

Article

Tunable Optical Spatial Differential Operations via Photonic Spin Hall Effect on Hexagonal Boron Nitride

Wenhao Xu ^{1,*} , Yunlan Zuo ^{1,2}, Shiyang Zhang ^{1,2}, Qianze Li ¹ and Lan Xu ^{1,2,*} 

¹ School of Physics and Chemistry, Hunan First Normal University, Changsha 410205, China; 201920111905@hunnu.edu.cn (Y.Z.); zhangsy@hnfnu.edu.cn (S.Z.); qianzeli@hnu.edu.cn (Q.L.)

² Key Laboratory of Low-Dimensional Quantum Structures and Quantum Control of the Ministry of Education, Hunan Normal University, Changsha 410081, China

* Correspondence: wenhaoxu@hnu.edu.cn (W.X.); xulan@hnfnu.edu.cn (L.X.)

Abstract

We propose a photonic spin Hall spatial differentiator based on a natural hexagonal boron nitride (hBN) interface in the mid-infrared Reststrahlen band. The optical-axis orientation is set by the crystal cut and lies in the incidence plane, while the operating mode is selected through the incidence angle, input polarization, and analyzer orientation. Near the ENZ-assisted Brewster-like condition, co-polarized reflection is strongly suppressed, enabling a high-contrast X-differentiation channel, whereas the Y-differentiation channel originates from first-order angular-spectrum spin-orbit coupling and remains background-free. At $\nu = 824.8 \text{ cm}^{-1}$ and fixed orientation $\phi = 20^\circ$, the effective Brewster-like minimum occurs at $\theta_B^{\text{eff}} \approx 69.28^\circ$ with $|r_p|^2 \approx 2.8 \times 10^{-4}$. Numerical simulations demonstrate switching among intensity-like imaging, X differentiation, and background-free Y differentiation. These results identify fixed-orientation hBN as a natural-crystal platform for polarization-selective mid-infrared optical spatial differentiation without subwavelength patterning.

Keywords: photonic spin Hall effect; spatial differentiation; hexagonal boron nitride; epsilon-near-zero; analog optical computing; hyperbolic materials

1. Introduction

Optical analog computing performs mathematical operations directly on optical fields, bypassing electronic conversion in selected image-processing tasks [1,2]. First-order spatial differentiation is a foundational operation for edge extraction, image enhancement, and real-time information processing [3–6]. Existing implementations employ metamaterials, metasurfaces, multilayer films, or interferometric systems that encode the required transfer function through engineered phase profiles, resonant nanostructures, or carefully designed optical paths [7–10]. Spin Hall-based and resonant-interface schemes provide another route to first-order spatial filtering [11–13]. These approaches are effective, yet they generally require nanofabrication or complex multilayer geometries.

Natural van der Waals crystals offer a complementary route in which the optical response is governed by intrinsic anisotropy rather than artificial patterning [14,15]. Hexagonal boron nitride (hBN) is particularly attractive in the mid-infrared band because it supports phonon-polaritonic Reststrahlen bands. In Reststrahlen band I ($760\text{--}825 \text{ cm}^{-1}$), hBN exhibits type-I hyperbolic dispersion with $\text{Re}(\epsilon_\perp) > 0$ and $\text{Re}(\epsilon_\parallel) < 0$ [16–19]. Near the upper edge of this band, $\epsilon_\parallel$ approaches an epsilon-near-zero (ENZ) condition, and the resulting anisotropic impedance can produce strong Brewster-like suppression



Received: 2 July 2026

Revised: 25 July 2026

Accepted: 28 July 2026

Published: 31 July 2026

Copyright: © 2026 by the authors.

Licensee MDPI, Basel, Switzerland.

This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

of the p -polarized reflection coefficient at an air–hBN interface [20]. The photonic spin Hall effect (PSHE) describes spin-dependent shifts arising from spin–orbit coupling and the angular dependence of Fresnel reflection [21–23]. As a representative manifestation of optical spin–orbit interaction, PSHE has been studied in metasurfaces, resonant interfaces, and topological photonic systems [24–27]. Recent extensions to optical microscopy, holography, and related photonic functionalities further highlight its potential for spatial-field manipulation [28–31]. In the angular-spectrum picture, the first-order dependence of the reflected field on the transverse wave vector can be mapped to a spatial transfer function, linking PSHE with optical spatial differentiation [11,12,32]. A practical differentiator requires both suppression of the zeroth-order background and a sufficiently large first-order slope in the spatial-frequency domain.

In this work, we present a photonic spin Hall spatial differentiator based on a fixed-orientation hBN interface. The optical-axis angle ϕ is set by the crystal cut and lies in the incidence plane; it is not treated as a continuously adjustable mechanical rotation. For the central incidence geometry considered here, the ordinary s wave and extraordinary p wave are decoupled at the central wave vector, so the Y -differentiation channel originates from the first-order angular-spectrum spin–orbit term and is background-free. The X -differentiation channel, in contrast, is governed by the ENZ-assisted Brewster-like response. Switching among intensity-like, X -differentiation, and background-free Y -differentiation modes is achieved by selecting the incidence angle and analyzer orientation. The calculations below focus on a representative fixed-orientation hBN sample at $\nu = 824.8 \text{ cm}^{-1}$ and $\phi = 20^\circ$.

2. Theoretical Model

2.1. Fixed-Orientation Dielectric Tensor of hBN

The dielectric response of hBN is described by two Lorentz oscillator components,

$$\varepsilon_{\perp}(\omega) = \varepsilon_{\infty,\perp} \frac{\omega_{\text{LO},\perp}^2 - \omega^2 - i\gamma_{\perp}\omega}{\omega_{\text{TO},\perp}^2 - \omega^2 - i\gamma_{\perp}\omega}, \quad (1)$$

$$\varepsilon_{\parallel}(\omega) = \varepsilon_{\infty,\parallel} \frac{\omega_{\text{LO},\parallel}^2 - \omega^2 - i\gamma_{\parallel}\omega}{\omega_{\text{TO},\parallel}^2 - \omega^2 - i\gamma_{\parallel}\omega}. \quad (2)$$

At the working wavenumber $\nu = 824.8 \text{ cm}^{-1}$, we use $\varepsilon_{\infty,\perp} = 4.52$, $\omega_{\text{LO},\perp} = 1610 \text{ cm}^{-1}$, $\omega_{\text{TO},\perp} = 1360 \text{ cm}^{-1}$, $\gamma_{\perp} = 5 \text{ cm}^{-1}$, $\varepsilon_{\infty,\parallel} = 4.95$, $\omega_{\text{LO},\parallel} = 825 \text{ cm}^{-1}$, $\omega_{\text{TO},\parallel} = 760 \text{ cm}^{-1}$, and $\gamma_{\parallel} = 5 \text{ cm}^{-1}$. These parameters give $\varepsilon_{\perp} \approx 7.39 + 0.01i$ and $\varepsilon_{\parallel} \approx -0.008 + 0.20i$. The material is therefore in the type-I hyperbolic regime, with $\text{Re}(\varepsilon_{\perp}) > 0$ and $\text{Re}(\varepsilon_{\parallel}) < 0$, while the parallel component is close to the epsilon-near-zero (ENZ) condition.

For an hBN sample with a fixed optical-axis orientation in the x – z plane, the unit vector along the optical axis is

$$\mathbf{a} = (\sin \phi, 0, \cos \phi), \quad (3)$$

where ϕ is determined by the crystal cut or sample mounting. The dielectric tensor in the laboratory frame is

$$\bar{\varepsilon}(\phi) = \varepsilon_{\perp} \mathbf{I} + (\varepsilon_{\parallel} - \varepsilon_{\perp}) \mathbf{a} \mathbf{a}^T = \begin{pmatrix} \varepsilon_{xx} & 0 & \varepsilon_{xz} \\ 0 & \varepsilon_{\perp} & 0 \\ \varepsilon_{xz} & 0 & \varepsilon_{zz} \end{pmatrix}, \quad (4)$$

with

$$\varepsilon_{xx} = \varepsilon_{\perp} \cos^2 \phi + \varepsilon_{\parallel} \sin^2 \phi, \quad (5)$$

$$\varepsilon_{zz} = \varepsilon_{\perp} \sin^2 \phi + \varepsilon_{\parallel} \cos^2 \phi, \quad (6)$$

$$\varepsilon_{xz} = (\varepsilon_{\parallel} - \varepsilon_{\perp}) \sin \phi \cos \phi. \quad (7)$$

The determinant of the x - z tensor block is

$$\Delta = \varepsilon_{xx}\varepsilon_{zz} - \varepsilon_{xz}^2 = \varepsilon_{\perp}\varepsilon_{\parallel}. \quad (8)$$

In the following analysis, ϕ is treated as a fixed sample-orientation parameter rather than an in situ mechanical tuning angle.

2.2. Zeroth-Order Fresnel Coefficients and First-Order Angular-Spectrum Coupling

The central incident wave vector is

$$\mathbf{k}_i = k_0(\sin \theta_i, 0, \cos \theta_i). \quad (9)$$

For this central wave vector, the ordinary s wave is polarized along y and is decoupled from the extraordinary p wave in the x - z subspace. The normalized longitudinal component of the ordinary transmitted wave is

$$q_s = \sqrt{\varepsilon_{\perp} - \sin^2 \theta_i}, \quad (10)$$

where the square-root branch is chosen such that the transmitted wave carries energy away from the interface or decays inside the hBN medium. The corresponding s -polarized reflection coefficient is

$$r_s = \frac{\cos \theta_i - q_s}{\cos \theta_i + q_s}. \quad (11)$$

For the extraordinary wave, we write the transmitted wave vector as $\mathbf{k}_t = k_0(\sin \theta_i, 0, q_p)$, where q_p is the normalized longitudinal component. The anisotropic dispersion relation gives

$$\varepsilon_{zz}q_p^2 + 2\varepsilon_{xz}\sin \theta_i q_p + \varepsilon_{xx}\sin^2 \theta_i - \Delta = 0, \quad (12)$$

and hence

$$q_p = \frac{-\varepsilon_{xz}\sin \theta_i + \sqrt{\Delta(\varepsilon_{zz} - \sin^2 \theta_i)}}{\varepsilon_{zz}}. \quad (13)$$

The branch is again selected by the radiation or decay condition inside hBN. Applying the electromagnetic boundary conditions at the air-hBN interface gives the p -polarized reflection coefficient

$$r_p = \frac{\cos \theta_i \Delta - (\sin \theta_i \varepsilon_{xz} + q_p \varepsilon_{zz})}{\cos \theta_i \Delta + (\sin \theta_i \varepsilon_{xz} + q_p \varepsilon_{zz})}. \quad (14)$$

Because the central s and p modes are decoupled in this geometry, the zeroth-order cross-polarized Fresnel coefficients vanish,

$$r_{ps}^{(0)} = r_{sp}^{(0)} = 0. \quad (15)$$

Thus, the transverse differentiation channel discussed below does not originate from a material-induced zeroth-order s - p conversion. It arises from the first-order rotation of the local polarization basis across the angular spectrum. The reflection geometry, fixed

optical-axis orientation, and local coordinate systems used in the angular-spectrum analysis are summarized in Figure 1.

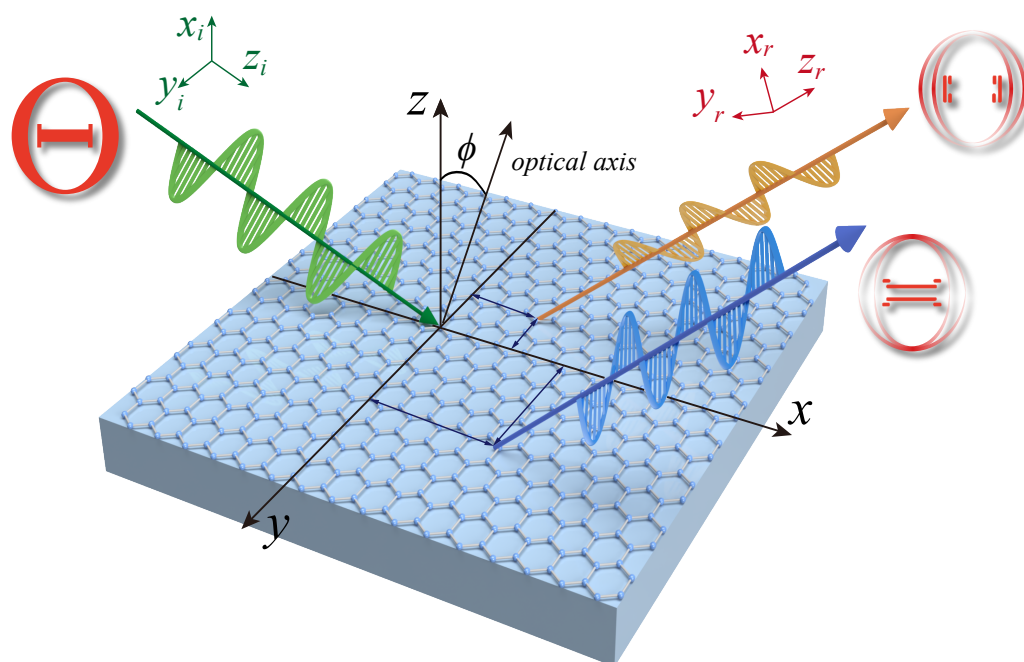


Figure 1. Geometry of angular-spectrum reflection at an air–hBN interface. A Gaussian beam is incident from air onto a fixed-orientation hBN sample with incidence angle θ_i . The hBN optical axis lies in the x – z plane and forms an angle ϕ with the interface normal, as determined by the crystal cut. The reflected field is described in the local reflected coordinate system (x_r, y_r, z_r) . The output sketches indicate representative polarization-selected X- and Y-differentiation responses. The green arrow denotes the incident beam, while the orange and blue arrows sketch representative polarization-selected reflected output channels.

For a paraxial Gaussian beam with input polarization angle α , the incident angular spectrum is written as

$$\tilde{\mathbf{E}}_{\text{in}}(k_{ix}, k_{iy}) = \frac{w_0}{\sqrt{2\pi}} \exp\left[-\frac{w_0^2(k_{ix}^2 + k_{iy}^2)}{4}\right] \begin{pmatrix} \cos \alpha \\ \sin \alpha \end{pmatrix}, \quad (16)$$

where w_0 is the beam waist. Expanding the reflected field to first order around the central incidence angle gives

$$\tilde{\mathbf{E}}_r = \mathbf{M}_R \tilde{\mathbf{E}}_{\text{in}}, \quad (17)$$

with

$$\mathbf{M}_R = \begin{pmatrix} r_p - \frac{k_{rx}}{k_0} r'_p & \frac{k_{ry}}{k_0} \chi_{ps} \\ \frac{k_{ry}}{k_0} \chi_{sp} & r_s - \frac{k_{rx}}{k_0} r'_s \end{pmatrix}. \quad (18)$$

Here $k_{rx} = -k_{ix}$, $k_{ry} = k_{iy}$, and $r'_p = \partial r_p / \partial \theta_i$, $r'_s = \partial r_s / \partial \theta_i$ are evaluated at the central incidence angle. The first-order coefficients induced by the transverse rotation of the local s/p basis are

$$\chi_{ps} = (r_p + r_s) \cot \theta_i, \quad \chi_{sp} = -(r_p + r_s) \cot \theta_i. \quad (19)$$

Their signs depend on the reflected-basis convention, whereas the magnitude and intensity response are unchanged.

2.3. Spin-Dependent Displacements and Polarization-Selected Transfer Functions

To connect the angular-spectrum model with the PSHE shift picture, we introduce the circular polarization basis in the local reflected coordinate system,

$$\mathbf{e}_{\pm} = \frac{1}{\sqrt{2}}(\mathbf{e}_x \pm i\mathbf{e}_y), \quad \mathbf{e}_x = \frac{\mathbf{e}_+ + \mathbf{e}_-}{\sqrt{2}}, \quad \mathbf{e}_y = \frac{i(\mathbf{e}_- - \mathbf{e}_+)}{\sqrt{2}}. \quad (20)$$

Here $\mathbf{e}_+$ and $\mathbf{e}_-$ denote two circularly polarized spin components with opposite handedness in the reflected frame. With the convention used here, they correspond to the left- and right-handed circular components, respectively. A different handedness convention only interchanges the labels $+$ and $-$ and does not affect the polarization-selected transfer functions below.

Projecting the reflected field onto this circular polarization basis gives

$$\begin{aligned} \tilde{E}_{\pm} = & \frac{\tilde{E}_0}{\sqrt{2}}(r_p \cos \alpha \pm ir_s \sin \alpha) \\ & \times \left[1 - \frac{k_{rx}}{k_0} \frac{r'_p \cos \alpha \pm ir'_s \sin \alpha}{r_p \cos \alpha \pm ir_s \sin \alpha} + \frac{k_{ry}}{k_0} \frac{\chi_{ps} \sin \alpha \pm i\chi_{sp} \cos \alpha}{r_p \cos \alpha \pm ir_s \sin \alpha} \right]. \end{aligned} \quad (21)$$

The associated complex spin-dependent shifts are

$$\Delta_x^{\pm} = \frac{i}{k_0} \frac{r'_p \cos \alpha \pm ir'_s \sin \alpha}{r_p \cos \alpha \pm ir_s \sin \alpha}, \quad (22)$$

$$\Delta_y^{\pm} = -\frac{i}{k_0} \frac{\chi_{ps} \sin \alpha \pm i\chi_{sp} \cos \alpha}{r_p \cos \alpha \pm ir_s \sin \alpha}. \quad (23)$$

These expressions connect the PSHE shift picture with the first-order spatial-filtering response. In the image simulations below, however, we use the polarization-selected transfer function directly rather than approximating the response by an exponential shift factor.

Let the analyzer vector be $\mathbf{u}_{\beta} = (\cos \beta, \sin \beta)^T$. For the transfer-function description, we use the transverse spatial frequencies of the output field,

$$k_x \equiv -k_{rx}, \quad k_y \equiv k_{ry}. \quad (24)$$

The scalar transfer function selected by the analyzer is defined as

$$\tilde{E}_{\text{out}} = H_{\alpha\beta}(k_x, k_y) \tilde{E}_0, \quad (25)$$

where, to first order,

$$H_{\alpha\beta}(k_x, k_y) = A_0 + B_x \frac{k_x}{k_0} + B_y \frac{k_y}{k_0}. \quad (26)$$

The coefficients are

$$A_0 = r_p \cos \alpha \cos \beta + r_s \sin \alpha \sin \beta, \quad (27)$$

$$B_x = r'_p \cos \alpha \cos \beta + r'_s \sin \alpha \sin \beta, \quad (28)$$

$$B_y = \chi_{ps} \sin \alpha \cos \beta + \chi_{sp} \cos \alpha \sin \beta. \quad (29)$$

In real space, Equation (26) corresponds to a superposition of a residual image term and first-order spatial derivatives,

$$E_{\text{out}} \approx A_0 E_{\text{in}} - i \frac{B_x}{k_0} \frac{\partial E_{\text{in}}}{\partial x} - i \frac{B_y}{k_0} \frac{\partial E_{\text{in}}}{\partial y}. \quad (30)$$

A practical differentiator therefore requires the first-order derivative term to dominate over the residual background within the paraxial spectral window of the incident beam.

For p -polarized incidence and co-polarized detection, namely $\alpha = 0^\circ$ and $\beta = 0^\circ$, the selected transfer function becomes

$$H_x(k_x) = r_p + r'_p \frac{k_x}{k_0}. \quad (31)$$

Near the Brewster-like minimum, $|r_p|$ is strongly suppressed while r'_p remains finite, producing X -direction differentiation with a finite residual background. For p -polarized incidence and cross-polarized detection, namely $\alpha = 0^\circ$ and $\beta = 90^\circ$, the selected transfer function is

$$H_y(k_y) = \chi_{sp} \frac{k_y}{k_0}. \quad (32)$$

This Y channel has $A_0 = 0$ and is therefore background-free at first order.

Equations (31) and (32) show that the fixed-orientation hBN interface can provide two polarization-selected first-order differentiation channels. The X channel is governed by the ENZ-assisted Brewster-like response of r_p , whereas the Y channel is governed by the first-order angular-spectrum coupling coefficient χ_{sp} . Function switching is achieved by changing the incidence angle and analyzer orientation, while the sample orientation ϕ remains fixed.

3. Results and Discussion

Unless otherwise specified, the operating spectral wavenumber is fixed at $\nu = 824.8 \text{ cm}^{-1}$ and the hBN optical-axis orientation is fixed at $\phi = 20^\circ$. The hBN region is treated as a semi-infinite anisotropic medium to isolate the air-hBN interface response. Finite thickness, substrates, and Fabry–Pérot effects may shift the exact operating point in experiments, but they do not change the symmetry result that $r_{ps}^{(0)} = r_{sp}^{(0)} = 0$ for the central x - z geometry.

3.1. ENZ-Assisted Brewster-like Response and Differential Metrics

Figure 2 shows the p - and s -polarized reflection responses at $\nu = 824.8 \text{ cm}^{-1}$ and $\phi = 20^\circ$. The Brewster-like operating point is defined by the minimum of the reflected p -polarized intensity $|r_p|^2$, rather than by the zero of $\text{Re}(r_p)$ alone. The calculated minimum occurs at

$$\theta_B^{\text{eff}} \approx 69.28^\circ, \quad |r_p|_{\min}^2 \approx 2.8 \times 10^{-4}. \quad (33)$$

In the vicinity of this minimum, the phase of r_p changes rapidly, while $|r_s|^2$ remains finite. This combination provides a small zeroth-order p -polarized background and a large angular slope for the X -differentiation channel.

The X -channel contrast is quantified by the relative differential enhancement

$$G_x^{\text{rel}} = \left| \frac{r'_p}{r_p} \right|. \quad (34)$$

This quantity is not an optical power gain. It measures the ratio between the first-order slope and the residual zeroth-order background. To quantify the residual background within a finite paraxial spectral window, we define

$$R_{\text{bg}}^x = \frac{|r_p|}{|r'_p|(k_{\max}/k_0)}, \quad (35)$$

where $k_{\max}/k_0$ is the half-width of the normalized spatial-frequency window. For $k_{\max}/k_0 = 0.03$, the present model gives $|r'_p| \approx 1.69$, $G_x^{\text{rel}} \approx 101$, and $R_{\text{bg}}^x \approx 0.33$ at the Brewster-like operating point. For the Y channel, the zeroth-order term vanishes by symmetry, so $R_{\text{bg}}^y = 0$. The corresponding differential metrics are summarized in Figure 3.

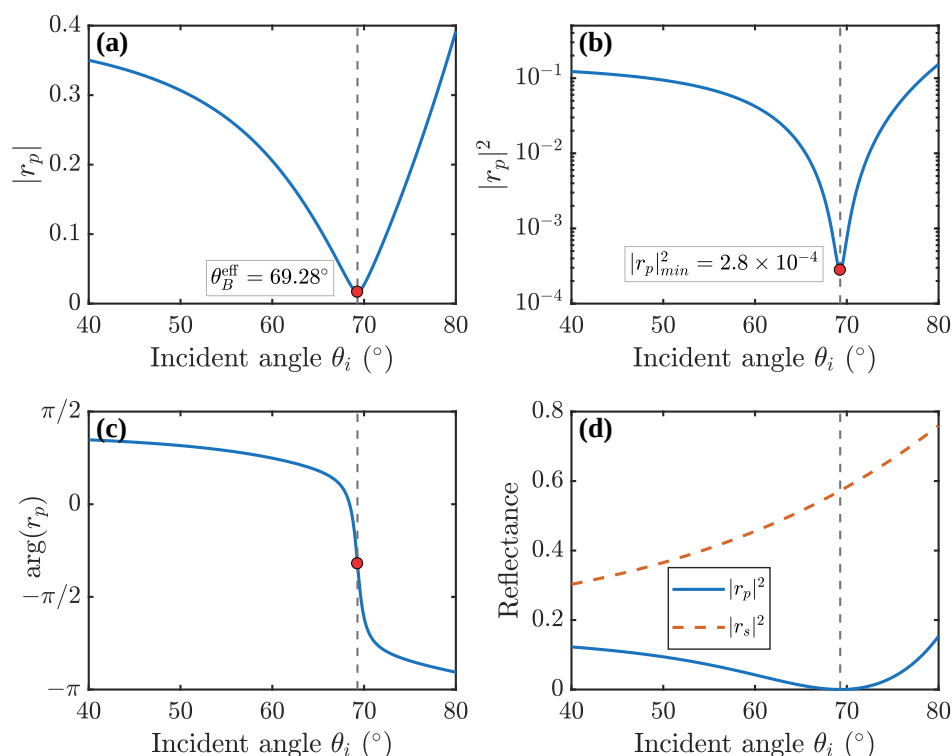


Figure 2. ENZ-assisted Brewster-like reflection response of the fixed-orientation hBN interface at $\nu = 824.8 \text{ cm}^{-1}$ and $\phi = 20^\circ$. (a) Amplitude $|r_p|$. (b) Reflected p -polarized intensity $|r_p|^2$ on a logarithmic scale. (c) Phase $\arg(r_p)$. (d) Comparison of $|r_p|^2$ and $|r_s|^2$. The dashed line marks the effective Brewster-like angle $\theta_B^{\text{eff}} \approx 69.28^\circ$, defined by the minimum of $|r_p|^2$.

3.2. First-Order Transfer-Function Validation and Throughput

The first-order transfer functions are valid within the paraxial angular-spectrum window of the incident beam. We therefore compare the first-order expressions with the corresponding exact angular-spectrum calculations over $|k_{x,y}/k_0| \leq 0.03$. For the X channel,

$$H_x^{\text{1st}}(k_x) = r_p + r'_p \frac{k_x}{k_0}, \quad (36)$$

and for the Y channel,

$$H_y^{\text{1st}}(k_y) = \chi_{sp} \frac{k_y}{k_0}. \quad (37)$$

The relative error is evaluated as

$$\text{Err} = \frac{|H^{\text{exact}} - H^{\text{1st}}|}{\max |H^{\text{exact}}|}. \quad (38)$$

Within the plotted range, the maximum errors are 6.4×10^{-2} for the X channel and 7.6×10^{-3} for the Y channel, confirming that the first-order approximation is adequate for the image simulations below.

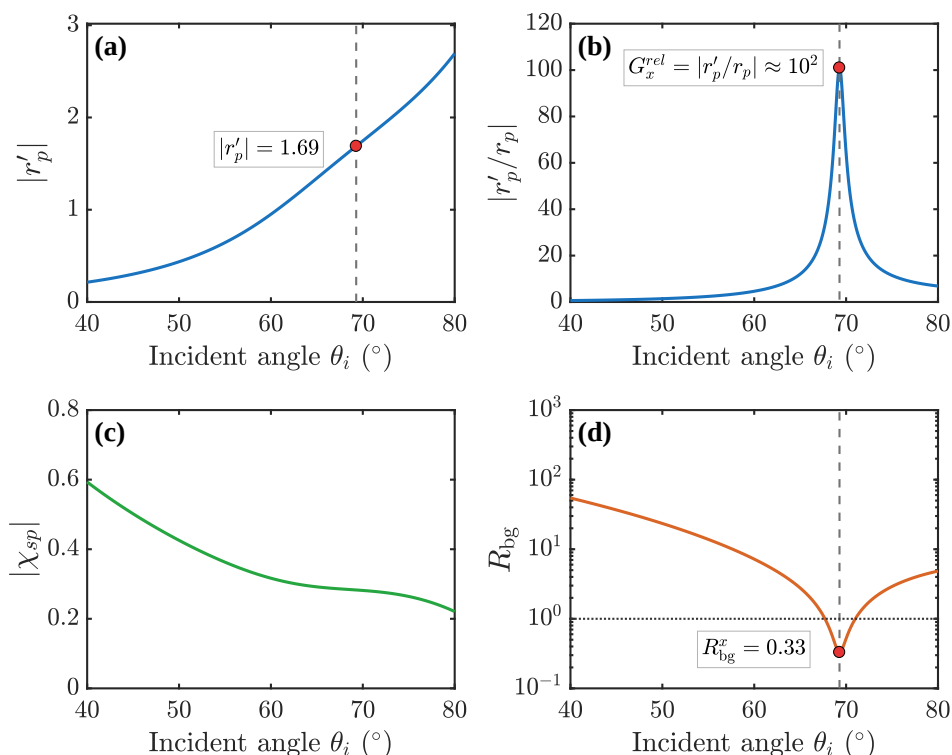


Figure 3. Differential metrics of the fixed-orientation hBN interface at $\nu = 824.8 \text{ cm}^{-1}$ and $\phi = 20^\circ$. (a) Absolute angular slope $|r'_p|$. (b) Relative differential enhancement $G_x^{\text{rel}} = |r'_p/r_p|$. (c) Transverse spin–orbit coupling coefficient $|\chi_{sp}|$. (d) Residual background ratio $R_{bg}^x = |r_p|/[|r'_p|(k_{\text{max}}/k_0)]$ for $k_{\text{max}}/k_0 = 0.03$. The dashed line marks $\theta_B^{\text{eff}} \approx 69.28^\circ$.

The absolute output throughput is estimated by weighting the transfer function with the Gaussian angular spectrum,

$$\eta = \frac{\iint |H(k_x, k_y)|^2 |\tilde{E}_{\text{in}}(k_x, k_y)|^2 dk_x dk_y}{\iint |\tilde{E}_{\text{in}}(k_x, k_y)|^2 dk_x dk_y}. \quad (39)$$

For a Gaussian beam, the characteristic normalized spectral width is

$$\frac{\sigma_k}{k_0} \sim \frac{1}{k_0 w_0} = \frac{1}{2\pi(w_0/\lambda_0)}. \quad (40)$$

Thus, the useful differential throughput decreases approximately as w_0^{-2} for a pure first-order channel. The total X-channel throughput also contains the finite residual background term $|r_p|^2$.

Table 1 clarifies the distinction between relative enhancement and absolute throughput. The large value $G_x^{\text{rel}} \sim 10^2$ originates from suppression of $|r_p|$ near the Brewster-like minimum and should not be interpreted as power amplification. The absolute throughput remains modest because the differentiating term is proportional to the paraxial spatial frequency $|k|/k_0 \ll 1$. The comparison between the exact and first-order transfer functions and the beam-waist-dependent throughput are shown in Figure 4.

Table 1. Representative calculated performance metrics of the hBN transfer functions. The background ratio is evaluated using $k_{\max}/k_0 = 0.03$, and the throughput values are estimated for $w_0 = 30\lambda_0$.

Channel	θ_i	Zeroth-Order Term	First-Order Coefficient	Representative Value
X total	69.28°	r_p	r'_p	$R_{\text{bg}}^x \approx 0.33, \eta_X \sim 3.6 \times 10^{-4}$
X differential part	69.28°	—	r'_p	$\eta_X^{\text{diff}} \sim 8.0 \times 10^{-5}$
Y channel	20°	0	χ_{sp}	$R_{\text{bg}}^y = 0, \eta_Y \sim 5.0 \times 10^{-5}$

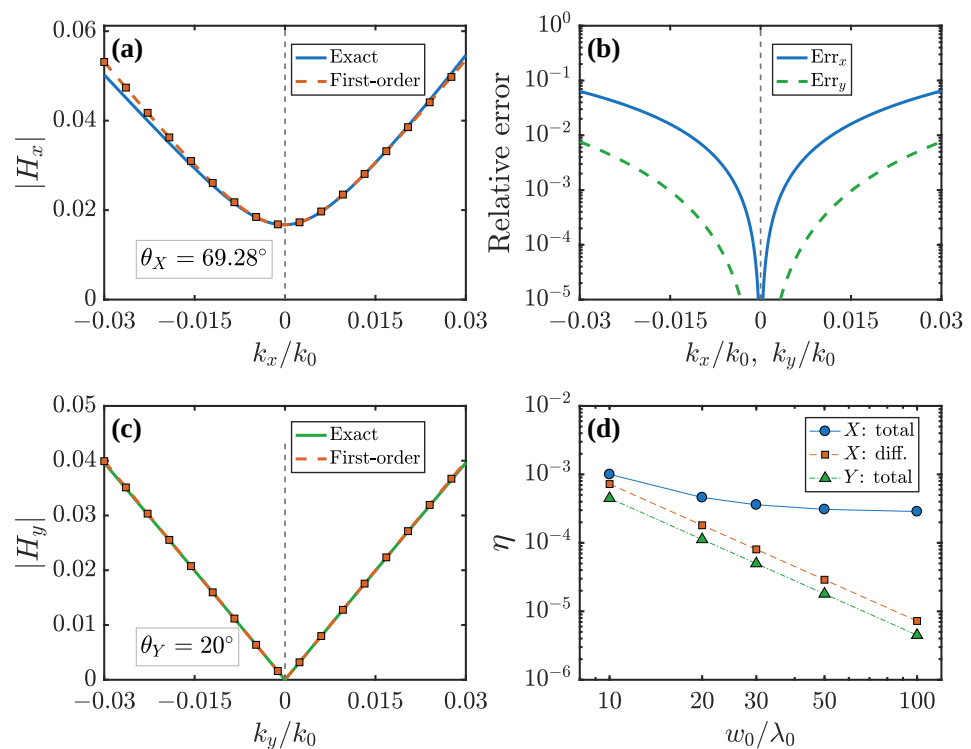


Figure 4. First-order validation and beam-waist-dependent throughput. (a) Exact and first-order X-channel transfer functions near $\theta_X = \theta_B^{\text{eff}} = 69.28^\circ$. (b) Relative errors of the first-order approximation for the X and Y channels. (c) Exact and first-order Y-channel transfer functions at $\theta_Y = 20^\circ$. (d) Gaussian spectrum-weighted integrated throughput as a function of w_0/λ_0 . The spatial-frequency range is restricted to $|k_{x,y}/k_0| \leq 0.03$.

3.3. Polarization-Selected Function Switching in Image Differentiation

Table 2 summarizes the representative operating modes used in the image simulations. The hBN sample orientation is fixed at $\phi = 20^\circ$. Switching among intensity-like imaging, X differentiation, and Y differentiation is achieved by selecting the incidence angle and analyzer orientation, rather than by rotating the sample.

Table 2. Representative operation modes for the fixed-orientation hBN sample at $\nu = 824.8 \text{ cm}^{-1}$ and $\phi = 20^\circ$.

Mode	θ_i	α	β	Transfer Function and Main Feature
Intensity-like	50°	0°	0°	$H \simeq A_0$; image-dominated output
X differentiation	69.28°	0°	0°	$H_x = r_p + r'_p k_x/k_0$; x gradient edge enhancement with residual background
Y differentiation	20°	0°	90°	$H_y = \chi_{sp} k_y/k_0$; background-free y gradient edge enhancement

For the image-domain simulations, the input object is sampled on a uniform 512×512 grid and encoded as a normalized scalar amplitude distribution. The reflected output is obtained by multiplying the Fourier spectrum of the input field by the corresponding transfer function and then applying an inverse Fourier transform. The spatial coordinates and output intensities are normalized for visualization. No additional free-space propagation distance is introduced after reflection; the results represent the polarization-selected reflected field at the image plane. Unless otherwise specified, the beam-waist-dependent throughput estimates use the Gaussian angular spectrum defined in Equation (16), with representative values reported for $w_0 = 30\lambda_0$.

Figure 5 shows the corresponding image-domain results. The intensity-like mode preserves the main shape of the input object because the output is dominated by the zeroth-order term A_0 . The X-differentiation mode enhances vertical edges through the transfer function $H_x = r_p + r'_p k_x/k_0$. Since r_p is small but nonzero, the output contains a residual zeroth-order background, consistent with the finite value of R_{bg}^x in Figure 3 and Table 1. In contrast, the cross-polarized Y channel has $A_0 = 0$ and produces a background-free first-order response $H_y = \chi_{sp} k_y/k_0$. The line profiles in Figure 5e,f show good agreement between the hBN outputs and the ideal first-order differentiation references. All displayed images and profiles are normalized for visual comparison; the absolute throughputs are reported in Figure 4d and Table 1.

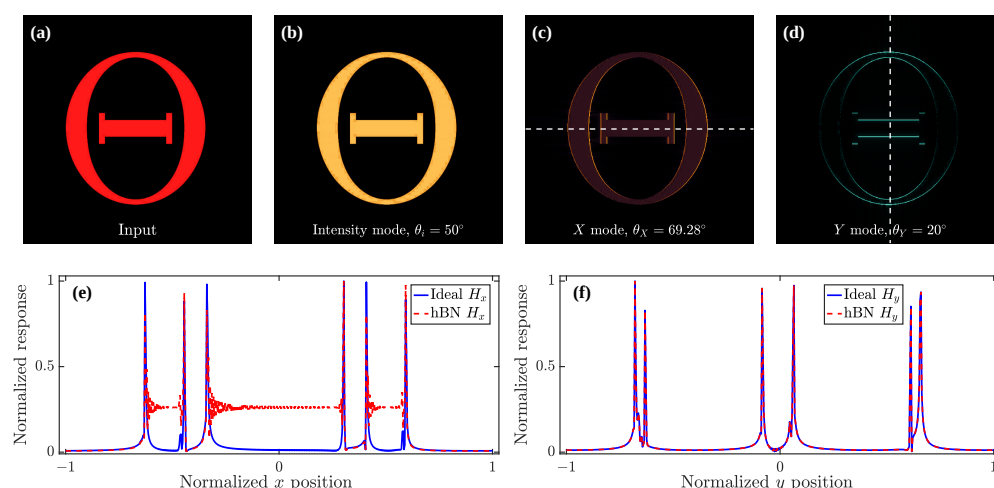


Figure 5. Function switching of the fixed-orientation hBN spatial-differentiation platform at $\nu = 824.8 \text{ cm}^{-1}$ and $\phi = 20^\circ$. (a) Input object. (b) Intensity-like output in an A_0 -dominated co-polarized mode. (c) X-differentiation output based on $H_x = r_p + r'_p k_x/k_0$ near $\theta_X = \theta_B^{\text{eff}} = 69.28^\circ$. (d) Background-free Y-differentiation output based on $H_y = \chi_{sp} k_y/k_0$ at $\theta_Y = 20^\circ$. (e,f) Normalized line profiles extracted along the dashed lines in (c,d), respectively, comparing the hBN outputs with ideal X- and Y-differentiation references. The displayed outputs are normalized for visual comparison.

3.4. Comparison and Experimental Considerations

The proposed hBN structure operates as a natural-crystal-based, reflection-mode mid-infrared differentiator. Its main advantage is not high absolute throughput, but the combination of ENZ-assisted background suppression, polarization-selective operation, and the absence of subwavelength patterning. Table 3 gives a qualitative comparison with representative differentiator platforms, including Brewster effect interfaces, ENZ or thin-film structures, band-degeneracy platforms, and Weyl semimetal-based systems [7–10,20,33,34].

In practical implementation, the optical-axis orientation would be fixed by crystal cutting, polishing, or sample mounting. Here ϕ denotes the angle between the hBN optical

axis and the surface normal of the prepared interface, rather than an independently tunable mechanical rotation. The incidence plane should be aligned with the projection of the optical axis on the sample surface. A small incidence-angle error mainly shifts the operation away from the measured Brewster-like minimum and can be compensated by retuning θ_i , whereas an azimuthal misalignment would introduce weak zeroth-order polarization leakage and reduce the contrast of the nominally background-free Y channel. Finite sample thickness and substrates would mainly modify the numerical operating angle, throughput, and contrast through additional propagation phases and multiple reflections. These effects can be included by replacing the half-space Fresnel coefficients with a multilayer transfer-matrix description. Variations in the hBN optical constants would shift θ_B^{eff} and modify the absolute throughput; in experiments, the dielectric tensor should therefore be calibrated from the same sample, for example by ellipsometry or infrared reflection measurements. Nonideal analyzer extinction ratios would add a small leakage background to the nominally background-free Y channel, but the polarization-selective switching mechanism remains the same.

Table 3. Qualitative comparison with representative optical spatial differentiator platforms. The entries summarize typical implementation features rather than absolute performance limits.

Platform	Main Mechanism	Tunability	Experimental Complexity
Air–glass Brewster interface	Brewster suppression	Angle/polarization	Very low
ENZ or thin-film structures	ENZ-enhanced reflection	Frequency/thickness/angle	Moderate
Dirac-point structures	Engineered dispersion	Structural/spectral tuning	High
Weyl semimetal-based systems	Chiral or topological response	Material/angle tuning	Moderate to high
Fixed-orientation hBN	Reststrahlen anisotropy and PSHE	Frequency, θ_i , α , β	No subwavelength patterning

4. Conclusions

We have presented the theoretical description of hBN-based PSHE spatial differentiation by separating the fixed optical-axis orientation of the hBN sample from externally tunable operating parameters. For the central geometry in which the optical axis and the incidence plane both lie in the x – z plane, the zeroth-order cross-polarized Fresnel coefficients vanish. The X channel is governed by the ENZ-assisted Brewster-like response $H_x = r_p + r'_p k_x/k_0$, while the Y channel is governed by the first-order angular-spectrum spin-orbit term $H_y = \chi_{sp} k_y/k_0$.

At $\nu = 824.8 \text{ cm}^{-1}$ and fixed sample orientation $\phi = 20^\circ$, the model yields an effective Brewster-like angle $\theta_B^{\text{eff}} \approx 69.28^\circ$ with $|r_p|^2 \approx 2.8 \times 10^{-4}$. This produces a large relative X -channel enhancement $G_x^{\text{rel}} \sim 10^2$, while the absolute throughput remains finite but modest because the useful differentiating term scales with the paraxial spatial frequency. First-order angular-spectrum calculations are validated within $|k_{x,y}/k_0| \leq 0.03$, and the throughput dependence on beam waist is quantified. Finally, image simulations demonstrate switching among intensity-like, X -differentiation, and background-free Y -differentiation modes through incidence-angle and polarization-analyzer selection. These results establish fixed-orientation hBN as a mid-infrared platform for polarization-selective optical spatial differentiation without subwavelength patterning.

Author Contributions: Conceptualization, W.X.; methodology, W.X.; software, W.X. and Y.Z.; validation, W.X., Y.Z. and S.Z.; formal analysis, W.X.; investigation, W.X. and Y.Z.; resources, W.X.; data curation, W.X.; writing—original draft preparation, W.X.; writing—review and editing, W.X., Y.Z., S.Z., Q.L. and L.X.; visualization, W.X. and Y.Z.; supervision, W.X.; project administration, W.X.; funding acquisition, W.X. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Natural Science Foundation of China (Grant No. 12304321, 12304104, 12575014) and the Natural Science Foundation of Hunan Province (Grant No. 2026JJ30116, 2025JJ60018, 2025JJ60032), the Key Laboratory of Low-Dimensional Quantum Structures and Quantum Control of Ministry of Education (Grant No. QSQC2405), and the Hunan Provincial Major Sci-Tech Program (Grant No. 2023ZJ1010).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data presented in this study are available within the article. Additional numerical data supporting the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest: The authors declare no conflicts of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript; or in the decision to publish the results.

References

1. Solli, D.R.; Jalali, B. Analog optical computing. *Nat. Photonics* **2015**, *9*, 704–706. [\[CrossRef\]](#)
2. Silva, A.; Monticone, F.; Castaldi, G.; Galdi, V.; Alù, A.; Engheta, N. Performing mathematical operations with metamaterials. *Science* **2014**, *343*, 160–163. [\[CrossRef\]](#) [\[PubMed\]](#)
3. Zhou, Y.; Zheng, H.; Kravchenko, I.I.; Valentine, J. Flat optics for image differentiation. *Nat. Photonics* **2020**, *14*, 316–323. [\[CrossRef\]](#)
4. Kwon, H.; Arbabi, E.; Kamali, S.M.; Faraji-Dana, M.; Faraon, A. Single-shot quantitative phase gradient microscopy using a system of multifunctional metasurfaces. *Nat. Photonics* **2020**, *14*, 109–114.
5. Zhou, J.; Qian, H.; Chen, C.F.; Zhao, J.; Li, G.; Wu, Q.; Luo, H.; Wen, S.; Liu, Z. Optical edge detection based on high-efficiency dielectric metasurface. *Proc. Natl. Acad. Sci. USA* **2019**, *116*, 11137–11140. [\[CrossRef\]](#) [\[PubMed\]](#)
6. Liu, J.; Yang, Q.; Chen, S.; Xiao, Z.; Wen, S.; Luo, H. Intrinsic optical spatial differentiation enabled quantum dark-field microscopy. *Phys. Rev. Lett.* **2022**, *128*, 193601. [\[CrossRef\]](#) [\[PubMed\]](#)
7. Zhou, J.; Liu, S.; Qian, H.; Li, Y.; Luo, H.; Wen, S.; Zhou, Z.; Guo, G.; Shi, B.; Liu, Z. Metasurface enabled quantum edge detection. *Sci. Adv.* **2020**, *6*, eabc4385. [\[CrossRef\]](#) [\[PubMed\]](#)
8. Pan, D.; Wan, L.; Ouyang, M.; Zhang, W.; Potapov, A.A.; Liu, W.; Liang, Z.; Feng, T.; Li, Z. Laplace metasurfaces for optical analog computing based on quasi-bound states in the continuum. *Photonics Res.* **2021**, *9*, 1758–1766. [\[CrossRef\]](#)
9. Bi, X.; Guo, X.; Wu, X.; Fan, X.; Wei, B.; Wen, D.; Liu, S.; Zhao, J.; Li, P. Wideband optical edge detection based on dielectric metasurface. *Appl. Phys. Lett.* **2023**, *123*, 061702. [\[CrossRef\]](#)
10. Xu, D.; He, S.; Zhou, J.; Chen, S.; Wen, S.; Luo, H. Optical analog computing of two-dimensional spatial differentiation based on the Brewster effect. *Opt. Lett.* **2020**, *45*, 6867–6870. [\[CrossRef\]](#) [\[PubMed\]](#)
11. Zhu, T.; Lou, Y.; Zhou, Y.; Zhang, J.; Huang, J.; Li, Y.; Luo, H.; Wen, S.; Zhu, S.; Gong, Q.; et al. Generalized spatial differentiation from the spin Hall effect of light and its application in image processing of edge detection. *Phys. Rev. Appl.* **2019**, *11*, 034043. [\[CrossRef\]](#)
12. Wang, R.; He, S.; Luo, H. Photonic spin-Hall differential microscopy. *Phys. Rev. Appl.* **2022**, *18*, 044016. [\[CrossRef\]](#)
13. Xiang, Y.; Jiang, X.; You, Q.; Guo, J.; Dai, X. Enhanced spin Hall effect of reflected light with guided-wave surface plasmon resonance. *Photonics Res.* **2017**, *5*, 467–472. [\[CrossRef\]](#)
14. Caldwell, J.D.; Kretinin, A.V.; Chen, Y.; Giannini, V.; Fogler, M.M.; Francescato, Y.; Ellis, C.T.; Tischler, J.G.; Woods, C.R.; Giles, A.J.; et al. Sub-diffractive volume-confined polaritons in the natural hyperbolic material hexagonal boron nitride. *Nat. Commun.* **2014**, *5*, 5221. [\[CrossRef\]](#) [\[PubMed\]](#)
15. Dai, S.; Ma, Q.; Liu, M.; Andersen, T.; Fei, Z.; Goldflam, M.D.; Wagner, M.; Watanabe, K.; Taniguchi, T.; Thiemens, M.; et al. Tunable phonon polaritons in atomically thin van der Waals crystals of boron nitride. *Science* **2014**, *343*, 1125–1129. [\[CrossRef\]](#) [\[PubMed\]](#)
16. Zhang, Y.; Wang, X.; Zhang, D.; Fu, S.; Zhou, S.; Wang, X.Z. Unusual spin and angular momentum of Dyakonov waves at the hyperbolic-material surface. *Opt. Express* **2020**, *28*, 19205–19217. [\[CrossRef\]](#) [\[PubMed\]](#)
17. Wang, X.G.; Zhang, Y.Q.; Fu, S.F.; Zhou, S.; Wang, X.Z. Goos-Hänchen and Imbert-Fedorov shifts on hyperbolic crystals. *Opt. Express* **2020**, *28*, 25048–25059. [\[CrossRef\]](#) [\[PubMed\]](#)
18. Li, Y.B.; Song, H.Y.; Zhang, Y.Q.; Zhou, S.; Fu, S.F.; Zhang, Q.; Wang, X.Z. Spatial shifts of reflected light beam on black phosphorus/hexagonal boron nitride structure. *Opt. Laser Technol.* **2023**, *159*, 108968. [\[CrossRef\]](#)

19. Lei, X.; Ren, Y.; Xu, W. Photonic spin Hall effect on the surface of two-dimensional black arsenic. *Phys. Lett. A* **2024**, *521*, 129754. [[CrossRef](#)]
20. Xu, W.; Ling, X.; Xu, D.; Chen, S.; Wen, S.; Luo, H. Enhanced optical spatial differential operations via strong spin-orbit interactions in an anisotropic epsilon-near-zero slab. *Phys. Rev. A* **2021**, *104*, 053513. [[CrossRef](#)]
21. Onoda, M.; Murakami, S.; Nagaosa, N. Hall effect of light. *Phys. Rev. Lett.* **2004**, *93*, 083901. [[CrossRef](#)] [[PubMed](#)]
22. Hosten, O.; Kwiat, P. Observation of the spin Hall effect of light via weak measurements. *Science* **2008**, *319*, 787–790. [[CrossRef](#)] [[PubMed](#)]
23. Luo, H.; Zhou, X.; Shu, W.; Wen, S.; Fan, D. Enhanced and switchable spin Hall effect of light near the Brewster angle on reflection. *Phys. Rev. A* **2011**, *84*, 043806. [[CrossRef](#)]
24. Bliokh, K.Y.; Rodríguez-Fortuño, F.J.; Nori, F.; Zayats, A.V. Spin–orbit interactions of light. *Nat. Photonics* **2015**, *9*, 796–808. [[CrossRef](#)]
25. Yin, X.; Ye, Z.; Rho, J.; Wang, Y.; Zhang, X. Photonic spin Hall effect at metasurfaces. *Science* **2013**, *339*, 1405–1407. [[CrossRef](#)] [[PubMed](#)]
26. Kort-Kamp, W.J.d.M. Topological phase transitions in the photonic spin Hall effect. *Phys. Rev. Lett.* **2017**, *119*, 147401. [[CrossRef](#)] [[PubMed](#)]
27. Fu, S.; Guo, C.; Liu, G.; Li, Y.; Yin, H.; Li, Z.; Chen, Z. Spin-orbit optical Hall effect. *Phys. Rev. Lett.* **2019**, *123*, 243904. [[CrossRef](#)] [[PubMed](#)]
28. Kim, M.; Yang, Y.; Lee, D.; Kim, Y.; Kim, H.; Rho, J. Spin Hall effect of light: From fundamentals to recent advancements. *Laser Photonics Rev.* **2023**, *17*, 2200046. [[CrossRef](#)]
29. Liu, S.; Chen, S.; Wen, S.; Luo, H. Photonic spin Hall effect: Fundamentals and emergent applications. *Opto-Electron. Sci.* **2022**, *1*, 220007. [[CrossRef](#)]
30. Zhou, J.; Wu, Q.; Zhao, J.; Posner, C.; Lei, M.; Chen, G.; Zhang, J.; Liu, Z. Fourier optical spin splitting microscopy. *Phys. Rev. Lett.* **2022**, *129*, 020801. [[CrossRef](#)] [[PubMed](#)]
31. Xu, X.; Liang, M.; Yang, Q.; Li, Q.; Chen, S.; Wen, S.; Luo, H. Photonic spin-Hall holography. *Photonics Res.* **2026**, *14*, 2611–2621. [[CrossRef](#)]
32. Deng, Y.; Xu, W.; Zhang, W.; Yang, Q.; Xu, D.; Luo, H. Rotational photonic spin Hall effect on twisted bilayer metasurfaces. *Opt. Commun.* **2024**, *560*, 130480. [[CrossRef](#)]
33. Xu, W.; He, T.; Zuo, Y.; Yang, Y.; Song, R.; Xu, L. Tunable optical spatial differential operations via photonic Dirac points. *Opt. Laser Technol.* **2025**, *192*, 113863. [[CrossRef](#)]
34. Wen, Z.; Xu, W.; Zhang, Y.; Jiang, T.; Luo, Z. Tunable optical spatial differential operation via photonic spin Hall effect in a Weyl semimetal. *Opt. Express* **2024**, *32*, 10022–10032. [[CrossRef](#)] [[PubMed](#)]

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.