

Article

Novel Spectrometer Designs for Laser-Driven Ion Acceleration

Antonia Morabito ^{1,2,*} , Kwinten Nelissen ² , Mauro Migliorati ³  and Sargis Ter-Avetisyan ^{2,4,5} ¹ Centro de Laseres Pulsados (CLPU), Edicio M5, Parque Cientco. C/ Adaja, 8, 37185 Villamayor, Spain² ELI-ALPS Research Institute, ELI-HU Non-Profit Ltd., Wolfgang Sandner utca 3, 6728 Szeged, Hungary; kwinten.nelissen@eli-alps.hu (K.N.); sargis.ter-avetisyan@eli-np.ro (S.T.-A.)³ INFN & University of Rome, Via Scarpa 14, 00161 Roma, Italy; mauro.migliorati@uniroma1.it⁴ National Laser-Initiated Transmutation Laboratory, University of Szeged, 6720 Szeged, Hungary⁵ Extreme Light Infrastructure-Nuclear Physics (ELI-NP), Horia Hulubei National Institute for R&D in Physics and Nuclear Engineering (IFIN-HH), 077125 Bucharest-Măgurele, Romania

* Correspondence: amorabito@clpu.es

Abstract: We propose novel spectrometer designs that aim to enhance the measured spectral range of ions on a finite-sized detector. In contrast to the traditional devices that use a uniform magnetic field, in which the deflection of particles increases inversely proportional to their momentum, in a gradient magnetic field, the deflection of particles will decrease due to the reduction of the magnetic field along their propagation. In this way, low-energy ions can reach the detector because they are deflected less, compared to the uniform field case. By utilizing a gradient magnetic field, the non-linear dispersion of ions in a homogeneous magnetic field approaches nearly linear dispersion behavior. Nonetheless, the dispersion of low-energy ions, using a dipole field, remains unnecessarily high. In this article, we discuss the employed methodology and present simulation results of the spectrometer with an extended ion spectral range, focusing on the minimum detectable energy (energy dynamic range) and energy resolution.

Keywords: laser-driven particle acceleration; Thomson parabola spectrometer; gradient magnetic field; diagnostics; detectors



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1. Introduction

The development of ultrahigh-power laser systems [1] in the last two decades has led to enormous scientific achievements in the field of laser-plasma-based research. Nowadays, laser intensities even above $10^{22} \text{ W} \cdot \text{cm}^{-2}$ [2] are available for experiments, presenting new perspectives for science and applications [3,4].

It is essential to develop advanced diagnostics that allow detailed measurements of plasma properties in this new interaction regime. It requires scaling up the range of available parameters in ion spectrometers, detectors, and calibrations in order to be ready to measure effects that are not even thought of today.

This paper focuses on spectrometric methods of particle diagnostics that allow extending the measured spectral range of accelerated ion phenomena. The analyses of multi-species ion emissions with the spectrometer are well described in the literature [5].

The Thomson parabola spectrometer (TPS) is one of the most used charge particle diagnostics. It consists of an ion pinhole, a sequence or a combination of parallel magnetic and electric fields, and an ion detector [6,7]. The magnetic field disperses the particles in the direction perpendicular to it, based on their velocities, while the electric field allows for discrimination of the particles' species. The spectrometer becomes uniquely valuable if the highly sensitive, active detector is used, such as a microchannel-plate (MCP) detector [8] coupled to a phosphor screen and a CCD camera [6]. It allows online recording of the particle spectrum on a single laser shot basis and at a repetition rate of more than 10 Hz [9,10]. However, there are some limitations, which can inhibit their use for experiments.

Consequently, several modifications have been made in their designs, such as variations in the magnetic [7,11–13] and electric sections [7,14,15] to overcome them.

Different approaches, e.g., the replacement of the ion pinhole with a horizontal slit or multiple pinhole arrays [7,13] have been implemented to obtain spatial information regarding the ion beam. The substitution of a standard H design of a dipole with C-shape magnetic Halbach structures [12] or variable gap permanent magnets [13] have also been employed to mitigate the magnetic fringe effects and increase the ion spectral information.

Trapezoidal-shaped electric field plates have been used to circumvent the spectral clipping at the low-energy end of the ion spectra. Some changes allowed temporally and spatially resolved detection of accelerated ion spectral distributions, the simultaneous measurement of ion and electron spectra along the same observation direction, precise measurements of the proton/ion trajectories for proton deflectometry [16] and tomography applications [17]. All these combinations [7,11–13,18] immensely support extensive and thorough research of relevant laser-plasma processes.

Nevertheless, in spectrometer designs, due to the finite size of the detector (e.g., MCP), the measured spectral range of ions and species resolution will be restricted. Thus, to achieve decent energy resolution at high particle energies (currently, laser-generated proton energies exceed 100 MeV [19,20]), the dispersion must be increased, which will reduce the spectral range covered by the detector.

In the article, we provide guidelines for new ion spectrometer designs that use a linear gradient magnetic field. This makes it possible to decrease the dispersion of ions at low energies and bring them back to the detector, while the dispersion at high energies will be almost as high as can be achieved with a dipole magnet.

The paper is structured as follows: Section 2.1 describes the conceptual design of the new spectrometer. Following that, Section 2.2 presents the implemented model system along with its initial assumptions. In Section 3, we delve into the investigation of spectrometer parameter tuning for two distinct proton energy range scenarios: (1) the low-energy range, ranging between a few keV up to 100 MeV, and (2) the high-energy range that goes from a few tens of MeV up to hundreds of MeV. In conclusion, we thoroughly discuss and summarize the advantages and limitations of our theoretical approach, and analyze the potential replacement of a dipole with various non-homogeneous magnetic profiles.

2. Methodology

2.1. Conceptual Approach

In the ion spectrometer, the particles are deflected in the homogeneous parallel magnetic (B) and electric (E) fields [21]. The particle's position on the trace depends on its energy; the higher the particle's energy, the less it is deflected from the so-called zero point, where undeflected plasma emission (mainly X-ray) hits the detector.

The spectra of ions with the same charge-to-mass ratio (Z/m) will trace a parabola and particles with different (Z/m) will follow different parabolic traces. In the approximation of small deflections of non-relativistic particles [7], the electric and magnetic deflections are as follows:

$$x = Ze \frac{EL_{el}}{2E_i} D_E \quad (1)$$

$$y = Ze \frac{BL_{magn}}{\sqrt{2m_0E_i}} D_B \quad (2)$$

In the case of non-relativistic velocities, the expression for kinetic energy is given by $E_i = (\gamma - 1)m_0c^2$, where γ represents the Lorentz factor. The variables m_0 and c correspond to the rest mass and the speed of light, respectively. We have $D_E = L_{det} + L_{el}/2$, which denotes the distance between the detector plane and the midpoint of the electric field, $D_B = L_{det} + L_{el} + L_{dis} + L_{magn}/2$ is the distance between the center of the magnetic field and the detector. L_{det} represents the distance between the electric field and the detector, while L_{el} is the length of the electric field. In this case, L_{magn} corresponds to the length of

the magnetic field, and L_{dis} is the distance between the electric field and the magnetic field (see Figure 1).

The deflection of particles from “zero point” is not-linearly increasing $y \sim 1/\sqrt{(E_i)}$ (see Equation (2)), as the particle’s energy is decreasing. This limits the measurable spectral range of ions due to the limited size of the detector [7,15].

Our conceptual approach aims to reduce the dispersion of “low-energy” ions, which is often excessively high. To achieve this, we suggest employing a magnetic field gradient instead of a homogeneous magnetic field.

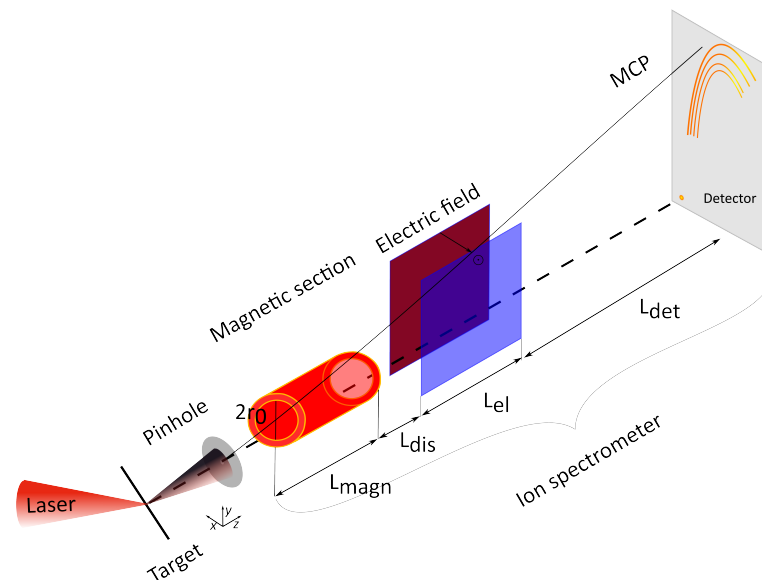


Figure 1. Scheme of the spectrometer, which is composed of a pinhole, a magnetic section, which is depicted as a cylinder with length L_{magn} and bore radius r_0 , separated by a distance L_{dis} from the electric field with length L_{el} . The distance between the exit of the electric field and the ion detector (MCP) is labeled as L_{det} .

In comparison to the homogeneous magnetic field, a gradient magnetic field results in the reduction of particle deflection due to the decreasing magnetic field strength along their path. This impacts low-energy ions, as they experience less deflection and are more likely to reach the detector rather than being pushed out when a uniform magnetic field is used. As a result, the use of a gradient magnetic field enables the achievement of an almost linear dispersion behavior.

2.2. Model System

The magnetic field of an inhomogeneous magnet can be expanded as a function of its multi-pole components [22,23] :

$$B_x(0, y) = B_0 \rho \sum_{n=0}^{\infty} K_n(s) y^n \quad (3)$$

The magnetic rigidity of a particle with momentum p_0 and charge e is given by the equation $B_0 \rho = p_0/e$, where ρ represents the gyroradius of the particle in the presence of the magnetic field. The term $K_n(s)$ is defined as follows:

$$K_n(s) = \left(\frac{1}{B\rho} \right) \left(\frac{1}{n!} \right) \left(\frac{\partial^n B_y}{\partial y^n} \right) \quad (4)$$

For pure multi-pole fields, the expression of the magnetic field is given by the following:

$$B_x(0, y) = \frac{1}{n!} \frac{\partial^n B_y}{\partial y^n} y^n \quad (5)$$

The term $\partial^n B_y / \partial y^n$ is referred to as the field gradient. In the case of a dipole $n = 0$, we have the magnetic field B_0 on the central y -axis, while $n = 1$ represents the linear field gradient achieved in a single segment of the magnetic quadrupole configuration. Throughout the rest of this article, we denote $G = \partial^n B_y / n! \partial y^n$.

The particles are injected along the z -axis into the magnetic element, perpendicular to the (xy) plane, at the point $y = r_0$, where the field is the highest. We ignore the width of the particle beam and assume that the magnetic field lines are perpendicular to the trajectory of the particle. These particles disperse perpendicular to the y -axis, where $B_x = 0$ (refer to Figure 1). It is assumed that a parallel proton beam is formed by the entrance pinhole of the spectrometer, neglecting its divergence. This is justified by the fact that ions are usually detected within the nsr solid angle [6,7]. Additionally, the charge density is considered low, so that the interactions between the charged particles and, therefore, the beam space charge effects can be safely ignored.

The force acting on a charged particle is given by the Lorentz force:

$$\vec{F} = \frac{d\vec{p}}{dt} = q(\vec{E} + \vec{v} \times \vec{B}), \quad (6)$$

where E is the electric field, B is the magnetic field. The particle traces from the source to the detector screen are obtained by integrating the equation of motion numerically by a second-order Runge–Kutta (RK) method [24]. The particle deflection in the magnetic field can be obtained by solving Hill's equations, taking into account relativistic effects and ignoring the space-charge effects. The electric and magnetic fields are assumed to have linear components (up to quadrupole field) while preserving the particle impulse. These calculations can be conducted analytically [22,25], as follows:

$$\begin{aligned} x'' + K_x(s)x &= 0 \\ y'' + K_y(s)y &= 0 \end{aligned} \quad (7)$$

where $x'' = d^2x/ds^2$ and $y'' = d^2y/ds^2$, respectively, due to the change in variable $d/dt = v_0 d/ds$.

Using the transfer matrix methodology, the transport matrices are defined as follows [22,25]:

$$\begin{bmatrix} u(s) \\ u(s') \end{bmatrix} = \begin{bmatrix} \cos(\sqrt{\kappa} \cdot L_{\text{magn}}) & \frac{1}{\sqrt{\kappa}} \sin(\sqrt{\kappa} \cdot L_{\text{magn}}) \\ -\sqrt{\kappa} \sin(\sqrt{\kappa} \cdot L_{\text{magn}}) & \cos(\sqrt{\kappa} \cdot L_{\text{magn}}) \end{bmatrix} \cdot \begin{bmatrix} 1 & D_B \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_0 \\ p_0 \end{bmatrix} \quad (8)$$

where $u(s)$ stands for the particle position, $u(s')$ denotes the particle momentum, y_0 denotes the injection point into the quadrupole field (see Figure 1), $\kappa = G \cdot Ze/p_0$, p_0 is the relativistic impulse of the particles.

The analytical expression of the electric field is taken from [15], whereas, the magnetic deflection on the detector plane for our quadrupole spectrometer is given by the following:

$$\begin{cases} x = Ze \frac{\gamma}{\gamma^2 - 1} \frac{E_0 L_{el}}{m_0 c^2} (L_{det} + L_{el}/2), \\ y = [\cos(\sqrt{\kappa} L_{\text{magn}}) - D_B \cdot \sqrt{\kappa} \cdot \sin(\sqrt{\kappa} \cdot L_{\text{magn}})] \cdot y_0 \end{cases} \quad (9)$$

From the magnetic field equation, one can see that changing D_B results in a linear re-scaling of the y -component, while the factor $\sqrt{\kappa} L_{\text{magn}}$ tailors the non-linear contribution

of the magnetic particle deflection. Therefore, the longer the dispersive field (or $\sqrt{G}$), the larger the non-linear dispersion.

3. Particle Dispersion in a Radially Inhomogeneous Magnetic Field

The proposed ion spectrometer is illustrated in Figure 1. The performance of the spectrometer with magnetic gradient fields is investigated for the proton beams with 100 MeV [19,20] and 700 MeV cutoff energies [25]. The proton spectra obtained in the detector plane for homogeneous and quadrupole magnetic fields are compared.

4. Results

4.1. Optimized Design for Proton Spectrum with 100 MeV Cut-Off Energy

A point source proton beam in the energy range from 0.5 MeV to 100 MeV is considered as input [19,20].

In experimental configurations (refer to, for example, [6,15]), the pinhole sizes are around 100 μm or 200 μm , with the source-pinhole distances being 20 cm and 58 cm, resulting in a beam divergence of less than half a mrad and solid angles of approximately 10^{-5} and 10^{-8} sr. This indicates that the proton beam had very low divergence, thereby making our initial conditions closely resemble realistic scenarios. Thus, the pinhole size was chosen as typical, about 100 μm . The size of the detector, specifically an MCP in our case, is usually around 8 cm [8]. The “zero point” is set about 5 mm from the edge of the detector to not lose the signal. The main spectrometer parameters are summarized in Table 1.

Table 1. Optimized design parameters for low-energy protons.

Element	B_0/G	L_{magn}	E	L_{el}	L_{det}
Dipole	0.9 T	10 cm	12 kV/cm	5 cm	5 cm
Quadrupole	45 T/m at $r_0 = 2.2$ cm	10 cm	12 kV/cm	5 cm	5 cm

The particles enter the magnetic field at a distance r_0 from the bottom edge of the magnet, where the magnetic field is the highest, $B_0 = 0.9$ T.

The retrieved value of G resembles the typical gradient values employed for the design of laser-driven proton beamlines ([26] and references therein). The distance to the detector plane L_{det} is tuned in order to obtain the magnetic deflection for 100 MeV protons at 1 cm on the detector. This would reasonably avoid the overlapping of the spectral trace with a “zero point”. This requirement is applied to all the cases [27]. Furthermore, we are able to specify a gap ranging from 5 mm to 1 cm between the electric plates in order to achieve the desired electric field strength [6].

Figure 2A illustrates the particle deflection in the detector plane for both the homogeneous (red) and linear-gradient (blue) cases. The detector screen is indicated by the horizontal black line. The double arrows point to the protons with the same energies.

The spectrometer equipped with a dipole will detect protons in the energy range from 2–100 MeV, compared with the linear-gradient design, which can record the entire proton spectrum starting from 0.5 MeV and even lower.

We evaluate the normalized resolution of the spectrometers in both cases and compare them in Figure 2B.

The uncertainties of the ion position on a detector limit the energy resolution of a TPS at a given size δR_p . The energy resolution is a function of the displacement of the particles from the zero point. This distance from the neutral point is calculated as $R = \sqrt{x^2(E) + y^2(E)}$, at $z = z_0$ (our propagation axis).

The energy uncertainty is, thus, given by the following:

$$Res = \frac{\delta E_{\text{res}}}{E} = \frac{dE}{\delta R} \frac{\delta R_p}{E}, \quad (10)$$

where δR_p depends on the pinhole size [6,11].

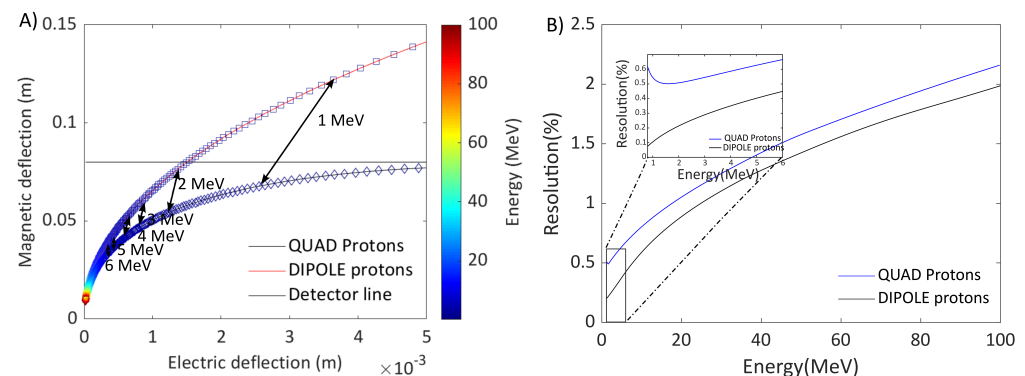


Figure 2. (A) The comparison between particle deflection in dipole and linear–gradient (quadrupole) magnetic fields with the parameters given in Table 1. The color bar on the right-hand side indicates the initial proton energy range. (B) The energy resolution is calculated according to Equation (10) for homogeneous and linear gradient cases. The inset shows a zoom of the lowest-energy particles.

The overall energy resolution for the total analyzed energy range, i.e., from a few keV up to 100 MeV, slightly decreases, as expected in the case of the linear gradient. We highlight in the inset of Figure 2B the energy resolution in the range between 0.5 and 6 MeV, which corresponds to the same energy shown by the arrows.

The difference is of the order of 0.4 % for the low-energy region of interest and has a value of 0.2% for the high-energy region.

Hence, the results in Figure 2 show that using a linear gradient (quadrupole) field enlarges the detectable dynamic energy range at a reasonable cost in terms of energy resolution.

4.2. Optimized Design for Proton Spectrum with 700 MeV Cut-Off Energy

For this set of simulations, a proton beam with 700 MeV cutoff energy is considered as input. The 1 cm deflection point is found for a magnetic field strength of $B_0 = 1.9$ T at a distance of $L_{det} = 11$ cm. Please note that a dipole magnet of 1.9 T is extremely bulky.

In the linear gradient case, the field gradient is $G \simeq 100$ T/m and the beam is injected in the magnetic field at $r_0 = 1.9$ cm. All design parameters are summarized in Table 2.

Table 2. Optimized design parameters for high-energy protons.

Element	B_0/G	L_{magn}	E	L_{el}	L_{det}
Dipole	1.9 T	10 cm	12 kV/cm	5 cm	11 cm
Quadrupole	100 T/m at $r_0 = 1.9$ cm	10 cm	12 kV/cm	5 cm	11 cm

In Figure 3A, the particle dispersion is shown for a dipolar field of 1.9 T (dashed black lines), for the quadrupole field (black line), and a reference dipole field of 0.9 T (red lines).

The dipole case of $B_0 = 0.9$ T has been added, tuning, accordingly, the L_{det} , which increases up to 16.5 cm, in order to take 5 MeV protons to the detector screen. The color bar on the right-hand side represents the correspondent energy range (from a few keV up to a maximum of 700 MeV [25]). The double arrows in Figure 3 are connecting points of equal proton energy shown for 10, 20, 30, 40, 60, and 80 MeV. Furthermore, the normalized energy resolutions were calculated using Equation (10).

As can be seen in Figure 3B, the energy resolution changes between a minimum of 0.2% up to a maximum of 0.8%, comparing the quadrupole gradient and the dipole cases. In this case, there is a significant increase in the dynamic range of the detected protons of almost 10 MeV, substituting the dipolar field $B_0 = 1.9$ T with a quadrupole field.

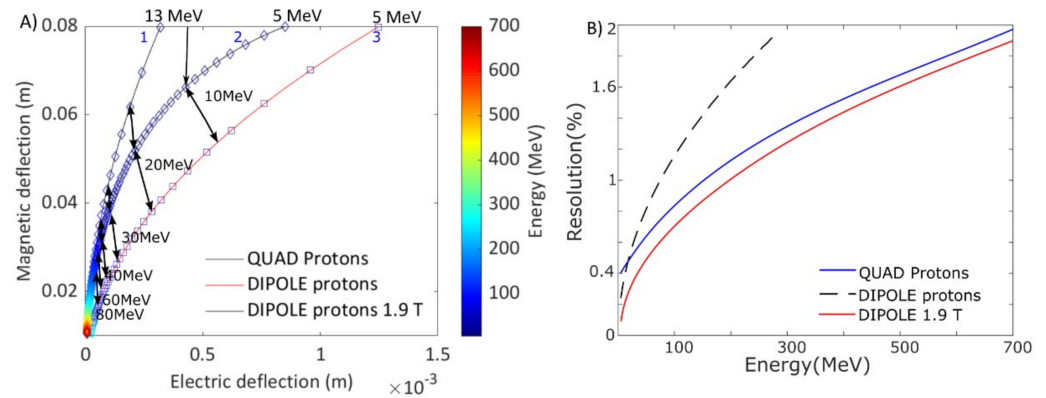


Figure 3. (A) A comparison of particle deflection in a dipole (line 1) and quadrupole (line 2) magnetic fields with the parameters given in Table 2 and a dipole (line 3) field with $B_0 = 0.9$ T, $L_{magn} = 10$ cm, $L_{det} = 16.5$ cm. On the x axis, one can see the electric deflection in m, whereas, on the y -axis, one can see the magnetic deflection, also in m. (B) The comparison between the energy resolutions (y axis) versus the proton energy range (x axis in MeV) for both the cases of the dipoles and the case of the quadrupole field.

In Figure 3A, it can be observed that in the case of the reference dipole with $B_0 = 0.9$ T, an increase in distance from the detector is needed, and the measurable proton energy range increases from 5 MeV (our lower limit) to 250 MeV, while in the case of the quadrupole field, we can extend it up to 700 MeV and keep the total spectrometer size more compact.

For completeness, we also studied the same three cases described before, fixing the proton energy cut-off at 700 MeV on the detector (indicated in red in Figure 4A).

As already conducted in refs. [12,18], it is possible to tune the drift between the electric field to the detector L_{det} to modify the detectable energy spectra. The value of $L_{det} = 30$ cm has been chosen in order to achieve the 1 cm magnetic deflection for 700 MeV protons (line 3 in Figure 4A). In Figure 4A, one can observe a larger energy dynamic range in the linear gradient design case compared to dipole ones. The energy resolutions are very close to each other and the difference between the three cases goes up to a maximum of 0.5%, as shown in Figure 4B.

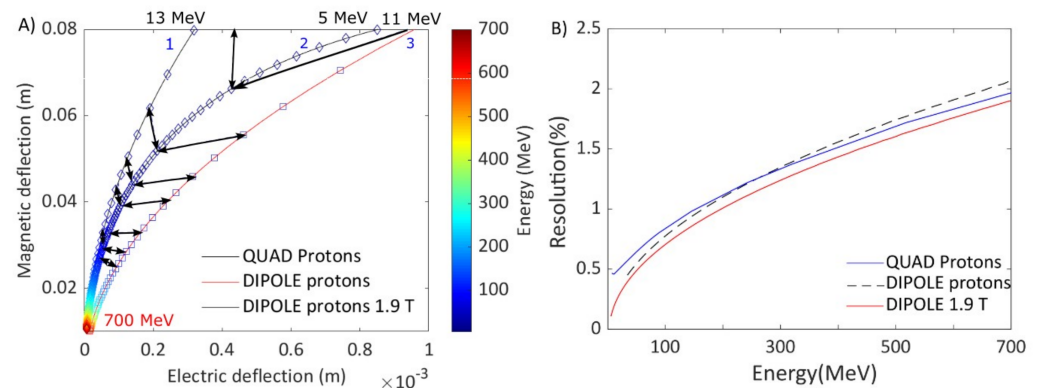


Figure 4. (A) A comparison of particle deflection in a dipole (line 1) and quadrupole magnetic fields (line 2) with the parameters given in Table 2 and the dipole (line 3) field with $B_0 = 0.9$ T, $L_{magn} = 10$ cm, $L_{det} = 30$ cm. On the x axis, we plot the electric deflection in m, while on the y -axis, we plot the magnetic deflection, which is also in m. (B) Comparison between the energy resolutions (y axis) versus proton energy range (x axis) expressed in MeV for both the cases of the dipoles and the case of the magnetic field profile of the quadrupole.

In summary, we show that the dispersion of high-energy particles is not affected by the use of quadrupole fields, while it significantly impacts lower-energy particles. By replacing

the dipole field with a linear-gradient design, the overall dynamic range of proton energy can be expanded, while still preserving accuracy.

4.3. High Order Magnetic Field Profiles

We also study the effects of the replacement of a homogeneous magnetic field with different magnetic field profiles, such as decapolar and octupolar profiles.

A set of simulations was implemented in order to investigate the minimum achievable detectable proton energy with these modifications. A source of protons that goes from a few keV up to 100 MeV was used as input. The quadrupole parameters are the same as reported in Section 4.1. The results are shown in Figure 5A.

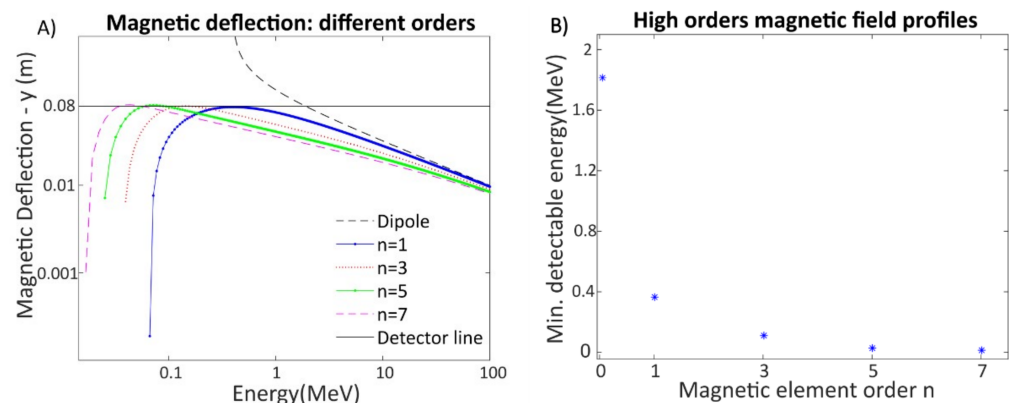


Figure 5. (A) The overlap of the different magnetic profiles compared to the case of the dipole is shown. The continuous black line represents the detector line. The dashed black line represents the magnetic deflection of the reference case of the dipole, while the blue, red, green, and magenta lines represent the cases of the non homogeneous magnetic profiles. Both the x axis and the y axis are on a logarithmic scale. On the x axis, the proton energy range is in MeV, while on the y axis, the magnetic deflection is in m units. (B) The scaling law is shown. We illustrate the different orders (n) of the magnetic profiles vs. the minimum detectable energy in MeV.

For consistency, the same distance between the spectrometer and the detector $L_{det} = 5$ cm and the same distance between the electric and magnetic field $L_{dis} = 1$ cm have been used for all the cases.

The cut-off minimal proton energy as a function of the gradient power, i.e., $n = 0, 1, 3, 5, 7$ is reported in Figure 5B to investigate the change in the lowest detectable energy. The values n are odd-integer numbers of the magnetic field profiles used in Equation (5). The magnetic field profiles were chosen to reach a maximum magnetic field of $B = 0.9$ T at the injection point. This corresponds to a quadrupole $G_q \simeq 45$ T/m, an octupolar $G_{oct} = 8.5 \times 10^4$ T/m³, a decapolar $G_{dec} = 1.7 \times 10^8$ T/m⁵, and multi-polar $G_{mult} = 3.6 \times 10^{11}$ T/m⁷ of order 7.

We are aware that these values are very demanding, but they align with our initial assumptions. In this way, the highest detectable energy is preserved for all magnetic field profiles, whereas the lowest detectable energy changes as a function of the field gradient power.

Please note that when a charged particle passes through a magnetic field profile with an odd power index n , particles are deflected back when crossing the zero point. In principle, the traces of the back-deflected particle could be used to determine its energy (see Figure 5A). These particles are excluded to determine the lowest detectable energy and avoid the potential crossing of different ion species on the detector screen. This is, in practice, easily achievable by blocking particles crossing the central axis.

In the case of the spectrometer composed by a dipole, the energy spectral information achievable and observable on the detector screen increases from 1.891 MeV (~ 2 MeV) to 100 MeV, while for the magnetic field profiles, the spectral information increases from

1.891–100 MeV to 0.392–100 MeV (case $n = 1$), 0.15–100 MeV (case $n = 3$), 0.008–100 MeV (case $n = 5$), and 0.0061–100 MeV (case $n = 7$).

The results of Figure 5B show that the replacement of a constant magnetic field with the considered magnetic field profiles increases the energy dynamic range, without introducing significant losses at high energy. Reducing the magnetic field of the dipole is clearly possible, but it inevitably leads to a loss in dispersion for high-energy particles. As expected, there is proportionality between the increase in the order of the elements and the respective increase in minimum detectable energy information.

However, after the third order (decapole), there is no significant improvement in terms of the capture of the low-energy particles compared to the previous orders, as can be seen in Figure 5B.

5. Discussion and Conclusions

In this paper, we present the theoretical study of novel types of spectrometers to investigate the enhancement of the detectable energy dynamic range, compared to existing ones.

We implemented different sets of simulations, studying the replacement of different gradient field profiles instead of a constant dipole field in a TPS design for two proton energy range scenarios. Our results offer valuable insights into the benefits and constraints of the proposed designs.

In Section 4.1, we show that a higher energy spectral range can be achieved without significant losses in terms of energy resolution. This “new” detectable low proton energy range is particularly relevant for applications, such as the ones in cultural heritage [28,29], laser-driven proton boron fusion [30,31], and toward the development of low-divergent MeV-class proton beams, which are generated from a micrometer-sized source by a few-cycle laser pulse [9].

For the proton energy range that increases from a few keV up to hundreds of MeV [25], in Section 4.2, we show that we can obtain considerable improvements in terms of the detectable proton energy dynamic range.

It is evident that this enhancement is easily scalable by tweaking the quadrupole field parameters. We also explore the possibility of using more sophisticated magnetic devices with strong gradients. However, after implementing a third-order magnetic field profile corresponding to a decapole element, there is no significant improvement.

In conclusion, among the analyzed options, the quadrupole magnetic field profile is the most reliable, suitable, and feasible (easy to manufacture and assemble). In laser-driven proton acceleration, the use of quadrupole configurations, such as doublets and/or triplets, is not new because they have been implemented (as reported in refs. [26,32]) downstream of the laser-plasma interaction point to manipulate and adapt laser proton sources for applications.

The versatility and tunability of the linear gradient field profile, close to the quadrupole, in our spectrometer designs, compared to existing combinations, e.g., the tuning of the length of the drift between the spectrometer and detector in combination with a TPS [13,18], allows for the enlargement of the detectable energy range without significant losses in energy resolution and results in a more compact overall spectrometer structure. All the modifications that can be applied to a TPS design are not mutually exclusive, i.e., the length of the drift between the spectrometer and detector and the tuning of both electric and magnetic sections can be implemented at the same time, as reported in [7].

In summary, we can conclude that, according to the constraints of the different experimental setups and the needed detectable proton energy of interest, the proposed spectrometer designs can represent alternative and versatile proton diagnostic devices for both laser applications and other radiation sources.

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References

1. Strickland, D.; Morou, G. Chirped pulse amplification. *Opt. Commun.* **1985**, *56*, 219. [CrossRef]
2. Yoon, J.W.; Jeon, C.; Shin, J.; Lee, S.K.; Lee, H.W.; Choi, I.W.; Kim, H.T.; Sung, J.H.; Nam, C.H. Achieving the laser intensity of 5.5×10^{22} W/cm² with a wavefront-corrected multi-PW laser. *Opt. Express* **2019**, *27*, 20412–20420. [CrossRef] [PubMed]
3. Daido, H.; Nishiuchi, M.; Pirozhkov, A.S. Review of laser-driven ion sources and their applications. *Rep. Prog. Phys.* **2012**, *75*, 056401. [CrossRef] [PubMed]
4. Macchi, A. A review of laser-plasma ion acceleration. *arXiv* **2017**, arXiv:1712.06443.
5. Bolton, P.; Borghesi, M.; Brenner, C.; Carroll, D.; De Martinis, C.; Fiorini, F.; Flacco, A.; Floquet, V.; Fuchs, J.; Gallegos, P.; et al. Instrumentation for diagnostics and control of laser-accelerated proton (ion) beams. *Phys. Medica* **2014**, *30*, 255–270. [CrossRef]
6. Jeong, T.W.; Singh, P.; Scullion, C.; Ahmed, H.; Kakolee, K.; Hadjisolomou, P.; Alejo, A.; Kar, S.; Borghesi, M.; Ter-Avetisyan, S. Experimental evaluation of the response of micro-channel plate detector to ions with 10s of MeV energies. *Rev. Sci. Instrum.* **2016**, *87*, 083301. [CrossRef] [PubMed]
7. Alejo, A.; Gwynne, D.; Doria, D.; Ahmed, H.; Carroll, D.; Clarke, R.; Neely, D.; Scott, G.; Borghesi, M.; Kar, S. Recent developments in the Thomson Parabola Spectrometer diagnostic for laser-driven multi-species ion sources. *J. Instrum.* **2016**, *11*, C10005. [CrossRef]
8. Microchannel Plates-Photonics Website. Available online: <https://www.photonis.com/products/microchannel-plates> (accessed on 10 January 2024).
9. Morrison, J.T.; Feister, S.; Frische, K.D.; Austin, D.R.; Ngirmang, G.K.; Murphy, N.R.; Orban, C.; Chowdhury, E.A.; Roquemore, W. MeV proton acceleration at kHz repetition rate from ultra-intense laser liquid interaction. *New J. Phys.* **2018**, *20*, 022001. [CrossRef]
10. Xu, N.; Streeter, M.; Ettlinger, O.; Ahmed, H.; Astbury, S.; Borghesi, M.; Bourgeois, N.; Curry, C.; Dann, S.; Dover, N.; et al. Versatile tape-drive target for high-repetition-rate laser-driven proton acceleration. *High Power Laser Sci. Eng.* **2023**, *11*, e23. [CrossRef]
11. Jung, D.; Hörlein, R.; Kiefer, D.; Letzring, S.; Gautier, D.; Schramm, U.; Hübsch, C.; Öhm, R.; Albright, B.; Fernandez, J.; et al. Development of a high resolution and high dispersion Thomson parabola. *Rev. Sci. Instrum.* **2011**, *82*, 013306. [CrossRef]
12. Teng, J.; He, S.; Deng, Z.; Zhang, B.; Hong, W.; Zhang, Z.; Zhu, B.; Gu, Y. A compact high resolution Thomson parabola spectrometer based on Halbach dipole magnets. *Nucl. Instrum. Methods Phys. Res. Sect. A Accel. Spectrom. Detect. Assoc. Equip.* **2019**, *935*, 30–34. [CrossRef]
13. Kojima, S.; Inoue, S.; Dinh, T.H.; Hasegawa, N.; Mori, M.; Sakaki, H.; Yamamoto, Y.; Sasaki, T.; Shiokawa, K.; Kondo, K.; et al. Compact Thomson parabola spectrometer with variability of energy range and measurability of angular distribution for low-energy laser-driven accelerated ions. *Rev. Sci. Instrum.* **2020**, *91*, 053305. [CrossRef]
14. Gwynne, D.; Kar, S.; Doria, D.; Ahmed, H.; Cerchez, M.; Fernandez, J.; Gray, R.; Green, J.; Hanton, F.; MacLellan, D.; et al. Modified Thomson spectrometer design for high energy, multi-species ion sources. *Rev. Sci. Instrum.* **2014**, *85*, 033304. [CrossRef]
15. Alejo, A.; Kar, S.; Tebartz, A.; Ahmed, H.; Astbury, S.; Carroll, D.; Ding, J.; Doria, D.; Higginson, A.; McKenna, P.; et al. High resolution Thomson Parabola Spectrometer for full spectral capture of multi-species ion beams. *Rev. Sci. Instrum.* **2016**, *87*, 083304. [CrossRef]
16. Sokollik, T.; Schnürer, M.; Ter-Avetisyan, S.; Nickles, P.; Risse, E.; Kalashnikov, M.; Sandner, W.; Priebe, G.; Amin, M.; Toncian, T.; et al. Transient electric fields in laser plasmas observed by proton streak deflectometry. *Appl. Phys. Lett.* **2008**, *92*, 091503. [CrossRef]

17. Ter-Avetisyan, S.; Schnürer, M.; Nickles, P.; Sandner, W.; Borghesi, M.; Nakamura, T.; Mima, K. Tomography of an ultrafast laser driven proton source. *Phys. Plasmas* **2010**, *17*, 063101. [CrossRef]
18. Torrisi, L.; Costa, G. Compact Thomson parabola spectrometer for fast diagnostics of different intensity laser-generated plasmas. *Phys. Rev. Accel. Beams* **2019**, *22*, 042902. [CrossRef]
19. Göthel, I.; Assenbaum, S.; Bernert, C.; Brack, F.E.; Cowan, T.; Dover, N.; Gaus, L.; Kluge, T.; Kraft, S.; Kroll, F.; et al. Laser-Driven High-Energy Proton Beams from Cascaded Acceleration Regimes. 2023. Available online: <https://www.researchsquare.com/article/rs-2841731/v1> (accessed on 18 May 2023).
20. Higginson, A.; Gray, R.; King, M.; Dance, R.; Williamson, S.; Butler, N.; Wilson, R.; Capdessus, R.; Armstrong, C.; Green, J.; et al. Near-100 MeV protons via a laser-driven transparency-enhanced hybrid acceleration scheme. *Nat. Commun.* **2018**, *9*, 724. [CrossRef]
21. Thomson, J.J. XXVI. Rays of positive electricity. *Lond. Edinb. Dublin Philos. Mag. J. Sci.* **1911**, *21*, 225–249. [CrossRef]
22. Reiser, M.; O'Shea, P. *Theory and Design of Charged Particle Beams*; Wiley Online Library: Hoboken, NJ, USA, 1994; Volume 312.
23. Jackson, J.D. *Classical Electrodynamics*; John Wiley & Sons: Hoboken: NJ, USA, 1999.
24. Butcher, J.C. *The Numerical Analysis of Ordinary Differential Equations: Runge-Kutta and General Linear Methods*; Wiley-Interscience: Hoboken, NJ, USA, 1987.
25. Wiedemann, H. *Particle Accelerator Physics*; Springer: Berlin/Heidelberg, Germany, 2007; Volume 314.
26. Morabito, A.; Scisciò, M.; Veltri, S.; Migliorati, M.; Antici, P. Design and optimization of a laser-PIXE beamline for material science applications. *Laser Part. Beams* **2019**, *37*, 354–363. [CrossRef]
27. Morabito, A. Transport and Manipulation of Laser Driven Proton Beams for Diagnostics and Applications. Ph.D. Thesis, Università la Sapienza di Roma, Roma, Italy, 2021. Available online: <https://iris.uniroma1.it/handle/11573/1695964> (accessed on 13 December 2023).
28. Barberio, M.; Veltri, S.; Scisciò, M.; Antici, P. Laser-Accelerated Proton Beams as Diagnostics for Cultural Heritage. *Sci. Rep.* **2017**, *7*, 40415. [CrossRef]
29. Mirani, F.; Maffini, A.; Casamichiela, F.; Pazzaglia, A.; Formenti, A.; Dellasega, D.; Russo, V.; Vavassori, D.; Bortot, D.; Huault, M.; et al. Integrated quantitative PIXE analysis and EDX spectroscopy using a laser-driven particle source. *Sci. Adv.* **2020**, *7*, eabc8660. [CrossRef]
30. Margarone, D.; Bonvalet, J.; Giuffrida, L.; Morace, A.; Kantarelou, V.; Tosca, M.; Raffestin, D.; Nicolai, P.; Picciotto, A.; Abe, Y.; et al. In-target proton–boron nuclear fusion using a PW-class laser. *Appl. Sci.* **2022**, *12*, 1444. [CrossRef]
31. Giuffrida, L.; Belloni, F.; Margarone, D.; Petringa, G.; Milluzzo, G.; Scuderi, V.; Velyhan, A.; Rosinski, M.; Picciotto, A.; Kucharik, M.; et al. High-current stream of energetic α particles from laser-driven proton-boron fusion. *Phys. Rev. E* **2020**, *101*, 013204. [CrossRef]
32. Milluzzo, G.; Petringa, G.; Catalano, R.; Cirrone, G. Handling and dosimetry of laser-driven ion beams for applications. *Eur. Phys. J. Plus* **2021**, *136*, 1170. [CrossRef]

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