

Article

A Novel Underlying Algorithm for Reducing Uncertainty in Process Industry Risk Assessment

Yuanyuan Zhang  and Long Zhao *

School of Environmental and Safety Engineering, Liaoning Petrochemical University, Fushun 113001, China; zhangyuanyuan@lnpu.edu.cn

* Correspondence: zhaolong@lnpu.edu.cn; Tel.: +86-13904934222

Abstract: Normal fuzzy fault tree is a classic model in the field of process industry risk assessment, and it can provide reliable prior knowledge for machine learning. However, it is difficult to adapt the traditional approximate calculation method to highly nonlinear problems, and this may introduce model uncertainty. To solve this problem, this study proposes an accurate calculation algorithm. In the proposed algorithm, first, an exact α -cut set of normal fuzzy fault tree is derived according to the exact calculation formula of normal fuzzy numbers and in combination with the cut-set theorem. Subsequently, the relationship between the membership function and the exact cut set is derived based on the representation theorem. Finally, according to the previous derivation, the coordinates of the point on the exact membership curve are found within the range of x from 0 to 1. Based on this, an accurate membership graph is drawn, the membership curve is evenly divided with the area enclosed by the x -axis, and the fuzzy median is obtained. The results of the two chemical accident cases demonstrate that the proposed algorithm has a strong ability to handle uncertainty and can significantly reduce the uncertainty of the process industry risk assessment results. The results also reveal that the superiority of the accurate calculation algorithms becomes more obvious when the mean failure probability of basic events is larger or the accident tree is more complex. This study provides a high-accuracy underlying algorithm for process industry risk assessment, and it is of great value for improving system security.

Keywords: uncertainty; process industry risk assessment; normal fuzzy fault tree; the underlying algorithm; system safety



Citation: Zhang, Y.; Zhao, L. A Novel Underlying Algorithm for Reducing Uncertainty in Process Industry Risk Assessment. *Processes* **2024**, *12*, 1292. <https://doi.org/10.3390/pr12071292>

Academic Editor: Carlos Sierra Fernández

Received: 23 May 2024

Revised: 12 June 2024

Accepted: 19 June 2024

Published: 21 June 2024



Copyright: © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

The advent of the artificial intelligence era has promoted the development of process industry risk assessment [1,2]. As a classic method in the field of process industry risk assessment, the fuzzy fault tree has been widely applied in various fields [3,4].

However, Yazdi et al. [5] pointed out that the existence of some factors leads to uncertainty in fuzzy fault tree risk assessment results, including the complexity of the object being evaluated, the insufficiency of the required data, the hesitation factors in the decision-making process, and the approximations and simplifications in the modeling process. Furthermore, uncertainties are transmitted from the lowest level of the fault tree to the highest level of the fault tree, seriously affecting the credibility of the fuzzy fault tree risk assessment results. Therefore, uncertainty has become a key issue that needs to be discussed in the fuzzy fault tree risk assessment process [6].

In order to better handle uncertainty in the case of insufficient failure data, the Bayesian theorem is widely used as an analytical tool to update probability distributions. Soltanali et al. [7] established a fuzzy fault tree–Bayesian model to optimize complex automobile manufacturing equipment and improve the accuracy of prediction based on two types of uncertainty factors: insufficient data and system complexity. Kaushik et al. [8] further proposed a similarity aggregation method based on intuitionistic fuzzy fault tree

and Bayesian network, which promptly update the failure probability of new evidence. Zhao et al. [9] used fuzzy sets to calculate the probability of basic events and transformed the fault tree into a Bayesian network using Noisy-OR gate, solving the problem of the absolute description of the relationship between basic events and intermediate events. Ding et al. [10] combined FMEA, FTA, and Bayesian analysis to fill the gap in risk assessments of preheating and exclusion systems under uncertain conditions, expressing the polymorphism of events and the uncertainty logic relationship between events. The combination of fuzzy fault tree and Bayesian network can allow for the calculation of the prior probability of each failure based on historical data, thus being able to handle objective uncertainty.

To resolve the uncertainty of information input in fuzzy fault tree analysis, evidence theory and expert opinion are also good choices. Pang et al. [11] analyzed the uncertainty of systems based on fuzzy fault trees and evidence theory, overcoming the uncertainty of imprecise evidence. Zaib et al. [12] used expert opinions to address the uncertainty of human error events.

However, while these obtain the probability of basic events, uncertainty related to knowledge is likely to occur. Therefore, the subjectivity of expert opinions has become an inevitable issue in the handling of uncertainty in fuzzy fault trees. Celik et al. [13] used fuzzy set theory to address the subjective uncertainty in expert heuristic processes, emphasizing that the use of linguistic terms is an appropriate choice for dealing with the uncertainty of basic event probabilities. Purba et al. [14] used the FPFTA method to quantify the uncertainty of experts' overall understanding.

For some complex systems, experts often cannot provide accurate membership functions for system component failures. In order to overcome the subjective uncertainty of experts in fuzzy fault tree risk assessment, Kaushik and Kumar [15] developed a new IF importance measurement method that effectively measures the uncertainty of basic events by providing cut intervals in an α -intuitionistic fuzzy environment. Hu et al. [16] considered the hesitant factors in uncertainty quantification and proposed an IF importance measurement method that is beneficial for identifying key basic events. Lu et al. [17] proposed an improved similarity aggregation method based on the butterfly optimization algorithm to address the uncertainty problem of basic event failure probability in three-dimensional EVAC systems and improve the accuracy of aggregation results. Garg et al. [18] proposed a fuzzy risk assessment method based on the granularity Z-number to improve the utilization of information and the degree of uncertainty in management decisions. Feng et al. [19] proposed a fuzzy importance sampling method to improve the computational efficiency of system failures with fuzzy uncertainty.

From the above analysis, it can be seen that the fuzzy fault tree analysis method produces two uncertainties in the process of outputting risk assessment results [20]: one is the uncertainty of the input quantity (parameter), which is related to the true value of the input x , and the other is structural model uncertainty, that is uncertainty related to model errors, which is related to fuzzy fault tree membership models and approximate calculations.

The above literature solves the uncertainty of information input in the fuzzy fault tree model from different perspectives. To a certain extent, the uncertainty of expert understanding in fuzzy fault tree risk assessment is reduced, and the uncertainty of model input parameters in the process of fuzzy fault tree risk assessment is improved.

However, there is currently little research on the uncertainty of the fuzzy fault tree's own structural model. The uncertainty of the fuzzy fault tree's own structural model stems from the gap between knowledge and practice. It may take the form of an insufficient understanding of phenomena that already occur in the system or the simplification of complex phenomena leading to "inappropriate" assumptions about the structure of the model.

The purpose of this study is to propose a new algorithm based on the cut-set theorem and the representation theorem, with the aim of solving the problems of difficulty in making accurate calculations and determining uncertainty in approximate calculations during the normal fuzzy fault tree risk assessment process. The formula derivation and

proof can provide theoretical support for the implementation of the algorithm. The method of drawing a precise membership graph can help to quickly obtain accurate risk assessment results. This will reduce the uncertainty of the process industry risk assessment and improve the accuracy of risk assessment. It will provide a high-precision underlying algorithm for process industry risk assessment.

The basic theory as well as Equations (A1)–(A12) are given in Appendix A, and the interested reader can read the literature [21–25].

2. Accurate Calculation Algorithms

To overcome the shortcomings of traditional approximate calculations, an accurate calculation algorithm is proposed in this section, and a theoretical proof process is provided.

2.1. Algorithm Principles

The principle of the accurate calculation algorithm is as follows: Firstly, convert the normal fuzzy numbers into classical sets based on the cut-set theorem. Secondly, discretize the normal fuzzy numbers based on the representation theorem, and draw accurate membership curves based on discrete data. Finally, calculate the area of the graph bounded by the membership curve and the x -axis, and further calculate the fuzzy normal median and fuzzy importance.

2.2. Theoretical Basis of the Algorithm

Normal fuzzy fault tree risk assessment is an effective method that can fully consider the opinions of multiple experts. Hence, m experts are invited to evaluate the possibility of basic event X_i , and then the failure probability Q_i (i.e., $Q_{i1}, Q_{i2}, \dots, Q_{im}$) of X_i is obtained. Based on this, the mean value of the failure probability is calculated as q_i , and the variance is σ_i . According to Equation (A1), the membership function $A_{Q_i}(x)$ of Q_i can be expressed as

$$A_{Q_i}(x) = \exp\left(-\left(\frac{x - q_i}{\sigma_i}\right)^2\right) \quad (1)$$

In production practice, the multiplication of failure probabilities represented by normal fuzzy numbers is too complicated, because the fault tree is complex and there are many basic events. Therefore, approximate calculation formulas are commonly used, as shown in Equation (2),

$$A_{\prod_{i=1}^n Q_i}(x) \approx \exp(-) \quad (2)$$

However, approximate calculations can lead to uncertainty in the assessment results.

Therefore, an accurate calculation algorithm is proposed in this paper. The derivation of the relevant formulas is carried out below in two cases:

2.2.1. A Normal Fuzzy Fault Tree with Only OR Gates

According to Equation (1), the normal fuzzy number $A_{1-Q_i}(x)$ of the probability of the non-occurrence of the basic event is obtained using

$$A_{1-Q_i}(x) = \exp\left(-\left(\frac{x - (1 - q_i)}{\sigma_i}\right)^2\right) \quad (3)$$

According to Equations (A2) and (3), let $A_{1-Q_i}(x) = \alpha$, and the α -cut set of $1 - Q_i$ is calculated as

$$(1 - Q_i)_\alpha = \left[1 - q_i - \sigma_i \times \sqrt{-\ln \alpha}, 1 - q_i + \sigma_i \times \sqrt{-\ln \alpha}\right] \quad (4)$$

Based on the fuzzy interval number operation rule, we have

$$(1 - Q_i)_\alpha \times (1 - Q_j)_\alpha = ((1 - Q_i) \times (1 - Q_j))_\alpha \quad (5)$$

According to Equations (4) and (5), the exact α -cut set is obtained for the probability of basic events not occurring simultaneously,

$$\left(\prod_{i=1}^n (1 - Q_i)\right)_\alpha = \left[\prod_{i=1}^n \left(1 - q_i - \sigma_i \times \sqrt{-\ln \alpha}\right), \prod_{i=1}^n \left(1 - q_i + \sigma_i \times \sqrt{-\ln \alpha}\right)\right] \quad (6)$$

Based on the concept of bounded closed fuzzy numbers and Equation (A5), it can be seen that the membership function in which n basic events do not occur at the same time can be expressed as

$$A_{\prod_{i=1}^n (1 - Q_i)}(x) = ([m_A, n_A], L_A(x), R_A(x)) \quad (7)$$

According to Equations (A6)–(A8), m_A and n_A are further derived

$$m_A = \lim_{n \rightarrow \infty} \left\{ \prod_{i=1}^n \left[\left(1 - q_i\right) - \sigma_i \times \sqrt{-\ln \left(1 - \frac{1}{n+1}\right)} \right] \right\} = \prod_{i=1}^n (1 - q_i) \quad (8)$$

$$n_A = \lim_{n \rightarrow \infty} \left\{ \prod_{i=1}^n \left[\left(1 - q_i\right) + \sigma_i \times \sqrt{-\ln \left(1 - \frac{1}{n+1}\right)} \right] \right\} = \prod_{i=1}^n (1 - q_i) \quad (9)$$

and then $L_A(x)$ and $R_A(x)$ are derived based on Equations (A9) and (A10).

$$L_A(x) = \bigvee_{0 < \alpha < 1} \left(\alpha \mid \left(\prod_{i=1}^n \left(1 - q_i - \sigma_i \times \sqrt{-\ln \alpha}\right) \right) \leq x \right) \quad (10)$$

$$R_A(x) = \bigwedge_{0 < \alpha < 1} \left(\alpha \mid \left(\prod_{i=1}^n \left(1 - q_i + \sigma_i \times \sqrt{-\ln \alpha}\right) \right) \geq x \right) \quad (11)$$

On the one hand, $\left(\prod_{i=1}^n \left(1 - q_i - \sigma_i \times \sqrt{-\ln \alpha}\right)\right)$ is a monotonically increasing function with α as the independent variable. The proof is as follows:

Because

$$\begin{aligned} & \frac{d\prod_{i=1}^n \left(1 - q_i - \sigma_i \times \sqrt{-\ln \alpha}\right)}{d\alpha} \\ &= \frac{\sigma_1}{2\alpha\sqrt{-\ln \alpha}} \frac{\prod_{i=1}^n \left(1 - q_i - \sigma_i \times \sqrt{-\ln \alpha}\right)}{1 - q_1 - \sigma_1 \times \sqrt{-\ln \alpha}} + \frac{\sigma_2}{2\alpha\sqrt{-\ln \alpha}} \frac{\prod_{i=1}^n \left(1 - q_i - \sigma_i \times \sqrt{-\ln \alpha}\right)}{1 - q_2 - \sigma_2 \times \sqrt{-\ln \alpha}} \\ & \quad + \dots + \frac{\sigma_n}{2\alpha\sqrt{-\ln \alpha}} \frac{\prod_{i=1}^n \left(1 - q_i - \sigma_i \times \sqrt{-\ln \alpha}\right)}{1 - q_n - \sigma_n \times \sqrt{-\ln \alpha}} \\ &= \frac{1}{2\alpha \times \sqrt{-\ln \alpha}} \sum_{i=1}^n \sigma_i \times \frac{\prod_{i=1}^n \left(1 - q_i - \sigma_i \times \sqrt{-\ln \alpha}\right)}{1 - q_i - \sigma_i \times \sqrt{-\ln \alpha}} \end{aligned}$$

we have

$$2\alpha > 0, \sqrt{-\ln \alpha} > 0, \sigma_i > 0, \prod_{i=1}^n \left(1 - q_i - \sigma_i \times \sqrt{-\ln \alpha}\right) > 0, \left(1 - q_i - \sigma_i \times \sqrt{-\ln \alpha}\right) > 0$$

calculated as

$$\frac{d\prod_{i=1}^n \left(1 - q_i - \sigma_i \times \sqrt{-\ln \alpha}\right)}{d\alpha} > 0$$

and thus, $\left(\prod_{i=1}^n \left(1 - q_i - \sigma_i \times \sqrt{-\ln \alpha}\right)\right)$ monotonically increases.

On the other hand, $\left(\prod_{i=1}^n \left(1 - q_i + \sigma_i \times \sqrt{-\ln \alpha}\right)\right)$ is a monotonically decreasing function with α as the independent variable. We prove that

$$\begin{aligned}
& \frac{d \prod_{i=1}^n (1 - q_i + \sigma_i \times \sqrt{-\ln \alpha})}{d\alpha} \\
&= - \left(\frac{\sigma_1}{2\alpha \sqrt{-\ln \alpha}} \frac{\prod_{i=1}^n (1 - q_i + \sigma_i \times \sqrt{-\ln \alpha})}{1 - q_1 + \sigma_1 \times \sqrt{-\ln \alpha}} + \frac{\sigma_2}{2\alpha \sqrt{-\ln \alpha}} \frac{\prod_{i=1}^n (1 - q_i + \sigma_i \times \sqrt{-\ln \alpha})}{1 - q_2 + \sigma_2 \times \sqrt{-\ln \alpha}} \right. \\
&\quad \left. + \cdots + \frac{\sigma_n}{2\alpha \sqrt{-\ln \alpha}} \frac{\prod_{i=1}^n (1 - q_i + \sigma_i \times \sqrt{-\ln \alpha})}{1 - q_n + \sigma_n \times \sqrt{-\ln \alpha}} \right) \\
&= - \left(\frac{1}{2\alpha \times \sqrt{-\ln \alpha}} \sum_{i=1}^n \sigma_i \times \frac{\prod_{i=1}^n (1 - q_i + \sigma_i \times \sqrt{-\ln \alpha})}{1 - q_i + \sigma_i \times \sqrt{-\ln \alpha}} \right)
\end{aligned}$$

Here,

$$2\alpha > 0, \sqrt{-\ln \alpha} > 0; \sigma_i > 0, \prod_{i=1}^n (1 - q_i + \sigma_i \times \sqrt{-\ln \alpha}) > 0; (1 - q_i + \sigma_i \times \sqrt{-\ln \alpha}) > 0$$

therefore

$$\frac{d \prod_{i=1}^n (1 - q_i + \sigma_i \times \sqrt{-\ln \alpha})}{d\alpha} < 0$$

and $\prod_{i=1}^n (1 - q_i + \sigma_i \times \sqrt{-\ln \alpha})$ monotonically decreases.

Thus, according to Equations (10) and (11), combined with the monotonicity of $\left(\prod_{i=1}^n (1 - q_i - \sigma_i \times \sqrt{-\ln \alpha}) \right)$ and $\prod_{i=1}^n (1 - q_i + \sigma_i \times \sqrt{-\ln \alpha})$, $A_{\prod_{i=1}^n (1-Q_i)}(x)$ is discretized, and based on this, an accurate membership curve is drawn.

2.2.2. A Normal Fuzzy Fault Tree with OR and AND Gates

Using Formula (A12) and the cut-set theorem, the exact cut set calculation formulas for $L_A'(x)$ and $R_A'(x)$ are derived. The derivation results are as follows:

The formula for the cut set of membership function $L_A'(x)$:

$$\sum_{k=1}^{n_G} \prod_{X_i \in G_k} (q_i - \sigma_i \times \sqrt{-\ln \alpha}) \quad (12)$$

The formula for the cut set of membership function $R_A'(x)$:

$$\sum_{k=1}^{n_G} \prod_{X_i \in G_k} (q_i + \sigma_i \times \sqrt{-\ln \alpha}) \quad (13)$$

The proof of the monotonic increasing property of the cut set of $L_A'(x)$ is as follows:
Because

$$\frac{d \prod_{X_i \in G_k} (q_i - \sigma_i \times \sqrt{-\ln \alpha})}{d\alpha} > 0$$

thus $\sum_{k=1}^{n_G} \prod_{X_i \in G_k} (q_i - \sigma_i \times \sqrt{-\ln \alpha})$ increases monotonically.

The proof of the monotonic decreasing property of the cut set of $R_A'(x)$ is as follows:
Because

$$\frac{d \prod_{X_i \in G_k} (q_i + \sigma_i \times \sqrt{-\ln \alpha})}{d\alpha} < 0$$

we have

$$\frac{d \sum_{k=1}^{n_G} \prod_{X_i \in G_k} (q_i + \sigma_i \times \sqrt{-\ln \alpha})}{d\alpha} < 0$$

thus, $\sum_{k=1}^{n_G} \prod_{X_i \in G_k} (q_i + \sigma_i \times \sqrt{-\ln \alpha})$ decreases monotonically.

Secondly, based on the derived Equations (14) and (15), as well as the monotonicity of the cut set, we obtain the coordinates of points on the exact membership curve.

$$L_A'(x) = \bigvee_{0 < \alpha < 1} \left(\alpha \left| \sum_{k=1}^{n_G} \prod_{X_i \in G_k} (q_i - \sigma_i \times \sqrt{-\ln \alpha}) \right| \leq x \right) \quad (14)$$

$$R_A'(x) = \bigvee_{0 < \alpha < 1} \left(\alpha \left| \sum_{k=1}^{n_G} \prod_{X_i \in G_k} (q_i + \sigma_i \times \sqrt{-\ln \alpha}) \right| \geq x \right) \quad (15)$$

2.3. Algorithm Execution Process

The specific execution process of the algorithm is shown in Figures 1 and 2.

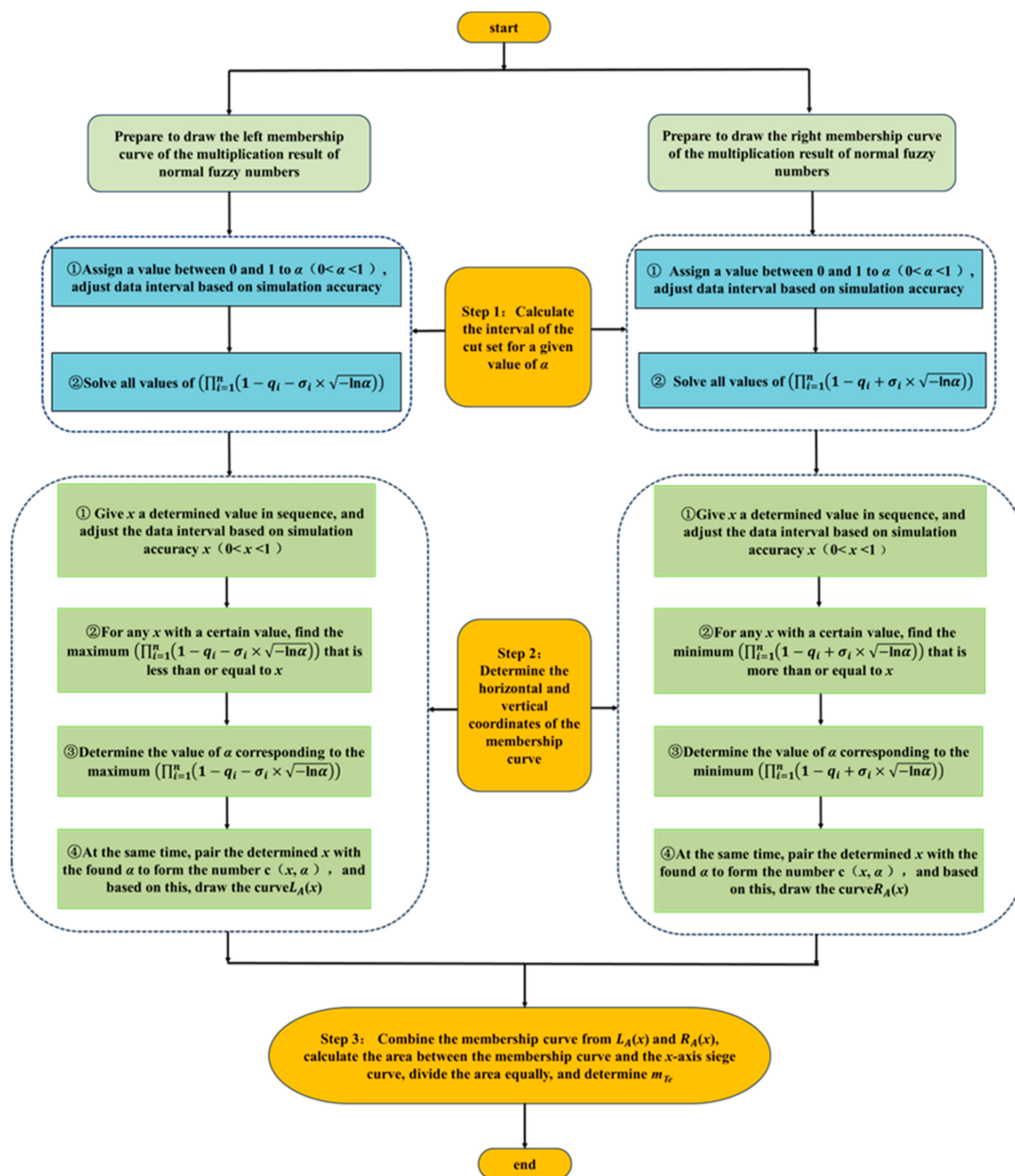


Figure 1. Accurate calculation algorithm for normal fuzzy fault tree (only including OR gates).

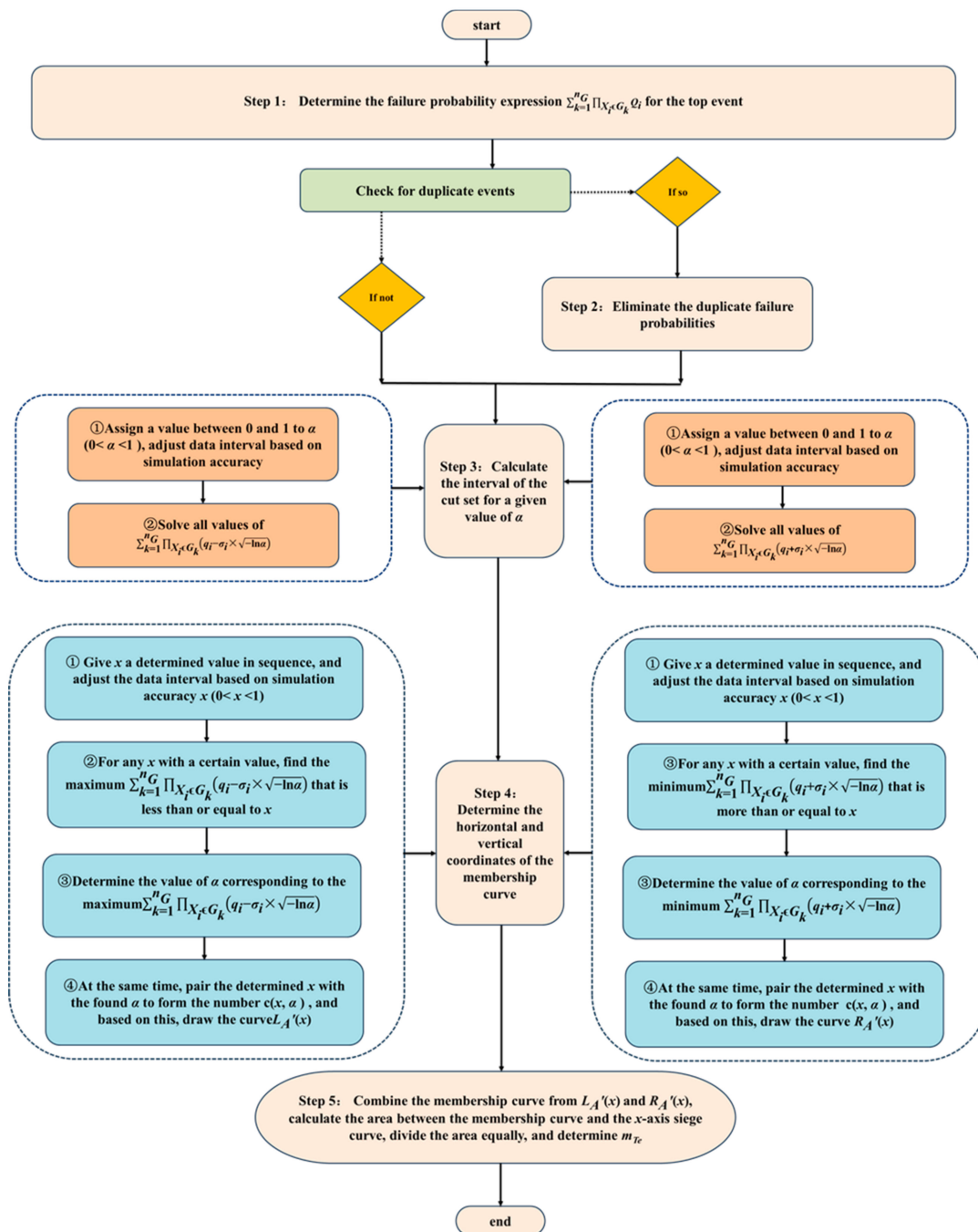


Figure 2. Accurate calculation algorithm for normal fuzzy fault tree (inclusion of OR gate and AND gate).

The algorithm is mainly divided into three steps. The first step is to calculate the cut-set interval under a given α . The second step is to discretize the result of multiplying the normal fuzzy numbers according to the cut-set interval and determine the ordered arrays. The third step is to draw a membership curve based on the ordered arrays and determine the fuzzy median value based on the area bounded by the membership curve and the x -axis.

Firstly, the algorithm in Figure 2 needs to compute the expression for the failure probability of the top event. Secondly, if there are duplicate probabilities in the probability expression, then the duplicate probabilities need to be eliminated. Thirdly, an accurate cut-set expression for the probability of the top event needs to be expressed. The remaining steps, shown in Figures 1 and 2, are essentially the same.

The new algorithm adopts a data equal-width discretization method. It is necessary to reasonably select the number of discretization intervals based on the value ranges of α and x . This can avoid information loss or redundancy due to too few or too many intervals.

3. Empirical Analysis

To verify the effectiveness and feasibility of the algorithm, this study selects two types of fault trees for analysis, with the top events being electrostatic accumulation and liquid ammonia leakage. The failure probability of basic events in the fault tree is scored by experienced experts in the chemical industry.

This section includes the following content: Firstly, the execution process of expert scoring is introduced. Then, after obtaining the expert scoring results, the normal characteristics of the failure probability of basic events are verified, and the mean and variance of basic events are obtained. Finally, a new algorithm and approximate calculation method are used to draw a membership curve of the failure probability of top events and intermediate events; following this, the risk assessment results are compared and analyzed.

3.1. Expert Introduction and Scoring Rules

The introduction of experts is shown in Table 1, and the scoring rules for experts are shown in Table 2.

Table 1. Expert introduction.

Expert	Major	Professional Title	Educational Background	Affiliation	Working Hours
1	safety engineering	professor	doctor	university	20 years
2	chemical safety engineering	associate professor of engineering	doctor	enterprise	22 years
3	chemical safety engineering	professor of engineering	doctor	enterprise	35 years
4	safety technology and engineering	associate professor of engineering	doctor	enterprise	14 years
5	safety engineering	senior engineer	master	enterprise	13 years
6	safety technology and engineering	senior engineer	doctor	enterprise	10 years

Table 2. Expert scoring rules.

Failure Probability	Possibility of Occurrence
$\alpha \times 10^{-1}$	very easy to occur
$\alpha \times 10^{-2}$	easy to occur
$\alpha \times 10^{-3}$	not easy to occur
$\alpha \times 10^{-4}$	more difficult to occur
$\alpha \times 10^{-5}$	extremely difficult to occur
$\alpha \times 10^{-6}$	almost never happens

In Table 2, α represents the membership degree of danger at the same probability level. The value of α is shown in Table 3.

Table 3. The value and meaning of α .

Evaluation Grades	Value
Very low	[1,2]
Low	(2,4]
Medium	(4,6]
High	(6,8]
Very High	(8,10]

Note that Tables 2 and 3 must be used jointly.

3.2. Empirical Case 1

3.2.1. Case 1 Introduction

Case 1 selects electrostatic accumulation during oil storage as the top event for analysis [26] and analyses the electrostatic accumulation due to the relative movement of oil flow, filtration, mixing, spraying, sprinkling, flushing, perfusion, shaking, etc. The fault tree has three layers, including two intermediate events and eight basic events, as shown in Figure 3. Based on this, considering the fire source factor, an accident tree with an electrostatic spark explosion as the top event is drawn, as shown in Figure 4.

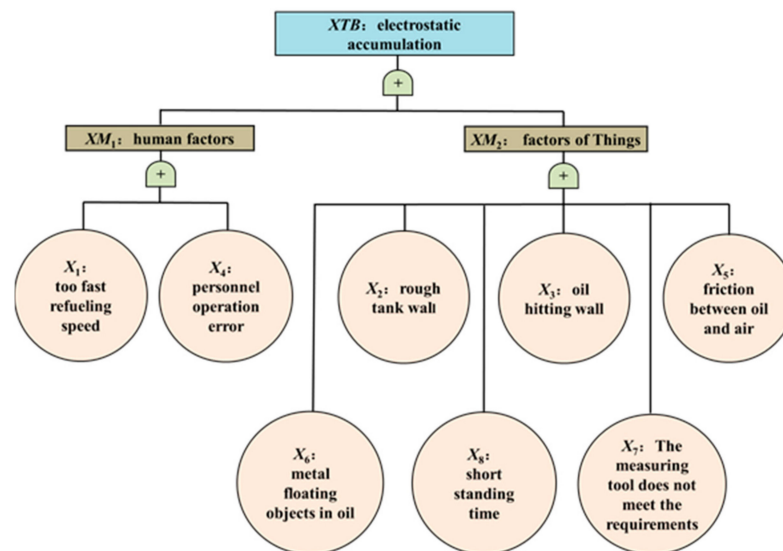


Figure 3. Fault tree of electrostatic accumulation.

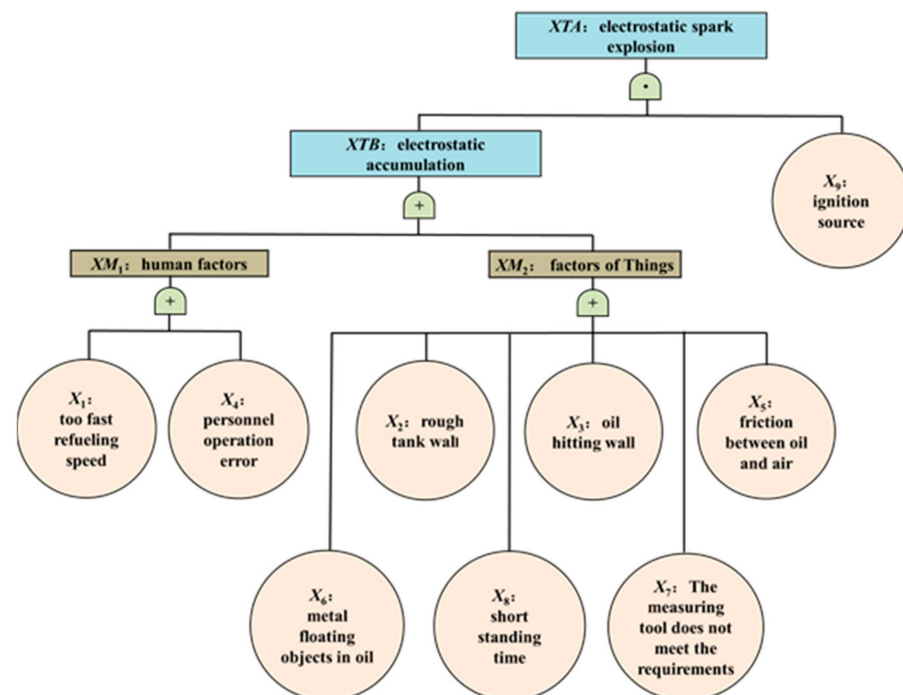


Figure 4. Fault tree of electrostatic spark explosion.

The nine basic events are as follows: X_1 —too fast refueling speed; X_2 —rough tank wall; X_3 —oil hitting wall; X_4 —personnel operation error; X_5 —friction between oil and air;

X_6 —metal floating objects in oil; X_7 —the measuring tool does not meet the requirements; X_8 —short standing time; and X_9 —ignition source.

3.2.2. Expert Opinions on Case 1

Based on Tables 2 and 3, six experts evaluated the possibility of basic events, and the results are shown in Tables 4 and 5.

Table 4. Expert opinions of Case 1 (a).

Basic Events	Expert 1		Expert 2		Expert 3	
	Possibility of Occurrence	Possibility Level and Value	Possibility of Occurrence	Possibility Level and Value	Possibility of Occurrence	Possibility Level and Value
X_1	very easy to occur	VL, 1	very easy to occur	L, 3	very easy to occur	VL, 1.5
X_2	not easy to occur	VL, 1.5	not easy to occur	L, 3.5	not easy to occur	VH, 9
X_3	very easy to occur	M, 5	very easy to occur	M, 4.5	very easy to occur	M, 6
X_4	very easy to occur	VL, 2	very easy to occur	L, 2.5	very easy to occur	H, 7.5
X_5	very easy to occur	VL, 2	very easy to occur	H, 8	very easy to occur	L, 2.5
X_6	not easy to occur	VL, 1.5	not easy to occur	VL, 1	not easy to occur	VH, 9
X_7	easy to occur	VL, 1	easy to occur	L, 2.5	easy to occur	L, 4
X_8	very easy to occur	M, 6	very easy to occur	VL, 1.5	very easy to occur	M, 5.5
X_9	very easy to occur	VL, 1	very easy to occur	VL, 1	easy to occur	VH, 9

Table 5. Expert opinions of Case 1 (b).

Basic Events	Expert 4		Expert 5		Expert 6	
	Possibility of Occurrence	Possibility Level and Value	Possibility of Occurrence	Possibility Level and Value	Possibility of Occurrence	Possibility Level and Value
X_1	very easy to occur	VL, 2	very easy to occur	L, 3.5	very easy to occur	L, 4
X_2	not easy to occur	H, 7	not easy to occur	VL, 1	not easy to occur	VL, 1
X_3	very easy to occur	H, 7.5	very easy to occur	H, 6.5	very easy to occur	M, 6
X_4	very easy to occur	L, 3	very easy to occur	H, 6.5	very easy to occur	L, 2.5
X_5	very easy to occur	H, 7.5	very easy to occur	VL, 1.5	very easy to occur	H, 8.5
X_6	not easy to occur	H, 8	not easy to occur	VL, 1	not easy to occur	VL, 1
X_7	easy to occur	VL, 1	easy to occur	L, 3.5	easy to occur	VL, 2
X_8	very easy to occur	M, 4.5	very easy to occur	L, 2.5	very easy to occur	VH, 8.5
X_9	easy to occur	VH, 9	very easy to occur	VL, 1	easy to occur	VH, 9

3.2.3. Case 1: Probability of Basic Events

According to Tables 2, 4 and 5, the failure probability of the basic event is obtained, as shown in Table 6.

Table 6. The failure probability of basic events.

Basic Events	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5	Expert 6
X_1	1×10^{-1}	3×10^{-1}	1.5×10^{-1}	2×10^{-1}	3.5×10^{-1}	4×10^{-1}
X_2	1.5×10^{-3}	3.5×10^{-3}	9×10^{-3}	7×10^{-3}	1×10^{-3}	1×10^{-3}
X_3	5×10^{-1}	4.5×10^{-1}	6×10^{-1}	7.5×10^{-1}	6.5×10^{-1}	6×10^{-1}
X_4	2×10^{-1}	2.5×10^{-1}	7.5×10^{-1}	3×10^{-1}	6.5×10^{-1}	2.5×10^{-1}
X_5	2×10^{-1}	8×10^{-1}	2.5×10^{-1}	7.5×10^{-1}	1.5×10^{-1}	8.5×10^{-1}
X_6	1.5×10^{-3}	1×10^{-3}	9×10^{-3}	8×10^{-3}	1×10^{-3}	1×10^{-3}
X_7	1×10^{-2}	2.5×10^{-2}	4×10^{-2}	1×10^{-2}	3.5×10^{-2}	2×10^{-2}
X_8	6×10^{-1}	1.5×10^{-1}	5.5×10^{-1}	4.5×10^{-1}	2.5×10^{-1}	8.5×10^{-1}
X_9	1×10^{-1}	1×10^{-1}	9×10^{-2}	9×10^{-2}	1×10^{-1}	9×10^{-2}

3.2.4. Case 1: Normal Distribution Characteristic Test of Raw Data

After obtaining the failure probability of the basic event in Case 1, the K–S (Kolmogorov–Smirnov) test is performed. It is checked whether the probability of failure conforms to a normal distribution, and the test results are shown in Table 7.

Table 7. K–S calculation results of Case 1.

Basic Events	Mean Value (q_i)	Variance (σ_i)	Accurate and Significant Consistency
X_1	0.2500	0.1183	0.988
X_2	0.0038	0.0034	0.758
X_3	0.5917	0.1069	0.937
X_4	0.4000	0.2366	0.434
X_5	0.5000	0.3317	0.666
X_6	0.0036	0.0038	0.294
X_7	0.0233	0.0125	0.953
X_8	0.4750	0.2525	0.997
X_9	0.095	0.00547	0.476

In Table 7, it can be seen that the exact significant consistency of X_i ($i = 1\sim 9$) is greater than 0.05. So, the assumption that the probability of all basic events occurring follows a normal distribution is accepted at a significance level of 0.05.

3.2.5. Case 1: Drawing of Membership Curve

Based on the new algorithm (Figures 1 and 2), a failure probability membership curve of the top event is drawn considering the following situations: all events occur; except X_i ($i = 1\sim 9$), all events occur; X_1 and X_4 occur simultaneously; and X_2 , X_3 , X_5 , X_6 , X_7 , and X_8 occur simultaneously. Based on the membership curve, m_{TX1e} , m_{TX2e} , m_{TX3e} , m_{TX4e} , m_{TX5e} , m_{TX6e} , m_{TX7e} , m_{TX8e} , m_{TX9e} , and m_{TX10e} are determined. The drawing results are shown in Figures 5–16.

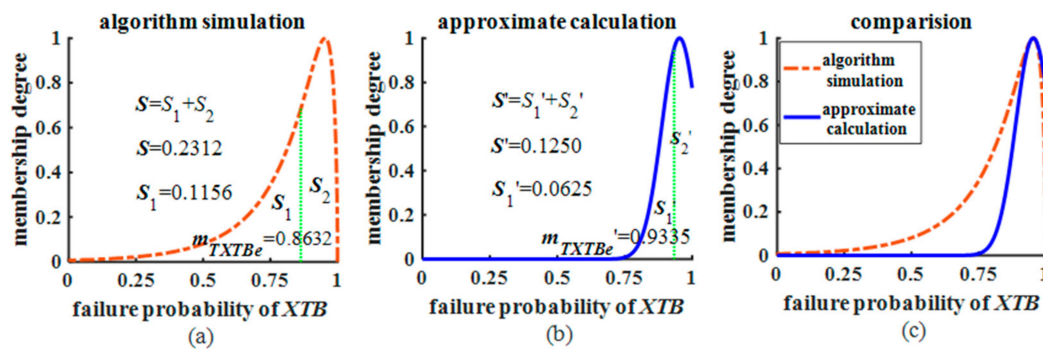


Figure 5. All basic events occurred (the top event is XTB).

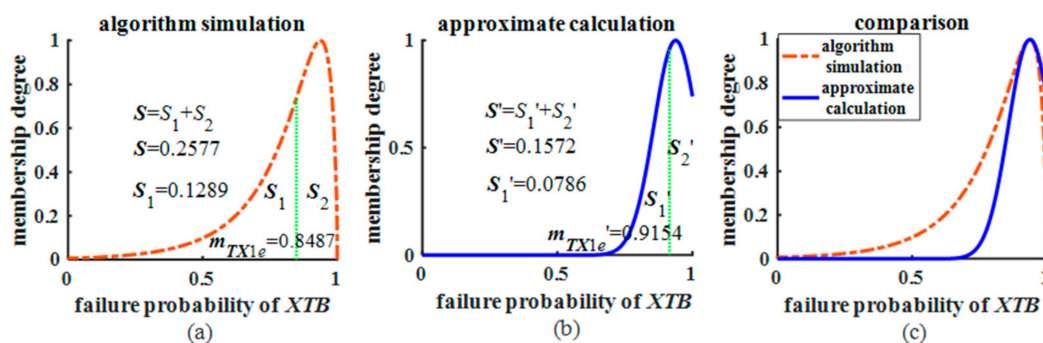
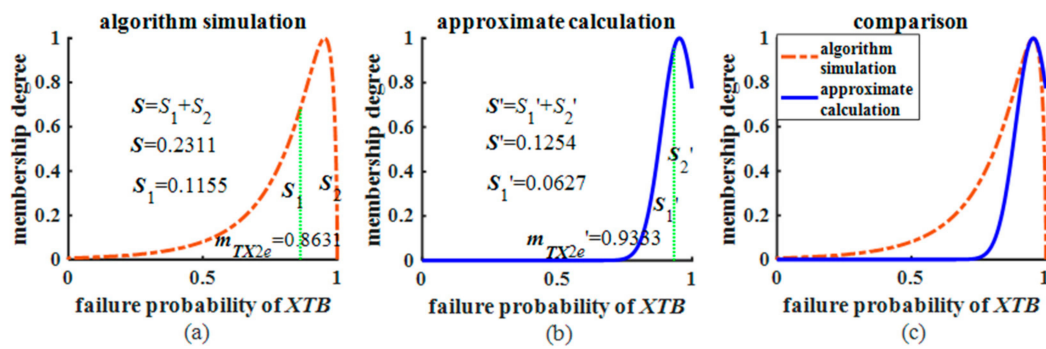
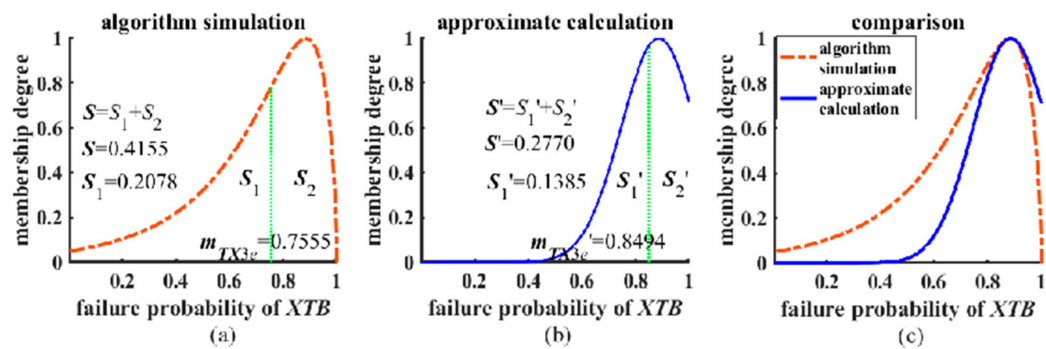
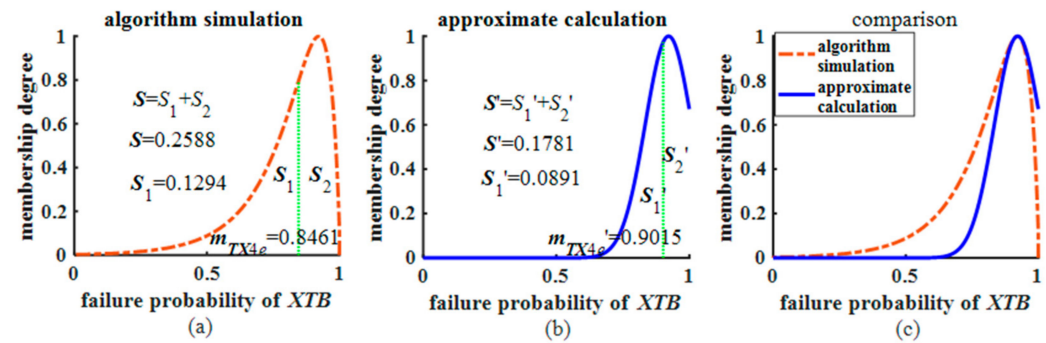
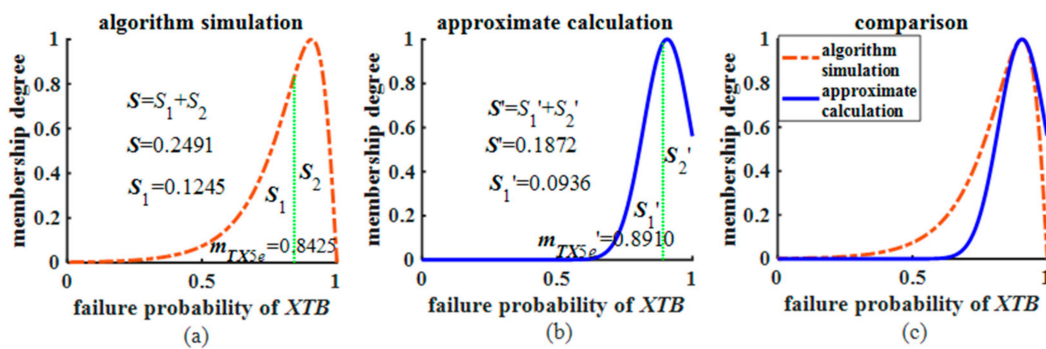


Figure 6. Except for X_1 , all basic events occurred (the top event is XTB).

Figure 7. Except for X_2 , all basic events occurred (the top event is XTB).Figure 8. Except for X_3 , all basic events occurred (the top event is XTB).Figure 9. Except for X_4 , all basic events occurred (the top event is XTB).Figure 10. Except for X_5 , all basic events occurred (the top event is XTB).

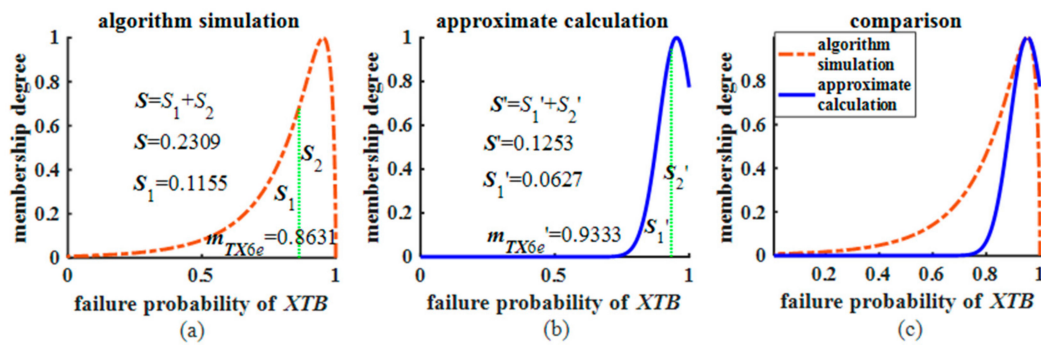


Figure 11. Except for X_6 , all basic events occurred (the top event is XTB).

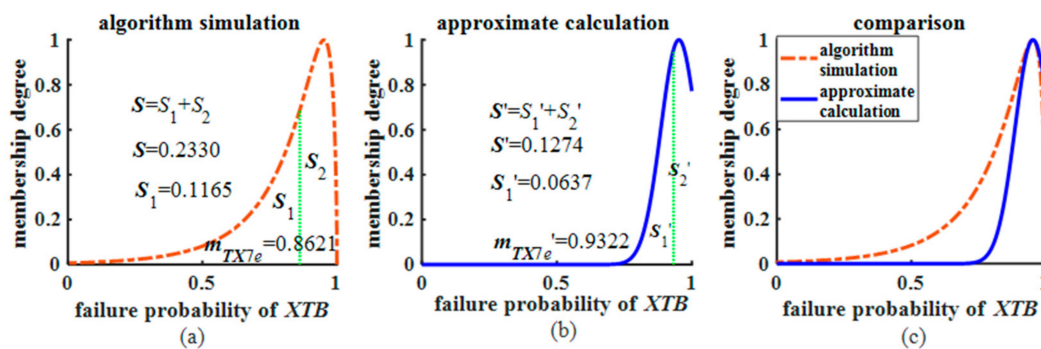


Figure 12. Except for X_7 , all basic events occurred (the top event is XTB).

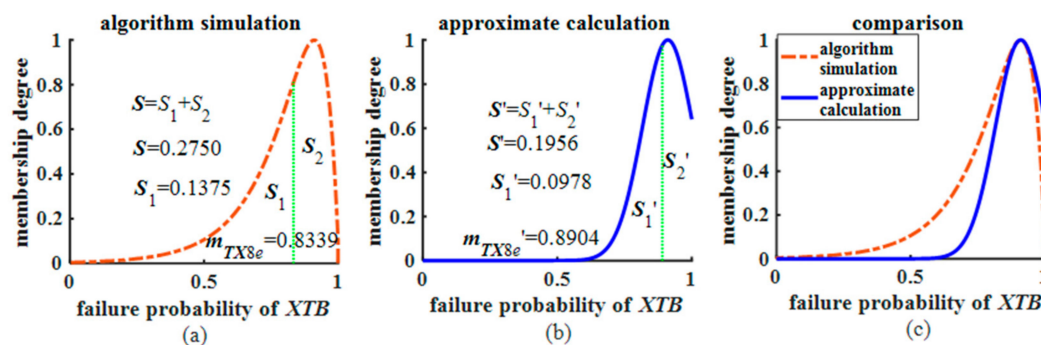


Figure 13. Except for X_8 , all basic events occurred (the top event is XTB).

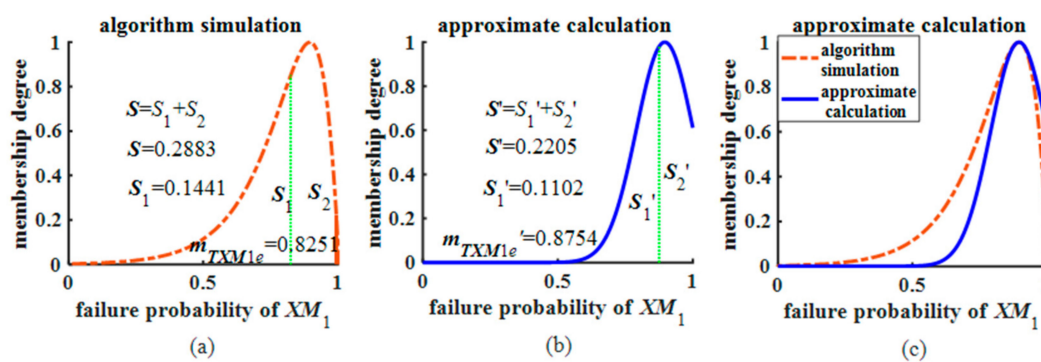


Figure 14. X_2 , X_3 , X_5 , X_6 , X_7 , and X_8 occurred simultaneously (XM_1 is the top event).

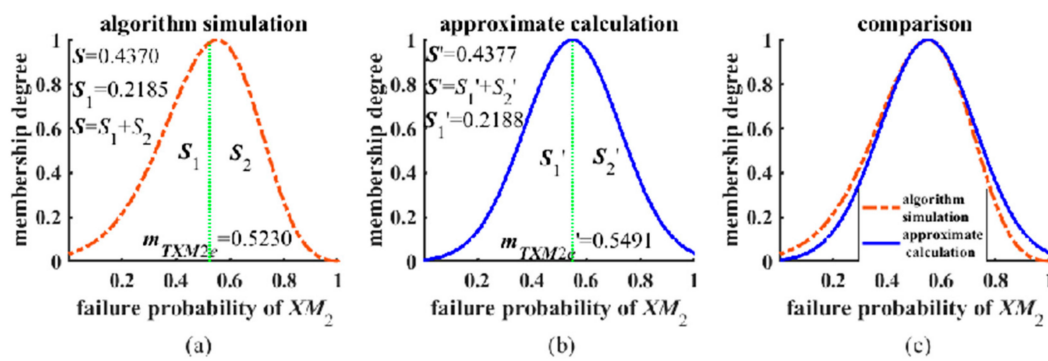


Figure 15. X_1 and X_4 occurred simultaneously (XM_2 is the top event).

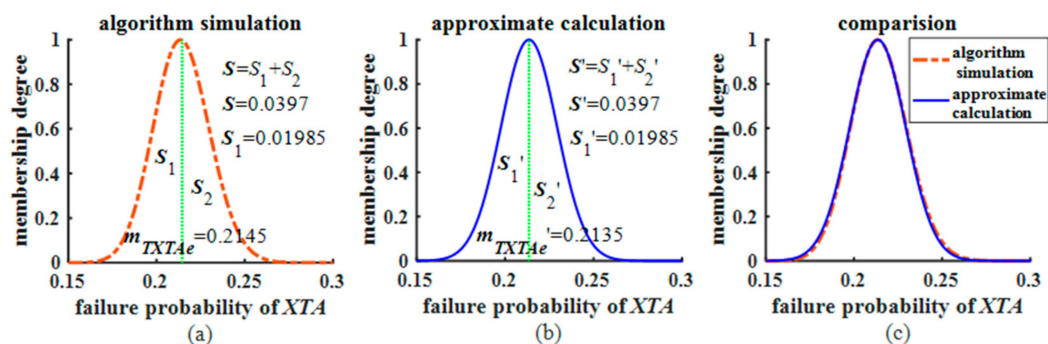


Figure 16. All basic events occurred (the top event is XTA).

Subfigure (a) is a graph of the membership curves obtained by simulation using the new algorithm; subfigure (b) is a graph of the membership curves obtained by the traditional approximation calculation; and subfigure (c) is a comparison of the results obtained by the two calculation methods. The meanings of the letters in the subfigures refer to the explanations in Figure A1 in the Appendix A.4.

3.2.6. Case 1: Discussion and Analysis

According to Figures 5–16, firstly, the fuzzy median value of the accurate algorithm simulation and approximate calculation is obtained. Secondly, the absolute uncertainty reduction value of the fuzzy median value is computed. Finally, the relative uncertainty reduction value is calculated. The results are shown in Table 8.

It can be seen in Table 8 that, after the simulation using accurate calculation algorithms, the uncertainty of both intermediate events and top events is reduced. The uncertainty reduction values of XM_1 and XM_2 are 4.75% and 5.75%, respectively. The uncertainty reduction value of the top event (electrostatic accumulation) is 7.53%. Considering all cases, the uncertainty reduction value is between 4.75% and 11.05%.

The above data illustrate that, by using approximate calculations, system uncertainty can be greatly propagated. After using the new algorithm, this uncertainty is significantly reduced. The uncertainty reduction value significantly increases from the intermediate layer to the top layer of the fault tree.

When the number of basic events occurring simultaneously is 2, 6, 7, or 8, the uncertainty of the fuzzy median value is significantly reduced. This conclusion can be reached by observing the fuzzy median of the top events and the intermediate events.

From the results, it can be seen that, in this case, the fault tree containing mixed gates has lower uncertainty than the accident tree containing only OR gates. This is because, after the transformation with AND gates, the number of elementary events that are multiplied becomes much less, and the probability of elementary events that are multiplied at the same time becomes much smaller.

Table 8. Fuzzy median of membership function (Case 1).

Status of Events	Accurate Calculation Algorithm (Improved)	Approximation Calculation (Traditional)	Absolute Uncertainty Reduction Value	Relative Uncertainty Reduction Value
All basic events occurred (the top event is XTA)	$M_{TXTAe} = 0.2145$	$M_{TAe}' = 0.2135$	0.001	0.47%
All basic events occurred (the top event is XTB)	$M_{XTBe} = 0.8632$	$M_{TBe}' = 0.9335$	0.0703	7.53%
XM_1 occurred	$M_{TXM1e} = 0.5230$	$M_{TXM1e}' = 0.5491$	0.0261	4.75%
XM_2 occurred	$M_{TXM2e} = 0.8251$	$M_{TXM2e}' = 0.8754$	0.0503	5.75%
except for X_1 , all basic events occurred	$M_{TX1e} = 0.8487$	$M_{TX1e}' = 0.9154$	0.0667	7.29%
except for X_2 , all basic events occurred	$M_{TX2e} = 0.8631$	$M_{TX2e}' = 0.9333$	0.0702	7.52%
except for X_3 , all basic events occurred	$M_{TX3e} = 0.7555$	$M_{TX3e}' = 0.8494$	0.0939	11.05%
except for X_4 , all basic events occurred	$M_{TX4e} = 0.8461$	$M_{TX4e}' = 0.9015$	0.0554	6.15%
except for X_5 , all basic events occurred	$M_{TX5e} = 0.8425$	$M_{TX5e}' = 0.8910$	0.0485	5.44%
except for X_6 , all basic events occurred	$M_{TX6e} = 0.8631$	$M_{TX6e}' = 0.9333$	0.0702	7.52%
except for X_7 , all basic events occurred	$M_{TX7e} = 0.8621$	$M_{TX7e}' = 0.9322$	0.0701	7.52%
except for X_8 , all basic events occurred	$M_{TX8e} = 0.8339$	$M_{TX8e}' = 0.8904$	0.0565	6.35%

In Figure 17, X_i' indicates that basic event X_i does not occur. The trend of the changes in the fuzzy median obtained from the algorithm simulation and approximate calculation is basically consistent. However, the fuzzy median calculated using the accurate algorithm is significantly smaller. This shows that there is large uncertainty in the approximate calculation.

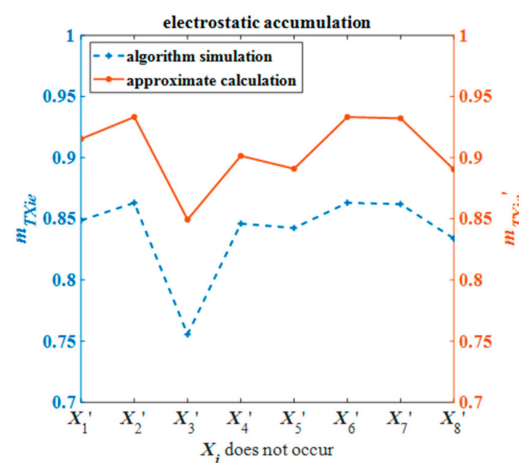


Figure 17. Comparison of fuzzy median.

According to Equation (A11) and the data in Table 8, the fuzzy importance of X_i ($i = 1 \sim 8$) is first calculated. This includes the traditional approximate calculation results and the improved accurate calculation results. Second, the absolute uncertainty reduction value of fuzzy importance is calculated. Finally, the fuzzy importance relative uncertainty reduction value is computed. The results are shown in Table 9.

Table 9. Fuzzy importance (Case 1).

Basic Events	Accurate Calculation Algorithm (Improved)	Approximation Calculation (Traditional)	Absolute Uncertainty Reduction Value	Relative Uncertainty Reduction Value
X_1	$Z_{TX1} = 0.0145$	$Z_{TX1}' = 0.0181$	0.0036	19.89%
X_2	$Z_{TX2} = 0.0001$	$Z_{TX2}' = 0.0002$	0.0001	50.00%
X_3	$Z_{TX3} = 0.1077$	$Z_{TX3}' = 0.0841$	0.0236	28.06%
X_4	$Z_{TX4} = 0.0171$	$Z_{TX4}' = 0.0320$	0.0149	46.56%
X_5	$Z_{TX5} = 0.0207$	$Z_{TX5}' = 0.0425$	0.0218	51.29%
X_6	$Z_{TX6} = 0.0001$	$Z_{TX6}' = 0.0002$	0.0001	50.00%
X_7	$Z_{TX7} = 0.0011$	$Z_{TX7}' = 0.0013$	0.0002	15.38%
X_8	$Z_{TX8} = 0.0293$	$Z_{TX8}' = 0.0431$	0.0138	32.02%

Based on the data in Table 9, a fuzzy importance comparison is drawn, as shown in Figure 18.

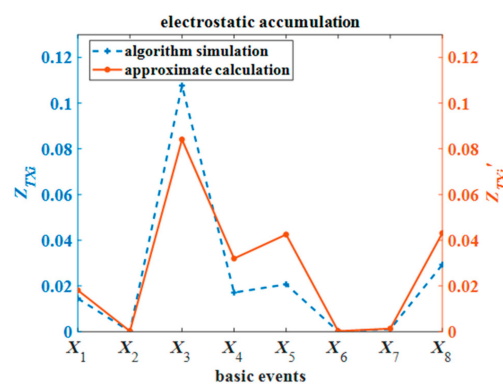


Figure 18. Comparison of fuzzy importance (Case 1).

It can be seen in Table 9 and Figure 19 that, after the simulation using the accurate algorithm, the uncertainty reduction effect of the fuzzy importance of X_2 , X_4 , X_5 , and X_6 is the most obvious. The relative uncertainty reduction values are 50%, 46.56%, 51.29%, and 50.00%, respectively.

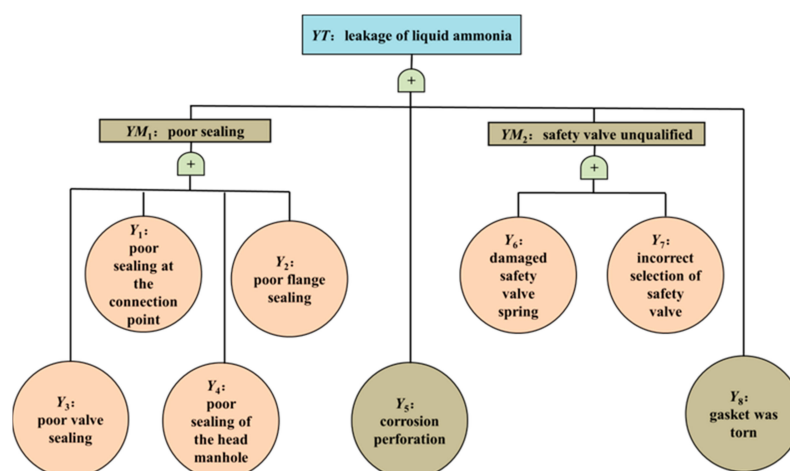


Figure 19. Fault tree of leakage of liquid ammonia.

3.3. Empirical Case 2

3.3.1. Case 2 Introduction

As shown in Figure 19, the top event of the fault tree is the leakage of liquid ammonia, which is divided into two layers. There are eight basic events: Y_1 —poor sealing at the connection point; Y_2 —poor flange sealing; Y_3 —poor valve sealing; Y_4 —poor sealing of the head manhole; Y_5 —corrosion perforation; Y_6 —damaged safety valve spring; Y_7 —incorrect selection of safety valve; and Y_8 —gasket was torn.

3.3.2. Expert Opinions on Case 2

Six experts evaluated the possibility of the basic events occurring, and the results are shown in Tables 10 and 11.

Table 10. Expert opinions of Case 2 (a).

Basic Events	Expert 1		Expert 2		Expert 3	
	Possibility of Occurrence	Possibility Level and Value	Possibility of Occurrence	Possibility Level and Value	Possibility of Occurrence	Possibility Level and Value
Y_1	easy to occur	L, 3.5	easy to occur	L, 3	easy to occur	L, 4
Y_2	easy to occur	H, 8	easy to occur	H, 7	easy to occur	H, 6.5
Y_3	easy to occur	M, 4.5	easy to occur	M, 5	easy to occur	M, 6
Y_4	easy to occur	VL, 2	easy to occur	VL, 1.5	easy to occur	L, 3.5
Y_5	easy to occur	L, 3	easy to occur	M, 5	easy to occur	M, 5.5
Y_6	easy to occur	H, 8	easy to occur	VH, 8.5	easy to occur	H, 6.5
Y_7	easy to occur	L, 3.5	easy to occur	L, 2.5	easy to occur	VL, 1
Y_8	not easy to occur	VL, 1.5	not easy to occur	VL, 2	not easy to occur	VL, 1

Table 11. Expert opinions of Case 2 (b).

Basic Events	Expert 4		Expert 5		Expert 6	
	Possibility of Occurrence	Possibility Level and Value	Possibility of Occurrence	Possibility Level and Value	Possibility of Occurrence	Possibility Level and Value
Y_1	easy to occur	M, 6	easy to occur	M, 5	easy to occur	M, 5.5
Y_2	easy to occur	H, 7.5	easy to occur	VH, 8.5	easy to occur	H, 8
Y_3	very easy to occur	M, 5	easy to occur	L, 2.5	easy to occur	H, 7
Y_4	easy to occur	L, 3.5	easy to occur	L, 3	easy to occur	L, 4
Y_5	easy to occur	L, 4	easy to occur	M, 4.5	easy to occur	L, 3
Y_6	easy to occur	H, 7.5	easy to occur	VH, 9	easy to occur	H, 6.5
Y_7	easy to occur	L, 3.5	easy to occur	VL, 1.5	easy to occur	L, 3
Y_8	not easy to occur	VL, 1.5	not easy to occur	L, 2.5	not easy to occur	VL, 1

3.3.3. Case 2: Probability of Basic Events

According to Tables 2, 10 and 11, the failure probability of the basic events is obtained, as shown in Table 12.

Table 12. The failure probability of the basic events.

Basic Events	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5	Expert 6
Y_1	3.5×10^{-2}	3×10^{-2}	4×10^{-2}	6×10^{-2}	5×10^{-2}	5.5×10^{-2}
Y_2	8×10^{-2}	7×10^{-2}	6.5×10^{-2}	7.5×10^{-2}	8.5×10^{-2}	8×10^{-2}
Y_3	4.5×10^{-2}	5×10^{-2}	6×10^{-2}	5×10^{-1}	2.5×10^{-2}	7×10^{-2}
Y_4	2×10^{-2}	1.5×10^{-2}	3.5×10^{-2}	3.5×10^{-2}	3×10^{-2}	4×10^{-2}
Y_5	3×10^{-2}	5×10^{-2}	5.5×10^{-2}	4×10^{-2}	4.5×10^{-2}	3×10^{-2}
Y_6	8×10^{-2}	8.5×10^{-2}	6.5×10^{-2}	7.5×10^{-2}	9×10^{-2}	6.5×10^{-2}
Y_7	3.5×10^{-2}	2.5×10^{-2}	1×10^{-2}	3.5×10^{-2}	1.5×10^{-2}	3×10^{-2}
Y_8	1.5×10^{-3}	2×10^{-3}	1×10^{-3}	1.5×10^{-3}	2.5×10^{-3}	1×10^{-3}

3.3.4. Normal Distribution Test of Raw Data of Case 2

The normal distribution test results of Case 2 are shown in Table 13.

Table 13. Calculation results of Case 2.

Basic Events	Mean Value (q_i)	Variance (σ_i)	Accurate and Significant Consistency
Y_1	0.0450	0.0118	0.988
Y_2	0.0758	0.0074	0.895
Y_3	0.1250	0.1843	0.451
Y_4	0.0292	0.0097	0.859
Y_5	0.0417	0.0103	0.923
Y_6	0.0767	0.0103	0.923
Y_7	0.0250	0.0105	0.964
Y_8	0.0016	0.0006	0.066

It can be seen in Table 13 that the probability of Y_i ($i = 1 \sim 8$) obeys the normal distribution.

3.3.5. Case 2: Drawing of Membership Curve

Using the new algorithms and traditional approximate calculation methods, a membership curve for the probability of the top event is drawn considering the following situations: all events occur; except for Y_i ($i = 1 \sim 8$), all events occur; Y_1 , Y_2 , Y_3 , and Y_4 occur simultaneously; and Y_6 and Y_7 occur simultaneously. According to the membership curve, m_{TYe} , m_{TXie} ($i = 1 \sim 8$), m_{TXM1e} , and m_{TXM2e} are determined, as shown in Figures 20–30.

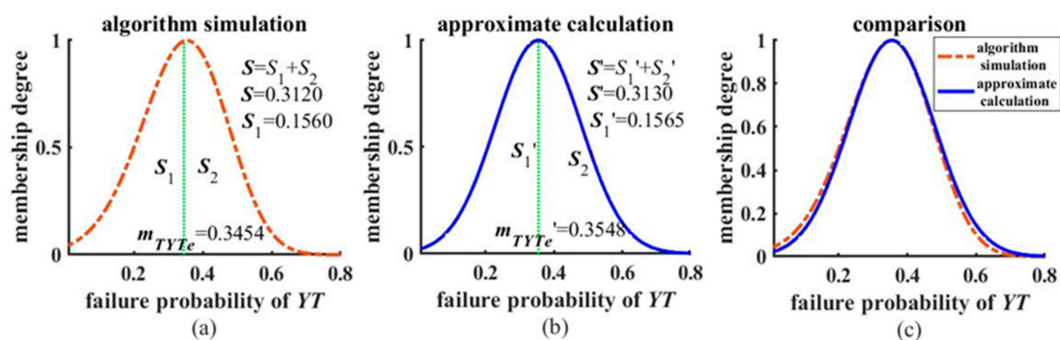


Figure 20. All basic events occurred (the top event is YT).

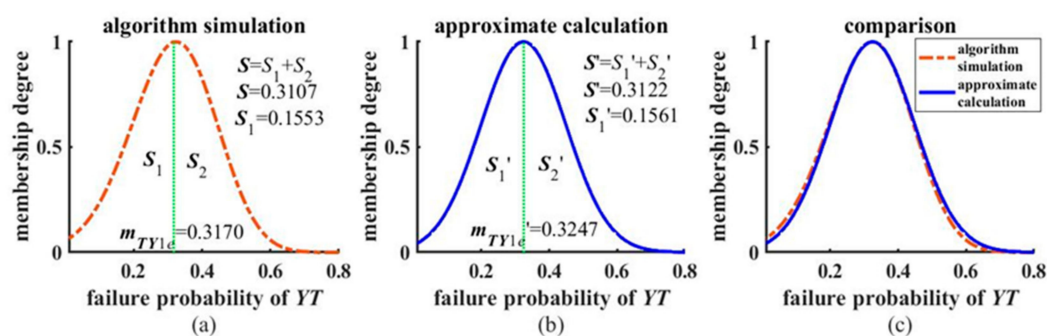
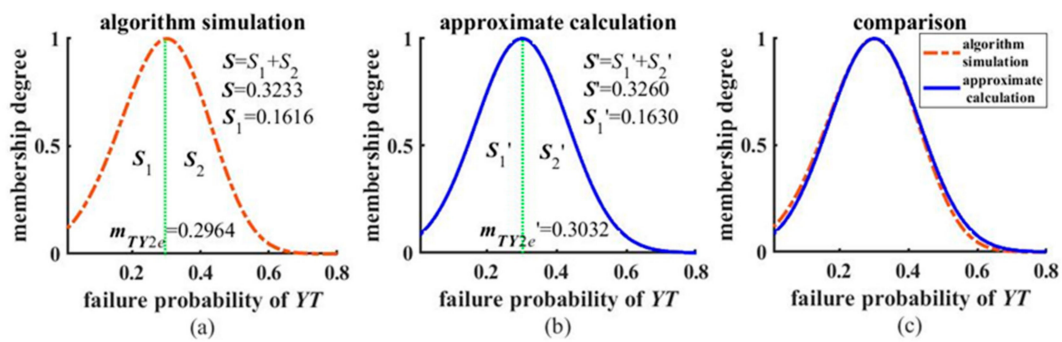
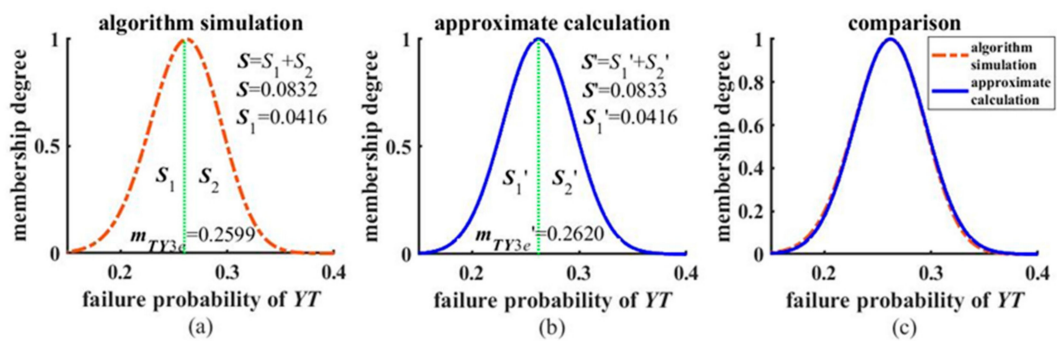
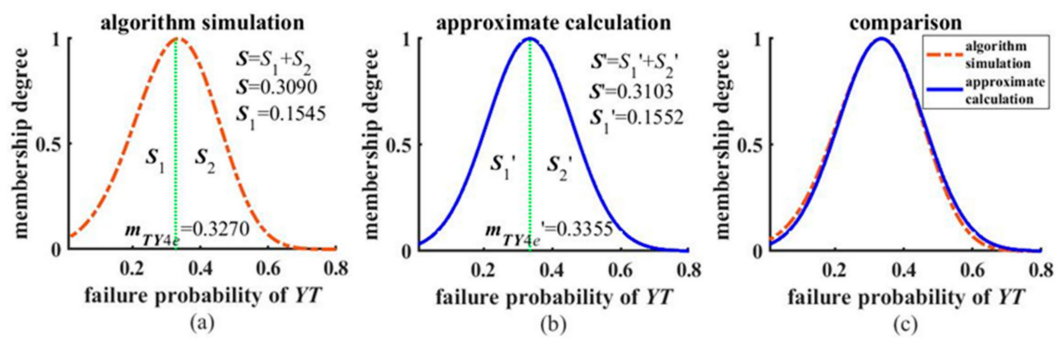
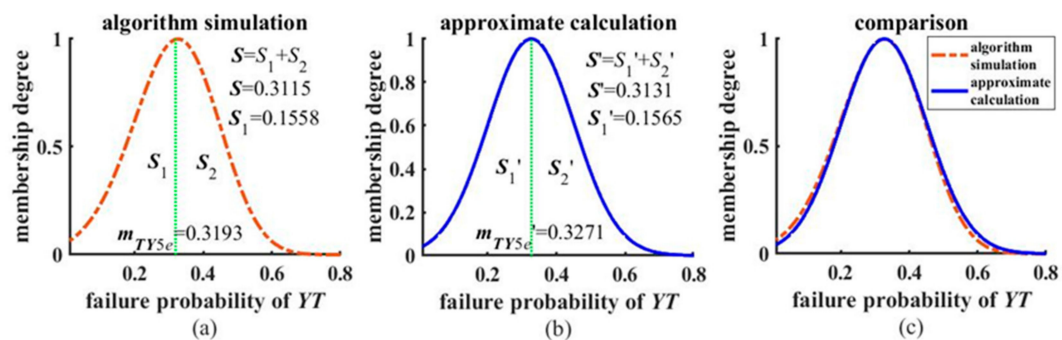


Figure 21. Except for Y_1 , all basic events occurred (the top event is YT).

Figure 22. Except for Y_2 , all basic events occurred (the top event is YT).Figure 23. Except for Y_3 , all basic events occurred (the top event is YT).Figure 24. Except for Y_4 , all basic events occurred (the top event is YT).Figure 25. Except for Y_5 , all basic events occurred (the top event is YT).

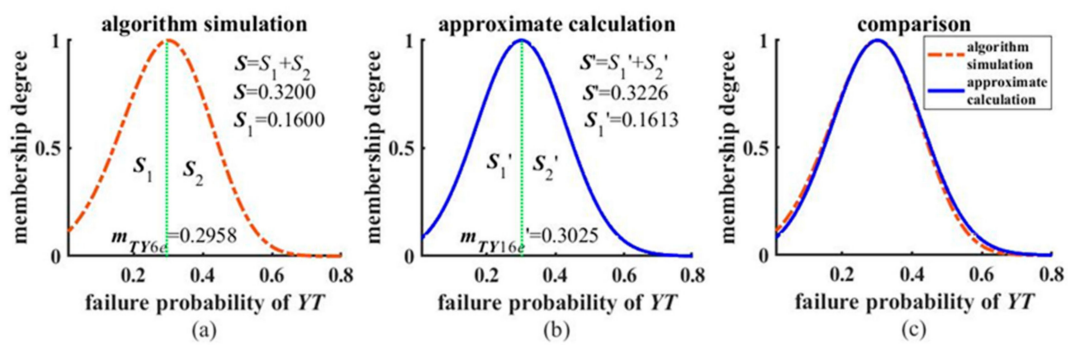


Figure 26. Except for Y_6 , all basic events occurred (the top event is YT).

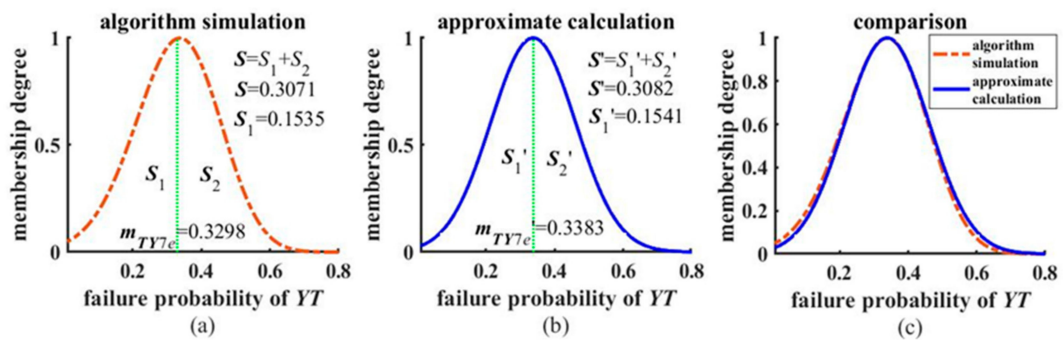


Figure 27. Except for Y_7 , all basic events occurred (the top event is YT).

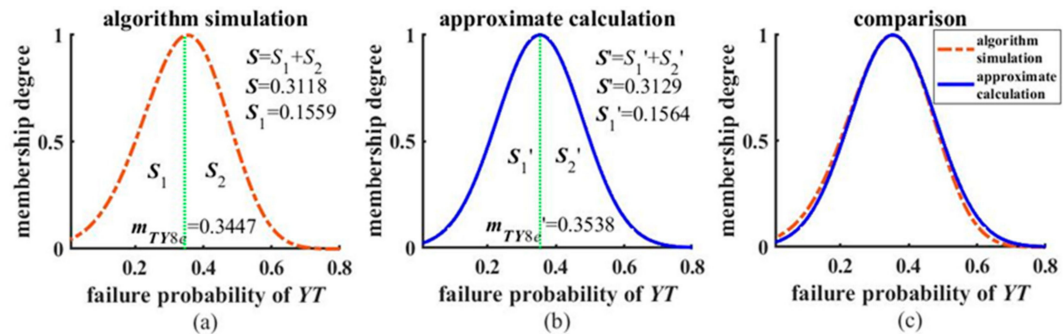


Figure 28. Except for Y_8 , all basic events occurred (the top event is YT).

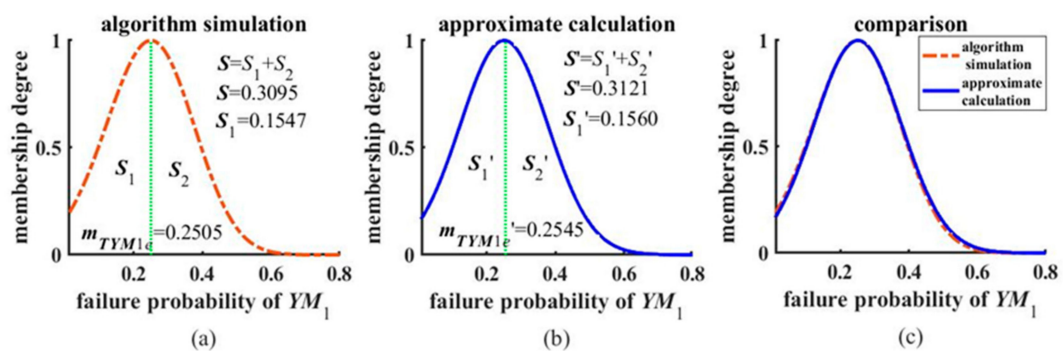


Figure 29. Y_1 , Y_2 , Y_3 , and Y_4 occurred simultaneously (YM_1 is the top event).

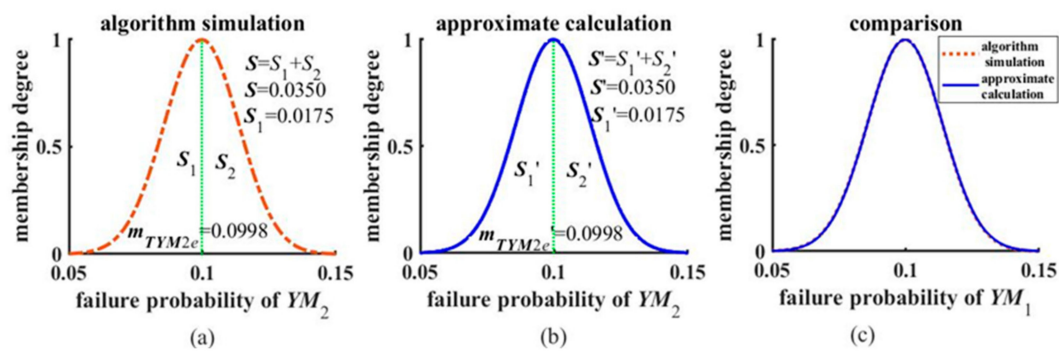


Figure 30. Y_6 and Y_7 occurred simultaneously (YM_2 is the top event).

Subfigure (a) is a graph of the membership curves obtained by simulation using the new algorithm; subfigure (b) is a graph of the membership curves obtained by the traditional approximation calculation; and subfigure (c) is a comparison of the results obtained by the two calculation methods. The meanings of the letters in the subfigures refer to the explanations in Figure A1 in the Appendix A.4.

3.3.6. Case 2: Discussion and Analysis

According to Figures 20–30, the fuzzy median values of the improved accurate calculation and traditional approximate calculation are obtained, as shown in Table 14.

Table 14. Fuzzy median of membership function (Case 2).

Status of Events	Accurate Calculation Algorithm (Improved)	Approximation Calculation (Traditional)	Absolute Uncertainty Reduction Value	Relative Uncertainty Reduction Value
all basic events occurred	$M_{TYTe} = 0.3454$	$M_{TYTe}' = 0.3548$	0.0094	2.65%
YM_1 occurred	$M_{TYM1e} = 0.2505$	$M_{TYM1e}' = 0.2545$	0.004	1.57%
YM_2 occurred	$M_{TXM2e} = 0.0998$	$M_{TXM2e}' = 0.0998$	0	0
except for Y_1 , all basic events occurred	$M_{TY1e} = 0.3170$	$M_{TY1e}' = 0.3247$	0.0077	2.37%
except for Y_2 , all basic events occurred	$M_{TY2e} = 0.2964$	$M_{TY2e}' = 0.3032$	0.0068	2.24%
except for Y_3 , all basic events occurred	$M_{TY3e} = 0.2599$	$M_{TY3e}' = 0.2620$	0.0021	0.80%
except for Y_4 , all basic events occurred	$M_{TY4e} = 0.3270$	$M_{TY4e}' = 0.3355$	0.0085	2.53%
except for Y_5 , all basic events occurred	$M_{TY5e} = 0.3193$	$M_{TY5e}' = 0.3271$	0.0078	2.38%
except for Y_6 , all basic events occurred	$M_{TY6e} = 0.2958$	$M_{TY6e}' = 0.3025$	0.0067	2.21%
except for Y_7 , all basic events occurred	$M_{TY7e} = 0.3298$	$M_{TY7e}' = 0.3383$	0.0085	2.51%
except for Y_8 , all basic events occurred	$M_{TY8e} = 0.3445$	$M_{TY8e}' = 0.3538$	0.0093	2.63%

In Table 14, it can be seen that, when intermediate event Y_{M2} occurs, the fuzzy importance obtained using the two calculation methods is completely consistent. This is because there are only two basic events that occur simultaneously, and the probability of occurrence is relatively small.

By comparing the reduction values of fuzzy importance uncertainty between the top event and the middle event, it can be seen that the new algorithm can reduce the uncertainty of propagation. Based on the data in Table 14, a fuzzy median uncertainty reduction effect diagram is drawn, as shown in Figure 31.

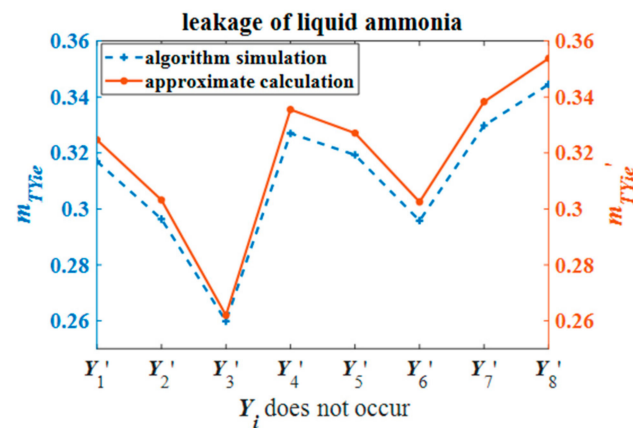


Figure 31. Comparison of fuzzy median (Case 2).

In Figure 32, Y_i' indicates that basic event Y_i does not occur. It can be seen that the fuzzy median value obtained using the improved accurate calculation and the fuzzy median value obtained using the approximate calculation have the same change trend. When Y_1 does not occur, the difference between the approximate calculation and the accurate calculation is the largest.

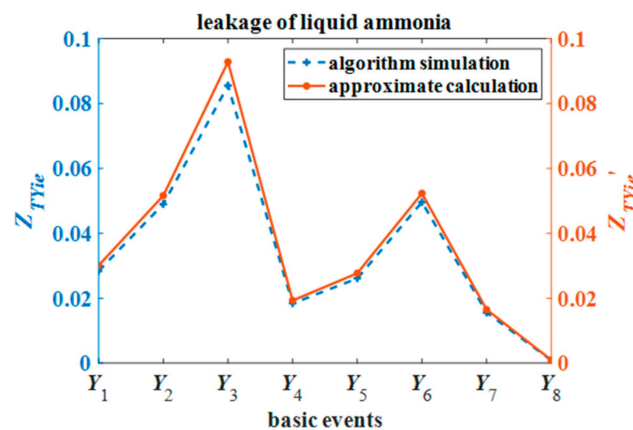


Figure 32. Comparison of fuzzy importance (Case 2).

From the intermediate layer to the top layer of the fault tree, as the number of layers increases, the uncertainty reduction value significantly increases. In actual production practice, fault trees are often quite complex. Using new algorithms to handle complex fault trees will have a more significant effect on reducing uncertainty.

As for Case 1, the approximate and accurate fuzzy importance values were calculated, as shown in Table 15.

Table 15. Fuzzy importance (Case 2).

Basic Events	Accurate Calculation Algorithm (Improved)	Approximation Calculation (Traditional)	Absolute Uncertainty Reduction Value	Relative Uncertainty Reduction Value
Y_1	$Z_{TY1} = 0.0284$	$Z_{TY1'} = 0.0301$	0.0017	5.65%
Y_2	$Z_{TY2} = 0.0490$	$Z_{TY2'} = 0.0516$	0.0026	5.04%
Y_3	$Z_{TY3} = 0.0855$	$Z_{TY3'} = 0.0928$	0.0073	7.87%
Y_4	$Z_{TY4} = 0.0184$	$Z_{TY4'} = 0.0193$	0.0009	4.66%
Y_5	$Z_{TY5} = 0.0261$	$Z_{TY5'} = 0.0277$	0.0016	5.78%
Y_6	$Z_{TY6} = 0.0496$	$Z_{TY6'} = 0.0523$	0.0027	5.16%
Y_7	$Z_{TY7} = 0.0156$	$Z_{TY7'} = 0.0165$	0.0009	5.45%
Y_8	$Z_{TY8} = 0.0009$	$Z_{TY8'} = 0.0010$	0.0001	10.00%

According to Table 15, a fuzzy importance comparison chart was drawn, as shown in Figure 32.

In Table 15 and Figure 32, it can be seen that the fuzzy importance of Y_i ($i = 1-8$) significantly decreases. The changing trends of fuzzy importance obtained using the two calculation methods are consistent. The fuzzy importance relative uncertainty reduction effect of Y_8 is the most obvious, with the relative uncertainty reduced by 10%.

4. Changing Rules of Uncertainty Reduction Effect

To further investigate the factors that affect the effectiveness of uncertainty reduction, Figures 33–35 are drawn for an analysis.

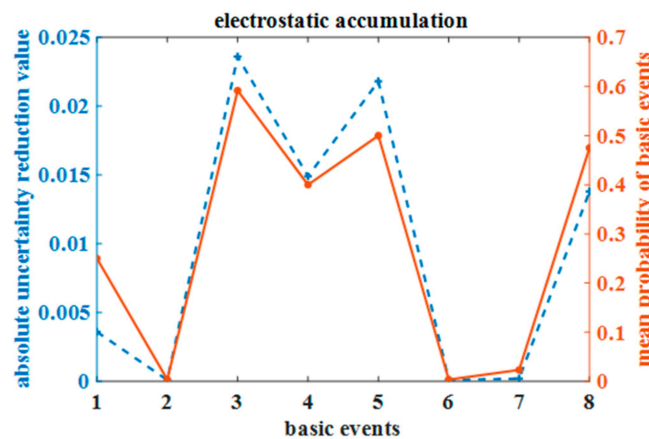


Figure 33. The relationship between the failure probability of basic events and the reduction in absolute uncertainty (Case 1).

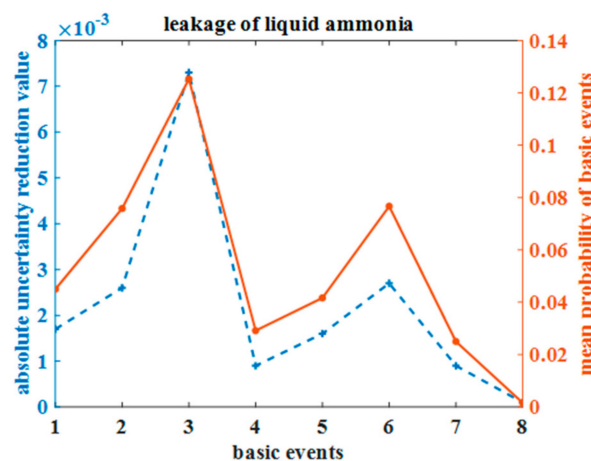


Figure 34. The relationship between the failure probability of basic events and the reduction in absolute uncertainty (Case 2).

By comparing Figures 33 and 34, it can be observed that the trend of the change in the mean of basic events is basically consistent with the trend of the change in the absolute uncertainty of fuzzy importance. The larger the mean of basic events, the more significant the reduction effect of fuzzy importance uncertainty after using optimization algorithms. The results of the risk assessments for electrostatic accumulation and the leakage of liquid ammonia conform to this pattern.

In Figure 35, it can be seen that, with electrostatic accumulation as the top event, the relative fuzzy importance of the basic event decreases by a maximum of 51.29%. With the leakage of liquid ammonia as the top event, the relative fuzzy importance of the basic event decreases by a maximum of 14.95%.

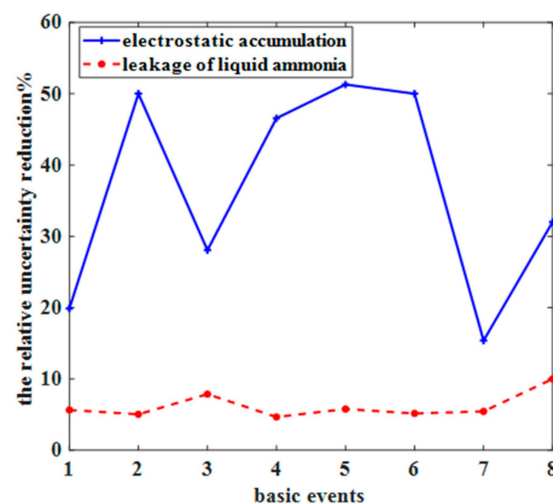


Figure 35. Comparison of uncertainty reduction effects between Case 1 and Case 2.

When the top event is electrostatic accumulation, compared with the top event being the leakage of liquid ammonia, the relative uncertainty reduction effect of the fuzzy importance of the basic event is obvious. This is because the fault tree with electrostatic accumulation as the top event has a larger overall mean. Therefore, in different fault trees, the greater the overall mean value of basic events, the more obvious the effect of reducing fuzzy importance.

Compared with X_7 (mean: 0.0333; variance: 0.0125) and Y_7 (mean: 0.0250; variance: 0.0105), the mean and variance are close, but the fuzzy importance reduction value of X_7 (15.38%) is significantly higher than the fuzzy importance reduction value of Y_7 (11.52%). It can be seen that the uncertainty reduction effect of low-probability events in fault trees with a large overall mean is often greater than that of high-probability events in fault trees with a small overall mean.

5. Conclusions

This study proposes a new algorithm that improves traditional normal fuzzy fault tree risk assessment results. First, the algorithm focuses on converting the precise membership curve into multiple cut-set intervals. Second, it further converts them into ordered arrays. Finally, an accurate membership graph is drawn based on this. Through this processing, the limitation where the number of basic events affects the risk assessment results of normal fuzzy fault tree is overcome. More importantly, shortcomings such as difficulty in obtaining accurate calculation results for complex systems and uncertainty in approximate calculations are solved. By combining two chemical accident cases for a verification analysis, the following conclusions are drawn:

The new algorithm can significantly reduce the uncertainty of process industry risk assessment. The maximum relative uncertainty reduction in the fuzzy importance of basic events is 50.29%. Considering that accidents in production practice often cause a large amount of property damage, the reliability of the process industry risk assessment results will be greatly improved if the improved results are applied to engineering practice.

This study found that the new algorithm can play a greater role in the following three situations: first, when the average probability of basic events in a single fault tree is high; second, when the overall mean and variance of basic events in different fault trees are large; and, third, when the complexity of the fault tree increases and the number of layers increases. The new algorithm has good adaptability, and it can maintain stable operation and expected performance when the model parameters change, such as the number of basic events and the mean and variance of the probability of basic events.

The contribution of this study is that it can provide a high-precision underlying algorithm for process industry risk assessment, and it improves the reliability of risk

assessment results for complex systems. Given the widespread application of fuzzy fault trees in the field of process industry risk assessment, the accurate calculation algorithm of normal fuzzy fault trees proposed in this paper, combined with other intelligent risk assessment algorithms such as neural networks and Bayesian analysis, is a direction worthy of in-depth research.

Author Contributions: Conceptualization, Y.Z. and L.Z.; methodology, Y.Z.; software, Y.Z.; validation, Y.Z. and L.Z.; formal analysis, Y.Z. and L.Z.; investigation, L.Z.; resources, L.Z.; data curation, Y.Z. and L.Z.; writing—original draft preparation, Y.Z.; writing—review and editing, Y.Z. and L.Z.; visualization, L.Z.; supervision, L.Z.; project administration, L.Z.; funding acquisition, L.Z. All authors have read and agreed to the published version of the manuscript.

Funding: This work was funded by the Science and Technology Research Project of the Educational Department of Liaoning Province (NO. JYTQN2023346).

Data Availability Statement: All data are presented in the study.

Acknowledgments: The authors are grateful to the scoring experts for their valuable suggestions.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

First, this section introduces the theoretical knowledge on which the new algorithm is based. Second, knowledge of the normal fuzzy fault tree risk assessment is introduced.

Appendix A.1. The Concept of Normal Fuzzy Numbers

The set composed of all fuzzy sets on the real number R is denoted as $F(R)$. If the fuzzy number $A \in F(R)$, then

$$A(x) = \exp\left(-\left(\frac{x-a}{\sigma}\right)^2\right) \quad (A1)$$

$A(x)$ is referred to as a normal fuzzy number or Gaussian fuzzy number [21]; a is an ordinary number; and σ denotes the degree of deviation of a , indicating the magnitude of ambiguity. The normal fuzzy number $A(x)$ is a bounded closed fuzzy number $(m, L(x), R(x))$, which can be expressed as

$$A(x) = \begin{cases} 1 & x = m \\ L(x) & x < m \\ R(x) & x > m \end{cases}$$

Here m represents the value on the x -axis; $L(x)$ is the increasing function, right continuous;

$0 \leq L(x) < 1$ and $\lim_{x \rightarrow -\infty} L(x) = 0$; $R(x)$ is the subtraction function, left continuous; $0 \leq R(x) < 1$ and $\lim_{x \rightarrow +\infty} R(x) = 0$.

Appendix A.2. Cut-Set Theorem

If fuzzy number $A(x)$ and parameter $\alpha (\alpha \in [0, 1])$ satisfy the expression

$$A_\alpha = \{x \in X | A(x) \geq \alpha\} \quad (A2)$$

then A_α is called the α -cut set of fuzzy set A or the α -level set of A . α represents the cut level, threshold value, or belief level [22].

Due to the fuzzy boundaries of the fuzzy set, different confidence levels α need to be selected to determine its membership relationship; the α -cut set is a method of converting a fuzzy set into a classical set.

Appendix A.3. Representation Theorem of Fuzzy Numbers

Assuming that there is a mapping $H: [0, 1] \rightarrow F(X)$ that satisfies the condition

$$\forall \alpha_1, \alpha_2 \in [0, 1], \forall \alpha_1 < \alpha_2 \Rightarrow H(\alpha_2) \subseteq H(\alpha_1)$$

then H is referred to as the nest of sets on X [23].

Assuming that $H: (0, 1] \rightarrow I_R, \alpha \rightarrow H(\alpha) = [m_\alpha, n_\alpha] \neq \emptyset$ meet

$$\alpha_1 < \alpha_2 \Rightarrow [m_{\alpha_2}, n_{\alpha_2}] \subseteq [m_{\alpha_1}, n_{\alpha_1}]$$

then

$$A = \bigcup_{\alpha \in [0,1]} \alpha H(\alpha) \in \tilde{R} \quad (A3)$$

$$A_\alpha = \bigcap_{n=1}^{\infty} H(\alpha_n), (\alpha > 0), (\alpha_n = (1 - \frac{1}{n+1})\alpha) \quad (A4)$$

$$A = ([m_A, n_A], L_A, R_A) \quad (A5)$$

where

$$m_A = \lim_{n \rightarrow \infty} m_{\alpha_n} \quad (A6)$$

$$n_A = \lim_{n \rightarrow \infty} n_{\alpha_n} \quad (A7)$$

$$\alpha_n = 1 - \frac{1}{n+1} \quad (A8)$$

$$L_A(x) = \bigvee_{0 < \alpha < 1} \{\alpha | m_\alpha \leq x\} \quad (A9)$$

$$R_A(x) = \bigvee_{0 < \alpha < 1} \{\alpha | n_\alpha \geq x\} \quad (A10)$$

Appendix A.4. The Concept of Normal Fuzzy Importance

The normal fuzzy importance represents the amount of the system fuzzy unreliability reduction when the basic event changes from a fuzzy fault state to another fuzzy functional state.

As shown in Figure A1, let $S_1 = \int_0^m L(x)dx, S_2 = \int_m^\infty R(x)dx$; the area enclosed by normal fuzzy numbers and the x -axis is denoted as “ S ”, and it can be computed as $S = S_1 + S_2$. The point m_e , with a straight line passing through it and perpendicular to the x -axis as the dividing line, $S_1 = S_2$ is obtained. m_e is known as the fuzzy median of the normal fuzzy number $A(x)$.

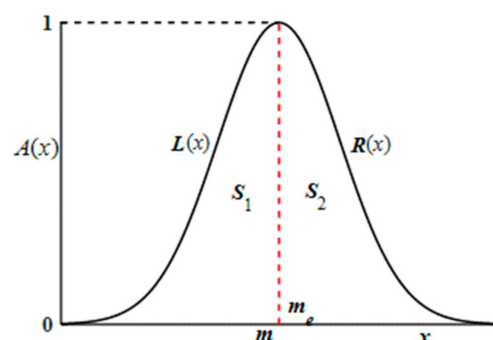


Figure A1. Normal fuzzy number.

The fault tree composed of basic event $X_1, X_2, \dots, X_n$ has the structural function of $\varnothing(X_1, X_2, \dots, X_n)$. If the failure probability of X_i is represented by a normal fuzzy number, then the probability distribution of the top event is

$$T = \varnothing(X_1, X_2, \dots, X_n) = (m_T, L_T(x), R_T(x))$$

This number is also a bounded closed fuzzy number, and its fuzzy median is denoted as m_{T_e} . If basic event X_i does not occur and all other events occur, the failure probability distribution of the top event T_i is

$$T_i = \varnothing(X_1, X_2, \dots, X_{i-1}, 0, X_{i+1}, \dots, X_n) = (m_{T_i}, L_{T_i}(x), R_{T_i}(x))$$

Moreover, if its fuzzy median is denoted as $m_{T_{ie}}$, then Z_{T_i} can be expressed by

$$Z_{T_i} = m_{T_e} - m_{T_{ie}} > 0 \quad (\text{A11})$$

In Equation (A11), Z_{T_i} is the normal fuzzy importance of x_i [24]. If $Z_{T_i} > Z_{T_j}$, it is considered that the impact of x_i on the system is greater than that of x_j on the system, and improving the system requires prioritizing the improvement of x_i .

Appendix A.5. Failure Probability of Top Event

The failure probability P_T of the top event is

$$P_T = \sum_{k=1}^{n_G} \prod_{X_i \in G_k} Q_i \quad (\text{A12})$$

Here n_G represents the number of minimum cut sets; k represents the sequence number of the minimum cut set; Q_i represents the failure probability of the basic event; G_k denotes the k th minimal cut sets [25].

References

1. Anwar, A.G.; Zhang, X. Deep reinforcement learning for intelligent risk optimization of buildings under hazard. *Reliab. Eng. Syst. Saf.* **2024**, *247*, 110118. [\[CrossRef\]](#)
2. Rocha, R.C.N.; Soares, R.A.; Santos, L.I.; Camargos, M.O.; Ekel, P.Y.; Libório, M.P.; dos Santos, A.C.G.; Vidoli, F.; D'Angelo, M.F.S.V. A New Fault Classification Approach Based on Decision Tree Induced by Genetic Programming. *Processes* **2024**, *12*, 818. [\[CrossRef\]](#)
3. Liu, C.; Li, J.; Zhang, D. Fuzzy Fault Tree Analysis and Safety Countermeasures for Coal Mine Ground Gas Transportation System. *Processes* **2024**, *12*, 344. [\[CrossRef\]](#)
4. Tan, X.; Xiao, S.; Yang, Y.; Khakzad, N.; Reniers, G.; Chen, C. An agent-based resilience model of oil tank farms exposed to earthquakes. *Reliab. Eng. Syst. Saf.* **2024**, *247*, 110096. [\[CrossRef\]](#)
5. Yazdi, M.; Kabir, S.; Walker, M. Uncertainty handling in fault tree based risk assessment: State of the art and future perspectives. *Process Saf. Environ. Prot.* **2019**, *131*, 89–104. [\[CrossRef\]](#)
6. Klinke, A.; Renn, O. A New Approach to Risk Evaluation and Management: Risk-Based, Precaution-Based, and Discourse-Based Strategies. *Risk Anal.* **2002**, *22*, 1071–1094. [\[CrossRef\]](#)
7. Soltanali, H.; Khojastehpour, M.; Farinha, J.T.; Pais, J.E.D.A.E. An Integrated Fuzzy Fault Tree Model with Bayesian Network-Based Maintenance Optimization of Complex Equipment in Automotive Manufacturing. *Energies* **2021**, *14*, 7758. [\[CrossRef\]](#)
8. Kaushik, M.; Kumar, M. An integrated approach of intuitionistic fuzzy fault tree and Bayesian network analysis applicable to risk analysis of ship mooring operations. *Ocean Eng.* **2023**, *269*, 113411. [\[CrossRef\]](#)
9. Zhao, C.; Yip, T.L.; Wu, B.; Lyu, J. Use of fuzzy fault tree analysis and Bayesian network for occurrence likelihood estimation of navigational accidents in the Qinzhou Port. *Ocean Eng.* **2022**, *263*, 112381. [\[CrossRef\]](#)
10. Ding, R.; Liu, Z.; Xu, J.; Meng, F.; Sui, Y.; Men, X. A novel approach for reliability assessment of residual heat removal system for HPR1000 based on failure mode and effect analysis, fault tree analysis, and fuzzy Bayesian network methods. *Reliab. Eng. Syst. Saf.* **2021**, *216*, 107911. [\[CrossRef\]](#)
11. Pang, J.; Dai, J.; Li, Y. An Intelligent Fault Analysis and Diagnosis System for Electromagnet Manufacturing Process Based on Fuzzy Fault Tree and Evidence Theory. *Mathematics* **2022**, *10*, 1437. [\[CrossRef\]](#)
12. Zaib, A.; Yin, J.; Khan, R.U. Determining Role of Human Factors in Maritime Transportation Accidents by Fuzzy Fault Tree Analysis (FFTA). *J. Mar. Sci. Eng.* **2022**, *10*, 381. [\[CrossRef\]](#)
13. Celik, M.; Lavasani, S.M.; Wang, J. A risk-based modelling approach to enhance shipping accident investigation. *Saf. Sci.* **2010**, *48*, 18–27. [\[CrossRef\]](#)
14. Purba, J.H.; Tjahyani, D.T.S.; Ekariansyah, A.S.; Tjahjono, H. Fuzzy probability based fault tree analysis to propagate and quantify epistemic uncertainty. *Ann. Nucl. Energy* **2015**, *85*, 1189–1199. [\[CrossRef\]](#)
15. Kaushik, M.; Kumar, M. An α -cut interval based IF-importance measure for intuitionistic fuzzy fault tree analysis of subsea oil and gas production system. *Appl. Ocean Res.* **2022**, *125*, 103229. [\[CrossRef\]](#)
16. Hu, L.; Kang, R.; Pan, X.; Zuo, D. Risk assessment of uncertain random system—Level-1 and level-2 joint propagation of un-617 certainty and probability in fault tree analysis. *Reliab. Eng. Syst. Saf.* **2020**, *198*, 106874. [\[CrossRef\]](#)

17. Lu, M.; Jin, Y.; Lin, J.; Liu, Q.; Du, Y.; Yang, Y. Fuzzy fault tree analysis of EVAC system based on improved SAMFFTA with butterfly optimization algorithm. *Eng. Fail. Anal.* **2023**, *154*, 107658. [[CrossRef](#)]
18. Garg, A.; Maiti, J.; Kumar, A. Granulized Z-OWA aggregation operator and its application in fuzzy risk assessment. *Int. J. Intell. Syst.* **2022**, *37*, 1479–1508. [[CrossRef](#)]
19. Feng, K.; Lu, Z.; Ling, C.; Wang, L. Fuzzy importance sampling method for estimating failure possibility. *Fuzzy Sets Syst.* **2021**, *424*, 170–184. [[CrossRef](#)]
20. Aven, T.; Zio, E. Model Output Uncertainty in Risk Assessment. *Int. J. Perform. Eng.* **2014**, *9*, 662–667.
21. Akraam, M.; Rashid-Zafar, T.S. An image encryption scheme proposed by modifying chaotic tent map using fuzzy numbers, Multimed. *Tools Appl.* **2023**, *82*, 16861–16879.
22. Li, Y.; He, X.; Qin, K.; Meng, D. An improved method to estimate the similarity between LR-type fuzzy numbers. *Fuzzy Sets Syst.* **2024**, *476*, 108770. [[CrossRef](#)]
23. Yuan, X.; Li, H.; Sun, K. The cut sets, decomposition theorems and representation theorems on intuitionistic fuzzy sets and interval valued fuzzy sets. *Sci. China Inf. Sci.* **2011**, *54*, 91–110. [[CrossRef](#)]
24. Furuta, H.; Shiraishi, N. Fuzzy Importance in Fault Tree Analysis. *Fuzzy Sets Syst.* **1984**, *12*, 205–213. [[CrossRef](#)]
25. Liu, J.; Yin, H.; Zhang, Y.; Li, X.; Li, Y.; Gong, X.; Wu, W. An Interval-Valued Intuitionistic Fuzzy Bow-Tie Model (IVIF-BT) for the Effectiveness Evaluation of Safety Barriers in Natural Gas Storage Tank. *Appl. Sci.* **2024**, *14*, 1586. [[CrossRef](#)]
26. Shi, L.; Shuai, J.; Xu, K. Fuzzy fault tree assessment based on improved AHP for fire and explosion accidents for steel oil storage tanks. *J. Hazard. Mater.* **2014**, *278*, 529–538. [[CrossRef](#)]

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.