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Distributed Economic Dispatch Control Method with Frequency Regulator for Smart Grid under Time-Varying Directed Topology

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Abstract: The paper studies a new distributed control method to solve the economic dispatch problem (EDP) under directed topology based on consensus protocol. Electrical equipment is closely related to frequency, and the frequency of each generator varies independently during operation. Therefore, it hinders the realization of economic dispatch. To solve the problem, we combine a frequency regulator with a consensus protocol, which eliminates the effect of frequency variation on the designed consensus algorithm. Meanwhile, considering the problem of excessive communication cost and low computational efficiency in large-scale power systems, an event-triggered mechanism is introduced into the designed algorithm. Furthermore, in order to overcome the unexpected loss of communication links, the time-varying topology mechanism is employed to develop the distributed economic dispatch (DED) algorithm to improve the robustness. Then, the stability of the above algorithm is proved by graph theory and convergence analysis. Finally, several simulations illustrate that our proposed methods are effective.

Keywords: economic dispatch; consensus protocol; frequency regulator; event-triggered; time-varying topology

1. Introduction

With the emergence of the smart grid, people’s power consumption has become more scientific and low consumption. How to solve the problem of power generation in power systems at low cost is an important topic of general concern in society. EDP is a conditional optimization problem, which mainly studies some dispatch methods to ensure reliable power supplies to users with the lowest generation cost [1]. At present, the economic dispatch methods of a smart grid are mainly divided into the centralized method and distributed method. The traditional centralized method requires a central processing unit to capture global resources for all generators and calculate large amounts of data [2,3], such as the Lagrange multiplier method and neural network Approach [4,5]. However, with the continuous expansion of power system scale, it leads to the over dispersion of power systems. In view of this situation, the centralized method has limitations so that the communication cost increases and the processing capacity decreases. Hence, a distributed algorithm is very suitable to solve EDP in the distributed power system [6], which does not need a centralized control center, and each generator only needs to obtain local information of adjacent generators to complete EDP.

In recent years, many DED methods have emerged. Generally distributed algorithms are designed based on consensus protocol [7]. Meanwhile, the solution of these methods
should also meet supply and demand balance constraints. At the beginning of the study, the DED method has some limitations. For instance, a consensus-based EDP algorithm for the microgrid is proposed in [8], which obtains the optimal solution by obtaining the global power mismatch value in advance. However, the global power mismatch value still needs a control center to calculate, so the algorithm is not completely distributed. In [9], an innovation term is introduced on the basis of the consensus term, which is used to make an incremental cost converge to a common value and ensure balance between supply and demand. Although [9] is a fully distributed algorithm, its scope of application is still limited. At present, considering the diversity of the communication environment, some DED algorithms had been proposed. In terms of communication topology, the algorithm mentioned in [10] makes up for the shortcoming of the traditional consensus-based algorithm that only after convergence can there be feasible solutions, and it is applicable for the jointly connected switching undirected topology. Close to reality, [11] combines the frequency control method and consensus protocol under undirected topology, which is introduced to balance the actual power between load and power generation in the process of economic dispatch. However, the above methods are carried out in the ideal communication and do not take into account the problem of communication asynchrony. Hence, [12] proposes a DED algorithm to overcome the difficulties of of asynchronous communication under unbalance directed topology. The algorithm needs only the row-stochastic weighted matrices, which assigns the weight of messages that each generator receives from its in-neighbors. The algorithm in [13] has higher convergence speed than the previous method. It presents an algorithm of distributed Lagrangian momentum, which uses two momentum terms to track the gradient and updates the Lagrange multipliers in non-uniform steps. Furthermore, since all of the above methods are real-time communication, it can consume more communication resources and have low computational efficiency when these methods are applied to a large-scale power system. Therefore, the event-triggered mechanisms are added into some algorithms [14–16]. The theta-logarithmic barrier-based method and consensus-based approach are proposed in [14], which introduce the event-triggered mechanism. To improve economic dispatch efficiency, a novel event-triggered distributed accelerated primal-dual algorithm with uncoordinated step sizes is proposed [16]. In addition, uncertain communication link failure may lead to continuous changes in the topology of the communication network. Therefore, introducing the time-varying topology mechanism into the algorithm can eliminate the adverse effects of topology changes. Ref. [17] introduces the dual gradient idea into the consensus algorithm and designs a DED algorithm under time-varying topology, which uses a fixed step size and column random mixing matrix to accurately guide the incremental cost of all generators to reach the optimal consensus value. Ref. [18] proposes the gradient push-sum method in time-varying topology, which ensures the robustness of the system and improves the convergence efficiency. In general, these algorithms are updated iteratively in a relatively ideal environment, but in the actual situation, many practical factors should be considered in the process of economic dispatch. Recently, some DED methods considering complex environments have emerged, such as transmission loss [19–23], time delay [24–28], cyber-attack [29,30], privacy protection [31], and so on. These methods greatly improve the adaptability of EDP in a complex environment. In addition, the economic dispatch problem of a smart grid can be extended to energy dispatching. For example, in [32], a day-ahead optimal dispatching method for a smart grid is proposed with the objective of minimizing the fuel energy input from the main grid. It is mentioned in [33] that increasing the share of renewable energy connected to the grid frequency converter can reduce the inertia of the power system, which is critical to the stability of the grid frequency. Therefore, we can take such factors into account in the algorithm. Ref. [34] proposed a fully distributed hierarchical control framework for an autonomous AC microgrid, which only requires local inter-agent interaction with a sparse communication network. Therefore, it is completely distributed. Finally, some articles on multi-intelligent systems are also helpful to the optimization of our economic scheduling algorithm [35,36].
Combined with the actual operating environment, the frequency will have a certain impact on the normal operation of the generator. However, some studies in the literature consider the change of frequency. Most algorithms add a frequency control term to the consensus protocol to balance the actual power of the generation and load [11]. This is a novel innovation, but it still has some aspects that are not considered enough. Inspired by the distributed algorithm in [11], in this paper, an event-triggered DED controlled algorithm with a frequency regulator under time-varying topology is proposed. The main innovations and improvements are summarized below: (1) Compared with [11], Ref. [11] considers that the frequency of each generator is always the same. However, in practice, the frequency of each generator may be different due to the uncertainty of frequency variation. Therefore, we introduce a frequency regulator into the consensus protocol to eliminate the impact of frequency change on the economic dispatching process, which is more suitable for the power generation process in the actual scenario, and ensure the balance of power supply and demand. (2) The existing work based on undirected topology [10,11,14,15] does not consider the case of communication disturbance or asynchronous communication; i.e., there is one-way communication between two generators. So, we use a row-stochastic and column-stochastic matrix to design an algorithm under directed topology to solve these problems. So, our algorithm has the characteristics of one-way communication. (3) The paper adds an event-triggered mechanism to the algorithm, which makes up for the disadvantages of consuming too many communication resources and low computing efficiency in a large-scale power system. Therefore, the communication frequency of our algorithm is greatly reduced. (4) The communication fault proposed in [11] refers to the situation in which a generator cannot measure the frequency, but it does not consider the change of the actual topology caused by the communication fault. We consider that the topology may change at every moment due to line fault in this paper. Therefore, a new time-varying topology mechanism is added into the algorithm to increase the robustness of the entire power system, which makes the algorithm we designed more practical. Different from the design in [17,18], we design a mapping topology function in the consensus algorithm to indicate the topology at any time.

The structure of this paper is divided several parts: we describe some preliminaries in Section 2. Section 3 give a DED formulation and consensus protocol related to economic dispatch. Several DED control methods are proposed in Section 4 to solve EDP. In Section 5, we prove the convergence of the design algorithm through theoretical analysis. Several simulations are carried out in Section 6. Finally, a brief conclusion is provided in Section 7.

2. Preliminary

This section describes the correlation graph theory used in the paper, which is necessary to describe the communication network. In a smart grid, a directed topology $G = (V, E)$ denotes the relationship between multiple generators, which is composed of node set $V = \{v_1, v_2, \ldots, v_n\}$ and edge set $E \subseteq V \times V$. A directed edge from $v_i$ to $v_j$ is represented as $(v_i, v_j) \in E$. To simplify the notation, it is assumed that there is no self-loop at any node, i.e., $(v_i, v_i) \notin E, v_i \in V$. The in-neighbor and out-neighbor of each node are represented as $N_i^+ = \{v_j \in V | (v_j, v_i) \in E\}$ and $N_i^- = \{v_j \in V | (v_i, v_j) \in E\}$, respectively. $d_{out}(i) = \sum_{j=1}^{n} a_{ij}$ and $d_{in}(i) = \sum_{j=1}^{n} a_{ji}$ denote the out-degree and in-degree of each node, respectively. $G$ is strongly connected if for each pair of $v_i, v_j \in V, v_i \neq v_j$, there are paths from $v_i$ to $v_j$ and from $v_j$ to $v_i$.

The Laplacian matrix of a digraph is elucidated as $L = D_{in} - A$. $D_{in} = \text{diag}\{d_{in}(1), d_{in}(2), \ldots, d_{in}(n)\}$ is a diagonal matrix, where $d_{in}(i)$ denotes the $i$-th diagonal element, for $i \in \{1, 2, \ldots, n\}$. $A = [a_{ij}]_{n \times n}$ indicates an $n \times n$ dimensional adjacency matrix. $A$ is a row-stochastic matrix if it satisfies $\sum_{j=1}^{n} a_{ij} = 1 \text{ for } \forall i \in \{1, 2, \ldots, n\}$. $A$ is a column-stochastic matrix if it satisfies $\sum_{i=1}^{n} a_{ij} = 1 \text{ for } \forall j \in \{1, 2, \ldots, n\}$. The paper introduces a row-
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where $\alpha$ and $\beta$ are coefficients of the cost function, respectively. Thus, we have

\[\min C(P_G) = \sum_{i=1}^{N} C_i(P_i)\]

s.t. $P_D = \sum_{i=1}^{N} P_i$

\[p_i^{\text{min}} \leq P_i \leq p_i^{\text{max}}\]

where $P_i$ represents the output power of the $i$-th generator. $P_G = [P_1, P_2, \ldots, P_N]^T \in \mathbb{R}_+^N \cdot C_i(P_i)$ is the cost function. $C(P_G)$ and $P_D$ represent the total cost and total power demand in the power system, respectively. $p_i^{\text{min}}$ and $p_i^{\text{max}}$ are defined as the minimum output power and the maximum output power of the $i$-th generator. (2) guarantees the balance of power supply and demand during the operation of the entire power system. By limiting the output of each generator by (3), we can ensure the normal operation of each generator. In most cases, $C_i(P_i)$ is generally a quadratic convex function, which is generally defined as

\[C_i(P_i) = \alpha_i P_i^2 + \beta_i P_i + \gamma_i\]

where $\alpha_i$, $\beta_i$, and $\gamma_i$ denote the coefficients of each generator’s cost function, respectively. To facilitate problem analysis, the following assumptions are provided.

**Assumption 1.** The communication topology in this paper is a directed connected graph. The corresponding adjacency matrix is row-stochastic matrix $A = [a_{ij}]_{n \times n}$ and column-stochastic matrix $W = [w_{ij}]_{n \times n'}$, which satisfy $\|A\| < 1, \|W\| < 1$.

**Assumption 2.** For generators with frequency regulators, the frequency of all generators can be monitored and initially run at a rated frequency $f_{\text{rated}}$. The frequency of each generator can be different and vary periodically within the normal range.
3.2. Centralized Lagrangian Method

In order to solve EDP, the traditional centralized methods mainly rely on some global information in the power system and even require the parameter information of each generator. At present, the Lagrange multiplier method is the main centralized EDP method. Then, the Lagrangian function is defined as follows:

\[ L(P_i, \lambda) = \sum_{i=1}^{N} C_i(P_i) - \lambda \left( \sum_{i=1}^{N} P_i - P_D \right) \]  

where \( \lambda \) is both incremental cost and Lagrange multiplier. Therefore, \( \lambda^* \) is the optimal incremental cost, which can be obtained by the Lagrange multiplier method.

\[ \frac{dL(P_i, \lambda)}{dP_i} = 2\alpha_i P_i + \beta_i - \lambda = 0 \]  

\[ \frac{dL(P_i, \lambda)}{d\lambda} = P_D - \sum_{i=1}^{N} P_i = 0 \]

According to (6) and (7), the optimal incremental cost \( \lambda^* \) can be calculated by Equation (8).

\[ \lambda^* = \frac{P_D + \sum_{i=1}^{N} \frac{\beta_i}{2\alpha_i}}{\sum_{i=1}^{N} \frac{1}{2\alpha_i}} \]  

(8)

Considering the constraints (2) and (3), the solution of (1) can be calculated through (6).

\[ P_i^*(k) = \begin{cases} \lambda^* - \frac{\beta_i}{2\alpha_i} & \text{if } P_i^\text{min} < P_i^*(k) < P_i^\text{max} \\ P_i^\text{min} & \text{if } P_i^*(k) \leq P_i^\text{min} \\ P_i^\text{max} & \text{if } P_i^*(k) \geq P_i^\text{max} \end{cases} \]  

(9)

where \( P_i^*(k) \) denotes the optimal output power for the \( i \)-th generator at the \( k \)-th iteration. The solution of (1) can be expressed as \( P^*(k) = \{ P_1^*(k), P_2^*(k), \ldots P_N^*(k) \} \).

Remark 1. The centralized economic dispatch method requires some global resources of the entire power system and some detailed parameters of each generator. However, if there are too many generators in the power system, the method is not feasible. Hence, we introduce a basic distributed consensus protocol to solve distributed EDP.

3.3. Distributed Consensus Protocol

The necessary condition to realize economic dispatch in a power system is that each incremental cost has the same value. For the sake of overcoming the shortcomings of the centralized approach, a discrete-time DED consensus protocol under undirected topology is proposed as follows [11]. However, due to the different ability of each generator to obtain the state information of neighbor nodes, the bidirectional communication between distributed generator units may not be realized. Therefore, we will consider the DED under directed communication.

\[ \lambda_i(k) = \lambda_i(k-1) + \tau \sum_{j \in N_i^+} a_{ij} [\lambda_j(k-1) - \lambda_i(k-1)] \]  

(10)

where \( \lambda_i(k) \) denotes the incremental cost of the \( i \)-th generator at the \( k \)-th iteration. \( \tau \) is a fixed step size. \( a_{ij} \) represents the weight coefficient from generator \( j \) to generator \( i \). Let \( \lambda(k) = [\lambda_1(k), \lambda_2(k), \ldots \lambda_N(k)] \), and (10) can be rewritten as

\[ \lambda(k) = \lambda(k-1) - \tau L \lambda(k-1) \]  

(11)
where $L_a$ is a Laplacian matrix. Using (11), it is more convenient for us to deduce the convergence of the algorithm.

**Lemma 1** ([11]). (11) can converge to the following as:

$$\lim_{k \to \infty} \lambda(k) = \left( \frac{1}{N} \sum_{j=1}^{N} \lambda_j(0) \right) v_1 = \frac{1}{N} 1^T \lambda(0)$$

(12)

if and only if the communication topology is connected and satisfies

$$\rho(L_a) \leq \frac{2}{\tau}$$

(13)

where $\lambda_j(0)$ is the initial incremental cost of generator $i$. $1 = [1, 1, \ldots, 1]^T$. $\rho(L_a)$ is the largest absolute value of the eigenvalue of $L_a$.

4. Algorithm Design and Description

4.1. DED Approach with Frequency Regulator

Based on (10), we propose a DED algorithm with a frequency regulator. Different from the algorithm proposed in [11], the frequency of each generator can be different and fluctuate periodically within the normal range. We add the frequency factor to the consensus protocol by introducing the objective attraction function [18] as follows

$$\lambda_i(k) = \lambda_i(k-1) + \tau \sum_{j \in N_i} a_{ij} [\lambda_j(k-1) - \lambda_i(k-1)] + \lambda^*_i(k)$$

(14)

where $\lambda^*_i(k)$ denotes a target attraction function, which helps $\lambda(k)$ achieve consensus. In the power system, if all $\lambda_i(k)$ reach consensus under the action of consensus protocol, then $\lambda(k)$ can be calculated for each generator, and the total generation cost is also the minimum; that is, economic dispatch is completed. The algorithm mentioned is summarized in Algorithm 1.

$$\Delta f_i(k) = \mu \left( \sum_{j \in N_i} a_{ij} [\Delta f_j(k-1) - \Delta f_i(k-1)] + \varepsilon \sum_{i=1}^{N} (f_i(k) - f_i(k-1)) \right)$$

(17)

$$\Delta \lambda_i(k) = \sigma \Delta f_i(k)$$

(18)
we introduce leader–follower mode into the algorithm in order to avoid communication which represents the new event-triggered instant when the event-triggered condition is where node \( i \) which may lead to communication congestion and even economic dispatch failure. To solve this problem, we introduce an event-triggered mechanism based on Algorithm 1.

In order to conveniently describe the event-triggered algorithm, a necessary introduction is given first. Event-triggered control means that information transmission is only allowed between generators after triggering conditions are satisfied, which effectively reduces the amount of computation, reduces the waste of communication bandwidth and improves the efficiency of economic dispatch. The notation specification is defined as follows: for \( \forall i, x_i(k) \) denotes the variable transmitted by the \( i \)-th node in the communication topology at the \( k \)-th iteration. \( x_i(k) = x_i(k_{1i}), k_{1i} < k < k_{1i+1} \) denotes the transmission variable of the \( i \)-th node at event-triggered time instant \( k_{1i} \). \( t_i \) is the number of events triggered at the current time instant for the \( i \)-th node. \( E_i(k, x_i(k)) \) denotes the event-triggered function. Then, we define the event-triggered condition as

\[
E_i(k, x_i(k)) > 0
\]  

where node \( i \) can receive \( \{ x_j(k) \mid j \in N_i^+ \} \) and send \( \xi_i(k) = x_i(k_{1i+1}) \) to the \( j \)-th \( (j \in N_i^-) \) neighbor node in the next iteration if (19) is satisfied. The event-triggered discrete instants for node \( i \) is defined iteratively as \( k_{1i+1} = \inf\{ k > k_{1i} | E_i(k, x_i(k)) > 0 \} (t_i = 0, 1, 2, \ldots) \), which represents the new event-triggered instant when the event-triggered condition is met. Moreover, the discrete time method naturally excludes zero behavior [15].

Consider that the event-triggered mechanism is suitable for large power systems, we introduce leader–follower mode into the algorithm in order to avoid communication failures as much as possible. In this mode, leaders with a frequency regulator can measure the frequency, and followers without a frequency regulator have no frequency measurement ability. Therefore, we introduce a new event-triggered-based DED approach.

Follower:

\[
\lambda_i(k) = \lambda_i(k-1) + \tau \sum_{j \in N_i^+} a_{ij}[\lambda_j(k-1) - \lambda_i(k-1)] + \xi(y_i(k) - y_i(k-1))
\]  

\[
y_i(k) = \sum_{j \in N_i^+} w_{ij} y_j(k-1) + \Delta P_i(k-1)
\]

Leader:

\[
\lambda_i(k) = \lambda_i(k-1) + \tau \sum_{j \in N_i^+} a_{ij}[\lambda_j(k-1) - \lambda_i(k-1)] + \xi(y_i(k) - y_i(k-1)) + \Delta \lambda_i(k)
\]
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\[ y_i(k) = \sum_{j \in N_i^{L+}} w_{ij} \hat{y}_j(k-1) + \Delta P_i(k-1) \]  

(23)

\[ \Delta P_i(k) = P_i(k) - P_i(k-1) \]  

(24)

\[ \Delta f_i(k) = \mu \left( \sum_{j \in N_i^{L+}} a_{ij} \Delta f_j(k-1) - \Delta f_i(k-1) + \frac{e}{N} \sum_{i=1}^{N} (f_i(k) - f_i(k-1)) \right) \]  

(25)

\[ \Delta \lambda_i(k) = \sigma \Delta f_i(k) \]  

(26)

where \( N_i^{L+} \) represents the set of neighbor leader generators adjacent to leader generator \( i \). \( \xi \) denotes the gain coefficient. \( y_i(k) \) is an auxiliary variable. \( \Delta P_i(k) \) is the difference of active power between two iterations. \( A = [a_{ij}]_{n \times n} \) is a row-stochastic matrix and \( W = [w_{ij}]_{n \times n} \) is a column-stochastic matrix. Event-triggered condition:

\[
\begin{align*}
E_i(k, \lambda_i(k)) &= \|\epsilon_i(k)\|_2 - \gamma \\
F_i(k, y_i(k)) &= \|\phi_i(k)\|_2 - \mu v
\end{align*}
\]

(27)

where \( \epsilon_i(k) = \lambda_i(k) - \lambda_i(k_i^L), \lambda_i(k) = \lambda_i(k_i^L), k_i^L \leq k \leq k_i^{L+1}, \phi_i(k) = \hat{y}_i(k) - y_i(k), k_i^{L+} \leq k \leq k_i^{L+1}, \gamma, \mu, v \in (0, 1) \). If and only if \( E_i(k, \lambda_i(k)) > 0 \) and \( F_i(k, y_i(k)) > 0 \), the event will be triggered.

**Remark 2.** It can be seen from (20)–(27) that the leader generator can only receive messages from the neighbor leader and can send messages to any neighbor generator it points to. Meanwhile, the follower can communicate with any neighbor generator. Then, under the action of the consensus term, each generator’s incremental cost can be the same. The algorithm mentioned is summarized in Algorithm 2. Figure 1 shows the flow of Algorithm 2.

**Algorithm 2 Event-triggered based DED approach (Leader-Follower)**

1. **Initialize:** \( P_i(0) \rightarrow \lambda_i(0) = \hat{\lambda}_i(0), y_i(0) = \hat{y}_i(0), t_i(0), \phi_i(0), \Delta P_i(0), f_i(0), \Delta f_i(0) \) for \( \forall i \).
2. **Set parameters:** \( \tau, \epsilon, \sigma, \xi, \gamma, \mu \in (0, 1), k_i^L, A[a_{ij}], W [w_{ij}]_{n \times n} \).
3. **for** \( k > 0 \) **do**
4. **Measure** \( f_i(k) \): the frequency of each generator at iteration \( k \).
5. **for** \( i = 1 : N \) **do**
6. **if** \( i \) **is Leader** **then**
7. The \( i \)-th generator receives \( \{ \hat{\lambda}_j(k-1), \hat{y}_j(k-1), \Delta f_j(k-1) \} \) \( j \in N_i^{L+} \) and sends \( \hat{\lambda}_i(k-1), \hat{y}_i(k-1) \) and \( \Delta f_i(k-1) \) to the \( j \)-th neighbor generator \( (j \in N_i^-) \).
8. **Through** (22)–(26), calculate the new value \( \lambda_i(k) \).
9. **else**
10. The \( i \)-th generator receives \( \{ \hat{\lambda}_j(k-1), \hat{y}_j(k-1), \Delta f_j(k-1) \} \) \( j \in N_i^{L+} \) and sends \( \hat{\lambda}_i(k-1), \hat{y}_i(k-1) \) and \( \Delta f_i(k-1) \) to the \( j \)-th neighbor generator \( (j \in N_i^-) \).
11. **Through** (20)–(21), calculate the new value \( \lambda_i(k) \).
12. **Through** (9), calculate the active power \( P_i(k) \) of the \( i \)-th generator.
13. **Through** (24), calculate the new value \( \Delta P_i(k) \).
14. **if** \( E_i(k, \lambda_i(k)) > 0 \) and \( F_i(k, y_i(k)) > 0 \) **then**
15. \( k_i^{L+} = \inf \{ k : k > k_i^L | E_i(k, x_i(k)) > 0 \ and \ F_i(k, y_i(k)) > 0 \} \) (\( t_i = 0, 1, 2, \ldots \)),
16. \( \hat{\lambda}_i(k) = \lambda_i(k_i^L) \hat{y}_i(k) = y_i(k_i^L) \),
17. **Let** \( k = k + 1 \).


**Assumption 3** ([18]). The time-varying directed topology $G(k)$ is uniformly jointly strongly connected, i.e., $G([k_0, k_0+t)) = \bigcup_{k \in [k_0,k_0+t]} G^{(k)}$ is strongly connected for any $k_0 \geq 0$ with some constant $t > 0$.

**Assumption 4.** In the case of time-varying topology, $\|A^{\delta(k)}\| < 1$ and $\|W^{\delta(k)}\| < 1$ can be guaranteed for any $A^{\delta(k)}$ and $W^{\delta(k)}$. Communication between leaders is always smooth.

**4.3. Robustness to Time-Varying Topology**

At present, most related work generally considers that the communication topology is unchanged. However, in practical applications, the communication link may have uncertain faults, which may lead to changes of communication topology. Therefore, for the sake of solving the problem of imperfect communication, we design a robust DED consensus algorithm for time-varying topology based on the above algorithm, which is very necessary for the future smart power grid. Since the topology is time-varying, $\mathcal{G} = \{G^1, G^2, \ldots, G^m\}$ is the set of all possible topologies. The mapping topology function is expressed as $G^{(k)} = (V, E(k))$, $G^{(k)} \in \mathcal{G}$, which represents the topology of the $k$-th iteration and its corresponding adjacency matrices are $A^{\delta(k)} = [a_{ij}^{\delta(k)}]_{n \times n}$ and $W^{\delta(k)} = [w_{ij}^{\delta(k)}]_{n \times n}$.

**Figure 1.** Algorithm flow chart.
\[ \Delta \lambda_i(k) = \sigma \Delta f_i(k) \] (34)

where the event-triggered condition remains unchanged and the communication topology of each iteration may change. \( \delta(k) \in \{1, 2, \ldots, m\} \) \((k = 0, 1, 2, \ldots)\).

5. Convergence Analysis

In this part, the convergence of the proposed algorithm is analyzed. First, the convergence of \( y_i(k) \) and \( \lambda_i(k) \) is proved under an event-triggered condition. Second, it is proved that the protocol can converge even considering the time-varying topology. To give the convergence analysis, we need the following lemmas.

Lemma 2 ([37]). If the graph is connected, its Laplacian matrix is \( L_a \) and the spectral radius \( \rho(L_a) \) satisfies \( \rho(L_a) < \frac{2}{\tau} \), then
\[ (1 - L_a)^\infty = \lim_{k \to \infty} (1 - L_a)^k = \frac{1}{N} 11^T \] (35)

Lemma 3. \( M_n \) is the entire \( n \)-order matrix, let \( A \in M_n \), \( \rho(A) \) be the spectral radius, then
\[
\begin{cases}
\sum_{k=1}^{\infty} A^k \text{ converges and } A^k = 0, & \rho(A) < 1 \\
\sum_{k=1}^{\infty} A^k \text{ diverges}, & \rho(A) > 1
\end{cases}
\] (36)

Theorem 1. In a directed connected graph, let \( \Psi_a = 1 - L_a \). As \( k \to \infty \), then \( \sum_{r=1}^{k} \| \Psi_a \|^r \) can converge by adjusting \( L_a \) so that \( \| \Psi_a \| < 1 \).

Proof. We can observe that \( \{ \| \Psi_a \|, \| \Psi_a \|^2, \| \Psi_a \|^3, \ldots, \| \Psi_a \|^k \} \) is a geometric sequence. Therefore, we can use the geometric sequence summation formula to calculate.
\[
\sum_{r=1}^{k} \| \Psi_a \|^r = \| \Psi_a \| + \| \Psi_a \|^2 + \| \Psi_a \|^3 + \cdots + \| \Psi_a \|^k
\]
\[
= \| \Psi_a \| \left(1 - \| \Psi_a \|^k \right) \frac{1}{1 - \| \Psi_a \|}
\]
(37)

So as \( k \to \infty \) and \( \| \Psi_a \| < 1 \), then \( \| \Psi_a \|^k = 0 \). We can conclude that
\[
\lim_{k \to \infty} \sum_{r=1}^{k} \| \Psi_a \|^r = \frac{\| \Psi_a \|}{1 - \| \Psi_a \|}
\]
(38)

then (37) converges to \( \frac{\| \Psi_a \|}{1 - \| \Psi_a \|} \) \( \blacksquare \)

5.1. Convergence under Event-Triggered Conditions

Proof. According to (21), \( y_i(k) = \sum_{j \in N_i^+} w_{ij} \hat{y}_j(k-1) + \Delta P_i(k-1) \). Let \( y(k) = [y_1(k), y_2(k), \ldots, y_N(k)] \) and \( \Delta P(k) = [\Delta P_1(k), \Delta P_2(k), \ldots, \Delta P_N(k)] \). So, (21) can be converted to the following form.
\[
y(k) = W \hat{y}(k-1) + \Delta P(k-1)
\]
(39)
From \( q_i(k) = \hat{y}_i(k) - y_i(k) \), we can derive that \( \hat{y}(k) = y(k) + \varphi(k) \), where 
\[
\hat{y}(k) = [\hat{y}_1(k), \hat{y}_2(k), \cdots, \hat{y}_N(k)]^T, \varphi(k) = [q_1(k), q_2(k), \cdots, q_N(k)]^T.
\]
Then, (39) can be written as
\[
y(k) = W(y(k-1) + \varphi(k-1)) + \Delta P(k-1)
= W^2 y(k-2) + W^2 \varphi(k-2) + W \varphi(k-1) + W \Delta P(k-2) + \Delta P(k-1)
= W^k y(0) + W^k \varphi(0) + W^{k-1} \varphi(1) + \cdots + W \varphi(k-1)
+ W^{k-1} \Delta P(0) + W^{k-2} \Delta P(1) + \cdots + \Delta P(k-1)
\]
\[= W^k y(0) + \sum_{r=0}^{k-1} W^{k-r} \varphi(r) + \sum_{r=0}^{k-1} W^{k-1-r} \Delta P(r) \tag{40}\]

According to Assumption 1 and Lemma 3, \( W^k y(0) = 0, \text{ if } k \to \infty \). The dynamics of the power system and event-triggered constraints, trigger error and local power error cannot be arbitrarily large. Therefore, we suppose \( \varphi(k) \) and \( \Delta P(k) \) have upper bounds \( \varphi_{\text{max}} \) and \( \Delta P_{\text{max}} \) for some norms, i.e., \( \| \varphi(k) \| \leq \| \varphi_{\text{max}} \|, \| \Delta P(k) \| \leq \| \Delta P_{\text{max}} \| \) for \( \forall k \). Assuming \( \varphi(k) = \varphi_{\text{max}} \) and \( \Delta P(k) = \Delta P_{\text{max}} \) in each iteration, under such extreme conditions, if we can prove that the proposed algorithm can converge, then the algorithm must always converge in general [11]. Let \( \sigma = k - r, k \to \infty \); then, it can be deduced
\[
\sum_{\sigma=1}^{\infty} W^\sigma \varphi_m = W^1 \varphi_m + W^2 \varphi_m + \cdots. \tag{41}\]

To prove the convergence of (41), we must find a matrix norm \( \| \cdot \| \) to satisfy geometric series \( \sum_{\sigma=1}^{\infty} \| W \|^{\sigma} \) converges [7]. According to Assumption 1, \( \| W \|^{\sigma} < 1 \), then the convergence condition of \( \sum_{\sigma=1}^{\infty} \| W \|^{\sigma} \) is satisfied. Therefore, (41) converges to
\[
\sum_{\sigma=1}^{\infty} W^\sigma \varphi_m = Q_w \varphi_m \tag{42}\]
where \( Q_w = \sum_{\sigma=1}^{\infty} W^\sigma \). The convergence of \( \sum_{r=0}^{k-1} W^{k-1-r} \Delta P(r) \) can be proved by using the same method, so it is omitted here. By analyzing (40)–(42), \( y(k) \) can be proved to converge. Next, we analyze the convergence of \( \lambda_i(k) \). Since \( y(k) \) converges, we just need to prove that (43) converges.
\[
\lambda_i(k) = \lambda_i(k-1) + \tau \sum_{j \in \mathcal{N}_i^+} a_{ij} [\hat{\lambda}_j(k - 1) - \hat{\lambda}_i(k - 1)] + \Delta \lambda_i(k) \tag{43}\]

According to (11), (43) can be rewritten as
\[
\lambda(k) = \lambda(k-1) - \tau \lambda \hat{\lambda}(k-1) + \Delta \lambda(k) \tag{44}\]
where \( \lambda(k) = [\lambda_1(k), \lambda_2(k) \ldots, \lambda_N(k)]^T \), \( \hat{\lambda}(k) = [\hat{\lambda}_1(k), \hat{\lambda}_2(k) \ldots, \hat{\lambda}_N(k)]^T \) and \( \Delta \lambda(k) = [\Delta \lambda_1(k), \Delta \lambda_2(k) \ldots, \Delta \lambda_N(k)]^T \). Let \( \mathbb{E}_a = 1 - \tau L_a, |\mathbb{E}_a| \neq 0, \lambda_i(k) = \lambda_i(k) + e_i(k) \) and \( e(k) = [e_1(k), e_2(k) \ldots, e_N(k)]^T \), then Equation (44) becomes

\[
\lambda(k) = \mathbb{E}_a \lambda(k) - \tau La e(k-1) + \Delta \lambda(k) \\
= \sum_{r=1}^{k} \mathbb{E}_a^{k-r} La e(r-1) + \sum_{r=0}^{k} \mathbb{E}_a^{k-r} \Delta \lambda(r)
\]

(45)

By Lemma 2, \( \mathbb{E}_a^{k} \lambda(0) \) converges to \( \hat{\lambda}(0) = \frac{1}{N} 1^T \lambda(0) \). Let \( e(k) = \bar{e}(k) + \eta(k) \), where \( \bar{e}(k) = \frac{1}{N} 1^T e(k) \) is the average vector of \( e(k) \). Then,

\[
\sum_{r=0}^{k} \mathbb{E}_a^{k-r} La \eta_{\max} = La \eta_{\max} + \sum_{r=0}^{k} \mathbb{E}_a^{k-r} La \eta_{\max} + \sum_{r=0}^{k} \mathbb{E}_a^{k-r} La \eta_{\max} \ldots \ldots \ldots \ldots
\]

(46)

(47)

According to Theorem 1, we can prove that \( \sum_{\sigma=0}^{\infty} \| \mathbb{E}_a^\sigma \|^{\gamma} \) converges. Therefore, it can be concluded that (47) converges [7]. Furthermore, since \( L_a \) is a Laplacian matrix, then (47) converges to 0. We can obtain \( \sum_{\sigma=0}^{\infty} \mathbb{E}_a^\sigma L_a = 0 \). So, (46) converges to 0. Next, let \( Q = \frac{1}{N} 11^T \)

and \( \Delta \lambda(0) = \Delta f(0) = f(0) - f_{\text{rated}} = 0 \), then

\[
\sum_{r=0}^{k} \mathbb{E}_a^{k-r} \Delta \lambda(r) = \mathbb{E}_a^{k} \Delta \lambda(0) + \sum_{r=1}^{k} \mathbb{E}_a^{k-r} \Delta \lambda(r)
\]

\[
= \mathbb{E}_a^{k} \Delta \lambda(0) + \sigma \sum_{r=1}^{k} \mathbb{E}_a^{k-r} \mu (La \Delta f(r-1) + eQ(f(r) - f(r-1)))
\]

\[
= \mathbb{E}_a^{k} \Delta \lambda(0) + \sigma e \sum_{r=1}^{k} \mathbb{E}_a^{k-r} (\mu La)^r \Delta f(0)
\]

\[
+ \mu La^{r-1} Q(f(1) - f(0)) + \cdots + \mu Q(f(t) - f(r-1))
\]

\[
= \mathbb{E}_a^{k} \Delta \lambda(0) + \sigma e \sum_{r=1}^{k} \mathbb{E}_a^{k-r} \sum_{t=1}^{r} (\mu La)^t Q(f(t) - f(t-1))
\]

(48)

According to Lemma 2, it converges to \( \Delta \overline{\lambda}(0) = \frac{1}{N} 11^T \Delta \lambda(0) \).

Let \( H(k) = \sum_{r=1}^{k} \mathbb{E}_a^{k-r} \sum_{t=1}^{r} (\mu La)^t Q(f(t) - f(t-1)) \); then, the convergence of (48) is proved by inductive reasoning.

when \( k = 1 \), \( H(1) = Q(f(t) - f(t-1)) \).

when \( k = n \), suppose \( H(n) = \sum_{r=1}^{n} \mathbb{E}_a^{n-r} \sum_{t=1}^{r} (\mu La)^t Q(f(t) - f(t-1)) \) converges.
when \( k = n + 1 \),

\[
H(n + 1) = \sum_{r=1}^{n+1} \mathcal{A}_{n+1-r} \sum_{i=1}^{r} (\mu L_a)^{-t} Q(f(t) - f(t-1))
\]

\[
= \mathcal{A}_n \sum_{r=1}^{n} \mathcal{A}_{n-r} \sum_{i=1}^{r} (\mu L_a)^{-t} Q(f(t) - f(t-1))
\]

\[
= \mathcal{A}_n \left( \sum_{r=1}^{n} \mathcal{A}_{n-r} \sum_{i=1}^{r} (\mu L_a)^{-t} Q(f(t) - f(t-1)) \right) + \mathcal{A}_n^{-1} \sum_{r=1}^{n+1} (\mu L_a)^{n+1-t} Q(f(t) - f(t-1))
\]

\[
= \mathcal{A}_n H(n) + \sum_{r=1}^{n+1} (\mu L_a)^{n+1-t} Q(f(t) - f(t-1))
\]

Considering the actual power system, the frequency variation cannot be arbitrarily large. It assumes that \( \Delta F_m \) is an upper bound of \( f(t) - f(t-1) \) for some norms, i.e., \( \|f(t) - f(t-1)\| \leq \|\Delta F_m\| \). According to the above similar ideas, we just need to make sure \( \|\mu L_a\| < 1 \) so that \( \sum_{r=0}^{n} \|\mu L_a\|^r \|\Delta F_m\| \) converges, where \( r = n + 1 - t \). Then, we can obtain that \( \sum_{r=1}^{n+1} (\mu L_a)^{n+1-t} Q(f(t) - f(t-1)) \) converges so that (49) converges to 0 because \( Q(f(t) - f(t-1)) \) guarantees that each entry of it is the same. Therefore, we can deduce that (48) converges to \( \Delta \lambda(0) = \xi^{-1} \Delta \lambda(0) \). By deducing that (46) and (48) converge, it can be concluded that each vector of \( \lambda(k) \) converges to the same value; that is, each incremental cost reaches consensus. \( \square \)

### 5.2. Convergence in Time-Varying Topology

Considering the time-varying topology, according to (23), \( y_i(k) = \sum_{j \in N_i^k} w^{ij}_k y_j(k-1) + \Delta P_i(k-1) \). Then, (23) can take the following form.

\[
y(k) = W^{\delta(k-1)} \hat{g}(k-1) + \Delta P(k-1)
\]

\[
= W^{\delta(k-1)} (y(k-1) + \varphi(k-1)) + \Delta P(k-1)
\]

\[
= W^{\delta(k-1)} W^{\delta(k-2)} \cdots W^{\delta(0)} y(0) + W^{\delta(k-1)} \varphi(k-1) + W^{\delta(k-1)} W^{\delta(k-2)} \varphi(k-1) + \cdots + W^{\delta(k-1)} W^{\delta(k-2)} \cdots W^{\delta(1)} \varphi(0)
\]

\[
+ \Delta P(k-1) + W^{\delta(k-1)} \Delta P(k-2) + \cdots + W^{\delta(k-1)} W^{\delta(k-2)} \cdots W^{\delta(1)} \Delta P(0)
\]

\[
= \sum_{r=0}^{k-1} ( \prod_{i=r}^{k-1} W^{\delta(i)} ) \varphi(r) + \sum_{r=1}^{k-1} ( \prod_{i=r}^{k-1} W^{\delta(i)} ) \Delta P(r-1) + \Delta P(k-1)
\]

Next, we need to prove that \( \sum_{r=0}^{k-1} ( \prod_{i=r}^{k-1} W^{\delta(i)} ) \varphi(r) \) converges. As mentioned above, there exists an \( \varphi_{\text{max}} \) that makes \( \|\varphi(r)\| < \|\varphi_{\text{max}}\| \). Under such extreme conditions, if we can prove that the proposed algorithm can converge, then the algorithm must always converge in general. Therefore, we need to prove that \( \sum_{r=0}^{k-1} ( \prod_{i=r}^{k-1} W^{\delta(i)} ) \varphi_{\text{max}} \) converges.

\[
\sum_{r=0}^{k-1} ( \prod_{i=r}^{k-1} W^{\delta(i)} ) = W^{\delta(0)} W^{\delta(1)} \cdots W^{\delta(k-1)} + W^{\delta(1)} W^{\delta(2)} \cdots W^{\delta(k-1)} + \cdots + W^{\delta(k-1)}
\]
Suppose
\[
\begin{align*}
\|W_{\text{max}}\| &= \max \{ \|W^{\delta(0)}\|, \|W^{\delta(1)}\|, \ldots, \|W^{\delta(k)}\| \} \\
\|W_{\text{min}}\| &= \min \{ \|W^{\delta(0)}\|, \|W^{\delta(1)}\|, \ldots, \|W^{\delta(k)}\| \}
\end{align*}
\]
where \(W_{\text{max}}, W_{\text{min}} \in \{W^{\delta(0)}, W^{\delta(1)}, \ldots, W^{\delta(k)}\}\). Set \(W^{\delta(k)} = W_{\text{max}}\) and \(W^{\delta(k)} = W_{\text{min}}\), respectively, \(k = 0, 1, 2, \ldots\).
\[
\begin{align*}
\sum_{r=0}^{k-1} (W_{\text{max}})^r &= W_{\text{max}}^k + W_{\text{max}}^{k-1} + \ldots + W_{\text{max}} \\
\sum_{r=0}^{k-1} (W_{\text{min}})^r &= W_{\text{min}}^k + W_{\text{min}}^{k-1} + \ldots + W_{\text{min}}
\end{align*}
\]

According to Assumption 4, we can obtain \(\|W_{\text{max}}\| < 1\) and \(\|W_{\text{min}}\| < 1\). Using Lemma 3, \(\sum_{r=0}^{k-1} (W_{\text{max}})^r\) and \(\sum_{r=0}^{k-1} (W_{\text{min}})^r\) are proved to be convergent. Since (51) can reach convergence under extreme conditions, then (51) must be convergent. Therefore, \(\sum_{r=0}^{k-1} (\prod_{i=r}^{k-1} W^{\delta(i)}) \varphi_m\) converges so that \(\sum_{r=0}^{k-1} (\prod_{i=r}^{k-1} W^{\delta(i)}) \varphi(r)\) converges. Similarly, \(\sum_{r=0}^{k-1} (\prod_{i=r}^{k-1} W^{\delta(i)}) \Delta P(r-1)\) can also be proved to be convergent, which is omitted here. To sum up, (50) is proved to converge.

According to Assumption 4, communication between leaders is always smooth; then, the connection between leader generators will not be interrupted in the process of time-varying topology. Therefore, the influence of topology change on the frequency regulator is negligible, which is equivalent to converging to 0 under a fixed topology. Combined with relevant conclusions of (48), we can ignore the value of the frequency regulator in the process of proving the convergence of (30). Let \(\Xi \delta^{(k)} = 1 - \pi L_a \delta^{(k)}, \|\Xi \delta^{(k)}\| \neq 0\). Since (50) converges, then (30) take the following form.

\[
\lambda(k) = \lambda(k-1) - \tau L_a \sigma^{(k-1)} \lambda(k-1)
\]

\[
\begin{align*}
&= \Xi \delta^{(k-1)} \lambda(k-1) - \tau L_a \hat{e}(k-1) \\
&= \Xi \delta^{(k-1)} \Xi \delta^{(k-2)} \ldots \Xi \delta^{(0)} \lambda(0) \\
&- (L_a \delta^{(k-1)} \hat{e}(k-1) + \Xi \delta^{(k-1)} L_a \delta^{(k-2)} \hat{e}(k-2) + \ldots + \Xi \delta^{(k-1)} \Xi \delta^{(k-2)} \ldots \Xi \delta^{(0)} L_a \hat{e}(0)) \\
&= \Xi \delta^{(k-1)} \Xi \delta^{(k-2)} \ldots \Xi \delta^{(0)} \lambda(0) + \Gamma(a) \hat{e}
\end{align*}
\]

where \(\Gamma(a) = L_a \delta^{(k-1)} \hat{e}(k-1) + \sum_{r=1}^{k-1} (\prod_{i=r}^{k-1} \Xi \delta^{(i)}) L_a \delta^{(i-1)} \hat{e}(t-1)\). The entries of \(\hat{e}(k)\) have the same upper bound, and every entry of \(\varepsilon_{\text{max}}\) is the same. So, \(\hat{e}(k)\) values have upper bounds \(\varepsilon_{\text{max}}\) for some norms, i.e., \(0 \leq \|\hat{e}(k)\| \leq \|\varepsilon_{\text{max}}\|\). When \(k \to \infty\), let \(\hat{e}(k) = \varepsilon_{\text{max}}\) and \(\hat{e}(k) = 0\), respectively. We can obtain \(\Gamma(a)(\varepsilon_{\text{max}}) = \Gamma(a)(0) = 0\), so \(\Gamma(a)(\eta) = 0\). Therefore, (54) becomes

\[
\lambda(k) = \Xi \delta^{(k-1)} \Xi \delta^{(k-2)} \ldots \Xi \delta^{(0)} \lambda(0)
\]

The convergence of (55) is proved by mathematical induction.

\[
\begin{align*}
k &= 1, \lambda(1) &= \Xi \delta^{(0)} \lambda(0), \\
k &= n, \text{Suppose } \lambda(n) &= \Xi \delta^{(n-1)} \Xi \delta^{(n-2)} \ldots \Xi \delta^{(0)} \lambda(0) \text{ converges.} \\
k &= n + 1, \lambda(n + 1) &= \Xi \delta^{(n)} \Xi \delta^{(n-1)} \ldots \Xi \delta^{(0)} \lambda(0) = \Xi \delta^{(n)} \lambda(n).
\end{align*}
\]
We deduced that \( \lambda(n + 1) \) converges. Therefore, (55) converges so that it is deduced that (54) converges. It has been shown above that (54) converges, but consensus is not guaranteed. Next, we analyze the consensus of (54) on the basis of (55).

\[
\left\{ \begin{array}{l}
\| a_{\text{max}} \| = \max \{ \| a_{\delta(n)} \|, \| a_{\delta(n-1)} \|, \ldots, \| a_{\delta(0)} \| \} \\
\| a_{\text{min}} \| = \min \{ \| a_{\delta(n)} \|, \| a_{\delta(n-1)} \|, \ldots, \| a_{\delta(0)} \| \}
\end{array} \right.
\]  

(56)

where \( a_{\text{max}}, a_{\text{min}} \in \{ a_{\delta(k-1)}, a_{\delta(k-2)}, \ldots, a_{\delta(0)} \} \). Set \( a_{\delta(t)} = a_{\text{max}} \) and \( a_{\delta(t)} = a_{\text{min}}, \) respectively, \( t = 0, 1, 2, \ldots, k - 1 \). According to Assumption 5 and (56), we know that \( \rho(a_{\text{max}}) \leq \| a_{\text{max}} \| < 1 \) and \( \rho(a_{\text{min}}) \leq \| a_{\text{min}} \| < 1 \). Thus, we can deduce the consensus of (54) under extreme conditions as follows. Using Lemma 2, (55) becomes

\[
\left\{ \begin{array}{l}
\lim_{k \to \infty} (a_{\text{max}})^{k} \lambda(0) = \frac{1}{N} 1^{T} \lambda(0) \\
\lim_{k \to \infty} (a_{\text{min}})^{k} \lambda(0) = \frac{1}{N} 1^{T} \lambda(0)
\end{array} \right.
\]  

(57)

It can be inferred from (57) that (55) can reach consensus under extreme conditions. Therefore, (54) or (55) can always reach consensus.

6. Simulation and Analysis

In this part, we conduct some simulations to verify the effectiveness of our algorithm. Frequency is variable in an electric power system, so we assume that the frequency changes periodically. According to the number of generators, we can divide them into four-generator systems and 10-generator systems. For a four-generator system, we study the economic dispatch algorithm with a frequency regulator under the circumstances of an event-triggered mechanism and the directed graph topology. Extending to 10-generator systems, we simulate economic dispatch algorithms with event-triggered mechanism under time-varying topology in leader–follower mode. Finally, the PI algorithm in [11] is compared with the algorithm we design to verify the superiority of our designed algorithm. Our parameters in the above algorithm are set as follows: \( \tau = 1, \xi = 0.0001, \mu = 0.1, \gamma = 0.00022, v = 0.5, \epsilon = 0.2, \sigma = 1 \).

6.1. Four-Generator System

We simulate a four-generator system, and Figure 2 shows the corresponding directed topology. The cost function coefficients and power limit for each generator in the system are listed in Table 1. Next, simulation experiments will be carried out. In this case, we add an event-triggered mechanism to the algorithm and we set the total power demand of the entire power system as 599 kw. Figures 3 and 4 illustrates the simulation results. From Figure 3a, we can see that the output of each generator remains stable Whenever \( \mu \), \( \gamma \), and \( \sigma \) are affected by the active power constraint and adopts the minimum time-varying topology in leader–follower mode. Finally, the PI algorithm in [11] is extended to 10-generator systems, we simulate economic dispatch algorithms with event-triggered mechanism under the circumstances of an event-triggered mechanism and the directed graph topology. Extending to 10-generator systems, we simulate economic dispatch algorithms with event-triggered mechanism under time-varying topology in leader–follower mode. Finally, the PI algorithm in [11] is compared with the algorithm we design to verify the superiority of our designed algorithm. Our parameters in the above algorithm are set as follows: \( \tau = 1, \xi = 0.0001, \mu = 0.1, \gamma = 0.00022, v = 0.5, \epsilon = 0.2, \sigma = 1 \).
mechanism improves the efficiency of calculation. Figure 4d shows the relationship between the power generation and total demand. It can be seen that the balance of supply and demand is achieved within the allowable range of error.

Figure 2. Four-generator system communication topology.

Figure 3. (a) The output power of each generator. (b) The total cost of power generation. (c) The frequency variation process. (d) The process of frequency regulator.
Figure 4. (a) The iteration of incremental cost $\lambda_i(k)$. (b) The iterative process of auxiliary variable $y_i(k)$. (c) Event sampling instants of four generators. (d) Total active power.

Table 1. Generator cost function coefficient.

<table>
<thead>
<tr>
<th>Generators</th>
<th>$\alpha_i$</th>
<th>$\beta_i$</th>
<th>$\gamma_i$</th>
<th>$p_{\text{min}}$</th>
<th>$p_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DG1</td>
<td>0.00118</td>
<td>3.08</td>
<td>531</td>
<td>30</td>
<td>550</td>
</tr>
<tr>
<td>DG2</td>
<td>0.00346</td>
<td>0.80</td>
<td>388</td>
<td>30</td>
<td>550</td>
</tr>
<tr>
<td>DG3</td>
<td>0.00322</td>
<td>1.65</td>
<td>256</td>
<td>30</td>
<td>550</td>
</tr>
<tr>
<td>DG4</td>
<td>0.00184</td>
<td>2.00</td>
<td>335</td>
<td>30</td>
<td>550</td>
</tr>
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Table 2. Generator event-triggered statistics.

<table>
<thead>
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<th>Generators</th>
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<th>3</th>
<th>4</th>
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<tr>
<td>Event-triggered</td>
<td>32</td>
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<td>30</td>
<td>31</td>
</tr>
<tr>
<td>Ratios</td>
<td>26.7</td>
<td>35.8</td>
<td>25.0</td>
<td>25.8</td>
</tr>
</tbody>
</table>

6.2. Ten-Generator System

In this part, we simulate a ten-generator system ($N = 10$). From Figure 5a, we obtain the corresponding directed topology $G$. The blue nodes are the leaders and the green nodes are the followers. The dotted line indicates the communication between leaders and the solid line indicates the communication between any two nodes.
In order to improve the robustness of topology, we add a time-varying topology mechanism to the event-triggered algorithm. In this case, the 10-generator system uses leader–follower mode, and only the leader–generator has a frequency regulator. From Figure 5b,c, all possible topology sets are set to $\tilde{G} = \{G^1, G^2\}$, and the time-varying topology $G^{\delta(k)}(k) \in \tilde{G}$ switches between two fixed topologies $G^1$ and $G^2$ at each iteration. $E^1$ and $E^2$ represent the edge set of $G^1$ and $G^2$, respectively. $G^{\delta(k)}(k) = (V, E(k))$, $G^{\delta(k)} \in \tilde{G}$, which represents the topology of the $k$-th iteration. Its corresponding adjacency matrix are $A^{\delta(k)}$ and $W^{\delta(k)}$. $G^{\delta(k)}(k)$ is defined as follows.

1. $G^{\delta(k)}(k) = \begin{cases} G^1 & \text{if } \mod(k, 2) = 0 \\ G^2 & \text{if } \mod(k, 2) = 1 \end{cases}$ (58)

In addition, we need to ensure that $G(k) = (V, E^1 \cup E^2)$ is uniformly jointly strongly connected. Then, we set the total power demand of the entire power system as 4085 KW. The cost function coefficients and power limit for each generator in the system are listed in Table 3. We can obtain the simulation results from Figures 6 and 7. Figure 6 is similar to the explanation of Figure 3; it shows the variable iteration in frequency variation. Figure 7a shows that our algorithm still achieves consensus under the time-varying topology and event-triggered mechanism. Figure 7b shows that the auxiliary variable $y_i(k)$ can still converge to a range in a time-varying topology. The information in Figure 7c about the event-triggered mechanism is summarized in Table 4. Figure 7d shows that the algorithm still balances supply and demand in the allowable range of error, reflecting the robustness of the algorithm.

![Figure 5](image_url)

**Figure 5.** (a) Topology $G$. (b) Topology $G^1$. (c) Topology $G^2$.

**Table 3.** Generator cost function coefficient.

<table>
<thead>
<tr>
<th>Generators</th>
<th>$a_i$</th>
<th>$\beta_i$</th>
<th>$\gamma_i$</th>
<th>$P_{\text{min}}$</th>
<th>$P_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DG1</td>
<td>0.00118</td>
<td>3.08</td>
<td>531</td>
<td>30</td>
<td>550</td>
</tr>
<tr>
<td>DG2</td>
<td>0.00346</td>
<td>0.80</td>
<td>388</td>
<td>30</td>
<td>550</td>
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<tr>
<td>DG3</td>
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<td>1.65</td>
<td>256</td>
<td>30</td>
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<td>DG4</td>
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<td>335</td>
<td>30</td>
<td>550</td>
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<tr>
<td>DG5</td>
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<td>478</td>
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<td>DG6</td>
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<td>DG7</td>
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<td>DG8</td>
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<td>DG10</td>
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**Table 4.** Generator event-triggered statistics.

<table>
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<th>Generators</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
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<th>10</th>
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<td>120</td>
<td>120</td>
<td>120</td>
<td>120</td>
<td>120</td>
<td>120</td>
<td>120</td>
<td>120</td>
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<tr>
<td>Event-triggered</td>
<td>34</td>
<td>35</td>
<td>43</td>
<td>45</td>
<td>50</td>
<td>49</td>
<td>48</td>
<td>44</td>
<td>45</td>
<td>53</td>
</tr>
<tr>
<td>Ratios</td>
<td>28.3</td>
<td>29.2</td>
<td>35.8</td>
<td>37.5</td>
<td>41.7</td>
<td>40.8</td>
<td>40.0</td>
<td>36.7</td>
<td>37.5</td>
<td>44.2</td>
</tr>
</tbody>
</table>
Figure 6. (a) The output power of each generator $P_i(k)$. (b) The total cost of power generation. (c) The frequency variation process $F_i(k)$. (d) The process of frequency regulator.

6.3. Comparison Experiment

The purpose of this paper is to confirm the validity, correctness and superiority of our designed algorithm; we compare it with the PI algorithm mentioned in [11] from the perspective of event-triggered times, convergence speed and total power generation cost.

After introducing the event-triggered mechanism, from Figure 8a,b and Table 5, we can obtain that the number of iterations in this algorithm is significantly reduced, which reduces the consumption of communication resources compared with the PI algorithm. It can be observed from Figure 8c,d that the PI algorithm promotes the incremental cost to achieve consensus in 18 iterations, while the event-triggered algorithm only needs 12 iterations to achieve consensus. Therefore, the convergence speed of the event-triggered algorithm is higher than that of the PI algorithm. Figure 9 shows that the total power generation cost of the PI algorithm and event-triggered algorithm is almost the same. Under the same total power generation cost, according to the above analysis, the event-triggered algorithm we designed is obviously better than the PI algorithm.
Figure 7. (a) The iteration of incremental cost $\lambda_i(k)$. (b) The iterative process of auxiliary variable $y_i(k)$. (c) Event sampling instants of four generators. (d) Total active power.

Table 5. Comparison of iteration times between two algorithms.

<table>
<thead>
<tr>
<th>Generators</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<td>120</td>
<td>120</td>
<td>120</td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td>Event-triggered</td>
<td>37</td>
<td>47</td>
<td>56</td>
<td>43</td>
<td>37</td>
<td>61</td>
<td>40</td>
<td>39</td>
<td>31</td>
<td>45</td>
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</tbody>
</table>
Figure 8. (a) PI without event-triggered. (b) Event-triggered. (c) PI DED. (d) Event-triggered DED.

Figure 9. Comparison of total cost of two algorithms.

7. Conclusions

In this paper, a novel DED control method is proposed to solve EDP under directed topology based on consensus protocol, which is suitable for a more general environment. The method introduces a frequency regulator into the consensus protocol to eliminate the
influence of the frequency difference change of each generator. Moreover, considering large-scale power systems, we added an event-triggered mechanism to the method to decrease the communication resources. This method only needs to update the current state after meeting the triggered conditions, so it improves the efficiency of our calculation. In addition, the robustness of the algorithm is improved by introducing a time-varying topology mechanism. The mechanism uses the mapping topology function to carry out economic dispatch under the topology at any time. Finally, numerical experiments verify the validity of the method. Nevertheless, the method is not perfect. It is worth emphasizing that the algorithm in this paper also has limitations. For example, our frequency changes periodically. The frequency change period simulated in the experiment is 50 s. In addition, we need to meet a condition when we introduce the time-varying topology mechanism; that is, the topology we design must be a joint uniformly connected graph to meet the convergence of the algorithm. Thus, future works will focus on network attack, transmission losses, ramp rate limits for power generators and other practical operational constraints.

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Abbreviations
The following abbreviations are used in this manuscript:

<table>
<thead>
<tr>
<th>Abbreviations</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>EDP</td>
<td>Economic Dispatch Problem</td>
</tr>
<tr>
<td>DED</td>
<td>Distributed Economic Dispatch</td>
</tr>
<tr>
<td>Nomenclature</td>
<td>symbolic variable describe</td>
</tr>
<tr>
<td>$P_i$</td>
<td>the output power of the i-th generator.</td>
</tr>
<tr>
<td>$C(P_G)$</td>
<td>total cost.</td>
</tr>
<tr>
<td>$P_D$</td>
<td>total power demand.</td>
</tr>
<tr>
<td>$P_{\text{min}}$</td>
<td>the minimum output power of the i-th generator.</td>
</tr>
<tr>
<td>$P_{\text{max}}$</td>
<td>the maximum output power of the i-th generator.</td>
</tr>
<tr>
<td>$\lambda_i(k)$</td>
<td>the incremental cost of the i-th generator at the k-th iteration.</td>
</tr>
<tr>
<td>$\tau$</td>
<td>a fixed step size.</td>
</tr>
<tr>
<td>$a_{ij}$</td>
<td>the weight coefficient from generator j to generator i.</td>
</tr>
<tr>
<td>$L_a$</td>
<td>a Laplacian matrix.</td>
</tr>
<tr>
<td>$T^*_i(k)$</td>
<td>a target attraction function.</td>
</tr>
<tr>
<td>$\Delta \lambda_i(k)$</td>
<td>not only a frequency regulator but also a local frequency error.</td>
</tr>
<tr>
<td>$f_i(k)$</td>
<td>the real-time discrete frequency of each generator.</td>
</tr>
<tr>
<td>$x_i(k)$</td>
<td>the variable transmitted by the i-th node in the communication topology.</td>
</tr>
<tr>
<td>$\hat{x}_i(k)$</td>
<td>the transmission variable of the i-th node at event-triggered time instant $k_i$.</td>
</tr>
<tr>
<td>$t_i$</td>
<td>the number of event-triggered.</td>
</tr>
</tbody>
</table>
References


28. Zhang, Y.; Ni, M.; Sun, Y. Fully distributed economic dispatch for cyber-physical power system with time delays and channel noises. *J. Mod. Power Syst. Clean Energy* 2021. [CrossRef]


