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A Novel Modulation Scheme Based on the Kramers–Kronig Relations for Optical IM-DD Systems

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Abstract

The increasing demand for higher data rates in optical communication systems, especially within data centers and backbone networks, calls for the development of advanced modulation formats that can significantly enhance system performance. In this work, we introduce a novel modulation format based on the Kramers–Kronig relations, designed to improve upon traditional techniques such as Pulse Amplitude Modulation (PAM) and Carrier-less Amplitude Phase (CAP) modulation. The novel modulation format was rigorously validated through experimental investigations using an optical wireless communication (OWC) link. The results demonstrate a notable improvement in bit error rate (BER) performance and receiver sensitivity when compared to the conventional PAM-4 modulation scheme and CAP-16 modulation schemes. Moreover, the proposed scheme effectively reduces the complexity of digital filtering required by CAP while lowering the demands on the Digital-to-Analog Converter (DAC), making it a more practical solution for high-speed optical communication. This advancement facilitates higher data transmission rates, proving the Kramers–Kronig relations modulation format as a promising alternative to existing methods. Its potential for enhancing the efficiency and capacity of optical communication systems is evident. Future research will focus on optimizing the modulation parameters and exploring their application in more complex scenarios, such as high-speed underwater visible light communication systems, where advanced modulation formats are crucial for overcoming bandwidth limitations.

Keywords: optical wireless communication; carrierless amplitude and phase modulation; Kramers–Kronig modulation; optical IM-DD systems; phase retrieval

1. Introduction

Optical communication systems are at the heart of modern telecommunications, enabling the high-speed data transmission necessary to support the rapidly growing global network infrastructure. As the demand for data rates continues to escalate, particularly within data centers and backbone networks, the choice of modulation formats plays a critical role in determining the efficiency and capacity of optical links. Intensity-modulated direct detection (IM-DD) systems, with their low-cost configurations, are well-suited for Ethernet, especially as speeds and the number of lanes increase through wavelength division multiplexing (WDM) and parallel single-mode fiber (PSM). Studies have shown that increasing the number of lanes can achieve higher overall transmission capacity, even

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at a relatively lower per-lane speed [1–6]. While IM-DD systems offer economic advantages, the emerging Ethernet standards must aim to reduce the number of multiplexing channels, thereby enabling fewer lanes with higher per-lane transmission rates. Therefore, advanced modulation formats have garnered substantial attention from both the research community and industry as promising approaches to reducing the number of required lanes while simultaneously increasing overall link capacity. M-PAM [7], DMT [8,9], and CAP [10] have been reported in a high-speed optical IM-DD communication system.

Turning to optical wireless communication (OWC) systems, bandwidth limitations arising from both the transmitter and receiver, coupled with inter-symbol interference (ISI), continue to represent significant constraints on overall link capacity. Besides advanced equalization techniques to mitigate ISI [11–17], advanced modulation schemes have also emerged as promising approaches to address these limitations and improve link capacity. Several studies have demonstrated the potential of these advanced techniques. For instance, in [11], PAM-4 modulation combined with a decision feedback equalizer was utilized on a VCSEL-based FSO link, cutting off the bandwidth in half compared to that of NRZ modulation. Similarly, CAP modulation is used to achieve 4.5 Gb/s [12], and a pre-equalized PAM-4 scheme enabled 1 Gb/s for free-space visible light communication [13]. Moreover, orthogonal frequency-division multiplexing (OFDM) was leveraged to mitigate bandwidth limitations, achieving a transmission rate of 2.32 Gb/s aided by a two-stage linear software equalizer [18]. Other OFDM/DMT systems can be found in [19,20]. Additional research focusing on the application of advanced modulation schemes to enhance OWC link performance can be found in [21–26]. These developments underscore the substantial potential of combining advanced modulation formats with equalization techniques to significantly improve the capacity of OWC links.

Despite these advancements, traditional advanced modulation formats still encounter several limitations. PAM, while simple and cost-effective, is prone to baseband wandering [27], which can negatively impact performance at higher data rates. OFDM/DMT, although resilient to frequency-selective fading, necessitates the application of Hermitian symmetry to convert the signal into the real value [8], which in turn reduces spectral efficiency and also suffers from the high peak-to-average power ratio (PAPR) [28]. Although various PAPR reduction techniques have been proposed for OFDM/DMT systems [29–31], single-carrier modulation inherently achieves much lower PAPR than OFDM/DMT. CAP modulation, while advantageous in minimizing baseline wander and maintaining spectral efficiency, complicates system design due to its reliance on multiple digital filters [32].

To address these challenges, we propose a novel modulation format based on the Kramers–Kronig (KK) relations. This new scheme is specifically designed to mitigate the shortcomings of existing formats, offering several key benefits. In comparison to OFDM, the proposed format enhances spectral efficiency by obviating the need for Hermitian symmetry. As a single-carrier-based modulation scheme, it does not generate high PAPR. Unlike conventional CAP modulation, four digital filters are required at the receiver, each typically consisting of dozens of taps, leading to relatively high computational complexity. In contrast, the proposed KK-based modulation scheme requires only a single digital filter for demodulation. In this work, a 15-tap filter is employed to perform the KK demodulation process. Therefore, compared with CAP modulation, the proposed scheme significantly reduces the digital filtering complexity while maintaining comparable demodulation performance. Additionally, the proposed scheme imposes lower demands on the Digital-to-Analog Converter (DAC) compared to CAP, making it a more efficient solution for high-speed optical communication systems.

This paper presents the first thorough evaluation of this Kramers–Kronig (KK) relation-based modulation format for optical links: by enforcing a minimum-phase constraint on the transmitted waveform, it enables the consolidation of traditional I/Q filter banks

into a single-filter structure. We demonstrate that a single 15-tap FIR filter can replace the entire four-filter CAP architecture while preserving the full modulation functionality. Compared to standard CAP systems, the proposed approach significantly reduces the demodulation-side computational complexity, offering substantial advantages in latency, power, and hardware requirements.

Given its compact structure and high spectral efficiency, the proposed scheme is particularly well-suited for bandwidth-limited and power-constrained optical communication scenarios, making it an attractive candidate for underwater wireless optical communication (UWOC) links. UWOC has garnered increasing research attention owing to its potential in marine environmental monitoring, oceanographic studies, and autonomous underwater systems [7–11]. By exploiting the low-attenuation blue-green optical window in seawater, UWOC systems can achieve orders-of-magnitude higher bandwidth and significantly lower latency compared with conventional acoustic or RF-based underwater links [12].

The remainder of this paper is organized as follows: Section 2 provides an in-depth exploration of the proposed modulation scheme and its theoretical foundation. Section 3 details the experimental setup used to assess the performance of the new format. Section 4 discusses the experimental results, highlighting significant improvements in link performance, ease of implementation, and reduced hardware requirements. Finally, Section 5 concludes the paper and outlines potential directions for future research.

2. Theoretical Analysis

The Kramers–Kronig (KK) relations are pervasive in various domains of physics and engineering, with applications spanning multiple areas [33–38]. In the context of direct-detection coherent receivers, their relevance arises from a fundamental property of minimum-phase signals, as detailed in [39,40]. Specifically, in [41], a Kramers–Kronig-based coherent receiver is proposed to recover complex data information directly from the current of the optical detector. In this paper, we take a different approach by constructing a minimum-phase signal in an intensity-based system and extracting its phase information from the signal itself. This method allows simultaneous transmission of both in-phase and quadrature-phase components. The central concept of our scheme is the identification of a minimum-phase condition, which ensures that the phase of the signal can be uniquely recovered from its intensity through the Hilbert transform. We define a baseband message signal $s(t)$ as a complex data-carrying signal, whose spectrum is constrained within the frequency range between $-B/2$ and $B/2$, and construct a single-sideband signal of the form:

$$h(t) = A + s(t)e^{i\omega t} \quad (1)$$

where A is constant, ω equals to πB . It has been established that $h(t)$ is a minimum-phase signal if and only if $|A| > |s(t)|_{\max}$ [30]. Under this condition, the phase $\phi(t)$ of the $h(t)$ is uniquely related to the Hilbert transform with its modulus $|h(t)|$, as described by the Kramers–Kronig relation:

$$\phi(t) = \frac{1}{\pi} p.v. \int_{-\infty}^{\infty} dt' \frac{\log|h(t')|}{t - t'} dt' \quad (2)$$

where $p.v.$ stands for principal value. Therefore, the minimum phase signal $h(t)$ can be expressed by:

$$h(t) = |h(t)|e^{\phi(t)} \quad (3)$$

In optical IM-DD systems, the modulated optical signal is inherently constrained to a real-valued waveform. We define the modulated signal $|h(t)|$ at the transmitter side expressed by:

$$|h(t)| = |h(t)e^{-i\omega t}| = |Ae^{-i\omega t} + s(t)| \quad (4)$$

The message signal $s(t)$ includes both in-phase and quadrature information and therefore let $s(t)$ be expressed as:

$$s(t) = I(t) + jQ(t) \quad (5)$$

where $I(t)$ are the in-phase transmitted symbols and $Q(t)$ are the quadrature transmitted symbols, whose bandwidth equals $s(t)$. The transmitted signal can be expressed by combining the real and imaginary parts of the equation and substituting ω with $2\pi f_c$:

$$|h(t)| = \sqrt{[I(t) + A\cos(2\pi f_c t)]^2 + [Q(t) - A\sin(2\pi f_c t)]^2} \quad (6)$$

This operation is referred to as KK mapping. For a roll-off factor of α and symbol rate R_s , the carrier frequency f_c can be expressed as:

$$f_c = \frac{\omega}{2\pi} = \frac{\pi B}{2\pi} = \frac{1}{2}(1 + \alpha)R_s \quad (7)$$

The message signal $s(t)$ is demodulated by:

$$s(t) = (|h(t)|e^{\phi(t)} - A)e^{-j2\pi f_c t} \quad (8)$$

Consequently, its in-phase and quadrature symbols can be demodulated as:

$$I(t) = \text{Re}[(|h(t)|e^{\phi(t)} - A)e^{-j2\pi f_c t}] \quad (9)$$

$$Q(t) = \text{Im}[(|h(t)|e^{\phi(t)} - A)e^{-j2\pi f_c t}] \quad (10)$$

where $\text{Re}[\cdot]$ and $\text{Im}[\cdot]$ represent the real and imaginary parts of the expression. Therefore, we could construct a transmitted signal $|h(t)|$ defined by Equation (6) and calculate the phase $\phi(t)$ of $h(t)$ by Hilbert transform. The in-phase and quadrature symbols of the message signal $s(t)$ can then be demodulated by Equations (9) and (10).

As demonstrated in [30], the amplitude A is critical for ensuring that the signal $h(t)$ meets the minimum phase condition. The phase of the signal can then be calculated using the Hilbert transform. The multilevel encoding is included in the following analysis. Figure 1 illustrates the modulation and demodulation process of the KK-16 modulation scheme. To satisfy the minimum phase condition, it is essential to select an optimal value for A to ensure accurate signal demodulation. Figure 2 presents theoretical demodulation diagrams of KK-16 for various values of A . The analysis shows that for $A = 0$, $A = 3$, and $A = 5$, the reconstructed signal fails to meet the minimum phase condition, leading to erroneous constellation recovery. In contrast, when $A = 8$, $A = 10$, and $A = 20$, the minimum phase condition is satisfied, resulting in successful constellation recovery.



Figure 1. Schematic representation of the KK based system (Demod.: Demodulation).

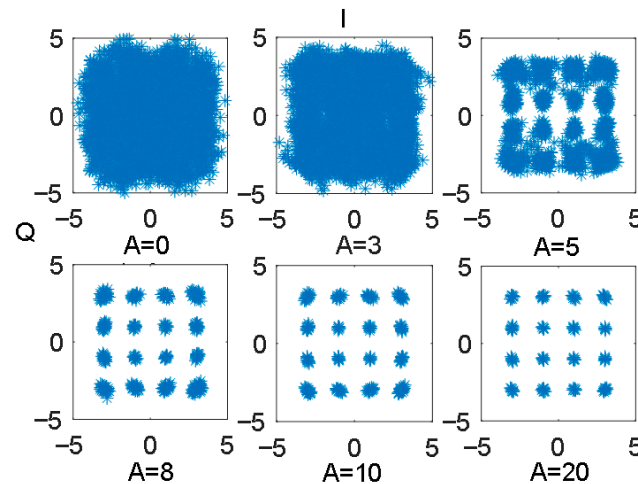


Figure 2. Theoretical demodulation diagrams illustrating the effect of varying the parameter A on constellation recovery.

We evaluate the demodulation error vector magnitude (EVM) of KK and standard CAP modulation under ideal conditions with varying total filter taps (spanning symbols for standard CAP) at the receiver. Note that the Hilbert filter employs an odd number of taps, while the CAP matched filters have taps in multiples of eight. An EVM threshold of 3% is used as the performance benchmark. Figure 3 illustrates the EVM as a function of the number of receiver filter taps. The results show that a 15-tap FIR filter achieves 2.8% EVM for KK, whereas the standard CAP requires 56 taps (across two filters) to reach 2.4% EVM. Consequently, a 15-tap FIR filter and 56-tap (7-symbol span corresponding to 28-tap for each channel) are chosen for KK and standard CAP owing to the comparable EVM performance, respectively, for computational complexity comparison. This substantial reduction in filter length directly translates into lower computational complexity and hardware implementation cost, underscoring the suitability of KK for practical low-complexity IM-DD systems.

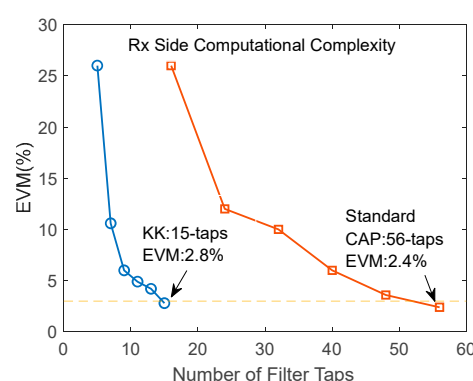


Figure 3. EVM of the KK scheme and standard CAP versus the number of Rx filter taps.

3. Experimental Configuration

In our experimental framework, we evaluate the performance of the KK scheme. Data transmission measurements were conducted over a 50 cm OWC link, as depicted in Figure 4. At the transmitter end, a 2.5 GSa/s arbitrary waveform generator (AWG, SIGLENT SDG7102A) with a 1 GHz analog bandwidth was employed to generate the high-speed modulated signal. The modulation was implemented using one arm (MZM A) of an I-Q modulator (SUMITOMO T.SBXH1.5-20PD-ADC) with a 20 GHz analog bandwidth. The

MZM A was biased at 0.5 V, with the peak-to-peak voltage of the modulated signal set at one volt to ensure linear operation, while the other arm (MZM B) was not utilized.

Each data pattern consisted of 30,000 randomly generated bits, and 10 such patterns were used for BER testing. The data was first encoded into the desired symbol format, and this encoded data stream was used to drive one channel output of the AWG, which was connected to the corresponding arms of the I-Q modulator.

The modulated optical signal was then coupled into free space through an optical collimator. The transmitted beam was then collected by an objective lens and coupled to the 0.2 mm-diameter active area of a free-space photodetector. A free-space variable optical attenuator (VOA) was employed to precisely control the received optical power (ROP) of the photodetector. This photodetector, with a 3 dB bandwidth of 1.5 GHz, converted the received optical signal into its electrical equivalent. The captured waveform was then recorded by a 50 GHz sampling oscilloscope (Tektronix MS064B) with a 2.5 GHz analog bandwidth. Subsequent offline signal processing, including demodulation, equalization, and BER computation, was performed using MATLAB R2022a.

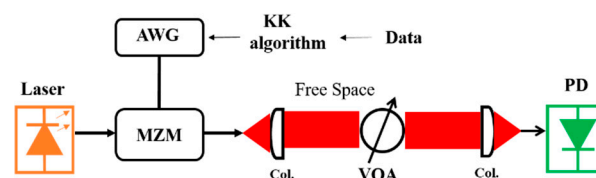


Figure 4. Experimental setup used in the data transmission test.

4. Experimental Results and Discussion

Due to the limitations of the AWG used in this setup (2.5 GSa/s), the objective is not to push the boundaries of data rate, but rather to experimentally validate the underlying principles of the KK scheme.

In this section, we present a comprehensive analysis of the experimental results obtained from the implementation of the proposed modulation scheme. The experiments were conducted over an OWC link to assess the performance of the novel modulation format, with a focus on comparing it to the PAM-4 and conventional CAP-16 modulation.

4.1. Impact of Parameter A on System Performance

Figure 5 illustrates the constellation diagrams of the proposed modulation scheme at a data rate of 2.5 Gb/s, under an ROP level of 1 μ W. The constellations were captured following the application of the CAP equalizer [10], utilizing a 2×2 FFE equalizer to simultaneously suppress the ISI and phase rotation impairments. The results reveal a progressive improvement in constellation clarity as the parameter A increases. Specifically, the constellations corresponding to $A = 8$, $A = 10$, and $A = 20$ demonstrate well-defined symbol separation, indicating successful demodulation.

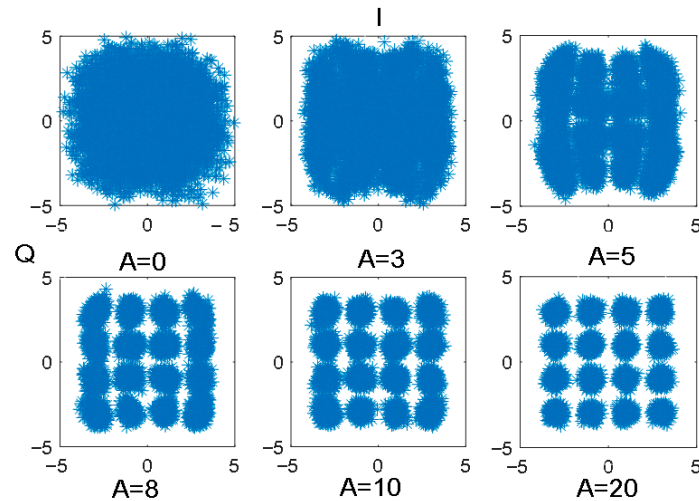


Figure 5. Constellation diagrams of the KK-16-based modulation scheme at 2.5 Gb/s with a ROP of 1 μ W.

The BER performance of the OWC link, as a function of the parameter A , is presented in Figure 6. The experimental data indicate that the proposed modulation scheme achieves a BER below the hard-decision forward error correction (HD-FEC) threshold of 3.8×10^{-3} for $A = 8$ and higher. In contrast, the configurations with $A = 3$ and $A = 5$ fail to meet the HD-FEC threshold, further corroborating the necessity of optimal parameter selection to ensure reliable transmission. The experimental results align well with the numerical simulation results presented in Section 2. It is noteworthy that the BER plot for $A = 0$ was omitted, as the corresponding constellation diagrams clearly indicate a failure in signal demodulation. Furthermore, the receiver sensitivity to achieve the HD-FEC limit was observed to be approximately -31.5 dBm, -32 dBm, and -33 dBm for $A = 8$, $A = 10$, and $A = 20$, respectively. Figure 7a systematically evaluates the relationship between feedforward equalizer (FFE) tap length and BER performance in the in-phase (I) channel under constrained receiver conditions ($A = 20$, ROP = 1 μ W, 2.5 Gb/s data rate), with equalization exclusively applied to the main channel. The BER curve exhibits characteristic saturation behavior beyond 20 taps, demonstrating diminishing returns when extending the equalizer span. Figure 7b reveals a critical system impairment through demodulated constellation diagrams: phase drift induced by quadrature channel interference. This crosstalk-induced rotation is effectively compensated through sparse equalizer configurations on the crosstalk channel. Figure 8a systematically examines BER evolution versus crosstalk channel equalizer span, exhibiting significant error reduction. The corresponding demodulation map with 60 taps applied to the crosstalk channel is also presented in Figure 8b. Even with just a single tap, the phase rotation effect is effectively mitigated, demonstrating the efficiency of the CAP equalizer in counteracting the crosstalk-induced distortion.

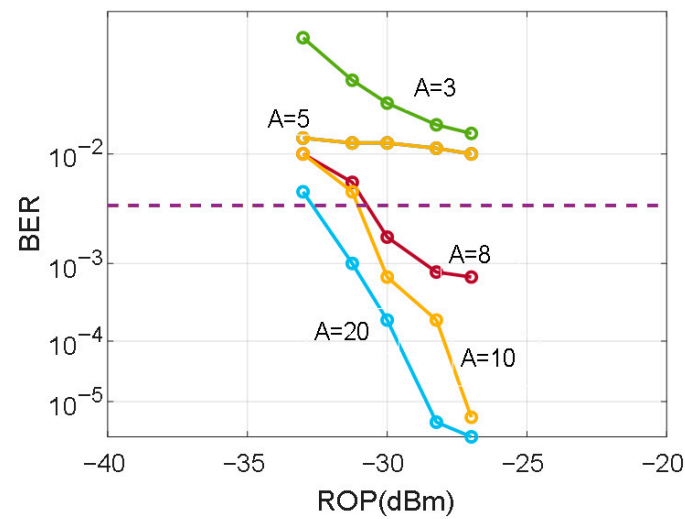


Figure 6. BER performance (dashed line: HD-FEC) of the KK-16 modulation scheme as a function of the parameter A at 2.5 Gb/s data transmission.

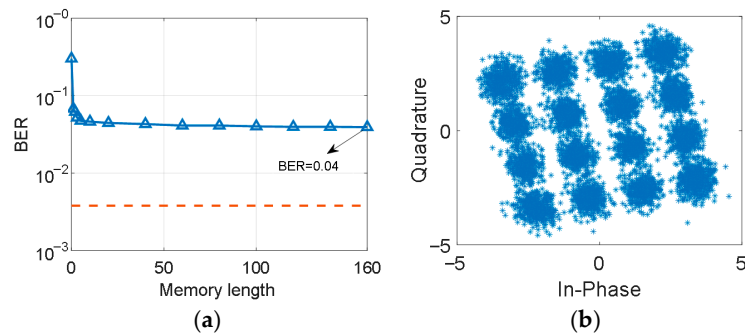


Figure 7. (a) BER dependence (dashed line: HD-FEC) on main channel tap length (b) demodulation constellation with 100 taps on the main channel for 2.5 Gb/s data transmission at ROP of 1 μ W.

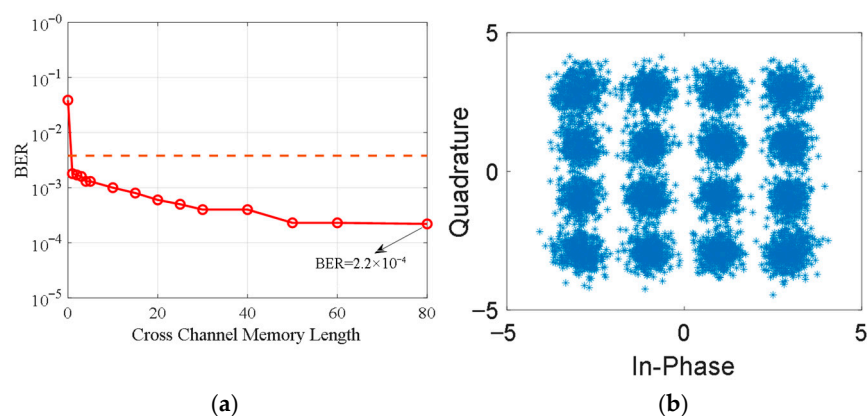


Figure 8. (a) BER dependence (dashed line: HD-FEC) on the crosstalk channel tap length, while the main channel tap length is fixed at 100 taps. (b) demodulation constellation with 60 taps on the crosstalk channel for 2.5 Gb/s data transmission at ROP of 1 μ W.

4.2. Comparative BER Analysis with PAM-4 Modulation and CAP-16 Modulation

To further validate the efficacy of the KK-16 scheme, we compared its BER performance against the PAM-4 and conventional CAP-16 modulation at 2.5 Gb/s. Figure 9 shows the demodulation results of CAP-16 and the eye diagram of the PAM-4 signal at

ROP of 1 μ W. Furthermore, the comparison, illustrated in Figure 10, reveals that the KK scheme outperforms PAM-4 and CAP-16 by a margin of 1.5 dB and 0.6 dB in receiver sensitivity, respectively.

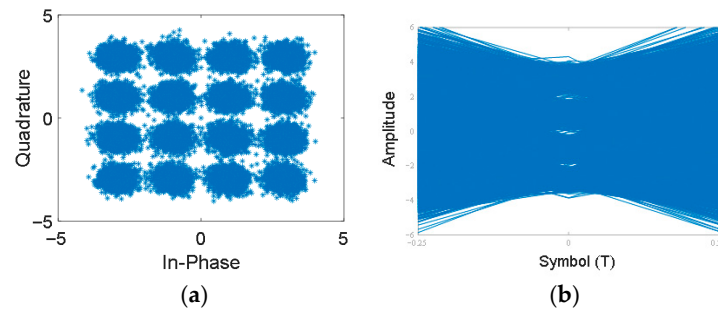


Figure 9. Demodulation constellation for the (a) CAP-16 modulation scheme and (b) eye diagram of the PAM4 modulation at 2.5 Gb/s under ROP of 1 μ W.

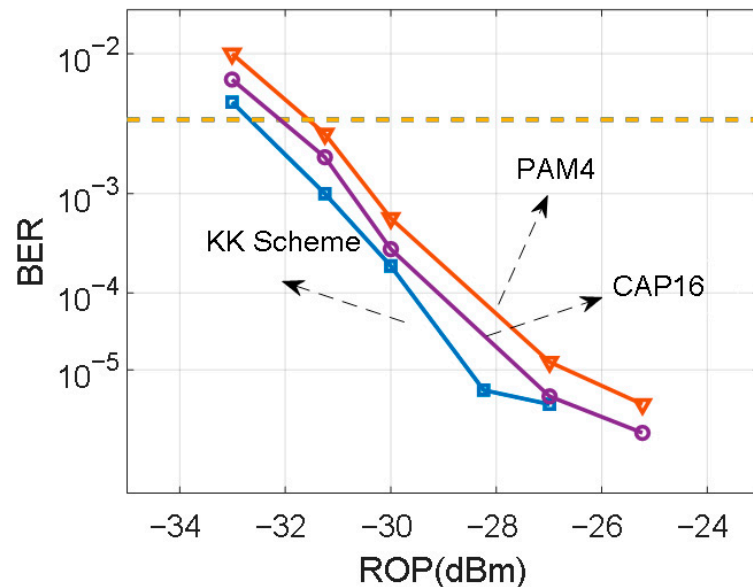


Figure 10. Comparative analysis of BER performance (dashed line: HD-FEC) among the KK modulation scheme, conventional CAP-16, and PAM-4 modulation at 2.5 Gb/s.

The KK provides a simpler implementation with one digital filter to operate the Hilbert transform than the CAP modulation, which requires four digital filters. In addition to this improvement, the proposed modulation format also lowers the required sampling rate of the DAC, thereby enabling more efficient and practical implementations in high-speed optical communication systems, which will be discussed in the next section. Figure 11 presents the BER performance of three modulation schemes—PAM-4, KK-16, and CAP-16—at various data rates, with a received optical power (ROP) of 500 nW. The results indicate that the KK-16 consistently outperforms both the PAM-4 and CAP-16 schemes across all data rates. Figure 12 displays the demodulation constellation diagrams for the KK-16 at an ROP of 500 nW, illustrating its stable performance even under low ROP conditions.

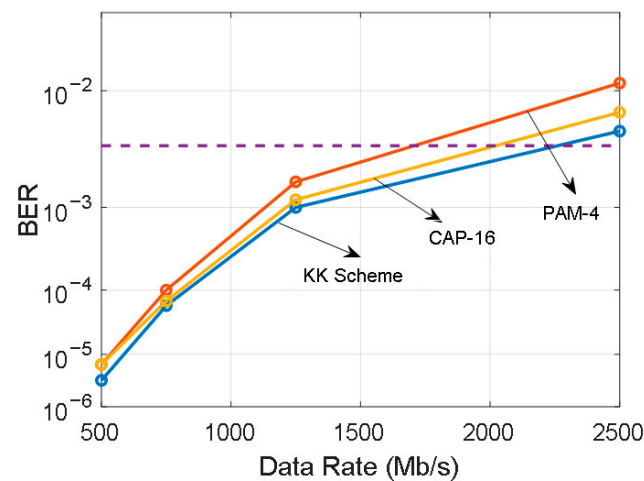


Figure 11. BER plot (dashed line: HD-FEC) of KK scheme, CAP-16, and PAM-4 at various data rates under 500 nW ROP.

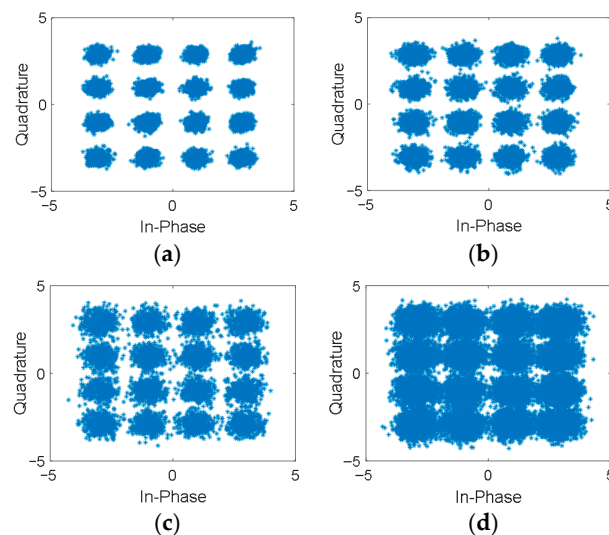


Figure 12. Demodulation constellations for KK-16 modulation under data rates of: (a) 500 Mb/s, (b) 750 Mb/s, (c) 1.25 Gb/s, (d) 2.5 Gb/s at ROP of 500 nW.

4.3. Oversampling Ratio Requirement for CAP and KK Based Modulation

The experimental data rate in this configuration is fundamentally limited by the arbitrary waveform generator's (AWG) maximum sampling rate of 2.5 GSa/s. When employing a 2.5 GSa/s sampling clock with an oversampling ratio (OSR) of 2, both conventional CAP and the proposed KK schemes achieve a symbol rate of 1.25 Gbaud, theoretically supporting 5 Gb/s transmission through 4 bits/symbol encoding. However, conventional CAP-16 exhibits critical limitations under this configuration: The insufficient OSR induces severe inter-symbol interference (ISI) in the quadrature channel. This impairment is quantitatively demonstrated in Figure 13, showing irreducible EVM in CAP-16 constellations at ROP levels of 1.5 μ W and 3 μ W.

In contrast, Figure 14 reveals the KK's inherent resilience to OSR constraints. Despite operating at the same 5-Gb/s rate and ROP conditions, the KK maintains well-defined constellation clusters compared to conventional CAP-16. This validates the scheme's ability to preserve orthogonality with OSR = 2, whereas conventional CAP requires at least OSR = 3 to achieve comparable stability.

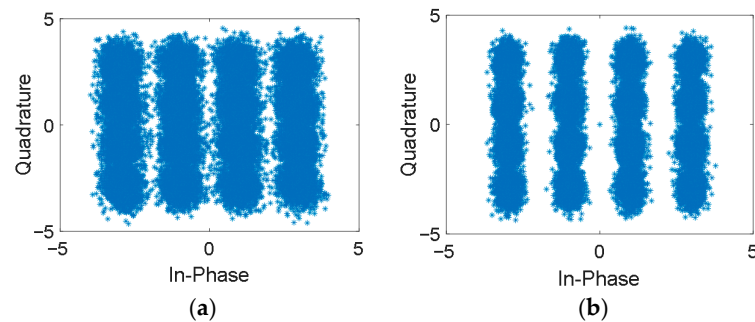


Figure 13. Constellation diagrams at 5 Gb/s data transmission for CAP-16 (oversampling ratio = 2) modulation under ROP level of: (a) 1.5 μ W and (b) 3 μ W.

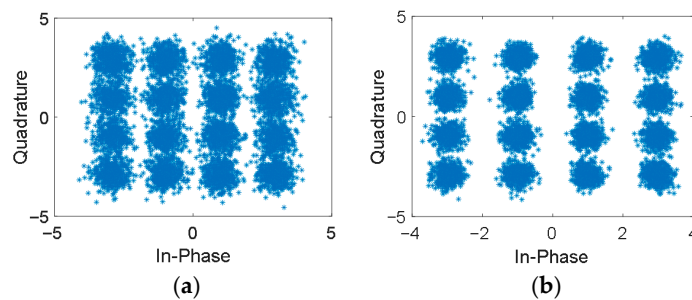


Figure 14. Constellation diagrams at 5 Gb/s data transmission for KK-16 scheme ($A = 20$, oversampling ratio = 2), under ROP levels of (a) 1.5 μ W and (b) 3 μ W.

Correspondingly, the BER performance at 5 Gb/s, shown in Figure 15, demonstrates that the receiver sensitivity at the HD-FEC limit is approximately -27.5 dBm, further highlighting the KK scheme's capacity for high-speed optical communication. These results also indicate that the proposed scheme can ease the DAC sampling rate in comparison with the conventional CAP modulation.

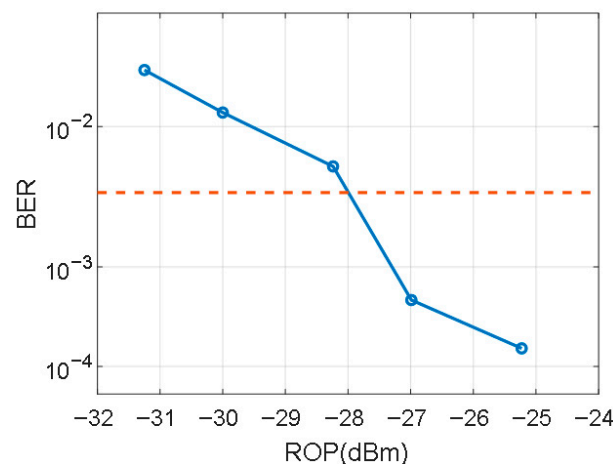


Figure 15. BER performance (dashed line: HD-FEC) for KK-16 at 5 Gb/s data transmission with the HD-FEC limit as a benchmark.

5. Conclusions

In this work, we proposed a Kramers–Kronig Relation based modulation format for IM-DD systems and experimentally validated its performance over an OWC link. The results demonstrate that the proposed scheme achieves improved BER performance and receiver sensitivity compared with conventional PAM-4, providing a measured sensitivity

gain of 1.5 dB while simultaneously relaxing DAC requirements. These features highlight its low-complexity implementation and suitability for high-speed optical links. Overall, this modulation format offers an efficient alternative to existing IM-DD techniques, enabling enhanced link performance without increasing system complexity. Future work will focus on parameter optimization and extension to more challenging scenarios, such as underwater visible light communication, where bandwidth limitations demand spectrally efficient and robust modulation formats.

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Conflicts of Interest: The authors declare no conflict of interest.

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