

## Article

# Particle-in-Cell Simulations of Laser Crossbeam Energy Transfer via Magnetized Ion-Acoustic Wave

Yuan Shi <sup>1,\*</sup>  and John D. Moody <sup>2</sup> <sup>1</sup> Department of Physics, Center for Integrated Plasma Studies, University of Colorado Boulder, Boulder, CO 80309, USA<sup>2</sup> Lawrence Livermore National Laboratory, Livermore, CA 94550, USA; moody4@llnl.gov

\* Correspondence: yuan.shi@colorado.edu

## Abstract

Magnetic fields, either imposed externally or produced spontaneously, are often present in laser-driven high-energy-density systems. In addition to changing plasma conditions, magnetic fields also directly modify laser–plasma interactions (LPI) by changing the participating waves and their nonlinear interactions. In this paper, we use two-dimensional particle-in-cell (PIC) simulations to investigate how magnetic fields directly affect crossbeam energy transfer (CBET) from a pump to a seed laser beam when the transfer is mediated by the ion-acoustic wave (IAW) quasimode. Our simulations are performed in the parameter space where CBET is the dominant process and in a linear regime, where pump depletion, distribution function evolution, and secondary instabilities are insignificant. We use a Fourier filter to separate out the seed signal and project the seed fields onto two electromagnetic eigenmodes, which become nondegenerate in magnetized plasmas. By comparing the seed energy before CBET occurs and after CBET reaches quasi-steady state, we extract the CBET energy gains for both eigenmodes in lasers that are initially linearly polarized. Our simulations reveal that, starting from a few MG fields, the two eigenmodes have different gains, and magnetization alters the dependence of the gains on laser detuning. The overall gain decreases with magnetization when the laser polarizations are initially parallel, while a nonzero gain becomes allowed when the laser polarizations are initially orthogonal. These findings qualitatively agree with theoretical expectations.

**Keywords:** magnetized laser–plasma interactions; crossbeam energy transfer; particle-in-cell simulations

## 1. Introduction

When laser-driven inertial confinement fusion (ICF) experiments are pre-magnetized using external coils, the additional magnetic confinement increases ion temperature and fusion yield [1–5]. Beyond existing experiments, simulations of magnetized ICF implosions [6–9] suggest that magnetic fields may also stabilize Rayleigh–Taylor instabilities and reduce mix, with plausible inhibition of fusion burn propagation [10,11]. When combining magnetization effects optimally, achieving more robust fusion ignition at reduced driver laser energy and relaxed target smoothness requirements may be possible.

Even when magnetic fields are not imposed externally, ICF experiments, and more broadly, high-energy density (HED) experiments, tend to be spontaneously magnetized. A well-studied source of spontaneous magnetization is the Biermann battery effect [12–15], where non-parallel density and temperature gradients drive the growth of magnetic fields.



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Additionally, the Poynting–Robertson effect [16,17] and kinetic effects [18] can also drive seed magnetic fields, which may be subsequently amplified by flux compression [19,20]. Near laser entrance holes of indirect-drive implosions, where fields are compressed by plasma inflow, megagauss spontaneous fields are common [21]. Moreover, near the center of a pre-magnetized capsule, kilotesla fields have been inferred in experiments using nuclear diagnostics [22].

In magnetized HED experiments, the field modifications to laser–plasma interactions (LPI) is still not well understood [23]. To estimate when magnetization directly affects LPI, as opposed to indirectly through changing plasma conditions, we can compare characteristic frequency scales. In about-1 MG fields, the electron gyro frequency  $\Omega_e$  becomes comparable to typical frequencies of ion-acoustic waves (IAW) that mediate Brillouin scattering and crossbeam energy transfer (CBET). In about-10 MG fields,  $\Omega_e$  becomes comparable to the frequency of electron plasma waves (EPW) that mediate Raman scattering and two-plasmon decay. In even stronger fields,  $\Omega_e$  becomes comparable to the laser frequency, where drastic changes to LPI occur [24,25]. Moreover, when the plasma is magnetized, a variety of additional plasma waves, which have no analogue in unmagnetized plasmas, can mediate additional resonant interactions [26–28]. Existing studies of magnetized LPI (MagLPI) mostly focus on how magnetic fields modify Raman- and Brillouin-type interactions [29], through modifying plasma conditions and hot electrons [30], changing collisionless wave damping [31], and splitting polarizations of electromagnetic eigenmodes [32].

Although the study of MagLPI is scarce, magnetized three- and four-wave interactions [33,34] and parametric instabilities [35,36], which are the more general overarching phenomena, have been investigated in other contexts. For example, in magnetic confinement fusion, antennas mounted on vacuum chamber walls launch radio-frequency waves and excite parametric instabilities [37]. In this near-perpendicular geometry, the pump wave can excite parametric instabilities and decay to upper- and lower-hybrid waves [38,39], as well as Bernstein waves [40,41], which can subsequently cascade to other waves [42]. As another example, in astrophysical plasmas, waves often propagate nearly parallel to the magnetic field. In this context, the pump wave is an Alfvén wave whose frequency is below ion cyclotron frequencies. The pump Alfvén wave can couple with sound waves and other Alfvén-type waves [43–45]. In addition to these examples, a wealth of nonlinear processes occur in magnetized plasmas.

This paper is dedicated to the 85th birthday of Professor Lennart Stenflo, who pioneered the study of magnetized wave-wave interactions starting from the late 60s and the early 70s [46–49]. Together with his collaborators, Stenflo has developed cold-fluid [50], warm-fluid [51], kinetic [52], and collisional [53,54] theories of magnetized three-wave interactions. Stenflo and colleagues derived fluctuation spectrum enhanced by pump waves [55,56], investigated effects of nonuniformity [57], plasma boundary [58,59], gravity potential [60,61], and nonstationary turbulence [62]. They studied many special cases [63–65], as well as made numerous attempts to simplify general expression of the coupling coefficients [66–69]. The theories developed have been applied to ionospheric experiments [70], free-electron lasers [71], THz radiation generation [72], and quantum plasmas [73].

In our companion paper [74], we develop a Vlasov theory of magnetized CBET (MagCBET) using a low-frequency approximation of magnetized ponderomotive force. In laser-driven implosions, CBET is an important LPI process where energy is transferred from one laser beam to another [75]. In unmagnetized plasmas [76], CBET is mediated by IAW quasimodes, and the resonant conditions are satisfied either due to a plasma flow, in which laser beams are Doppler shifted by different amounts, or due to an intentional laser

wavelength detuning  $\Delta\lambda$  at the lens. In the plasma rest frame, the resonance conditions are  $\omega_0 = \omega + \omega_{IAW}$  and  $\mathbf{k}_0 = \mathbf{k} + \mathbf{k}_{IAW}$ , where the wave 4-vector  $(\omega_0, \mathbf{k}_0)$  is for the pump laser,  $(\omega, \mathbf{k})$  is for the seed laser, and  $(\omega_{IAW}, \mathbf{k}_{IAW})$  is for the mediating IAW quasimode. In direct-drive implosions, CBET often leads to loss of coupling and symmetry, when incoming lasers are consumed to amplify backscattered beams [77–79]. In indirect-drive implosions, CBET is used to control the implosion symmetry, whereby energy is redistributed between polar and equatorial beams to achieve spherical implosions [80–82]. Our MagCBET theory suggests three new physical effects at fixed plasma conditions. First, the coupling between initially parallel polarized lasers is reduced by magnetization. Second, orthogonally polarized lasers, which do not couple in unmagnetized plasmas, become coupled due to magnetization. Third, the gain curve, namely, the CBET gain as a function of  $\Delta\lambda$ , is altered by magnetization, with additional resonances away from the IAW resonance. When plasma is magnetized, the two electromagnetic eigenmodes are nondegenerate and have different gains. The total gain of a linearly polarized seed laser is between the eigenmode gains.

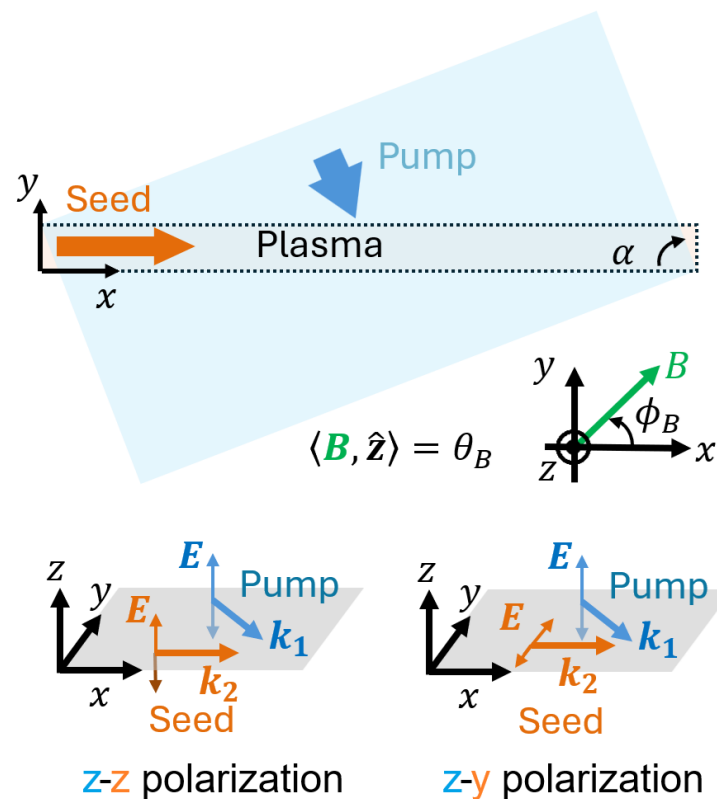
In this paper, we verify these theoretical findings for MagCBET using particle-in-cell (PIC) simulations in the plasma rest frame, and we focus on the IAW resonance. The simulations are performed using the EPOCH code [83], which allows for an arbitrary background magnetic field and is fully relativistic and electromagnetic. The PIC simulations are two-dimensional (2D) in space with all three field components and three velocity (3V) directions (2D-3V). To compare with our Vlasov theory, we turn off collisions, which may nevertheless have effects, especially in lower temperature experiments [84,85]. To avoid complications due to additional nonlinear effects [86,87], we choose simulation parameters to stay within the linear regime of CBET, namely, when pump depletion and distribution function evolution are insignificant and when secondary instabilities are weak. In this linear regime, our simulations qualitatively confirm our theoretical findings. However, quantitative differences are observed, suggesting additional physics beyond our Vlasov theory.

The paper is organized as follows. In Section 2, we describe our simulation setup and our considerations for choosing simulation parameters. In Section 3, we describe how we extract CBET gain from raw PIC data. In Section 4, we present representative results of our simulations, and additional results can be found in Ref. [88]. A summary and discussion are given in Section 5.

## 2. Simulation Setup

To reduce computational cost, we use an elongated rectangular simulation domain centered around the seed laser beam, as shown in Figure 1. The domain is a two-dimensional box with  $0 < x < L_x$  and  $-L_y/2 < y < L_y/2$ , where  $L_x = 500\lambda_0$ ,  $L_y = 10\lambda_0$ , and  $\lambda_0 = 0.351 \mu\text{m}$  is the pump laser wavelength in vacuum. A domain of length  $L$  gives a finite wavevector resolution  $\Delta k = 2\pi/L$ . The seed laser enters from the  $-x$  domain boundary, propagates with unit wavevector  $\hat{\mathbf{k}} = \hat{\mathbf{x}}$ , and then exits the  $+x$  domain boundary. All boundaries use Convolutional Perfectly Matched Layer (CPML) to realize absorbing boundary conditions for electromagnetic waves [89]. For particles, all boundaries are open, namely, particles are removed from the simulation when they reach the boundaries. In free space, the seed laser is a Gaussian beam, focused at the center of the domain with a half divergence angle of  $4^\circ$ . However, due to a finite aperture, the seed laser is diffracted, and the Rayleigh angle is  $1.22\lambda/L_y \approx 7^\circ$ , where  $\lambda \approx \lambda_0$  is the seed laser wavelength in vacuum. Since the Rayleigh angle is larger than the Gaussian focusing angle, the seed beam is slightly divergent, leading to a loss from  $\pm y$  domain boundaries. A larger  $L_y$  reduces the loss but increases the computational cost, so is not used here. Additionally, because the domain is filled with a plasma, the seed laser undergoes refraction when entering and exiting the domain. Nevertheless, because the plasma density we use is much

lower than the critical density of the laser, the refraction angle is minuscule. The overall effect, with diffraction overcoming focusing, is that the seed laser is approximately a plane wave, with a Gaussian intensity envelope along the  $y$  direction and an approximately  $y$ -independent envelope along the  $x$  direction.



**Figure 1.** Setup of 2D-3V PIC simulations, where the simulation domain (dashed rectangle) is centered around the seed laser and the magnetic field  $\mathbf{B}$  is at oblique angles. For  $z$ - $z$  polarization, the pump and seed lasers have initially parallel polarizations, while for  $z$ - $y$ , their polarizations are initially orthogonal.

To observe CBET, which changes the seed envelope along the  $x$  direction, we turn on a pump laser and fill the domain with a magnetized plasma. The pump laser is a plane wave with unit wavevector  $\hat{\mathbf{k}}_0 = \hat{\mathbf{x}} \cos \alpha - \hat{\mathbf{y}} \sin \alpha$ , where  $\alpha = 30^\circ$  is the crossing angle between the pump and seed lasers. The pump laser enters from both the  $-x$  and  $+y$  boundaries, and exits from the  $+x$  and  $-y$  boundaries. Because of a considerable incidence angle to the CPML, which is designed to absorb waves with near-normal incidence, about 5% pump energy is reflected from the exit boundaries when the pump laser is linearly polarized along the  $\hat{\mathbf{z}}$  direction at the entrance boundaries. At the entrance boundaries, we fix the pump laser to  $\hat{\mathbf{z}}$  polarization, and simulate two seed laser polarizations: In the  $z$ - $z$  polarization setup, the seed laser is also linearly polarized along the  $\hat{\mathbf{z}}$  direction at the entrance boundary, so the pump and seed lasers have initially parallel polarizations. On the other hand, in the  $z$ - $y$  polarization setup, the seed laser is linearly polarized along the  $\hat{\mathbf{y}}$  direction at the entrance boundary, so the pump and seed lasers have initially orthogonal polarizations. As the lasers propagate through the magnetized plasma, their polarization vectors rotate and gain ellipticity. The entire domain is filled with a plasma of electron density  $n_e = 3.5 \times 10^{20} \text{ cm}^{-3}$ . We use a fully ionized carbon plasma with initially Maxwellian distribution functions at electron temperature  $T_e = 0.8 \text{ keV}$  and ion temperature  $T_i = 0.2 \text{ keV}$ . The  $T_e/T_i$  ratio affects the CBET gain. The ratio we choose here is an upper bound for our recent MagCBET experiments at the OMEGA laser facility, where

plasma conditions are measured using Thomson scattering. The plasma is embedded in a uniform magnetic field. We specify the direction of magnetic field  $\mathbf{B}$  in standard spherical coordinates, with polar angle  $\theta_B$  and azimuthal angle  $\phi_B$ .

We choose simulation parameters so that linear CBET is the dominant process in our PIC simulations, which also captures other competing processes. Notice that the plasma has a sheath electric field  $E_S \sim \phi_S/\lambda_D \sim 10^{11}$  V/m, where  $\phi_S \sim k_B T/e$  is the sheath potential with  $k_B$  the Boltzmann constant and  $e$  the elementary charge, and  $\lambda_D$  is the Debye length. The sheath field is comparable to the laser electric field  $E_L = \sqrt{2I/\epsilon_0 c} \sim 10^{11} \sqrt{I_{15}}$  V/m, where  $I$  denotes the laser intensity,  $\epsilon_0$  is the permittivity in the vacuum,  $c$  denotes the speed of light, and  $I_{15}$  is the laser intensity in units of  $10^{15}$  W/cm<sup>2</sup>. Because  $\lambda_D$  is nonnegligible compared to  $L_y$ , plasma expansion has an appreciable effect due to the sheath field unless  $I_{15} = \mathcal{O}(1)$ . We choose pump ( $I_p$ ) and seed ( $I_s$ ) laser intensities to be  $I_p = I_s = 0.5 \times 10^{15}$  W/cm<sup>2</sup> at the entering boundaries. When we double  $I_s$ , noticeable spontaneous backscattering is observed for the seed laser. When we halve  $I_s$  or double  $I_p$ , instabilities seeded by PIC noise start to compete. When we halve  $I_p$ , a larger  $L_x$  is needed to achieve a moderate gain for CBET. However, a larger  $L_x$  also allows more growth of PIC-noise seeded instabilities, and is thus unfavorable. Therefore, our data analysis protocol and simulation results are applicable when the pump and seed laser intensity is within the range  $10^{14}$  W/cm<sup>2</sup>  $\ll I \ll 10^{16}$  W/cm<sup>2</sup>. We scan numerical resolutions and find that using twenty particles per cell and twenty cells per pump wavelength is often sufficient to achieve convergence.

### 3. Data Analysis

From raw PIC data, we extract the seed laser fields using a Fourier filter. Suppose  $E(x, y)$  is the data of an electric field component; we take a 2D discrete Fourier transform to find  $\hat{E}(k_x, k_y)$ . Apart from peaks corresponding to the seed laser, the complex-valued  $\hat{E}$  includes peaks due to the incident pump laser, the reflected pump, and noise-seeded instabilities. We select the seed laser peaks using a bandpass filter, which applies a weight  $w(k_x, k_y)$  to  $\hat{E}$ . The weight is  $w = 1$  within two square windows centered at  $k_x = \pm k_0 \equiv \pm 2\pi/\lambda_0$  and  $k_y = 0$ . The windows have widths  $\Delta k_x = 5\%k_0$  and  $\Delta k_y = 2\pi/L_y$ . Outside the windows, the weight has a Gaussian profile, which falls off from 1 to 0 with standard deviations  $\sigma_{k_x} = \Delta k_x$  and  $\sigma_{k_y} = \Delta k_y$ . This Gaussian bandpass filter removes most waves related to the pump laser, as well as the sheath electric field and background magnetic field, while retaining waves related to the seed laser. After filtering, we apply the inverse discrete 2D Fourier transform on  $w\hat{E}$  to reconstruct  $E_s(x, y)$ , the real-valued seed field in the configuration space. Because the seed field is approximately a plane wave, we reduce the 2D data to 1D data by integrating along the  $y$  direction. The 1D seed field is  $\tilde{E}(x) = \int_{-h}^h dy E_s(x, y)$ , where  $h = f_y L_y/2$  and we vary the fraction  $0 < f_y < 1$  to estimate uncertainties.

Using 1D seed fields  $\tilde{E}_y$  and  $\tilde{E}_z$ , we synthesize fields of two underlying eigenmodes, which are non-degenerate in magnetized plasma. When the laser frequency is much larger than the electron cyclotron frequency, the two eigenmodes are approximately transverse. Because the seed propagates along the  $x$  direction, complex unit polarization vectors of the two orthogonal eigenmodes can be parameterized as

$$\hat{\mathbf{e}}_1 = (0, \cos \phi, e^{i\psi} \sin \phi), \quad (1)$$

$$\hat{\mathbf{e}}_2 = (0, \sin \phi, -e^{i\psi} \cos \phi), \quad (2)$$

where the polarization angles  $\phi$  and  $\psi$  are computed using a warm-fluid theory [90,91]. At  $B = 0$ , the eigenmodes  $\hat{\mathbf{e}}_1$  and  $\hat{\mathbf{e}}_2$  are right- and left-handed circularly polarized with

respect to  $\mathbf{k}$ . At  $B \neq 0$ , we identify  $\hat{\mathbf{e}}_1$  with the X mode, and  $\hat{\mathbf{e}}_2$  with the O mode. These two mode branches reduce to the standard X and O modes when the wave propagates perpendicular to the magnetic field, where the X-mode electric field is perpendicular to the background magnetic field  $\mathbf{B}$  while the O-mode electric field is parallel to  $\mathbf{B}$ . On the other hand, when the wave propagates parallel to  $\mathbf{B}$ , the X-mode branch becomes the R wave, which is right-handed circularly polarized with respect to  $\mathbf{B}$ , while the O-mode branch becomes the L wave, which is left-handed circularly polarized with respect to  $\mathbf{B}$ . From the unit polarization vectors, the complex electric field at a snapshot in time is  $\mathbf{E}_C = \mathcal{E}_1 \hat{\mathbf{e}}_1 e^{ik_1 x} + \mathcal{E}_2 \hat{\mathbf{e}}_2 e^{ik_2 x}$ . Denoting the complex envelope as  $\mathcal{E} = A e^{i\alpha}$ , where  $A$  and  $\alpha$  are amplitude and phase of the envelope, the real field  $\mathbf{E} = \text{Re } \mathbf{E}_C$  has components

$$\tilde{E}_y = A_1 \cos \phi \cos \theta_1 + A_2 \sin \phi \cos \theta_2, \quad (3)$$

$$\tilde{E}_z = A_1 \sin \phi \cos(\theta_1 + \psi) - A_2 \cos \phi \cos(\theta_2 + \psi), \quad (4)$$

where  $\theta_j \equiv k_j x + \alpha_j$  and  $j = 1, 2$ . If the simulation domain is long enough to resolve the difference of the two eigenmode wavevectors, namely, when  $\Delta k = (k_1 - k_2)/2 \gg 1/L_x$ , Fourier transforms of simulation data will show two distinct peaks, and the two eigenmodes are separable using a Fourier filter. However, our simulations are in the regime  $\Delta k \sim 1/L_x$ , where the two Fourier peaks merge. In this regime, a different scheme is needed to separate the two eigenmodes: We approximate the two modes as having a common  $\bar{k} = (k_1 + k_2)/2$ . We then reconstruct the eigenmodes from linear combinations of  $\tilde{E}_y$  and  $\tilde{E}_z$ . Because  $\tilde{E}_y$  and  $\tilde{E}_z$  are not in phase, to linearly combine them, we shift  $\tilde{E}_z \rightarrow \tilde{E}'_z(x) = \tilde{E}_z(x - \Delta x)$ , where  $\bar{k} \Delta x = \psi$ . After the shift, the field components become in-phase, so the eigenmode fields can be reconstructed by

$$\tilde{E}_1 \equiv A_1 \cos \theta_1 \approx \cos \phi \tilde{E}_y + \sin \phi \tilde{E}'_z, \quad (5)$$

$$\tilde{E}_2 \equiv A_2 \cos \theta_2 \approx \sin \phi \tilde{E}_y - \cos \phi \tilde{E}'_z. \quad (6)$$

While  $\tilde{E}_y$  and  $\tilde{E}_z$  are subject to effects like Faraday rotation, envelopes of the eigenmode fields  $\tilde{E}_1$  and  $\tilde{E}_2$  change along the  $x$  direction primarily due to CBET.

Using eigenmode fields  $\tilde{E}_1$  and  $\tilde{E}_2$ , we estimate CBET energy gain when simulations have reached quasi-steady state. For eigenmode  $j$ , the energy gain at time  $t$  is defined as

$$g_j(t) = \ln \frac{U_j(t)}{U_j(t_0)}, \quad (7)$$

where the electric-field areal energy density is  $U_j = \frac{\epsilon_0}{2} \int_0^{L_x} dx \tilde{E}_j^2$ , and  $t_0$  is the time when the seed laser has completely filled the simulation domain, but the pump laser has just begun to enter. In our setup, the seed laser begins to enter at  $t = 0$ , so  $t_0 \approx 0.6$  ps, which is the time for the front the seed laser to reach the other end of the simulation box. A quasi-steady state is reached around  $t_1 = 3$  ps, and  $U_j$  remains approximately the same around  $t_2 = 3.6$  ps. In PIC simulations, because the distribution functions evolve on a slower time scale, there is no true-steady state, and we use both  $g_j(t_1)$  and  $g_j(t_2)$  to estimate uncertainties. Similar to eigenmode energy gains, we also compute the total gain, which is defined using the total areal energy density  $U = U_1 + U_2 = U_y + U_z$ . We verify that these two ways of computing  $U$  give consistent results. The energy gain  $g$  is closely related to the amplitude gain  $G$  in CBET theory, in which one finds the imaginary part of the wavevector at a given real frequency. Suppose the complex wavevector is  $k = k_R + ik_I$ , then the complex field is  $E_c \propto e^{ikx} = e^{ik_R x} e^{-k_I x}$ . When  $k_I < 0$ , the envelope grows exponentially in  $+x$ . The envelope

gain is defined as  $G = -2k_I L_x$ . Integrating an exponential envelope in space, the energy gain is  $g = \ln F(G)$ , which is related to the amplitude gain  $G$  by

$$F(G) = (e^G - 1)/G. \quad (8)$$

In the limit  $G \rightarrow 0$ , the energy gain is  $g \simeq G/2$ . In our PIC simulations,  $g \simeq 1$  is moderate. In this regime, pump depletion is insignificant, partly also because the pump freshly enters and exits from the transverse domain boundaries with a transit length smaller than  $1/k_I$ . However, we notice that the seed eigenmode envelopes have noticeable differences from exponential profiles, possibly due to competing instabilities in PIC simulations, as well as artifacts from our data analysis method, which introduces distortions especially close to boundary points due to Fourier filtering. We report the energy gain, which is well-defined regardless of the profiles. The energy gain is also what is typically measured in experiments.

To compare theory with PIC simulations and experiments, we also take into account finite plasma size effects that cause blurring of the gain curve, namely,  $g$  as a function of the detuning  $\Delta\lambda = \lambda - \lambda_0$ . To understand the blurring, suppose a laser enters from a vacuum with frequency  $\omega$ . In an infinite plasma, the laser wavevector would be  $k$  from the linear dispersion relation. However, in a plasma of a finite size  $L_x$ , the modes are discrete with  $\Delta k = 2\pi/L_x$ . When  $k$  does not match the wavevector of one discrete mode, the laser excites multiple modes. When  $\Delta k \ll k$ , the laser approximately excites two adjacent modes with  $k_- < k < k_+$ . The complex laser field is a linear superposition  $E_C \approx \mathcal{E}(f_+ e^{ik_+ x} + f_- e^{ik_- x})$ , where the amplitudes are partitioned as

$$f_+ = \frac{k - k_-}{\Delta k}, \quad f_- = \frac{k_+ - k}{\Delta k}, \quad (9)$$

with  $f_+ + f_- = 1$ . In particular, when  $k = k_-$ , the partition is  $f_+ = 0$  and  $f_- = 1$ , so only the  $k_-$  mode is excited, and similarly for  $k = k_+$ . Using Equation (8), the areal energy density of the superimposed excitation is

$$U = \frac{\epsilon_0}{4} |\mathcal{E}|^2 L_x \left[ f_+^2 F(G_+) + f_-^2 F(G_-) + 2f_+ f_- F\left(\frac{G_+ + G_-}{2}\right) \langle \cos(\delta k x) \rangle \right], \quad (10)$$

where  $\delta k = k_R^+ - k_R^-$  is on the order of  $\Delta k$ , but is modified by the nonlinear dispersion relation due to CBET. The average is defined as  $\langle \dots \rangle \equiv \frac{1}{L_x} \int_0^{L_x} dx \dots$ . Because  $\delta k = \mathcal{O}(1/L_x)$ , the average  $\langle \dots \rangle$  is not zero, but is rather sensitive to values at the boundary points. Nevertheless, the average is bounded  $|\langle \dots \rangle| \leq \frac{2}{L_x \delta k}$ . Because two discrete modes are excited, the total gain is different from their respective single-mode gains due to an averaging effect. Therefore, compared to the gain curve  $g(\Delta\lambda)$  in the continuum,  $g(\Delta\lambda)$  in a finite-size plasma is broadened. This finite-size broadening effect can be estimated using Equation (10) when comparing numeric  $g$  with analytic  $G_{\pm}$ .

To map out CBET gain curves, we fix the pump laser vacuum wavelength  $\lambda_0$ , and scan the seed laser vacuum wavelength  $\lambda$ . In our simulations, the seed is launched from the boundary using an antenna driven at frequency  $\omega = 2\pi c/\lambda$ . Because of a fine time resolution,  $\Delta\omega = \omega_0 - \omega$  is resolved, even though the spatial domain size can only resolve  $\Delta k = 2\pi/L_x$ , which limits  $\Delta\lambda = \lambda\lambda_0/L_x = \lambda/500 \approx 7\text{\AA}$ . As mentioned above, increasing  $L_x$ , which is already on par with interaction lengths typically used in experiments, is unfavorable due to competing PIC processes.

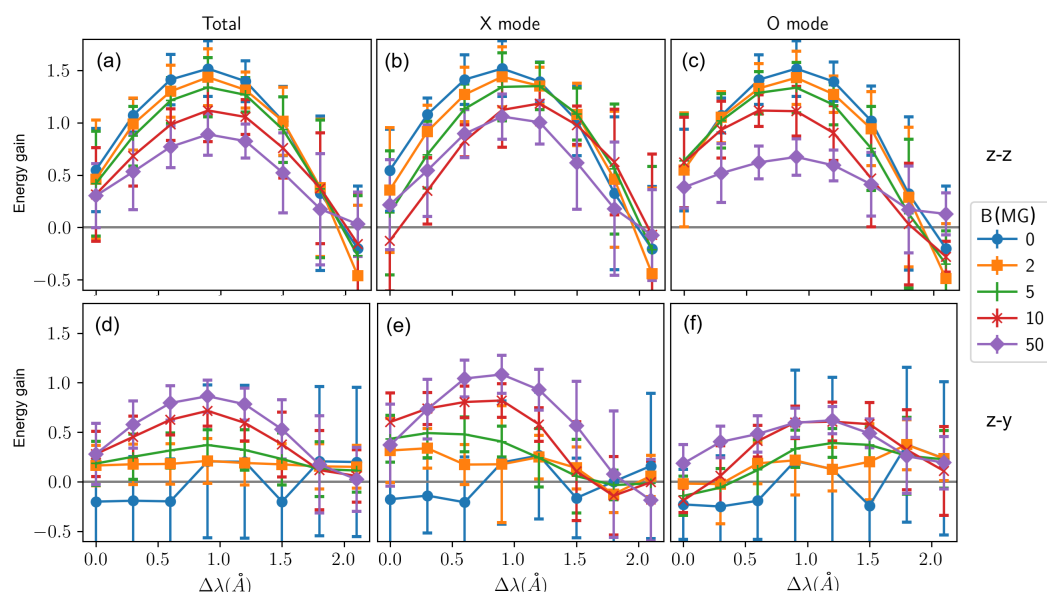
In the following Section 4, we report  $g$  with error bars that include two effects. First, due to the competition of CBET with other PIC processes, the gain is not in a true steady state. To estimate the uncertainty due to slow deviations away from the steady state, we

measure the gains at two simulation time  $t_1 = 3$  ps and  $t_2 = 3.6$  ps. Second, due to the competition between Gaussian focusing and finite-aperture diffraction, the seed laser is not a plane wave. To estimate the uncertainty due to the weak  $y$ -dependence of the wave field  $\tilde{E}(x)$ , we vary the fraction  $f_y = 1, 0.75, 0.5$  when computing the energy gain. The reported error bar for  $g$  is the total spread due to these variations. The significant spreads in  $t$  and  $f_y$  lead to large error bars, which are nevertheless surpassed by the trends we observe. The data analysis code and default parameters are available in Ref. [92].

#### 4. Results

In this paper, we only show a representative subset of our data, and a more complete set of post-processed PIC data is available in Ref. [88]. First, we check our simulation setup and data analysis method using the  $B = 0$  case, which serves as a baseline for comparing magnetized cases. For simulations using the  $z$ - $y$  polarization setup, we verify that  $g$  is nearly flat as a function of  $\Delta\lambda$ , which agrees with the expectation that orthogonally polarized lasers have zero CBET in unmagnetized plasmas. However,  $g(\Delta\lambda)$  on average has a negative offset  $g_0 \approx -0.4$  due to losses from competing PIC processes, such as back and side scattering of the seed laser. To estimate how much competing processes depend on magnetization, we measure side-scattered light that leaves from the  $+y$  domain boundary. The scattered energy varies between 0.05% and 0.17% when  $B$  changes from 0 to 20 MG. Because CBET is the dominant process, all gain values we report hereafter have the common offset  $g_0$  removed. For the  $z$ - $z$  polarization, the gain curve shows a peak near the IAW resonances. In the small-gain regime, we verify that  $g(-\Delta\lambda) \approx -g(\Delta\lambda)$  is approximately an odd function, as expected from CBET theory. When the gain is larger, because of Equation (8), the energy gain is no longer an odd function, but  $g(\Delta\lambda = 0)$  remains close to zero. We verify that when the pump intensity  $I_p$  is halved,  $g$  is halved for  $\Delta\lambda > 0$ , because  $G$  is proportional to the pump intensity. On the other hand,  $g$  remains unchanged for  $\Delta\lambda < 0$ , because for negative  $\Delta\lambda$ , the roles of pump and seed lasers are swapped. Likewise, we verify that when the seed intensity  $I_s$  is doubled,  $g$  remains unchanged for  $\Delta\lambda > 0$ , whereas  $g$  is doubled for  $\Delta\lambda < 0$ , consistent with CBET theory. After these basic checks, we now focus on the positive half of the gain curve when discussing magnetized cases.

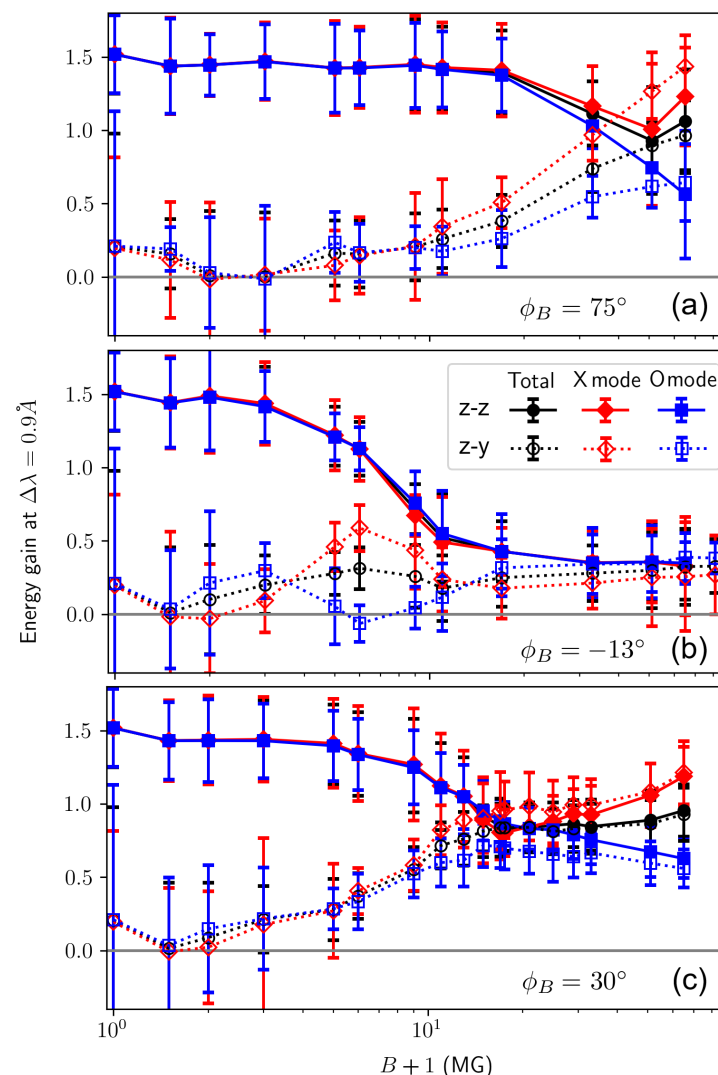
Second, we map out CBET gain curves for three magnetized cases for both polarizations. In all cases, we use  $\theta_B = 90^\circ$ , which means that  $\mathbf{B}$  is in the  $x$ - $y$  plane, namely, the crossing plane of the pump and seed lasers. (i) The case  $\phi_B = 75^\circ$  is when the plasma wavevector  $\Delta\mathbf{k} = \mathbf{k}_0 - \mathbf{k}$  is nearly parallel to the background magnetic field. In this case, the plasma wave that mediates CBET is essentially unmagnetized. Magnetization effects thus arise primarily from modifying the pump and seed lasers. The modifications are smaller than the error bars until  $B \sim 10$  MG, when the electron cyclotron frequency is no longer negligible compared to the laser frequency. As  $B$  increases, the gain for  $z$ - $z$  polarization reduces, while the gain for  $z$ - $y$  polarization increases. (ii) The case  $\phi_B = -13^\circ$  is when the angle  $\langle \Delta\mathbf{k}, \mathbf{B} \rangle \approx 88^\circ$  is near perpendicular. We avoid the exact perpendicular case, because it is a special case where some interactions may be exactly suppressed [26,93,94]. In this near-perpendicular case, the plasma wave is magnetized, and magnetization effects on CBET start to become discernible for  $B \sim 1$  MG. When  $B$  increases, the peak of the gain curve shifts and the gain curve generally becomes broader. While the gain for  $z$ - $z$  polarization reduces, for the  $z$ - $y$  polarization, the gain remains close to zero. (iii) The case  $\phi_B = 30^\circ$  is at an intermediate angle  $\langle \Delta\mathbf{k}, \mathbf{B} \rangle \approx 45^\circ$ . The gain curves in this intermediate case are shown in Figure 2, which have features that resemble the two extreme cases: similar to the  $\Delta\mathbf{k} \parallel \mathbf{B}$  case, the  $z$ - $z$  gain decreases, while the  $z$ - $y$  gain increases with  $B$ . Additionally, similar to the  $\Delta\mathbf{k} \perp \mathbf{B}$  case, magnetization effects begin to manifest at a few MG, and the gain curves shift and broaden at higher  $B$ .



**Figure 2.** Energy gain  $g(\Delta\lambda)$  for MagCBET at  $\theta_B = 90^\circ$  and intermediate angle  $\phi_B = 30^\circ$ . (a–c) For  $z$ - $z$  polarization, the gain reduces with  $B$ , and the IAW peak broadens and shifts. (d–f) For  $z$ - $y$  polarization, the gain increases with  $B$ . The total gain (a,d) is within the X-mode gain (b,e) and O-mode gain (c,f). The gain at other  $\theta_B$  and  $\phi_B$  are qualitatively similar.

Third, we scan the CBET gain as a function of the magnetic field strength at fixed  $\Delta\lambda = 0.9 \text{ \AA}$ , which is near the peak of the gain curves. The scans more clearly illustrate the dependence of  $g$  on  $B$ . To show the unmagnetized case on a log scale, we plot  $B + 1 \text{ MG}$  for the abscissa, so the leftmost points correspond to  $B = 0$ . We again focus on the same three cases. (i) When  $\Delta\mathbf{k}$  and  $\mathbf{B}$  are near parallel ( $\phi_B = 75^\circ$ ), the gains remain approximately constant until  $B \sim 10 \text{ MG}$ , as shown in Figure 3a. At weaker  $B$ , the X-mode gain (red diamonds) and O-mode gain (blue squares) are the same, and equal to the total gain (black circles). In other words, the pump laser transfers the same amount of energy to both eigenmodes, which are nearly degenerate in weak magnetic fields. At larger  $B$ , the two eigenmodes become non-degenerate, and their gains split from the total gain. Gains for the X mode, which is right-handed with respect to  $\mathbf{B}$ , is larger than gains for the O mode, which is left-handed with respect to  $\mathbf{B}$ . This is expected because the X mode couples more strongly to electron motion, which is also right-handed and has a larger response. The gains for the two polarizations meet at  $B \sim 50 \text{ MG}$  and around half the unmagnetized  $z$ - $z$  gain. The crossing point depends on plasma conditions and the interaction length. (ii) When  $\Delta\mathbf{k}$  and  $\mathbf{B}$  are near perpendicular ( $\phi_B = -13^\circ$ ), the gain for  $z$ - $z$  polarization decreases more rapidly with  $B$ , while the gain for  $z$ - $y$  polarization remains close to zero, as shown in Figure 3b. Beyond  $B \sim 10 \text{ MG}$ , the gains drop below  $\sim 30\%$  of the unmagnetized gain. In this near-perpendicular case, magnetization effects primarily arise from modifying the plasma wave, which provides a smaller coupling and is damped more heavily because cyclotron motion introduces additional phase mixing effects into the wave motion. The gain is insensitive to either X or O modes of the laser, whose gains are close to the total gain. Kinetic theories [29,74] predict that at near perpendicular angles, additional resonances due to, for example, Bernstein waves also arise. We will report PIC simulations of these non-IAW-mediated CBET in a separate paper. (iii) When  $\Delta\mathbf{k}$  and  $\mathbf{B}$  are at an intermediate angle ( $\phi_B = 30^\circ$ ), magnetization effects arise both from modifying the lasers and the mediating plasma wave, and the gains have an intermediate behavior, as shown in Figure 3c. Similar to the perpendicular case, magnetization effects begin to manifest at a few MG. Moreover, similar to the parallel case, the  $z$ - $z$  gain decreases, while the  $z$ - $y$  gain increases with  $B$ .

The crossing point is at a smaller  $B$  compared to the parallel case. The X-mode gain is larger than the O-mode gain, which sandwich the total gain.



**Figure 3.** Energy gain near the peak of the gain curves at  $\theta_B = 90^\circ$ . (a) When the plasma wave  $\Delta \mathbf{k}$  is nearly parallel to magnetic field  $\mathbf{B}$ , the gains remain nearly constant until  $B \approx 10$  MG, where laser modes become magnetized. (b) When  $\Delta \mathbf{k}$  is near perpendicular to  $\mathbf{B}$ , the plasma wave is magnetized. The gains are small at large  $B$  and are insensitive to laser modes. (c) When  $\Delta \mathbf{k}$  is at an intermediate angle to  $\mathbf{B}$ , both effects are at play, and the gains have features that resemble the two extreme cases.

Finally, we also perform simulations under different plasma conditions and interaction geometries and find qualitatively similar behavior. The additional simulations are for fully ionized hydrogen plasmas at density  $n_e = 3 \times 10^{20} \text{ cm}^{-3}$ . The CBET crossing angle is  $\alpha = 24^\circ$ . The field angles are  $\theta_B = 45^\circ$  and  $\phi_B = 15^\circ$ , so that  $\mathbf{B}$  has a component out of the crossing plane, and is at a general angle with respect to all waves participating in CBET. We map out gain curves at two temperatures. The gain at a lower temperature  $T_e = 4T_i = 0.4 \text{ keV}$  is overall larger than the gain at a higher temperature  $T_e = 4T_i = 1.2 \text{ keV}$ , similar to the unmagnetized case. Near the peak of the gain curves, the  $z$ - $z$  gain decreases while the  $z$ - $y$  gain increases with increasing  $B$ . At the lower temperature, the peak unmagnetized  $z$ - $z$  gain is  $g \approx 2.1$ , and  $B = 5$  MG already noticeably suppresses the gain to a peak value of  $g \approx 1.8$ . The  $z$ - $z$  and  $z$ - $y$  gains cross near  $B \approx 18$  MG. At  $B = 30$  MG, the gains are approximately 30% of the unmagnetized  $z$ - $z$  gain. In comparison, at the higher temperature, a larger  $B = 10$  MG is needed for noticeably suppressing

the already-small gain, where the peak unmagnetized gain  $g \approx 0.8$  is suppressed to  $g \approx 0.6$ . These additional data, not shown in this paper, can be found in Ref. [88].

## 5. Discussion

We perform 2D-3V PIC simulations of MagCBET at fixed plasma density and temperature, which isolate direct magnetization effects on CBET. Starting from MG fields, direct magnetization effects arise from modifying the mediating IAW quasimode when it propagates at an angle with  $\mathbf{B}$ . Additionally, starting from about 10 MG, direct magnetization effects arise from modifying the electromagnetic eigenmodes that underlie the initially linearly polarized lasers commonly used in experiments. When two eigenmodes become nondegenerate, they mediate CBET at different rates, and the overall CBET is a superposition of eigenmode interactions. In experiments, turning on an external magnetic field often changes plasma conditions. Appreciable indirect magnetization effects on CBET start at a lower field, and we will report our experimental findings in a forthcoming paper.

In this paper, we choose simulation parameters so that IAW-mediated CBET is the dominant process and remains within the linear regime, where pump depletion and distribution-function evolution are negligible. Outside this regime, CBET coexists with other LPI processes, and the interactions are more complicated. The linear regime allows for a direct comparison with theory, which we will report in more detail in our companion paper [74]. At a qualitative level, our PIC simulations are consistent with the theory, which predicts three general trends as the magnetic field strength increases. First, when pump and seed laser polarizations are initially parallel, the gain decreases, and the gain curve broadens and shifts. Second, when the polarizations are initially perpendicular, in which case unmagnetized CBET is forbidden, we find that the gain increases beyond zero. The magnetic field at which the gains of the two polarizations cross depends on plasma conditions and interaction geometry. Third, at a given polarization, X-mode gain is often larger than the O-mode gain, which is split from the total gain. Overall, magnetization has a tendency to suppress CBET.

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**Data Availability Statement:** The MATLAB (version R2025b) code for performing data analysis is openly available [92], along with example EPOCH input decks and Linux shell scripts for setting up batch simulations. The data underlying Figures 2 and 3 are openly available on Zenodo [88], along with additional data discussed but not shown in this paper.

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