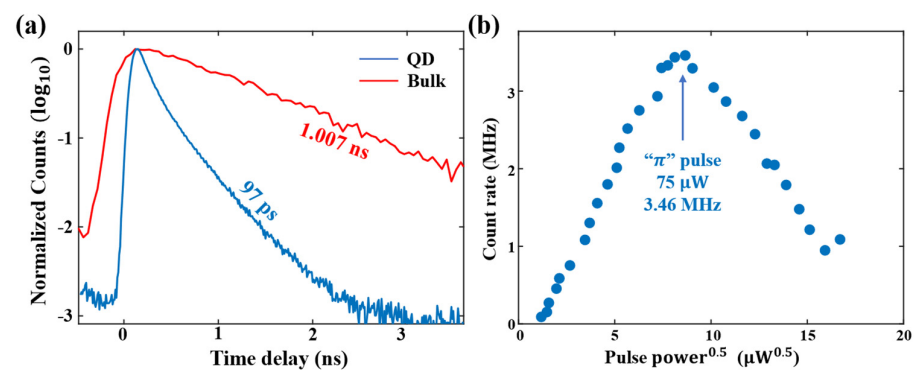


Figure 2. (a) Characterization of a resonant single-photon lifetime: To assess the magnitude of the Purcell factor, we probed the lifetimes of quantum dots in a planar cavity (black curve). Under the influence of the Purcell effect, the lifetime of a single-photon coupled in the defect cavity was shortened by nearly an order of magnitude (red curve). (b) Rabi oscillations of resonant fluorescence: The count rate of single photons exhibited oscillations with increasing excitation pulse power, reaching a maximum of 3.46 MHz under “ π ”—pulse excitation. An increasing trend in the single-photon count rate was observed when the square root of the pulse power was twice that of the “ π ” pulse.

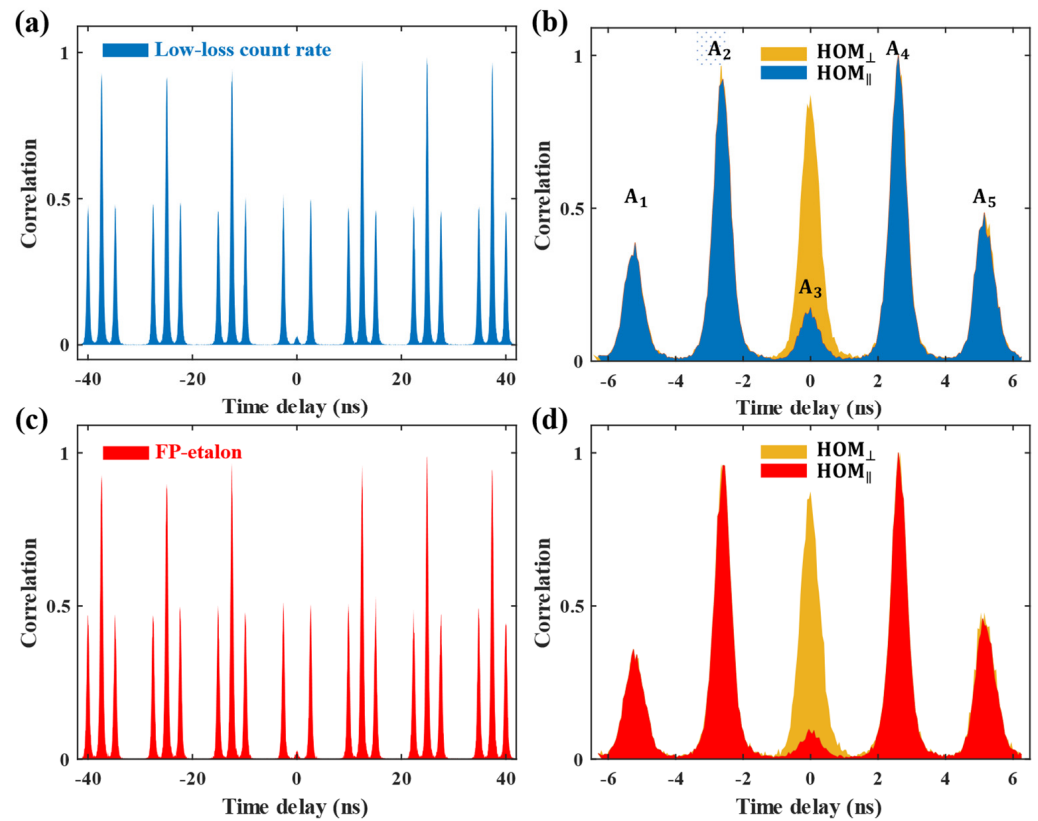


Figure 3. Characterization of single-photon purity and indistinguishability. (a,c) Single-photon purity characterization: For the condition of low-loss photon emission, $g^{(2)}(0) = 0.0387$ corresponded to a purity of 96.13% (blue). With the addition of an FP-etalon, $g^{(2)}(0) = 0.0309$ corresponded to a purity of 96.91% (red). (b,d) Characterization of single-photon indistinguishability: Emitting two single photons at an interval of 2.5 ns every 12.48 ns, the second-order autocorrelation function under the HOM experiment showed the standard five peaks. For the orthogonal polarization, a clear contrast at zero time delay was observed, resulting in an indistinguishability of $V_{raw, low-loss} = 1 - \frac{g_{\parallel}^{(2)}(0)}{g_{\perp}^{(2)}(0)} = 1 - \frac{0.0750}{0.3562} = 78.94\%$ (blue) for the low-loss condition, while $V_{raw, FP-etalon} = 1 - \frac{0.0410}{0.3612} = 88.84\%$ (red).

Under the effect of two-photon interference [30], two identical single photons merged at the first beamsplitter (BS₁ in Figure 4a), traveling along the same path to create the path-entangled NOON state ($N = 2$). When using the NOON state generated within the QD system for optical phase measurement, the precision relied on the fidelity of the entanglement state. To obtain a high-performance NOON state, we further optimized the indistinguishability of single photons. We used an FP-type Etalon with a linewidth of 50 GHz to further filter out the laser at the expense of a 50% single-photon count loss, thereby reducing the resonance fluorescence linewidth from 7.18 GHz to 6.28 GHz and mitigating the influence of laser sidebands. This refinement resulted in an enhanced purity of 96.91% ($g^{(2)}(0) = 0.0309$) (Figure 3c), while concurrently significantly boosting the indistinguishability of single photons, where $V_{raw-FP} = 88.65\%$ (Figure 3d) and $V_{HOM-FP} = 98.32\%$ after correction, signifying the successful preparation of a high-performance NOON state.

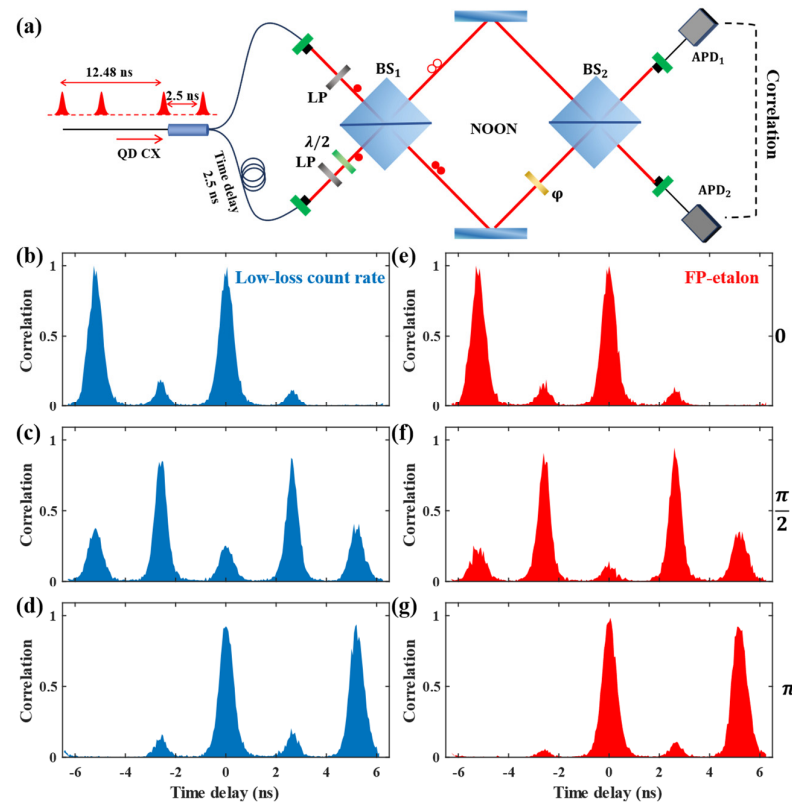


Figure 4. Phase measurement with a NOON state. The second–order autocorrelation function of MZI output: Peak A3 exhibited oscillations with a period of π , while A1 and A5 exhibited oscillations with a period of 2π . (a) Optical setup of phase measurement: Identical photon pairs met at BS₁ forming the NOON state, and after passing through the MZI, the relative phase evolution of the two paths of the interferometer could be estimated. The second–order correlation function at the two outputs of BS₁ could be used to assess the indistinguishability of single photons. (b–d) Interference patterns for the low–loss condition: For the phase shift of $\pi/2$, we obtained a value of 0.1252 for the minimal coincident count $I_{\min}$, while the maximal coincident count $I_{\max}$ was 0.4857. (e–g) With the FP–etalon, the HOM interference contrast and the indistinguishability of single photons increased ($I_{\max} = 0.4894$, $I_{\min} = 0.0550$).

3.2. Phase Measurement

We proceeded to introduce the NOON state into a phase-dependent Mach–Zehnder interferometer (MZI) constructed with two beam splitters, incorporating a phase shifter to induce continuous relative phase variation in two paths (see Figure 4a). When the input was a classical state (such as a coherent state) for phase measurement, the output intensity following the interferometer satisfied the following:

$$I_{out} = \frac{I_{in}}{2} [1 + \cos(\varphi)] \quad (11)$$

The interference fringes exhibited oscillations with a period of 2π .

In the case of a NOON state, the number of resources undergoing phase evolution in a single detection becomes N . For an ideal NOON state, the coincidence events at the two outputs of the MZI interferometer comply with Equation (9). When the interferometer receives input with the NOON ($N = 2$) state generated by QDs, the oscillation fringe period of the coincidence is halved, a phenomenon recognized as phase super-resolution. Figure 4b–g depict the variation in the second-order correlation function at various phase shifts φ . Consistent with Equation (9) for phase shifts of 0 and π , the central peak area reached its maximum while attaining a minimum at $\varphi = \pi/2$. The remaining four peaks, aside from the central one, also displayed periodic oscillations. The outermost peaks,

A1 and A5, corresponded to the reaction of single photons to phase variation, exhibiting a period of 2π . Peaks A2 and A4 represented the reaction of non-entangled photon pairs to phase variation, with a period of π . Nevertheless, the absence of entanglement significantly diminished the contrast of the interference fringe.

With a phase shift of $\pi/2$, the second-order correlation function of the output of the MZI exhibited the same pattern observed in the HOM interferometer, but with a larger central peak area (Figure 4c,f). Considering the impact of the imperfect indistinguishability of single photons on NOON state preparation and the spatial overlap at BS₂, we were able to change the form of Equation (10) to [8] the following:

$$P_{D1, D2} = \frac{1}{4} \left[(2 + \eta^2 (1 - \eta^2 V_{raw})) + \eta^2 (1 + \eta^2 V_{raw}) \cos(2\varphi) \right] \quad (12)$$

Here, η represents the overlap of the photons at two BSs, which can be determined via classical interference (Figure 5b). Therefore, the increase in the area of the central peak mainly arose from η and V_{raw} . For the low-loss excitation with a V_{raw} of 78.97%, we obtained a value of 59.01% for the contrast of the interference fringes, defined as $C = (I_{max} - I_{min}) / (I_{max} + I_{min})$. After filtering with the etalon, the contrast of the interference fringes C_{FP} increased to 79.79% (Figure 5a).

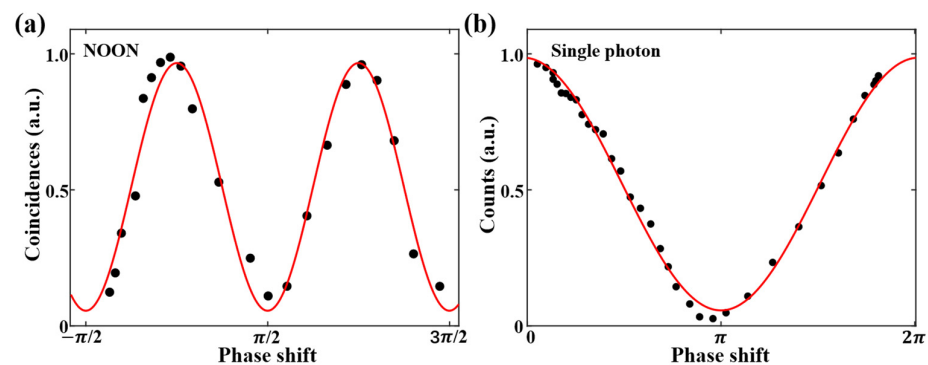


Figure 5. (a) The phase detection with high-performance NOON state: With the FP etalon, we achieved a high contrast of 79.79%, surpassing the SQL by a value of $1/\sqrt{2}$. (b) The phase detection with single photon: This curve was obtained by blocking one input port of the BS₁ and detecting the single-photon count rate from one of the output ports of BS₂, from which we were able to obtain a contrast of 98%.

4. Discussion

The quantum dot (QD) used in this study is located 500 μm from the cleaved edge of the sample, leading to a longer laser propagation distance within the waveguide. Compared to a QD situated 40 μm from the edge (reference [26]), we required higher pump power to achieve a “ π ” pulse, which in turn enhanced filtering ability. However, due to the proximity of the QD to the marker, the single-photon purity at saturation was 92.49%, higher than the 90% purity for the QD 40 μm from the edge. Since single-photon purity impacts the Hong–Ou–Mandel (HOM) interference contrast, we conducted phase measurements at half the saturation intensity ($0.5I_{\text{sat}}$) to ensure sufficient single-photon purity along with a high single-photon count rate.

The five peaks in the HOM interference pattern, which vary with phase shift, clearly demonstrated the improved measurement precision enabled by quantum resources. For non-entangled photon pairs, the maximum normalized coincidence events were approximately 0.36 (A2 and A4), which is less than the maximum value of 0.49 for A3 (NOON state). By contrast, the normalized maximum peak values for A1 and A5, representing single-photon evolution, matched those of A3, with a contrast close to 1. The asymmetry of the four peripheral peaks mainly resulted from the asymmetric reflectivity and transmissiv-

ity of the beam splitter [29]. Apart from the high-performance NOON state, the other four peaks did not exhibit measurement-enhanced phase estimation.

By examining the correlation between the phase shift and coincidence events, we were able to estimate phase variation through the detection of coincidences. Correspondingly, when considering errors in phase measurement, it is essential to account for error propagation. Merely increasing the frequency of oscillation of interference fringes does not suffice to signify an enhancement in phase measurement precision. To surpass the standard quantum limit, the condition $\Delta\varphi = \Delta P / \frac{\partial P}{\partial \varphi} < 1/\sqrt{N}$ must be fulfilled, which translates to $C > 1/\sqrt{N}$ [31]. Hence, by incorporating the FP-etalon, we achieved phase super-sensitivity ($C_{FP} = 79.79\% > 1/\sqrt{2}$) with a raw HOM interference contrast of 88.65%. Nevertheless, in the absence of the FP etalon and confronted with a low-loss HOM interference contrast, we encountered challenges in effectively capitalizing on the benefits of quantum effects ($C_{poor-V_{HOM}} = 59.01\% < 1/\sqrt{2}$). Extrapolating these findings to future research, we anticipate that charge stabilization using electrical gated-devices [32] can be directly incorporated in our devices to achieve low-noise and high indistinguishable single photon emission.

5. Conclusions

In summary, we achieved high-performance NOON states and realized phase super-sensitivity in phase measurements using a high-indistinguishability single photon generated from Purcell-enhanced deterministic single photon sources. The introduction of side excitation significantly reduced losses during collection, further enhancing the advantages of QD in multi-photon interference. The concept of coupling our SPS with fiber [33] offers potential for low-loss resonant fluorescence, thus expanding the scope of on-chip phase measurement experiments. Additionally, high-performance quantum dot single photon sources can generate various entangled states, such as a polarization-entangled NOON state [34] and a multi-photon cluster state [35], which can serve as quantum resources for quantum metrology.

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