

Article

Design of Highly Compact Catadioptric Freeform Linear-Scanning Optical Systems

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Abstract

Laser scanning systems are adopted for diverse applications in imaging, printing, and material processing, and there is a growing trend toward volume miniaturization without compromising optical performance. In this work, a highly compact optical configuration is proposed by combining catadioptric architecture with freeform optics. A multi-stage design method, from starting point acquisition to fine-tuning, is proposed, with particular attention to concurrently controlling physical layout and optical performance. Two systems were designed, demonstrating an average RMS wavefront error below 0.006λ and a linearity error under 0.3% over a 216 mm scanning width. The spot size deviation is below $\pm 9\%$, and the position deviation is kept within half the diffraction-limited spot diameter over a ± 2 mm depth of field. Compared to a conventional freeform lens design, the proposed systems reduce the total length by over 37.7%. The significant reduction in size achieved paves the way for next-generation portable and embedded laser scanning devices.

Keywords: freeform optics; laser scanning system; lens design

1. Introduction

Laser scanning units (LSUs) serve as critical optical components in high-precision laser printing platforms, additive manufacturing systems, and laser micromachining workstations [1–8]. Their performance hinges on generating a linear field-of-view (FOV) while maintaining high spot quality and a good f - θ scan relationship across large scanning widths [6–8]. To achieve these requirements, the scanning lenses in LSU systems inherently require large linear FOVs, relatively small apertures, and long focal lengths. To overcome the problem of field curvature and maintain a good scan linearity in a large FOV, freeform optics is adopted in the optical design. In terms of lens design, the introduction of freeform optics breaks the limitation of rotational symmetry and delivers superior aberration correction, significantly simplifying the optical structure of f - θ lenses and reducing the number of lenses required. Park et al. designed and tested the fabrication performance of a freeform f - θ lens in 2021 [6]. Wang et al. reported a point-by-point design for a double freeform telecentric f - θ lens, which could be applied to laser measurement or laser detection [7]. In 2025, Xu et al. proposed a freeform singlet f - θ lens for 3D printing LSUs [8]. Although superior aberration correction enables one- or two-element designs, freeform lens systems still suffer from long track lengths when large scanning widths are required. Notably, the long back focal length remains the dominant factor in total LSU system length, which prevents the



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miniaturization of related devices. In parallel with the development of freeform optics, metasurfaces have emerged as a complementary but promising paradigm for optical field manipulation and beam scanning [9–11], which may introduce new optical configurations for compact laser scanning systems. Despite their advantages in wavefront shaping and integration, their application in high-precision scanning systems is still constrained by efficiency and fabrication challenges. In this work, we focus on improving freeform design, which provides a practical and manufacturable solution for achieving a compact LSU with good performance.

Many efforts have been devoted to reducing the total track lengths in optical design. Off-axis reflective systems that fold the optical path have shown wide applications in improving system compactness [12–19]. Regarding the linear FOV, the tangential plane is defined as the cross-section containing principal rays of different field angles. Yang et al. designed linear field-of-view freeform two-mirror systems with a zig-zag optical path inside the tangential plane, which greatly reduces the total system length [15]. In 2020, Chen et al. proposed the design of a $\pm 10.4^\circ$ linear FOV freeform mirror scanning system through a multiple surfaces expansion and mixing method [16]. In 2022, Xu et al. showed the capability of distortion correction for large oblique fields of freeform mirror system designs for linear FOV cameras [17]. Although such a folded optical arrangement has shown its ability in reducing system length, the total width in the tangential plane will experience a dramatic increase and physical constraints prevent such designs from large linear FOV conditions. Perpendicular optical path folding (relative to the tangential plane) avoids such excessive width. Meng et al. demonstrated this approach with a $30^\circ \times 1^\circ$ FOV three-mirror anastigmat system [18], and later proposed an $80^\circ \times 4^\circ$ ultra-wide FOV design in 2019 [19]. Nevertheless, in LSUs, mirror tilting or displacement for obscuration avoidance complicates assembly and expands space perpendicular to scanning angles, which is not suitable for the intended LSU application. Catadioptric optics adopts both reflective and refractive surfaces in the optical system that shows the potential for high compactness. This approach is well adopted in head-mounted displays with a large FOV by using a refractive combiner and a lens group [20]. And ultra-wide-angle lens prototypes have been demonstrated with such optical configuration [21,22]. In recent years, pancake optical lenses, a special kind of catadioptric lens that adopts back-and-forth reflections between internal lens elements to decrease the back focal length, have appeared in eyepieces for compact virtual reality applications [23–27]. Notably, Hou et al. prototyped a pancake eyepiece with a total length of 20 mm, enabling a glass-like, near-eye-display system with a FOV of 96° [27]. Pancake catadioptric configuration may maintain a co-axial setting for the LSU's scanning lens, which helps to limit the length, width, and thickness of the system simultaneously. But the drawback of the pancake configuration lies in the low light efficiency and the stray light issue due to the use of the half-transmissive film and the polarization beam splitter coating. This makes them unsuitable for energy-sensitive applications like laser printing or laser fabrication. Consequently, a solution simultaneously achieving LSU size reduction and high light efficiency remains undiscovered.

To address these challenges, this paper proposes a partial-mirror-coated catadioptric optical structure that eliminates beam-splitter losses while enabling improved system compactness. Section 2 introduces the novel optical configuration and design methodology for this highly compact catadioptric linear-scanning system. Section 3 then presents two implemented designs with performance analysis, including a comparative benchmarking against the conventional freeform lens solution. And conclusions are drawn in Section 4.

2. Design Method

2.1. High-Compactness System Model

An LSU comprises four major components: a laser source, an auxiliary lens for light-source light shaping, a scanning element, and an f - θ lens to focus the ray bundle on the target scanning plane. The laser source often adopts a laser diode (LD), and the scanning element changes the ray bundle angle, which is often implemented by a rotating polygon mirror or a galvanometer. The f - θ lens establishes a linear relationship between scanning angle (θ) and image point height (y_i). This relationship ensures uniform linear velocity of the scanning spot when the scanning element maintains constant angular velocity. Crucially, f - θ lenses provide field-independent F/# uniformity, resulting in consistent spot quality across the scan field [8].

With the help of freeform optics, the number of elements in f - θ lens systems can be reduced to one or two. However, the total length from the scanning element to the scanning plane is still long. According to the principle of f - θ scanning, the focal length, f , is determined if both the maximum scanning angle (θ_{max}) and the scanning width (w_d) are provided. And it is difficult to reduce the total length with one or two lenses while maintaining the position of the entrance pupil and the f - θ imaging property. In this paper, we propose a pair of partially mirror-coated lenses to form a catadioptric system to fold the optical path, as illustrated in Figure 1. This partially mirror-coated surface may be potentially fabricated using the physical masking deposition method [28], in which a custom shield covers the uncoated zones during vacuum coating, thereby producing a sharp, straight boundary on the lens surface. The f - θ lens, as shown in the dashed box in this figure, consists of two elements, with its four surfaces denoted as S1 to S4. The side view of the f - θ lens in Figure 1b shows the transmissive and reflective ray paths on S2 and S3 with the partially mirror-coated surface shown in Figure 1c. In this way, the system is folded to achieve a smaller total length with negligible energy loss or stray light issues commonly encountered in pancake lens designs using half-transmissive film and a quarter-wave plate [27]. To guarantee the fabrication and smoothness of the partial-mirrored surfaces, the reflective and the transmissive regions share the same surface expression.

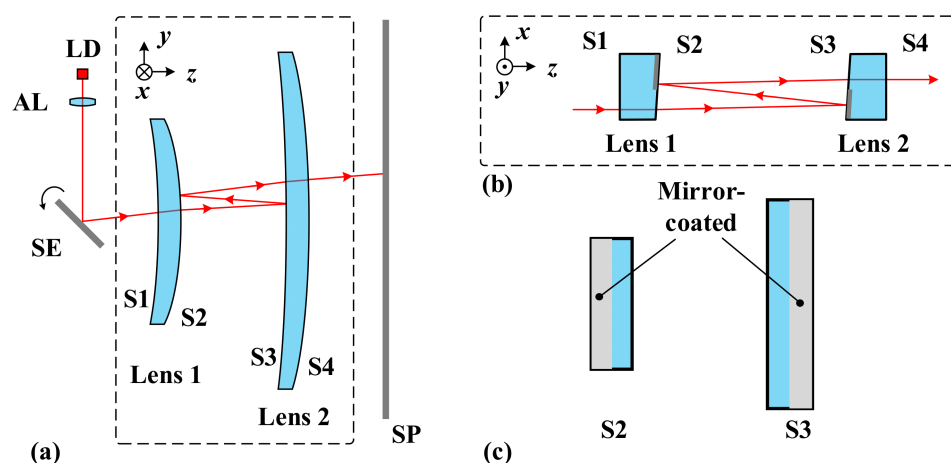


Figure 1. The optical configuration of the high-compactness catadioptric linear-scanning freeform optical system. (a) Optical layout. LD: laser diode, AL: auxiliary lens, SE: scanning element, SP: scanning plane. (b) Side view of the ray path within the f - θ lens. (c) Front view of the partially reflective-coated surfaces.

To ensure the separation of the optical regions on the partially mirror-coated surface, a basic model of the lens pair is established as illustrated in Figure 2a. In this model, all optical surfaces are assumed to be planar and a pair of tilted surfaces with the same

tilt angle is adopted to fold the optical path. The two prisms are of the same material. A parallel ray bundle first passes through the first prism and is reflected by the front surface of the second prism (S3). It then gets reflected by the rear surface of the first prism (S2) and finally passes through the second prism. Due to the same tilt angle on the prism pair, the resultant ray bundle is kept parallel, and the ray bundle experiences a transverse displacement on the same partially reflective-mirrored surface, d_t , in the vertical direction that is given by

$$d_t = 2 \cos^2 \alpha \cdot \tan(\arcsin(n_1/n_0 \cdot \sin \alpha)) \cdot t \approx 2n_1/n_0 \cdot (\alpha - \alpha^3) \cdot t \quad (1)$$

where α denotes the tilt angle, t denotes the axial distance between the two partially reflective surfaces, and n_0 and n_1 denote the refractive index of the air and the prism pair, respectively. An approximation is made with the assumption that α is a small angle, and thus $\sin \alpha \approx \tan \alpha \approx \alpha$, omitting the α^2 and higher terms in the equation. Figure 2c illustrates the transverse displacement with respect to different t and α values, and it can be noted that millimeter-level displacement can be achieved with a small tilt angle and an axial distance of 10 mm or greater. To briefly assess the reduction in total length and the improvement of compactness, a first-order analysis with the assumption of thin-lens is performed, as shown in Figure 2b. In this condition, the thickness of each lens and the off-axis tilt of the surfaces are both omitted. Assume the target focal length of the co-axial, two-lens system, f , and the focal length of the first lens, f_1 , are known, and the physical distance between the two thin lenses, d , is constant. Then, the focal length of the second lens is derived by focal length match. Hence, the back focal length, f_{bfl} , can be determined in Equation (2), as

$$f_{bfl} = \frac{f(f_1 - \Delta)}{f_1} \quad (2)$$

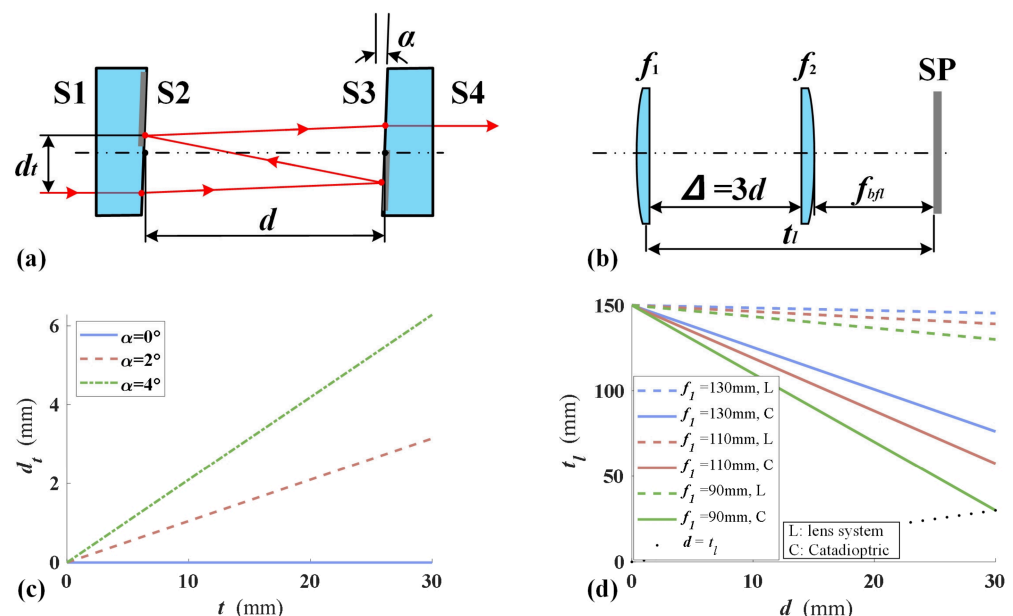


Figure 2. The basic model for analysis. (a) The basic model for ray translation with the prism pair. (b) The model for first-order analysis of the back focal length. (c) Translation distance d_t corresponding to different d and α . (d) System length from the first lens to the scanning plane, t_l , with respect to different d and f_1 .

The imposing of partially refractive-coated surfaces is equivalent to an increase of Δ . And it is noted that Δ is enlarged by 3 times in the proposed catadioptric sys-

tem, which brings a remarkable decrease in back focal length. Figure 2d shows the total lengths from the first lens to the scanning plane (i.e., $d + f_{bfl}$) with respect to d and f_1 , which indicates a dramatic decrease in system length with a catadioptric configuration (solid curves) compared with the lens system (dashed curves). It should be noted that a positive back focal length is needed to form a real scanning image, indicating that d cannot be excessively large and only the line segments above $t_1 = d$ can be physically feasible. As stated above, the proposed catadioptric optical configuration has the potential to both shorten the total length and separate the optical regions on lenses, which avoids the energy loss from using a semi-reflective, semi-transmissive film.

2.2. Design Strategy

To achieve the design of the proposed catadioptric lens, the following design strategy is adopted. As illustrated in Figure 3, the design strategy contains four stages: starting point acquisition, preliminary optimization, fine-tuning optimization, and design result analysis.

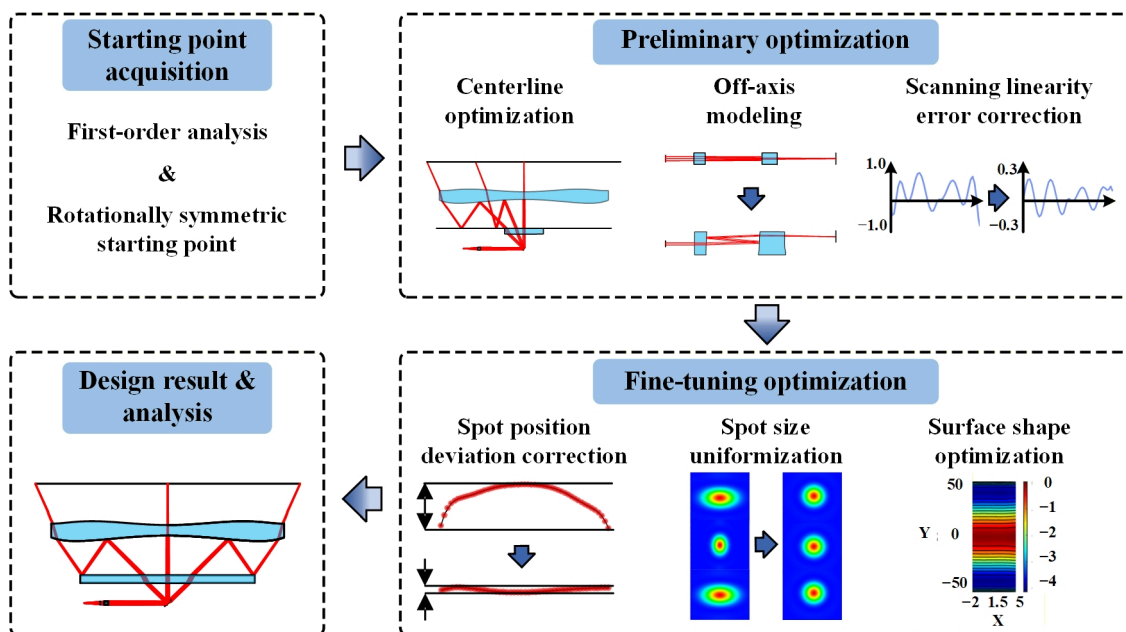


Figure 3. Flowchart of the design process.

The design process starts with a co-axial configuration. To obtain a good starting point, first-order analysis is performed with the assumption of a rotationally symmetric system. For the rough analysis of the first-order properties, the effect of surface tilt and the different regions on the surfaces are omitted. The transfer matrix ray-tracing method [29] is adopted for paraxial ray-tracing analysis. And the transfer matrix of the whole system can be obtained by

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ (n-1)/R_4 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & t_2/n \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ (1-n)/R_3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -2/R_2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 2/R_3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ (n-1)/R_2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & t_1/n \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ (1-n)/R_1 & 1 \end{bmatrix} \quad (3)$$

where M is the transfer matrix of the whole catadioptric system, R_i denotes the curvature radius of the i th surface, t_i denotes the physical distance from surface S_i to S_{i+1} , and the refractive index of both lenses is n . From the above equation, the effective focal length of the catadioptric system, f' , and the total length, d_l , can be obtained as

$$\begin{cases} f' = -1/M_{21} \\ d_l = t_1 + d + t_2 - M_{11}/M_{21} \end{cases} \quad (4)$$

By constraining the effective focal length and achieving a small d_l with a large absolute value of R_1 to R_4 , the structure parameters can be determined either by manual adjustment or by function optimization methods such as those proposed in [30]. All surfaces are spherical and the system is co-axial at this stage. When modeling the LSU system, the long side of the lens is set along the y -axis, which also corresponds to the M -direction of the scanning system. For each optical surface, its optical axis is along the z -direction, and the x -direction is defined based on the established Cartesian coordinate system, and it aligns with the S -direction. Different scanning angles are modeled by multi-configuration settings with a tilted mirror, and the system is to be optimized to roughly meet the f - θ linearity requirement, and the starting point for optimization is thus obtained.

In the preliminary optimization process, the centerlines of the lens in the yoz plane are obtained. By only considering the meridional geometric aberration, the co-axial system's centerline is first optimized. In the preliminary optimization stage, the incoming rays for the f - θ lens are simplified as a collimated ray bundle. To further decrease the image distance, the object distance can then be set to a finite value, producing a slightly convergent incoming ray bundle. Surface pick-up settings are used to ensure a uniform surface expression for the transmissive and reflective regions of the same partially reflective coated surfaces. A straight interface is designed to separate the transmissive and reflective regions. By adopting a surface tilt term on S3, the reflected rays are tilted so that they are incident on the upper portion of S2, where the x -coordinates are larger than those of all transmitted rays on S2. The x -coordinates of the extreme feature rays on S2 are controlled in the design phase to achieve a straight interface while preserving sufficient manufacturing margin. Similarly, such an interface can be achieved on S3.

A surface upgrade design strategy is utilized in the design, where the spherical surfaces are first upgraded to aspheric or odd-aspheric surfaces and then converted to freeform surfaces. In this paper, two freeform surface expressions are used, namely

$$z(x, y) = \frac{c(x^2 + y^2)}{1 + \sqrt{1 - (1 + k)c^2(x^2 + y^2)}} + \sum_{i=2}^8 \sum_{j=2}^{10} P_{i,j} x^i |y|^j \quad (5)$$

and

$$\begin{cases} z(x, y) = \frac{C_1 |y|^2}{1 + \sqrt{1 - C_1^2 |y|^2}} + \sum_{n=1}^{10} A_n |y|^n + \frac{C_2 |x|^2}{1 + \sqrt{1 - C_2^2 |x|^2}} + \sum_{i=2}^8 \sum_{j=2}^{10} D_{i,j} x^i y^j \\ C_2 = C \left(1 + \sum_{n=2}^{10} B_n |y|^n \right) \end{cases} \quad (6)$$

Equation (5) is based on the standard XY-polynomials but the absolute values of y are used, which makes it possible to be directly upgraded from an odd-term asphere type [29] without any fitting error, whereas Equation (6) is modified from the Freeform-3D expression that is widely used in the designs of f - θ lenses for laser printers [5], with additional polynomials of $\{D_{i,j}\}$ for aberration correction in the off-axis regions. When the whole optical model is obtained and freeform surfaces are applied, the fine-tuning optimization stage takes place. Since the off-axis region on surfaces is used, a shape change in the x -direction will cause a change in image height in the y -direction, which means linearity error will change during the spot position deviation correction process and the spot uniformity correction process. Hence, an optimization with all the variables and all the constraints is performed at the end of the fine-tuning stage to obtain the final design.

2.3. Design Considerations and Constraints

To achieve optimal optical performance, various constraints need to be added to the optimization process, including physical constraints, scanning linearity, diffractive spot size uniformity, and spot deviation.

(i) Physical constraints

Physical constraints are adopted to achieve optical region separation of the reflective and refractive areas and maintain a proper shape for the f - θ lens. Tilt angles in the β direction (tilting around the y -axis) separate the optical regions. Feature rays corresponding to the uppermost and lowest points in the x -direction are traced and their heights on S2 and S3 are constrained to maintain the width of the gap. Lens shape control is imposed in the y - and x -directions. To maintain manufacturability, 1st- and 2nd-order surface derivatives are constrained on the centerline in the y -direction. In the x -direction, surface sag is controlled and surface extreme points are avoided by controlling derivatives in this direction. The maximum optical path in the material is controlled to avoid excessively thick lens elements, especially at edge positions.

(ii) Scanning linearity

Under ideal conditions, the scanning length on the image plane is proportional to the rotated angle. The difference between the actual scanning length, δL , and the ideal length, δL_i , over a certain rotated angle, $\delta \alpha$, is denoted as

$$RE = \frac{\delta L - \delta L_i}{\delta L_i} \times 100\% \quad (7)$$

where RE denotes the relative linearity error.

(iii) Diffractive spot size uniformity

Diffractive spot size is denoted by the $1/e^2$ threshold in energy and is evaluated in the M - and S - directions, respectively. For a certain design, the diffractive spot size is determined by the $F/\#$ when geometric aberration is well corrected. The system is designed to have an oval aperture whose sizes are utilized to balance the diameter of the spot size in the two directions. The f - θ relationship in the y -direction inherently brings good spot size uniformity in the M -direction, but the diffractive spot size in the S -direction may vary significantly over the scanning width. We maintain a stable $F/\#$ in the S -direction across scanning angles by controlling the lens' local curvatures. As illustrated in Figure 4, for a pair of conjugate object and image points, the shape of the front and rear surface of the lens can result in different $F/\#$ values in the image space. The biconvex lens converges the ray bundle twice with its two surfaces and results in a greater $F/\#$ at the image point, but the meniscus lens first diverges the ray and then converges it, making a smaller $F/\#$. This difference caused by the lens shape will result in a change in diffractive spot size. Hence, the x -curvature distribution of the lens along the y -direction is controlled for uniformity improvement. Feedback in constraints is imposed in optimization to balance the diffractive spot diameters in the S -direction, and constraints on the curvature in the x -direction at the center and edge points in the y -direction are controlled.

(iv) Spot position deviation

Like keystone distortion [29], the scanned line may exhibit undesired curvature in the S -direction. This deviation is controlled by constraints on image point positions in the S -direction. Odd terms of x -components in surface coefficients are added into variable sets for image position control. In the proposed system, S2 and S3 are tilted surfaces that lack symmetry in the x -direction and can therefore be used to correct spot deviation without

introducing assembly difficulties. Additionally, since the incident height in the x -direction on S4 tends to vary across the scanning width, odd-order terms of the X components in S4 are also employed for spot deviation correction.

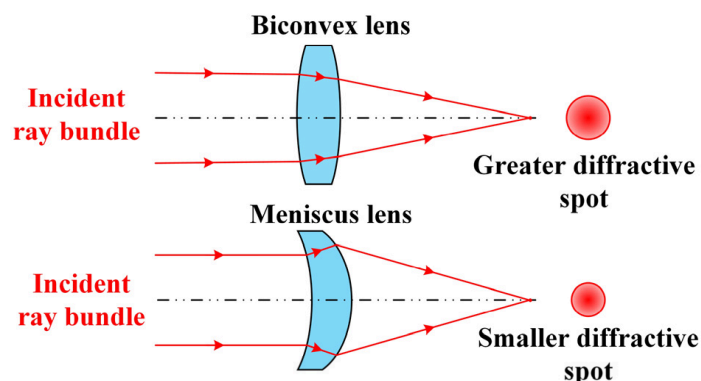


Figure 4. Diffractive spot size difference caused by the shape of the lens.

As the system is not symmetric in the S -direction and the incident angles of different scanning positions are not 0 in the S -directions, defocus may cause spot position deviation in this direction. If the maximum defocus distance is $\pm\delta$, and the allowable deviation limit is $\pm e_S$, then the incident angle deviation in the S -direction can be approximated by e_S/δ . Particularly, when $\delta = 2$ mm and $e_S = 30$ μm , such angle deviation needs to be controlled within 0.859° in optimization. If a residual line curve exists in the nominal plane, a stricter constraint is needed for defocus spot position deviation control.

3. Optical Designs and Analysis

Following the previous design strategy, two highly compact catadioptric f - θ lens systems were designed: one featuring a planar symmetric lens for a scanning galvanometer, and the other employing a fully asymmetric lens to fit a rotating polygon mirror.

3.1. Planar Symmetric Lens System with the Galvanometer

The first design example is a linear f - θ scanning system consisting of a laser diode, an auxiliary singlet lens, a scanning galvanometer, and a catadioptric f - θ lens. A cover glass is also placed behind the lenses to prevent contaminants in the scanning plane from reaching the optical system. The system specifications are provided in Table 1, and the layout and key performance are illustrated in Figure 5. The structural data of the design result can be found in the Supplementary Materials (Tables S1–S3). The total rotation angle of the galvanometer is 46° . It should be noted that the scanning effective focal length (EFL) of the f - θ lens, defined as $y = f \cdot \theta$, may differ from the paraxial focal length, particularly when the incident beam is designed to be slightly convergent. The EFL value listed in Table 1 refers to the former. Since the rotation center lies on the galvanometer mirror surface, the system exhibits planar symmetry along the y -direction. In this design, reflective surface S2 was initially defined as planar, while S3 was made concave to reduce the optical region size on S2. During optimization, S2 was upgraded to the XY-polynomial form, retaining only terms from x^2 to x^8 to maintain no curvature in the y -direction. In the final design, S1 is an odd-aspheric surface, S2 and S3 are XY-polynomial surfaces, and S4 is a segmented XY-polynomial surface with mirror symmetry about the xoz -plane. The total length from the galvanometer center to the scanning plane is 97.1 mm. All surfaces are planar symmetric about the xoz -plane. The lens material has a refractive index of 1.5406 at 788 nm wavelength, and the cover glass has a refractive index of 1.5110. The angle of incidence at the edge of the scanning plane is 24.28° . The average diffractive spot

diameter is $80.2\ \mu\text{m}$ in the S -direction and $73.3\ \mu\text{m}$ in the M -direction. The maximum spot size deviation is $2.64\ \mu\text{m}$ and $1.25\ \mu\text{m}$ in the S - and M - directions, respectively, indicating good size uniformity. The average RMS wavefront error over the scanning field is 0.002λ . The spot position deviation in the S -direction is $\pm 18.7\ \mu\text{m}$ over the $\pm 2\ \text{mm}$ depth of focus, corresponding to $\pm 23.3\%$ of the spot diameter in that direction. The scanning linearity error is controlled within 0.3% . Surfaces S2 and S3 incorporate a 2° beta tilt angle. The gap between the optical regions at the edges of the reflective and the transmissive parts is $2.26\ \text{mm}$ on S2 and $2.61\ \text{mm}$ on S3, respectively. The optical aperture on S1 is $33\ \text{mm}$ along the y -direction, with a planar surface stitched to the surrounding area to match the aperture of S2.

Table 1. System specifications for the first design.

Parameters	Value
Scanning EFL (mm)	134.52
F/#	58
Scanning angle ($^\circ$)	± 23
Scanning width (mm)	216
Wavelength (nm)	788

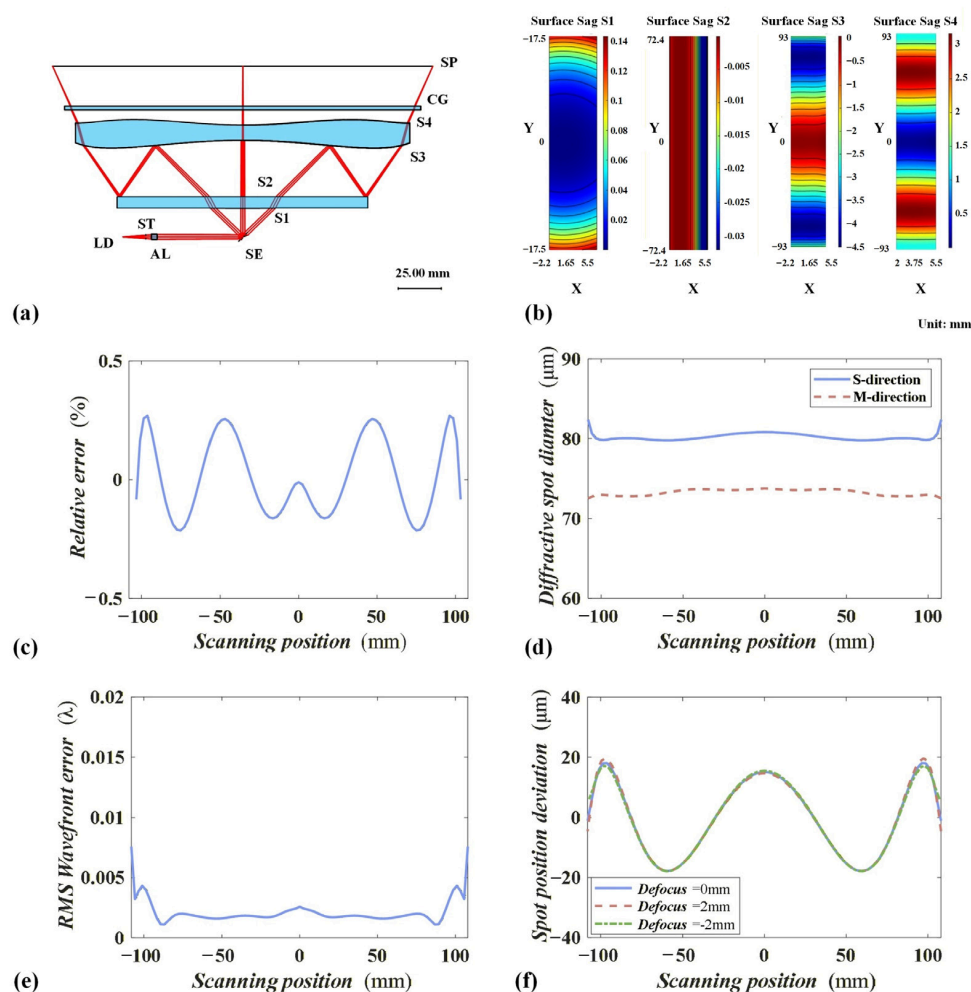


Figure 5. Design result of the first example. (a) Optical layout. (b) Surface sag of the f - θ scanning lens (note: unequal axis scaling). (c) Scanning linearity error. (d) Diffractive spot diameter. (e) RMS wavefront error. (f) The spot position deviation. LD: laser diode; ST: stop; AL: auxiliary lens; SE: scanning element; CG: cover glass; SP: scanning plane.

3.2. Asymmetric Lens System with the Rotating Polygon Mirror

The second design is a linear f - θ scanning system employing a polygon mirror, and its system specifications are listed in Table 2. As the center of rotation is offset from the mirror surface, the system exhibits no planar symmetry, necessitating the use of asymmetric surface types during optimization. Given that polygon mirrors typically have a facet width (in the x -direction) of approximately 1 mm, the auxiliary lens in this design is anamorphic, collimating the light in the y -direction while converging it in the x -direction to match the polygon mirror's geometry. The rotating polygon is square with a circumscribed circle diameter of 10 mm, and the total scanning angle used is 50° . The layout and key performance of the design are illustrated in Figure 6, and the structural data can be found in the Supplementary Materials (Tables S4–S6). The axial distance between the two lenses is 25 mm, and the catadioptric configuration results in a total track length of 85 mm from the incident point on the polygon mirror to the scanning plane. The linearity error is controlled within 0.3% over the entire 216 mm scanning width. The diffractive spot sizes are $77.2\ \mu\text{m}$ and $80.2\ \mu\text{m}$ in the M - and S -directions, respectively. The spot size deviations are $\pm 3.1\ \mu\text{m}$ (M -direction) and $\pm 7.0\ \mu\text{m}$ (S -direction), both within specification. The angle of incidence at the edge of the scanning plane is 36.7° . In this design, S2 and S3 are both tilted by 2.5° , and S4 is displaced in the x -direction to achieve an off-axis optical region. Surface sag contours are plotted in Figure 6b. To minimize complexity and limit the number of surface coefficients in Equation (6), surfaces with planar symmetry are preferred. Accordingly, S1, S3, and S4 are defined by Equation (6) with yo -plane symmetry. To better control spot position deviation in the x -direction, S2 adopts an XY-polynomial surface that is asymmetric in the x -direction but remains symmetric about the xoz plane. Thus, each surface is planar symmetric about either the xoz plane or the yo -plane if displacements or angular tilts are omitted. The average RMS wavefront error over the scanning field is 0.006λ . The spot position deviation is $\pm 16.7\ \mu\text{m}$ over the $\pm 2\ \text{mm}$ depth of field, corresponding to only $\pm 20.8\%$ of the diffractive spot size, which results in negligible mismatch in the scanning pattern. The gap between the optical regions on S2 and S3 is 1.86 mm and 1.11 mm, respectively. Under the condition that no coatings are applied to the lens surfaces except for the partially mirror-coated regions, and assuming a 90% reflectivity for all reflective surfaces, the simulated system transmittance is 54.6% at the center and 52.8% at the edge, which is much higher than the theoretical maximum of the pancake lens type (25%).

Table 2. System specifications for the second design.

Parameters	Value
Scanning EFL (mm)	123.76
F/#	58
Scanning angle ($^\circ$)	± 25
Scanning width (mm)	216
Wavelength (nm)	788

3.3. Comparison with the Single Freeform Lens Design

A freeform lens design was implemented using the same specifications outlined in Section 3.2, with the design result illustrated in Figure 7a. A comparative analysis between the proposed catadioptric lens and the freeform lens design is presented in Figure 7b, evaluating wavefront error, scanning linearity, and total length to cover both optical performance and compactness. The freeform lens design yields an average wavefront error of 0.0119λ and a maximum linearity error of 0.3%, with a total system length of 136.46 mm. In comparison, the second design achieves comparable scanning linearity and superior wavefront error correction, while notably reducing the total length by 37.7%. Thus, the

proposed optical configuration significantly improves system compactness while maintaining comparable optical performance. Compared with the freeform LSU design reported in Ref. [31], the proposed design results also show smaller wavefront errors across the scanning plane, comparable linearity error, and a 43% reduction in total length.

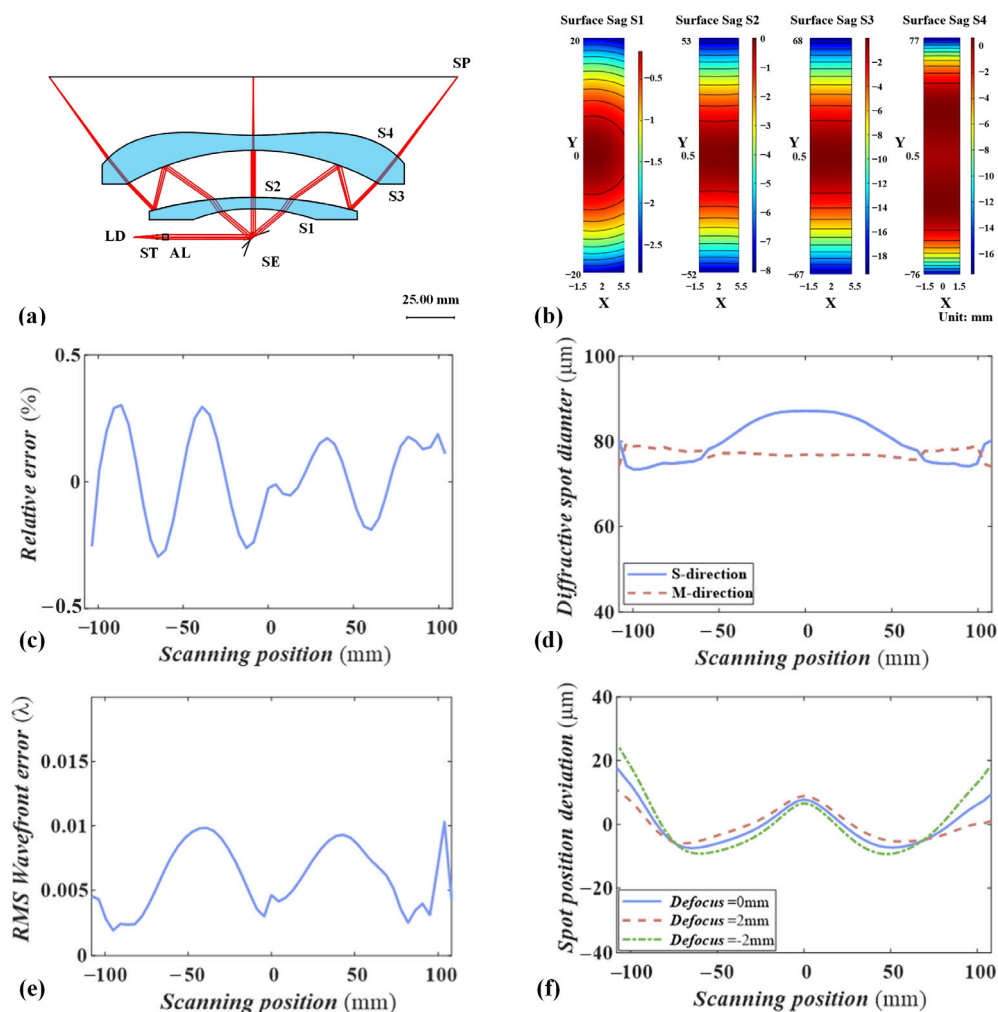


Figure 6. Design result of the second example. (a) Optical layout. (b) Surface sag of the f - θ scanning lens (note: unequal axis scaling). (c) RMS wavefront error. (d) Linearity error and the spot position deviation. (e) Spot uniformity. (f) The spot position deviation. LD: laser diode; ST: stop; AL: auxiliary lens; SE: scanning element; SP: scanning plane.

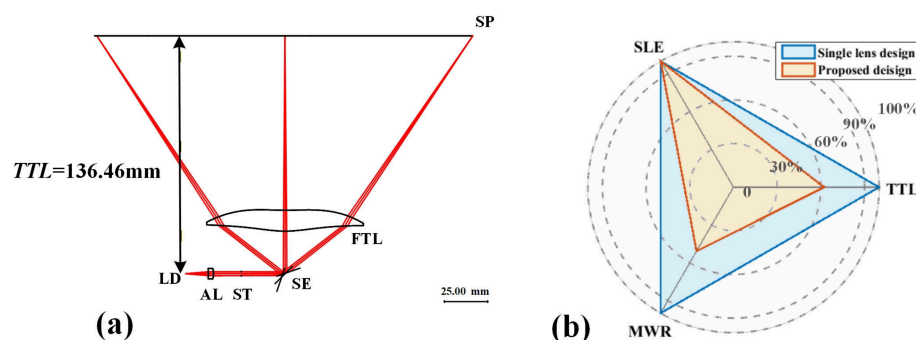


Figure 7. Comparison between the single freeform lens design and the proposed design. (a) Optical layout of the single lens design. (b) Comparison results in scanning linearity error (SLE), mean RMS wavefront error (MWR), and total length (TTL) from the ray incident point on SE to SP.

4. Conclusions

In this paper, a highly compact catadioptric linear f - θ scanning system is proposed. By adopting coating on partial regions of freeform lenses to form the catadioptric optical path, the total length can be reduced by 37.7% as compared to conventional freeform-lens-type design results. A multi-stage design method based on surface upgrade is developed and two examples are demonstrated, namely, a planar symmetric freeform system design for the galvanometer and a fully asymmetric freeform system design for the rotating polygon mirror. For both design results, the scanning linearity error is reduced to 0.3%, the spot position deviation is controlled within half of the diffractive spot size, and spot size non-uniformity is below $\pm 7.0\ \mu\text{m}$, indicating good optical performance along with high compactness. The proposed system may facilitate the development of next-generation miniaturized laser scanning equipment, with applications including laser printing, scanning displays, and 3D printing.

Supplementary Materials: The following supporting information can be downloaded at <https://www.mdpi.com/article/10.3390/photonics13070661/s1>: Table S1: Structure parameters for Design 1; Table S2: Off-axis settings for Design 1; Table S3: Additional surface coefficients for Design 1; Table S4: Structure parameters for Design 2; Table S5: Off-axis settings for Design 2; Table S6: Additional surface coefficients for Design 2.

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