

Article

Design Method of Cam Steering Mechanism Based on Path Fitting

Xiaofei Zheng ¹, Hantao Zhao ², Songhui Zhang ¹, Dan Liu ³ and Binrui Wang ^{3,*}¹ Engineering Training Center, China Jiliang University, Hangzhou 310018, China; zhengxiaofei.xyz@126.com (X.Z.); zsh@cjlu.edu.cn (S.Z.)² College of Quality and Standardization, China Jiliang University, Hangzhou 310018, China; zht@126.com³ College of Mechanical and Electrical Engineering, China Jiliang University, Hangzhou 310018, China; danliu712@126.com

* Correspondence: wangbrpaper@163.com

Abstract: In order to improve the accuracy of a solar-powered punch card car's movement on a designated route and reduce positional deviations during its operation, a solar-powered punch card car with a single cam as the steering guidance mechanism was designed. The car adopts a three-wheel structure. The transmission mechanism, steering mechanism, driving mechanism, and regulating mechanism of the car were analyzed. The kinematics model of the car was established and the motion characteristics of the car were obtained. By analyzing the relationship between the steering angle of the car and the curvature radius of its travel route, the front wheel angle of the car at each position was calculated using MATLAB R2020a. This allowed us to establish the relationship between the front wheel angle and the displacement of the steering push rod, which was further converted into the theoretical contour line of the cam. Subsequently, the theoretical contour line of the cam was completed and envelope correction was performed. Finally, through mechanical analysis and experimental verification using a prototype, the results indicated that the single-cam steering guidance mechanism calculated using this fast path fitting method exhibited excellent mechanical performance and a smooth and accurate trajectory, and the traveling path of the theoretical cam contour curve was basically consistent with the actual trajectory route.

Keywords: new energy vehicles; cam mechanism; structural design; path analysis; motion simulation



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1. Introduction

Cam mechanisms are commonly used in mechanical structure design because of their excellent performance in operating speed, motion accuracy, structural rigidity, and production costs [1]; they are widely used in the fields of light industry, textiles, food, transportation, mechanical transmission, and other fields [2]. Therefore, in order to achieve an accurate and complex trajectory movement control, the cam is often used [3]. The solar electric car is a well-known national extracurricular science and technology product. An electric motor is needed as the sole active part to drive the car in accordance with the prescribed route, and the route needs to be reasonably designed according to the given coordinate points so that the car can be driven according to the trajectory of the design. Because the driving path of the car is randomly given in the game, and a series of operations such as design and simulation are required within 6 h, the driving path of the car is complicated and steep. This means that the speed of the car should not be too great; otherwise, it may cause motion distortion, slipping, and other situations. This imposes strict requirements on the steering design of the car [4]. The cam mechanism is the best choice for this solution. But there are many problems with the existing cam design methods. For example, Liu Y. [5] and Fu P. [6] used SolidWorks Motion to draw a trajectory to reverse the simulation of the cam. This method cannot complete the trajectory with an intersecting curve, and it cannot adapt to a more flexible route. And the results after simulation need to

be further screened. Zhou [7] and Wu D. [8] designed the cycle, amplitude value, and other parameters of the positive string curve, and the trajectory design was completed. However, this method takes a lot of time to calculate the more complicated and flexible trajectory. Zeng D. [9] used MATLAB to fit the trajectory into a polynomial and drew a cam through the function calculation. But this method cannot be applied to the coordinate point without regular. Huang H. [10] used NX 12.0 and MATLAB to design closed-loop trajectories, but this method cannot be designed for a non-closed trajectory.

Some scholars abroad have analyzed the mechanical properties of the cam. For example, Sundar [11] calculated the contact stiffness at the contact point of Hertz's contact theory. They used numerical methods to analyze the recovery coefficient. Hertz's law was also used with damping items to adapt to energy loss during the impact process [12]. Kuang and others studied the values and experiments of the relationship between the dynamic response of the curved cam system and the motor driving speed with grid stiffness [13]. In order to suppress the high contact at high speed, a cam-type line with uneven surface contact between the cam and the port of the dynamic parts was used. In addition, the use of Hertz's contact theory for stress analysis is of great significance to improve the fatigue life of the cam system. Hua [14] found that the smooth surface of the cam can be reduced by 53% maximum equivalent Von Mises stress. In the experiment, by considering using the constant speed in the staying itinerary, the contact stress was reduced to 41% [15]. Hus [16] adopted a cam with double-concave surface translation from the moving part to reduce the contact stress of the cam's equal pathway. Based on the above problems, this paper studies an efficient and fast cam contour fitting method to adapt to the complex trajectory design needs, and also analyzes the mechanical properties of the cam to ensure that the performance of the cam mechanism can meet the motion needs.

The main contributions of this paper are as follows:

- (1) We propose a method to fit the trajectory path based on B-spline curve. By using MATLAB to solve the cam contour curve of the non-closed trajectory, the design of any complex trajectory path can be completed efficiently.
- (2) We propose a method to complete and envelope cam profiles using MATLAB. The reliability and accuracy of the cam profile design were verified through a dynamic and static analysis of the cam in ADAMS View 2020 and ANSYS 2020R2.

This paper is organized as follows. Section 2 introduces the four mechanical structures of the car. Section 3 describes the principle of the NURBS curve. The route trajectory is drawn and the mathematical model of the car is established. The section also describes how to solve the contour and envelope curve of the cam theory with MATLAB. And the maximum pressure angle and maximum curvature radius are calculated. Section 4 solves the cam according to the parameters of the car. The kinematics and dynamics of the steering structure are simulated. Finally, the feasibility and accuracy of the design are verified through physical verification. The last section concludes this paper.

2. Structural Design

2.1. Transmission Mechanism

For a three-wheel solar-powered punch card car, the fewer power-consuming components it has, the longer its battery life and driving range will be [17]. In order to make the vehicle structure simple, a gear motor is used as the only drive source. The motor shaft is directly connected to the driving wheel shaft by means of a coupling. The driving wheel is used to move the trolley forward. At the same time, single direction bearings are installed on the driving wheel shaft to facilitate the adjustment of the cam position. In order to achieve a small energy loss and an accurate transmission ratio, the gear transmission mechanism is selected. And the two-stage transmission is adopted to realize the larger transmission ratio movement.

The power is provided by a DC motor with a drive voltage of 3 V. If the power supply is directly connected to the motor, it will cause voltage fluctuations and lead to unstable power output. Therefore, it is necessary to design a booster voltage regulator circuit to

drive the motor to stabilize the output power. After testing, the boost structure is selected to make the boost voltage regulator module, and the efficiency can reach more than 80%.

Due to the motor's starting characteristics, the motor reaches its maximum speed immediately upon power-on. This rapid acceleration results in an excessively large instantaneous driving force on the driving wheel during vehicle startup. The maximum friction force of the driving wheel is insufficient to counterbalance the torque, leading to the car slipping. To prevent skidding when starting, it is necessary to make the reducer motor start slowly. There are two primary methods to achieve a slow start of the motor. The first option is mechanical. The motor shaft and the driving wheel shaft are belt-driven or the gear clutch structure [18] is used to transmit power. After the motor rotates, the force is gradually applied to the driving wheel until the maximum static friction is surpassed and slowly started. The second solution uses electrical circuits. The scheme uses the NTC thermistor soft start circuit [19], and the NTC resistor is connected to the motor in series. When the circuit is running, the NTC resistor gradually heats up and the resistance value decreases. The temperature change is proportional to time, so the NTC resistance change is inversely proportional to time. Therefore, the partial voltage of the resistor is inversely proportional to the time, which effectively promotes slow starts. The transmission structure is shown in Figure 1.

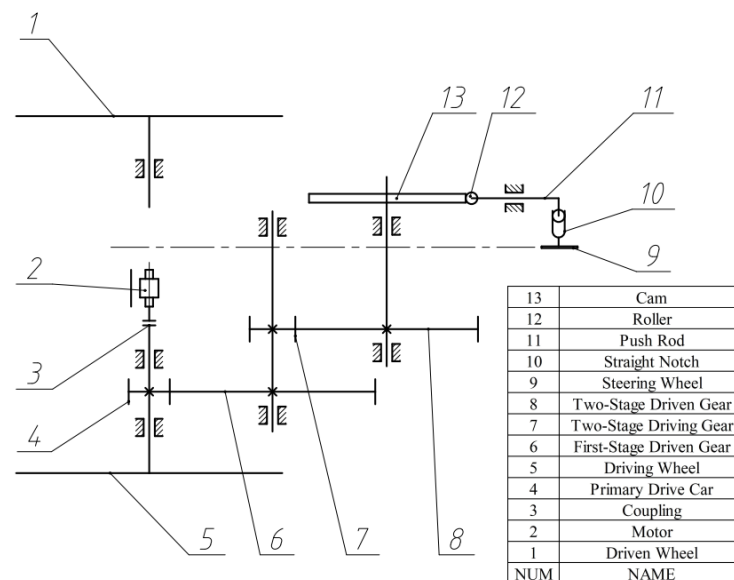


Figure 1. Transmission structure.

2.2. Knuckle Mechanic

The steering mechanism requires high control precision, low friction, and adjustable flexibility. This design uses a cam push rod mechanism to achieve the steering. The mechanism is composed of a cam, a push rod, and a slider. One end of the push rod maintains line contact with the cam and converts the rotation of the cam into a straight-line movement of the push rod. The rotation of the cam is converted into a straight-line movement of the push rod, and the other end is connected to the front wheel by a swinging rod. The push rod acts in linear motion on the swing rod that is fixed to the front wheel. The angle of the front wheel is changed by pushing forward and pulling back of the push rod, thereby controlling the change in the movement direction of the car. Thus, autonomous direction control is realized.

2.3. Drive Mechanism

From the point of view of cost and operability, single-wheel drive is more suitable. That is, only one side of the wheel is used as the driving wheel, and the other side is used as the driven wheel. The friction between the driving wheel and the driven wheel needs

to rely on the ground to achieve the differential structure. The structure is convenient for adjustment, assembly, and high efficiency.

In a tricycle of this structure, when the driving wheel begins to rotate under the drive of the motor, it pushes the vehicle forward as a whole. Since the vehicle is rigid, the front and driven wheels will also begin to move due to the push of the drive wheel. At this point, the speed of the driven wheel will be affected by the speed of the driving wheel and the ground friction. If the ground friction is large enough and the vehicle structure is stable, then the right rear wheel will maintain the same speed as the drive wheel as much as possible to ensure that the vehicle can move steadily in a straight line. However, in the actual process of driving, due to road conditions, vehicle load, tire wear, and other factors, the speed of the right rear wheel may be slightly different from the drive wheel. However, this error is within the acceptable range and usually does not have much impact on the stability of the linear motion of the vehicle, because the steering wheels of the vehicle are responsible for controlling the direction of travel, ensuring that the vehicle can follow the predetermined trajectory.

2.4. Precision Regulating Mechanism

Due to manufacturing errors between the theoretical data and the actual parts [20], an accurate fine-tuning mechanism is needed to compensate for these errors. The push rod is equipped with a screw splitter at its front end, which can be used to compensate for the error of cam cutting. The upper end of the steering wheel is also equipped with a deviation trimmer. The offset trimmer screw can adjust the transverse distance from the cam to the steering wheel.

The steering and adjusting mechanism is shown in Figure 2, and the overall structure is shown in Figure 3.

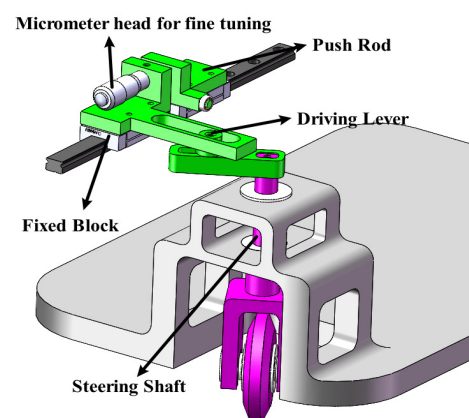


Figure 2. Steering and regulating mechanism.

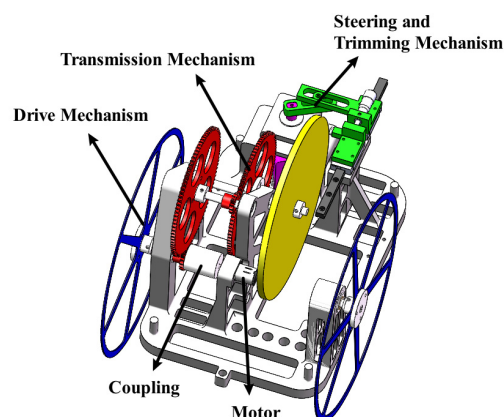


Figure 3. Overall structure of car.

3. Cam Design

3.1. Travel Path Analysis

The coordinates of the checking points are listed in Table 1. According to the competition question, the map boundary is 4000 mm $\times$ 4000 mm and the site boundary is 5000 mm $\times$ 5000 mm. The car needs to traverse the checking points from 1 to 10 in sequence. Premature or repeated clocking is not allowed. And after traversing three punch points three consecutive times, no light is regarded as the end of the operation. The car needs to be punched in sequence, as shown in Figure 4.

Table 1. Trolley check-in point.

Num	X (mm)	Y (mm)
1	3725	475
2	2975	250
3	1950	550
4	1450	1000
5	250	525
6	400	1700
7	250	1825
8	600	2625
9	1150	3500
10	2125	3750

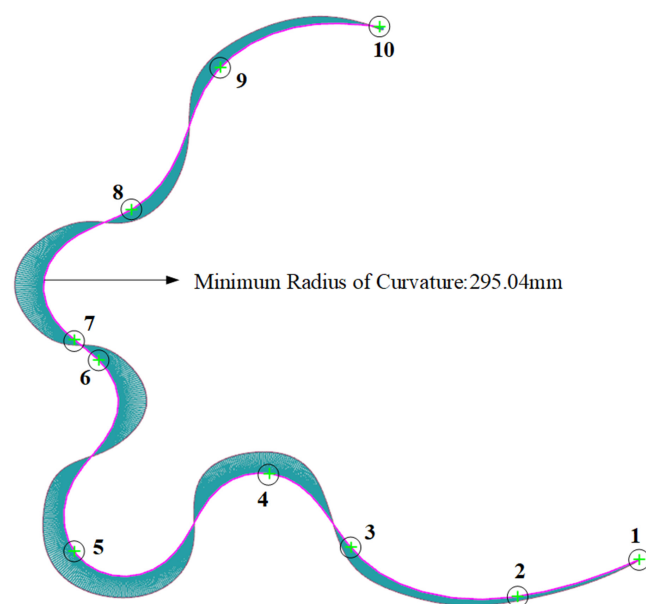


Figure 4. Trolley trajectory planning (1–10: the point where the car needs to pass).

After analysis, the trajectory of the route is long and the curvature of the coordinate points changes greatly, making it difficult to use the simple cosine curve and straight line to complete the path planning. If the number of punch points and coordinates change, the previous methods cannot complete the design or need to consume more time to calculate and verify the route.

3.2. Path Fitting Based on Polynomial Curvature Radius

In computer graphics, Bessel Curve and B-spline curve are typically used to draw curves. However, due to the complexity of splicing Bessel Curve, and due to the fact that when parts of it are modified, the other parts of the curve will deviate from the expected curve, it can cause inconvenience in path fitting. The B-spline curve effectively overcomes these shortcomings [21]. In the 3D software NX 1987, the generated splines are NURBS

curves, which can pass through any number of control points. Compared with the other two curves, NURBS curves can be adjusted more flexibly [22]. Therefore, to enhance the speed and efficiency of path planning, this paper employs NURBS curve for path planning and analysis.

Given the d-primary function of the B-spline $B_i^d(u)$ and the knot vector $U = \{U_0, U_1, \dots, U_n, U_{n+1}, \dots, U_{n+d+1}\}$, the fitted vehicle driving track is shown in Formula (1) [23]:

$$P(u) = \sum_{i=0}^n B_i^d(u) P_i \quad (1)$$

In the formula, P_i is the control point of the NURBS curves connection.

The k -order derivative of this curve is shown in Formula (2):

$$P^k(u) = \sum_{i=0}^{n-k} B_i^{d-k}(u) P_i^{(k)} \quad (2)$$

In order to ensure that the car can move smoothly and the cam has no abrupt points, at least a G2 curve should be constructed. This requires the existence of a second derivative of the curve. The G2 curve refers to curvature continuity. In order to make the path smoother, based on continuous curvature, the G3 curve is used, which means that the rate of change of curvature is continuous [24]. Therefore, NURBS curves higher than cubic are used to draw trajectories to ensure the curvature continuity.

Enter coordinate points in the UG software and use a cubic NURBS spline to fit the route. Add control points appropriately between each punching point and control the minimum curvature to be as large as possible. Make the curve changes more smooth at the inflection point, and the curvature changes tend to be smooth at the non-inflection point section. Then, open the curvature graph to verify the reasonableness of the curve drawing. The final route planning and path curvature analysis results are shown in Figure 4.

3.3. Car Mathematical Model Establishment

The simplified geometric model of solar electric vehicles is shown in Figure 5.

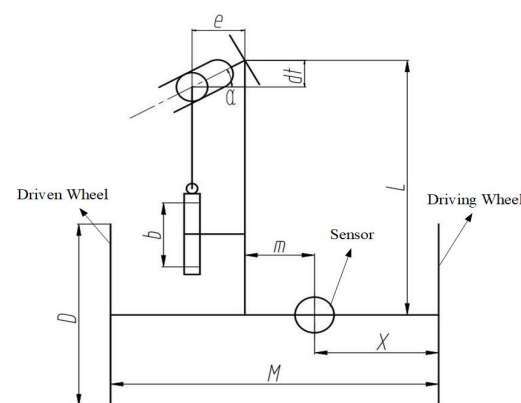


Figure 5. Trolley parameters.

In Figure 5, M is the distance between the driving wheel and the driven wheel; m is the deviation distance between the front wheel and the sensor's horizontal coordinate direction; L is the distance from the front wheel to the rear wheel; e is the cam offset; D is the diameter of the drive wheel; b is the prime circle of the cam; X is the distance from the sensor to the driving wheel, and the sensor is placed on a certain point of the connection between the driving wheel and the driven wheel.

In order to facilitate the analysis, the dynamics of the car during actual operation, such as friction, are not considered. The motion of the car is analyzed only on a two-dimensional plane. The kinematic analysis of a car can be idealized as a bicycle model [25].

During the analysis, the position and diameter of the driven wheel have no influence on the theoretical trajectory analysis. The position of the sensor can be regarded as the central axis of the car, and the distance X from the sensor to the driving wheel can be equivalent to half the width of the car.

3.4. The Pitch Curve Calculation

The running state of the car is shown in Figure 6. In the motion analysis, the planned route is the trajectory scanned by the projection of the car sensor, rather than the actual path traveled by the car's drive wheel. Therefore, the path of the fitted trajectory cannot be directly calculated as the path of the driving wheel.

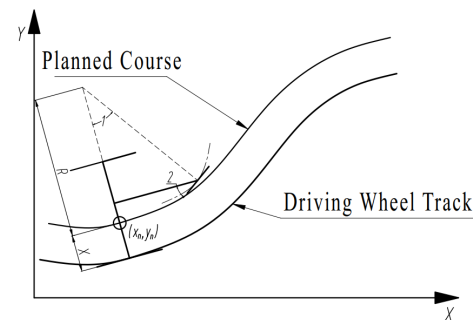


Figure 6. Running status.

The curve is equally divided into n segments to obtain the coordinate values of the beginning or end point of each segment. When n takes a maximum value, each segment can be treated as a straight line. The length of each segment can be calculated by differential method, as shown in Formula (3):

$$ds_n = \sqrt{(x_{n+1} - x_n)^2 + (y_{n+1} - y_n)^2} \quad (3)$$

when driving to the NTH point, the trajectory of the driving wheel is calculated according to Formula (4).

$$s' = \sum \left(ds_n \frac{X + R}{R} \right) \quad (4)$$

In the formula, the maximum value of n can be adjusted by modifying the split value of the curve. Theoretically, the larger the value of n , the more accurate the calculated value will be.

To calculate the theoretical profile of the cam, it is necessary to find the relationship between the steering angle of the front wheel and the displacement of the push rod [26].

Since the line from the front wheel to the center of the radius of curvature is always perpendicular to the front wheel, an expression for the angle can be represented by Formula (5).

$$\alpha = \angle 1 = \arctan \frac{L}{R - m} \quad (5)$$

In the formula, R is the radius of curvature of the car at this point. The radius of curvature can be derived directly from the curve analysis in NX. It can also be calculated according to the point coordinates of the known trajectory, using the principle of determining a circle through three points. However, the direction of the radius needs to be distinguished. In the above formula, the radius is calculated using clockwise as negative and counterclockwise as positive.

When the cam rotation is complete, the total distance traveled by the car is calculated according to Formula (6):

$$s = i\pi D \quad (6)$$

In the formula, i is transmission ratio.

When the car is driven to a certain point, the angle of rotation of the cam relative to the initial position is calculated by Formula (7):

$$\theta = \frac{s}{s'} \times 2\pi \quad (7)$$

In the formula, s' is the length of distance that the driving wheel of the car has traveled at that point.

The displacement of the push rod at this point can be calculated according to Formula (8):

$$dt = e \sin \alpha \quad (8)$$

Thus, after the cam has rotated past θ , the push or return expression of the cam at that point is shown in Formula (9).

$$\rho = dt + r = r + e \sin(\arctan(L/(R - m))) \quad (9)$$

In the formula, r is the radius of the base circle, so $r = b/2$.

Since the trajectory of the competition problem is in the form of a non-closed loop, the effective part of the calculated cam cannot be connected. Therefore, the remaining unused angle of the cam is $360^\circ - \theta$. In principle, the smaller the value, the better to make full use of the cam. But ultimately, in order to obtain a complete cam, the end of the cam is required to be completed.

Firstly, according to the number of effective part points, the average number of points of the whole cam is calculated, and the number of spare part points of the cam is calculated. The following is expressed in a MATLAB expression, as shown in Formula (10).

$$num = \text{floor}(imax \times \pi \times i \times D/s) - imax \quad (10)$$

In the formula, $imax$ is the number of valid partial points, and s is the track length.

Then, generate num equidistant points between the end and start of the effective curve to complete the cam, as shown in Formula (11).

$$\begin{cases} \theta_{add} = \text{linspace}(2\pi n, 2\pi, num) \\ \rho_{add} = \text{linspace}(2\pi n, 2\pi, num) \end{cases} \quad (11)$$

The newly generated array is superimposed onto the original arrays ρ and θ , and the starting position is added to the last row to make the cam closed.

Finally, the polar parameters are converted to Cartesian parameters, as shown in Formula (12).

$$\begin{cases} X = \rho \cos(\theta) \\ Y = \rho \sin(\theta) \end{cases} \quad (12)$$

By using the above calculation method, the column vector of the cam theoretical contour X and Y can finally be calculated.

By using MATLAB, the curvilinear coordinates of the trajectory are imported, the code is written for calculation and drawing, and the coordinates of the pitch curve can be quickly calculated.

3.5. Cam Envelope

Because the cam mechanism employs a roller follower [27], the cam profile obtained through the above calculation is the pitch curve. Envelope processing is necessary to acquire the correct cam contour. Typically, the actual cam profile can be obtained by using the offset curve instruction in the drawing software to shift the theoretical profile inward by the distance equal to the roller radius. In MATLAB, the cam contour can be obtained by finding the normal vector at a point and then moving a distance equal to the cam roller radius in the direction of that normal vector. The specific calculation principle is as follows.

The column vector coordinates $X[i], Y[i]$ of the processed cam theoretical profile are known.

Firstly, the radius of curvature of each point on the cam is calculated. For a set of discrete points, the curvature can be calculated according to Formula (13).

$$\rho_i = \frac{\arctan\left(\frac{y_{i+2}-y_{i+1}}{x_{i+2}-x_{i+1}}\right) - \arctan\left(\frac{y_{i+1}-y_i}{x_{i+1}-x_i}\right)}{\sqrt{(x_{i+1}-x_i)^2 + (y_{i+1}-y_i)^2}} \quad (13)$$

In the formula, x_i, y_i are discrete points on the theoretical contour of the cam. Then, the radius of curvature is calculated according to Formula (14).

$$R_i = \frac{1}{\rho_i} \quad (14)$$

To avoid movement distortion caused by the intersection of actual profiles, the minimum curvature radius of the convex part must be greater than the radius of the roller, that is, $r < R_{min}$, and the position of the minimum curvature radius should be marked.

To calculate the tangential and normal vectors at the first point, the last row of the calculated theoretical cam vector is removed, and the penultimate row is moved to the first row to form a new vector. Due to the large number of points and the dense distribution of points, according to Lagrange's mean value theorem, the slope of the tangential vector at this point is equal to the vector direction of the line between the two adjacent points, as shown in Formula (15).

$$\bar{n} = (x(i+1) - x(i-1), y(i+1) - y(i-1)) \quad (15)$$

In order to obtain the direction of the envelope, it is necessary to rotate the tangential vector by 90° counterclockwise, as shown in Figure 7. Since the vector verticals are perpendicular, their dot product is equal to 0, and the normal vector can be deduced from this, as shown in Formula (16).

$$\bar{n}' = (-y(i+1) + x(i-1), x(i+1) - x(i-1)) \quad (16)$$

The unit length of the algorithm vector in MATLAB is shown in Formula (17).

$$\bar{e} = \bar{n}' / \text{norm}(\bar{n}') \quad (17)$$

To complete the envelope, offset the distance of the roller radius in the direction of the pitch curve to the normal vector, as shown in Formula (18):

$$\begin{cases} \text{cam}(i, 1) = x(i) + \bar{e}r \\ \text{cam}(i, 2) = y(i) + \bar{e}r \end{cases} \quad (18)$$

Finally, the starting point coordinates of the cam are added to the end, so that the cam contour is closed and the cam envelope is completed.

In order to find out the position of maximum pressure angle, the pressure angle is analyzed and the pressure angle of the cam is calculated.

The direction of the pressure is the normal vector of the roller at that point, as shown in Formula (19).

$$\bar{n}1 = \bar{n}' \quad (19)$$

The velocity direction is the vector from the center of the cam to this point, and Formula (20) can be used for calculation.

$$\bar{n}2 = (\text{cam}(i, 1), \text{cam}(i, 2)) \quad (20)$$

The pressure angle is the angle between two vectors, and the calculation formula is shown in Formula (21).

$$\alpha = \arccos \frac{\overline{n1} \cdot \overline{n2}}{|\overline{n1}| |\overline{n2}|} \quad (21)$$

When using MATLAB to calculate the path of a solar electric vehicle traveling on the path fitted in NX, the calculated path of the driving wheel of the car is 9265 mm. Therefore, the transmission calculation formula is:

$$i \geq s/\Pi/D = 9265/3.14/190 = 15.5$$

In order to leave a certain margin for the cam, the transmission ratio is temporarily set to 20.

The specific parameter design of the car is shown in Table 2:

Table 2. Car parameter design.

First Transmission Ratio	Second Transmission Ratio	Front Wheel Offset	Rear Wheel Diameter	Rear Wheel Width/M	Front and Rear Wheel Distance/L
5	4	0 mm	190 mm	197 mm	200 mm

The above parameters were substituted into the calculation formula, and MATLAB was used to calculate the angle of the trolley and the push of the cam. The curves of the calculated angle and push are shown in Figure 8. According to the curve, the degree of change in the path can be directly analyzed. If the change in the angle is drastic, and the minimum curvature radius is less than the roller radius, the route needs to be readjusted.

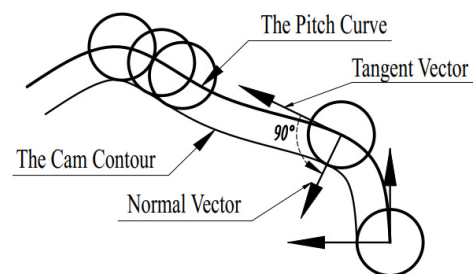


Figure 7. Cam envelope.

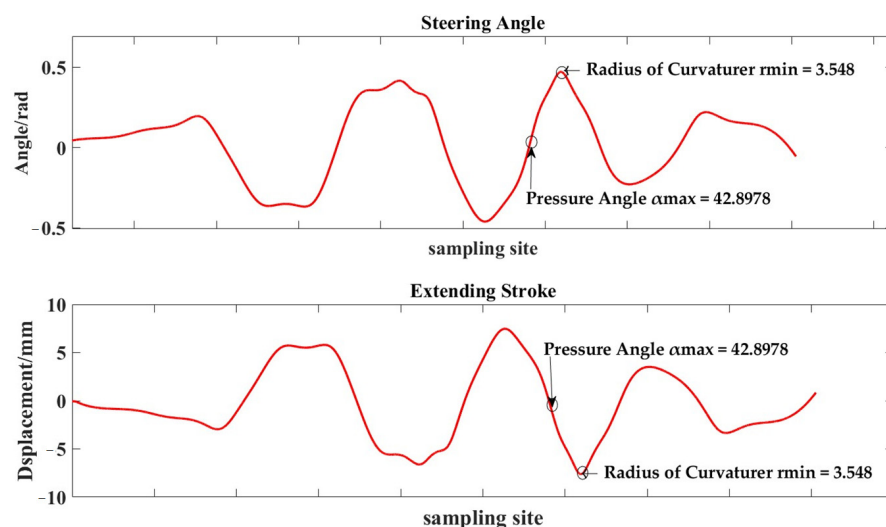
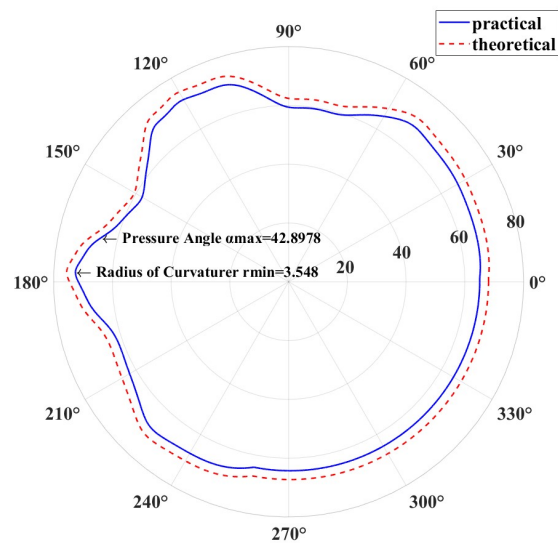
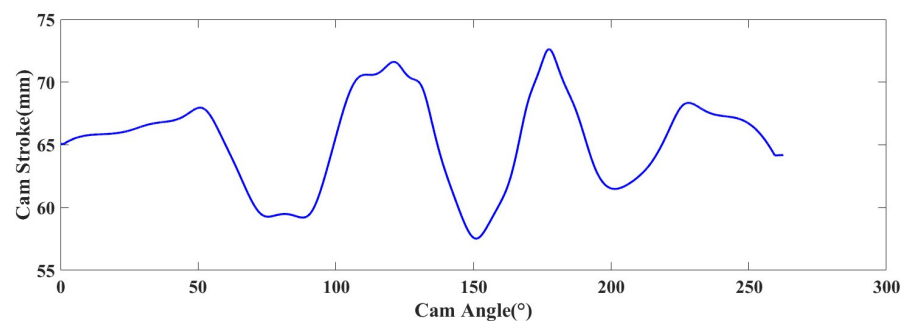


Figure 8. Trolley corner and cam push curve.

Finally, a cam with a base circle diameter of 130 is calculated based on the push. The final calculated cam, position of maximum pressure angle, and minimum curvature radius are shown in Figure 9a.



(a) Cam profile



(b) Cam angle and push curve

Figure 9. Cam design results.

By analyzing the cam angle and cam push curve, as shown in Figure 9b, there are some chaotic points and bumps at certain angles, which may cause distortion in the cam motion. Therefore, the trajectory of the cam can be smoothed by increasing the radius of the base circle and reducing the offset e . The larger the radius of the base circle or the smaller the offset e , the less the cam's stroke will rise and fall, thus reducing the impact. Additionally, before the actual manufacturing of the cam, the theory of equal acceleration and deceleration can be used to optimize the cam trajectory, so as to reduce the impact of the cam.

4. Emulation Proof

4.1. Trajectory Simulation

After analyzing the simulation trajectory results, the feasibility and reliability of the cam design results can be directly assessed, and any issues of the body parameter design can be identified. which reduces the time of real car debugging and reduces the cost of cutting the cam multiple times. After the basic parameters of the solar electric vehicle are determined, the theoretical cam is calculated using MATLAB, and the solar electric vehicle model is simplified in SolidWorks, retaining only the driving, steering parts, and key dimensions. The SolidWorks Motion plugin is then used to reasonably set the parameters

such as contact, gravity, motor, and others. Additionally, add path tracking to facilitate the verification of vehicle trajectory.

The simulation results of the cam are shown in Figure 10.

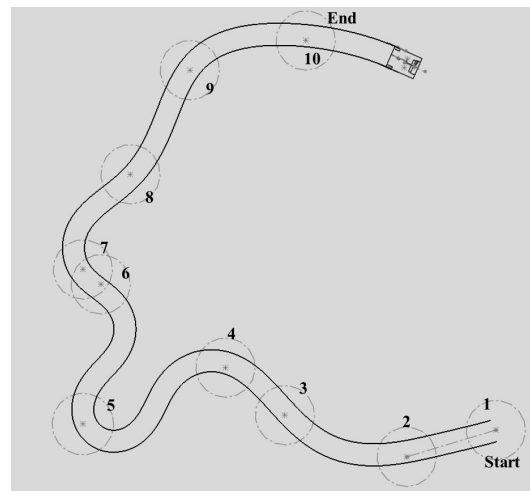


Figure 10. Motion result (1–10: the point where the car needs to pass).

After analyzing the path of the motion simulation, it can be seen that the actual running route of the car is basically consistent with the designed route. The car can pass through the punching points in sequence without repeating or missing any punches. During the running process, the sensor can basically cover the punching points with only minor deviation, which can realize the punching points and meet the requirements of the competition question. Therefore, the cam designed by this method satisfies the requirements and is basically consistent with the designed route.

4.2. Dynamics Simulation

To verify whether the designed cam still maintains a high degree of consistency with the design results under the loading and contact conditions, the multi-body dynamics software ADAMS is used to conduct the dynamic simulation analysis of the steering mechanism.

In the simulation model, the connection pair and contact force are correctly set, the motor is set as the driving source, and the speed is set to 60D, causing the first stage pinion to engage in mesh movement, thereby driving the cam to rotate. Additionally, a tension spring with a stiffness coefficient of 0.004 N/mm is added to the chute and frame.

After the simulation, the change in the Y coordinate of the push rod, the change in the rotation angle of the front wheel, and the contact force between the roller and the cam were extracted and analyzed. Figure 11 shows the push rod displacement curve, Figure 12 depicts the front wheel angle curve, and Figure 13 illustrates the contact force curve between the cam and the roller. In Figure 13, X represents the displacement of the push rod, and F indicates the contact force between the cam and the roller.

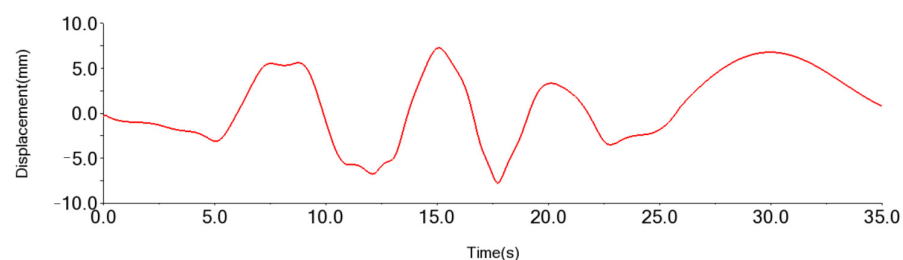


Figure 11. Push rod displacement simulation results.

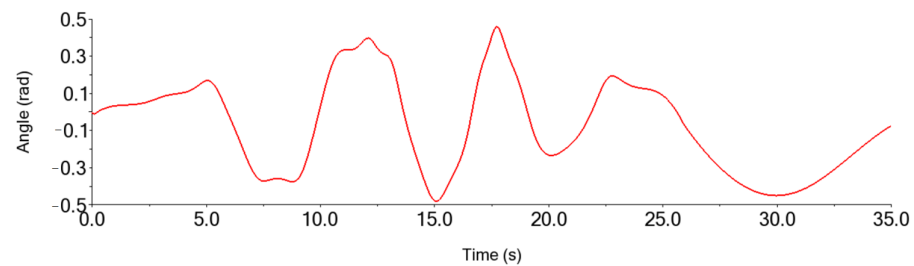


Figure 12. Front wheel angle simulation results.

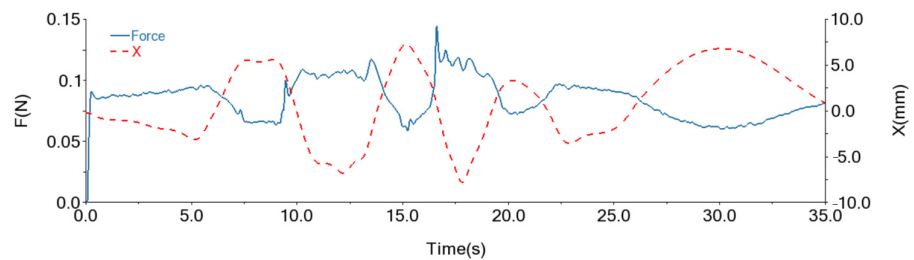


Figure 13. Simulation results of contact force between roller and cam.

The results obtained by ADAMS simulation are highly consistent with the design results within the effective stroke. The maximum contact force between the rubber band and the cam is 0.144 N, the minimum contact force is 0.059 N, and the maximum contact force occurs at the position with the maximum pressure angle during the design, which meets the requirements. The smooth motion curve shows that the design is reasonable and reliable under low speed and light load, and thus verifies the feasibility of the design.

Different compression loads will affect the contact force between the roller and the cam [28]. To verify whether the choice of the elastic coefficient is reasonable, simulations are conducted with $K = 0.04 \text{ N/mm}$ and $K = 0.0008 \text{ N/mm}$. The results are shown in Figures 14 and 15.

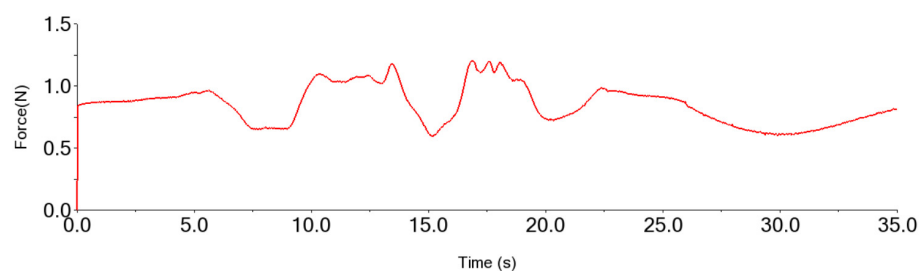


Figure 14. Contact force curve when $k = 0.04 \text{ N/mm}$.

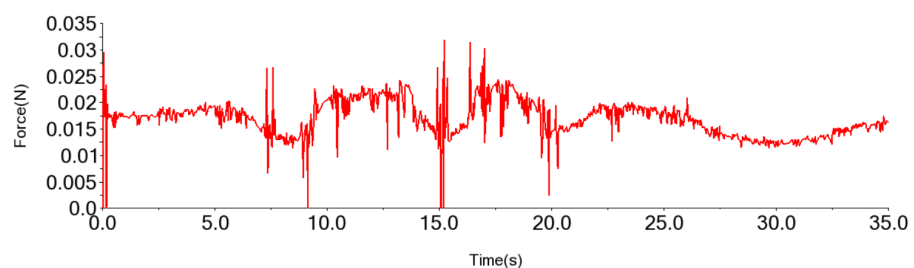


Figure 15. Contact force curve when $k = 0.0008 \text{ N/mm}$.

When $K = 0.04 \text{ N/mm}$, the generated contact force is too large, which will affect the normal operation of the cam. When $K = 0.0008 \text{ N/mm}$, the contact force is unstable and fluctuates significantly, with a portion close to zero, causing the cam to separate from the

roller and leading to trajectory distortion. Therefore, it is reasonable to select a stiffness coefficient of $K = 0.004 \text{ N/mm}$.

4.3. Ansys Mechanical Simulation

Basis on dynamic simulation, in order to verify the suitability of the contact stress between the cam and the pusher, it is necessary to check whether the cam deforms significantly under the pusher's pressure, resulting in motion distortion. In addition, it is necessary to examine the stiffness and strength of the cam mechanism to validate the rationality of the choice of cam materials [29]. Static Structural in ANSYS Workbench is utilized for static analysis.

The kinematic simulation results indicate that at a time of 16.5 s, the contact force between the cam and the roller also reaches a maximum of 0.144 N. Therefore, the working condition at 16.5 s should be taken as the basis for determining the boundary conditions and loads when conducting stress analysis of the cam mechanism in the trolley.

First, it is necessary to adjust the cam mechanism to the working condition when the contact force is maximum, and then import the push rod, roller, and cam in that condition into the Static Structure analysis module of ANSYS Workbench.

The material property of the cam is set to PMMA, with an elastic modulus $E = 3160 \text{ MPa}$ and Poisson ratio $\nu = 0.32$.

After adding the correct material, a force of 0.144 N is applied in the normal direction of the contact point between the roller and the cam. Simultaneously, a Cylindrical Support is added in the center of the cam and the tangential direction is set to be free. The mesh is divided using the MultiZone method, with a mesh size set to 1.5 mm, and the mesh is locally encrypted at the contact point between the roller and the cam. The result of the meshing is shown in Figure 16.

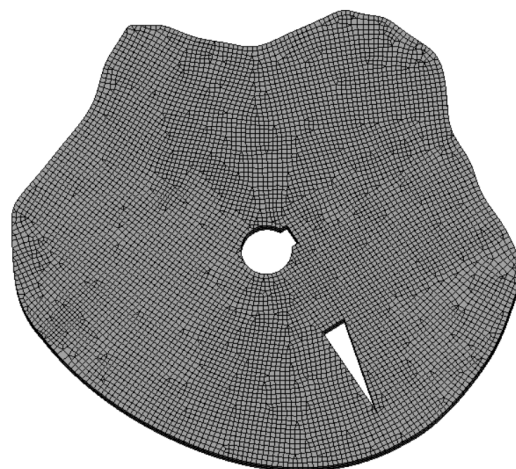


Figure 16. Meshing result.

The maximum contact force obtained by ADAMS dynamics simulation was loaded at the position with the maximum cam pressure angle. The final displacement results on the cam are shown in Figure 17a, and the equivalent Von Mises stress analysis results are shown in Figure 17b.

The results showed that the maximum Von Mises stress on the cam occurred at the maximum contact point of the pressure angle between the roller and the cam, with the maximum stress of $\sigma = 3.08 \times 10^{-2} \text{ MPa}$ and the maximum displacement of $x = 2.91 \times 10^{-4} \text{ mm}$. The deformation and stress values of the cam were minimal, which did not cause serious failure, indicating that the cam path was smooth. There were no mutation points, indicating the accuracy of the proposed path fitting method.

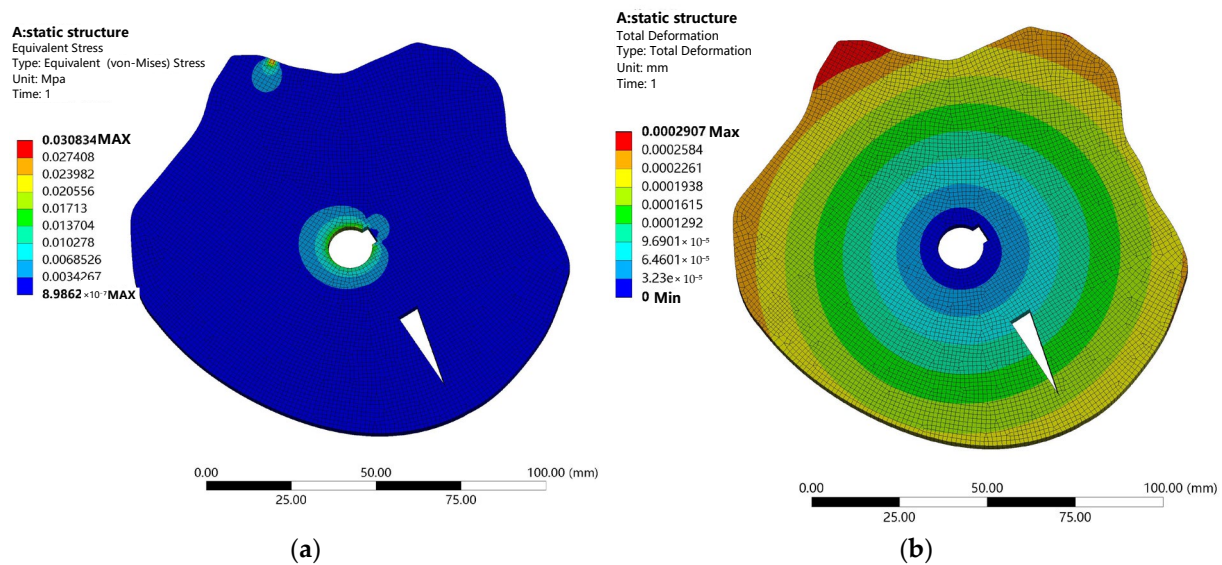


Figure 17. Finite element analysis.

4.4. Experimental Verification

In order to verify the accuracy and reliability of the design, it is necessary to conduct a physical verification. After fabrication and assembly in accordance with the design parameters, the designed cam is positioned to the same position as the simulation.

Additionally, in order to verify the influence of speed on the driving result of the car, two speeds are set for the test. The first is low speed, achieved by using PWM to control the motor speed, ensuring that the motor reaches 10 RPM, which corresponds to the simulation speed. The second is high speed, achieved by applying the rated voltage to the motor, enabling it to reach its maximum speed of 60 revolutions per minute.

The upper angle displacement sensor is connected to the front axle of the car. The model is the Omron rotary encoder E6B2-CWZ6CCWZ5BCWZ1X 100–2500 P/R incremental encoder. The pulse per turn is 2000, which can be converted into angle output with high precision. The laser displacement sensor, model mti instruments LTS-025-02, has a measuring range of 2 mm, dynamic resolution of 0.12 μm , and sampling period of 200 μs . The data collected by the STM32F103C8T6 are calculated based on the information returned by the sensor. The front wheel angle and push rod displacement data are returned to the PC through the serial port. After a cycle of data collection is completed, the calculated data are imported into MATLAB to draw the push rod push and front wheel angle curves and compare them with the designed parameters. The construction of experimental equipment is shown in Figure 18.

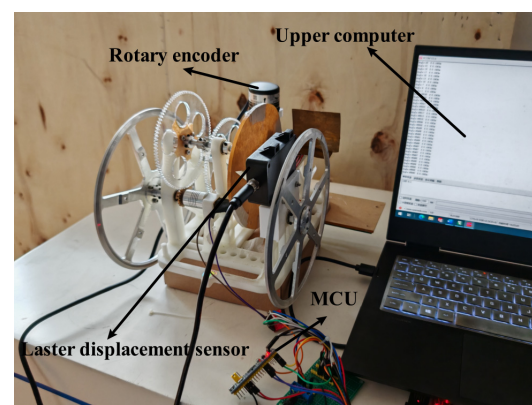


Figure 18. Experimental setup.

After processing the data using MATLAB, the angle of the front wheel of the trolley is shown in Figure 19, and the displacement of the push rod is shown in Figure 20. Through comparative analysis of the experimental results and the theoretical simulation design results, it can be seen from the chart that the actual motion trend of the cam is basically consistent with the design, regardless of whether it is at low speed or high speed, and the deviation is small. This indicates that the cam design is reasonable, and the cam motion trajectory is significantly affected by the speed. However, when running at full speed at rated voltage, the stability of the car is poorer, and the rotation direction is all biased towards the same direction. This is because the driving wheel and the driven wheel of the car are independent. When the car is running at high speed, the driven wheel is more likely to slip relative to the ground, resulting in a trajectory offset. Therefore, the car should be driven under low-speed conditions to ensure more accuracy and stability through the checkpoint. And during operation, the low-speed car can start smoothly and drive more accurately according to the designated route, which verifies the accuracy and reliability of the design.

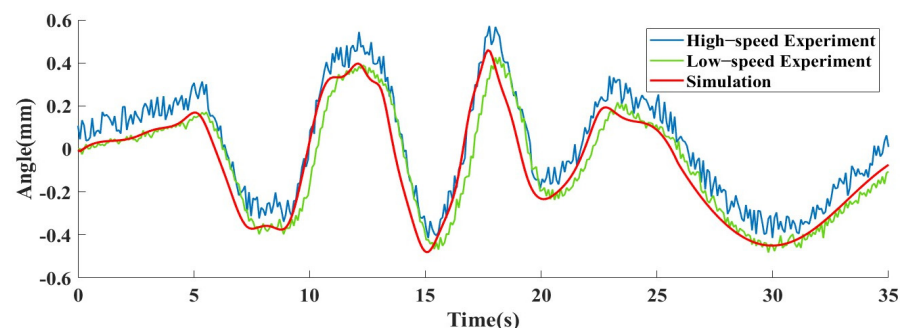


Figure 19. Comparison of front wheel angle results.

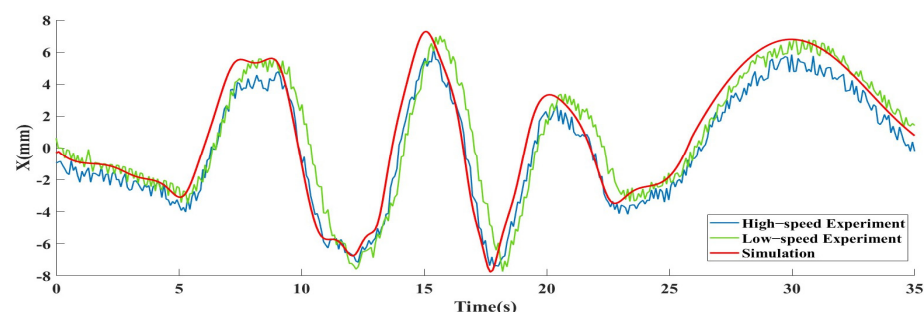


Figure 20. Comparison of push rod displacement results.

5. Conclusions

In this paper, the method of drawing B-spline curves using UG is discussed, and the theoretical contour of the cam is analyzed through an analytical method. The theoretical cam contour is solved using MATLAB programming, and the actual cam contour is drawn through completing and enveloping algorithms. Using this method for cam design can avoid the long time to fit the route, resulting in high efficiency and accuracy. After simulation using ADAMS and ANSYS, the rationality of the design is verified. By analyzing the results of the final operation and the simulation outcomes, the similarity between the front wheel angle and the push rod displacement curve in actual operation can reach 90%. Finally, experimental verification demonstrates that the cam designed with this method closely resembles the intended design and can accurately fulfill the requirements of punching through the punching point, significantly reducing the time for cam design, thus verifying the reliability of this method. The research findings can provide theoretical support for the structural design of solar-powered punch card cars and the design of arbitrary path cams.

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Nomenclature

DC	Direct Current
NURBS	Non-Uniform Rational B-Splines
NTC	Negative Temperature Coefficient
PPMA	Polymethyl Methacrylate

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