Perfect Off-Axis Optical Vortex Lattice

Yuping Tai 1,2, Xueyun Qin 1, Chenying Li 1, Wenjun Wei 1, Hao Zhang 1,* and Xinzhong Li 1,2,*

1 School of Chemistry and Chemical Engineering & School of Physics and Engineering, Henan University of Science and Technology, Luoyang 471023, China; yptai@haust.edu.cn (Y.T.); qinxueyun1998@163.com (X.Q.); lichenying2401@163.com (C.L.); 220320090787@stu.haust.edu.cn (W.W.)
2 State Key Laboratory of Transient Optics and Photonics, Xi’an Institute of Optics and Precision Mechanics of CAS, Xi’an 710119, China
* Correspondence: haozhang@haust.edu.cn (H.Z.); xzli@haust.edu.cn (X.L.)

Abstract: Optical vortex lattices (OVLs) with diverse modes show potential for a wide range of applications, such as high-capacity optical communications, optical tweezers, and optical measurements. However, vortices in typical regulated OVLs often exhibit irregular shapes, such as being narrow and elongated. The resulting increase in asymmetry negatively impacts the efficiency of particle trapping. Additionally, the vortex radii expand with an increase in topological charge (TC), limiting the TC value of the vortices and hindering their ability to fully utilize orbital angular momentum (OAM). Herein, we propose an alternative approach to custom OVLs using off-axis techniques combined with amplitude modulation. Amplitude modulation enables the precise generation of an OVL with perfect vortex properties, known as a perfect off-axis OVL. Further, the number of vortices in the perfect off-axis OVL, the off-axis distances, and the TC can be freely modulated while maintaining a circular mode. This unique OVL will promote new applications, such as the complex manipulation of multi-particle systems and optical communication based on OAM.

Keywords: physical optics; perfect optical vortex; off-axis optical vortex; optical vortex lattice; orbital angular momentum

1. Introduction

A vortex beam is a “doughnut-shaped” beam with a helical phase wavefront and a central core of zero intensity, carrying an additional degree of freedom in its orbital angular momentum (OAM). They are essential tools in the fields of optical trapping [1,2], optical communication [3], and quantum information [4,5]. As research progresses, optical vortex lattices (OVLs), which are two- or three-dimensional structures composed of multiple vortices, have gained significant attention owing to their complex phase and intensity distributions. These lattices have potential applications in multi-particle trapping [6–9] and high-capacity optical communication based on OAM [10,11]. Therefore, research on the generation, regulation, and application of new OVLs is a leading topic in optics and related sciences.

In recent years, significant progress has been made in the study of OVLs, which are known for their rich physical properties and broad application prospects. From the initial optical Ferris wheels with specific parameters [12] to circular OVLs with multi-parameter modulation [13] and anomalous ring-connected OVLs [14], the degrees of freedom in OVL modulation have consistently increased. To diversify the spatial pattern distributions of these optical fields, arbitrary curve OVLs with sub-vortices arranged along various parametric curved paths were proposed [15]. Subsequently, high-order OVLs based on phase multiplication technology [6] and switchable hybrid-order OVLs based on hot-swapping concepts were developed [9], improving the ability of OVLs for use in optical tweezers. However, a detailed review of the development and research of OVLs reveals that, despite their broad application prospects, these optical fields have limitations that need to be
urgently resolved. First, owing to the modulation of special spatial modes, the shape of the sub-vortices in typical OVLs often becomes irregular or elongated instead of maintaining a circular mode. This increases the asymmetry of optical traps, affecting their particle trapping efficiency. Second, the radii of the OVL sub-vortices increase with topological charge (TC), limiting the range of achievable TCs and restricting the full utilization of OAM. Therefore, realizing OVLs with perfect characteristics where the radii of the sub-vortices do not change with the TC while maintaining a circular mode is of great importance.

Thus, inspired by the perfect Laguerre–Gaussian beam [16,17] and combined with off-axis vortex techniques [18,19], we propose a perfect off-axis OVL in which the radius of the sub-vortices in the lattice does not change with the TC, maintaining perfect characteristics and a circular mode. Specifically, our method combines amplitude modulation with off-axis technology to achieve precise control over the number, TC, and off-axis distance of sub-vortices while maintaining the circular mode of the sub-vortices. This enhances the precision and flexibility of optical field regulation, promoting the application and development of OVLs. Furthermore, we numerically and experimentally verified the generation of various customized perfect off-axis OVLs that are significant for high-capacity optical communications and complex multi-particle manipulation.

2. Methods

The proposed perfect off-axis OVL is generated by modifying the traditional off-axis OVL formula to achieve amplitude modulation. The perfect off-axis OVL can be generated and regulated rapidly and in real time by combining computational holography and a spatial light modulator (SLM). This results in a lattice in which the radii of the sub-vortices are independent of the TC. First, the expression of the diffracted field for a traditional off-axis OVL is as follows [18,19]:

\[
E_{\text{tra}}(r, \theta) = A_0 \exp \left(-\frac{r^2}{w^2}\right) \prod_{n=1}^{N} \left[r \exp(\pm i \theta) - r_n \exp(\pm i \theta_n)\right]^{m_n},
\]  

where \(A_0\) is an amplitude constant, \(r\) and \(\theta\) are the polar coordinates, \(w\) is the waist radius of the incident Gaussian beam, \(N\) is the number of off-axis vortices in the incident Gaussian beam, and \(m_n\) represents the TC of the \(n\)th off-axis vortex at position \(r_n\). When \(m_n\) is positive, \(\theta\) and \(\theta_n\) are positive; when \(m_n\) is negative, \(\theta\) and \(\theta_n\) are negative. Numerically simulated intensity patterns of traditional off-axis OVLs are shown in Figure 1(a1,a2), with the corresponding phase patterns in Figure 1(b1,b2). The yellow curves represent the vortex distributions at the position indicated by the white dashed line within the lattice. And the white arrow indicates the direction in which the TC of the lattice sub-vortices is assigned, similarly hereinafter. Here, the sub-vortex radii in the lattice increase with the TC, which hinders the full utilization of the OAM and limits the utility of this type of optical field. To achieve a vortex radius independent of the TC and realize perfect characteristics, Equation (1) is modified:

\[
E_{\text{per}}(r, \theta) = A_0 \exp \left(-\frac{r^2}{w^2}\right) \prod_{n=1}^{N} \left[r \exp(\pm i \theta) - r_n \exp(\pm i \theta_n)\right]^{m_n} \exp(\pm i \theta_n). \tag{2}
\]

The proposed method decouples the Gaussian distribution of the vortices in the lattice from the TC, thereby constructing the perfect off-axis OVL. Specifically, the TC is associated only with the phase, effectively separating it from the amplitude term associated with the vortex radius. This separation ensures that changes in the TC do not affect the amplitude distribution, enabling precise phase modulation and providing the perfect vortex properties within the lattice. The corresponding numerical-simulation light intensity and phase patterns are shown in Figure 1(a3,a4,b3,b4), respectively. These figures illustrate the perfect properties of the vortex in the lattice; specifically, the radius of the vortex remains constant regardless of an increase in TC.
3. Results and Discussion

The experimental setup for the generation of the perfect off-axis OVL is illustrated in Figure 2. A solid-state Nd:YAG laser with a wavelength $\lambda = 532$ nm was expanded and collimated through a pinhole filter (PF) and a lens (L1) with focal length of $f_1 = 200$ mm, respectively, to obtain parallel light. To ensure alignment with the directional axis of the reflective SLM (HOLOEYE, PLUTO-VIS-016; pixel size: $8 \times 8 \mu m^2$; resolution: $1920 \times 1080$ pixels), the beam was horizontally polarized using a circular aperture (A1) and a polarizer (P). The beam then illuminated the phase-only SLM loaded with a predesigned computer-generated hologram, as illustrated in Figure 2a. And the hologram is produced by multiplying the amplitude pattern of the on-demand mode and the phase component, which is a superposition of the phase distribution of the on-demand mode and the blazed grating. Subsequently, the reflected diffraction beams passed through a 4f imaging system consisting of two lenses (L2 and L3) with a focal length of $f_2 = f_3 = 150$ mm. A circular aperture (A2) located in the Fourier plane was used to select the positive or negative first-order diffraction beam, resulting in the desired perfect off-axis OVL. Finally, the experimental light intensity pattern was recorded by a complementary metal oxide semiconductor camera (CMOS, Basler acA1600-60gc; pixel size: $4.5 \times 4.5 \mu m^2$; resolution: $1600 \times 1200$ pixels), as illustrated in Figure 2b. A reference beam was split from the primary beam by beam splitter BS1 and was reflected by two mirrors (M2 and M3). The intensity distribution of the interference with the target beam was captured by a CMOS camera. By analyzing the number and orientation of interference fringe forks, both the magnitude and sign of the TC of the generated perfect off-axis OVL can be intuitively detected [1].

Using this experimental setup, a plane wave is selected as an incident field; we generated perfect off-axis OVLs with $m_1 = m_2 = 1, 2, 3, 4$ as examples of the two vortices in Figure 2. The experimental light intensity patterns are shown in Figure 3(a1–a4). The dashed rectangle results indicate that the vortex radii in the lattice remain constant and do not increase with the TC, demonstrating perfect properties. The agreement with the theoretical predictions confirms the effectiveness of the amplitude modulation method for traditional off-axis OVLs. The realization of perfect off-axis OVLs is significant for optical tweezers and high-capacity optical communication based on OAM. This advancement will further expand the application and scope of OVLs, enhancing their practical utility and enabling more precise and versatile manipulation of light and particles. To verify the

![Figure 1. Comparison between traditional and perfect off-axis optical vortex lattices. (a1,a2) Traditional off-axis optical vortex lattice simulation light intensity patterns and (b1,b2) the corresponding phase patterns. (a3,a4) Perfect off-axis optical vortex lattice simulation light intensity patterns and (b3,b4) the corresponding phase patterns.](image-url)
existence of vortices and determine the OAM, a reference beam was introduced to measure the TC.

![Figure 2](image_url)

**Figure 2.** Schematic of the experimental setup for generating perfect off-axis OVLs. PF: pinhole filter; L1–L3: convex lenses; M1–M3: mirrors; P: polarizer; BS1 and BS2: beam splitters; A1 and A2: circular apertures; SLM: spatial light modulator; CMOS: complementary metal-oxide semiconductor. Insets: (a) Hologram of the amplitude-modulated phase information encoded onto the SLM; (b) The intensity of the perfect off-axis OVL recorded by CMOS.

![Figure 3](image_url)

**Figure 3.** Perfect off-axis OVLs with different TCs. (a1–a4) Experimental light intensity patterns of perfect off-axis OVLs with $m_1 = m_2 = 1, 2, 3, 4$, respectively. (b1–b4) Experimental interference patterns of perfect off-axis OVLs and a plane wave. (c1–c4) The corresponding experimental phases. The white circular arrows are used to mark the singularities in the experimental phases.

As illustrated in Figure 2, a plane wave was selected for this purpose. By comparing the interference patterns between the vortex and reference beams, the presence and charac-
teristics of vortices were confirmed. The corresponding experimental interference patterns are shown in Figure 3(b1–b4). The magnitude and sign of the TC in the vortices can be obtained according to the number and direction of the interference fringes; a downward-facing interference fringe fork represents a positive vortex, and an upward-facing fringe fork represents a negative vortex. The phase retrieval technique was used to extract the experimental phase from the plane wave interferogram, as shown in Figure 3(c1–c4). From the experimental phase, the positions and TCs of the vortices can be determined. The phase increased from 0 to $2\pi$ at the singularity in a counterclockwise direction, indicating a positive TC. The number of phase increments indicates the TC magnitude [20,21]. The experimental results align with the theoretical predictions, confirming the effectiveness of the method. Note that the largest TC of the sub-vortex can reach 15; after more than 15, the radius will slowly increase. In the future, with careful design and arrangement of the hologram and experimental elements, it is possible to obtain a perfect off-axis OVL with higher TCs.

Moreover, different applications require different optical field spatial mode distributions, necessitating precise control and adjustment of the optical field. The perfect OVL generated using off-axis techniques provides abundant and tunable degrees of freedom, such as the number of vortices $N$, off-axis distance $d$, and TC $m_n$. These adjustments are difficult to achieve using traditional interference methods to generate such optical fields [22].

First, we regulated the number of vortices in the perfect off-axis OVL. Figure 4(a1–a4) show the experimental light intensity patterns for vortex numbers $N = 3, 4, 5, 6$. The number and direction of plane wave fringes in the experimental interferogram and the number and direction of experimental phase (0 to $2\pi$) changes determine the OAM of the vortices, matching the preset value $m_1 = m_2 = m_3 = 2$, as shown in Figure 4(b1–b4,c1–c4).

![Figure 4](image-url)

Figure 4. Perfect off-axis OVLs with different numbers of sub-vortices. (a1–a4) Experimental light intensity patterns of perfect off-axis OVLs with $N = 3, 4, 5, 6$, respectively. (b1–b4) Experimental interference patterns of the perfect off-axis OVLs and a plane wave. (c1–c4) The corresponding experimental phases.

To demonstrate the flexibility of the perfect off-axis OVLs in independently modulating vortices, we will use the three vortices in Figure 4(a1) as an example. The experimental light intensity patterns for a single vortex with different off-axis distances $d = 0, 0.4, 0.8, 1.2, 1.6$ are shown in Figure 5(a1–a5). The modulated vortex gradually moves horizontally...
to the right from the central position, consistent with the preset values. Moreover, the experimental interferograms shown in Figure 5(b1–b5) and the experimental phase patterns shown in Figure 5(c1–c5) confirm this phenomenon and verify the existence and values of the vortices. This method allows precise control over the position of individual vortices without altering the lattice structure of the perfect off-axis OVL, significantly broadening its application potential across various fields.

![Experimental interference patterns](image)

**Figure 5.** Perfect off-axis OVLs with different off-axis distances for a single vortex. (a1–a5) Perfect off-axis OVL experimental light intensity patterns with \( d = 0, 0.4, 0.8, 1.2, \) and 1.6, respectively. (b1–b5) Experimental interference patterns of the perfect off-axis OVLs and a plane wave. (c1–c5) The corresponding experimental phases.

Finally, to verify the potential application advantages of perfect off-axis OVLs in multiparticle manipulation, we demonstrated the ability to independently modulate TC. This verification confirms the versatility and robustness of perfect off-axis OVLs, underscoring their suitability for advanced applications in various scientific and technological domains. Here, we use a sub-vortex (white dashed box in Figure 6) as an example. The distribution of perfect off-axis OVL experimental modes under different TCs \( m_3 = -4, -1, 1, \) and 4 are shown in Figure 6(a1–a4). The experimental results show that the radius of a vortex in a perfect off-axis OVL does not change with the TC, maintaining its perfect characteristics. Furthermore, the TC of the vortex at any position in the perfect off-axis OVL can be freely customized. Similarly, the corresponding experimental interferograms and phase patterns were used to verify and identify vortices of different values in the perfect off-axis OVL, as shown in Figure 6(b1–b4,c1–c4), wherein the clockwise and counterclockwise arrows denoted by black and white circles, respectively, represent the positive and negative OVs.

In addition, the distribution patterns for the energy flow and OAM of vortices at specific locations were calculated. The calculations confirmed the stability and consistency of the physical properties of the vortices within the perfect off-axis OVL, reinforcing the effectiveness of the method for potential applications in particle manipulation and other fields. The corresponding energy flow distributions of the specific-unit OV marked by the white dashed boxes in Figure 6(a1–a4) are shown in Figure 6(d1–d4), where the directions of the red arrows represent the energy flow direction, and the size of the arrows indicate the magnitude. The direction of energy flow is consistent with that of the phase growth, and its magnitude increases with the absolute value of the TC, with positive and negative energy flows increasing in opposite directions. Moreover, the energy flow distribution is not uniform owing to the influence of the other OVs with \( m_1 = m_2 = 2. \) The insets in Figure 6(d1–d4) depict four-fold magnifications of the white box areas to facilitate the observation of the size and direction of the arrows. Figure 6(e1–e4) show the
distribution patterns of the OAM. As the absolute value of the TC increases, the OAM also gradually increases. This demonstrates that a perfect off-axis OVL can realize vortex order customization while maintaining the vortex radius and circular mode. This capability should expand the range of applications for this type of optical field.

![Figure 6](image_url)

**Figure 6.** Perfect off-axis OVLs under the hybrid TC of sub-vortices. (a1–a4) Perfect off-axis OVL experimental light intensity patterns with $m_3 = -4, -1, 1, \text{ and } 4$, respectively. (b1–b4) Experimental interference patterns of the perfect off-axis OVLs and a plane wave. (c1–c4) The corresponding experimental phases. (d1–d4) The energy flow of a specific-unit OV, highlighted by the white dashed boxes in the top row. (e1–e4) The corresponding experimental OAM densities.

### 4. Conclusions

In summary, we proposed a novel method of amplitude modulation combined with an off-axis vortex technique to achieve a perfect off-axis OVL. In this lattice, the radius of any vortex remains unchanging with TC, maintaining the perfect characteristics. This method allows precise control over the number of vortices, off-axis distances, and TC customization in two-dimensional space, with a wealth of adjustable parameters. It ensures that the circular structure of vortices is preserved, providing an effective scheme for accurate lattice shaping and multi-parameter control. This advancement will enhance the utilization of OAM in OVLs, thereby expanding their application utility.
Author Contributions: Y.T. and X.Q. contributed equally to this work. Conceptualization, Y.T. and H.Z.; data curation, Y.T.; formal analysis, X.Q. and W.W.; funding acquisition, X.L.; investigation, C.L.; methodology, X.Q.; project administration, X.L.; resources, Y.T.; software, C.L.; supervision, Y.T., H.Z. and X.L.; validation, X.Q., C.L. and W.W.; writing—original draft, Y.T. and C.L.; writing—review and editing, Y.T. and C.L. All authors have read and agreed to the published version of the manuscript.

Funding: Natural Science Foundation of Henan Province (232300421019); National Natural Science Foundation of China (12274116); Key Scientific Research Projects of Institutions of Higher Learning of the Henan Province Education Department (21zx002); State Key Laboratory of Transient Optics and Photonics (SKLST202216).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

Conflicts of Interest: The authors declare no conflicts of interest.

References


5. Wang, Z.; Lu, X.; Gao, J.; Zhao, X.; Zhan, Q.; Cai, Y.; Zhao, C. Coherence phase spectrum analyzer for a randomly fluctuated fractional vortex beam. Photonics Res. 2024, 12, 33–39. [CrossRef]


**Disclaimer/Publisher’s Note:** The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.