

Article

LED-Based Polar Coded Wireless Quantum Optical Communications for 6G and Beyond [†]

Kushtrim Dini *, Hamza Almujaheed and Peter Jung 

Lehrstuhl für Kommunikationstechnik, Universität Duisburg-Essen, 47057 Duisburg, Germany; hamza.almujaheed@kommunikationstechnik.org (H.A.); peter.jung@kommunikationstechnik.org (P.J.)

* Correspondence: kushtrim.dini@kommunikationstechnik.org

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Abstract

Wireless communication above 300 GHz requires highly sophisticated analog circuit design due to severe frequency dependent ohmic losses. The complexity of such electronic hardware motivates exploring wireless quantum optical communication approaches even for the 6G “terahertz (THz) range” [300 GHz, 10 THz]. In this work, the classical radio frequency (RF)-based inner physical layer (PHY) transceiver blocks of channel coded wireless communication systems are replaced by wireless quantum optical transceiver blocks. Short range concepts employing LEDs as transmitters are particularly attractive, owing to their low implementation cost and practical simplicity. In contrast to laser based wireless quantum optical transmission over multipath channels, the quantum mechanical density operator $\rho_{\text{RX},[\mathbf{s}_i, \mathbf{b}_i]}$ and the transition probability $\gamma(\mathbf{s}_i, \mathbf{s}_{i+1})$ required by the quantum data detection must be revised accordingly. Furthermore, the novel interpretation introduced here, in which the extrinsic information is treated as a diversity branch rather than as an estimate of the a priori information, facilitates turbo equalization that still can accommodate varying a priori information. However, due to the limited uncoded transmission performance achievable with such systems, the incorporation of sophisticated channel coding schemes appears imperative. The authors therefore investigate the combination of sophisticated channel coding techniques, such as polar coding, with LED based wireless quantum optical transmission technologies. All numerical results assume a cryogenically cooled receiver front-end (approximately 10 K), yielding thermal noise levels. Operation at room temperature in the 6G THz range [300 GHz, 10 THz] would require an average number $\bar{N}_\alpha$ of thermal noise photon values of approximately 5 to 20, which is beyond the scope of this feasibility study. The results show that the proposed paradigm enables simple, robust, and practically viable wireless quantum optical communication systems with favorable transmission performance. Additional gains are achieved through iterative turbo equalization. The results also suggest that the proposed approach can pave the way toward robust and economically viable future communication solutions.

Keywords: 6G; log likelihood ratio (LLR); quantum data detection; polar codes; turbo equalization; wireless quantum optical communications



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1. Introduction

Future wireless communication systems are expected to increasingly incorporate quantum communication technologies, motivated by the demonstration of quantum

supremacy [1] and supported by strategic initiatives such as the “Quantum Europe Strategy” [2], the “Draghi report” ([3], part B), and the upcoming “Quantum Act”. In September 2024, the Third Generation Partnership Project (3GPP) approved a study on use cases and service requirements for 6G systems, aiming to support societal progress in a secure, resilient, and sustainable manner [4]. To enable these use cases, carrier frequencies from 100 GHz to 10 THz are envisioned [5]. Since conventional RF front-end design becomes increasingly challenging at very high frequencies due to frequency-dependent losses and device limitations, the 6G “terahertz (THz) range” [300 GHz, 10 THz] has mainly been studied for photonic systems [6]. This motivates the investigation of wireless quantum optical communication as a possible pathway for replacing conventional mixed-signal and analog RF transceiver components with quantum optical alternatives. Recently, both laser based and LED based wireless quantum optical communication concepts have been considered [7,8].

LED based wireless quantum optical systems are of particular interest because LEDs enable comparatively simple transmitters. Unlike lasers, however, LEDs likely emit incoherent thermal states, which significantly degrade error performance [8]. Advanced channel coding is therefore required. Among modern channel codes, polar codes [9] are especially relevant because they were adopted in the 5G/New Radio (5G/NR) standard [10] and may also be applicable to future 6G systems. Accordingly, this work applies polar codes to LED based wireless quantum optical communication.

Since the 1960s, quantum communication has evolved into a major research field, exploiting physical effects such as entanglement and leading to concepts like quantum teleportation and quantum computing [11–18]. From an engineering perspective, however, communication remains the transmission of information between points via electromagnetic waves ([19], pp. 218, 642). Since human generated messages are classical, while quantum states act as carriers of this information, quantum communication can be viewed as the transmission of classical information through quantum states ([20], Figure 4.8, p. 114). Consequently, communication between a transmitter, “Alice,” and a receiver, “Bob,” should not be regarded merely as the transfer of qubits, but as part of a broader communication process that begins and ends in the classical domain.

To illustrate so-called “quantum enhancements,” consider the well-known quantum teleportation protocol [11,14,20,21], cf. Figure 1, using an entangled EPR pair and a third qubit $|\psi\rangle$, representing a superposition of $|0\rangle$ and $|1\rangle$. Since a qubit has two eigenstates, it can encode at most one classical bit b . “Alice,” applies a CNOT gate, a Hadamard gate H , and measurements to her half of the EPR pair and the third qubit, causing what appears to be the “collapse” of the state of “Bob’s” half of the EPR pair. The obtained two classical bits are transmitted to “Bob” who applies the Pauli gates X and Z when required, cf. Figure 1. Although this protocol is often described as the transfer of a qubit, it also relies on classical communication and therefore concerns the physical layer (PHY) of the communication stack [22,23], where “raw bits” are transmitted via physical signals, including quantum mechanical states. In classical systems, the PHY also includes forward error correction (FEC), modulation and demodulation, synchronization, detection, and multiplexing [23].

Quantum teleportation also raises practical challenges for mobile communications. Reliable and low cost generation and distribution of entangled qubits remain difficult, particularly for handheld devices. Moreover, the idealized protocol in Figure 1 neglects environmental interactions and quantum noise, whose mitigation may require quantum error correction (QEC), increasing the required number of qubits N_q beyond the idealized three-qubit description. The protocol also assumes ideal classical links. In practice, reliable wireless transmission requires classical FEC with code rates R_c often below $1/2$ ([24], Sect. 6.1.4, pp. 297–299), ([25], Sect. 3.3, pp. 153–170), ([26], p. 43; pp. 113–188). If $(N_q - 2)$

entangled qubits are shared between “Alice” and “Bob,” the effective code rate of the teleportation protocol becomes $1/(N_q - 2 + 2/R_c)$. For N_q equal to 3 and R_c equal to $1/2$, the overall rate will become as low $1/5$, and perfectly reliable transmission is still not guaranteed.

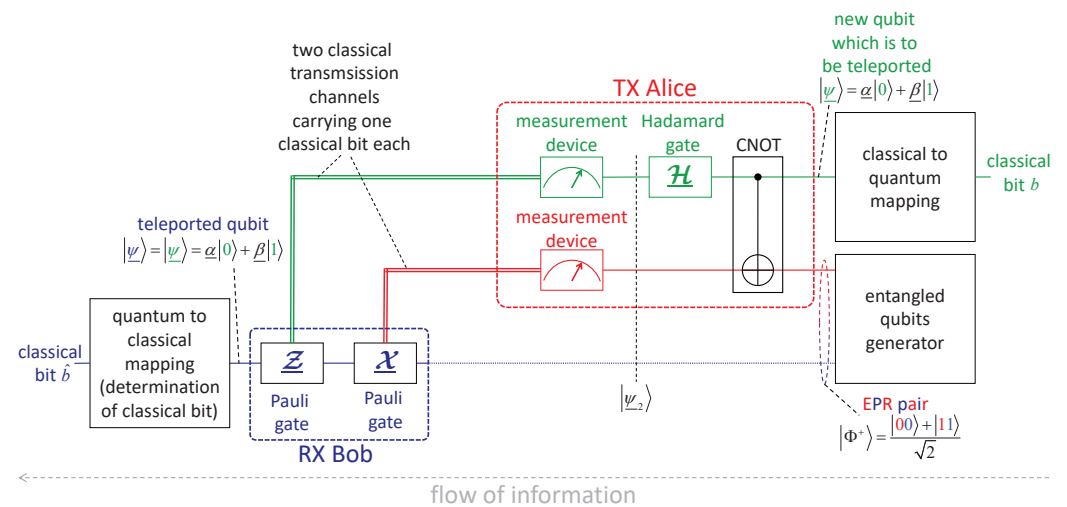


Figure 1. Schematic of the quantum teleportation of a single qubit neglecting interactions with the environment, cf., e.g., ([11], Figure 1.13, p. 27), ([14], Figure 6.3, p. 194), ([20], Figure 13.9, p. 661), ([21], Sect. 1.8.2).

Furthermore, the energy per information bit is also dominated by the classical transmissions, whereas the photon energy associated with the N_q qubits can be comparatively small. Consider a classical transmission carrier frequency in the far infrared domain at f equal to 4.85 THz, i.e., the center of the prospective 6G “terahertz (THz) range” [300 GHz, 10 THz] ([21], p. 6), with an average transmit power of 100 mW and a target data rate of 5 Gbit/s. The emitted average energy per classical information bit b is 20 pJ/bit, corresponding to

$$\bar{N}_\mu \approx 6.2 \cdot 10^9, \quad (1)$$

i.e., more than six billion signal photons emitted per classical information bit on average, with each photon carrying the energy

$$E_{\text{photon}} = \hbar 2\pi f \approx 3.2 \cdot 10^{-9} \text{ pJ}. \quad (2)$$

The extremely large average numbers of emitted signal photons, $\bar{N}_\mu$, explain why classical wireless systems are usually modeled with continuous waves rather than discrete photons. As the carrier frequency increases, however, each photon carries more energy, reducing the number of photons required per classical information bit. This shows why classical radio systems are largely insensitive to quantum effects, whereas optical and quantum communication systems may operate in a photon counting regime. Therefore, a transition from classical transmission models toward a quantum mechanical framework is well motivated.

The need for classical transmissions, as in Figure 1, is widely recognized [27–32]. However, classical PHY impairments have received comparatively little attention. These include hardware imperfections such as I/Q mismatch, imperfect image rejection, DC offsets, thermal noise, and shot noise [33], as well as channel impairments like co-channel interference (CCI), adjacent channel interference (ACI), and intersymbol interference (ISI) [34]. Such impairments often require sophisticated FEC techniques.

From an engineering perspective, the central objective is efficient communication, i.e., achieving the required throughput or data rate with limited resources, especially system bandwidth per cell, while satisfying quality of service (QoS) constraints such as the target bit error ratio (BER) P_b or block error ratio (BLER) P_{block} [34]. In mobile communications, this led to spectral efficiency as a key performance metric [34,35], with medium code rates, typically above 1/4, often providing favorable spectral efficiency [24–26]. Since quantum enhancements are unlikely to fully replace existing communication systems in the near future, established QoS criteria should remain considerable. Demonstrable improvements over competing classical communication concepts are therefore required to justify the practical relevance of quantum enhanced approaches.

The discussion above suggests that the schematic in Figure 1 should be revised. In particular, eliminating classical transmissions appears advantageous because it may reduce the average number of emitted signal photons and thus the overall energy consumption. Achieving this requires abandoning entanglement in the quantum communication links that replace the classical links in Figure 1. The corresponding system aspects are discussed in Section 2. Based on these considerations, Section 2 identifies the quantum data detector as a central component because it interfaces the received quantum states with the classical decision variables required for digital communication.

In classical communications, channel coded systems can be further improved by turbo equalization, an iterative process that combines data detection and channel decoding by exchanging soft information between the detector and decoder [36]. Turbo equalization for classical polar coded communications was first studied in [37]. In [7], the novel extended concept was applied to laser based wireless quantum optical communications, while turbo equalization for LED based polar coded wireless quantum optical communications was first considered in [8]. This manuscript considerably extends the findings reported in [8], cf. Section 3.

Section 4 presents simulation results demonstrating the benefits of polar coding. The results also show that the revised communication concept of Section 2, which abandons entanglement and classical links, significantly reduces the number of emitted signal photons compared with classical transmission.

Finally, Section 5 concludes the manuscript.

In what follows, complex values are underlined. Operators, matrices and vectors are in bold face italics. $\mathbb{F}_2$ equal to $\{0, 1\}$ denotes the Galois field with the two elements 0 and 1. $\mathbb{K}$ equal to $\{0, 1 \cdots (K - 1)\}$, $K \in \mathbb{N}^*$, is the set of message bit indices, and $\mathbb{B}$ equal to $\{0, 1 \cdots (N - 1)\}$, $N \in \mathbb{N}^*$, $N \geq K$, is the set of codebit indices. We will apply the Dirac notation.

2. System Model

Frequently used symbols are listed in Table 1. The PHY schematic of a possible wireless quantum optical communication system is shown in Figure 2, illustrating the basic transmitter and receiver connected via the quantum channel (see also [7], Figure 1, p. 3, ref. [20], Figure 13.9, p. 661, and [21], Figure 2.1, p. 450). Referring to Figure 1, the proposed wireless quantum optical communication system, exemplarily illustrated in Figure 2, can be employed to replace the two classical transmission channels shown in Figure 1, each conveying one bit of information. Consequently, the wireless quantum optical communication system shown in Figure 2 becomes an integral component of the quantum teleportation scheme depicted in Figure 1. In addition, the proposed approach enables the sequential teleportation of large numbers of qubits with comparatively low transmitter-side energy consumption, as discussed above. Furthermore, quantum teleportation may be dispensed with altogether in favor of the wireless quantum optical communication

system considered herein. In this case, the communication concept illustrated in Figure 1 can be replaced entirely by the wireless quantum optical communication system shown in Figure 2. The architecture depicted in Figure 2 combines classical PHY transceiver functionality, widely used in wireless and mobile radio systems, with a quantum PHY front-end. At the transmitter, the quantum PHY front-end replaces the modulator and mixed-signal front-end, while at the receiver it replaces the mixed-signal front-end and adaptive data detector used in classical systems [34]. Thus, Figure 2 illustrates a natural integration of quantum communication functionality into future wireline and wireless systems, e.g., 6G. Layer 2 is assumed to provide control messages for the transmitter and receiver blocks. However, their detailed structure is beyond the scope of this PHY focused manuscript.

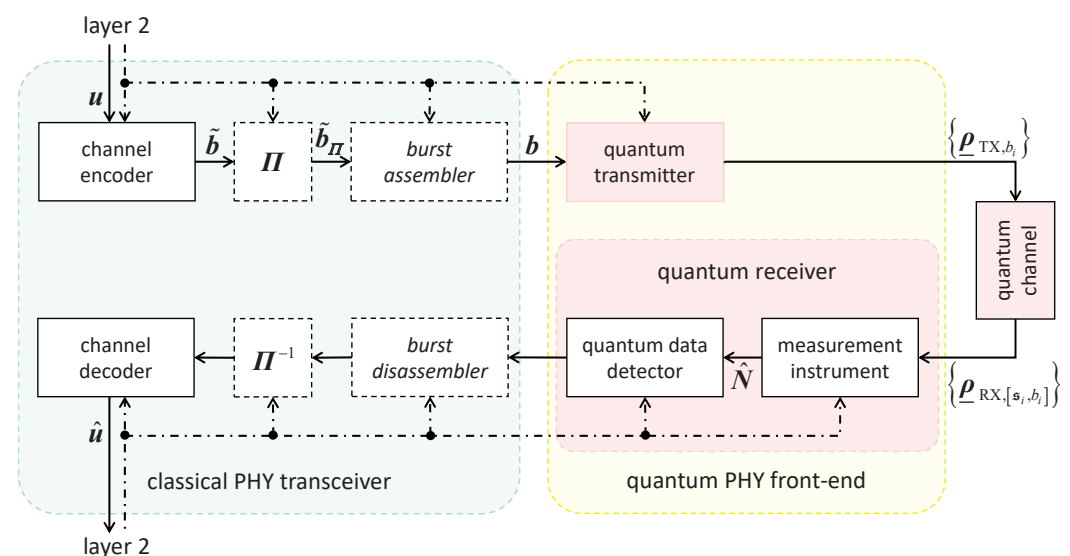


Figure 2. Schematic of a quantum communication system, which renounces on entanglement and on classical transmissions, cf., e.g., ([7], Figure 1, p. 3), ([20], Figure 13.9, p. 661), ([21], Figure 2.1, p. 450).

To demonstrate the viability of the concept in Figure 2, particularly the integration of the quantum PHY front-end as a replacement for classical modulation, demodulation, and mixed-signal functionality in mobile radio transceivers, the authors employ the 3GPP polar code used for 5G/NR control channels [10,38]. To the best of the authors' knowledge, quantum communication using this polar code was first discussed in 2025, see, e.g., ref. [7]. Since the 5G/NR polar code is well established and widely deployed, only the parameters relevant to the present work are specified, while further details are available in, e.g., ref. [7,10,38]. The reader is referred to, e.g., ref. [39], for further details on polar codes.

At the transmitter, messages are generated, each having the form

$$\mathbf{u} = (u_0, u_1 \cdots u_{N-1}), \quad N \in \mathbb{N}^*. \quad (3)$$

The said message $\mathbf{u}$ contains $(N - K)$ frozen bits, $K \in \mathbb{N}$, each one fixed to 0, and, in addition, 24 cyclic redundancy check (CRC) bits generated according to the 5G/NR specification ([38], Sect. 6.13.2.1, pp. 140–142), neither of which carries information. Hence, there are only $(K - 24)$ message bits carrying information created by the user.

Table 1. Frequently used symbols.

$\alpha(\mathbf{s}_i)$	probability of the occurrence of $\mathbf{s}_i$ in the forward recursion of the BCJR algorithm
$\underline{\alpha}$	complex amplitude
$ \underline{\alpha}\rangle$	state vector of a coherent state
$\beta(\mathbf{s}_{i+1})$	probability of the occurrence of $\mathbf{s}_{i+1}$ in the backward recursion of the BCJR algorithm
$\tilde{\mathbf{b}}$	codeword
$\tilde{\mathbf{b}}_{\Pi}$	interleaved codeword
b	information bit
b_i	codebit
$\tilde{b}_{\Pi,i}$	interleaved codebit
C_S	nonnegative scaling factor $\in \mathbb{R}_0^+$
$\mathcal{D}_{[\mathbf{s}_i]}$	detection operator
$\mathbb{F}_2$	$\{0, 1\}$, Galois field with the two elements 0 and 1
$\gamma(\mathbf{s}_i, \mathbf{s}_{i+1})$	transition probability of the transition from $\mathbf{s}_i$ to $\mathbf{s}_{i+1}$
$\underline{I}$	identity operator
L_c	log likelihood ratio (LLR) vector for the codebits $b_i \in \mathbb{F}_2, i \in \mathbb{B}$
L_e	extrinsic log likelihood ratio (LLR) vector
L_{in}	input log likelihood ratio (LLR) vector
L_u	log likelihood ratio (LLR) vector for the message bits $u_i \in \mathbb{F}_2, i \in \mathbb{B}$
ΠL_e	interleaved extrinsic log likelihood ratio (LLR) vector
$L_{\Pi e,i}$	extrinsic LLR contained in the interleaved extrinsic log likelihood ratio (LLR) vector
L_i	log likelihood ratio (LLR)
L	number of candidate paths (“list length”) evaluated by the CA-SCL polar code decoder
$\{ n\rangle\}$	Fock basis, i.e., the basis of the number (Fock) states
$\bar{N}_\alpha$	average number of thermal noise photons
$\bar{N}_\mu$	average number of emitted signal photons
N_q	number of qubits
N_{tail}	number of tail bits
P_b	bit error ratio
P_{block}	block error ratio
$\mathcal{O}$	zero operator
$\mathcal{P}_{[\mathbf{s}_i, b_i]}, \mathcal{P}_{[\mathbf{s}_i]}$	measurement operators
$PM_i^{(l)}$	path metric
p_{b_i}	a priori probability of transmitting b_i
Π	interleaver matrix
Π^{-1}	deinterleaver matrix
R_c	code rate
ρ_{TX, b_i}	quantum mechanical density operator at the transmitter output
$\rho_{RX, [\mathbf{s}_i, b_i]}$	quantum mechanical density operator at the receiver input
$\mathbf{s}_i$	state vector of the FSM
T_s	duration of a codebit
$\mathbf{u}$	message
u_i	message bit
W_p	number of paths
$\mathcal{X}, \mathcal{Z}$	Pauli gates

Each message $\mathbf{u}$, cf. (3), is processed by a classical channel encoder to mitigate transmission errors. The resulting codeword

$$\tilde{\mathbf{b}} = (\tilde{b}_0, \tilde{b}_1 \cdots \tilde{b}_{N-1}), \quad N \in \mathbb{N}^*, \quad (4)$$

contains N codebits $\tilde{b}_i \in \{0, 1\}$. The code rate is given by

$$R_c = \frac{K - 24}{N}, \quad K, N \in \mathbb{N}^*. \quad (5)$$

The message bit period and the codebit period are assumed to be equal, denoted by T_s , such that transmission of the N message bits, respectively, N codebits, requires a duration of NT_s .

The codeword $\tilde{\mathbf{b}}$ is then interleaved using the $N \times N$ permutation matrix $\mathbf{\Pi}$ [40]. The matrix $\mathbf{\Pi}$ is obtained by row-wise permutation of the canonical unit row vectors

$$\mathbf{e}^{(m)} = (0 \cdots 0, \underbrace{1}_{m\text{th position}}, 0 \cdots 0), \quad m \in \{1, 2 \cdots N\}, \quad (6)$$

and therefore contains exactly N ones ([40], pp. 24–31). At the interleaver output, the interleaved codeword

$$\tilde{\mathbf{b}}_{\mathbf{\Pi}} = \tilde{\mathbf{b}} \mathbf{\Pi} = (\tilde{b}_{\mathbf{\Pi},0}, \tilde{b}_{\mathbf{\Pi},1} \cdots \tilde{b}_{\mathbf{\Pi},(N-1)}) \quad (7)$$

is obtained. More advanced interleaving schemes are also possible (see e.g., ref. [41], Sect. 4.3.2, pp. 238–241), but this is not considered further here.

The interleaved codebits $\tilde{b}_{\mathbf{\Pi},i}$, $i \in 0, 1, \cdots (N-1)$, are grouped into transmission bursts $\mathbf{b}$, composed of the burst bits b_i , and passed to the quantum transmitter using an LED like in, e.g., ref. [8]. The transmitter maps the burst bits onto quantum states represented by the density operators ρ_{TX,b_i} , $i \in \{0, 1 \cdots (N-1)\}$. Besides the codebits $\tilde{b}_{\mathbf{\Pi},i}$, $i \in 0, 1, \cdots (N-1)$, the bursts $\mathbf{b}$ may also contain system information such as training sequences and tail bits [34]. These are neglected here for simplicity.

The LED is assumed to operate in a single mode with fixed angular frequency ω . It emits radiation for b_i equal to 1 and remains off for b_i equal to 0, corresponding to on-off keying (OOK) modulation [20] (see also [8]).

The emitted radiation propagates through a linear multipath quantum channel [8]. Signal photons undergo uncorrelated scattering caused by independent scatterers [42,43], leading to decoherence [44,45]. As maintaining phase coherence between transmitter and receiver may therefore be infeasible, no phase synchronization is assumed [43,46]. It is well known that, without phase coherence, the optimum receiver for coherent signals with random phase relies on photon counting [47].

The transmitter and receiver are assumed to be time synchronized. Hence, the receiver in Figure 2 processes mutually disjoint observation time intervals $[iT_s, (i+1)T_s)$ of duration T_s , each associated with one burst bit b_i [7,43].

Due to multipath propagation and temporal spreading of the W_p paths, leading to decoherence, the multipath transmission channel becomes a discrete-time W_p -paths quantum channel at the measurement device output, where typically $W_p \in \{2, 3 \cdots\}$ ([21], Sect. 2.4.1, pp. 522–543), ref. [7,43]. The discrete-time W_p -paths quantum channel is characterized by a channel impulse response (CIR) with W_p nonnegative channel coefficients s_w , $w \in \{0, 1 \cdots (W_p - 1)\}$ ([21], Sect. 2.4.1, pp. 522–543), ref. [7,43].

Let $\bar{N}_\mu$ denote the average number of signal photons transmitted during the observation time interval $[iT_s, (i+1)T_s)$ when b_i is equal to 1. The corresponding average number $\bar{N}_{\mu,i}$ of received photons in the observation time interval $[iT_s, (i+1)T_s)$ is

$$\bar{N}_{\mu,i} = \bar{N}_{\mu} b_i s_0 + \underbrace{\bar{N}_{\mu} \sum_{w=1}^{W_p-1} b_{i-w} s_w}_{\text{intersymbol interference (ISI) term}} \quad (8)$$

(ref. [21], Sect. 2.4.1, pp. 522–543). For $W_p > 1$, the ISI term in (8) generally does not vanish ([21], Sect. 2.4.1, pp. 522–543). Hence, the average number $\bar{N}_{\mu,i}$ of received signal photons depends not only on the current transmission burst bit b_i , but also on the preceding $(W_p - 1)$ bits $b_{i-W_p+1} \cdots b_{i-1}$. This dependence can be modeled by a finite state machine (FSM) representing a Markov chain [43,48].

Using a Mealy-type representation [48], at each time instant i , this FSM has 2^{W_p-1} states described by the state vectors $\mathbf{s}_i$ equal to $(b_{i-W_p+1}, \dots, b_{i-1})$. Since $\bar{N}_{\mu} b_i s_0$ can take the two values 0 and $\bar{N}_{\mu} s_0$, each state generates two transitions labeled by $b_i \in \mathbb{F}_2$, yielding a total of 2^{W_p} transitions terminating in the FSM states with the state vectors $\mathbf{s}_{i+1}$ at time instant $(i + 1)$ [43]. The FSM operation can be represented by a trellis diagram, cf., e.g., ref. [48]. The said 2^{W_p} transitions correspond to the 2^{W_p} possible values of $\bar{N}_{\mu,i}$ in (8) during the observation time interval $[iT_s, (i + 1)T_s)$. Accordingly, we denote $\bar{N}_{\mu,i}$ by $\bar{N}_{\mu, [\mathbf{s}_i, b_i]}$, thereby associating the 2^{W_p} values with the 2^{W_p} hypotheses used in the quantum data detection scheme, see, e.g., ref. [43].

The error performance of trellis based data detectors improves when proper trellis initialization and termination are enforced. Accordingly, we assume the initial and terminal states

$$\mathbf{s}_0 = \mathbf{s}_{N+W_p-1} = (1, 1 \cdots 1). \quad (9)$$

To ensure this, N_{tail} initial and N_{tail} terminal tail bits with $N_{\text{tail}} \geq W_p - 1$ are included into the burst $\mathbf{b}$ by appending them to the interleaved codeword $\tilde{\mathbf{b}}_{\Pi}$ of (7), cf. Figure 2. Hence, the LED remains on for N_{tail} intervals of duration T_s before and after transmission of $\tilde{\mathbf{b}}_{\Pi}$. For clarity, we assume that the LED is off prior to and after each $\tilde{\mathbf{b}}_{\Pi}$, using appropriately designed guard periods to avoid interburst interference.

At the input of the quantum receiver, the LED radiation field is linearly superimposed with the omnipresent thermal background radiation field. The latter is in a thermal state characterized by Bose–Einstein distributed thermal noise photons with average number $\bar{N}_{\alpha}$. Following the formalism of, e.g., ref. [46], and neglecting any phase coherence between the quantum transmitter and the quantum receiver, the corresponding quantum mechanical density operator must be determined. Denote by $\underline{\alpha}$ the complex amplitude and by the following state vector the corresponding coherent state emitted by a thermally agitated harmonic oscillator ([21], eq. (1.1365), p. 350):

$$|\underline{\alpha}\rangle = \exp\left\{-\frac{\bar{N}_{\alpha}}{2}\right\} \sum_{n=0}^{\infty} \frac{\underline{\alpha}^n}{\sqrt{n!}} |n\rangle. \quad (10)$$

In Glauber’s P -representation [46,47], the quantum mechanical density operator under discussion can be expressed as

$$\rho_{\text{RX}, [\mathbf{s}_i, b_i]} = \frac{1}{\pi (b_i \bar{N}_{\mu, [\mathbf{s}_i, b_i]} + \bar{N}_{\alpha})} \int_{\mathbb{C}} \exp\left\{-\frac{|\underline{\alpha}|^2}{b_i \bar{N}_{\mu, [\mathbf{s}_i, b_i]} + \bar{N}_{\alpha}}\right\} |\underline{\alpha}\rangle \langle \underline{\alpha}| d^2 \underline{\alpha}, \quad (11)$$

yielding

$$\rho_{\text{RX},[\mathbf{s}_i,b_i]} = \sum_{n=0}^{\infty} \frac{(b_i \bar{N}_{\mu,[\mathbf{s}_i,b_i]} + \bar{N}_{\alpha})^n}{(1 + b_i \bar{N}_{\mu,[\mathbf{s}_i,b_i]} + \bar{N}_{\alpha})^{n+1}} |n\rangle \langle n| \quad (12)$$

in the Fock basis $\{|n\rangle\}$ for the considered case of using an LED at the transmitter. The quantum mechanical density operator $\rho_{\text{RX},[\mathbf{s}_i,b_i]}$ of (12), derived for the LED based quantum transmission, differs substantially from that obtained for the laser based case considered in, e.g., ref. [7]. Whereas the latter is governed by the Laguerre distribution, $\rho_{\text{RX},[\mathbf{s}_i,b_i]}$ in (12) embodies the Bose–Einstein distribution characteristic of thermal states. Clearly, the quantum mechanical density operator $\rho_{\text{RX},[\mathbf{s}_i,b_i]}$ is diagonal in the Fock basis $\{|n\rangle\}$, showing that it has only populations but no coherences.

The optimum quantum data detection considered in this work is based on the BCJR (Bahl Cocke Jelinek Raviv) algorithm, which was already proposed in 1974 [49]. The BCJR has been well-known.

Setting out from the paradigm that quantum communication means the transmission of classical information through quantum states ([20], Figure 4.8, p. 114), we will consider the detection of the classical codewords, composed of the codebits $b_i \in \mathbb{F}_2$, $i \in \mathbb{B}$. Let the a priori probability of transmitting codebit b_i with value 0 be p_0 . The a priori probability of transmitting b_i with value 1 is then

$$p_1 = 1 - p_0. \quad (13)$$

Let

$$\mathbb{E}_i = \left\{ \text{all } 2^{W_p-1} \text{ FSM states with state vectors } \mathbf{s}_i \right\} \quad (14)$$

denote the set of all 2^{W_p-1} FSM states with state vectors $\mathbf{s}_i$. Furthermore, let

$$\mathbb{E}_{i+1}(b_i = \nu) = \left\{ \begin{array}{l} \text{all } 2^{W_p-2} \text{ FSM states with} \\ \text{state vectors } \mathbf{s}_{i+1} = \\ (b_{i-W_p+2}, b_{i-W_p+3} \cdots b_i = \nu) \end{array} \right\} \quad (15)$$

denote the set of all 2^{W_p-2} FSM states whose state vectors contain the fixed bit b_i equal to $\nu \in \{0, 1\}$.

With the probability $\alpha(\mathbf{s}_i)$ of the occurrence of the FSM state with the state vector $\mathbf{s}_i$ determined in the forward recursion of the BCJR algorithm, with the probability $\beta(\mathbf{s}_{i+1})$ of the occurrence of the FSM state with the state vector $\mathbf{s}_{i+1}$ determined in the backward recursion of the BCJR algorithm and with the transition probability $\gamma(\mathbf{s}_i, \mathbf{s}_{i+1})$ of the transition from the FSM state with the state vector $\mathbf{s}_i$ to the FSM state with the state vector $\mathbf{s}_{i+1}$ the optimum maximum a posteriori (MAP) symbol by symbol detection is based on the log likelihood ratios (LLRs)

$$L_i = \ln \left\{ \frac{\sum_{\mathbf{s}_i \in \mathbb{E}_i} \sum_{\mathbf{s}_{i+1} \in \mathbb{E}_{i+1}(0)} \alpha(\mathbf{s}_i) \gamma(\mathbf{s}_i, \mathbf{s}_{i+1}) \beta(\mathbf{s}_{i+1})}{\sum_{\mathbf{s}_i \in \mathbb{E}_i} \sum_{\mathbf{s}_{i+1} \in \mathbb{E}_{i+1}(1)} \alpha(\mathbf{s}_i) \gamma(\mathbf{s}_i, \mathbf{s}_{i+1}) \beta(\mathbf{s}_{i+1})} \right\}. \quad (16)$$

Equation (16) leads to the following detection rule:

- If L_i from (16) is greater than 0, decide in favor of hypothesis H_0 and output the detected bit $\hat{b}_i$ equal to 0.

- If L_i from (16) is less than or equal to 0, decide in favor of hypothesis H_1 and output the detected bit $\hat{b}_i$ equal to 1.

Therefore, (16) becomes

$$\frac{\sum_{\mathbf{s}_i \in \mathbb{E}_i} \sum_{\mathbf{s}_{i+1} \in \mathbb{E}_{i+1}(0)} \alpha(\mathbf{s}_i) \gamma(\mathbf{s}_i, \mathbf{s}_{i+1}) \beta(\mathbf{s}_{i+1})}{\sum_{\mathbf{s}_i \in \mathbb{E}_i} \sum_{\mathbf{s}_{i+1} \in \mathbb{E}_{i+1}(1)} \alpha(\mathbf{s}_i) \gamma(\mathbf{s}_i, \mathbf{s}_{i+1}) \beta(\mathbf{s}_{i+1})} \underset{\hat{b}_i=1}{\overset{\hat{b}_i=0}{\geq}} 1 \quad (17)$$

and we yield

$$\sum_{\mathbf{s}_i \in \mathbb{E}_i} \alpha(\mathbf{s}_i) \sum_{\mathbf{s}_{i+1} \in \mathbb{E}_{i+1}} \left\{ \gamma(\mathbf{s}_i, \mathbf{s}_{i+1})|_{b_i=0} - \gamma(\mathbf{s}_i, \mathbf{s}_{i+1})|_{b_i=1} \right\} \beta(\mathbf{s}_{i+1}) \underset{\hat{b}_i=1}{\overset{\hat{b}_i=0}{\geq}} 0. \quad (18)$$

The notation $\gamma(\mathbf{s}_i, \mathbf{s}_{i+1})|_{b_i=0}$ in (18) indicates that only terms containing b_i equal to 0 in $\mathbf{s}_{i+1}$ contribute to the sum, while all others are zero. Similarly, $\gamma(\mathbf{s}_i, \mathbf{s}_{i+1})|_{b_i=1}$ indicates that only terms containing b_i equal to 1 contribute to the sum.

Next, the quantities $\gamma(\mathbf{s}_i, \mathbf{s}_{i+1})|_{b_i=0}$ and $\gamma(\mathbf{s}_i, \mathbf{s}_{i+1})|_{b_i=1}$ are transferred to quantum communication. Suppose that, during the observation time interval $[iT_s, (i+1)T_s)$, the received radiation field is in a specific but unknown state. To identify this state, quantum communication employs measurement operators at the receiver input. Assuming binary transmission of the codebits $b_i \in \mathbb{F}_2$, $i \in \mathbb{B}$, and that the FSM describing the W_p -paths quantum channel is in trellis state $\mathbf{s}_i$ at the beginning of the interval, the measurement operator $\mathcal{P}_{[\mathbf{s}_i, \hat{b}_i=0]}$ tests the hypothesis $\hat{b}_i$ equal to 0, while $\mathcal{P}_{[\mathbf{s}_i, \hat{b}_i=1]}$ tests $\hat{b}_i$ equal to 1. Since both hypotheses cannot occur simultaneously and no further operators are needed for binary transmission, the following relation with the identity operator $\mathcal{I}$ holds:

$$\mathcal{P}_{[\mathbf{s}_i, \hat{b}_i=0]} + \mathcal{P}_{[\mathbf{s}_i, \hat{b}_i=1]} = \mathcal{P}_{[\mathbf{s}_i]} \quad (= \mathcal{I}). \quad (19)$$

Furthermore, the following relation holds with the zero operator $\mathcal{O}$:

$$\mathcal{P}_{[\mathbf{s}_i, \hat{b}_i=0]} \mathcal{P}_{[\mathbf{s}_i, \hat{b}_i=1]} = \mathcal{P}_{[\mathbf{s}_i, \hat{b}_i=1]} \mathcal{P}_{[\mathbf{s}_i, \hat{b}_i=0]} = \mathcal{O}. \quad (20)$$

Since the measurement operators $\mathcal{P}_{[\mathbf{s}_i, \hat{b}_i=0]}$ and $\mathcal{P}_{[\mathbf{s}_i, \hat{b}_i=1]}$ commute according to (20), the receiver measurement instrument is realized by their combination, i.e., by $\mathcal{P}_{[\mathbf{s}_i]}$.

It follows from the above discussion that $\gamma(\mathbf{s}_i, \mathbf{s}_{i+1})|_{b_i}$ represents the best possible transition probability obtained

- during the transition
 - from the FSM state with the state vector $\mathbf{s}_i$ at the beginning of the observation time interval $[iT_s, (i+1)T_s)$
 - to the FSM state with the state vector $\mathbf{s}_{i+1}$ at the beginning of the next observation time interval $[(i+1)T_s, (i+2)T_s)$, if
- the codebit b_i is transmitted during the observation time interval $[iT_s, (i+1)T_s)$ and
- the measurement in the observation interval $[iT_s, (i+1)T_s)$ with the operators $\mathcal{P}_{[\mathbf{s}_i, \hat{b}_i=0]}$ and $\mathcal{P}_{[\mathbf{s}_i, \hat{b}_i=1]}$, or equivalently $\mathcal{P}_{[\mathbf{s}_i]}$, shows that the received radiation field at the receiver input, described by the quantum mechanical density operator $\rho_{\text{RX}, [\mathbf{s}_i, b_i]}$ of (12), was in a given measured state, which is an eigenstate of the measurement instrument.

Using the trace operator $\text{tr}\{\cdot\}$, we hence yield

$$\gamma(\mathbf{s}_i, \mathbf{s}_{i+1})|_{b_i} = p_{b_i} \text{tr} \left\{ \rho_{\text{RX}, [\mathbf{s}_i, b_i]} \left(\mathcal{P}_{[\mathbf{s}_i, \hat{b}_i=0]} + \mathcal{P}_{[\mathbf{s}_i, \hat{b}_i=1]} \right) \right\} \quad (21)$$

$$= p_{b_i} \text{tr} \left\{ \rho_{\text{RX}, [\mathbf{s}_i, b_i]} \mathcal{P}_{[\mathbf{s}_i]} \right\}. \quad (22)$$

The quantity $\text{tr} \left\{ \rho_{\text{RX}, [\mathbf{s}_i, b_i]} \mathcal{P}_{[\mathbf{s}_i, \hat{b}_i]} \right\}$ is the likelihood, i.e., the conditional transition probability, that the measurement instrument outputs the aforementioned given measured state and yields the hypothesis $\hat{b}_i$, given the FSM state with the state vector $\mathbf{s}_i$ and transmitted codebit b_i .

Setting out from (22), we immediately obtain

$$\gamma(\mathbf{s}_i, \mathbf{s}_{i+1})|_{b_i=0} - \gamma(\mathbf{s}_i, \mathbf{s}_{i+1})|_{b_i=1} = p_0 \text{tr} \left\{ \left[\rho_{\text{RX}, [\mathbf{s}_i, b_i=0]} - \frac{p_1}{p_0} \rho_{\text{RX}, [\mathbf{s}_i, b_i=1]} \right] \mathcal{P}_{[\mathbf{s}_i]} \right\}. \quad (23)$$

Obviously, quantum data detection based on $\left(\gamma(\mathbf{s}_i, \mathbf{s}_{i+1})|_{b_i=0} - \gamma(\mathbf{s}_i, \mathbf{s}_{i+1})|_{b_i=1} \right)$ requires 2^{W_P-1} measurement operators $\mathcal{P}_{[\mathbf{s}_i]}$.

In incoherent quantum reception, the detection operators $\rho_{\text{RX}, [\mathbf{s}_i, b_i=0]}$ and $\rho_{\text{RX}, [\mathbf{s}_i, b_i=1]}$ are diagonal in the Fock basis $\{|n\rangle\}$. Hence, the 2^{W_P-1} detection operators are given by

$$\begin{aligned} \mathcal{D}_{[\mathbf{s}_i]} &= \rho_{\text{RX}, [\mathbf{s}_i, b_i=0]} - \frac{p_1}{p_0} \rho_{\text{RX}, [\mathbf{s}_i, b_i=1]} \\ &= \sum_{n=0}^{\infty} \left(\frac{\bar{N}_\alpha^n}{(1 + \bar{N}_\alpha)^{n+1}} - \frac{p_1}{p_0} \cdot \frac{\left(\bar{N}_{\mu, [\mathbf{s}_i, b_i=1]} + \bar{N}_\alpha \right)^n}{\left(1 + \bar{N}_{\mu, [\mathbf{s}_i, b_i=1]} + \bar{N}_\alpha \right)^{n+1}} \right) |n\rangle \langle n| \end{aligned} \quad (24)$$

with eigenvalues

$$\eta_{[\mathbf{s}_i], n} = \frac{\bar{N}_\alpha^n}{(1 + \bar{N}_\alpha)^{n+1}} - \frac{p_1}{p_0} \cdot \frac{\left(\bar{N}_{\mu, [\mathbf{s}_i, b_i=1]} + \bar{N}_\alpha \right)^n}{\left(1 + \bar{N}_{\mu, [\mathbf{s}_i, b_i=1]} + \bar{N}_\alpha \right)^{n+1}} \quad (25)$$

depending on the FSM states with the state vectors $\mathbf{s}_i$. Clearly, the 2^{W_P-1} detection operators $\mathcal{D}_{[\mathbf{s}_i]}$ in (24) are pairwise commensurable, i.e., they commute, sharing the Fock basis $\{|n\rangle\}$ as a common eigenbasis with orthonormal eigenvectors. Therefore, a simultaneous, uncertainty-free measurement with all 2^{W_P-1} measurement operators $\mathcal{P}_{[\mathbf{s}_i]}$ is possible. Moreover, since the eigenvectors of all 2^{W_P-1} detection operators $\mathcal{D}_{[\mathbf{s}_i]}$ are taken from the Fock basis $\{|n\rangle\}$, the 2^{W_P-1} optimal measurement operators $\mathcal{P}_{[\mathbf{s}_i]}$ are identical and take the following form

$$\mathcal{P} = \mathcal{P}_{[\mathbf{s}_i]} = \sum_{n=0}^{\infty} |n\rangle \langle n|. \quad (26)$$

In analogy to ([47], right column, p. 1588) we set

$$\mathcal{P}_{[\mathbf{s}_i, \hat{b}_i=0]} = \sum_{\substack{n=0 \\ \eta_{[\mathbf{s}_i], n} > 0}}^{\infty} |n\rangle \langle n|, \quad (27)$$

$$\mathcal{P}_{[\mathbf{s}_i, \hat{b}_i=1]} = \sum_{\substack{n=0 \\ \eta_{[\mathbf{s}_i], n} \leq 0}}^{\infty} |n\rangle \langle n|. \quad (28)$$

If the received radiation field at the receiver input is in the unknown pre-measurement state $|\hat{N}_i\rangle$, then (24) leads to

$$\langle \hat{N}_i | \mathcal{D}_{[\mathbf{s}_i]} | \hat{N}_i \rangle = \langle \hat{N}_i | \rho_{\text{RX},[\mathbf{s}_i,b_i=0]} | \hat{N}_i \rangle - \frac{p_1}{p_0} \langle \hat{N}_i | \rho_{\text{RX},[\mathbf{s}_i,b_i=1]} | \hat{N}_i \rangle = \eta_{[\mathbf{s}_i],\hat{N}_i}. \quad (29)$$

With (12) and (22), we obtain

$$\gamma(\mathbf{s}_i, \mathbf{s}_{i+1}) = p_{b_i} \cdot \langle \hat{N}_i | \rho_{\text{RX},[\mathbf{s}_i,b_i]} | \hat{N}_i \rangle \quad (30)$$

$$= p_{b_i} \cdot \frac{\left(b_i \bar{N}_{\mu,[\mathbf{s}_i,b_i]} + \bar{N}_\alpha\right)^{\hat{N}_i}}{\left(1 + b_i \bar{N}_{\mu,[\mathbf{s}_i,b_i]} + \bar{N}_\alpha\right)^{\hat{N}_i+1}}, \quad (31)$$

which is the product of

- the a priori information p_{b_i} and
- the likelihood $\langle \hat{N}_i | \rho_{\text{RX},[\mathbf{s}_i,b_i]} | \hat{N}_i \rangle$, which is the conditional probability of the measured number of received photons $\hat{N}_i$ characterizing the number state $|\hat{N}_i\rangle$, given the FSM state with the state vector $\mathbf{s}_i$ and the transmitted codebit b_i .

The transition probability $\gamma(\mathbf{s}_i, \mathbf{s}_{i+1})$ of (31), derived for the LED based quantum transmission, differs substantially from that obtained for the laser based case considered in, e.g., ref. [7]. Whereas the latter is governed by the Laguerre distribution, $\gamma(\mathbf{s}_i, \mathbf{s}_{i+1})$ in (31) embodies the Bose–Einstein distribution characteristic of thermal states.

In terms of probability theory, the two events implied above, i.e., the occurrence of b_i at the transmitter and the measurement of $\hat{N}_i$, characterizing the number state $|\hat{N}_i\rangle$, given the FSM state with the state vector $\mathbf{s}_i$ and the transmitted codebit b_i , at the receiver, form a joint event. Using (31) in (16), respectively, in (18), yields the generalization of the result developed by Helstrom et al. in ([47], right column, p. 1588) for the single path quantum channel.

Now suppose that a third event, independent of the likelihood $\langle \hat{N}_i | \rho_{\text{RX},[\mathbf{s}_i,b_i]} | \hat{N}_i \rangle$, occurs, e.g., represented by the extrinsic information, namely the interleaved extrinsic LLR vector $\mathbf{LL}_e$. This vector $\mathbf{LL}_e$ consists of the extrinsic LLRs $L_{\Pi e,i}$ generated by the polar decoder as discussed in Section 3. Each extrinsic LLR $L_{\Pi e,i}$ is associated with a distinct codebit b_i . “Extrinsic” refers to information, which is independent of other input signals at the input of the quantum data detector [7]. Then, an additional conditional probability term arises given by

$$p\{L_{\Pi e,i} | b_i\} = \frac{\exp\{(1 - b_i)L_{\Pi e,i}\}}{\exp\{L_{\Pi e,i}\} + 1}, \quad b_i \in \{0, 1\}, \quad (32)$$

fulfilling

$$L_{\Pi e,i} = \ln \left\{ \frac{p\{L_{\Pi e,i} | b_i = 0\}}{p\{L_{\Pi e,i} | b_i = 1\}} \right\} \quad (33)$$

and

$$p\{L_{\Pi e,i} | b_i = 0\} + p\{L_{\Pi e,i} | b_i = 1\} = 1, \quad (34)$$

that must be included in the transition probability $\gamma(\mathbf{s}_i, \mathbf{s}_{i+1})$ yielding

$$\gamma(\mathbf{s}_i, \mathbf{s}_{i+1}) = p_{b_i} \cdot p\{L_{\Pi e,i} | b_i\} \cdot \frac{\left(b_i \bar{N}_{\mu,[\mathbf{s}_i,b_i]} + \bar{N}_\alpha\right)^{\hat{N}_i}}{\left(1 + b_i \bar{N}_{\mu,[\mathbf{s}_i,b_i]} + \bar{N}_\alpha\right)^{\hat{N}_i+1}}. \quad (35)$$

Since the denominator ($\exp\{L_{\Pi e,i}\} + 1$) in (32) does not depend on the codebit b_i , it may be neglected in (35). The third event, characterized by the occurrence of the extrinsic LLR $L_{\Pi e,i}$, constitutes a second diversity branch. This second diversity branch may be regarded as virtual relative to the likelihood described above, because it is not associated with an actual signal reception through the W_p -paths quantum channel. Note that the extrinsic LLRs $L_{\Pi e,i}$ must be subtracted from the LLR L_i , cf. (16), before processing by the polar code decoder that generated the extrinsic LLRs $L_{\Pi e,i}$, as illustrated in Figure 3.

To the best of the authors' knowledge, the interpretation of the extrinsic LLRs $L_{\Pi e,i}$ entering the quantum data detector via a further diversity branch, namely as a second diversity branch, has not been discussed in the literature. In contrast to the widely used view of $L_{\Pi e,i}$ as estimates of the a priori information, the formulation proposed in this manuscript leaves the a priori information unchanged, thus facilitating data detection that still can accommodate varying a priori probabilities in combination with turbo equalization.

In summary, the key element of optimum quantum data detection is embodied in the transition probabilities $\gamma(\mathbf{s}_i, \mathbf{s}_{i+1})$ given in (30) and (31). Once these quantities are determined, optimum quantum data detection schemes reduce to standard, well-established sequence and symbol detection schemes, and even suboptimum detectors can be implemented in a straightforward manner.

Since data detection is performed after the quantum measurement, this concept may be termed "post-measurement decision", following the nomenclature of Cariolaro ([20], pp. 189–192).

The authors are aware of the fact that the technical aspects of quantum transmission presented here may not yet be widely known. Nevertheless, it is undisputed that the fundamental structure of the wireless quantum transmission system under consideration, including the optimum quantum data detectors for multipath reception employed, has already been described in the relevant literature. In order to avoid unnecessary repetition of background information that is already available, the discussion provided in this section, together with the explicit reference to the specialist literature, for example, to [7,21], should suffice.

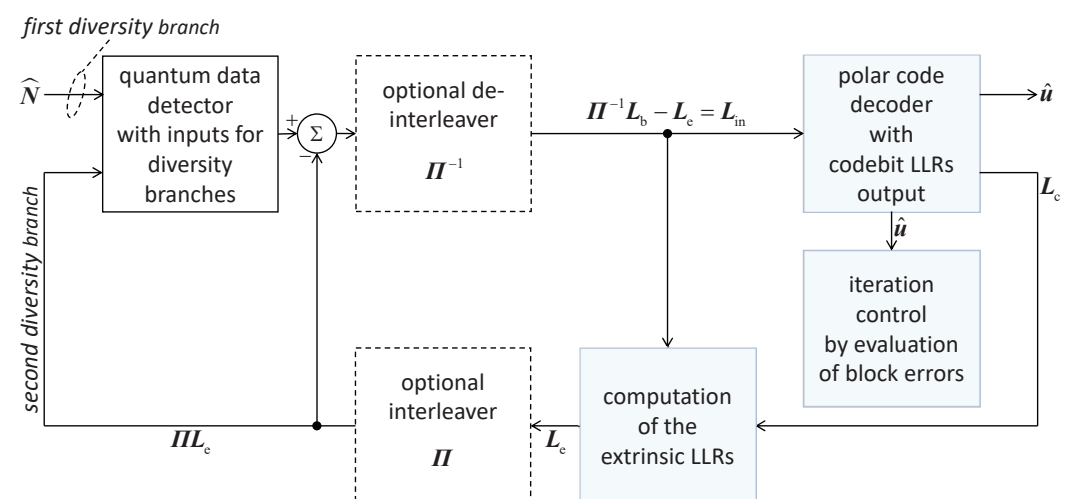


Figure 3. Generic receiver structure for data transmission in polar coded wireless quantum optical communications via multipath channels with uncorrelated scattering and intersymbol interference (ISI) including the turbo equalization concept, cf. also ([7], Figure 10, p. 33).

3. Turbo Equalization

Turbo equalization is an iterative process combining data detection and channel decoding to progressively reduce errors in the decoded message $\hat{\mathbf{u}}$. It was first proposed in

1995 [36] for additive white Gaussian noise (AWGN) channels with intersymbol interference (ISI), with and without fading, where both the detector and decoder employed the soft output Viterbi algorithm (SOVA). Related work on source controlled channel decoding [50,51] highlighted the importance of exchanging soft information such as LLRs between source and channel decoders. These concepts were first applied to polar coded wireless quantum optical communications over multipath quantum channels with uncorrelated scattering and ISI in 2025 [7] in the case of laser based wireless quantum optical communication. First results on applying turbo equalization to LED based wireless quantum optical communication were published in 2026 [8]. Since [7] provides a detailed discussion of the turbo equalization applied to polar coding, only a concise overview is presented here.

Figure 3 illustrates the turbo equalization principle for polar coded wireless quantum optical communications over multipath channels with uncorrelated scattering and ISI. The receiver consists of the quantum data detector, which can process more than one discrete-time input signal, the polar code decoder, which is capable of generating one LLR per code bit, the iteration control unit, which evaluates the occurrence of block errors by exploiting CRC decoding, using e.g., the CRC code according to the 5G/NR specification ([38], Sect. 6.13.2.1, pp. 140–142), and the computation unit determining one extrinsic LLR per code bit. Since interleaving is common in wireless systems and relevant for future quantum optical wireless communications, an optional interleaver Π and deinterleaver Π^{-1} are included in Figure 3. In this work, simple square block interleaving is considered: Let the interleaver matrix Π have size $N_I \times N_I$, giving an interleaver length of N_I^2 . Using the ceiling function $\lceil \cdot \rceil$, choose N_I equal to $\lceil \sqrt{N} \rceil$ and form the extended codeword

$$\tilde{\mathbf{b}} = \left(b_0, b_1 \cdots b_{N-1}, \underbrace{0, 0 \cdots 0}_{\substack{(N_I^2 - N) \\ \text{zeros}}} \right) \quad (36)$$

of length $N_I^2 \geq N$. The extended codeword $\tilde{\mathbf{b}}$ in (36) is written row-wise into the square block interleaver and read out column-wise to produce the interleaved extended codeword. Deinterleaving is performed by the inverse operation.

Turbo equalization requires the quantum data detector to process information, e.g., a priori information, which is initially unavailable at the receiver [7]. Therefore, this information must be iteratively estimated and updated during turbo equalization [7]. The said estimate is generated by a modified channel decoder that, besides providing the decoded message $\hat{\mathbf{u}}$, also computes the LLR vector $\mathbf{L}_c$ for the codebits $b_i \in \mathbb{F}_2, i \in \mathbb{B}$.

It has been known since the seminal 2009 paper by Erdal Arıkan that successive cancellation (SC) decoding of polar codes operates on the input LLR vector $\mathbf{L}_{\text{in}}$, which is formed from the transition probabilities, see, e.g., (ref. [9], eqs. (75) and (76), pp. 3065f.). Moreover, SC decoding can generate the LLR vector $\mathbf{L}_u$ associated with the message bits $u_i \in \{0, 1\}, i \in \mathbb{B}$, u_i being a frozen bit with the bit value 0 or not, cf. ([9], eqs. (11) and (12), p. 3054). Processing of the input LLR vector $\mathbf{L}_{\text{in}}$ has also been employed in CRC aided successive cancellation list (CA-SCL) decoding of polar codes, as discussed, for example, by Balatsoukas-Stimming et al. in their 2015 paper [52]. Consequently, in the following, we restrict our attention to polar code decoders that operate on $\mathbf{L}_{\text{in}}$ at their input. If the decoder is appropriately modified to generate the LLR vector $\mathbf{L}_c$ corresponding to the code bits $b_i \in \mathbb{F}_2, i \in \mathbb{B}$, then the extrinsic LLR vector $\mathbf{L}_e$ can be obtained by processing both $\mathbf{L}_c$ and the input LLR vector $\mathbf{L}_{\text{in}}$ [7]. “Extrinsic” refers to information newly generated during decoding in addition to the input vector $\mathbf{L}_{\text{in}}$, which is independent of other input signals at the input of the quantum data detector [7].

As shown in Figure 3, the quantum data detector processes the photon count vector $\hat{N}$, which is composed of the photon counts $\hat{N}_i$, measured in the observation time intervals $[iT_s, (i+1)T_s)$ together with the interleaved extrinsic LLR vector ΠL_e and outputs the LLR vector L_b for the codebits $b_i \in \mathbb{F}_2, i \in \mathbb{B}$ [7]. To improve the mutual information exchange during turbo equalization, the interleaved extrinsic LLR vector ΠL_e is subtracted from L_b before the optional deinterleaving [7]. At the deinterleaver output, the input LLR vector [7]

$$L_{in} = \Pi^{-1} L_b - L_e \quad (37)$$

serves as the input to the polar code decoder. Besides the decoded message $\hat{u}$, the polar code decoder outputs the LLR vector L_c for the codebits $b_i \in \mathbb{F}_2, i \in \mathbb{B}$.

The iteration control unit then checks $\hat{u}$ for errors, e.g., using a CRC according to the 5G/NR specification ([38], Sect. 6.13.2.1, pp. 140–142). If an error is detected, another turbo equalization iteration is performed. Otherwise, the process terminates for the message u . This adaptive iteration control reduces computational effort and power consumption.

The determination of the extrinsic LLR vector L_e is likely the most critical aspect of turbo equalization. Since the extrinsic LLR vector L_e requires the LLR vector L_c for the codebits $b_i \in \mathbb{F}_2, i \in \mathbb{B}$. The treatment of the availability of extrinsic information of good quality, collected in the extrinsic LLR vector L_e , seems still to be in its infancy as far as turbo equalization using polar codes is concerned.

According to Hagenauer's 1997 analysis ([53], Figure 4, p. 7), given the input LLR vector L_{in} and the LLR vector L_c for the codebits $b_i \in \mathbb{F}_2, i \in \mathbb{B}$, the optimum determination of the extrinsic LLR vector L_e in the context of optimum MAP based symbol by symbol data detection and optimum MAP based channel decoding is given by

$$L_e = L_c - L_{in}. \quad (38)$$

The approach of (38) has thus governed turbo code decoding. However, optimum polar code decoding has not yet been deployed. Therefore, the well motivated approach (38) is not likely to provide the best possible performance when turbo equalization using polar codes is considered.

In what follows, the authors will concisely discuss known approaches of turbo equalization using polar codes. To the best knowledge of the authors, the soft cancellation (SCAN) polar decoding scheme proposed by Fayyaz and Barry [54] in 2014 was the first approach for generating extrinsic LLRs in a polar code decoder. It should be noted that, despite the claim made in [54], the SCAN polar decoding scheme does not realize turbo equalization but rather constitutes an elegant adaptation of the SPA to polar codes. Due to the limited performance gains of SCAN decoding [7,54], we do not consider this aspect further in this work.

In their paper [37] of 2024, Eltohamy et al. use the polar code decoding concept first suggested by Xiang et al. [55] in 2020 to generate the LLR vector L_c for the codebits $b_i \in \mathbb{F}_2, i \in \mathbb{B}$. The polar code decoding scheme employed in [37,55] first performs CA-SCL polar code decoding with a list length of L of 8, thereby generating eight competing candidate messages and selecting the candidate associated with the best path metric as the most likely message. Subsequently, the most likely message produced by the CA-SCL decoder is re-encoded, after which belief propagation (BP) decoding is applied to the re-encoded codeword in order to determine the LLR vector L_c of the codebits $b_i \in \mathbb{F}_2, i \in \mathbb{B}$, as detailed, for example, in ([39], Sect. 10.1, pp. 254–261). The corresponding extrinsic LLR vector L_e is then computed. Requiring the operation of not just one, but two complete polar code decoders with an intermediate polar encoding step appears neither efficient nor elegant. Regrettably, the schemes proposed in ([55], Figure 4, p. 7) do not specify the exact

computation of the extrinsic LLR vector L_e , rendering a rigorous discussion impossible. Fortunately, (ref. [37], Figure 6, pp. 146f.) illustrates the computation of the extrinsic LLR vector L_e , which is obtained by

$$L_e = C_S(L_c - L_{in}), \quad C_S \in \mathbb{R}_0^+. \quad (39)$$

using the empirically optimized nonnegative scaling factor $C_S \in \mathbb{R}_0^+$, which is not necessarily 1 in all cases as suggested by Hagenauer's 1997 approach ([53], Figure 4, p. 7), cf. (38). Surprisingly, C_S may be chosen arbitrarily ([37], Figure 8, p. 147), suggesting a departure from earlier approaches, such as the original proposal by Douillard et al. ([36], Figure 5, p. 510), as well as from the extrinsic LLR computation rule introduced by Berrou et al. in their seminal 1993 turbo coding paper ([56], p. 1096). Although [37] does not specify the number of turbo equalization iterations employed, this number appears to be fixed rather than adapted to the prevailing transmission conditions. Similarly, ref. [55] also seems to rely on a fixed number of turbo equalization iterations. Moreover, the limited performance gains reported in ([37], Figures 8 and 9, pp. 147f.) indicate residual correlations between L_c and L_{in} . In light of the above discussion, the authors do not pursue the schemes proposed in [37] and its predecessor [55] any further.

For polar coded transmissions, the dynamic adaptation of the number of turbo equalization iterations had apparently not been proposed prior to the publication of [7]. In the context of laser based polar coded wireless quantum optical communications, ref. [7] demonstrated for the first time that adaptive control of the number of turbo equalization iterations is feasible, achieving, on average, fewer than two iterations while ensuring a desirably high transmission quality. As noted at the beginning of this Section 3, the requirement for additional turbo equalization iterations to successfully decode a particular codeword is determined by CRC decoding. In the scenario considered in this work, the CRC code specified for 5G/NR systems is employed ([38], Sect. 6.13.2.1, pp. 140–142). This CRC code is assumed to be transmitted irrespective of whether turbo equalization is applied, because it is also required for CA-SCL polar code decoding. Furthermore, to prevent receiver processing deadlocks, the number of turbo equalization iterations is constrained by a predefined maximum value.

In [7], the systematic computation of the nonnegative scaling factor $C_S \in \mathbb{R}_0^+$ was suggested. An explicit computation rule was found as follows ([7], eq. (94), p. 37)

$$L_e = \left(1 - \frac{L_{in} L_c^T}{\|L_c\|^2} \cdot \frac{L_c (L_c - L_{in})^T}{\|L_c - L_{in}\|^2} \right) \{L_c - L_{in}\} \quad (40)$$

$$\propto \{L_c - L_{in}\}, \quad \frac{L_{in} L_c^T}{\|L_c\|^2} \cdot \frac{L_c (L_c - L_{in})^T}{\|L_c - L_{in}\|^2} \in \mathbb{R}.$$

Using (40), the real nonnegative scaling factor C_S of (39) becomes ([7], eq. (95), p. 37)

$$C_S = 1 - \frac{L_{in} L_c^T}{\|L_c\|^2} \cdot \frac{L_c (L_c - L_{in})^T}{\|L_c - L_{in}\|^2}. \quad (41)$$

A simplified but viable alternative to (39) is given by ([7], eq. (93), p. 36)

$$L_e = \left(1 - \frac{L_{in} L_c^T}{\|L_c\|^2} \right) L_c \propto L_c, \quad \frac{L_{in} L_c^T}{\|L_c\|^2} \in \mathbb{R}. \quad (42)$$

Both approaches, (40) and (42), proposed in ([7], pp. 36f.), are fully adaptive, theoretically founded, and free of empirical optimization. In order to avoid repetitions regarding

the theoretical foundation of the two approaches (40) and (42), the reader is kindly referred to the detailed discussion given in ([7], pp. 36f.).

Unlike the approaches by other authors, these novel solutions (40) and (42) of ([7], pp. 36f.) use the exactly known real valued coefficients ([7], p. 37)

- $L_{\text{in}} L_{\text{c}}^T / \|L_{\text{c}}\|^2$ and
- $\left(L_{\text{in}} L_{\text{c}}^T / \|L_{\text{c}}\|^2\right) \left(L_{\text{c}}(L_{\text{c}} - L_{\text{in}})^T / \|L_{\text{c}} - L_{\text{in}}\|^2\right)$,

respectively. It was shown in [7] that employing $L_{\text{in}} L_{\text{c}}^T / \|L_{\text{c}}\|^2$ yields a transmission performance similar to that obtained with $\left(L_{\text{in}} L_{\text{c}}^T / \|L_{\text{c}}\|^2\right) \left(L_{\text{c}}(L_{\text{c}} - L_{\text{in}})^T / \|L_{\text{c}} - L_{\text{in}}\|^2\right)$. Consequently, only (42) will be considered in the remainder of this paper.

Turbo equalization employing BP polar code decoding has been discussed in ([39], Sect. 10.1, pp. 254–261). In their analysis, the authors found that first generating the LLR vector L_{u} of the message bits $u_i \in \{0, 1\}$, $i \in \mathbb{B}$, u_i being a frozen bit with the bit value 0 or not, and then applying the algorithm introduced in ([7], eqs. (83)–(87), p. 35) to form the LLR vector L_{c} of the codebits $b_i \in \mathbb{F}_2$, $i \in \mathbb{B}$, yields a better transmission performance than the conventional method used in, e.g., ref. [37,39,55]. Hence, the method of [37,39,55] is not considered further in this work. The extrinsic LLR vector L_{e} is generated with (42), using the LLR vector L_{c} of the codebits $b_i \in \mathbb{F}_2$, $i \in \mathbb{B}$.

Turbo equalization employing SC polar code decoding to generate the LLR vector L_{c} was first introduced and discussed in detail in [7]. To avoid repetition, the reader is referred to this earlier work for a comprehensive presentation.

Turbo equalization employing CA-SCL polar code decoding requires that the CA-SCL polar code decoder is also capable of providing the LLR vector L_{c} at its output. The determination of the LLR vector L_{c} will be briefly illustrated in the following. Let the time instant $i \in \mathbb{B}$ be associated with the message bit $u_i \in \{0, 1\}$, u_i being a frozen bit with the bit value 0 or not. At each time instant $i \in \mathbb{B}$, the CA-SCL polar code decoder determines $2L$, $L \in \mathbb{N}^*$, candidate sequences called paths, each one having a distinct path metric $\text{PM}_i^{(l)}$ ([39], p. 219). Before moving on to the next time instant $(i + 1)$, those distinct L candidate paths with the largest path metric values are pruned while the CA-SCL polar code decoder keeps only the best $L \in \mathbb{N}^*$ candidate paths, which are ordered in a list labeled by $l \in \{1, 2 \dots L\}$ in the order of ascending path metric values ([39], p. 219). Consequently, at time instant $i \in \mathbb{B}$, the best candidate path has the path metric $\text{PM}_i^{(1)}$, the second best candidate path has the path metric $\text{PM}_i^{(2)}$ and the L th best candidate path has the path metric $\text{PM}_i^{(L)}$, the path metric values complying with ([39], p. 219)

$$\text{PM}_i^{(1)} \leq \text{PM}_i^{(2)} \leq \dots \leq \text{PM}_i^{(L)}, \quad L \in \mathbb{N}^*. \quad (43)$$

After the processing of all N time instants $i \in \mathbb{B}$, the best candidate path has the path metric $\text{PM}_{N-1}^{(1)}$ and its associated message is selected as the decoded message $\hat{\mathbf{u}}$, which is related to the decoded codeword $\hat{\mathbf{b}}$. Obviously, the CA-SCL polar code decoding is a sequential decoding scheme, which is closely related to, e.g., the M algorithm being a hot topic until about the early 1990s, see, e.g., ref. [48,57–63] for further details.

The partial candidate message of the l th path at time instant i is denoted by $[\hat{\mathbf{u}}^{(l)}]_0^i$. With the candidate bits $\hat{u}_0^{(l)}, \hat{u}_1^{(l)} \dots \hat{u}_i^{(l)}$, we have ([39], p. 218)

$$[\hat{\mathbf{u}}^{(l)}]_0^i = \left(\hat{u}_0^{(l)}, \hat{u}_1^{(l)} \dots \hat{u}_i^{(l)}\right). \quad (44)$$

Of course, the partial candidate message $[\hat{\mathbf{u}}^{(l)}]_0^i$ of the l th path at time instant i is uniquely related to the partial candidate codeword $[\hat{\mathbf{b}}^{(l)}]_0^i$ of the l th path at time instant i , which consists of $\hat{b}_0^{(l)}, \hat{b}_1^{(l)} \dots \hat{b}_i^{(l)}$:

$$[\hat{\mathbf{b}}^{(l)}]_0^i = (\hat{b}_0^{(l)}, \hat{b}_1^{(l)} \dots \hat{b}_i^{(l)}). \quad (45)$$

Setting out from the above notation, the path metric $\text{PM}_{N-1}^{(l)}$ of the l th path at the end of the polar code decoding, i.e., at the time instant $(N-1)$, is defined as the negative natural logarithm of the conditional probability $P\{\hat{\mathbf{u}}^{(l)} | \mathbf{L}_{\text{in}}\}$ of the candidate message $\hat{\mathbf{u}}^{(l)}$ equal to $[\hat{\mathbf{u}}^{(l)}]_0^{N-1}$ associated with l th path given the input LLR vector $\mathbf{L}_{\text{in}}$ ([52], eq. (13), p. 5170), (ref. [39], eq. (8.3), p. 219), which is given by

$$\text{PM}_{N-1}^{(l)} = -\ln\{P\{\hat{\mathbf{u}}^{(l)} | \mathbf{L}_{\text{in}}\}\} \quad (46)$$

$$= -\ln\{P\{\hat{\mathbf{b}}^{(l)} | \mathbf{L}_{\text{in}}\}\}. \quad (47)$$

Setting $P\{\hat{\mathbf{u}}^{(l)} | \mathbf{L}_{\text{in}}\}$ equal to $P\{\hat{\mathbf{b}}^{(l)} | \mathbf{L}_{\text{in}}\}$ is correct because the polar encoding represents a one-to-one mapping of the candidate message $\hat{\mathbf{u}}^{(l)}$ and the candidate codeword $\hat{\mathbf{b}}^{(l)}$ equal to $[\hat{\mathbf{b}}^{(l)}]_0^{N-1}$, both associated with the l th path ([64], p. 1010). However, the identity of $P\{[\hat{\mathbf{u}}^{(l)}]_0^i | \mathbf{L}_{\text{in}}\}$, considering the partial candidate message $[\hat{\mathbf{u}}^{(l)}]_0^i$ of (44), and $P\{[\hat{\mathbf{b}}^{(l)}]_0^i | \mathbf{L}_{\text{in}}\}$, considering the partial candidate codeword $[\hat{\mathbf{b}}^{(l)}]_0^i$ of (45), is invalid in general for arbitrary time index $k < (N-1)$. Instead, the path metric $\text{PM}_i^{(l)}$ of the l th path at the time instant k is defined by

$$\text{PM}_i^{(l)} = -\ln\{P\{[\hat{\mathbf{u}}^{(l)}]_0^i | \mathbf{L}_{\text{in}}\}\}. \quad (48)$$

The CA-SCL polar code decoding capable of providing the extrinsic LLR vector $\mathbf{L}_e$ first determines the LLR value $L_{c,i}$ associated with the i th codebit, at each time instant $i \in \mathbb{B}$, which is defined by the natural logarithm of the ratio of the conditional probability $P\{b_i = 0 | \mathbf{L}_{\text{in}}\}$ that the transmitted i th codebit b_i has the value 0 given the input LLR vector $\mathbf{L}_{\text{in}}$ and the conditional probability $P\{b_i = 1 | \mathbf{L}_{\text{in}}\}$ that the transmitted i th codebit b_i has the value 1 given the input LLR vector $\mathbf{L}_{\text{in}}$, i.e.,

$$L_{c,i} = \ln\left\{\frac{P\{b_i = 0 | \mathbf{L}_{\text{in}}\}}{P\{b_i = 1 | \mathbf{L}_{\text{in}}\}}\right\}, \quad (49)$$

See also, e.g., (ref. [64], p. 1009) and ([65], eq. (6), p. 654). Owing to the fact that the considered CA-SCL polar code decoding, representing a sequential decoding method, does usually not carry out an exhaustive search but only considers L competing paths, taking the approach of the 2025 paper [65] by Gao et al. into account, (49) is approximately given by ([65], eq. (6), p. 654)

$$L_{c,i} \approx \ln\left\{\frac{\sum_{l=1}^L P\{b_i = 0 | \mathbf{L}_{\text{in}}, \hat{\mathbf{b}}^{(l)}\} P\{\hat{\mathbf{b}}^{(l)} | \mathbf{L}_{\text{in}}\}}{\sum_{l=1}^L P\{b_i = 1 | \mathbf{L}_{\text{in}}, \hat{\mathbf{b}}^{(l)}\} P\{\hat{\mathbf{b}}^{(l)} | \mathbf{L}_{\text{in}}\}}\right\}. \quad (50)$$

With (47), Equation (50) becomes ([65], eq. (6), p. 654)

$$L_{c,i} \approx \ln \left\{ \frac{\sum_{l=1}^L P\{b_i = 0 \mid L_{in}, \hat{b}^{(l)}\} \exp\{-PM_{N-1}^{(l)}\}}{\sum_{l=1}^L P\{b_i = 1 \mid L_{in}, \hat{b}^{(l)}\} \exp\{-PM_{N-1}^{(l)}\}} \right\}. \quad (51)$$

According to ([65], p. 654), the conditional probability $P\{b_i = 0 \mid L_{in}, \hat{b}^{(l)}\}$ that the transmitted i th codebit b_i has the value 0 given the input LLR vector L_{in} and the candidate codeword $\hat{b}^{(l)}$ and the conditional probability $P\{b_i = 1 \mid L_{in}, \hat{b}^{(l)}\}$ that the transmitted i th codebit b_i has the value 1 given the input LLR vector L_{in} and the candidate codeword $\hat{b}^{(l)}$ are related with the coded a posteriori LLR under the condition that the candidate codeword along the l th survived path is $\hat{b}^{(l)}$, which could be well approximated by the L SC polar code decoders operating in parallel in the CA-SCL polar code decoder. In the opinion of the authors, this approach adds further complexity to the polar code decoding, which can easily become prohibitively large. As Equation (51) represents an approximation, the additional complexity can, e.g., be avoided by replacing $P\{b_i = c \mid L_{in}, \hat{b}^{(l)}\}$, $c \in \{0, 1\}$, by the simple quantized versions

$$P\{b_i = c \mid L_{in}, \hat{b}^{(l)}\} \approx \begin{cases} 1 & \text{if } \hat{b}_i^{(l)} = b_i = c, \\ 0 & \text{otherwise.} \end{cases} \quad (52)$$

Then, (51) yields

$$L_{c,i} \approx \ln \left\{ \frac{\sum_{l=1, \hat{b}_i^{(l)}=0}^L \exp\{-PM_{N-1}^{(l)}\}}{\sum_{l=1, \hat{b}_i^{(l)}=1}^L \exp\{-PM_{N-1}^{(l)}\}} \right\}, \quad (53)$$

which is equivalent to ([64], eq. (7), p. 1010). Note that (53) was first proposed by Seshadri and Höher in their 1993 paper ([66], p. 742) as a method of post-processing for providing reliability information at the output of an arbitrary decoder. With L_c formed by the LLR values $L_{c,i}$ of (51), respectively (53), the extrinsic LLR vector L_e proposed by [64,65] is computed according to Hagenauer's 1997 approach given by (38). This scheme, which is considered by [64,65], will be termed "SCL, Yuan & Gao" in what follows.

A novel alternative to the above approach proposed by, e.g., ref. [64,65], termed "SCL, D&J" in what follows, is as follows. Setting out from ([65], eq. (6), p. 654) and using (48), the approximation

$$L_{u,i} \approx (1 - 2\hat{u}_i) \cdot \ln \left\{ \frac{\sum_{l=1, \hat{u}_i^{(l)}=0}^L \exp\{-PM_i^{(l)}\}}{\sum_{l=1, \hat{u}_i^{(l)}=1}^L \exp\{-PM_i^{(l)}\}} \right\}, \quad (54)$$

is introduced. Using (54), the LLR vector L_u associated with the message bits $u_i \in \{0, 1\}$, $i \in \mathbb{B}$, u_i being a frozen bit with the bit value 0 or not, is established. This LLR vector L_u associated with the message bits $u_i \in \{0, 1\}$, $i \in \mathbb{B}$, is processed as discussed in ([7],

p. 35) to determine the LLR vector L_c of the codebits $b_i \in \mathbb{F}_2$, $i \in \mathbb{B}$, which is then used to generate the extrinsic LLR vector L_e by applying (42), respectively.

It has long been known that the simulated transmission quality can occasionally exceed that which is achievable in actually implemented systems if no limits are imposed on the magnitudes of the LLRs. Fortunately, this effect sometimes occurs only when the BER P_b measured at the output of the quantum data detector decreases below a value of approximately 0.1 and occurs only very slowly as the average number of transmitted signal photons $\bar{N}_\mu$ increases, as is the case in LED based wireless quantum optical communication. In laser based wireless quantum optical communication, this is not the case and, hence, the phenomenon has not been observed so far. The aforementioned effect is promoted by the nonlinear combination of the LLRs, for example, in functions of the type

$$L_{u,i} \boxplus L_{u,k} = 2 \operatorname{artanh} \left\{ \tanh \left(\frac{L_{u,i}}{2} \right) \cdot \tanh \left(\frac{L_{u,k}}{2} \right) \right\} \quad (55)$$

or any approximation thereof, applied in the decoding of LDPC (low density parity check) codes and polar codes, see, e.g., [39]. In order to nevertheless retain the exact calculation according to Equation (55) in simulations while at the same time obtaining implementation realistic results for the transmission quality, we recommend to limit the magnitudes of the LLRs to a predetermined maximum value. The authors use the limit value 29.7. This choice results in the probability values associated with the LLRs being restricted to the range between $\approx 1.263146978246283 \cdot 10^{-13}$ and ≈ 0.99999999999874 , so that the theoretical values 0 and 1 are only closely approximated, but never actually reached.

4. Selected Simulation Results

In this Section 4, selected simulation results obtained using the log-MAP symbol-by-symbol quantum data detector with $\gamma_{\hat{N}_i}(\mathbf{s}_i, \mathbf{s}_{i+1})$ from (35) are presented. The used software is entirely developed in-house by the authors and does not rely on pre-existing components such as log-MAP detectors, polar code decoders, or similar modules. The log-MAP symbol-by-symbol quantum data detection approach applies the logarithmic version of the quantum data detection discussed in Section 2, cf. e.g., ref. [7,43]. Turbo equalization with up to 16 iterations is considered. As performance metrics, both the BER P_b at the output of the considered polar code decoder, termed coded BER in what follows, and the BLER P_{block} , which are of primary importance in wireless communications, are investigated.

The codebit values 0 and 1 are assumed to be equiprobable. Assuming a reasonably short link distance, transmission is modeled by a two-path quantum communication channel, in which the coefficient of the second path, s_1 , is considerably smaller than the coefficient of the first path, s_0 , these being the only two nonzero channel coefficients. In this study, the channel coefficients are selected as s_0 equal to 8/9 and s_1 equal to 1/9, referred to as “CIR #7.”

The average number $\bar{N}_\mu$ of emitted signal photons varies from 5 to 140. Since the simulated BER P_b and BLER P_{block} results are evaluated at the output of the considered polar decoders, the corresponding code rate R_c is explicitly taken into account. Hence, for polar coded transmission, $\bar{N}_\mu$ is larger by a factor of $1/R_c$ than in the uncoded case.

In this case study, the average number $\bar{N}_\alpha$ of thermal noise photons is selected from the set $\{0, 0.1, 0.3\}$. The case $\bar{N}_\alpha$ equal to 0 represents the ideal scenario with the lowest achievable coded BER P_b and BLER P_{block} . Furthermore, the values $\bar{N}_\alpha$ equal to 0.1 and $\bar{N}_\alpha$ equal to 0.3 are typical for the far infrared frequency range between approximately 50 THz and 100 THz, when room temperature is considered ([21], eq. (1.122), p. 50; Figure 1.41, p. 406).

It should be noted that $\bar{N}_\alpha$ will increase correspondingly when moving toward the 6G “terahertz (THz) range” [300 GHz, 10 THz] which in turn necessitates either a corresponding

increase in the average number $\bar{N}_\mu$ of emitted signal photons or cooling of the receiver front-end. In the opinion of the authors, both approaches appear feasible in principle, provided that suitable optical transmitters exhibiting thermal states, such as LEDs, are employed.

In the Monte Carlo simulations conducted by the authors, the relative accuracy of the parameters $\bar{N}_\mu$ and $\bar{N}_\alpha$ was approximately $2 \cdot 10^{-3}$. In their seminal textbook [67], Jeruchim, Balaban, and Shanmugan analyzed the expected confidence levels of simulation results for error ratios. Following the guidelines provided in [67], the authors ensured that

- for an estimated error ratio of 10^{-4} , the 95% confidence interval is approximately $[7 \cdot 10^{-5}, 1.3 \cdot 10^{-4}]$,
- for an estimated error ratio of 10^{-3} , the 95% confidence interval is approximately $[9 \cdot 10^{-4}, 1.1 \cdot 10^{-3}]$,
- for an estimated error ratio of 10^{-2} , the 95% confidence interval is approximately $[9.7 \cdot 10^{-3}, 1.03 \cdot 10^{-2}]$, and
- for an estimated error ratio of 10^{-1} , the 95% confidence interval is approximately $[9.9 \cdot 10^{-2}, 1.01 \cdot 10^{-1}]$.

These confidence intervals apply to both the simulated BER P_b and BLER P_{block} .

The employed polar code originally proposed by Arikan [9] and denoted as “ARIKAN PC” in the figures presented in Section 4 is considered. As representative examples, three different polar code configurations are considered, namely

- C1:** code length N equal to 256, code dimension K equal to 88 and code rate R_c equal to $(88 - 24)/256$, i.e., $1/4$,
- C2:** code length N equal to 512, code dimension K equal to 204 and code rate R_c equal to $(204 - 24)/512$, i.e., ≈ 0.352 , and
- C3:** code length N equal to 512, code dimension K equal to 280 and code rate R_c equal to $(280 - 24)/512$, i.e., $1/2$.

For configuration **C1**, the following decoding schemes are considered for comparison:

- the SC polar decoder with extrinsic LLR generation according to the approach proposed by the authors employing either an LLR limit of 29.7 or no limit at all, the latter included for comparison with the results reported in [8],
- the BP polar decoder with 10 internal left-right BP iterations, employing an LLR limit of 29.7,
- the CA-SCL polar decoder with list length L equal to 8 and extrinsic LLR generation according to the approaches proposed by Yuan [64] and Gao [65], using Hagenauer’s 1997 approach for computing the extrinsic LLR vector L_e given by (38) and employing an LLR limit of 29.7, and
- the CA-SCL polar decoder with list length L equal to 8 and extrinsic LLR generation according to the approach proposed by the authors, likewise employing an LLR limit of 29.7.

Naturally, CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by Yuan [64] and Gao [65], using Hagenauer’s 1997 approach for computing the extrinsic LLR vector L_e given by (38), and CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by the authors achieve identical performance when turbo equalization is not employed. Note that for configurations **C2** and **C3**, only the performance of these two CA-SCL polar decoder variants is investigated.

In general, for a given value of $\bar{N}_\alpha$ and a particular configuration, both the coded BER P_b and the BLER P_{block} decrease monotonically with increasing $\bar{N}_\mu$. Conversely, for a given $\bar{N}_\mu$, both performance metrics deteriorate with increasing $\bar{N}_\alpha$. Note that the polar coded schemes significantly outperform the uncoded case, as already reported in [8].

Considering configuration **C1**, we observed that the SC polar decoder and the BP polar decoder achieve similar coded BER P_b and BLER P_{block} performance (see Figures 4–9). Moreover, turbo equalization provides a considerable performance improvement for these two decoders. The somewhat idealized case of SC polar decoding without any LLR limitation also yields remarkably good performance, although such behavior may not be achievable in a practical implementation. A similar observation is made for configurations **C2** and **C3**. However, these results are omitted for brevity.

Compared with SC or BP polar decoding, turbo equalization provides a smaller additional gain when CA-SCL polar decoding is employed. Nevertheless, for configuration **C1**, CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by Yuan [64] and Gao [65], using Hagenauer’s 1997 approach for computing the extrinsic LLR vector L_e given by (38), tends to achieve a slightly better coded BER P_b performance than the approach proposed by the authors. However, the performance advantage of the former approach over the latter remains relatively small.

It is noteworthy that CA-SCL polar decoding significantly outperforms both SC and BP polar decoding even without turbo equalization. For this reason, SC and BP polar decoding are disregarded in the following discussion.

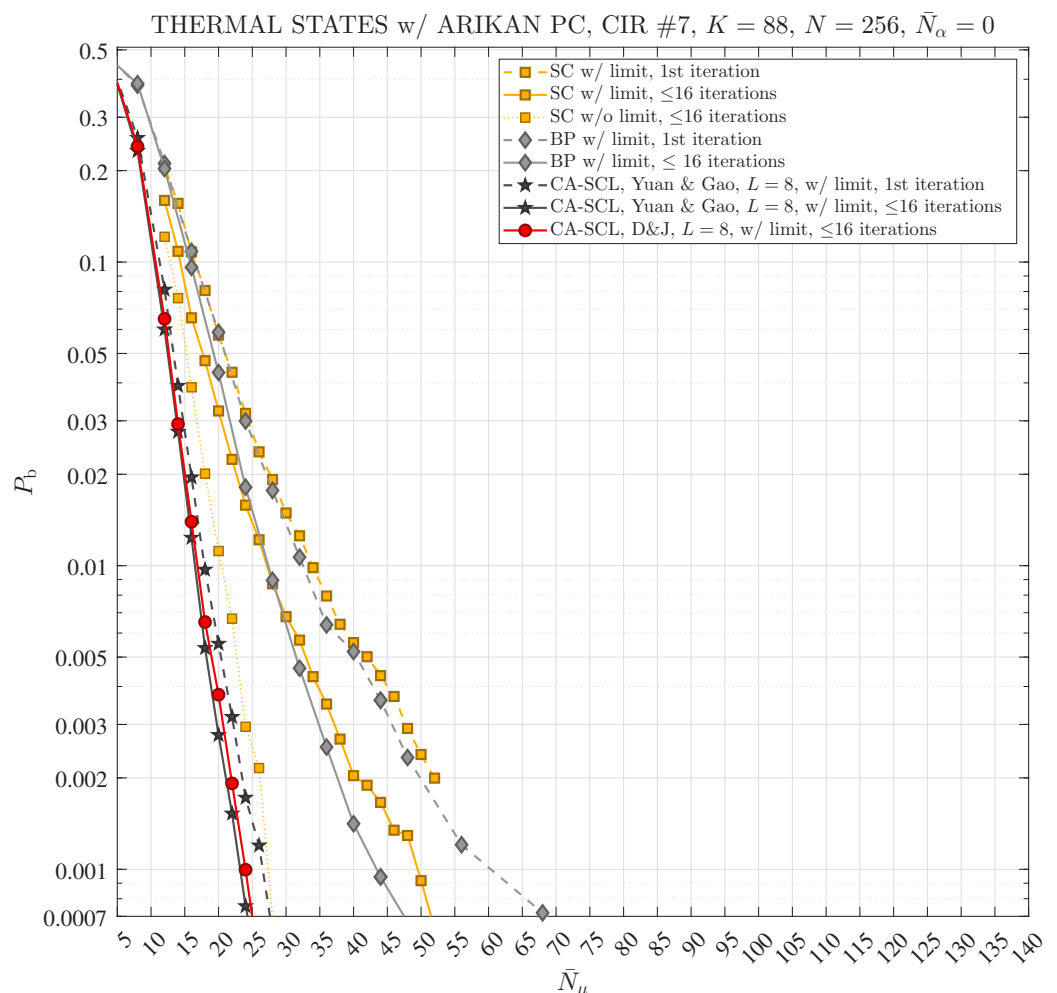


Figure 4. Detail of the polar coded BER P_b versus the average number $\bar{N}_\mu$ of emitted signal photons per message bit for the transmission over the two-paths quantum channel CIR #7, having the channel coefficients s_0 equal to 8/9 and s_1 equal to 1/9, and with the average number $\bar{N}_\alpha$ of thermal noise photons equal to 0; log-MAP symbol-by-symbol quantum data detection with equiprobable codebit values is applied; configuration **C1** forms the basis of the simulations; SC, BP, and CA-SCL polar code decoding approaches with and without turbo equalization are compared.

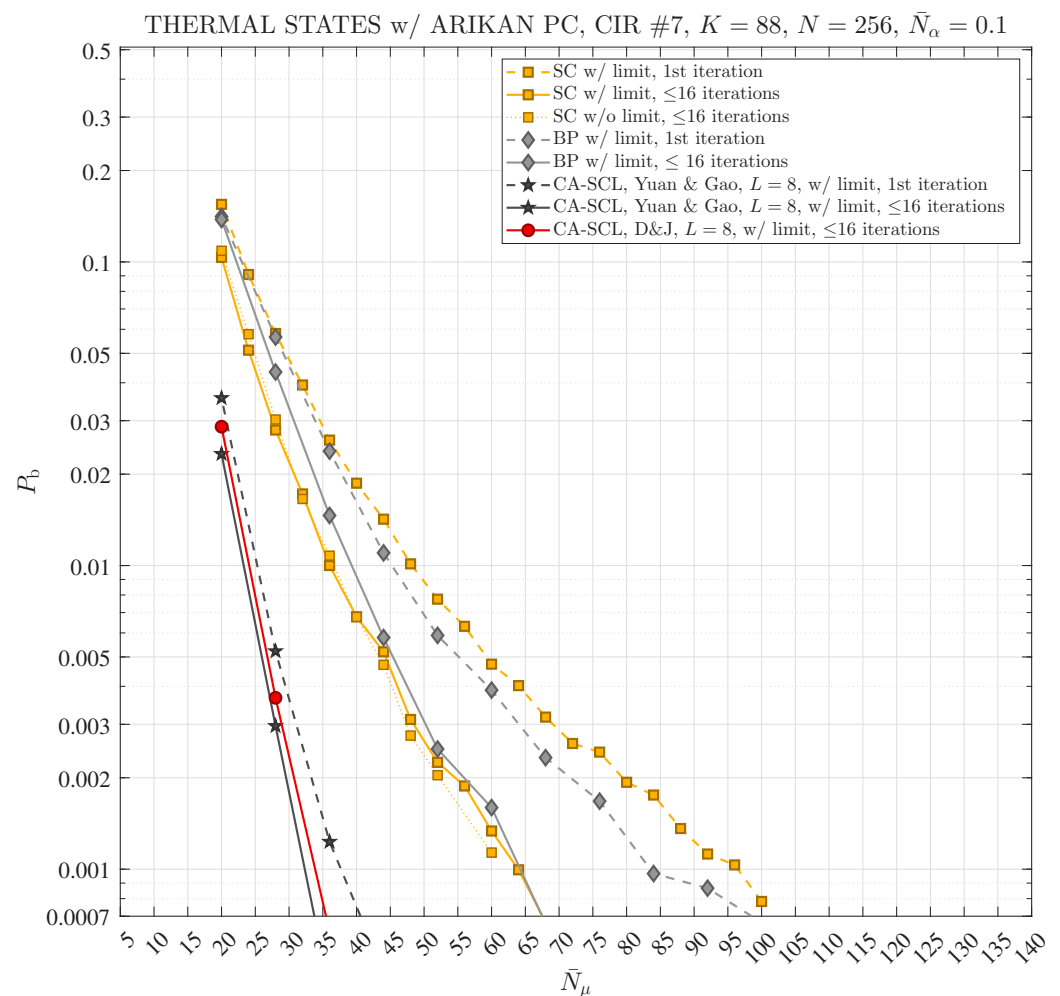


Figure 5. Detail of the polar coded BER P_b versus the average number $\bar{N}_\mu$ of emitted signal photons per message bit for the transmission over the two-paths quantum channel CIR #7, having the channel coefficients s_0 equal to 8/9 and s_1 equal to 1/9, and with the average number $\bar{N}_\alpha$ of thermal noise photons equal to 0.1; log-MAP symbol-by-symbol quantum data detection with equiprobable codebit values is applied; configuration C1 forms the basis of the simulations; SC, BP, and CA-SCL polar code decoding approaches with and without turbo equalization are compared.

The performance results for configuration C2 are presented in Figures 10 and 11, whereas those for configuration C3 are shown in Figures 12 and 13.

For both configurations C2 and C3, CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by the authors tends to outperform the approach proposed by Yuan [64] and Gao [65], using Hagenauer's 1997 approach for computing the extrinsic LLR vector L_e given by (38), as both $\bar{N}_\alpha$ and the code rate R_c increase. By contrast, for low code rates, the performance difference between the two decoding approaches is remarkably small. Table 2 summarizes relevant results that support this analysis.

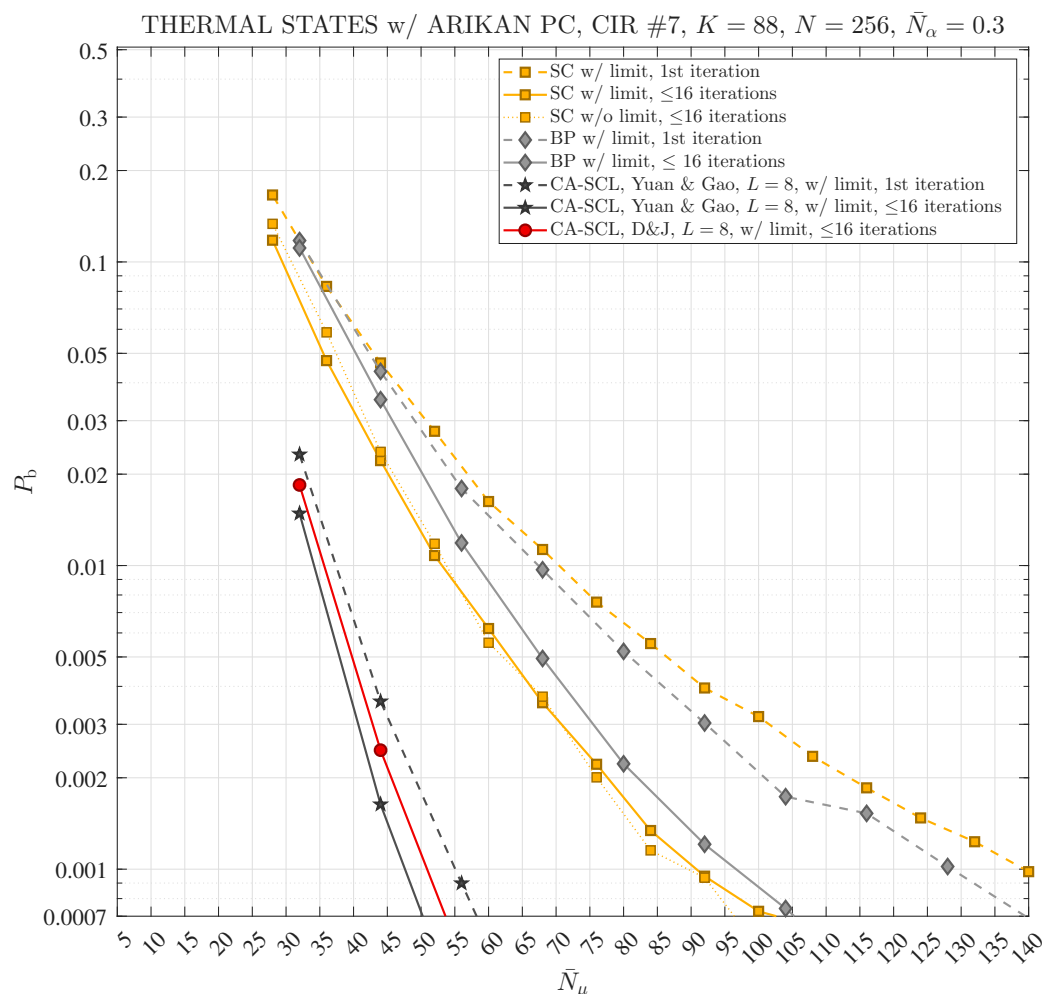


Figure 6. Detail of the polar coded BER P_b versus the average number $\bar{N}_\mu$ of emitted signal photons per message bit for the transmission over the two-paths quantum channel CIR #7, having the channel coefficients s_0 equal to 8/9 and s_1 equal to 1/9, and with the average number $\bar{N}_\alpha$ of thermal noise photons equal to 0.3; log-MAP symbol-by-symbol quantum data detection with equiprobable codebit values is applied; configuration C1 forms the basis of the simulations; SC, BP, and CA-SCL polar code decoding approaches with and without turbo equalization are compared.

Figure 14 shows the average number of turbo equalization iterations together with the associated standard deviation versus the average number $\bar{N}_\mu$ of emitted signal photons per message bit for the transmission over the two-paths quantum channel CIR #7, with the average number $\bar{N}_\alpha$ of thermal noise photons taken from the set $\{0, 0.1, 0.3\}$ when configuration C1 is considered and an SC polar decoder is used with extrinsic LLR generation according to the approach proposed by the authors employing either an LLR limit of 29.7. The average number of turbo equalization iterations is well below 2 when the BER P_b drops below 0.01. This shows that the additional average effort of turbo equalization is low when appropriate iteration control is accomplished.

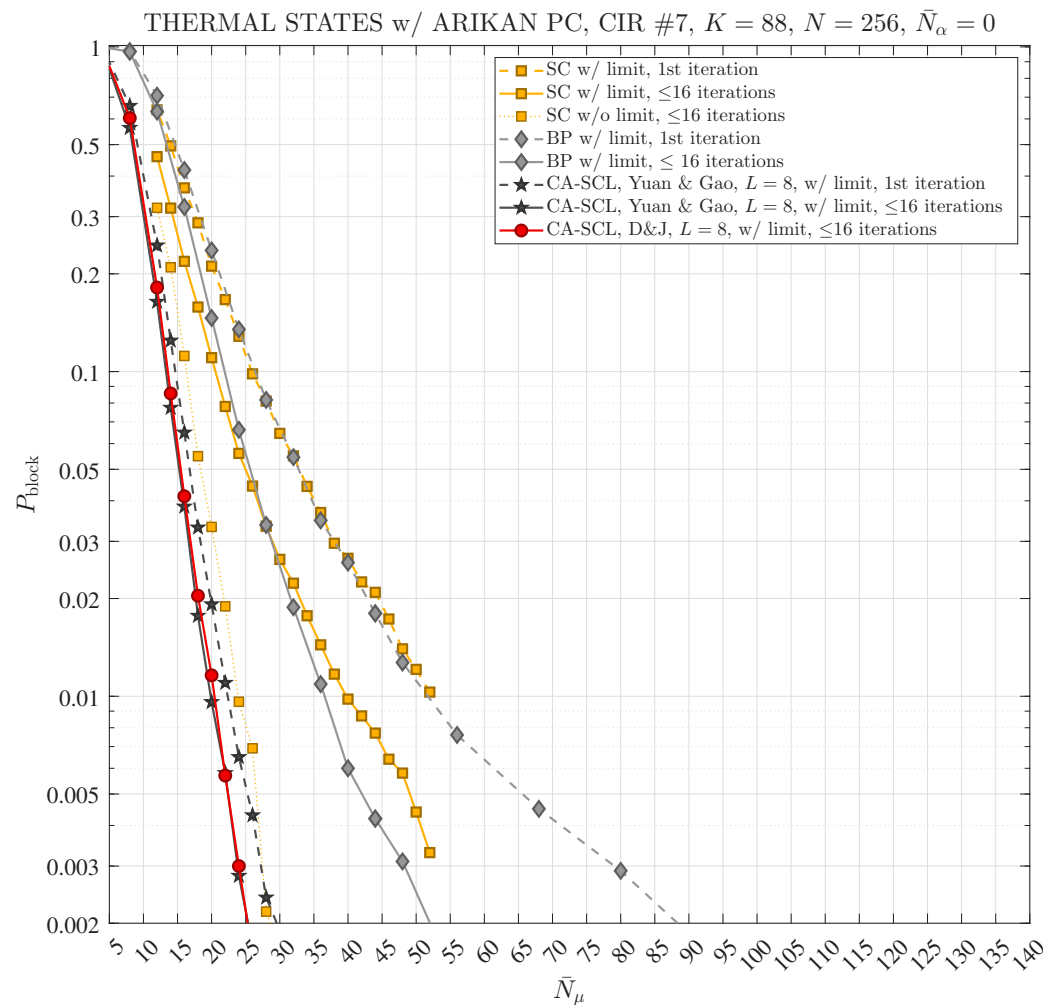


Figure 7. Detail of the polar coded BLER P_{block} versus the average number $\bar{N}_\mu$ of emitted signal photons per message bit for the transmission over the two-paths quantum channel CIR #7, having the channel coefficients s_0 equal to 8/9 and s_1 equal to 1/9, and with the average number $\bar{N}_\alpha$ of thermal noise photons equal to 0; log-MAP symbol-by-symbol quantum data detection with equiprobable codebit values is applied; configuration C1 forms the basis of the simulations; SC, BP, and CA-SCL polar code decoding approaches with and without turbo equalization are compared.

Finally, Figure 15 illustrates the average factor C_S equal to $L_{\text{in}} L_{\text{c}}^T / \|L_{\text{c}}\|^2$ together with the associated standard deviation versus the average number $\bar{N}_\mu$ of emitted signal photons per message bit for the transmission over the two-paths quantum channel CIR #7 with the average number $\bar{N}_\alpha$ of thermal noise photons taken from the set $\{0, 0.1, 0.3\}$ when configuration C1 is considered and an SC polar decoder is used with extrinsic LLR generation according to the approach proposed by the authors employing either an LLR limit of 29.7. C_S depends both on $\bar{N}_\mu$ and $\bar{N}_\alpha$. In general, for a given value of $\bar{N}_\alpha$, C_S decrease monotonically with increasing $\bar{N}_\mu$. Conversely, for a given $\bar{N}_\mu$, C_S increases monotonically with increasing $\bar{N}_\alpha$.

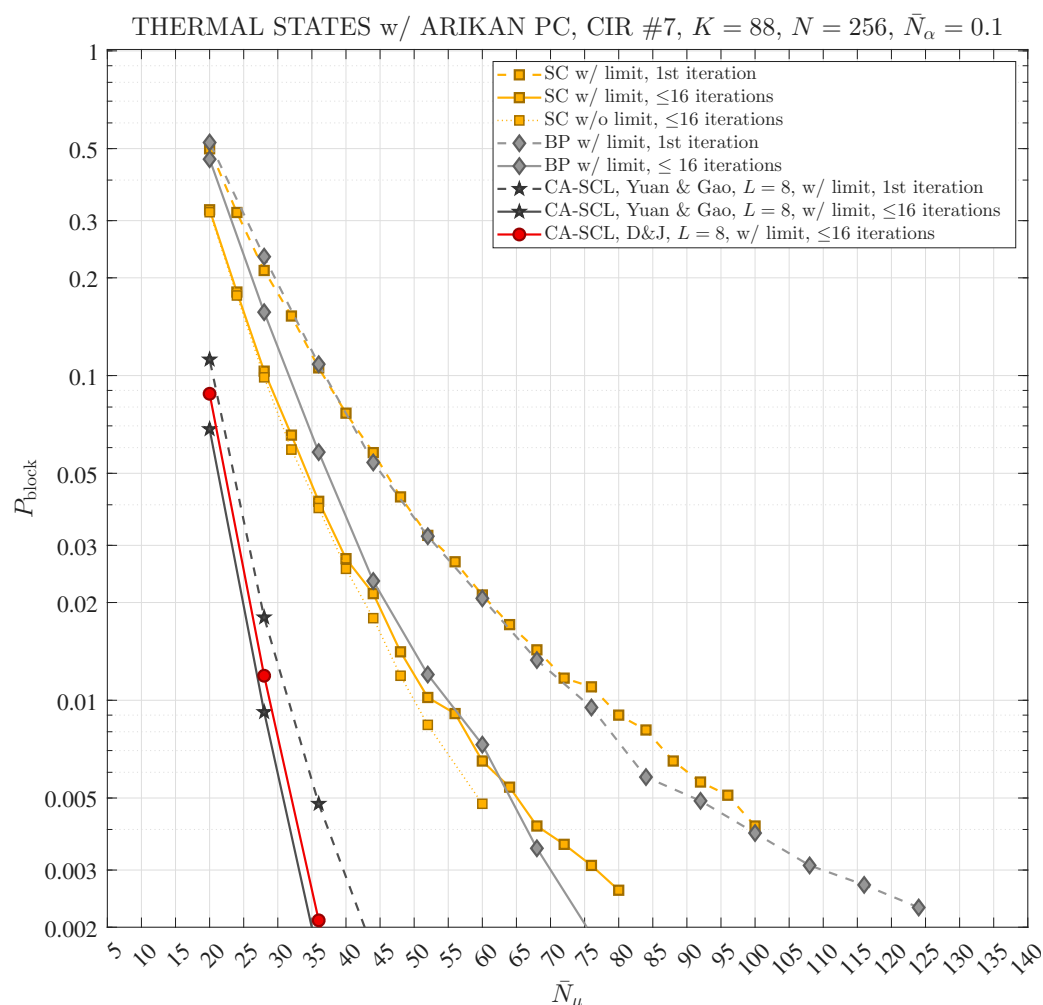


Figure 8. Detail of the polar coded BLER P_{block} versus the average number $\bar{N}_\mu$ of emitted signal photons per message bit for the transmission over the two-paths quantum channel CIR #7, having the channel coefficients s_0 equal to 8/9 and s_1 equal to 1/9, and with the average number $\bar{N}_\alpha$ of thermal noise photons equal to 0.1; log-MAP symbol-by-symbol quantum data detection with equiprobable codebit values is applied; configuration C1 forms the basis of the simulations; SC, BP, and CA-SCL polar code decoding approaches with and without turbo equalization are compared.

In this study, the LED employed in the quantum transmitter is assumed to operate in a single optical mode with a fixed angular frequency ω . Unfortunately, the resulting narrow bandwidth gives rise to Bose–Einstein photon count statistics, associated with comparatively high error ratios at low and moderate values of the average number $\bar{N}_\mu$ of emitted signal photons [8,47]. The present work, however, demonstrates that the application of sophisticated channel coding can effectively mitigate this performance limitation (see also [8] for selected results in this direction).

Moreover, the narrow bandwidth considered herein enables the deployment of a large number of terminals within a confined geographical area, with each terminal operating in its own dedicated single mode. Consequently, the operating frequencies can readily be assigned in a pairwise distinct manner to the various terminals. This naturally leads to a frequency division multiplexing scheme, which may equivalently be interpreted as wavelength division multiplexing. Such an approach greatly simplifies network planning and resource allocation in future cellular wireless quantum optical communication networks, as anticipated for 6G and beyond.

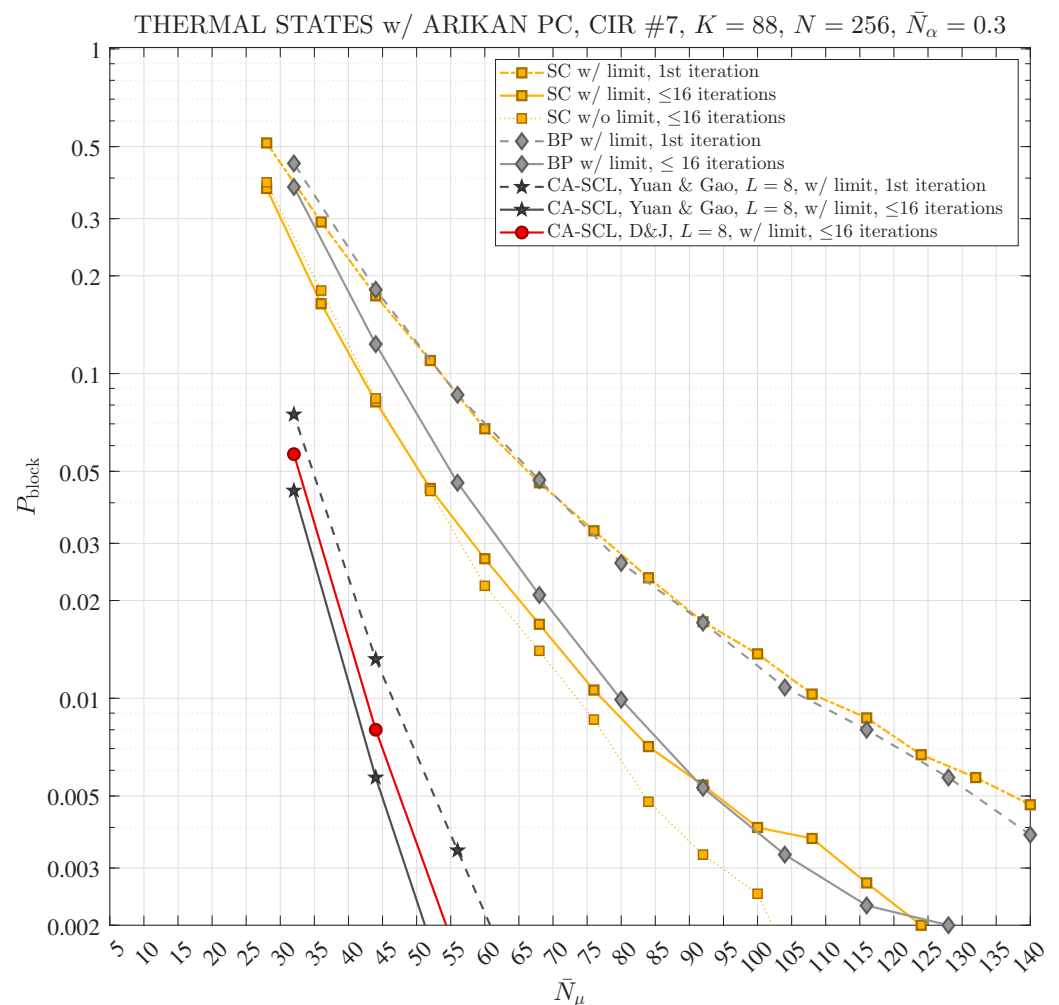


Figure 9. Detail of the polar coded BLER P_{block} versus the average number $\bar{N}_\mu$ of emitted signal photons per message bit for the transmission over the two-paths quantum channel CIR #7, having the channel coefficients s_0 equal to 8/9 and s_1 equal to 1/9, and with the average number $\bar{N}_\alpha$ of thermal noise photons equal to 0.3; log-MAP symbol-by-symbol quantum data detection with equiprobable codebit values is applied; configuration C1 forms the basis of the simulations; SC, BP, and CA-SCL polar code decoding approaches with and without turbo equalization are compared.

In conventional LED based optical communication systems, only multimode, i.e., wideband, operation has typically been considered [20,68]. As a result, the considerably more favorable Poisson photon count statistics arise, yielding substantially lower error ratios in the absence of channel coding than the Bose–Einstein photon count statistics. Similar to laser based wireless quantum optical communication systems, this enables high communication performance; however, it comes at the cost of a large bandwidth allocation for each communication link ([68], table 3.3, p. 81).

When a multitude of such wideband terminals is deployed within a confined geographical area, severe co-channel multiple access interference may arise, necessitating sophisticated coordination and synchronization mechanisms as well as appropriately designed multiplexing schemes, such as, e.g., time division multiplexing or code division multiplexing. Furthermore, in the context of cellular wireless quantum optical communication networks, such large bandwidth requirements render frequency allocation and, consequently, network planning significantly more challenging. These additional requirements increase both the operational complexity of the network and the associated implementation overhead.

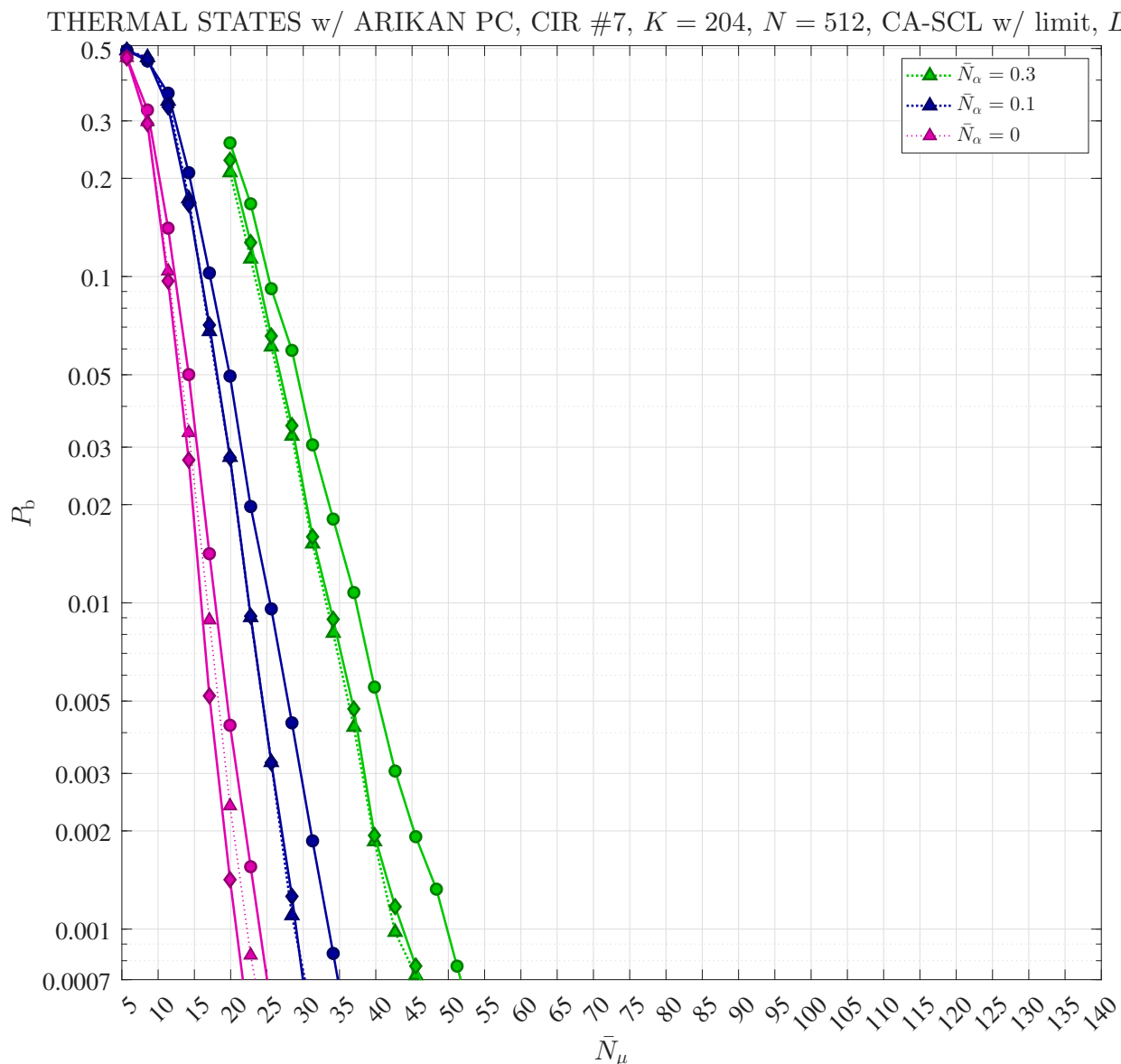


Figure 10. Detail of the polar coded BER P_b versus the average number $\bar{N}_\mu$ of emitted signal photons per message bit for the transmission over the two-paths quantum channel CIR #7, having the channel coefficients s_0 equal to $8/9$ and s_1 equal to $1/9$, and with the average number $\bar{N}_\alpha$ of thermal noise photons taken from the set $\{0, 0.1, 0.3\}$; log-MAP symbol-by-symbol quantum data detection with equiprobable codebit values is applied; configuration **C2** forms the basis of the simulations; the performance results obtained with CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by Yuan [64] and Gao [65], using Hagenauer's 1997 approach for computing the extrinsic LLR vector L_e given by (38), and CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by the authors with and without turbo equalization are compared: —•— CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by the authors after the first turbo equalization iteration; ...▲... CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by Yuan [64] and Gao [65] after at most 16 turbo equalization iterations; —◆— CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by the authors after at most 16 turbo equalization iterations.

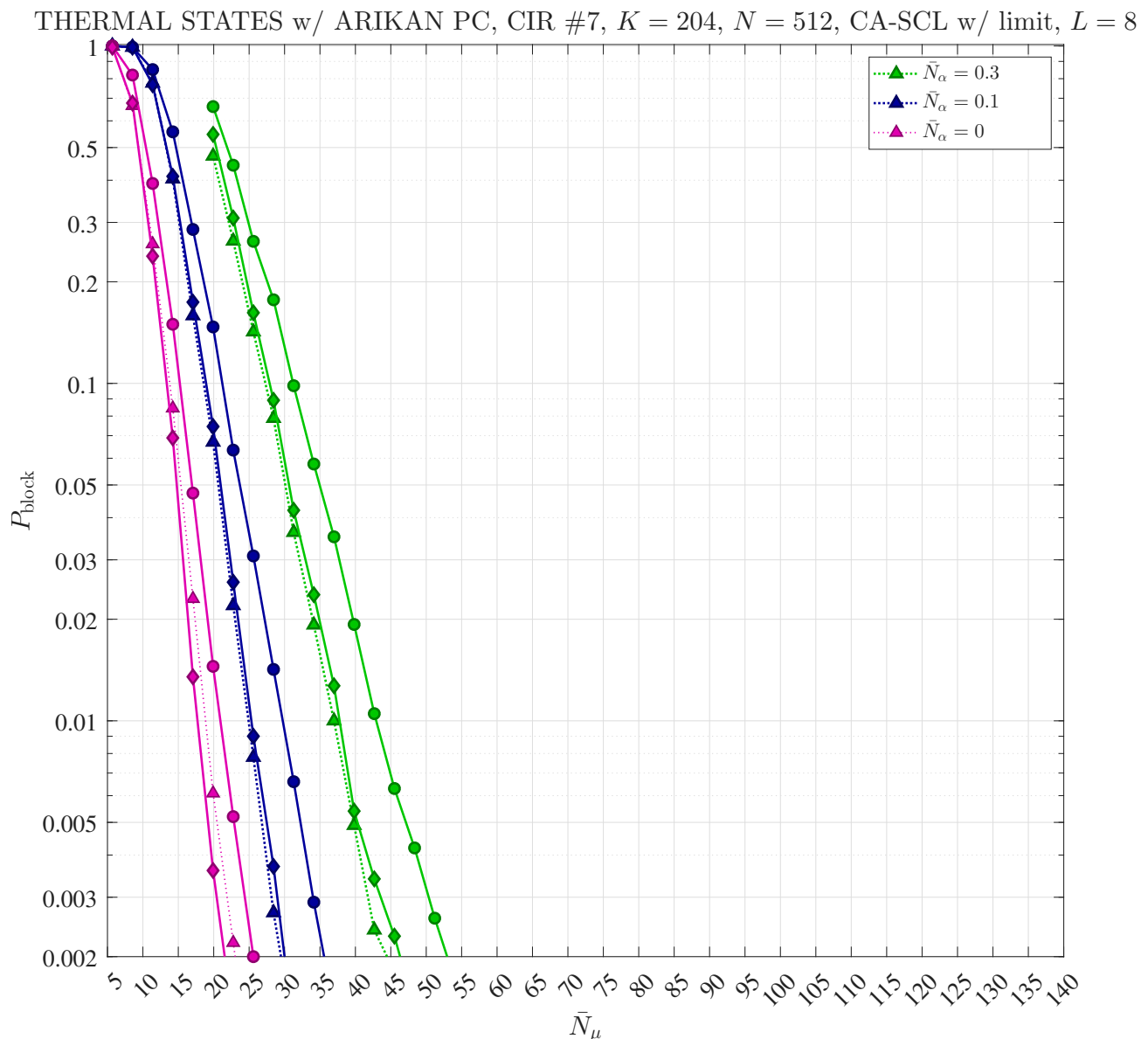


Figure 11. Detail of the polar coded BLER P_{block} versus the average number $\bar{N}_\mu$ of emitted signal photons per message bit for the transmission over the two-paths quantum channel CIR #7, having the channel coefficients s_0 equal to $8/9$ and s_1 equal to $1/9$, and with the average number $\bar{N}_\alpha$ of thermal noise photons taken from the set $\{0, 0.1, 0.3\}$; log-MAP symbol-by-symbol quantum data detection with equiprobable codebit values is applied; configuration C2 forms the basis of the simulations; the performance results obtained with CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by Yuan [64] and Gao [65], using Hagenauer's 1997 approach for computing the extrinsic LLR vector L_e given by (38), and CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by the authors with and without turbo equalization are compared: —●— CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by the authors after the first turbo equalization iteration; ···▲··· CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by Yuan [64] and Gao [65] after at most 16 turbo equalization iterations; —◆— CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by the authors after at most 16 turbo equalization iterations.

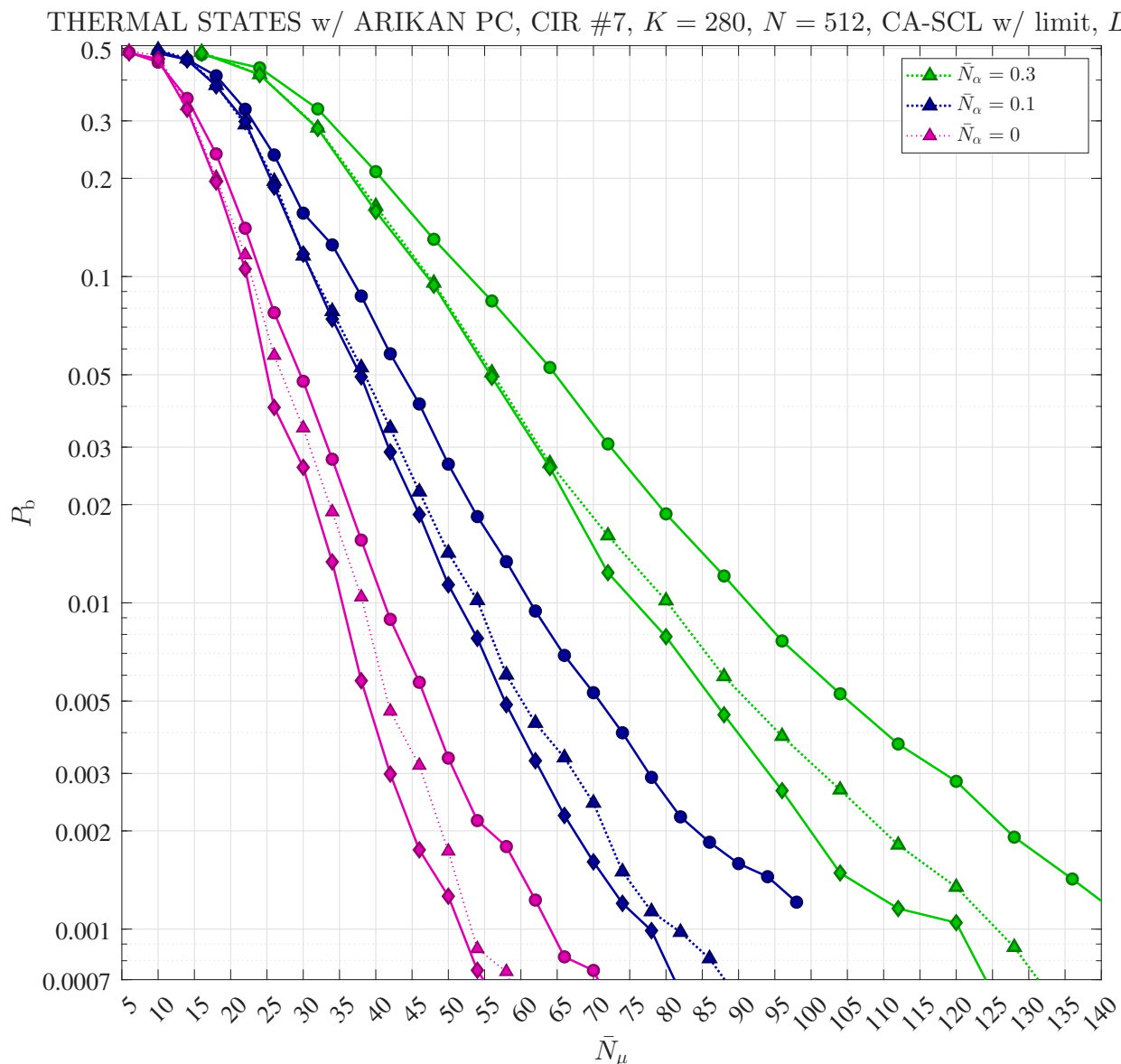


Figure 12. Detail of the polar coded BER P_b versus the average number $\bar{N}_\mu$ of emitted signal photons per message bit for the transmission over the two-paths quantum channel CIR #7, having the channel coefficients s_0 equal to $8/9$ and s_1 equal to $1/9$, and with the average number $\bar{N}_\alpha$ of thermal noise photons taken from the set $\{0, 0.1, 0.3\}$; log-MAP symbol-by-symbol quantum data detection with equiprobable codebit values is applied; configuration C3 forms the basis of the simulations; the performance results obtained with CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by Yuan [64] and Gao [65], using Hagenauer's 1997 approach for computing the extrinsic LLR vector L_e given by (38), and CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by the authors with and without turbo equalization are compared: —•— CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by the authors after the first turbo equalization iteration; ...▲... CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by Yuan [64] and Gao [65] after at most 16 turbo equalization iterations; —◆— CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by the authors after at most 16 turbo equalization iterations.

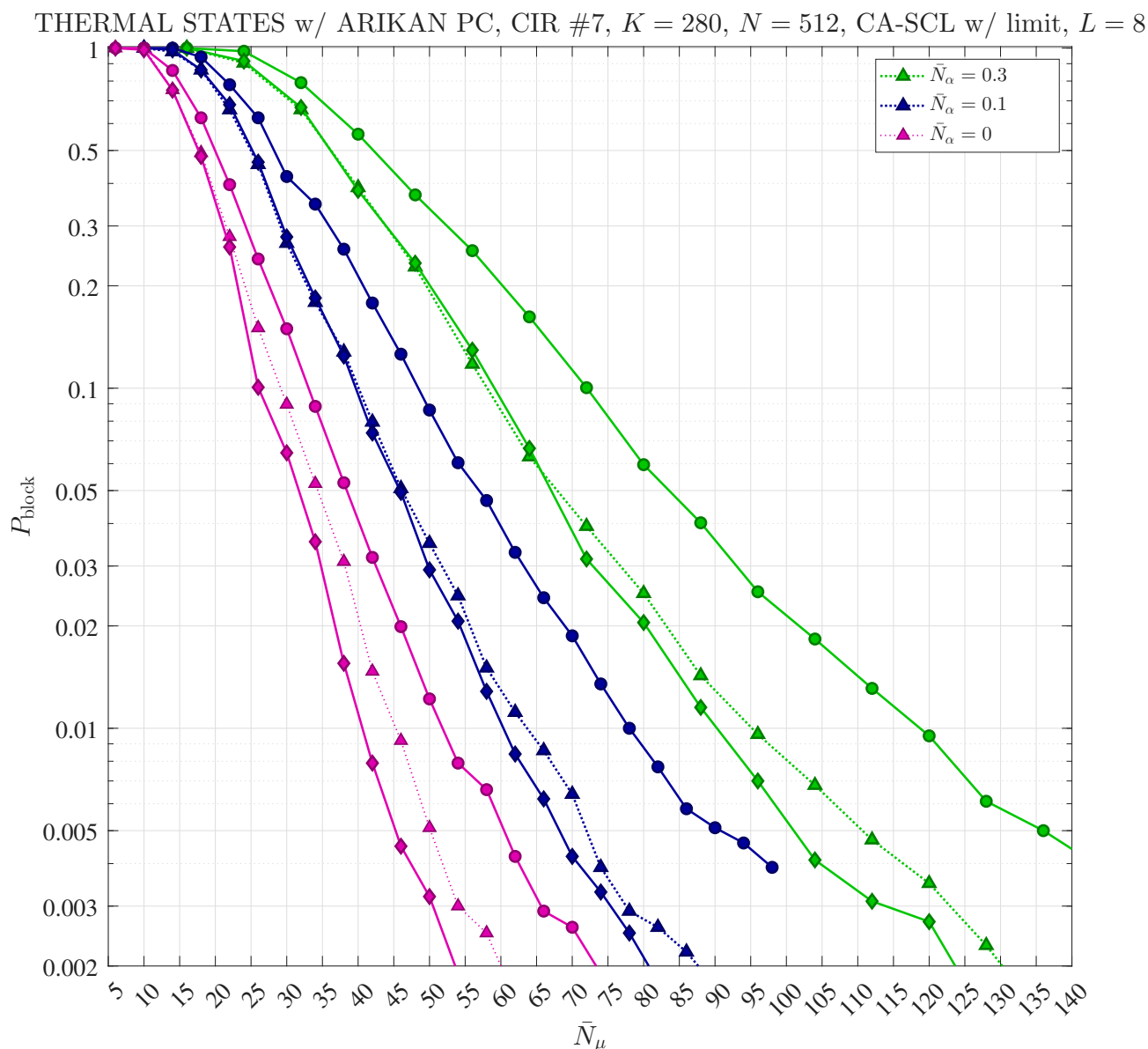


Figure 13. Detail of the polar coded BLER P_{block} versus the average number $\bar{N}_\mu$ of emitted signal photons per message bit for the transmission over the two-paths quantum channel CIR #7, having the channel coefficients s_0 equal to $8/9$ and s_1 equal to $1/9$, and with the average number $\bar{N}_\alpha$ of thermal noise photons taken from the set $\{0, 0.1, 0.3\}$; log-MAP symbol-by-symbol quantum data detection with equiprobable codebit values is applied; configuration C3 forms the basis of the simulations; the performance results obtained with CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by Yuan [64] and Gao [65], using Hagenauer's 1997 approach for computing the extrinsic LLR vector L_e given by (38), and CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by the authors with and without turbo equalization are compared: —●— CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by the authors after the first turbo equalization iteration; ...▲... CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by Yuan [64] and Gao [65] after at most 16 turbo equalization iterations; —◆— CA-SCL polar decoding with extrinsic LLR generation according to the approach proposed by the authors after at most 16 turbo equalization iterations.

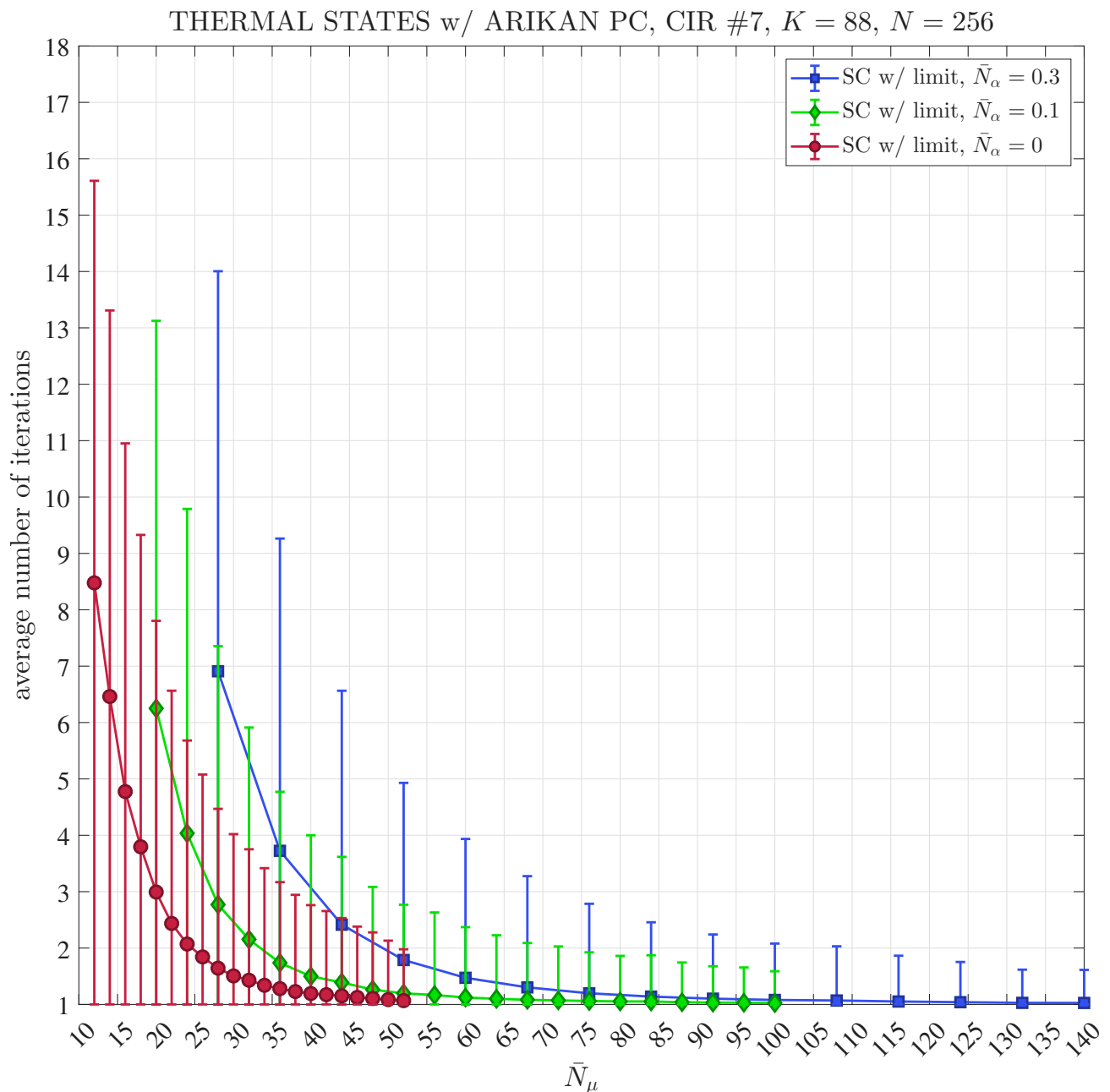


Figure 14. Detail of the average number of turbo equalization iterations together with the associated standard deviation versus the average number $\bar{N}_\mu$ of emitted signal photons per message bit for the transmission over the two-paths quantum channel CIR #7, having the channel coefficients s_0 equal to $8/9$ and s_1 equal to $1/9$, and with the average number $\bar{N}_\alpha$ of thermal noise photons taken from the set $\{0, 0.1, 0.3\}$; configuration C1 and SC polar decoder with extrinsic LLR generation according to the approach proposed by the authors employing either an LLR limit of 29.7 from the basis of the simulations.

Furthermore, the narrowband approach investigated in this study supports the joint operation of a dynamically configurable number of distinct modes, offering exceptional flexibility in resource allocation. This capability enables, for instance, a flexible multicarrier framework with dynamically adjustable bandwidth assignment applying LED based quantum transmitters, facilitating efficient adaptation to changing network conditions and user requirements.

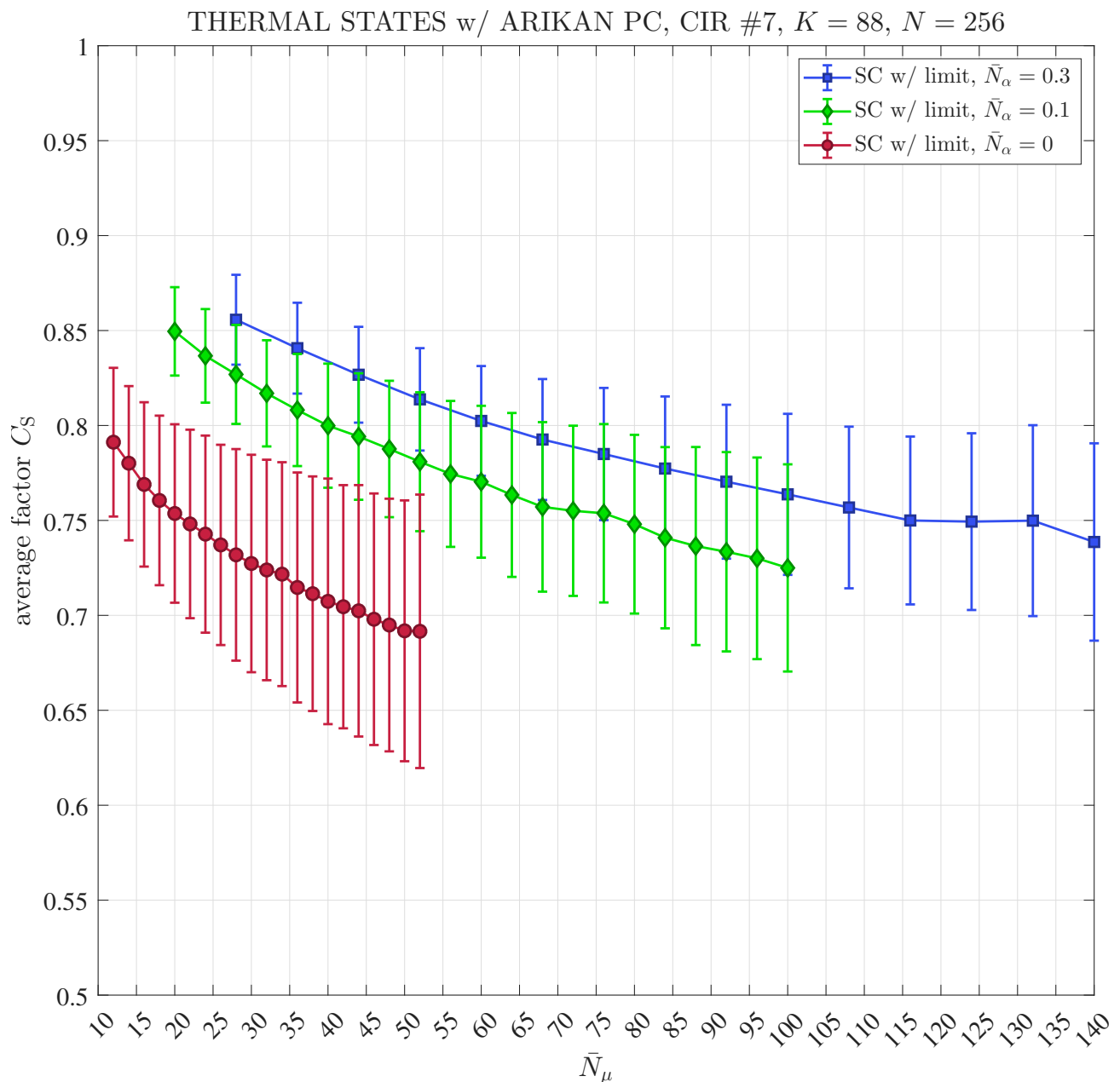


Figure 15. Detail of the average factor C_S equal to $L_{in}L_c^T/\|L_c\|^2$ together with the associated standard deviation versus the average number $\bar{N}_\mu$ of emitted signal photons per message bit for the transmission over the two-paths quantum channel CIR #7, having the channel coefficients s_0 equal to $8/9$ and s_1 equal to $1/9$, and with the average number $\bar{N}_\alpha$ of thermal noise photons taken from the set $\{0, 0.1, 0.3\}$; configuration C1 and SC polar decoder with extrinsic LLR generation according to the approach proposed by the authors employing either an LLR limit of 29.7 form the basis of the simulations.

In summary, the narrowband approach adopted in this study constitutes a viable alternative and should therefore be included in future detailed comparative investigations, especially when cellular network level considerations are taken into account. A direct comparison with classical LED based OOK transmission using a positive intrinsic negative (PIN) photodiode receiver under identical channel conditions is beyond the scope of this work and remains a topic for future investigation. Finally, a comprehensive assessment of cellular optical communication networks, whether quantum or classical, lies beyond the scope of the present manuscript.

Table 2. Estimated average number $\bar{N}_\mu$ of emitted signal photons per message bit required to obtain $P_b = 10^{-3}$ and $P_{\text{block}} = 10^{-2}$ associated with the application of CA-SCL polar decoding; — means “ L_e generation is not relevant”; the best performance for a given configuration is printed in **red color**.

	R_c	L_e Generation	Iteration	$\bar{N}_\alpha = 0$	$\bar{N}_\alpha = 0.1$	$\bar{N}_\alpha = 0.3$
BER	$(88 - 24)/256 = 1/4$	—	1	$\bar{N}_\mu \approx 26.5$	$\bar{N}_\mu \approx 37.7$	$\bar{N}_\mu \approx 55.1$
BER	$(88 - 24)/256 = 1/4$	(38), cf. [64,65]	≤ 16	$\bar{N}_\mu \approx 23.2$	$\bar{N}_\mu \approx 32.3$	$\bar{N}_\mu \approx 47.6$
BER	$(88 - 24)/256 = 1/4$	(42)	≤ 16	$\bar{N}_\mu \approx 24.0$	$\bar{N}_\mu \approx 33.9$	$\bar{N}_\mu \approx 50.9$
BLER	$(88 - 24)/256 = 1/4$	—	1	$\bar{N}_\mu \approx 22.4$	$\bar{N}_\mu \approx 31.6$	$\bar{N}_\mu \approx 46.5$
BLER	$(88 - 24)/256 = 1/4$	(38), cf. [64,65]	≤ 16	$\bar{N}_\mu \approx 19.9$	$\bar{N}_\mu \approx 27.7$	$\bar{N}_\mu \approx 40.7$
BLER	$(88 - 24)/256 = 1/4$	(42)	≤ 16	$\bar{N}_\mu \approx 20.4$	$\bar{N}_\mu \approx 28.8$	$\bar{N}_\mu \approx 42.6$
BER	$(204 - 24)/512 \approx 0.352$	—	1	$\bar{N}_\mu \approx 24.0$	$\bar{N}_\mu \approx 33.5$	$\bar{N}_\mu \approx 49.8$
BER	$(204 - 24)/512 \approx 0.352$	(38), cf. [64,65]	≤ 16	$\bar{N}_\mu \approx 22.3$	$\bar{N}_\mu \approx 28.8$	$\bar{N}_\mu \approx 42.6$
BER	$(204 - 24)/512 \approx 0.352$	(42)	≤ 16	$\bar{N}_\mu \approx 20.8$	$\bar{N}_\mu \approx 29.0$	$\bar{N}_\mu \approx 43.7$
BLER	$(204 - 24)/512 \approx 0.352$	—	1	$\bar{N}_\mu \approx 20.9$	$\bar{N}_\mu \approx 29.7$	$\bar{N}_\mu \approx 42.9$
BLER	$(204 - 24)/512 \approx 0.352$	(38), cf. [64,65]	≤ 16	$\bar{N}_\mu \approx 18.9$	$\bar{N}_\mu \approx 24.9$	$\bar{N}_\mu \approx 37.0$
BLER	$(204 - 24)/512 \approx 0.352$	(42)	≤ 16	$\bar{N}_\mu \approx 17.7$	$\bar{N}_\mu \approx 25.3$	$\bar{N}_\mu \approx 37.8$
BER	$(280 - 24)/512 = 1/2$	—	1	$\bar{N}_\mu \approx 64.0$	(not simulated)	$\bar{N}_\mu \approx 145.1$
BER	$(280 - 24)/512 = 1/2$	(38), cf. [64,65]	≤ 16	$\bar{N}_\mu \approx 53.2$	$\bar{N}_\mu \approx 81.4$	$\bar{N}_\mu \approx 125.6$
BER	$(280 - 24)/512 = 1/2$	(42)	≤ 16	$\bar{N}_\mu \approx 51.8$	$\bar{N}_\mu \approx 77.8$	$\bar{N}_\mu \approx 120.5$
BLER	$(280 - 24)/512 = 1/2$	—	1	$\bar{N}_\mu \approx 51.8$	$\bar{N}_\mu \approx 78.0$	$\bar{N}_\mu \approx 118.7$
BLER	$(280 - 24)/512 = 1/2$	(38), cf. [64,65]	≤ 16	$\bar{N}_\mu \approx 45.3$	$\bar{N}_\mu \approx 63.7$	$\bar{N}_\mu \approx 95.2$
BLER	$(280 - 24)/512 = 1/2$	(42)	≤ 16	$\bar{N}_\mu \approx 40.6$	$\bar{N}_\mu \approx 60.4$	$\bar{N}_\mu \approx 90.3$

5. Conclusions

Motivated by the trend toward carrier frequencies far beyond 300 GHz, this work investigated LED based wireless quantum optical transmission as a practically attractive concept for future short-range wireless communication systems. Since the uncoded performance of such systems is limited, sophisticated channel coding techniques, especially polar coding and turbo equalization were applied to improve the transmission reliability. The results show that the proposed approach can reduce the average number $\bar{N}_\mu$ of emitted signal photons by several orders of magnitude, while maintaining comparatively simple transmitter and receiver structures. Topics such as entanglement, quantum teleportation, and nonclassical light sources, e.g., squeezed light, were intentionally excluded to focus on practically suitable wireless communication architectures. Overall, the presented results indicate that LED based wireless quantum optical transmission combined with advanced channel coding is a promising candidate for robust and economically viable future wireless communication systems.

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