

Article

Discontinuous Adoption Dynamics and Token Valuation in Blockchain Platforms

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Abstract

This paper studies platform environments in which participation incentives change discontinuously when valuation crosses salient thresholds. Empirical research on digital and tokenised platforms documents valuation plateaus, abrupt participation shifts, and weak short-run links between price stability and usage dynamics, patterns that are difficult to reconcile with smooth adjustment models. We capture these features structurally by modelling adoption and valuation as a coupled equilibrium system with regime-dependent incentives, formulated as a Filippov differential inclusion. Token prices are determined endogenously through market clearing between usage-driven demand and regime-dependent speculative demand. We establish global well-posedness and identify conditions under which valuation thresholds become attracting manifolds. In these regimes, prices remain anchored at the threshold while adoption continues to evolve, generating persistent valuation plateaus and path dependence. When threshold crossings are regular and the induced dynamics are uniformly dissipative, endogenous boom–bust cycles are ruled out. The framework yields design-relevant insights for digital platforms. Valuation anchoring and volatility depend on governance primitives such as effective circulating supply and regime-dependent speculative depth. A diagnostic numerical analysis links time-at-anchor and sliding intensity to adoption volatility and platform risk, providing empirically interpretable indicators of stability in tokenised platforms.

Keywords: dynamic token valuation; discontinuous adoption dynamics; network effects; non-smooth dynamical systems; sliding modes; speculative behaviour; cryptocurrency volatility; behavioural finance; threshold switching; platform economics

1. Introduction

Digital platforms are a dominant organisational form in the contemporary economy. Their economic significance derives not merely from the provision of digital services but from their role as institutional environments that govern interaction among heterogeneous participants. Platform architecture, pricing rules, token design, and governance arrangements jointly shape incentives, expectations, and patterns of coordination, thereby influencing how value is created, allocated, and sustained within the ecosystem [1].

A defining characteristic of platform economies is their networked structure. Economic activity takes place on interaction networks whose topology influences information transmission, adoption dynamics, innovation diffusion, and systemic resilience. Classical results on random, small-world, and scale-free networks demonstrate that distinct network architectures generate fundamentally different aggregate outcomes [2–4]. Digital platforms



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should therefore be viewed not simply as markets but as evolving network economies whose performance depends jointly on institutional design and network organisation.

Within such environments, prices perform a coordination function in addition to their conventional allocative role. In two-sided markets, the structure of prices rather than their absolute level determines the depth and composition of platform participation [5,6]. Subsequent research shows that pricing policies, participation decisions, and ecosystem expansion evolve jointly over time [7,8], while platform design influences developer entry, innovation incentives, and user engagement [9]. Valuation is therefore not merely an outcome of platform activity but an endogenous determinant of future platform development.

This feedback between valuation and economic behaviour has close parallels in financial markets, where prices influence beliefs and beliefs influence subsequent participation decisions [10]. Tokenised platforms amplify this mechanism because valuation simultaneously serves as a market price, a signal of ecosystem viability, and a public indicator of governance credibility. As a consequence, token valuation can affect not only the state of the platform economy but also the incentives governing future participation.

Tokenised and blockchain-based platforms intensify the interaction between valuation and coordination. In decentralised ecosystems, trust, governance, and economic activity are mediated through a tradable token whose price aggregates information about usage demand, expected future participation, governance credibility, and speculative beliefs. The unity property identified by [11] formalises this feature by showing that the same token simultaneously prices access to platform services and provides incentives for infrastructure provision. As a consequence, valuation, participation, and security become jointly determined equilibrium objects rather than separable dimensions of platform performance [12,13]. Empirical evidence further suggests that network activity, staking behaviour, and usage intensity are systematically capitalised into token valuations [11,14].

This economic role places tokens at the intersection of platform economics and monetary theory. Like money, tokens facilitate transactions, provide liquidity services, and support coordination among decentralised agents. Token demand therefore inherits important features of classical theories of money demand, transaction balances, and liquidity provision [15–18]. Unlike traditional monetary assets, however, token valuations are often directly linked to governance rights, staking incentives, and ecosystem participation, creating additional feedback between market valuation and economic activity.

Recent research has consequently expanded beyond token pricing to examine governance design, protocol competition, decentralised organisational structures, and token-mediated coordination [19–25]. At the same time, a broader finance literature emphasises how valuation dynamics depend on liquidity conditions, intermediary constraints, balance-sheet capacity, and participation incentives [26–31]. Together, these contributions highlight a common theme: valuation is not merely an outcome of economic activity but an object that can influence the subsequent evolution of the system itself.

Existing models provide powerful frameworks for understanding platform adoption, token valuation, governance incentives, and ecosystem growth. However, they typically maintain a common behavioural adjustment mechanism throughout the state space. Participation, valuation, and liquidity may evolve endogenously, but the law governing adjustment remains smooth. What these approaches generally do not accommodate is the possibility that the participation-response mechanism itself changes once a publicly observed valuation threshold is crossed. In such environments, the relevant object is no longer a single adjustment law but a collection of regime-dependent behavioural responses connected by endogenous threshold switching.

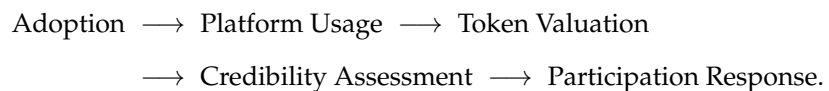
Empirically, token markets frequently exhibit valuation plateaus, price clustering near salient levels, abrupt participation reversals, and weak short-run relationships between

usage and valuation [32,33]. Adoption may continue to evolve while prices remain nearly unchanged. Such patterns are consistent with coordination environments in which incentives are revised discretely once commonly observed credibility or viability thresholds are crossed [34,35].

This paper develops an equilibrium theory of tokenised platforms in which valuation thresholds operate as endogenous coordination boundaries. We study environments in which crossing a governance-sensitive valuation threshold P_c changes the participation-response mechanism itself. The threshold therefore affects not only the state of the platform economy but also the transition rule governing its subsequent evolution.

The economic mechanism is straightforward. Market participants use publicly observable token prices as signals of ecosystem viability, governance credibility, treasury sustainability, validator profitability, and future development capacity. Because these assessments are inherently forward-looking, gradual changes in fundamentals may trigger discrete revisions in participation incentives once a commonly recognised threshold is crossed. Valuation thus becomes a coordination device that partitions the state space into regions governed by different behavioural responses.

The mechanism operates through a feedback loop that is distinctive to tokenised platform ecosystems:



Because token valuation simultaneously functions as a market outcome, a governance signal, and a publicly observable indicator of ecosystem viability, it influences how participants assess the future prospects of the platform. Crossing a credibility threshold P_c may therefore alter staking incentives, validator participation, liquidity provision, development activity, and speculative engagement. Valuation consequently affects not only the current state of the platform economy but also the behavioural responses that govern its subsequent evolution.

The central implication is that participation incentives need not be governed by a single adjustment mechanism throughout the state space. Instead, distinct behavioural laws may operate on either side of P_c , generating regime-dependent equilibrium dynamics. The contribution of the paper is to characterise the economic and dynamic consequences of such threshold-dependent participation responses.

Formally, we represent the resulting equilibrium law as a piecewise-smooth dynamical system in which behavioural responsiveness changes discretely when valuation crosses P_c . The induced evolution is described by a Filippov differential inclusion [36,37]. More broadly, the analysis connects platform economics with the theory of nonlinear, hybrid, and discontinuous dynamical systems [38], providing a tractable framework for studying endogenous regime switching and threshold-dependent equilibrium adjustment.

The framework accommodates abrupt re-optimisation while preserving well-defined forward trajectories and economically meaningful state constraints. The resulting equilibrium dynamics exhibit three qualitatively distinct regimes.

1. Valuation anchoring. The credibility threshold P_c becomes an attracting manifold. Valuation remains pinned at P_c while participation continues to adjust.
2. Regime switching. Trajectories repeatedly cross the threshold, generating endogenous switching between valuation regimes and potentially producing persistent boom–bust dynamics.
3. Governance stabilisation. Appropriate governance parameters eliminate recurrent threshold crossings and restore convergence to a stable equilibrium.

A central result is the existence of an explicit anchoring window

$$(a_-, a_+),$$

within which P_c is attracting. The width of this window depends on governance fundamentals, including effective circulating supply $\bar{S}$ and regime asymmetry (β_L, β_R) . Consequently, issuance policy, treasury management, liquidity provision, market-making intensity, and speculative depth influence the geometry of the anchoring region and the conditions under which valuation remains attached to the credibility threshold.

This mechanism differs fundamentally from existing models of platform growth and token valuation. Equilibrium tokenomics frameworks characterise valuation under a common incentive structure [11–13], while platform economics models typically analyse how participation and adoption evolve within a fixed behavioural environment [5,6,9]. By contrast, the present framework allows the participation-response mechanism itself to depend on valuation. The resulting object is therefore not merely an equilibrium valuation rule but a regime-dependent equilibrium law whose qualitative properties are determined by endogenous threshold interactions. Valuation plateaus, anchoring, and recurrent switching arise from threshold-dependent incentive revision rather than from exogenous regime changes, externally imposed shocks, or continuous adjustment dynamics.

The paper makes three contributions. First, it introduces threshold-dependent participation responses as structural equilibrium features of tokenised platforms, yielding a globally well-posed non-smooth dynamical system. Second, it characterises how governance parameters determine whether the platform operates in an anchored, switching, or stabilised regime, thereby linking token design, liquidity conditions, and governance architecture to dynamic stability. Third, it derives operational diagnostics, including threshold-crossing frequencies and time-at-anchor measures, that generate empirically observable implications for valuation persistence and regime transitions.

Because credibility thresholds may be latent and participation is not directly observable, empirical implementation necessarily relies on proxies such as active users, staking participation, transaction activity, wallet counts, validator activity, and related ecosystem measures. The proposed diagnostics should therefore be interpreted as theoretically motivated indicators of the underlying coordination dynamics rather than direct observations of the state variables.

More broadly, the paper contributes to a growing literature examining how valuation, institutional credibility, and endogenous structural change jointly shape economic outcomes. In doing so, it complements related work on dynamic coordination, institutional evolution, and endogenous economic transformation [39–43].

While motivated by blockchain platforms, the underlying mechanism is considerably more general. Any environment in which valuation serves as a publicly observable coordination signal and incentives adjust discretely at salient thresholds may exhibit similar regime-dependent dynamics.

The remainder of the paper proceeds as follows. Section 2 develops the platform ecosystem model and derives the threshold-dependent equilibrium dynamics. Section 3 establishes the main theoretical results on valuation anchoring, regime switching, and governance stabilisation. Section 4 discusses the economic, governance, and empirical implications of the framework. Section 5 concludes. Appendix A documents the alternative adoption specifications used throughout the analysis. Appendix B contains the technical derivations and proofs. Appendix C reports the numerical diagnostics and simulation results.

2. The Platform Ecosystem Model

We model a token-governed digital platform as a governance system that jointly determines ecosystem participation, token valuation, and incentive structure. The platform is not treated as a passive technology but as an institutional architecture in which protocol design, supply policy, and market-access rules shape how participants coordinate and create value.

The ecosystem consists of three tightly coupled layers: First, a network of participants whose engagement exhibits complementarities (developer entry, validator support, user adoption) and congestion (blockspace limits, governance overhead, fee pressure). Second, a protocol-level token economy that encodes access rights, staking commitments, vesting rules, and supply discipline. Third, a token market that aggregates usage demand and portfolio demand into a publicly observed valuation signal.

Let $a_i(t) \in [0, 1]$ denote participant i 's engagement intensity and let $P(t) > 0$ denote the endogenous token price. Participation generates transactional demand for tokens, while governance-controlled supply and market-access conditions shape speculative demand. The resulting valuation feeds back into participation incentives through fee exposure, collateral requirements, and credibility perceptions. Adoption and valuation therefore co-evolve as equilibrium outcomes of platform design.

A central feature of tokenised platforms is that participation incentives may change discontinuously at salient valuation levels. When token valuation crosses a credibility threshold P_c , participants can revise staking decisions, liquidity provision, development activity, and market engagement. The threshold therefore acts as a coordination boundary separating regions of the state space with distinct behavioural responses. Its location is shaped by platform governance, including issuance policy, vesting schedules, staking requirements, disclosure commitments, market-access conditions, and other institutional features.

The joint evolution of participation and valuation, $(a(t), P(t))$, is therefore naturally represented as a piecewise-smooth equilibrium system. We formalise the resulting regime-dependent dynamics through a Filippov differential inclusion [36,37,44–46]. The non-smooth structure is not imposed for mathematical generality but emerges directly from the economic mechanism: once valuation affects the participation-response rule itself, equilibrium adjustment is governed by different laws on either side of the credibility threshold. The Filippov framework accommodates such endogenous regime switching while preserving well-defined forward trajectories and economically meaningful state constraints.

2.1. Platform Participants, Ecosystem Architecture, and Adoption

A tokenised platform coordinates a heterogeneous ecosystem of participants: end-users, developers, validators, liquidity providers, and integrators. Let $i \in \{1, \dots, N\}$ index these actors. Each participant chooses an engagement intensity $A_i(t) \in [0, 1]$, interpreted as effective platform reliance: active usage, staking commitment, developer contribution, integration depth, liquidity provision, validator activity, or transactional dependence. The state A_i summarises repeated participation decisions relative to outside opportunities and should be interpreted as a reduced-form participation index rather than a single observable action.

2.1.1. Ecosystem Architecture and Complementarities

Platform architecture shapes how participants benefit from one another. Interoperability, communication channels, governance exposure, and complementor linkages are

represented by a weighted directed network $G = (V, E)$ with influence weights $w_{ij} \geq 0$. Let $W = (w_{ij})$ denote the adjacency matrix and define the local complementor-activity index

$$S_i(A) = \sum_{j=1}^N w_{ij} A_j, \quad A \in [0, 1]^N. \quad (1)$$

Expression (1) captures ecosystem complementarities. Developer activity increases user value, validator participation strengthens security, liquidity provision improves transaction efficiency, and integration activity expands ecosystem reach. Platform design enters through W , which reflects governance structure, interoperability choices, communication architecture, and institutional connectivity. To ensure bounded exposure, we impose uniformly bounded outgoing influence,

$$\bar{w} = \max_{1 \leq i \leq N} \sum_{j=1}^N w_{ij} < \infty, \quad (2)$$

so $S_i(A) \in [0, \bar{w}]$ for all feasible states.

2.1.2. Participation Value and Governance Channels

Participation depends on a comparison between ecosystem benefits and the costs of maintaining platform involvement. We summarise this trade-off through a reduced-form participation value that aggregates complementarities, congestion effects, and valuation-linked participation costs.

Participant i 's net platform value is

$$V_i(A, P) = \beta_i + \theta S_i(A) - \xi c(\bar{A}) - \chi u(P), \quad (3)$$

where aggregate ecosystem engagement is

$$\bar{A} = \frac{1}{N} \sum_{i=1}^N A_i. \quad (4)$$

Each term in (3) corresponds to a platform design channel. The baseline term β_i captures heterogeneous fit, capabilities, strategic alignment, or mission compatibility. The complementarity term $\theta S_i(A)$ links individual incentives to ecosystem exposure through (1), formalising the network effects central to platform economics. The congestion term $\xi c(\bar{A})$ depends on aggregate load (4) and captures scalability limits such as blockspace congestion, fee pressure, governance coordination costs, or attention constraints. Improvements in throughput, governance efficiency, or protocol architecture shift this channel. The valuation-mediated term $u(P)$ captures how token valuation feeds back into participation. Higher valuations may increase effective participation costs through fee exposure, collateral requirements, staking opportunity costs, regulatory compliance burdens, or perceived systemic risk. Through (3), tokenomics and market outcomes directly influence ecosystem engagement.

2.1.3. Dynamic Adjustment and Regime Dependence

Participation is not assumed to adjust instantaneously. Users, developers, validators, and liquidity providers revise their exposure to the ecosystem gradually as incentives change. We therefore model engagement as a bounded adjustment process in which

participation strengthens or weakens over time rather than jumping immediately between states. Engagement evolves according to

$$\dot{A}_i = A_i(1 - A_i)\Gamma_k(V_i(A, P)), \quad (5)$$

where $k \in \{L, R\}$ indexes the valuation regime. Equation (5) embeds the participation index (3) into a bounded adjustment rule. The normalisation $\Gamma_k(0) = 0$ makes $V_i(A, P) = 0$ the indifference locus. The logistic factor ensures $A_i \in [0, 1]$ and reflects gradual strengthening or weakening of reliance rather than instantaneous entry or exit.

Regime dependence arises through a governance-salient valuation threshold $P_c > 0$,

$$k = \begin{cases} L, & P < P_c, \\ R, & P > P_c. \end{cases} \quad (6)$$

Crossing P_c represents a discrete revision of participation heuristics and market conditions. In practice, such thresholds may correspond to audit completion, compliance milestones, decentralisation targets, security events, treasury sustainability assessments, exchange-access thresholds, or broader reassessments of platform viability.

The location of P_c is itself influenced by governance choices. Changes in treasury management, vesting schedules, exchange access, staking requirements, disclosure commitments, security certification, or institutional credibility may shift the threshold and thereby alter the conditions under which regime transitions occur. The adjustment functions satisfy

$$\Gamma'_L(v) \geq \Gamma'_R(v) \geq 0 \quad \text{a.e. } v \in \mathbb{R}, \quad \Gamma_L \neq \Gamma_R, \quad (7)$$

so marginal responsiveness is weaker in the high-valuation regime. Combined with (3), this captures environments in which elevated valuation amplifies congestion, collateral constraints, governance burdens, or regulatory scrutiny, thereby dampening incremental engagement.

The network formulation permits substantial heterogeneity in local interaction patterns and exposure structures. The aggregate reductions developed later in the paper are exact on the symmetric manifold used in the analytical derivations. More generally, they should be interpreted as governance-level mean-field approximations in which network topology, participant heterogeneity, and diffusion speeds may affect quantitative outcomes without altering the threshold mechanism that drives the analysis.

For each fixed regime k , the vector field defined by (5) is locally Lipschitz on $[0, 1]^N \times (0, \infty)$. Together with bounded exposure in (2), this prepares the ground for the global well-posedness result in Theorem 1.

2.2. Platform Governance, Token Market Design, and Valuation Dynamics

Token valuation in a decentralised platform is not an exogenous market outcome. It is jointly shaped by governance-controlled supply, ecosystem-generated usage demand, and market-access conditions that determine liquidity and speculative depth. The resulting valuation serves not only as a market price but also as a publicly observable coordination signal that influences future participation decisions. Understanding how governance affects valuation is therefore essential for understanding the dynamics of the platform ecosystem itself.

2.2.1. Governance-Controlled Effective Supply

Let $\bar{S} > 0$ denote the effective circulating supply of tokens available for trade. This quantity reflects platform design choices, including issuance schedules, vesting cliffs,

lockups, staking lock requirements, treasury releases, token burns, and listing or custody constraints. Although treated as fixed over the horizon studied, $\bar{S}$ is a direct governance lever rather than an exogenous market parameter. Through the market-clearing mechanism defined below, changes in $\bar{S}$ alter equilibrium valuation, the location of regime boundaries, and the conditions under which threshold-induced transitions occur.

2.2.2. Demand Decomposition and Regime Dependence

Token demand serves two conceptually distinct functions. Participants require tokens to access platform services, execute transactions, stake within the protocol, or interact with ecosystem applications. At the same time, investors, validators, liquidity providers, and speculators may hold tokens for portfolio, collateral, governance, or liquidity purposes. Distinguishing these channels is important because governance interventions can affect usage demand and speculative demand through different mechanisms. Total token demand combines ecosystem usage demand and portfolio demand,

$$D(A, P) = D_U(A, P) + D_{S,k}(P), \quad k \in \{L, R\}, \quad (8)$$

where regime dependence follows the valuation rule (6).

Transactional demand.

Usage demand arises from ecosystem engagement and scales with aggregate participation $\bar{A}$ defined in (4),

$$D_U(A, P) = \alpha \bar{A} \ell(P), \quad \alpha > 0. \quad (9)$$

The function $\ell(P)$ is C^1 , strictly decreasing, and satisfies $\lim_{P \rightarrow \infty} \ell(P) = 0$. It captures how valuation translates into effective participation cost. Higher token prices increase fee exposure, collateral requirements, staking opportunity costs, or required token holdings per unit of activity. Through (9) and the adoption dynamics (5), ecosystem engagement generates endogenous token demand.

Speculative, liquidity, and collateral demand.

In addition to transactional usage, tokens are held for portfolio allocation, liquidity provision, collateral services, governance participation, and speculative purposes. These motives are represented through

$$D_{S,k}(P) = \beta_k s(P), \quad k \in \{L, R\}. \quad (10)$$

The function $s(P)$ is C^1 and strictly decreasing, representing downward-sloping asset demand around perceived fundamental, collateral, or governance value. The regime parameter β_k scales effective market depth and summarises liquidity conditions, market-making intensity, exchange accessibility, institutional participation, collateral acceptance, and related determinants of secondary-market demand. We assume

$$\beta_L > \beta_R > 0, \quad (11)$$

so speculative support is stronger in the low-valuation regime. The ordering (11) captures environments in which high valuations induce greater caution among liquidity providers, tighter collateral standards, reduced speculative support, or more restrictive compliance requirements. Economically, crossing the credibility threshold P_c may reduce risk appetite, alter liquidity provision, tighten collateral constraints, or trigger governance and regulatory reassessments. Through (11) and (6), valuation itself reshapes market depth, creating a second feedback channel between governance, valuation, and participation.

The parameters β_L and β_R are theoretical objects rather than directly observable quantities. Empirical applications would require proxies such as order-book depth, bid–ask spreads, market liquidity measures, trading volume, exchange participation, validator activity, or market-maker presence. The model therefore interprets regime depth as an underlying structural characteristic of the market rather than a directly measured variable.

2.2.3. Token Market Adjustment

Token markets typically adjust through liquidity provision, inventory rebalancing, arbitrage activity, and order-flow absorption rather than through instantaneous Walrasian clearing. We therefore represent price formation through an excess-demand adjustment mechanism. Market clearing operates through

$$g_k(A, P) = \kappa_P [\alpha \bar{A} \ell(P) + \beta_k s(P) - \bar{S}], \quad \kappa_P > 0. \quad (12)$$

Price evolves according to

$$\dot{P} = g_k(A, P), \quad k = \begin{cases} L, & P < P_c, \\ R, & P > P_c. \end{cases} \quad (13)$$

The parameter κ_P is a reduced-form liquidity-adjustment coefficient governing the speed with which excess demand is translated into price movements. Rather than modelling market microstructure explicitly, κ_P summarises the aggregate responsiveness of the token market to order imbalances. Its determinants may include order-book depth, arbitrage activity, market-maker inventory capacity, liquidity provision, trading infrastructure, and related mechanisms. The model does not distinguish these channels individually. Instead, κ_P captures their combined effect on the rate of valuation adjustment.

Equation (13) therefore formalises valuation as a coordination signal that aggregates ecosystem demand, liquidity conditions, and effective token supply. Together with the adoption dynamics (5), Equation (13) closes the participation–valuation feedback loop that underlies the subsequent analysis.

2.2.4. Regime-Wise Equilibrium Valuation

For fixed $\bar{A}$ and regime k , the regime-specific clearing price $P_k^*(\bar{A})$ solves

$$\alpha \bar{A} \ell(P) + \beta_k s(P) = \bar{S}. \quad (14)$$

Equation (14) summarises the balance between governance-controlled effective supply and the two sources of token demand. It therefore provides the valuation benchmark around which the dynamic adjustment process operates. Because $\ell'(P) < 0$ and $s'(P) < 0$, the left-hand side of (14) is strictly decreasing in P , so $P_k^*(\bar{A})$ is unique whenever demand at $P \downarrow 0$ exceeds supply. Implicit differentiation of (14) yields

$$\frac{dP_k^*(\bar{A})}{d\bar{A}} = \frac{\alpha \ell(P_k^*(\bar{A}))}{-\alpha \bar{A} \ell'(P_k^*(\bar{A})) - \beta_k s'(P_k^*(\bar{A}))} > 0.$$

Within each regime, greater ecosystem participation increases transactional demand and therefore supports higher equilibrium valuation. Combined with (3), valuation feeds back into participation incentives, establishing the endogenous participation–valuation coordination loop that drives the platform dynamics. The assumption $s'(P) < 0$ ensures regime-wise uniqueness of (14). Allowing $s'(P) > 0$ locally would capture momentum trading, trend-following behaviour, or speculative feedback effects that can generate self-

reinforcing valuation dynamics within a regime. We maintain monotonicity in order to isolate the threshold-driven coordination effects induced by governance design, liquidity conditions, and effective token supply.)

2.3. Filippov Formulation and Valuation Anchoring

The valuation threshold P_c introduced in Equation (6) is simultaneously a market valuation level and a governance-sensitive coordination boundary. Crossing the threshold may alter staking incentives, liquidity provision, validator participation, governance credibility, and perceived ecosystem viability. Because behavioural responses depend on the valuation regime, equilibrium adjustment cannot generally be represented by a single globally smooth law.

The appropriate equilibrium object is therefore a piecewise-smooth dynamical system whose evolution depends on the side of the threshold occupied by the platform. We formalise this regime dependence using a Filippov differential inclusion [36,37,46]. The non-smooth structure is not imposed for mathematical generality but emerges directly from the economics: once valuation affects the participation-response mechanism itself, equilibrium dynamics are governed by distinct behavioural laws across regimes. This perspective is closely related to analyses of state-dependent adjustment and endogenous structural change in which the evolution of the system depends on the configuration it generates [40–43]. The central object of interest is therefore the equilibrium geometry induced by transitions across valuation regimes rather than equilibrium behaviour within a fixed regime. Let

$$x = (A_1, \dots, A_N, P) \in \mathcal{X} := [0, 1]^N \times (0, \infty),$$

where participation evolves according to (5) and valuation according to (13). Define the switching function

$$h(x) := P - P_c,$$

so that the switching manifold induced by the governance rule (6) is

$$\Sigma := \{x \in \mathcal{X} : h(x) = 0\} = \{P = P_c\}.$$

The manifold Σ represents the set of platform states at which the credibility threshold is exactly attained. At these states, the platform lies precisely at the boundary separating the two valuation regimes. The interaction between the geometry of Σ and the orientation of the regime vector fields determines whether the threshold acts as a switching surface, a repelling boundary, or a valuation anchor. For each regime $k \in \{L, R\}$, define the regime-specific vector field

$$f^{(k)}(x) = \begin{pmatrix} F_k(A, P) \\ g_k(A, P) \end{pmatrix},$$

where $F_k(A, P)$ is determined by the engagement dynamics (5) and $g_k(A, P)$ by the market-adjustment rule (12). The vector field combines two economic forces. The first is ecosystem adjustment generated by the participation incentives in (3). The second is token-market adjustment generated by the interaction of usage demand, speculative demand, and effective circulating supply through (8). Because regime selection in (13) depends discontinuously on valuation, crossing the credibility threshold may induce a discrete change in the governing adjustment law even when underlying fundamentals vary continuously.

We formalise this threshold dependence through the Filippov set-valued map

$$F : \mathcal{X} \rightrightarrows \mathbb{R}^{N+1},$$

defined by

$$F(x) = \begin{cases} \{f^{(L)}(x)\}, & h(x) < 0, \\ \text{co}\{f^{(L)}(x), f^{(R)}(x)\}, & h(x) = 0, \\ \{f^{(R)}(x)\}, & h(x) > 0, \end{cases} \quad (15)$$

where co denotes the convex hull.

A Filippov solution is an absolutely continuous trajectory $x(\cdot)$ satisfying

$$\dot{x}(t) \in F(x(t)) \quad \text{for almost every } t \geq 0.$$

The convexification on the switching manifold has a direct economic interpretation. At the credibility threshold, users, developers, validators, liquidity providers, and speculative traders may revise their behaviour in different directions as the platform transitions between valuation regimes. Simultaneously, market depth, liquidity conditions, and collateral acceptance may adjust discontinuously. The Filippov construction captures these competing adjustment pressures while preserving well-defined forward trajectories through threshold regions.

2.3.1. Governance Geometry at the Credibility Threshold

The central question is whether the credibility threshold attracts trajectories, repels them, or generates repeated switching. This depends on how the token market responds to excess demand on either side of the threshold. At points $x \in \Sigma$, the outward normal to the switching manifold is

$$\nabla h = (0, \dots, 0, 1),$$

so the normal component of the regime vector field is

$$\nabla h \cdot f^{(k)}(x) = g_k(A, P).$$

Because $g_k(A, P)$ is determined by the excess-demand adjustment rule (12), the sign of the normal component is governed by the balance between effective circulating supply $\bar{S}$, participation-generated usage demand, and regime-dependent speculative depth. A point $x \in \Sigma$ belongs to an attracting sliding region if

$$g_L(A, P_c) > 0 \quad \text{and} \quad g_R(A, P_c) < 0. \quad (16)$$

Condition (16) admits a transparent economic interpretation. Below the threshold, excess demand remains positive, creating upward valuation pressure. Above the threshold, excess demand becomes negative, generating downward valuation pressure. Both adjustment mechanisms therefore direct trajectories toward the same governance-sensitive valuation level P_c . The condition also has a direct governance interpretation. Through (12), the signs of g_L and g_R depend on effective circulating supply $\bar{S}$, ecosystem participation, and the regime-depth parameters (β_L, β_R) . Treasury releases, token burns, vesting schedules, staking requirements, market-making arrangements, exchange-access conditions, and liquidity programmes may therefore influence whether the threshold becomes attracting, repelling, or neutral. The existence of a valuation anchor is consequently not purely a market outcome; it is partially shaped by governance design. When (16) holds, Filippov trajectories remain on Σ for nontrivial time intervals. Along such segments,

$$P(t) \equiv P_c,$$

and (13) implies

$$\dot{P} = 0.$$

Valuation therefore ceases to adjust even though participation continues to evolve through (5). The platform economy remains dynamically active, but valuation becomes pinned to the credibility threshold.

2.3.2. Valuation Anchoring as a Governance Outcome

Sliding on Σ corresponds to valuation anchoring. The mechanism does not rely on exogenous intervention, externally imposed price targets, or discretionary stabilisation policies. Instead, it emerges endogenously from the interaction between participation incentives, effective token supply, and regime-dependent liquidity conditions.

From (12), $g_k(A, P_c) = 0$ implies that total token demand in (8) exactly equals effective circulating supply $\bar{S}$ at the credibility threshold. Transactional demand generated through ecosystem participation in (9) and speculative demand generated through (10) offset supply precisely. In this configuration, the governance-defined threshold acts as a coordination anchor. Valuation remains fixed at P_c while ecosystem participation, network exposure, validator commitment, developer activity, liquidity provision, and other forms of engagement continue to evolve. The threshold therefore separates valuation adjustment from ecosystem adjustment. This distinction is economically important. During anchoring, participation remains informative about the underlying condition of the ecosystem, whereas valuation ceases to reveal marginal changes in ecosystem activity. Significant shifts in adoption, developer engagement, staking participation, or network utilisation may occur while the token price remains effectively unchanged.

The model therefore provides a structural explanation for valuation plateaus and persistent price clustering frequently observed in tokenised platform ecosystems. In contrast to smooth adjustment frameworks, these outcomes emerge from endogenous threshold-dependent coordination mechanisms rather than from exogenous shocks, externally imposed regime changes, or temporary market frictions.

Because P_c , β_L , and β_R are theoretical objects, empirical implementation requires observable proxies. Potential indicators include time spent near salient valuation levels, threshold-crossing frequency, order-book depth, bid-ask spreads, liquidity measures, staking participation, validator activity, developer engagement, transaction intensity, or other indicators of ecosystem participation. The theory therefore generates empirically interpretable diagnostics while remaining primarily a methodological and conceptual framework for understanding threshold-dependent platform dynamics.

2.4. Planar Mean-Field Reduction

Platform governance and market participants rarely observe the full participant-level state of the ecosystem. Instead, they monitor aggregate indicators such as active users, transaction volume, developer activity, staking participation, validator engagement, or total value secured. Governance decisions are therefore typically made using aggregate measures of ecosystem health rather than detailed micro-level diffusion paths. To align the analytical framework with this governance perspective, we develop a planar mean-field reduction of the full network system defined by (5) and (13).

The purpose of the reduction is analytical rather than ontological. The underlying platform remains a heterogeneous network of interacting participants governed by (1) and (3). The planar representation does not eliminate heterogeneity, topology, or local interaction effects. Instead, it aggregates these influences into a governance-level description of ecosystem participation and valuation. This allows the analysis to focus on the

regime transitions, anchoring mechanisms, and stability properties that are most relevant for platform design and governance.

We summarise ecosystem participation by a single state variable

$$a(t) \in [0, 1],$$

interpreted as average platform engagement, corresponding to the population mean of A_i in (4). Depending on the empirical context, a may be proxied by active-user ratios, staking participation, developer activity, validator engagement, transaction intensity, or related measures of ecosystem involvement. The variable should therefore be interpreted as a latent aggregate participation index rather than a directly observed quantity.

The reduction corresponds to a homogeneous-mixing approximation in which local exposure in (1) is replaced by its population-average effect and heterogeneity in baseline propensities β_i and network weights w_{ij} is summarised through aggregate parameters. Proposition A1 in the Appendix B shows that this reduction is exact on the symmetric homogeneous-exposure manifold. More generally, it should be interpreted as a governance-level mean-field approximation. Network topology, participant heterogeneity, and diffusion speeds may affect quantitative outcomes without changing the threshold mechanism, valuation anchoring conditions, or regime structure that are the focus of the analysis.

Economically, this shift moves the object of analysis from participant-level diffusion to ecosystem-level coordination. The central question is no longer how a particular participant adopts or exits the platform, but how aggregate participation interacts with token valuation and governance design to generate anchoring, switching, or stabilisation at the ecosystem level.

Crucially, the structural mechanisms encoded in (3) are preserved under aggregation. Network complementarities continue to operate through scale effects, congestion remains linked to aggregate ecosystem load, valuation-mediated participation costs continue to enter through $u(P)$, and governance-sensitive regime shifts remain governed by the threshold rule (6). The reduction therefore simplifies dimensionality without altering the underlying economic mechanisms.

Aggregate participation evolves according to

$$\dot{a} = a(1 - a) \Gamma_k(V(a, P)), \quad a \in [0, 1], \quad (17)$$

which is the aggregate counterpart of (5). Equation (17) describes how ecosystem-wide engagement responds to the net incentives generated by platform participation. As in the full model, adjustment is gradual rather than instantaneous. The logistic factor ensures that participation remains economically admissible and reflects the fact that adoption cannot expand beyond full participation or fall below complete disengagement.

The reduced participation index is

$$V(a, P) = \bar{\beta} + \theta a - \xi c(a) - \chi u(P), \quad (18)$$

obtained from (3) by replacing $S_i(A)$ and $\bar{A}$ with a . Expression (18) preserves the four governance-relevant incentive channels. The parameter $\bar{\beta}$ captures average ecosystem fit and alignment with platform objectives. The complementarity term θa represents aggregate network effects through which broader participation increases ecosystem value. The congestion term $c(a)$ captures scalability constraints, governance overhead, coordination costs, and related frictions that grow with ecosystem size. The valuation channel $u(P)$ captures how token-market conditions influence participation incentives through fees, collateral requirements, staking costs, regulatory burdens, or perceived platform risk.

Token valuation continues to evolve through the governance-controlled excess-demand rule

$$\dot{P} = g_k(a, P) = \kappa_P [\alpha a \ell(P) + \beta_k s(P) - \bar{S}], \quad (19)$$

which is the aggregate counterpart of (12). Equation (19) summarises how ecosystem engagement, market liquidity, and governance-controlled token supply jointly determine valuation dynamics. The parameter $\bar{S}$ represents effective circulating supply and remains directly influenced by issuance schedules, treasury policy, vesting arrangements, lockups, and token burns. The parameters (β_L, β_R) capture regime-dependent liquidity depth and reflect market-making activity, exchange accessibility, institutional participation, and speculative support. The coefficient κ_P governs the speed with which order imbalances are translated into price movements and summarises market responsiveness. Together, (17) and (19) form a closed two-dimensional participation–valuation feedback system in which governance design, market liquidity, and ecosystem engagement jointly determine the evolution of the platform economy.

Regimes remain governed by the credibility threshold

$$k = \begin{cases} L, & P < P_c, \\ R, & P > P_c, \end{cases} \quad (20)$$

which induces the switching manifold

$$\Sigma = \{(a, P) \in [0, 1] \times (0, \infty) : P = P_c\}.$$

The manifold Σ retains the same economic interpretation as in the full model. It represents the governance-sensitive valuation boundary at which participation incentives and market depth may change discretely. Consequently, the planar system inherits the piecewise structure formalised through the Filippov map (15).

The reduced model retains the governance levers most relevant for platform design: effective circulating supply $\bar{S}$, regime-dependent liquidity parameters (β_L, β_R) , and the credibility threshold P_c . These variables correspond directly to observable governance instruments such as issuance policy, treasury management, staking requirements, lockup provisions, market-making programmes, exchange-access conditions, and credibility-enhancing institutional commitments.

The analytical value of the planar reduction is that the geometry of the system can be characterised completely. This permits a precise classification of the possible governance regimes generated by the participation–valuation feedback loop. In particular, the model allows the analysis of regular threshold crossing, valuation anchoring through attracting sliding, endogenous boom–bust switching, and stabilisation through dissipativity conditions. The resulting classifications are expressed directly in terms of governance parameters, making the subsequent results interpretable as statements about platform design rather than merely properties of an abstract dynamical system.

3. Platform Model Analysis and Results

The previous sections developed a threshold-dependent equilibrium model of a token-governed platform in which participation incentives, liquidity conditions, and token valuation interact through a participation–valuation feedback loop. The central economic question is not whether participation and valuation are linked—this follows directly from the platform architecture—but how the geometry of this feedback changes when valuation crosses a governance-sensitive credibility threshold.

The threshold P_c is economically significant because it alters the behavioural and liquidity environment within which adjustment occurs. Crossing the threshold may change staking incentives, liquidity provision, validator participation, speculative support, governance credibility, or perceived ecosystem viability. The resulting platform economy is therefore governed not by a single adjustment law but by a collection of regime-specific dynamics connected through the switching manifold

$$\Sigma = \{(a, P) \in \Omega : P = P_c\}.$$

The objective of the analysis is to classify the equilibrium regimes generated by this threshold structure. The classification is expressed in terms of governance-relevant primitives: effective circulating supply $\bar{S}$, regime-dependent liquidity depth (β_L, β_R) , participation responsiveness through (17), and the credibility threshold rule (20). These quantities determine whether the platform converges smoothly, becomes anchored at the credibility threshold, or exhibits persistent threshold-driven instability.

The analysis is conducted on the state space

$$\Omega = [0, 1] \times (0, \infty),$$

where aggregate participation evolves according to (17) and valuation according to (19). The switching manifold Σ inherits the piecewise structure formalised through the Filippov construction (15), allowing valuation thresholds to function as active coordination mechanisms rather than passive reference levels.

The results establish a complete equilibrium classification of the reduced platform economy. Theorem 1 establishes global well-posedness and forward invariance. This result shows that threshold-dependent governance rules preserve economic admissibility and generate globally defined trajectories within the participation–valuation state space. Theorem 2 identifies conditions under which the credibility threshold becomes an attracting sliding region. In this regime, valuation remains pinned to P_c while ecosystem participation continues to evolve. The threshold therefore acts as an endogenous coordination anchor generated by the interaction of governance-controlled supply, participation demand, and regime-dependent liquidity conditions. Theorem 3 characterises the opposite regime. When the threshold does not absorb adjustment pressures and neither valuation regime contains a stabilising equilibrium, the platform may exhibit persistent threshold-crossing dynamics. Endogenous boom–bust fluctuations then arise from regime switching itself rather than from exogenous shocks. Theorem 4 identifies governance conditions that eliminate such instability. Restrictions on effective supply, liquidity asymmetry, and participation responsiveness restore convergence and prevent threshold-induced amplification.

The results classify token-governed platforms into three qualitatively distinct equilibrium configurations:

- Anchoring regimes, in which credibility thresholds coordinate expectations and stabilise valuation.
- Switching regimes, in which thresholds organise persistent endogenous volatility.
- Stabilised regimes, in which governance design restores global convergence.

The classification is structural rather than event-driven. Equilibrium outcomes are determined by platform design primitives, governance choices, liquidity conditions, and participation incentives rather than by exogenous shocks or ad hoc behavioural assumptions. The geometry of (17)–(19) therefore provides a governance-relevant mapping from design choices to ecosystem outcomes. Proofs are provided in Appendix B. Appendix C illustrates the resulting equilibrium geometries through simulated phase portraits, time-at-

anchor statistics, threshold-crossing frequencies, and robustness diagnostics. The analytical results supply the theoretical foundation for interpreting those diagnostics as structural equilibrium phenomena rather than numerical artefacts.

Before studying valuation anchoring, threshold-induced switching, or governance stabilisation, we first establish that the discontinuous participation–valuation system is globally well defined.

Theorem 1 (Global well-posedness and forward invariance (Filippov)). *Consider the planar switching system (17)–(19) on $\Omega := [0, 1] \times (0, \infty)$ with regime rule (20) and switching manifold $\Sigma = \{P = P_c\}$. Assume the following:*

- (i) Γ_L, Γ_R are globally Lipschitz, strictly increasing, and satisfy $\Gamma_k(0) = 0$.
- (ii) $V(a, P)$ is given by (18) with $c \in C^1([0, 1])$ and $u \in C^1((0, \infty))$ such that $u'(P) > 0$.
- (iii) $\ell, s \in C^1((0, \infty))$ in (19) are strictly decreasing and positive, and $\beta_L > \beta_R > 0$.
- (iv) $\lim_{P \downarrow 0} s(P) = +\infty$.
- (v) There exists $P_{\max} > P_c$ such that

$$\alpha \ell(P_{\max}) + \beta_L s(P_{\max}) - \bar{S} < 0.$$

Then, for every initial condition $(a(0), P(0)) \in \Omega$ there exists a global absolutely continuous Filippov solution to (17)–(19). Moreover,

$$\Omega_{\max} := [0, 1] \times (0, P_{\max}]$$

is forward-invariant.

Theorem 1 establishes that threshold-dependent governance rules preserve the internal coherence of the platform economy. Participation remains economically admissible, valuation remains bounded, and regime switching generates globally defined equilibrium trajectories. As a result, valuation anchoring, threshold-induced fragility, and governance-stabilised convergence arise from the economic mechanism itself rather than from pathologies of the underlying dynamical system. The theorem therefore provides the analytical foundation for the equilibrium classification developed in the remainder of the paper.

The parameters $\bar{S}$, β_L , β_R , and P_c should not be interpreted as permanently fixed quantities. Effective circulating supply may evolve through vesting schedules, staking lockups, treasury operations, and token burns. Liquidity conditions may vary with market depth, investor sentiment, and exchange access, while the credibility threshold itself may shift as information, governance decisions, or market perceptions change.

Theorems 2–4 should therefore be interpreted as regime-classification and comparative-static results rather than permanent policy rules. At any point in time, the prevailing values of $\bar{S}$, β_L , β_R , and P_c determine whether the platform operates in an anchoring, fragility, or stabilisation regime. Governance influences these outcomes both directly through protocol design and indirectly through its effects on liquidity, participation incentives, and credibility. The central policy problem is therefore one of regime management rather than the selection of fixed parameter values.

Theorem 2 (Sliding interval and valuation anchoring). *Assume the hypotheses of Theorem 1. Let the switching manifold be*

$$\Sigma := \{(a, P) \in \Omega : P = P_c\},$$

and define

$$g_k(a, P) = \kappa_P [\alpha a \ell(P) + \beta_k s(P) - \bar{S}], \quad k \in \{L, R\}.$$

Set

$$a_- := \frac{\bar{S} - \beta_L s(P_c)}{\alpha \ell(P_c)}, \quad a_+ := \frac{\bar{S} - \beta_R s(P_c)}{\alpha \ell(P_c)}.$$

Then $a_- < a_+$ and the attracting Filippov sliding region on Σ is

$$\Sigma_s = \{(a, P_c) \in \Sigma : a \in (a_-, a_+)\}.$$

In particular, Σ_s is nonempty whenever $(a_-, a_+) \cap (0, 1) \neq \emptyset$. For any Filippov trajectory entering Σ_s , there exists a nontrivial time interval on which

$$P(t) \equiv P_c,$$

while $a(t)$ evolves according to the induced sliding dynamics.

Theorem 2 characterises the first equilibrium configuration generated by the participation–valuation feedback system, namely *valuation anchoring*. The result shows that the credibility threshold is not merely a reference valuation. Under

$$g_L(a, P_c) > 0, \quad g_R(a, P_c) < 0,$$

the threshold becomes an endogenous attractor of the dynamics. Valuation pressures on either side of P_c point toward the threshold, causing trajectories to converge to the switching manifold while participation continues to evolve.

The central economic implication is a separation between valuation adjustment and ecosystem adjustment. Once trajectories enter the sliding region, valuation ceases to absorb marginal changes in participation. Information generated by changes in adoption, transaction activity, staking participation, validator commitment, or developer engagement is no longer transmitted through continuous price adjustment. Instead, the credibility threshold absorbs these forces and maintains valuation at P_c while participation evolves along the manifold. The interval

$$(a_-, a_+)$$

provides a complete characterisation of the conditions under which anchoring is feasible. The theorem therefore identifies a precise equilibrium object rather than a qualitative market phenomenon. The existence and size of the anchoring region are determined jointly by liquidity conditions, effective circulating supply, and the responsiveness of participation demand. Changes in these primitives alter the geometry of the sliding region and therefore the range of ecosystem states for which valuation anchoring can be sustained.

The result differs fundamentally from standard smooth valuation models. In smooth systems, valuation adjusts continuously in response to changes in participation. Here, the discontinuity induced by the credibility threshold creates an attracting sliding manifold on which valuation remains fixed while participation continues to adjust. Anchoring is therefore a property of the system’s non-smooth equilibrium geometry rather than a consequence of exogenous stabilisation, temporary market frictions, or imposed price controls.

The theorem also generates empirically testable implications. Anchoring predicts prolonged concentration of valuations near salient threshold levels together with continued variation in participation measures. Observable manifestations include persistent price clustering near P_c , low threshold-crossing frequencies, extended residence times on the switching manifold, and ongoing adjustment in adoption, transaction activity, staking, governance participation, or developer engagement. The model therefore provides a struc-

tural explanation for valuation plateaus as equilibrium outcomes generated by platform architecture and participation feedback.

Figure 1 illustrates the global phase geometry implied by Theorem 2. Across a wide range of initial conditions, trajectories are drawn toward the credibility manifold $\Sigma = \{P = P_c\}$, indicating that valuation anchoring is robust to initial participation levels. Figure 2 reports the proportion of time spent in the sliding regime across initial conditions. Consistent with Theorem 2, the dominant outcome is persistent anchoring rather than recurrent threshold crossing.

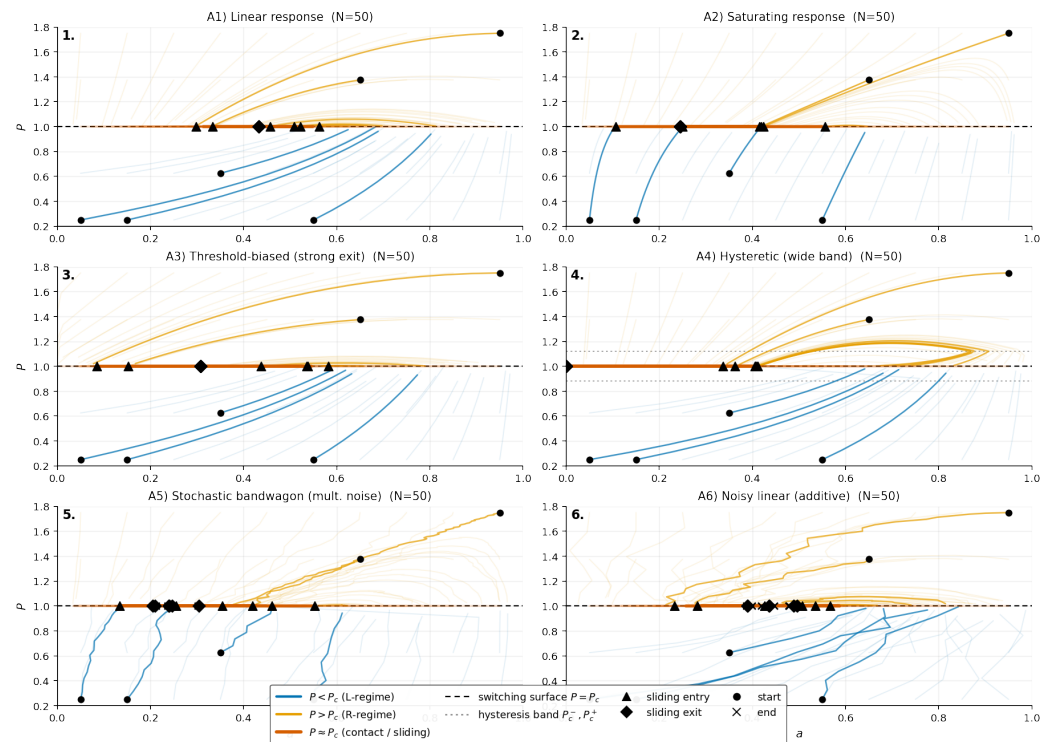


Figure 1. Phase portraits across 50 initial conditions (A1–A6). Trajectories converge toward $\Sigma = \{P = P_c\}$. Markers indicate entry and exit from sliding. Anchoring dominates under base-line parameters, consistent with Theorem 2.

Theorem 3 (Existence of a non-smooth limit cycle). *Assume the hypotheses of Theorem 1. Consider the associated planar Filippov inclusion*

$$\dot{x} \in F(x),$$

induced by (17)–(19) with switching manifold

$$\Sigma = \{(a, P) \in \Omega : P = P_c\}.$$

Suppose there exists a compact rectangle

$$\mathcal{R} = [a_1, a_2] \times [P_1, P_2] \subset \Omega, \quad 0 < a_1 < a_2 < 1, \quad 0 < P_1 < P_c < P_2,$$

such that

- (i) *Positive invariance.* The set $\mathcal{R}$ is positively invariant under the Filippov inclusion.
- (ii) *Absence of stationary objects.* There are no equilibria of the smooth regime vector fields associated with (17)–(19) in $\mathcal{R} \setminus \Sigma$, and there are no pseudo-equilibria of the induced sliding dynamics on $\mathcal{R} \cap \Sigma$.

(iii) *Regular switching geometry.* For every $(a, P_c) \in \mathcal{R} \cap \Sigma$,

$$g_L(a, P_c) \neq 0, \quad g_R(a, P_c) \neq 0.$$

Thus, each threshold point in $\mathcal{R} \cap \Sigma$ is either a transversal crossing point ($g_L g_R > 0$) or an attracting sliding point ($g_L > 0 > g_R$).

(iv) *Forward uniqueness and non-Zeno regularity.* Forward Filippov trajectories in $\mathcal{R}$ are unique and do not exhibit accumulation of switching times in finite time.

Then the Filippov dynamics admit at least one periodic orbit contained in $\mathcal{R}$. Any such orbit that intersects Σ is piecewise smooth and non-smooth at the switching manifold, consisting of smooth regime segments connected by transversal crossings and, where applicable, attracting sliding arcs.

Remark 1. Condition (iv) is imposed as an explicit regularity assumption. It is not a consequence of the standard existence theory for Filippov differential inclusions. While existence of Filippov solutions follows under mild boundedness and upper semicontinuity conditions, forward uniqueness and exclusion of Zeno accumulation generally require additional assumptions on the switching geometry and the associated vector fields. The purpose of condition (iv) is to rule out pathological chattering and nonunique forward continuations, so that the planar Poincaré–Bendixson argument applies to economically meaningful threshold-switching trajectories.

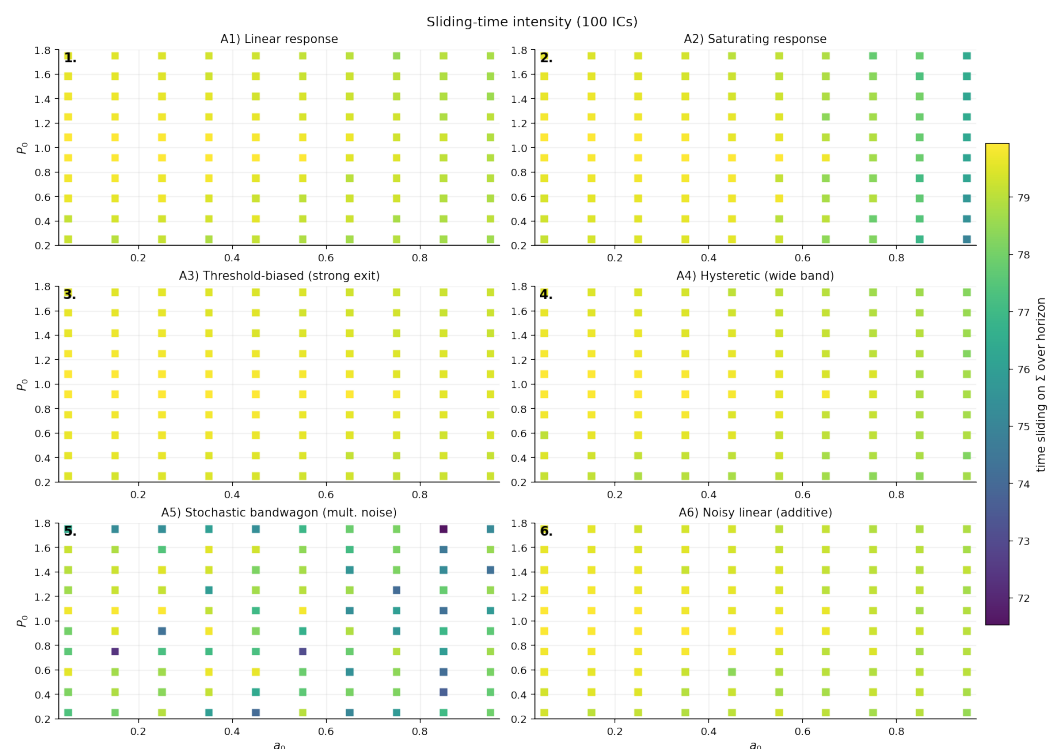


Figure 2. Sliding-time intensity across initial conditions (A1–A6). High sliding shares indicate persistent valuation anchoring. Results align with Theorem 2 and exclude sustained boom–bust dynamics in the baseline region.

Theorem 3 identifies a second equilibrium configuration of the platform economy, namely threshold-induced fragility. In contrast to the anchoring regime characterised by Theorem 2, the credibility threshold does not absorb valuation adjustment. Instead, it repeatedly redirects the equilibrium trajectory, generating persistent transitions between participation and valuation regimes.

The result demonstrates that sustained platform volatility need not be driven by exogenous shocks, changes in fundamentals, or unstable equilibria. Cyclical dynamics

emerge endogenously from the interaction between participation incentives and threshold-dependent valuation responses. Each threshold crossing alters the incentive environment faced by market participants, thereby changing the subsequent evolution of participation and valuation. The credibility threshold thus operates as a switching mechanism that continually reorients equilibrium adjustment.

The trapping region $\mathcal{R}$ provides a precise characterisation of this phenomenon. Trajectories remain bounded within $\mathcal{R}$, yet fail to converge to either an equilibrium or a pseudo-equilibrium. The participation–valuation feedback mechanism therefore generates recurrent expansions and contractions despite the absence of explosive behaviour. The resulting dynamics are globally stable in the sense of boundedness but locally unstable in the sense that equilibrium adjustment is never fully dissipated.

This mechanism differs fundamentally from standard explanations of persistent volatility in platform and asset-pricing models. Existing frameworks typically attribute recurrent fluctuations to external disturbances, parameter variation, or unstable equilibria. Here, persistent oscillations arise from the geometry of the credibility threshold itself. Regime transitions organised by the switching manifold generate endogenous cyclical behaviour even when the underlying equilibrium system remains bounded.

The existence and extent of the oscillatory regime depend on the structural primitives of the platform economy, including effective circulating supply $\bar{S}$, regime asymmetries (β_L, β_R) , participation responsiveness through Equation (17), and the location of the credibility threshold P_c . Changes in these primitives may enlarge, contract, or eliminate the trapping region supporting recurrent threshold crossings. Governance therefore influences platform fragility through its effect on the equilibrium geometry of $\mathcal{R}$.

The theorem also yields clear empirical implications. Threshold-induced fragility predicts elevated crossing-frequency statistics, clustered valuation fluctuations, alternating phases of participation expansion and contraction, and relatively short residence times near anchoring regions. Observed boom–bust dynamics may therefore reflect endogenous participation–valuation feedback generated by credibility thresholds rather than purely external sources of volatility.

Theorem 4 (Governance stabilisation). *Assume the hypotheses of Theorem 1 and let $\Omega_{\max} = [0, 1] \times (0, P_{\max}]$ denote the forward-invariant set. Write the regime vector fields as*

$$f^{(k)}(a, P) = \begin{pmatrix} a(1-a)\Gamma_k(V(a, P)) \\ \kappa_P [\alpha a \ell(P) + \beta_k s(P) - \bar{S}] \end{pmatrix}, \quad k \in \{L, R\},$$

with switching manifold $\Sigma = \{P = P_c\}$. Suppose the following:

(i) *Regular crossing at the threshold. For all $a \in [0, 1]$,*

$$g_L(a, P_c) g_R(a, P_c) > 0. \quad (21)$$

A sufficient condition is

$$\bar{S} > \alpha \ell(P_c) + \beta_L s(P_c) \quad \text{or} \quad \bar{S} < \beta_R s(P_c). \quad (22)$$

(ii) *Regime-wise differentiability. $\Gamma_L, \Gamma_R \in C^1(\mathbb{R})$ with bounded derivatives.*

(iii) *Uniform dissipativity. There exists $\eta > 0$ such that for each $k \in \{L, R\}$,*

$$\operatorname{div} f^{(k)}(a, P) \leq -\eta \quad \text{for all } (a, P) \in \Omega_{\max} \setminus \Sigma. \quad (23)$$

Then the Filippov system admits no periodic orbits in $\Omega_{\max}$. Every ω -limit set intersects the set of Filippov equilibria. If all Filippov equilibria are isolated, every trajectory converges to an equilibrium and endogenous boom–bust oscillations are excluded.

Corollary 1 (Regional stabilisation). *Let $K \subset \Omega_{\max}$ be a compact positively invariant set for the Filippov dynamics, with piecewise C^1 boundary, and suppose that $K \cap \Sigma$ contains only regular crossing points. Assume that for each regime $k \in \{L, R\}$,*

$$\operatorname{div} f^{(k)}(a, P) \leq -\eta \quad \text{for all } (a, P) \in K \setminus \Sigma$$

for some $\eta > 0$. Then the Filippov system admits no periodic orbit contained in K . If all Filippov equilibria in K are isolated, then every trajectory remaining in K has an ω -limit set contained in the set of Filippov equilibria.

Proof. The proof is the Bendixson argument applied on the positively invariant region K . On each smooth regime component, the negative-divergence condition implies strict contraction of oriented area. Regular crossing at Σ prevents the creation of sliding segments or singular switching arcs that could invalidate the piecewise-smooth Bendixson argument. Hence, a closed orbit contained in K would enclose a region whose area must be strictly decreasing after one return, which is impossible for a periodic orbit. Therefore no periodic orbit is contained in K . The remaining conclusion follows from compactness of K , positive invariance, and the planar Filippov Poincaré–Bendixson alternative. $\square$

Theorem 4 characterises the third and final equilibrium configuration of the platform economy, which we term governance-stabilised convergence. Unlike the anchoring regime of Theorem 2, where the credibility threshold absorbs valuation adjustment, and unlike the oscillatory regime of Theorem 3, where repeated threshold crossings generate endogenous boom–bust dynamics, the present regime is characterised by the dominance of stabilising forces throughout the relevant state space.

The economic significance of the result is that threshold-induced instability is not an inevitable consequence of participation complementarities and credibility-sensitive valuation. When the dissipativity condition

$$\operatorname{div} f^{(k)}(a, P) \leq -\eta$$

holds, adjustment pressures are progressively absorbed rather than amplified. Participation and valuation therefore converge toward equilibrium instead of generating persistent threshold-crossing cycles. The credibility threshold retains its informational role, but ceases to act as a source of dynamic amplification.

The divergence condition provides a geometric characterisation of stabilisation. Negative divergence implies systematic compression of nearby trajectories and excludes the persistence of self-sustaining oscillatory dynamics. The theorem therefore identifies a class of platform architectures in which stabilising feedbacks dominate the mechanisms capable of generating endogenous boom–bust behaviour.

The theorem should be interpreted as a global stabilisation benchmark. The condition is intentionally strong and provides a sufficient criterion under which recurrent threshold interaction disappears and all trajectories converge toward equilibrium. For this reason, Corollary 1 is likely to be more relevant empirically. The corollary shows that stabilisation need only hold on a compact forward-invariant region containing the realised trajectory. Positive feedback effects may therefore remain important elsewhere without generating persistent instability along the states actually visited by the platform.

Together, Theorem 4 and Corollary 1 establish a hierarchy of stabilisation outcomes. The theorem characterises globally stabilised platform economies, whereas the corollary characterises regionally stabilised economies. Both results demonstrate that credibility thresholds need not generate recurrent boom–bust dynamics when stabilising forces dominate the participation–valuation feedback system.

The model generates clear empirical implications. Stabilised regimes are characterised by low threshold-crossing frequencies, rapid attenuation of valuation disturbances, convergence of participation indicators, and the absence of recurrent cyclical dynamics. In contrast to the oscillatory regime of Theorem 3, fluctuations do not organise into self-sustaining cycles even though participation and valuation remain jointly determined.

4. Discussion

The central contribution of the paper is to show that credibility thresholds are not passive valuation markers but structural equilibrium objects capable of organising qualitatively distinct platform dynamics. The analysis identifies three equilibrium configurations generated by the interaction of participation incentives, liquidity conditions, and effective token supply through the adoption–valuation system (17)–(19). Depending on the underlying economic environment, the same credibility threshold may function as an anchor, a source of endogenous fragility, or a stabilising boundary.

This perspective differs fundamentally from standard platform and token-valuation models. In smooth adjustment frameworks, valuation responds continuously to changes in participation and liquidity conditions, while regime changes emerge through sufficiently large shocks or gradual parameter variation [6,47–49]. In contrast, the present framework introduces a governance-sensitive threshold P_c through (20) that alters the adjustment law itself. The resulting dynamics are therefore governed not only by the state of the system but also by its position relative to the credibility boundary. Credibility thresholds consequently become active coordination mechanisms that reshape equilibrium behaviour rather than passive reference points around which valuation fluctuates.

Theorem 1 provides the analytical foundation for this interpretation. The theorem establishes the existence of global Filippov solutions that remain confined to the economically meaningful domain $\Omega_{\max}$, with the formal barrier arguments reported in Appendix B. As a consequence, the qualitative behaviour generated by the switching manifold

$$\Sigma = \{P = P_c\},$$

including sliding, transversal crossing, and non-smooth periodic motion, is an intrinsic property of equilibrium rather than a numerical artefact. The theorem therefore ensures that the regime configurations studied below emerge directly from the economic structure of the model.

This interpretation is consistent with empirical evidence showing that users and investors frequently adjust participation, liquidity provision, and speculative demand discontinuously when valuation crosses salient credibility levels [11,14,33]. The practical importance of Theorem 1 extends beyond mathematical well-posedness. Because trajectories remain confined to the feasible state space, governance interventions cannot generate economically meaningless outcomes such as negative participation, negative valuation, or pathological switching behaviour. The theorem therefore provides a minimal consistency condition for the equilibrium and governance analysis that follows.

The practical importance of Theorem 1 extends beyond mathematical well-posedness. By ensuring that all trajectories remain confined to the economically admissible domain $\Omega_{\max}$, the theorem guarantees that the regime configurations identified below arise from economically meaningful interactions between participation and valuation rather than

from pathological states of the model. Governance interventions may alter the dynamics of the ecosystem, but they cannot generate negative participation, negative valuations, or other economically infeasible outcomes. The theorem therefore provides the analytical foundation on which the subsequent equilibrium classification rests.

The first equilibrium configuration identified by the theory is valuation anchoring. Theorem 2 shows that the credibility threshold enters the system structurally through (20) and organises the geometry of the excess-demand mechanism in (19). The resulting anchoring interval

$$(a_-, a_+)$$

depends on effective circulating supply $\bar{S}$, speculative depth (β_L, β_R) , and the usage-demand channel $\ell(P_c)$. Whether the switching manifold becomes an attracting set is therefore an endogenous property of the platform architecture rather than an exogenous market characteristic. Changes in token supply design, liquidity conditions, and participation incentives alter the geometry of the threshold and thereby influence the existence and extent of the anchoring region. Empirical evidence that supply rules and market-access frictions affect volatility regimes [32,50] is consistent with this mechanism.

When

$$(a_-, a_+) \cap (0, 1) \neq \emptyset,$$

trajectories entering

$$\Sigma = \{P = P_c\}$$

satisfy

$$P(t) \equiv P_c$$

over nontrivial intervals while participation continues to evolve according to (17). Valuation therefore becomes partially decoupled from ecosystem activity. Adoption, transaction intensity, staking participation, validator commitment, and developer engagement may continue to evolve even though valuation remains concentrated at the credibility threshold. Valuation plateaus emerge endogenously from the interaction of transactional demand, speculative asymmetry, and supply design rather than from exogenous intervention or imposed stabilisation mechanisms.

The numerical evidence reported in Appendix C supports this interpretation. The phase portraits in Figure 1 and the sliding-intensity heat maps in Figure 2 show that trajectories are frequently attracted toward the switching manifold across a broad range of initial conditions and adoption specifications. Quantitative evidence reported in Tables A2 and A3 demonstrates that anchoring persists across a substantial region of the economically relevant state space rather than arising as a knife-edge parameter configuration.

The anchoring mechanism is also robust to changes in the behavioural microfoundations governing participation. Appendix A introduces alternative adoption dynamics incorporating nonlinear saturation, exit asymmetry, hysteresis, and stochastic amplification. The robustness analysis summarised in Table A7 shows that sliding behaviour persists across all six adoption specifications. The geometry induced by (20) and (19) therefore appears to be the primary determinant of valuation anchoring, whereas the behavioural assumptions embedded in (17) mainly influence the level and composition of participation. Late-time participation outcomes reported in Table A3 and Figure A2 illustrate this distinction.

The anchoring interval also provides a governance diagnostic. Theorem 2 implies that valuation anchoring persists only while participation remains within (a_-, a_+) . Governance does not choose this interval directly. Instead, it influences the structural primitives that determine its location and width. Effective circulating supply, liquidity conditions, and participation incentives jointly determine whether the economy remains inside the

anchoring region or approaches one of its boundaries. A stable token price near P_c is therefore not sufficient evidence of ecosystem stability. Participation may simultaneously be moving toward the edge of the anchoring region, altering the long-run regime of the platform without producing any immediate change in valuation. The theorem consequently transforms the problem of platform stability from one of valuation monitoring to one of regime monitoring.

Importantly, valuation anchoring does not imply uniform participation outcomes. Appendix A introduces alternative adjustment specifications incorporating nonlinear saturation, exit asymmetry, hysteresis, and stochastic amplification. Although these behavioural mechanisms generate substantially different participation trajectories, the anchoring mechanism itself remains remarkably robust. The robustness results reported in Table A7 show that sliding behaviour persists across all six adoption specifications considered. The existence of the anchoring region is therefore not a knife-edge consequence of a particular behavioural assumption.

This distinction is economically important. The geometry induced by (20) and (19) governs valuation stability, whereas the behavioural microfoundations embedded in (17) primarily determine the level, composition, and persistence of participation. Consequently, two ecosystems may display similar valuation dynamics while exhibiting markedly different participation patterns. Late-time participation outcomes reported in Table A3 and Figure A2 illustrate this separation between valuation stabilisation and ecosystem evolution. The credibility threshold organises the valuation regime, but it does not uniquely determine the path of participation.

Additional numerical evidence reinforces this conclusion. Figure A4 shows that substantial sliding shares persist across adoption specifications, while Figures A5 and A6 illustrate how the prevalence of anchoring varies with the underlying participation and liquidity environment. Sensitivity analyses reported in Figures A7 and A8 further demonstrate that speculative depth and effective circulating supply influence the intensity of anchoring without altering the underlying regime structure. Taken together, the evidence suggests that valuation anchoring is a robust consequence of threshold geometry rather than a fragile artefact of a particular behavioural specification.

The second equilibrium configuration identified by the theory is threshold-induced fragility. Theorem 3 shows that the same threshold rule (20) can generate endogenous boom–bust dynamics when trajectories are confined to a compact trapping region that straddles the switching manifold Σ and neither regime contains a stabilising equilibrium nor a stabilising sliding pseudo-equilibrium. In this configuration, the credibility threshold ceases to function as an anchor and instead becomes a source of dynamic amplification. The Filippov Poincaré–Bendixson argument developed in Appendix B establishes that recurrent threshold interaction can generate persistent cyclical behaviour even in the absence of exogenous shocks.

The significance of Theorem 3 is that instability need not originate from external disturbances. Boom–bust dynamics emerge endogenously from the interaction between participation incentives and threshold-dependent valuation responses. Each threshold crossing alters the adjustment environment faced by participants, which in turn changes future participation and valuation incentives. The threshold therefore operates as a switching amplifier rather than a coordination device. The resulting volatility is a property of the equilibrium geometry itself.

Baseline simulations operate outside this fragility window, as shown in Appendix C, where sliding dominance prevents the trapping-region geometry required for persistent oscillations. Figure A1 illustrates how alternative adoption dynamics affect regime transitions, while the stochastic evidence reported in Table A4 shows that moderate random

disturbances do not eliminate the underlying regime classification. Although noise increases trajectory dispersion, the distinction between anchoring, fragility, and stabilisation remains intact. Additional diagnostics reported in Figures A9–A11 characterise the duration, timing, and directional properties of threshold episodes. Together, these results indicate that recurrent switching is a structural consequence of the participation–valuation feedback system rather than a numerical artefact.

The significance of Theorem 3 is that instability need not originate from external shocks, fundamental disturbances, or exogenous changes in market sentiment. Recurrent boom–bust dynamics emerge endogenously from the interaction between participation incentives and threshold-dependent valuation responses. Each threshold crossing alters the adjustment environment faced by market participants, which in turn modifies subsequent participation and valuation incentives. The credibility threshold therefore operates as a switching amplifier rather than a coordination anchor. Instability arises from the geometry of the participation–valuation feedback system itself rather than from an external disturbance process.

The theorem consequently provides a structural interpretation of platform fragility. Governance arrangements that increase participation sensitivity, amplify speculative concentration, generate abrupt supply adjustments, or reinforce liquidity asymmetries increase the likelihood of entering a threshold-switching regime. The result therefore suggests that governance should be evaluated not only by its ability to stimulate growth and participation, but also by its effect on the amplification channels that govern repeated threshold interaction.

The third equilibrium configuration identified by the theory is governance-stabilised convergence. Theorem 4 provides sufficient conditions under which recurrent threshold interaction ceases to generate persistent cyclical dynamics. Regular crossing at Σ combined with regime-wise dissipativity eliminates periodic motion and restores convergence toward equilibrium. Uniformly negative divergence of the regime vector fields associated with (17)–(19) yields a Filippov extension of the classical Bendixson criterion. Instability generated by threshold switching is therefore not an inevitable consequence of the model. Sufficiently strong stabilising forces can dominate the amplification mechanism and eliminate self-sustaining cycles.

The numerical evidence is consistent with this interpretation. Table A4 and Figure A3 show that the qualitative regime classification remains robust under stochastic perturbations. Although random disturbances increase trajectory dispersion, they do not eliminate the distinction between anchoring, fragility, and stabilisation. Numerical experiments satisfying (21)–(23) display rapid attenuation of threshold interaction and convergence toward equilibrium. Long-run participation outcomes reported in Figure A12 further demonstrate that stabilisation affects not only valuation dynamics but also the distribution of ecosystem participation.

The stabilisation result should not be interpreted as requiring governance to directly control divergence. Dissipativity is an aggregate property emerging from the interaction of participation adjustment, liquidity provision, token supply design, treasury management, staking incentives, and market structure. The theorem therefore identifies a class of platform architectures in which stabilising forces dominate destabilising feedbacks. Governance influences stability indirectly through its effect on the underlying economic primitives rather than through direct manipulation of the dynamical system.

The three equilibrium configurations should not be interpreted as isolated outcomes. Together, they define a regime map linking credibility formation, participation incentives, liquidity conditions, and token supply architecture to distinct long-run platform dynamics. Changes in effective circulating supply, speculative depth, participation responsiveness,

governance credibility, or market access may move an ecosystem between anchoring, fragility, and stabilisation. Governance therefore operates primarily through regime selection and regime management rather than through direct control of valuation.

Together, Theorems 1–4 establish that credibility thresholds are structural equilibrium objects. The same threshold may function as an anchor, an amplifier, or a stabiliser depending on the surrounding participation, liquidity, and supply architecture. The planar framework analysed in the main text is supported by the exact mean-field reduction established in Proposition A1, while the robustness and numerical analyses reported in Appendices A and C demonstrate that the qualitative regime classification persists across alternative behavioural specifications and stochastic environments. The framework therefore provides a unified equilibrium theory of threshold-induced coordination, endogenous instability, and governance design in tokenised platform ecosystems.

4.1. Endogenous Design Variables and Moving Governance Regimes

The equilibrium configurations characterised by Theorems 2–4 should be interpreted as a regime map rather than a collection of static outcomes. The location of a platform within this map depends on effective circulating supply $\bar{S}$, liquidity conditions (β_L, β_R) , participation incentives, and the credibility threshold P_c . Because these quantities evolve over time, platform governance continuously manages movement through the regime map rather than selecting a permanent equilibrium configuration.

This interpretation is important because the theorems treat $\bar{S}$, (β_L, β_R) , and P_c as fixed parameters in order to characterise the local geometry of the participation–valuation system. The analysis should not be read as the stronger claim that these quantities are immutable governance choices. Instead, they are state–design variables whose values evolve jointly through governance decisions, market conditions, and ecosystem development.

In practice, effective circulating supply is a moving object. Vesting schedules, token unlocks, staking and unstaking flows, treasury operations, token-burning mechanisms, bridge activity, exchange listings, and changes in secondary-market float all influence the quantity of tokens available for trade. Similarly, speculative depth is only partially determined by protocol design. It depends on liquidity provision, market-maker balance-sheet capacity, investor sentiment, risk appetite, exchange access, and broader market conditions. The credibility threshold P_c is likewise endogenous in a wider institutional sense. Governance reforms, security audits, major partnerships, regulatory developments, protocol upgrades, and shifts in perceived platform viability may all alter the valuation level at which participants revise expectations.

The practical implication is that platforms move through the regime map as these variables evolve. Governance therefore operates primarily through regime management rather than direct control of valuation. Policies influence whether the platform approaches anchoring, fragility, or stabilisation by affecting the structural variables that shape the geometry of the system.

This interpretation sharpens the meaning of Theorem 2. The anchoring interval

$$(a_-, a_+) = \left(\frac{\bar{S} - \beta_L s(P_c)}{\alpha \ell(P_c)}, \frac{\bar{S} - \beta_R s(P_c)}{\alpha \ell(P_c)} \right)$$

is not a once-and-for-all design object. It shifts whenever supply conditions, liquidity depth, or credibility perceptions change. A large token unlock may move the anchoring interval away from the current participation level. A deterioration in liquidity provision may narrow the range over which valuation anchoring is feasible. Conversely, smoother vesting schedules, staking lockups, liquidity support programmes, and credible governance

communication may slow the movement of the relevant parameters and increase the persistence of anchoring.

The broader governance implication is therefore dynamic. Platform designers should monitor how supply events, liquidity conditions, participation incentives, and credibility shocks alter the platform's position within the regime map. The model does not imply that governance controls $\bar{S}$, β_k , or P_c completely. Rather, it identifies these quantities as the channels through which governance and market conditions jointly reshape platform stability.

4.2. Empirical Identification of Valuation Anchoring

Theorem 2 provides a structural characterisation of valuation anchoring. An important question is whether the corresponding mechanism can be distinguished empirically from alternative sources of price stability.

The central difficulty is that the defining object of the theory is an unobserved sliding motion on the credibility manifold

$$\Sigma = \{P = P_c\}.$$

Observed data reveal realised prices and participation outcomes, but not the underlying vector fields governing the dynamics. Consequently, price stability near a salient valuation level is not, by itself, evidence of valuation anchoring.

The contribution of the theory is therefore not a direct empirical test of Filippov sliding, but a set of identifying restrictions. The model predicts persistent valuation concentration near a credibility threshold together with continued adjustment in participation. Anchoring is therefore characterised by a decoupling of valuation dynamics from ecosystem dynamics rather than by price stability alone.

The numerical evidence developed in Appendix C illustrates these identifying implications. The high sliding shares reported in Tables A2 and A3, together with the phase portraits and sliding-intensity heat maps, demonstrate that valuation may remain concentrated near the credibility threshold while participation continues to evolve. Moreover, the robustness results reported in Table A7 show that this qualitative pattern persists across all six adoption specifications considered in Appendix A. The numerical analysis therefore supports the theoretical prediction that anchoring is fundamentally a threshold phenomenon rather than a consequence of a particular behavioural specification.

These observations suggest a natural empirical strategy. First, the relevant credibility threshold should be identified using threshold, regime-switching, or structural-break methods. Second, valuation should remain concentrated near the estimated threshold over a sustained interval. Third, participation indicators should continue to evolve during the same period. Evidence satisfying all three conditions would be consistent with the anchoring mechanism implied by Theorem 2.

The theory also highlights important measurement challenges. Participation is not directly observable and must be inferred from proxies such as transaction activity, staking participation, validator engagement, liquidity provision, developer activity, or user adoption. No single proxy provides a complete measure of ecosystem participation. Empirical identification should therefore rely on multiple indicators rather than a single activity measure.

The principal contribution of the model is thus theoretical. It establishes a set of observable implications associated with threshold-induced valuation anchoring and provides a framework through which future empirical work can evaluate whether coordination effects of the type characterised by Theorem 2 are present in tokenised platform ecosystems.

The analysis also yields broader implications for the governance of tokenised platform ecosystems. Theorems 2–4 show that credibility thresholds may perform fundamentally different economic functions. Depending on the interaction between participation incen-

tives, liquidity conditions, and effective token supply, the same threshold may operate as an anchor, a switching amplifier, or a stabilising boundary.

A central implication is that valuation alone provides an incomplete measure of ecosystem health. In the anchoring regime, valuation may remain concentrated near a credibility threshold while participation continues to evolve. In the fragility regime, recurrent fluctuations emerge endogenously through threshold interaction. In the stabilised regime, participation and valuation converge jointly toward equilibrium. Observed valuations therefore cannot be interpreted independently of the underlying participation dynamics.

The numerical evidence reinforces this interpretation. The anchoring results reported in Tables A2 and A3 contrast sharply with the instability diagnostics reported in Figures A9–A11 and the stochastic results summarised in Table A4. Together, these results demonstrate that qualitatively different long-run outcomes can emerge from the same underlying platform once the economy crosses a credibility threshold.

The theory also provides a dynamic perspective on governance. Participation incentives, token issuance, liquidity support, staking design, and treasury management affect more than short-run valuation. By altering the participation–valuation feedback system, these mechanisms influence the equilibrium regime in which the platform operates. Governance therefore affects ecosystem outcomes primarily through its impact on the geometry of the underlying dynamics.

This observation is particularly relevant for smaller ecosystems. Large platforms often benefit from deeper liquidity and broader participation, whereas smaller platforms may operate closer to regime boundaries. In such environments, relatively small changes in participation, liquidity conditions, or effective circulating supply may generate large changes in long-run dynamics.

The broader implication is that tokenised platforms should be viewed as dynamic coordination systems rather than purely financial assets. The regime map developed in this paper, supported by the robustness results in Appendix A and the numerical evidence in Appendix C, provides a framework linking governance design, participation incentives, liquidity conditions, and credibility formation to distinct equilibrium configurations of platform development.

5. Conclusions

This paper develops an equilibrium theory of tokenised platforms in which valuation thresholds act not merely as price levels but as structural coordination objects. Participation incentives and speculative demand adjust endogenously when valuation crosses the governance-sensitive threshold P_c . Price therefore performs a dual function: it clears the token market while simultaneously serving as a public coordination signal that feeds back into participation decisions. The resulting discontinuity changes the law of motion governing platform participation rather than merely the state of the system itself. Modelling this mechanism through a Filippov differential inclusion captures threshold-dependent incentive adjustments while preserving global well-posedness and economically meaningful state constraints.

The analysis demonstrates that the credibility threshold P_c organises the global geometry of participation–valuation dynamics. Three qualitatively distinct regimes emerge. First, when the sliding conditions of Theorem 2 hold, the threshold becomes an attracting manifold and valuation remains anchored at P_c while participation continues to evolve. Second, when dissipativity is insufficient, Theorem 3 shows that the same threshold-based switching mechanism can generate endogenous boom–bust cycles without requiring exogenous shocks. Third, when regular threshold crossing combines with sufficiently strong dissipative forces, Theorem 4 and, more generally, Corollary 1, identify conditions under which oscillatory dynamics disappear and the participation–valuation system converges toward equilibrium.

These regimes possess a direct governance interpretation. Issuance schedules, vesting structures, lockups, staking mechanisms, treasury operations, liquidity provision, and market design influence the participation–valuation feedback system through their effects on effective circulating supply, liquidity conditions, and speculative participation. The analysis should not be interpreted as assuming that effective circulating supply $\bar{S}$, speculative depth (β_L, β_R) , or the credibility threshold P_c are fixed objects under complete governance control. Rather, the theory provides a regime map that characterises how the platform behaves conditional on the prevailing configuration of these variables. Governance influences movement within this regime map, but does not determine it unilaterally.

The framework also generates empirically observable implications. Periods of valuation anchoring, threshold-crossing frequency, plateau duration, and participation adjustment during price stability provide potential diagnostics of the underlying mechanism. At the same time, these quantities should not be viewed as direct measurements of Filippov sliding. The credibility threshold P_c is generally latent, and commonly used participation proxies—including transaction counts, active addresses, staking participation, and liquidity measures—may be noisy or strategically distorted. Consequently, the proposed diagnostics are best interpreted as indicators consistent with threshold anchoring rather than definitive tests of the theory.

Several limitations clarify the scope of the framework. The planar participation–valuation system is a governance-level mean-field approximation rather than a general aggregation theorem for arbitrary heterogeneous networks. While the approximation preserves the aggregate threshold geometry of the platform economy, it abstracts from topology-specific diffusion effects, heterogeneous exposure patterns, and local cascade dynamics. In addition, the analysis treats effective supply, speculative depth, and credibility thresholds as locally fixed over the horizon of analysis, even though these variables may evolve through governance decisions, liquidity conditions, regulation, or broader market forces. Finally, the paper develops a theoretical rather than empirical contribution and does not estimate the model using platform-level data.

Future work could extend the framework in several directions, including network-specific formulations, endogenous evolution of supply and liquidity conditions, and empirical estimation using on-chain activity, staking behaviour, governance events, and market microstructure data.

Despite these limitations, the central insight remains robust. The contribution of the paper is primarily methodological and conceptual. Methodologically, it introduces a Filippov framework for analysing threshold-dependent participation dynamics in tokenised platforms. Conceptually, it reinterprets valuation thresholds as active coordination mechanisms that reshape the geometry of platform evolution. Governance-sensitive valuation thresholds do not merely influence equilibrium outcomes; they alter the dynamic structure through which participation, liquidity, and valuation interact. Depending on the prevailing participation incentives, liquidity conditions, and effective supply configuration, the same threshold can act as an attracting anchor, a switching amplifier, or a stabilising boundary.

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Appendix A. Adoption Update Specifications

Table A1. Adoption update specifications used in the simulations (A1–A6). Each rule defines an adoption drift (or stochastic differential) that is regime-conditioned on the valuation threshold: the governing regime switches at $P = P_c$ (or, for A4, within a two-threshold band). These specifications are chosen to be structurally distinct while remaining comparable: A1–A3 are deterministic benchmarks (linear, saturating, and exit-biased), A4 introduces genuine hysteresis and regime persistence, and A5–A6 introduce stochastic participation dynamics (multiplicative “bandwagon” noise versus additive idiosyncratic shocks). Together, they allow controlled comparisons of how nonlinearity, asymmetry, memory, and stochasticity shape valuation anchoring (Filippov sliding), regime interaction, and basin geometry in the coupled adoption–valuation system.

ID	Name	Functional Form	Platform Interpretation
A1	Linear response	$\dot{a} = \begin{cases} \alpha_L(1-a), & P < P_c, \\ -\alpha_R a, & P > P_c \end{cases}$	Baseline participation adjustment. Representative participation increases linearly when valuation lies below the credibility threshold (entry/expansion region) and decays proportionally when valuation exceeds the threshold (contraction/exit region). This rule serves as a benchmark with monotone incentives and no saturation effects.
A2	Saturating response	$\dot{a} = \begin{cases} \alpha_L a(1-a), & P < P_c, \\ -\alpha_R a(1-a), & P > P_c \end{cases}$	Endogenous saturation under ecosystem crowding. Logistic adjustment captures diminishing marginal incentives near full participation (capacity limits, congestion, or diminishing incremental network benefits) and near zero participation (limited peer reinforcement). Relative to A1, the dynamics slow down near the boundaries even within the same valuation regime.
A3	Threshold-biased	$\dot{a} = \begin{cases} \alpha_L(1-a), & P < P_c, \\ -\alpha_R a - \beta, & P > P_c \end{cases}$	Exit amplification above the threshold. An additional negative drift $\beta > 0$ accelerates exit once valuation exceeds P_c , capturing loss sensitivity, trust breakdown, compliance frictions, or reputational cascades that make contraction sharper than expansion.
A4	Hysteretic (two-threshold)	$k(t^+) = \begin{cases} L, & P \leq P_c^-, \\ R, & P \geq P_c^+, \\ k(t^-), & P_c^- < P < P_c^+, \end{cases}$ $\dot{a} = \begin{cases} \alpha_L(1-a), & k(t) = L, \\ -\alpha_R a, & k(t) = R \end{cases}$	Regime persistence and path dependence. Separate entry and exit thresholds ($P_c^- < P_c^+$) generate genuine hysteresis, representing persistence in user beliefs, switching costs, governance frictions, or delayed coordination. Within the band, the ecosystem “remembers” its last regime, so valuation changes need not immediately reverse participation incentives.
A5	Stochastic bandwagon	$da = \begin{cases} \alpha_L a(1-a) dt + \sigma a(1-a) dW_t, & P < P_c, \\ -\alpha_R a(1-a) dt + \sigma a(1-a) dW_t, & P > P_c \end{cases}$	Social amplification under uncertainty. Multiplicative noise captures endogenous dispersion in participation updates that grows with the “room to move” (i.e., strongest at intermediate adoption). This represents experimentation, peer-driven amplification, and endogenous uncertainty in ecosystem participation that interacts with valuation anchoring.
A6	Noisy linear	$da = \begin{cases} \alpha_L(1-a) dt + \sigma dW_t, & P < P_c, \\ -\alpha_R a dt + \sigma dW_t, & P > P_c \end{cases}$	Idiosyncratic shocks around a linear benchmark. Additive noise represents exogenous news shocks, heterogeneous idiosyncratic participation changes, or measurement noise. Relative to A5, uncertainty does not vanish at the boundaries, so small fluctuations can persist even near $a \approx 0$ or $a \approx 1$.

Table A1 summarises the adoption update rules used in the numerical analysis. All rules share a common valuation-based switching mechanism at the threshold $P = P_c$ (or the band $[P_c^-, P_c^+]$ for A4), which represents a governance-relevant credibility threshold in the platform ecosystem. The alternative rules capture distinct platform design mechanisms. They allow controlled comparisons between (i) linear versus saturated adjustment in participation incentives, (ii) symmetric participation responses versus exit-biased dynamics arising from liquidity frictions or loss of confidence, (iii) instantaneous adjustment versus hysteresis generated by reputation, switching costs, or user memory, and (iv) multiplicative social noise reflecting network amplification versus additive idiosyncratic shocks. These design choices map directly into observable ecosystem outcomes. They are reflected in the diagnostics reported in Appendix C, including time spent at the valuation anchor, the frequency and duration of sliding episodes, late-time participation distributions, and basin geometry. Together, these diagnostics illustrate how alternative platform design choices influence the stability, fragility, or persistence of valuation coordination regimes.

Appendix B. Technical Analysis and Proofs

Proof of Theorem 1. We verify the hypotheses of standard existence results for Filippov differential inclusions (see [37], Ch. 2, and [44], Ch. 1).

Step 1: Local regularity away from Σ . For $P \neq P_c$, the system coincides with one of the smooth vector fields $f^{(L)}$ or $f^{(R)}$. Because $V(a, P)$ is C^1 on Ω , Γ_k is globally Lipschitz, and ℓ, s are C^1 on $(0, \infty)$, each $f^{(k)}$ is locally Lipschitz on Ω . Classical ODE theory therefore guarantees local existence and uniqueness of solutions away from Σ .

Step 2: Filippov regularisation on Σ . On the switching manifold $\Sigma = \{P = P_c\}$, the Filippov set-valued map

$$F(x) = \text{co}\{f^{(L)}(x), f^{(R)}(x)\}$$

is upper semicontinuous with nonempty compact convex values. Moreover, on every compact subset of $\Omega_{\max}$, both $f^{(L)}$ and $f^{(R)}$ are bounded; hence, F is locally bounded. It follows from Filippov's existence theorem ([37], Th. 1, p. 106) that for every initial condition in $\Omega_{\max}$ there exists at least one absolutely continuous trajectory satisfying $\dot{x}(t) \in F(x(t))$ almost everywhere.

Step 3: Forward invariance of $a \in [0, 1]$. At the boundary points $a = 0$ and $a = 1$,

$$\dot{a} = a(1 - a)\Gamma_k(V(a, P)) = 0,$$

for both regimes $k \in \{L, R\}$. Thus, the vector field is tangent to the boundary, and standard viability arguments imply that trajectories cannot exit the interval $[0, 1]$.

Step 4: Lower barrier for the price process. By assumption (iv) of Theorem 1,

$$\lim_{P \downarrow 0} s(P) = +\infty.$$

Since $\beta_k > 0$ and ℓ is bounded on $(0, P_c]$,

$$\alpha a \ell(P) + \beta_k s(P) - \bar{S} \longrightarrow +\infty \quad \text{as } P \downarrow 0,$$

uniformly in $a \in [0, 1]$ and $k \in \{L, R\}$. Hence, there exists $\delta > 0$ such that

$$\dot{P} > 0 \quad \text{whenever } 0 < P < \delta.$$

Therefore, $P(t)$ cannot reach zero in finite time.

Step 5: Upper barrier for the price process. By assumption (v) of Theorem 1 and monotonicity of ℓ and s , for any $P \geq P_{\max}$ and any $a \in [0, 1]$,

$$\alpha a \ell(P) + \beta_L s(P) - \bar{S} \leq \alpha \ell(P_{\max}) + \beta_L s(P_{\max}) - \bar{S} < 0.$$

Since $\beta_R < \beta_L$, the same inequality holds with β_R . Consequently,

$$\dot{P} < 0 \quad \text{whenever } P \geq P_{\max}.$$

Thus $P(t)$ cannot exceed $P_{\max}$ once inside $\Omega_{\max}$.

Step 6: Global existence. Because $a(t) \in [0, 1]$ and $P(t) \in (0, P_{\max}]$ for all forward times, solutions remain in the compact forward-invariant set $\Omega_{\max}$. Local existence together with boundedness of the vector field implies that solutions extend to all $t \geq 0$. $\square$

Proof of Theorem 3. Fix an initial condition $x_0 \in \mathcal{R}$. By Theorem 1, there exists a global Filippov solution $x(\cdot)$ with $x(0) = x_0$. By assumption (iv), the forward solution in $\mathcal{R}$ is unique.

Step 1: Compactness of the forward orbit. By positive invariance of $\mathcal{R}$,

$$x(t) \in \mathcal{R} \quad \text{for all } t \geq 0.$$

Since $\mathcal{R}$ is compact, the forward orbit is precompact. Hence, the ω -limit set

$$\omega(x_0) := \bigcap_{T>0} \overline{\{x(t) : t \geq T\}}$$

is nonempty, compact, connected, and invariant under the forward Filippov semiflow.

Step 2: Regular switching structure. By assumption (iii), every point of $\mathcal{R} \cap \Sigma$ is either a transversal crossing point, where

$$g_L(a, P_c)g_R(a, P_c) > 0,$$

or an attracting sliding point, where

$$g_L(a, P_c) > 0 > g_R(a, P_c).$$

Thus $\mathcal{R} \cap \Sigma$ contains no tangencies, no visible or invisible two-fold singularities, and no escaping sliding segments. By assumption (iv), switching times do not accumulate in finite time. Consequently, trajectories in $\mathcal{R}$ are concatenations of smooth regime arcs and, where applicable, attracting sliding arcs, with only finitely many switching events on each compact time interval.

Step 3: Exclusion of stationary objects. Assumption (ii) rules out equilibria of the smooth regime vector fields in $\mathcal{R} \setminus \Sigma$ and pseudo-equilibria of the sliding vector field on $\mathcal{R} \cap \Sigma$. Therefore, $\omega(x_0)$ contains no stationary object of the Filippov dynamics.

Step 4: Application of the planar Filippov Poincaré–Bendixson alternative. The dynamics on $\mathcal{R}$ are planar, forward unique by assumption (iv), positively invariant by assumption (i), and regular at the switching manifold by assumptions (iii)–(iv). The planar Filippov Poincaré–Bendixson alternative for regular piecewise-smooth systems therefore applies [36,37]. Since $\omega(x_0)$ is nonempty, compact, invariant, and contains no stationary object, it must be a periodic orbit. Thus, the Filippov dynamics admit at least one periodic orbit contained in $\mathcal{R}$.

Step 5: Non-smooth structure of the orbit. If this periodic orbit intersects Σ , its segments away from Σ are governed by either $f^{(L)}$ or $f^{(R)}$. At Σ , the orbit either crosses transversally

or follows the Filippov sliding vector field on an attracting sliding segment. Hence, any such orbit is piecewise smooth and non-smooth at the switching manifold.

□

Proof of Theorem 4. Suppose, for contradiction, that a periodic Filippov orbit $\gamma \subset \Omega_{\max}$ exists.

Step 1: Regular crossing geometry. By assumption (i) of Theorem 4,

$$g_L(a, P_c)g_R(a, P_c) > 0 \quad \text{for all } a \in [0, 1].$$

Hence, Σ is a regular crossing manifold: no point on Σ is a sliding or tangency point. Therefore, γ is a closed, piecewise C^1 curve that intersects Σ transversally a finite number of times (see [36]).

Step 2: Decomposition of the enclosed region. Let $D \subset \Omega_{\max}$ denote the compact region enclosed by γ . Decompose

$$D = D_L \cup D_R, \quad D_L := D \cap \{P < P_c\}, \quad D_R := D \cap \{P > P_c\}.$$

On each D_k , the vector field $f^{(k)}$ is C^1 . The boundary of D_k consists of (i) the portion of γ lying in regime k , and (ii) transversal interface segments along Σ .

Step 3: Divergence integration. Applying Green's theorem on each D_k gives

$$\iint_{D_k} \operatorname{div} f^{(k)} dA = \oint_{\partial D_k} f^{(k)} \cdot n_k ds, \quad k \in \{L, R\},$$

where n_k is the outward unit normal on ∂D_k . Summing over $k \in \{L, R\}$ yields

$$\iint_{D_L} \operatorname{div} f^{(L)} dA + \iint_{D_R} \operatorname{div} f^{(R)} dA = \oint_{\gamma} f \cdot n ds + \int_{\Sigma \cap D} (f^{(L)} - f^{(R)}) \cdot n_{\Sigma} ds.$$

Because γ is a periodic orbit, the vector field is tangent to γ almost everywhere, so $f \cdot n = 0$ on γ . Along Σ , assumption (i) guarantees regular crossing and absence of sliding. The outward normals of D_L and D_R along the interface are opposite, so the flux contributions across Σ cancel exactly. Therefore,

$$0 = \iint_{D_L} \operatorname{div} f^{(L)} dA + \iint_{D_R} \operatorname{div} f^{(R)} dA.$$

Step 4: Dissipativity contradiction. By assumption (iii) of Theorem 4, there exists $\eta > 0$ such that

$$\operatorname{div} f^{(k)}(a, P) \leq -\eta \quad \text{for all } (a, P) \in \Omega_{\max} \setminus \Sigma, \quad k \in \{L, R\}.$$

Since D has positive Lebesgue measure,

$$\iint_{D_L} \operatorname{div} f^{(L)} dA + \iint_{D_R} \operatorname{div} f^{(R)} dA \leq -\eta|D| < 0,$$

which contradicts the previous equality. Hence, no periodic orbit exists in $\Omega_{\max}$.

Step 5: Asymptotic behaviour. Compactness and forward invariance of $\Omega_{\max}$ imply that every Filippov trajectory admits a nonempty ω -limit set. In the absence of periodic orbits, the planar Filippov Poincaré–Bendixson alternative (see [36]) implies that every ω -limit set contains an equilibrium or pseudo-equilibrium. If equilibria are isolated, convergence is to a single equilibrium point. □

Proof of Theorem 2. Define the switching function

$$h(a, P) = P - P_c,$$

so that the switching manifold is $\Sigma = \{(a, P) \in \Omega : h(a, P) = 0\}$ and

$$\nabla h(a, P) = (0, 1).$$

For each regime $k \in \{L, R\}$, the normal component of the vector field at a point $(a, P_c) \in \Sigma$ is

$$\nabla h(a, P_c) \cdot f^{(k)}(a, P_c) = f_2^{(k)}(a, P_c) = g_k(a, P_c).$$

By the standard Filippov sliding criterion (see [37], Ch. 2, or [36]), Σ contains an attracting sliding region precisely at those points where the vector fields on either side point toward the manifold, i.e.,

$$g_L(a, P_c) > 0 \quad \text{and} \quad g_R(a, P_c) < 0.$$

Because $\kappa_P > 0$ and $\ell(P_c) > 0$, these inequalities are equivalent to

$$\alpha \ell(P_c) + \beta_L s(P_c) - \bar{S} > 0, \quad \text{and} \quad \alpha \ell(P_c) + \beta_R s(P_c) - \bar{S} < 0.$$

Solving the first inequality yields

$$a > \frac{\bar{S} - \beta_L s(P_c)}{\alpha \ell(P_c)} =: a_-,$$

while the second yields

$$a < \frac{\bar{S} - \beta_R s(P_c)}{\alpha \ell(P_c)} =: a_+.$$

Since $\beta_L > \beta_R$ and $s(P_c) > 0$, we have

$$\bar{S} - \beta_L s(P_c) < \bar{S} - \beta_R s(P_c),$$

which implies $a_- < a_+$. Hence, the attracting sliding region on Σ is

$$\Sigma_s = \{(a, P_c) \in \Sigma : a \in (a_-, a_+)\}.$$

The sliding region is nonempty whenever $(a_-, a_+) \cap (0, 1) \neq \emptyset$. Finally, by the definition of Filippov solutions, if a trajectory reaches a point in Σ_s , the convexified vector field contains directions tangent to Σ whose normal component vanishes. The induced sliding vector field therefore keeps the trajectory on Σ for a nontrivial time interval (see [37]). Along such segments,

$$P(t) \equiv P_c,$$

which corresponds to a valuation plateau. $\square$

Proposition A1 (Exact mean-field reduction under homogeneous exposure). *Consider the heterogeneous platform system (1)–(13). Assume the following:*

(i) *Participants have identical baseline platform fit,*

$$\beta_i = \bar{\beta} \quad \text{for all } i.$$

(ii) The network has homogeneous total exposure,

$$\sum_{j=1}^N w_{ij} = \bar{w} \quad \text{for all } i.$$

(iii) Initial participation is symmetric,

$$A_i(0) = a_0 \quad \text{for all } i.$$

Then, the diagonal manifold

$$\mathcal{D} = \{(A_1, \dots, A_N, P) : A_1 = \dots = A_N\}$$

is forward-invariant for the Filippov system. Along this manifold, $A_i(t) = a(t)$ for all i , and the heterogeneous system reduces exactly to the planar adoption–valuation system

$$\dot{a} = a(1-a)\Gamma_k(\bar{\beta} + \theta\bar{w}a - \zeta c(a) - \chi u(P)), \quad (\text{A1})$$

$$\dot{P} = \kappa_P [\alpha a \ell(P) + \beta_k s(P) - \bar{S}], \quad (\text{A2})$$

with regime rule

$$k = \begin{cases} L, & P < P_c, \\ R, & P > P_c. \end{cases}$$

Hence, under homogeneous exposure and symmetric initial conditions, the planar system is not merely an approximation: it is the exact restriction of the heterogeneous network economy to the invariant aggregate participation manifold.

Proof. Suppose that $A_1 = \dots = A_N = a$. By assumption (ii),

$$S_i(A) = \sum_{j=1}^N w_{ij} A_j = a \sum_{j=1}^N w_{ij} = \bar{w}a \quad \text{for every } i.$$

Moreover,

$$\bar{A} = \frac{1}{N} \sum_{i=1}^N A_i = a.$$

Using assumption (i), the participant value satisfies

$$V_i(A, P) = \bar{\beta} + \theta\bar{w}a - \zeta c(a) - \chi u(P) \quad \text{for every } i.$$

Therefore, every participant has the same right-hand side in (5):

$$\dot{A}_i = a(1-a)\Gamma_k(\bar{\beta} + \theta\bar{w}a - \zeta c(a) - \chi u(P)).$$

Thus, if $A_i(0) = a_0$ for all i , uniqueness of the regime-wise ordinary differential equations implies $A_i(t) = a(t)$ for all i between threshold events. At the switching manifold $P = P_c$, the Filippov set is the convex hull of the left and right regime vector fields. Since both regime vector fields are tangent to the diagonal manifold $\mathcal{D}$, their convex hull is also tangent to $\mathcal{D}$. Hence, $\mathcal{D}$ is forward invariant for the Filippov inclusion. Finally, substituting $\bar{A} = a$ into the excess-demand rule (12) yields (A2). The reduced adoption equation is (A1). This proves the claim. $\square$

Proposition A1 clarifies the precise sense in which the planar system can be obtained from the heterogeneous network model. The reduction is exact on the symmetric homogeneous-exposure manifold. Outside this case, the planar model should be interpreted as a governance-level mean-field approximation rather than as a lossless aggregation theorem for arbitrary networks. Network topology may affect diffusion speed, local cascades, and the timing of threshold crossings, but the aggregate threshold mechanism isolated in (A1)–(A2) remains the analytical object studied below.

Appendix C. Numerical Analysis

The numerical analysis examines how alternative platform design choices affect participation–valuation dynamics around the credibility threshold P_c . In the model, design primitives such as effective circulating supply $\bar{S}$, liquidity asymmetry (β_L, β_R) , and the structure of participation incentives determine how the ecosystem responds when valuation approaches the threshold. The simulations therefore interpret different adoption rules as stylised governance mechanisms that shape user adjustment, network amplification, and market responsiveness. The diagnostics reported below—such as time spent at the valuation anchor, sliding episode persistence, exit patterns, and late-time participation distributions—illustrate how these design features influence whether the platform ecosystem exhibits anchoring, volatility, or stable convergence.

Table A2. Time allocation and state ranges for single-trajectory simulations (A1–A6). For each adoption law, the table reports the fractions of the horizon spent in the left regime ($P < P_c$), in attracting Filippov sliding on the valuation threshold $\Sigma := \{P = P_c\}$, and in the right regime ($P > P_c$), together with the observed ranges of adoption $a(t)$ and price $P(t)$. All quantities are computed from a single trajectory over horizon $T = 120$ (fixed step size) using the sliding and contact definitions described in this appendix.

Law	T_L/T	$T_{\Sigma}^{\text{slide}}/T$	T_R/T	$a_{\min}-a_{\max}$	$P_{\min}-P_{\max}$
(A1) Linear response	0.0025	0.9965	0.0010	0.250–0.633	0.550–1.002
(A2) Saturating response	0.0032	0.9968	$<10^{-4}$	0.245–0.329	0.550–1.000
(A3) Threshold-biased (strong exit)	0.0026	0.9973	0.0002	0.250–0.595	0.550–1.001
(A4) Hysteretic (wide band)	0.0025	0.9855	0.0121	$<10^{-6}$ –0.871	0.550–1.184
(A5) Stochastic bandwagon (mult. noise)	0.0032	0.9968	$<10^{-4}$	0.153–0.380	0.550–1.001
(A6) Noisy linear (additive)	0.0026	0.9973	0.0001	0.247–0.592	0.550–1.001

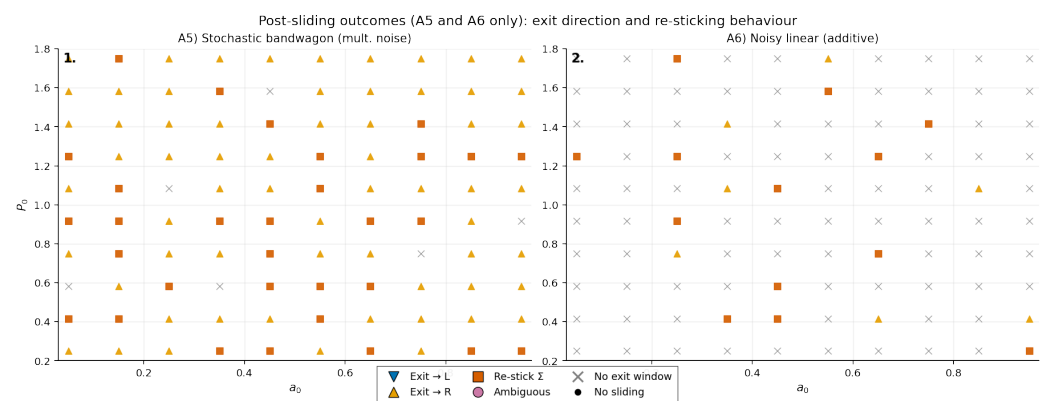


Figure A1. Post-sliding outcomes under stochastic adoption (A5 and A6): exit direction and re-anchoring. For each initial condition, the first detected attracting sliding episode on Σ is identified, if it exists. Subsequent behaviour is classified over a fixed post-window after the episode ends as Exit to L if $P < P_c - \text{tol}_{\text{contact}}$, Exit to R if $P > P_c + \text{tol}_{\text{contact}}$, Re-stick Σ if the trajectory remains within the contact band for at least 85% of the post-window, and Ambiguous otherwise. Initial conditions with no detected sliding are labelled No sliding, and cases where the episode ends too close to the horizon are labelled No exit window.

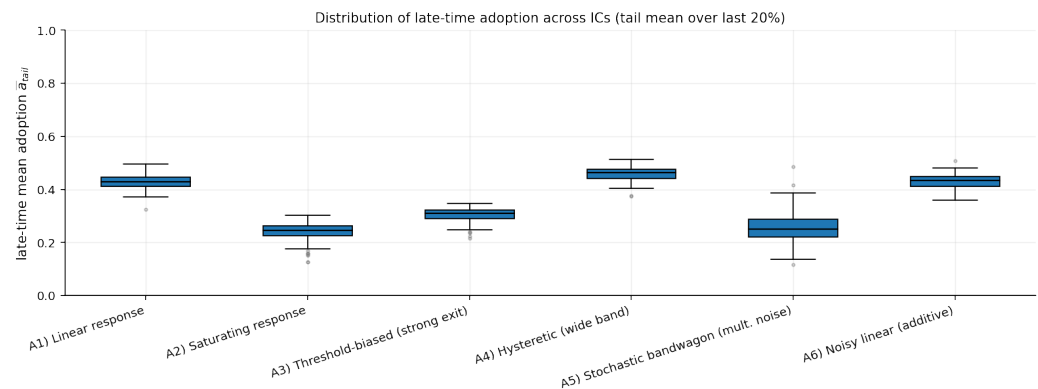


Figure A2. Late-time adoption across initial conditions: tail-mean participation under A1–A6. For each adoption law, the boxplot summarises the distribution (over initial conditions) of the late-time mean adoption $\bar{a}_{tail}$, computed as the average of $a(t)$ over the final 20% of the horizon. This diagnostic complements regime and sliding statistics by quantifying which participation levels are associated with the observed anchoring and switching patterns.

Table A3. Anchoring dominance and late-time adoption outcomes (deterministic and hysteretic rules A1–A4). Reported are the mean share of time spent in attracting sliding on the valuation threshold Σ , together with late-time adoption statistics (mean and standard deviation over the final 20% of the horizon) computed across initial conditions. Exit frequencies from Σ are also reported.

Law	Mean Slide Share	Mean a_{late}	SD a_{late}	Exit to L	Exit to R
(A1) Linear response	0.992	0.429	0.027	0.00	0.00
(A2) Saturating response	0.985	0.237	0.036	0.00	0.00
(A3) Threshold-biased (strong exit)	0.994	0.304	0.026	0.00	0.00
(A4) Hysteretic (wide band)	0.991	0.460	0.028	0.00	0.00

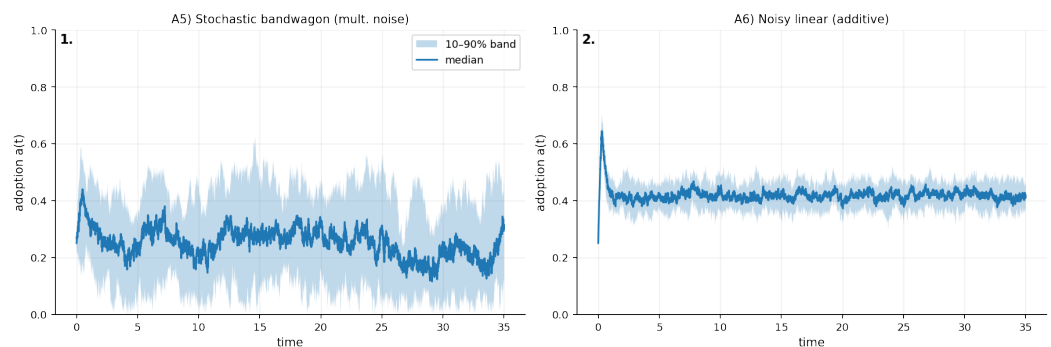


Figure A3. Monte Carlo adoption envelopes under stochastic adoption (A5 and A6) from a common initial condition. For each stochastic adoption rule, the system is simulated from the same initial condition $(a_0, P_0) = (0.25, 0.55)$. Across Monte Carlo replications, each run uses an independent noise realisation and draws mild price-side heterogeneity in $(\beta_R, \bar{S})$. The shaded band reports pointwise 10–90% quantiles of $a(t)$ and the central curve reports the pointwise median. The envelopes isolate joint variability induced by adoption noise and market heterogeneity, highlighting differences in dispersion and drift between multiplicative (A5) and additive (A6) noise.

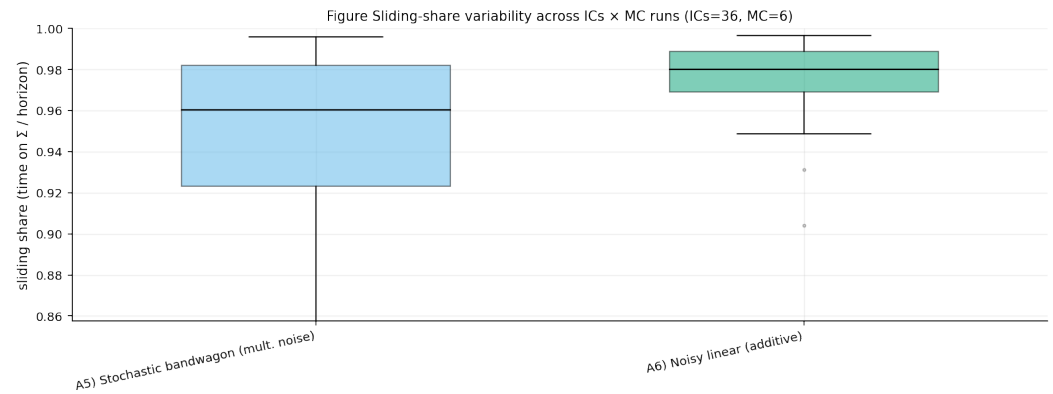


Figure A4. Distribution of sliding intensity pooled over initial conditions and Monte Carlo runs (A5 vs. A6). For each stochastic rule, the boxplot summarises the distribution of the sliding-share statistic $T_{\Sigma}^{\text{slide}}/T$ pooled across all initial conditions and Monte Carlo replications. Sliding is detected within a tight contact band and must satisfy the attracting inequalities $g_L(a, P_c) > 0$ and $g_R(a, P_c) < 0$, with a minimum episode duration to suppress spurious contact. Robust quantile-based axis limits are used to improve visibility of between-rule differences.

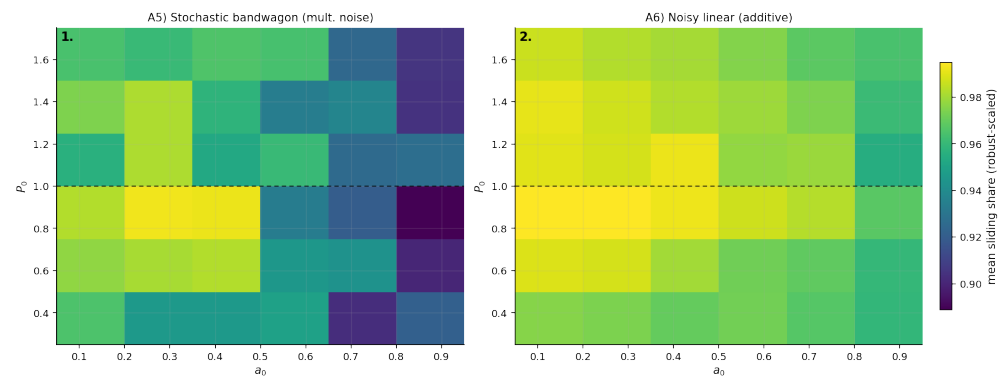


Figure A5. Initial-condition dependence of mean sliding intensity under stochastic adoption (A5 vs. A6). Heat maps report the mean sliding share $\mathbb{E}[T_{\Sigma}^{\text{slide}}/T \mid (a_0, P_0)]$ computed by averaging across Monte Carlo replications at each initial condition on the (a_0, P_0) grid. Colour limits use robust clipping to prevent a small number of extreme cells from saturating the scale. The dashed horizontal line marks the valuation threshold $P = P_c$.

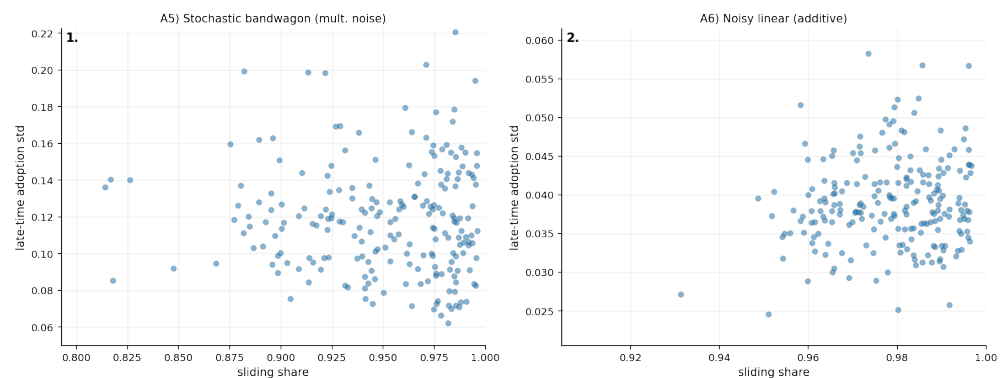


Figure A6. Trade-off between valuation anchoring and late-time participation volatility (stochastic rules). Each point corresponds to one simulation run (one initial condition and one Monte Carlo replication). The x-axis reports sliding share $T_{\Sigma}^{\text{slide}}/T$ (valuation anchoring intensity), and the y-axis reports the standard deviation of adoption over the final 20% of the horizon (late-time participation volatility). Robust quantile-based axis limits reduce empty space while preserving the distributional pattern.

Table A4. Stochastic anchoring intensity, volatility, and post-plateau exits (A5–A6). Reported are the mean and standard deviation of the sliding share $T_{\Sigma}^{\text{slide}}/T$, late-time adoption statistics, late-time adoption volatility (standard deviation over the final 20%), and post-sliding exit behaviour. Fractions are computed over the full ensemble of initial conditions and Monte Carlo replications.

Law	Mean Slide	SD Slide	Mean a_{late}	SD a_{late}	Mean Volatility	No Slide	Exit L	Exit R
(A5) Stochastic bandwagon (mult. noise)	0.948	0.044	0.255	0.084	0.119	0.00	0.00	0.54
(A6) Noisy linear (additive)	0.978	0.013	0.426	0.042	0.039	0.00	0.00	0.04

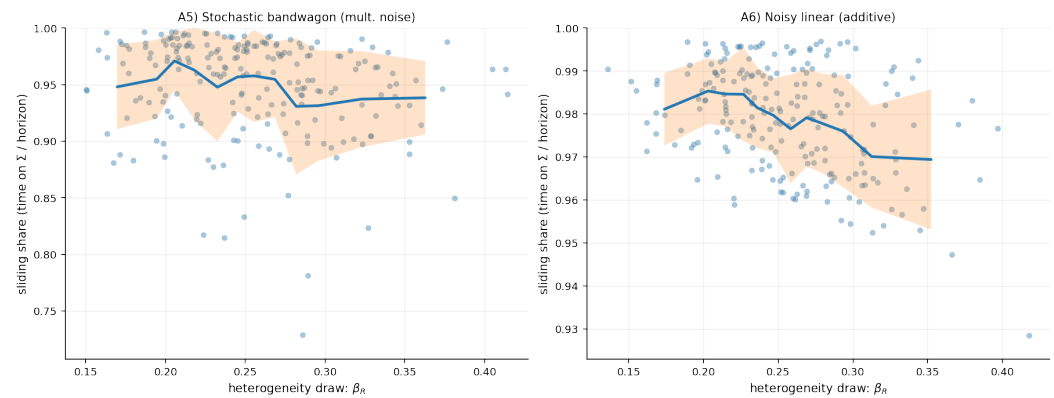


Figure A7. Sensitivity of valuation anchoring to heterogeneity in β_R (A5 and A6). Each point represents one Monte Carlo realisation across initial conditions and heterogeneity draws, plotting sliding share $T_{\Sigma}^{\text{slide}}/T$ against the realised β_R . The solid curve reports equal-count binned means and the shaded band reports ± 1 standard deviation within bins. The vertical axis is restricted to $[0, 1]$ because sliding share is a proportion.

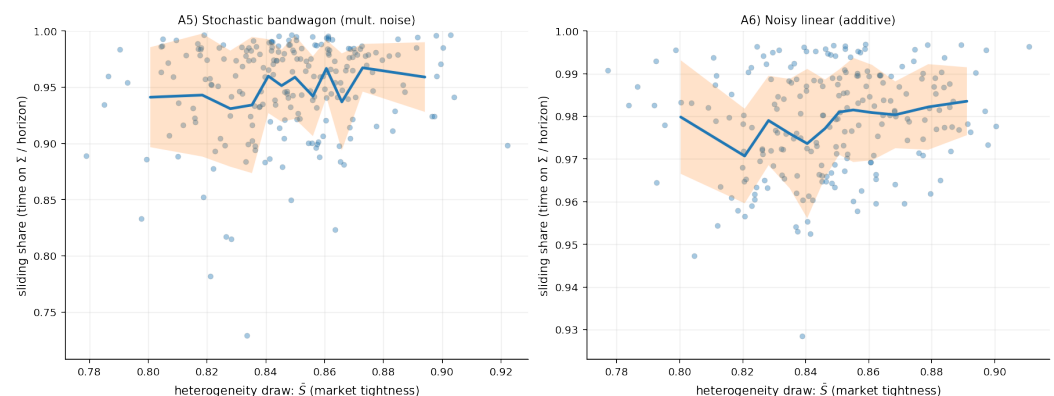


Figure A8. Sensitivity of valuation anchoring to market tightness heterogeneity $\bar{S}$ (A5 and A6). Points show Monte Carlo realisations across initial conditions, plotting sliding share $T_{\Sigma}^{\text{slide}}/T$ against the realised draw of $\bar{S}$. The solid curve reports equal-count binned means and the shaded band reports ± 1 standard deviation within bins. The vertical axis is restricted to $[0, 1]$.

Table A5. Sensitivity of valuation anchoring to market heterogeneity (stochastic adoption rules). Reported are Pearson and Spearman correlations between sliding share $T_{\Sigma}^{\text{slide}}/T$ and heterogeneity draws in β_R and $\bar{S}$, computed across all initial conditions and Monte Carlo replications for the stochastic adoption rules.

Law	Slide_Share vs. β_R		Slide_Share vs. $\bar{S}$	
	Pearson	Spearman	Pearson	Spearman
(A5) Stochastic bandwagon (mult. noise)	−0.185	−0.234	+0.149	+0.118
(A6) Noisy linear (additive)	−0.387	−0.338	+0.164	+0.175

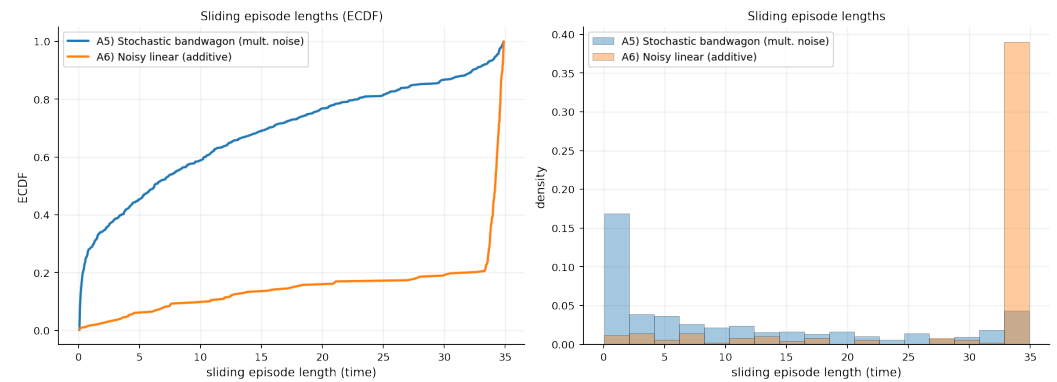


Figure A9. Sliding episode lengths under stochastic adoption (A5 vs. A6): plateau persistence. A sliding episode is a maximal contiguous time interval during which the trajectory remains in attracting Filippov sliding on Σ . The left panel shows empirical cumulative distribution functions of episode lengths, and the right panel shows the corresponding normalised histograms. Differences between A5 and A6 quantify how the noise structure affects the persistence of valuation plateaus.

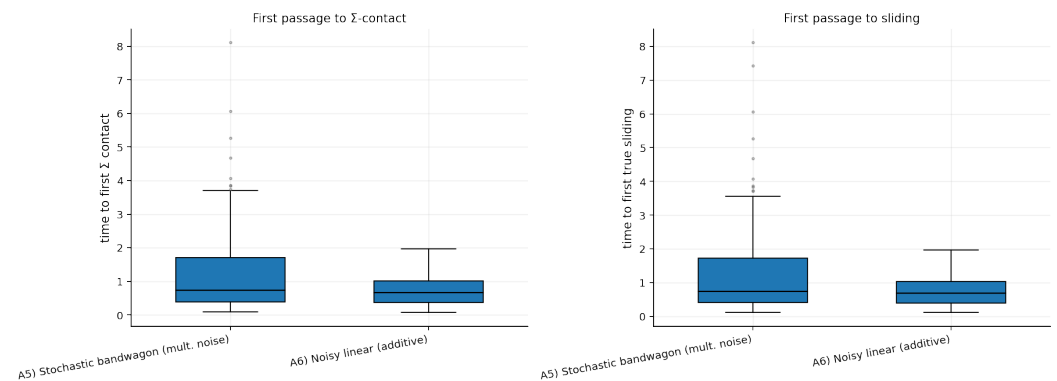


Figure A10. First-passage times to threshold contact and to genuine sliding. Boxplots compare (left) time to first contact with the switching manifold Σ and (right) time to first entry into an attracting Filippov sliding segment. The comparison highlights that geometric contact with Σ is not sufficient for valuation anchoring; anchoring requires the attracting inequalities defining sliding.

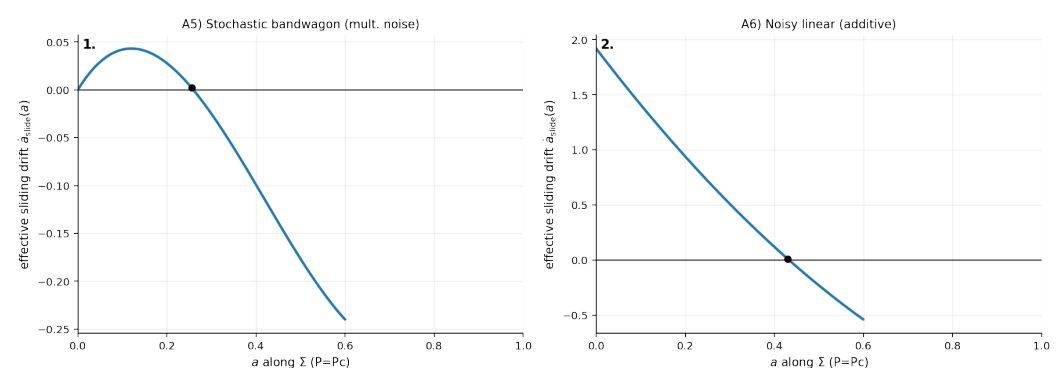


Figure A11. Induced sliding drift along the valuation threshold Σ . For each stochastic adoption rule, the Filippov sliding drift $\hat{a}_{\text{slide}}(a)$ induced on Σ is evaluated as a function of a and plotted only on the attracting sliding subset. Zeros of $\hat{a}_{\text{slide}}$ identify candidate sliding equilibria (pseudo-equilibria) that organise the long-run stochastic motion and the stationary adoption distributions reported below.

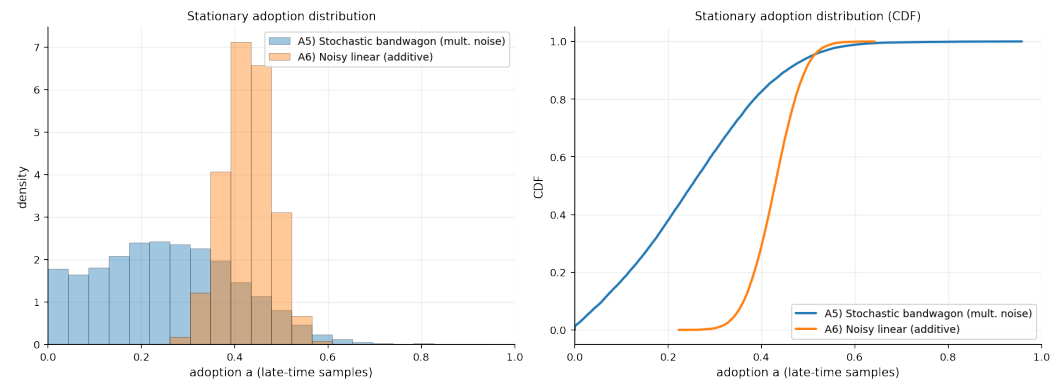


Figure A12. Stationary adoption distributions induced by stochastic sliding (A5 vs. A6). Late-time samples of $a(t)$ are pooled across initial conditions and Monte Carlo replications. The left panel reports normalised histograms and the right panel reports the corresponding cumulative distribution functions. These distributions summarise long-run participation outcomes generated by stochastic Filippov dynamics when valuation anchoring dominates.

Table A6. Threshold contact, sliding entry, and episode counts under stochastic adoption (A5 and A6). Reported are empirical probabilities of contacting Σ and entering attracting sliding, mean first-passage times to Σ and to sliding, and the mean number of distinct sliding episodes per trajectory, computed across all initial conditions and Monte Carlo replications.

Adoption Rule	p_{contact}	p_{sliding}	$\mathbb{E}[t_{\text{contact}}]$	$\mathbb{E}[t_{\text{sliding}}]$	$\mathbb{E}[N_{\text{episodes}}]$
(A5) Stochastic bandwagon (mult. noise)	1.00	1.00	1.202	1.241	3.014
(A6) Noisy linear (additive)	1.00	1.00	0.725	0.734	1.148

Appendix C.1. Relation Between Alternative Adoption Specifications and Theorem 2

The simulations reported in Appendix C employ six alternative participation-response specifications (A1–A6) in order to assess the robustness of the anchoring mechanism identified in Theorem 2. An important distinction should be emphasised.

Theorem 2 is not a statement about a specific functional form for participation dynamics. Rather, it is a statement about the geometry of the vector fields in a neighbourhood of the switching manifold

$$\Sigma = \{P = P_c\}.$$

The existence of a sliding interval depends on the relative orientation of the left- and right-regime vector fields along the threshold. Consequently, alternative adoption specifications need not reproduce the exact participation equation used in the baseline model in order to remain consistent with the theorem. What matters is whether the modified participation response preserves the sign structure required by the sliding conditions.

This observation provides a useful interpretation of the robustness exercises. The simulations are not intended as independent proofs of Theorem 2. Rather, they examine whether the anchoring mechanism survives when the participation response is modified while preserving the qualitative threshold geometry of the model.

Proposition A2 (Robustness of valuation anchoring). *Let*

$$\dot{a} = \tilde{g}_k(a, P), \quad k \in \{L, R\},$$

denote an alternative participation specification satisfying the regularity assumptions of the baseline model. Suppose that there exists an interval

$$I = (a_-, a_+)$$

such that along the switching manifold

$$P = P_c$$

the modified participation dynamics satisfy

$$\tilde{g}_L(a, P_c) > 0, \quad \tilde{g}_R(a, P_c) < 0, \quad \forall a \in I.$$

Then the switching manifold remains attracting on I , and the Filippov inclusion admits a sliding region on

$$\Sigma_I = \{(a, P_c) : a \in I\}.$$

Conversely, if either sign condition fails on a subset of I , then the sliding region may shrink, fragment, or disappear. In this case, trajectories generally exhibit regular crossing rather than persistent anchoring.

Proof. The proof follows directly from the standard Filippov sliding conditions.

Along the switching manifold, the left-regime vector field points toward Σ whenever

$$\tilde{g}_L(a, P_c) > 0,$$

while the right-regime vector field points toward Σ whenever

$$\tilde{g}_R(a, P_c) < 0.$$

Hence, both vector fields are directed inward on the interval I . The Filippov convexification therefore generates a sliding vector field on

$$\Sigma_I.$$

If either sign condition is violated, the inward-pointing condition fails. The corresponding segment of the switching manifold ceases to be attracting and trajectories cross the threshold instead. The sliding interval is therefore reduced or eliminated.

□

Proposition A2 clarifies the role of the alternative participation specifications. The anchoring result of Theorem 2 depends primarily on threshold geometry rather than on a particular adoption function. Consequently, different participation specifications may generate similar anchoring behaviour provided that they preserve the sign conditions underlying the theorem. Table A7 summarises the relationship between the alternative participation specifications and the assumptions of Theorem 2.

Table A7. Relation between alternative adoption specifications and Theorem 2. The table classifies whether each alternative participation specification preserves the threshold geometry underlying the sliding conditions of Theorem 2. The reported outcomes summarise the simulation evidence reported in Tables A3, A4, and A6.

Specification	Threshold Geometry	Expected Anchoring	Simulation Evidence
A1	Preserved	High	Anchoring dominant
A2	Preserved	High	Anchoring dominant
A3	Preserved	High	Anchoring dominant
A4	Partly preserved	Intermediate	Reduced anchoring
A5	Preserved in expectation	Probabilistic	Frequent re-anchoring
A6	Weakened by noise	Reduced	Lower persistence

Finally, the robustness exercises should be interpreted together with the theoretical results. The simulations are not designed merely to reproduce the baseline anchoring regime. Their purpose is to investigate whether the threshold mechanism identified analytically survives under alternative participation responses. The results indicate that anchoring remains prominent whenever the threshold geometry implied by Theorem 2 is preserved, whereas departures from the sign conditions weaken or eliminate persistent sliding.

References

- Ben Youssef, A. Introducing Platforms: A transdisciplinary journal on platform management, services and policy and all related research. *Platforms* **2022**, *1*, 1–4. [\[CrossRef\]](#)
- Barabási, A.-L.; Albert, R. Emergence of scaling in random networks. *Science* **1999**, *286*, 509–512. [\[CrossRef\]](#) [\[PubMed\]](#)
- Erdős, P.; Rényi, A. On random graphs I. *Publ. Math.* **1959**, *6*, 290–297. [\[CrossRef\]](#)
- Watts, D.J.; Strogatz, S.H. Collective dynamics of ‘small-world’ networks. *Nature* **1998**, *393*, 440–442. [\[CrossRef\]](#) [\[PubMed\]](#)
- Rochet, J.-C.; Tirole, J. Platform competition in two-sided markets. *J. Eur. Econ. Assoc.* **2003**, *1*, 990–1029. [\[CrossRef\]](#)
- Rochet, J.-C.; Tirole, J. Two-sided markets: A progress report. *RAND J. Econ.* **2006**, *37*, 645–667. [\[CrossRef\]](#)
- Sun, Y. A dynamic analysis of cross-category innovation in digital platform ecosystems. *J. Theor. Appl. Electron. Commer. Res.* **2025**, *20*, 229. [\[CrossRef\]](#)
- Zhou, Y.; Wang, L.; Chen, M. Innovation in platform ecosystems: Roles of complementors and sustainability. *Sustainability* **2025**, *17*, 2279. [\[CrossRef\]](#)
- Parker, G.G.; Van Alstyne, M.W.; Jiang, X. Platform ecosystems: How developers invert the firm. *MIS Q.* **2017**, *41*, 255–266. [\[CrossRef\]](#)
- Barberis, N.; Shleifer, A.; Vishny, R. A model of investor sentiment. *J. Financ. Econ.* **1998**, *49*, 307–343. [\[CrossRef\]](#)
- Pagnotta, E.S. Decentralizing money: Bitcoin prices and blockchain security. *Rev. Financ. Stud.* **2022**, *35*, 866–907.
- Biais, B.; Bisière, C.; Bouvard, M.; Casamatta, C.; Menkveld, A.J. Equilibrium Bitcoin pricing. *J. Financ.* **2022**, *78*, 967–1014.
- Cong, L.W.; Li, Y.; Wang, N. Tokenomics: Dynamic adoption and valuation. *Rev. Financ. Stud.* **2021**, *34*, 1105–1155. [\[CrossRef\]](#)
- Liu, Y.; Tsyvinski, A. Risks and returns of cryptocurrency. *Rev. Financ. Stud.* **2021**, *34*, 2689–2727. [\[CrossRef\]](#)
- Baumol, W.J. The transactions demand for cash: An inventory theoretic approach. *Q. J. Econ.* **1952**, *66*, 545–556. [\[CrossRef\]](#)
- Feenstra, R.C. Functional equivalence between liquidity costs and the utility of money. *J. Monet. Econ.* **1986**, *17*, 271–291. [\[CrossRef\]](#)
- Tobin, J. The interest-elasticity of transactions demand for cash. *Rev. Econ. Stat.* **1956**, *38*, 241–247. [\[CrossRef\]](#)
- Walsh, C.E. *Monetary Theory and Policy*, 2nd ed.; MIT Press: Cambridge, MA, USA, 2003.
- Han, Y.; Xie, L. Sustainable governance of digital platform ecosystem: A life cycle perspective through multiple governance parties. *Sustainability* **2025**, *17*, 3628. [\[CrossRef\]](#)
- Morkunas, V.J.; Paschen, J.; Boon, E. How blockchain technologies impact your business model. *Bus. Horiz.* **2019**, *62*, 295–306. [\[CrossRef\]](#)
- Panori, A. Platforms enhancing proximity in the digital era. *Platforms* **2024**, *2*, 1. [\[CrossRef\]](#)
- Saleh, F. Blockchain without waste: Proof-of-stake. *Rev. Financ. Stud.* **2021**, *34*, 1156–1190. [\[CrossRef\]](#)
- Schilling, L.; Uhlig, H. Some simple bitcoin economics. *J. Monet. Econ.* **2019**, *106*, 16–26. [\[CrossRef\]](#)
- Shams, A. What drives the covariation of cryptocurrency returns. In Proceedings of the Association of Financial Economists and American Economic Association Beyond Bitcoin Paper Session Conference, Atlanta, GA, USA, 4–6 January 2019.
- Tiscini, R.; Testarmata, S.; Ciaburri, M.; Ferrari, E. The blockchain as a sustainable business model innovation. *Manag. Decis.* **2020**, *58*, 1621–1642. [\[CrossRef\]](#)
- Brunnermeier, M.K.; Sannikov, Y. A macroeconomic model with a financial sector. *Am. Econ. Rev.* **2014**, *104*, 379–421. [\[CrossRef\]](#)
- Goldstein, I. *Initial Coin Offerings as a Commitment to Competition*; Working paper; SSRN: Rochester, NY, USA, 2020.
- He, Z.; Krishnamurthy, A. A model of capital and crises. *Rev. Econ. Stud.* **2011**, *79*, 735–777. [\[CrossRef\]](#)
- He, Z.; Krishnamurthy, A. Intermediary asset pricing. *Am. Econ. Rev.* **2013**, *103*, 732–770. [\[CrossRef\]](#)
- Krishnamurthy, A.; Vissing-Jorgensen, A. The aggregate demand for treasury debt. *J. Political Econ.* **2012**, *120*, 233–267. [\[CrossRef\]](#)
- Nagel, S. The liquidity premium of near-money assets. *Q. J. Econ.* **2016**, *131*, 1927–1971. [\[CrossRef\]](#)
- Makarov, I.; Schoar, A. Trading and arbitrage in cryptocurrency markets. *J. Financ. Econ.* **2020**, *135*, 293–319. [\[CrossRef\]](#)
- Osler, C.L. Currency orders and exchange-rate dynamics: Explaining the success of technical analysis. *J. Financ.* **2003**, *58*, 1791–1820. [\[CrossRef\]](#)
- Carlsson, H.; Van Damme, E. Global games and equilibrium selection. *Econometrica* **1993**, *61*, 989–1018. [\[CrossRef\]](#)
- Morris, S.; Shin, H.S. Unique equilibrium in a model of self-fulfilling currency attacks. *Am. Econ. Rev.* **1998**, *88*, 587–597.

36. di Bernardo, M.; Budd, C.J.; Champneys, A.R.; Kowalczyk, P. *Piecewise-Smooth Dynamical Systems: Theory and Applications*; Springer: London, UK, 2008. [\[CrossRef\]](#)
37. Filippov, A.F. *Differential Equations with Discontinuous Righthand Sides*; Kluwer Academic Publishers: Dordrecht, The Netherlands, 1988.
38. Sontag, E. Contractive systems with inputs. In *Perspectives in Mathematical System Theory, Control, and Signal Processing*; Lecture Notes in Control and Information Sciences, 398; Springer-Verlag: Berlin, Germany, 2010; pp. 217–228.
39. Stiefenhofer, P.; Giesl, P. Economic periodic orbits: A theory of exponential asymptotic stability. *Nonlinear Anal. Differ. Equ.* **2019**, *7*, 9–16. [\[CrossRef\]](#)
40. Stiefenhofer, P. Economic stability of non-smooth periodic orbits in the plane: Omega limit sets part I. *Nonlinear Anal. Differ. Equ.* **2020**, *8*, 41–52. [\[CrossRef\]](#)
41. Stiefenhofer, P. Economic stability of non-smooth periodic orbits in the plane: Omega limit sets Part II. *Nonlinear Anal. Differ. Equ.* **2020**, *8*, 53–65. [\[CrossRef\]](#)
42. Stiefenhofer, P. Exponential stability of nonsmooth periodic orbits in the complex space. *Phys. Lett. A* **2025**, *560*, 130935. [\[CrossRef\]](#)
43. Stiefenhofer, P. Spectral contraction framework for planar Filippov systems: Existence and stability of nonsmooth periodic orbits. *Commun. Adv. Math. Sci.* **2025**, *8*, 189–209. [\[CrossRef\]](#)
44. Aubin, J.-P.; Cellina, A. *Differential Inclusions: Set-Valued Maps and Viability Theory*; Springer-Verlag: Berlin, Germany, 1984.
45. Cortés, J. Discontinuous dynamical systems. *IEEE Control Syst. Mag.* **2008**, *28*, 36–73. [\[CrossRef\]](#)
46. Filippov, A.F. On certain questions in the theory of optimal control. *J. Soc. Ind. Appl. Math. Ser. A Control* **1963**, *1*, 76–84.
47. Cong, L.W.; Li, Y.; Wang, N. *Token-Based Platform Finance*; NBER Working Paper No. 27810; National Bureau of Economic Research: Cambridge, MA, USA, 2020. [\[CrossRef\]](#)
48. Pagnotta, E.; Buraschi, A. An Equilibrium Valuation of Bitcoin and Decentralized Network Assets. Working Paper, SSRN, 2018. Available online: <https://ssrn.com/abstract=3142022> (accessed on 14 May 2026) .
49. Sockin, M.; Xiong, W. *A Model of Cryptocurrencies*; NBER Working Paper No. 26816; National Bureau of Economic Research: Cambridge, MA, USA, 2020. Available online: <http://www.nber.org/papers/w26816> (accessed on 14 May 2026).
50. Howell, S.T.; Niessner, M.; Yermack, D.; Wei, J. Initial coin offerings: Financing growth with cryptocurrency token sales. *Rev. Financ. Stud.* **2020**, *33*, 3925–3974. [\[CrossRef\]](#)

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