

Article

Numerical Simulation of Cross-Layer Hydraulic Fracture Propagation in Interbedded Sandstone Reservoirs of the Lianggaoshan Formation

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Abstract

Vertical cross-layer propagation of hydraulic fractures is critical for the efficient stimulation of interbedded sandstone reservoirs in the Lianggaoshan Formation. To investigate the vertical fracture propagation mechanisms and controlling factors in this complex lithological setting, a three-dimensional (3D) numerical model was established using the continuum–discontinuum element method (CDEM) based on the typical “mudstone–sandstone–mudstone” geological structure of the Lianggaoshan Formation. The effects of geological parameters, including interlayer stress contrast, vertical stress contrast, elastic modulus ratio, and reservoir thickness, as well as engineering parameters, including fluid viscosity and injection rate, were systematically evaluated. The results show that interlayer stress contrast is the primary factor restricting vertical fracture growth. As the interlayer stress contrast increases from 2 MPa to 8 MPa, the fracture morphology gradually changes from effective cross-layer propagation to complete containment within the sandstone layer, while the injection pressure at 300 s increases from 55.91 MPa to 58.90 MPa and the fracture width increases from 4.58 mm to 5.05 mm. In contrast, vertical stress contrast has a limited influence under the horizontal-stratification conditions investigated. Increasing fluid viscosity and injection rate can enhance intra-fracture net pressure and promote interface breakthrough. When the fluid viscosity increases from 5 mPa·s to 50 mPa·s, the breakdown pressure increases from 61.05 MPa to 69.59 MPa and the fracture width increases from 4.79 mm to 6.37 mm. When the injection rate increases from 0.6 m³/min to 3.6 m³/min, the breakdown pressure increases from 58.57 MPa to 63.35 MPa and the fracture width increases from 4.28 mm to 5.17 mm. Based on the Effective Vertical Propagation Index (EVI), three vertical propagation modes were identified: restricted vertical propagation, partially effective cross-layer propagation, and effective vertical propagation. Gray Relational Analysis (GRA) revealed the following sensitivity ranking: interlayer stress contrast > injection rate > fluid viscosity > elastic modulus ratio > reservoir thickness > vertical stress contrast. For reservoirs with a typical 4 MPa stress barrier, effective vertical breakthrough can be achieved when the fluid viscosity exceeds 25 mPa·s or the injection rate reaches 3.6 m³/min. These



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findings provide quantitative guidance for optimizing fracturing parameters in interbedded sandstone reservoirs.

Keywords: hydraulic fracturing; cross-layer propagation; interbedded sandstone; CDEM

1. Introduction

With the rapid advancement of global unconventional oil and gas exploration, shale oil in the Jurassic Lianggaoshan Formation of the Sichuan Basin has garnered significant attention due to its immense resource potential [1–3]. Distinct from typical North American marine shales, the Lianggaoshan reservoir exhibits characteristic “sandwich-type” features, where high-quality sandstone reservoirs are frequently interbedded with mudstone barriers in the vertical direction, forming a complex “soft–hard–soft” mechanical structure [4,5]. In this geological context, hydraulic fractures are prone to confinement within the thin sandstone layers, making it difficult to achieve effective vertical cross-layer propagation. This containment limits the Stimulated Reservoir Volume (SRV) and hinders breakthroughs in single-well production [6–8]. Consequently, elucidating the vertical propagation mechanism and primary controlling factors of fractures in such sandwich-type reservoirs holds significant engineering importance for optimizing fracturing treatments and achieving “cross-layer volumetric stimulation” [9,10].

The vertical propagation behavior of hydraulic fractures is governed by the complex coupling of geological and engineering factors [11,12]. From the perspective of rock mechanics, the interlayer stress contrast is widely regarded as the primary factor restricting fracture height growth. Lyu et al. [13] conducted true triaxial tests on samples extracted from representative outcrops of the Lianggaoshan Formation in China. Their results indicated that the interlayer stress contrast plays a critical role in fracture initiation and propagation. Zhang et al. [14] quantitatively investigated the influence of key geological and engineering factors on vertical fracture propagation in layered formations using true triaxial hydraulic fracturing experiments. The experimental results demonstrated that the brittleness of the interlayer exerts a substantial influence on propagation behavior; specifically, interlayers with lower brittleness inhibit the hydraulic fracture’s ability to penetrate the interface and the barrier layer. Furthermore, Guo et al. [15] employed finite element simulations to demonstrate that an increase in reservoir thickness leads to an expansion of the fracture area within the target reservoir, which subsequently affects the efficacy of cross-layer propagation. Additionally, Xie et al. [16] established a 3D hydro-mechanical coupled model to analyze the impacts of interface dip angle and horizontal stress contrast. Their results revealed that the bedding dip angle and horizontal stress contrast jointly govern the initiation and propagation of hydraulic fractures.

Regarding the optimization of engineering parameters, fracturing fluid properties and operational parameters directly determine the driving energy within the fracture [17]. Liu et al. [6] investigated the initiation and propagation of hydraulic fractures under varying viscosities, injection rates, and injection schemes using synthetic rock samples representative of interlayered reservoir characteristics. The results indicate that a higher fracturing fluid viscosity is critical for ensuring vertical cross-layer propagation. Conversely, a lower viscosity facilitates the activation of weak bedding interfaces, promoting extensive horizontal propagation along these planes and the formation of branch fractures. Experimental studies by He et al. [17] demonstrated that hydraulic fractures induced by low-viscosity fluids tend to be restricted within specific lithologies, whereas those induced by high-viscosity fluids exhibit a stronger capability to penetrate through complex sandy–muddy

shale sequences. Furthermore, Huang et al. [18] revealed the evolution patterns of multiple fractures in sandwich-type reservoirs and elucidated the mechanisms by which increased fluid viscosity and injection rates enhance vertical fracture propagation.

Although extensive research has explored fracture propagation patterns in multi-lithological layered formations, existing models are predominantly simplified to 2D plane-strain conditions, making it difficult to accurately capture the complex stress interference within the “mudstone–sandstone–mudstone” multilayer system in three-dimensional space [19]. Furthermore, the traditional Finite Element Method (FEM) faces challenges regarding mesh dependency when addressing the progressive evolution from continuous deformation to discontinuous rupture [20]. In contrast, the continuum–discontinuum element method (CDEM) integrates continuum mechanics with the Discrete Element Method (DEM), demonstrating significant advantages in simulating fracture initiation, complex path reorientation, and cross-layer behaviors [21–23]. Recent CDEM-based studies have further improved the understanding of hydraulic fracture height growth in layered and heterogeneous reservoirs. Zhang et al. [24] established a three-dimensional hydro-mechanical coupling model based on CDEM to investigate the effects of bedding planes and interlayer property contrasts on fracture height propagation in shale reservoirs. Wang et al. [25] combined true triaxial hydraulic fracturing experiments, computed tomography scanning, and three-dimensional CDEM simulations to study hydraulic fracture growth in multi-lithology and multi-layered shale reservoirs. Huang et al. [26] developed a CDEM-based hydraulic fracturing model for heterogeneous multi-lithology stacked reservoirs and systematically analyzed the effects of heterogeneity, vertical stress difference, interlayer stress difference, fluid viscosity, and injection rate on multiple-fracture longitudinal propagation.

In light of this, focusing on the typical sandwich-type sandstone reservoirs of the Lianggaoshan Formation, this study established a fully three-dimensional hydraulic fracturing numerical model based on the continuum–discontinuum element method (CDEM). The controlling effects of geological parameters (interlayer stress contrast, modulus ratio, and reservoir thickness) and engineering parameters (fluid viscosity and injection rate) on the vertical fracture propagation morphology were systematically analyzed. Furthermore, the sensitivity weights of these factors were quantitatively revealed based on Gray Relational Analysis (GRA). The findings of this study aim to provide theoretical support for the efficient development of shale oil in the Lianggaoshan Formation.

2. Method

The continuum–discontinuum element method (CDEM) employs a continuum–discontinuum algorithm as its core kernel, integrating the distinct advantages of both the Discrete Element Method (DEM) and the Finite Element Method (FEM) [27,28]. By accounting for fracturing along block boundaries and within block interiors, CDEM effectively analyzes the progressive failure of materials and simulates the entire evolution process from continuous deformation to crack initiation and propagation. The CDEM framework comprises two fundamental components: block elements and contact elements [29]. Each block consists of one or more deformable finite element, while the interface is characterized by a series of normal and tangential springs. During computation, the solution domain is discretized into block elements, where each block can either represent a single finite element or be subdivided into multiple finite elements. Based on the element stiffness matrix, the stiffness matrix for any arbitrary block can be derived; notably, a global stiffness matrix is not assembled across the blocks. Instead, the dynamic relaxation technique is employed to iteratively solve for variables such as displacement, strain, and stress within each block element. Furthermore, the seepage field within the fractures is solved using the Finite Volume Method (FVM) [30].

2.1. Solution of Block Elements

In this study, each lithological layer is treated as an equivalent continuous and homogeneous material, while different mechanical parameters are assigned to the sandstone reservoir, mudstone barrier, and lithological interface. The block elements are governed by a linear elastic constitutive model. Under this assumption, the mechanical response of each block element is described by the dynamic equilibrium equation [28]:

$$M\ddot{u}^e + C\dot{u}^e + Ku^e = F^e, \quad (1)$$

where $\ddot{u}^e$, $\dot{u}^e$, and u^e are the acceleration matrix, velocity matrix, and displacement matrix of all nodes in the element, respectively, and M , C , K , and F^e are the element mass matrix, damping matrix, stiffness matrix, and external force, respectively. The external force vector includes the equivalent nodal forces generated by the boundary in situ stresses and the fluid pressure acting on fracture surfaces.

In the time domain, the forward Euler difference method is adopted for the explicit iterative solution. At each time step, the nodal velocity and displacement are updated according to the calculated acceleration. The iterative scheme can be expressed as [27]:

$$\begin{cases} \dot{u}^{n+1} = \dot{u}^n + \ddot{u}^n \Delta t \\ u^{n+1} = u^n + \dot{u}^{n+1} \Delta t \end{cases}, \quad (2)$$

where n is the iteration step, and Δt is the time step.

The strain and stress of each block element are updated from the displacement field and the linear elastic constitutive relationship [29]:

$$\sigma = D : \varepsilon, \quad (3)$$

where σ and ε are the stress and strain tensors, respectively, and D is the elastic stiffness matrix determined by Young's modulus and Poisson's ratio.

2.2. Solution of Fracture Propagation

CDEM simulates the continuity and discontinuity of materials by introducing virtual springs at the element interfaces. The failure criteria adopted are the Maximum Tensile Stress Criterion and the Mohr–Coulomb Criterion. The relationship between the relative displacement at the contact points of adjacent blocks and the spring force satisfies Hooke's Law [18].

$$\Delta u_n = \frac{F_n}{K_n} = \frac{(\sigma_{n1} + \sigma_{n2})A}{2K_n}, \quad (4)$$

$$\Delta u_t = \frac{F_t}{K_t} = \frac{(\sigma_{t1} + \sigma_{t2})A}{2K_t}, \quad (5)$$

where Δu_n and Δu_t are the normal displacement and tangential displacement, F_n and F_t are the normal force and tangential force, K_n and K_t are the normal stiffness and tangential stiffness of the spring, A is the contact area, σ_{n1} and σ_{n2} are the normal stresses at the contact point, and σ_{t1} and σ_{t2} are the tangential stresses at the contact point.

The computational procedure for the transition from a continuous element interface to a discontinuous fracture surface is described as follows. First, the stresses of the elements on both sides of the interface are calculated to obtain the normal and tangential stresses acting on the contact surface. Then, the contact element is checked using both the maximum tensile stress criterion and the Mohr–Coulomb shear failure criterion [31]. These two criteria

are used as complementary failure criteria. Once either criterion is satisfied, the contact element is considered to fail and is converted into a discontinuous fracture surface [27,28].

$$\sigma_n \geq T, \quad (6)$$

where σ_n is the normal stress and T is the tensile strength. This criterion is mainly used to describe the tensile opening failure of contact elements, which controls Mode-I fracture initiation and propagation.

The Mohr–Coulomb criterion can be expressed as:

$$\sigma_t \geq c + \sigma_n \tan \varphi, \quad (7)$$

where σ_t is the tangential stress, and c and φ are the cohesion and internal friction angle, respectively. In the numerical implementation, the Mohr–Coulomb criterion is applied to the contact elements between adjacent block elements. At each calculation step, the local normal and tangential stresses on the contact surface are used to determine whether shear failure occurs. Once this criterion is satisfied, the corresponding contact element fails and is converted into a discontinuous fracture surface, allowing the model to capture shear failure, interface slip, and shear-dominated fracture propagation during hydraulic fracturing.

2.3. Solution of Fracture Seepage

The fluid flow within the fracture exerts pressure on the adjacent solid elements. Under the combined influence of this fluid pressure and external loads, the adjacent solid elements undergo opening and closing deformations. Consequently, the variation in relative displacement between solid elements governs the fracture aperture. According to the cubic law, the flow rate in the hydraulic fracture is controlled by both fracture aperture and fluid viscosity. The fracture flow can be expressed as [32]:

$$\mathbf{q}_f = -\frac{w^3}{12\mu} \nabla p_f, \quad (8)$$

where $\mathbf{q}_f$ is the fracture flow rate, w is the fracture aperture, μ is the dynamic viscosity of the fracturing fluid, and p_f is the fracture fluid pressure. Therefore, fluid viscosity is explicitly incorporated into the fracture flow calculation through the cubic law relationship. Under the same fracture aperture and pressure gradient, a higher fluid viscosity reduces fluid mobility within the fracture.

In the finite volume seepage calculation, the above relationship can be written in terms of the fracture hydraulic conductivity coefficient:

$$k^F = \frac{w^2}{12\mu}, \quad (9)$$

where k^F is the fracture hydraulic conductivity coefficient. Thus, fracture aperture evolution caused by mechanical deformation and the viscosity of the fracturing fluid jointly determine the pressure redistribution inside the fracture.

When the cumulative saturation of a node reaches 1, the variables can be calculated using the following equation [22,24]:

$$p_p^F = -\sum_{t=0}^t (k^F \frac{(Q^F + Q_{app})}{nV} \Delta t), \quad (10)$$

where p_p^F is the fracture fluid pressure, k^F is the fracture permeability coefficient, Q^F is the fracture nodal flow rate, and Q_{app} is the external flow boundary condition.

3. Model Verification

To verify the reliability of the CDEM hydraulic fracturing model established in this study in calculating fracture propagation and fracture aperture, a two-dimensional KGD plane-strain hydraulic fracture propagation validation model was developed. The numerical simulation results were compared with the classical KGD analytical solution. The validation model had dimensions of 50 m × 50 m. An initial fracture was horizontally arranged along the middle of the model, and the injection point was located at the midpoint of the left boundary. The rock material was assumed to be a homogeneous linear elastic medium. The rock mechanical parameters and fluid parameters used in the validation model were consistent with those adopted in the numerical model. The elastic modulus of the rock was 45 GPa, Poisson's ratio was 0.24, the injection rate was 0.0005 m³/s, and the fracturing fluid viscosity was 5 mPa·s. According to the KGD analytical solution, the total length of the complete bi-wing fracture and the maximum fracture aperture can be expressed as

$$L(t) = 0.539 \left[\frac{Eq^3}{\mu(1-\nu^2)} \right]^{1/6} t^{2/3}; \quad (11)$$

$$w_{\max}(t) = 2.36 \left[\frac{\mu q^3 (1-\nu^2)}{E} \right]^{1/6} t^{1/3}, \quad (12)$$

where $L(t)$ is the fracture half-length, $w_{\max}(t)$ is the maximum fracture aperture, E is the elastic modulus, ν is Poisson's ratio, μ is the dynamic viscosity of the fracturing fluid, q is the injection rate supplied to one fracture wing, and t is the injection time.

The comparison between the CDEM numerical results and the KGD analytical solution is shown in Figure 1. Because fracture propagation in the CDEM model is realized through the progressive failure of interface elements, the simulated fracture half-length shows a stepwise growth pattern. Nevertheless, the overall evolution agrees well with the analytical solution. At $t = 40$ s, the fracture half-length obtained from the CDEM simulation is 21.5 m, while the KGD analytical solution gives 20.55 m, corresponding to a relative error of approximately 4.7%. The maximum fracture aperture calculated by the CDEM model is 1.19 mm, while the KGD analytical solution gives 1.24 mm, corresponding to a relative error of approximately 4.3%. Overall, the CDEM model can accurately reproduce the fracture half-length and aperture evolution predicted by the KGD analytical solution. The good agreement between the numerical and analytical results demonstrates that the proposed hydro-mechanical coupling model is reliable for simulating hydraulic fracture propagation.

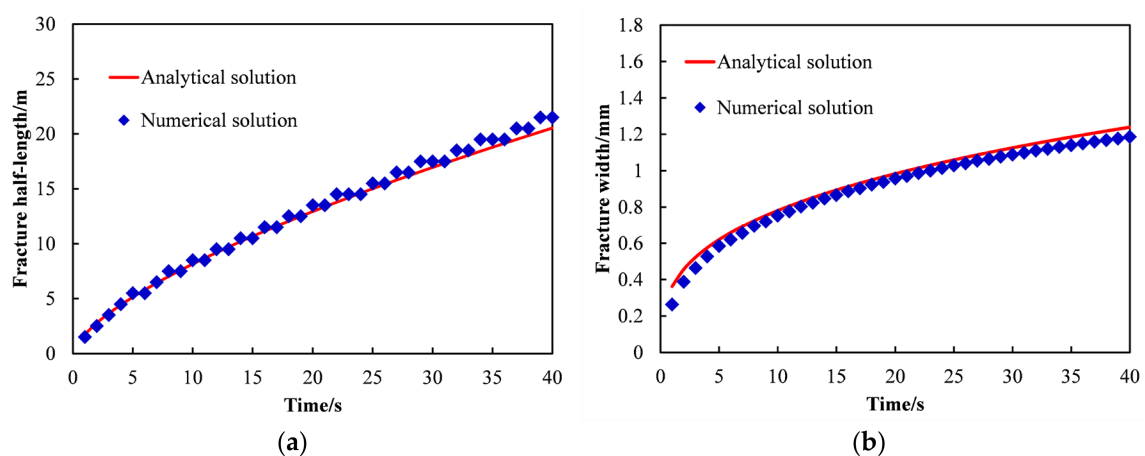


Figure 1. Comparison between the CDEM numerical results and the KGD analytical solution: (a) fracture half-length; (b) fracture width.

4. Model Description and Solution Procedure

To investigate the vertical propagation behavior and controlling factors of hydraulic fractures in the sandwiched sandstone reservoirs of the Liang 2 Member of the Liang-gaoshan Formation in the Sichuan Basin, a simplified three-dimensional “mudstone–sandstone–mudstone” layered model was established. The model was constructed based on the typical local geological characteristics of alternating sandstone and mudstone layers, including bottom fine sandstone, siltstone, and muddy siltstone, with a channel sand thickness of 4–8 m, as shown in Figure 2. The model dimensions were set to 80 m × 40 m × 20 m, and the domain was discretized using a structured mesh consisting of 32,000 hexahedral elements and 4800 quadrilateral elements, as shown in Figure 3. Based on well logging and field test data, different mechanical parameters and in situ stress conditions were assigned to the sandstone reservoir, mudstone barrier, and lithological interfaces. The sandstone reservoir has an average Young’s modulus of 45 GPa and a Poisson’s ratio of 0.24, whereas the mudstone barrier has an average Young’s modulus of 35 GPa and a Poisson’s ratio of 0.29. For the sandstone reservoir, the minimum horizontal principal stress, maximum horizontal principal stress, and vertical stress are 52 MPa, 66 MPa, and 62 MPa, respectively. In the simulation, the maximum horizontal principal stress, minimum horizontal principal stress, and vertical stress were aligned with the X-, Y-, and Z-directions, respectively. Normal displacement constraints were applied to the corresponding external boundaries. Before fluid injection, the model was equilibrated under these boundary and initial stress conditions to obtain a stable geostress state. The injection point was located at the geometric center of the model, i.e., (40, 20, 10), and a constant-rate injection condition was imposed. The detailed mechanical parameters of each lithological layer are listed in Table 1.

The hydraulic fracturing simulations were conducted using GDEM software (version 1.0) based on the continuum–discontinuum element method (CDEM). The governing equations for block deformation, contact element failure, and fracture flow were solved within the GDEM framework. The numerical solution procedure was as follows. First, the three-dimensional geological model was established, and the mechanical parameters of the sandstone reservoir, mudstone barrier, and lithological interfaces were assigned. Second, the in situ stress field was applied and equilibrated to obtain the initial stress state before fluid injection. Third, fracturing fluid was injected at the prescribed injection point under a constant injection rate. During hydraulic fracturing, block deformation and fracture fluid pressure were updated in a coupled manner at each calculation step. The local normal and tangential stresses of contact elements were then checked using the maximum tensile stress criterion and the Mohr–Coulomb criterion. Once either criterion was satisfied, the corresponding contact element failed and was converted into a discontinuous fracture surface. Finally, the fracture geometry, injection pressure, fracture width, and cross-layer propagation behavior were recorded for subsequent analysis.

Table 1. Basic parameters of the physical model.

Lithology	Parameters	Values
Sandstone	Reservoir thickness/m	8
	Vertical stress/MPa	62
	Maximum horizontal principal stress/MPa	66
	Minimum horizontal principal stress/MPa	52
	Elastic modulus/GPa	45
	Poisson’s ratio	0.24

Table 1. Cont.

Lithology	Parameters	Values
Mudstone	Vertical stress /MPa	62
	Maximum horizontal principal stress/MPa	63
	Minimum horizontal principal stress/MPa	56
	Elastic modulus/GPa	35
	Poisson's ratio	0.29
Lithological interface	Tensile strength/MPa	0.5
	Cohesion/MPa	1
	Internal friction angle/°	20
Injection rate/(m ³ /min)		1.8
Fluid viscosity/mPa·s		5

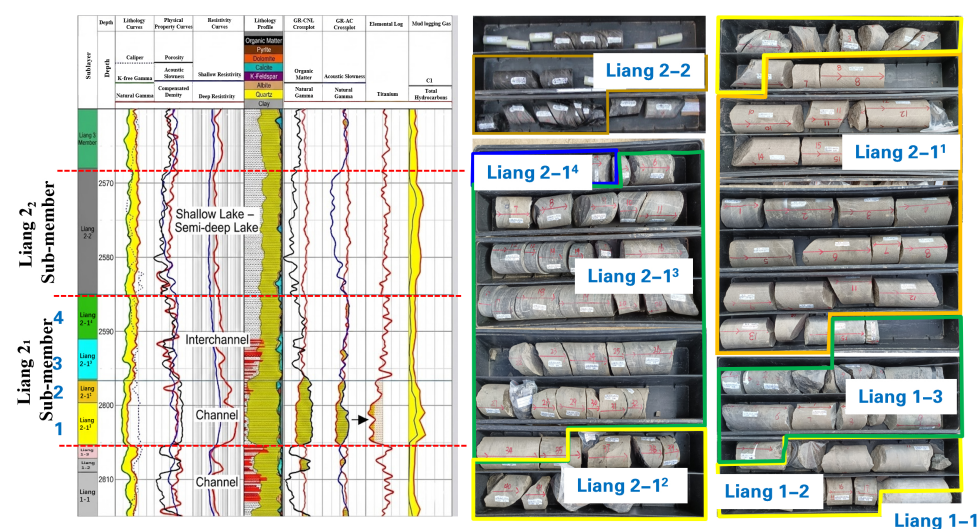


Figure 2. Comprehensive lithological distribution chart of the Liang 2 Member in Well X.

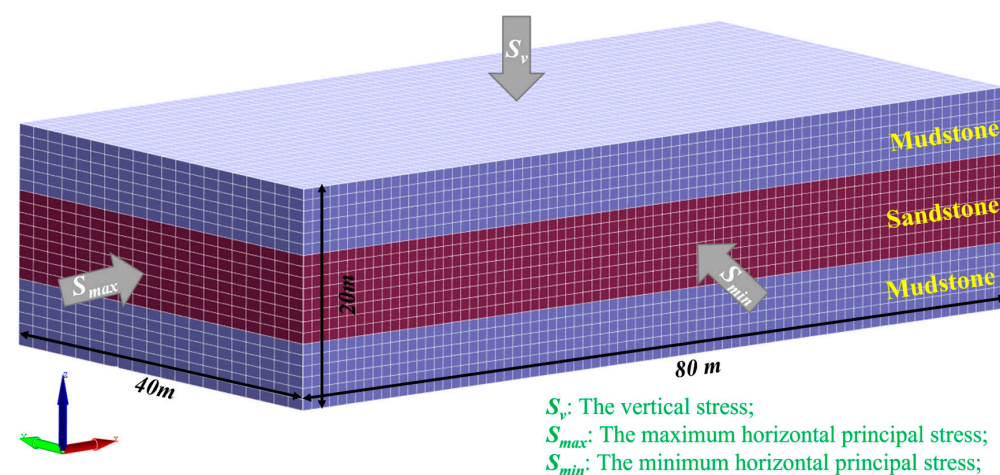


Figure 3. The physical model of the interbedded sandstone reservoir.

5. Results and Analysis

5.1. The Impact of Interlayer Stress Contrast

To investigate the impact of interlayer stress contrast on vertical fracture propagation, four scenarios with stress contrasts of 2, 4, 6, and 8 MPa were simulated, and the vertical fracture morphologies are shown in Figure 4. The results indicate that at a low stress contrast of 2 MPa, the hydraulic fracture successfully breaks through the lithological inter-

face and fully penetrates the mudstone barriers, achieving effective vertical propagation. However, as the stress contrast increases to 4 MPa, the shielding effect of the mudstone layer becomes apparent, resulting in only partial penetration and a noticeable discrepancy in propagation velocities between the sandstone and mudstone layers. With a further increase in stress contrast to 6 MPa, vertical propagation is significantly inhibited, with only limited fracture branches overcoming the interface resistance. Ultimately, at a high stress contrast of 8 MPa, vertical propagation is completely contained due to the strong stress barrier, strictly confining the hydraulic fracture within the sandstone reservoir without any cross-layer propagation.

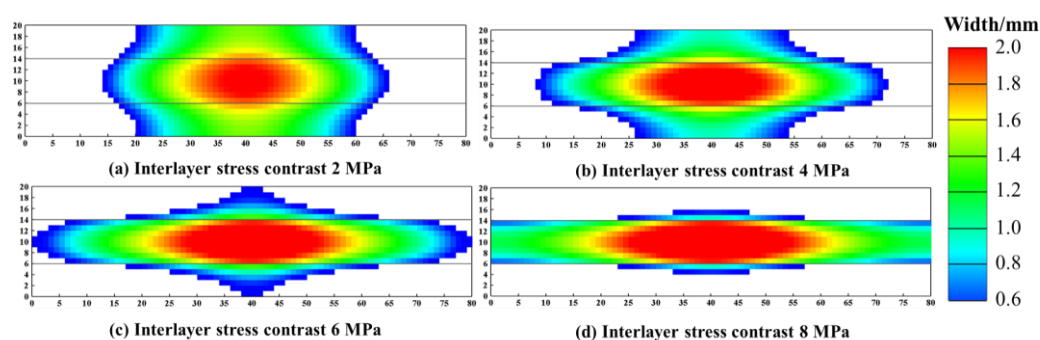


Figure 4. The vertical propagation morphologies under different interlayer stress contrasts.

In this study, fracture initiation pressure and breakdown pressure are distinguished [31]. Fracture initiation pressure refers to the injection pressure at which the first contact element satisfies the tensile or shear failure criterion and a new fracture surface is generated. Breakdown pressure refers to the peak value of the injection–pressure curve, which is generally associated with rapid fracture opening and unstable fracture propagation. Therefore, fracture initiation may occur before the breakdown pressure is reached. In the following analysis, the peak value of the injection–pressure curve is referred to as the breakdown pressure. Figure 5a shows the time-dependent evolution of injection pressure under different interlayer stress contrasts. Overall, the injection pressure exhibits a similar evolution pattern in all cases: it rapidly increases to the breakdown peak during the initial injection stage and then decreases sharply after fracture breakdown and unstable propagation. The subsequent pressure increase after the initial post-breakdown pressure drop does not indicate a second fracture initiation event. Previous studies have shown that fracture initiation pressure, unstable propagation pressure, and breakdown pressure are not necessarily identical, and the pressure response may continue to evolve after fracture initiation [31,33]. In the present model, this pressure recovery corresponds to the propagation-stage extension pressure. Under constant-rate injection, fluid continuously enters the fracture system after breakdown. However, the sandstone–mudstone interfaces and interlayer stress contrast restrict vertical fracture growth, which reduces the effective fracture storage capacity and increases resistance to further fracture extension. Therefore, the injection pressure gradually rebuilds during the constrained propagation stage. The breakdown pressures are comparable among the different cases, with values of 61.06, 61.05, 61.06, and 61.00 MPa, respectively, indicating that the interlayer stress contrast has a negligible influence on the breakdown pressure. However, during the propagation stage, the extension pressure increases with increasing interlayer stress contrast. As a higher stress contrast progressively restricts vertical fracture growth, higher fluid pressure is required to maintain fracture extension. Consequently, at $t = 300$ s, the injection pressures reach 55.91, 56.97, 58.00, and 58.90 MPa, respectively. Figure 5b shows the corresponding evolution of fracture width at the injection point. The fracture width increases nonlinearly with injection time, but the growth rate gradually decreases. A positive correlation is observed between

interlayer stress contrast and fracture width. This is because the restriction of vertical height growth under higher stress contrast causes more injected fluid to contribute to fracture aperture expansion. At $t = 300$ s, the fracture widths at the injection point are 4.58, 4.79, 4.95, and 5.05 mm, respectively.

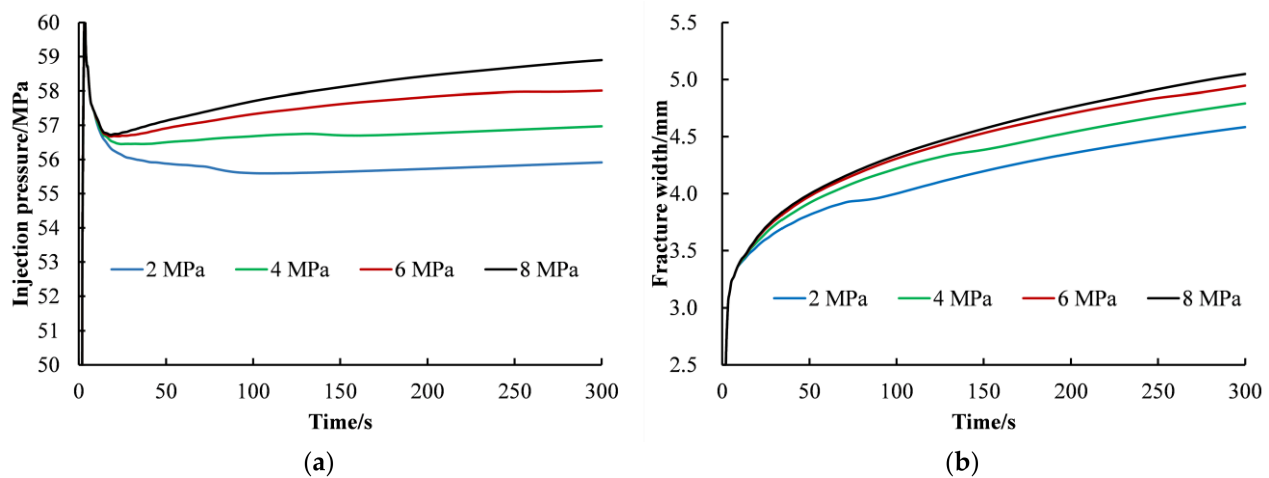


Figure 5. The evolution curves of (a) injection pressure and (b) fracture width under different interlayer stress contrasts.

5.2. The Impact of Vertical Stress Contrast

The vertical stress contrast, defined as the difference between the vertical stress and the minimum horizontal principal stress, is generally regarded as a critical parameter governing interface opening behavior and fracture reorientation. To investigate its influence, four scenarios with vertical stress contrasts of 4, 6, 8, and 10 MPa were simulated while maintaining a constant interlayer stress contrast of 4 MPa. Figure 6 shows the vertical fracture morphologies with different vertical stress contrasts. The simulation results indicate that within the investigated parameter range, variations in the vertical stress contrast have a negligible impact on the macroscopic propagation pattern. Across all scenarios, the hydraulic fractures exhibit consistent geometric characteristics, characterized by only partial penetration into the mudstone barriers without significant reorientation or morphological changes. This suggests that under these conditions, the interlayer stress contrast exerts a more dominant control over vertical propagation than the vertical stress contrast.

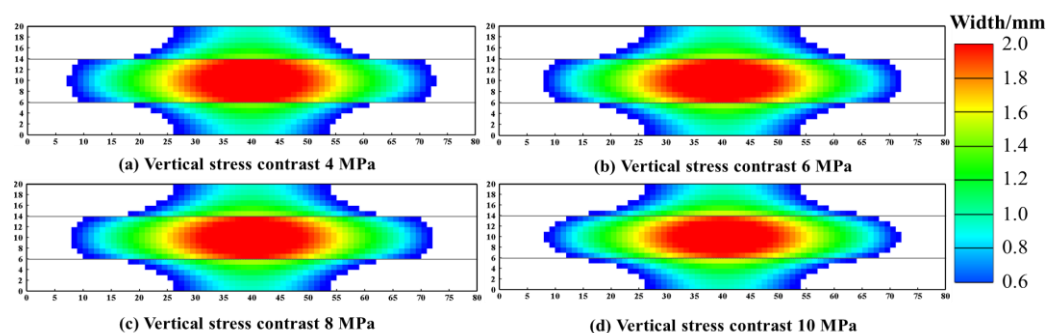


Figure 6. The vertical propagation morphologies under different vertical stress contrasts.

Figure 7a shows the time-dependent evolution of injection pressure under varying vertical stress contrasts. Consistent with the morphological analysis discussed in the previous section, the injection pressure curves exhibit a high degree of uniformity across different vertical stress contrasts, with negligible discrepancies between the scenarios. Figure 7b depicts the corresponding evolution of fracture width at the injection point. As

the fracturing fluid injection proceeds, the fracture width increases monotonically. Similar to the pressure response, the variation in vertical stress contrast has a very limited impact on the fracture width. At $t = 300$ s, the fracture widths are 4.81, 4.80, 4.79, and 4.79 mm, respectively. These comparable values further confirm that within the investigated range, the vertical stress contrast is not a dominant controlling factor.

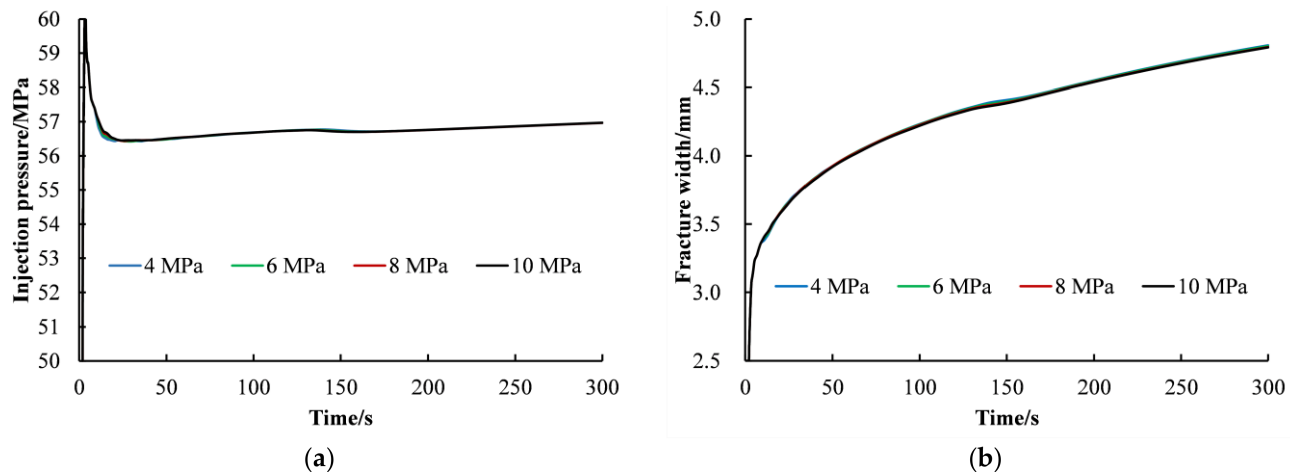


Figure 7. The evolution curves of (a) injection pressure and (b) fracture width under different vertical stress contrasts.

5.3. The Impact of Elastic Modulus Ratios

Figure 8 shows the vertical fracture morphologies under varying barrier-to-reservoir elastic modulus ratios. The difference in brittleness between lithologies, primarily characterized by the contrast in elastic modulus, is a critical factor governing vertical fracture propagation. In this study, while keeping the elastic modulus of the sandstone reservoir constant, the mechanical parameters of the mudstone barrier were adjusted to establish four distinct barrier-to-reservoir modulus ratios: 1:3, 5:9, 7:9, and 1:1. The simulation results indicate that as the modulus ratio increases—implying enhanced brittleness within the mudstone barrier—the hindrance effect of the interlayer interface diminishes. Consequently, the vertical propagation capability of the hydraulic fracture is significantly strengthened, leading to an increased stimulated reservoir volume and improved fracture coverage within the mudstone sections.

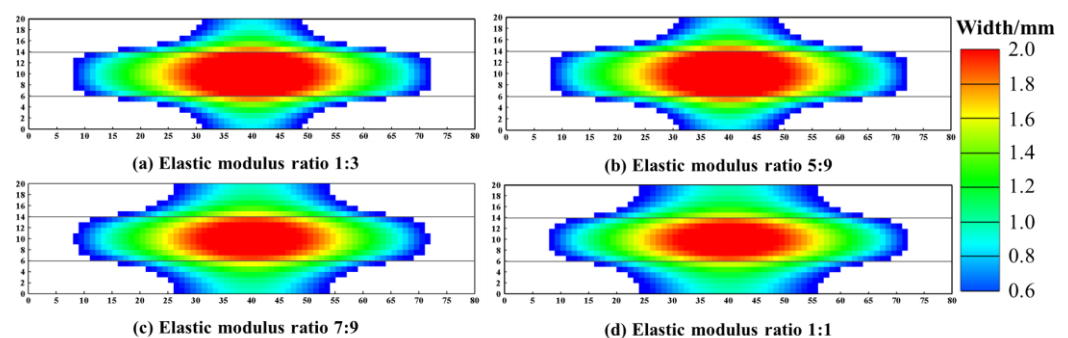


Figure 8. The vertical propagation morphologies under different elastic modulus ratios.

Figure 9a shows the time-dependent evolution of injection pressure under varying barrier-to-reservoir modulus ratios. Since the fracture initiates within the sandstone layer where properties remain constant, the breakdown pressure is negligibly affected by the modulus ratio, exhibiting only a marginal increase from 61.01 to 61.08 MPa. However, during the propagation stage, the extension pressure shows an upward trend with increasing

modulus ratio, which corresponds to enhanced stiffness and brittleness of the mudstone barrier; at $t = 300$ s, the extension pressures reach 56.54, 56.76, 56.97, and 57.17 MPa, respectively. Figure 9b shows the corresponding evolution of fracture width at the injection point. The results indicate an inverse relationship between the modulus ratio and fracture width: while a higher modulus in the barrier facilitates vertical propagation, it simultaneously restricts fracture aperture opening. Consequently, the fracture width decreases gradually as the modulus ratio increases. At $t = 300$ s, the fracture widths are recorded at 4.92, 4.83, 4.79, and 4.78 mm, respectively.

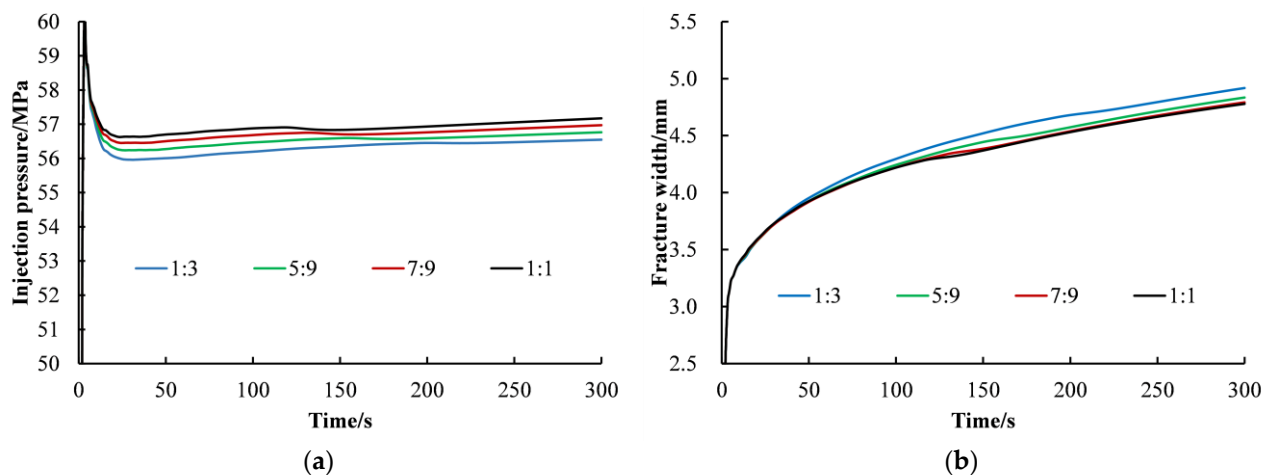


Figure 9. The evolution curves of (a) injection pressure and (b) fracture width under different elastic modulus ratios.

5.4. The Impact of Reservoir Thickness

To further investigate the impact of reservoir geometry on vertical fracture propagation, four scenarios with varying reservoir thicknesses of 4, 6, 8, and 10 m were simulated. Figure 10 shows the resulting vertical fracture morphologies under these different thickness conditions. The simulation results reveal an inverse relationship between reservoir thickness and the vertical cross-layer capability of the fracture. In the scenario with a thinner reservoir (4 m), the hydraulic fracture can more rapidly contact the upper and lower interfaces and overcome interlayer resistance, thereby achieving superior vertical breakthrough and stimulation effectiveness. However, as the reservoir thickness increases, the distance from the initiation point to the lithological boundaries increases, causing hydraulic energy to be predominantly dissipated in lateral propagation and fracture aperture expansion within the reservoir. Consequently, the difficulty of breaking through the interlayer interface increases, leading to a significant reduction in vertical cross-layer capability and a gradual transition in fracture morphology from a “breakthrough type” to a “contained type”.

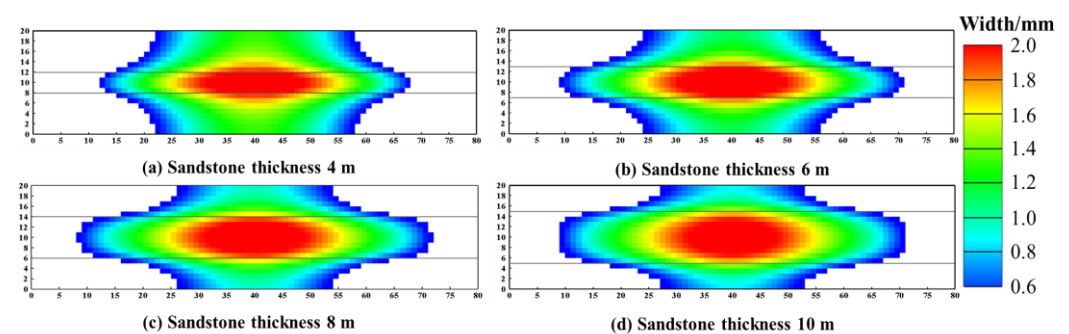


Figure 10. The vertical propagation morphologies under different reservoir thicknesses.

Figure 11a shows the time-dependent evolution of injection pressure under varying reservoir thicknesses. As previously mentioned, since the rock properties at the injection point remain constant, the breakdown pressures are comparable across all cases, recorded at 61.19, 61.05, 61.05, and 61.06 MPa, respectively. However, distinct variations are observed during the propagation stage: the fracture extension pressure decreases as the reservoir thickness increases. This is primarily because, in thicker reservoirs, the fracture tip is situated further from the high-stress barrier interfaces. Consequently, the fracture propagates predominantly within the low-stress, low-strength sandstone layer, experiencing weaker interlayer interface confinement effects and thus encountering lower resistance to fluid flow and fracture propagation. Figure 11b shows the evolution of fracture width at the injection point under different reservoir thicknesses. During the initial injection stage ($t = 6$ s), the fracture widths are identical (3.25 mm) across all scenarios as the fractures have not yet interacted with the boundaries. As fluid injection continues, the degree of vertical containment diverges, resulting in an inverse relationship between fracture width and reservoir thickness: thicker reservoirs exhibit weaker boundary confinement and a reduced passive ballooning effect. By $t = 50$ s, the fracture widths at the injection point are 4.01, 3.97, 3.92, and 3.86 mm, respectively. At the end of the simulation ($t = 300$ s), the fractures in the 4 m and 6 m reservoirs exhibit comparable, larger widths (~ 4.82 mm) due to strong vertical containment. In contrast, the fracture widths in the 8 m and 10 m reservoirs decrease to 4.79 mm and 4.74 mm, respectively.

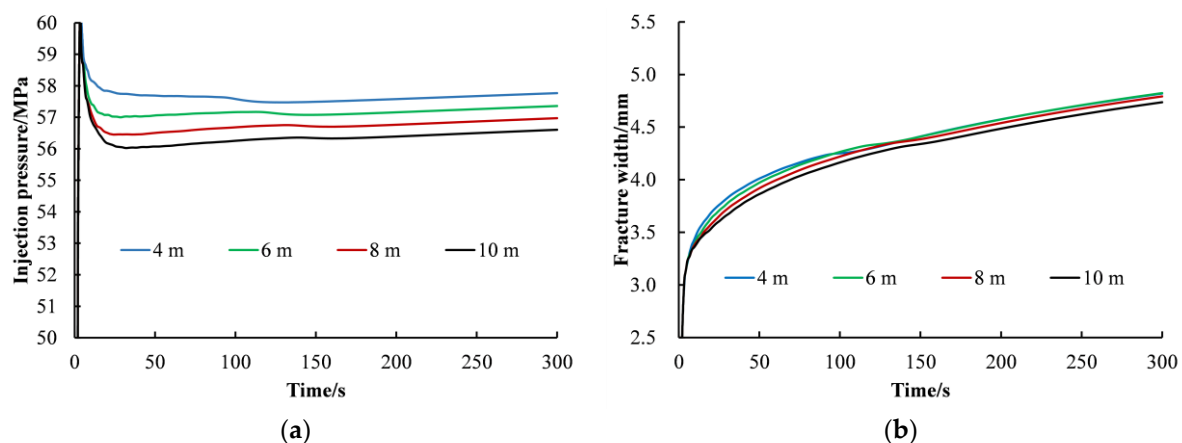


Figure 11. The evolution curves of (a) injection pressure and (b) fracture width under different reservoir thicknesses.

5.5. The Impact of Fluid Viscosity

Fracturing fluid viscosity is a critical controllable parameter for optimizing vertical cross-layer propagation in hydraulic fracturing. To investigate its impact on the vertical penetration capability within the Lianggaoshan reservoir, four fluid viscosity scenarios (5, 15, 25, and 50 mPa·s) were established for comparative simulation, with the corresponding vertical fracture morphologies illustrated in Figure 12. The simulation results indicate that an increase in fluid viscosity significantly reduces fluid mobility within the fracture and increases flow resistance, thereby substantially elevating the net pressure. This elevated net pressure effectively enhances the capability of the fracture to overcome interlayer resistance. Specifically, when the fluid viscosity reaches 25 mPa·s, the accumulated energy within the fracture is sufficient to break through the 4 MPa interlayer stress barrier, achieving effective vertical cross-layer propagation.

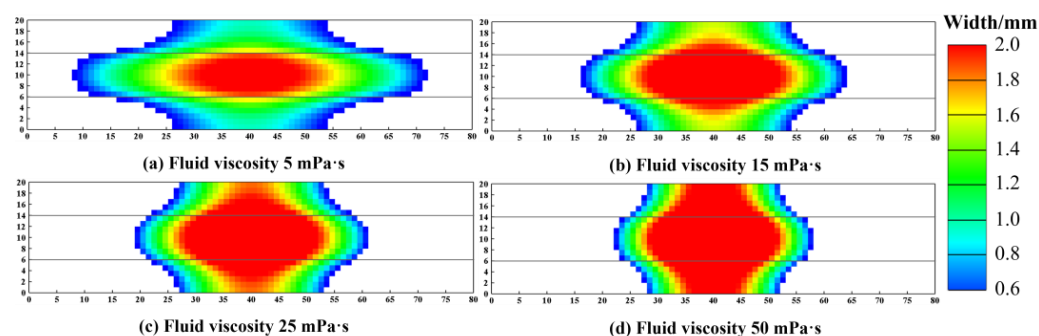


Figure 12. The vertical propagation morphologies under different fluid viscosities.

Figure 13a shows the time-dependent evolution of injection pressure under varying fluid viscosity conditions. The results indicate that increasing fluid viscosity significantly elevates both the formation breakdown pressure and the fracture propagation pressure. Specifically, the breakdown pressures for the respective cases were 61.05, 64.16, 65.65, and 69.59 MPa; meanwhile, at $t = 300$ s, the propagation pressures reached 56.97, 58.57, 59.76, and 62.15 MPa, respectively. Correspondingly, the elevated net pressure generated by the high-viscosity fluid leads to a substantial increase in fracture width. Figure 13b shows the evolution of fracture width at the injection point. A distinct upward trend in fracture width is observed with increasing viscosity; by $t = 300$ s, the fracture widths at the injection point reach 4.79, 5.43, 5.79, and 6.37 mm for the different fluid viscosity scenarios.

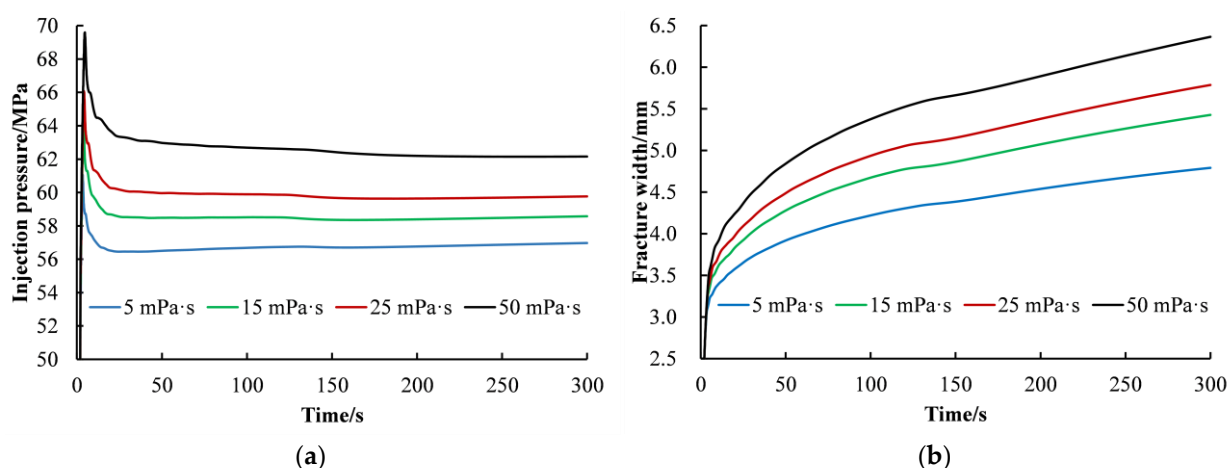


Figure 13. The evolution curves of (a) injection pressure and (b) fracture width under different fluid viscosities.

5.6. The Impact of Injection Rate

Injection rate serves as another critical operational parameter for regulating the net pressure within the fracture during hydraulic fracturing treatments. To clarify its controlling effect on vertical cross-layer behavior, comparative simulations were conducted under four injection rates: 0.6, 1.8, 3.0, and $3.6 \text{ m}^3/\text{min}$. The resulting vertical fracture morphologies under these different rates are shown in Figure 14. The simulation results indicate a positive correlation between the injection rate and the cross-layer propagation capability. Increasing the injection rate significantly accelerates the fluid pressurization rate within the fracture, thereby elevating the net pressure. This provides the fracture tip with a stronger driving force to overcome interlayer resistance. Notably, when the injection rate increases to $3.6 \text{ m}^3/\text{min}$, the accumulated energy within the fracture is sufficient to overcome the 4 MPa interlayer stress barrier, achieving effective vertical propagation and cross-layer breakthrough.

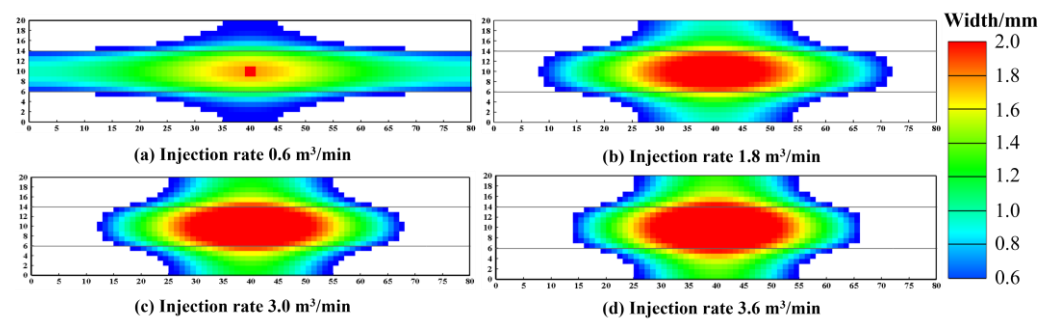


Figure 14. The vertical propagation morphologies under different injection rates.

Figure 15a shows the time-dependent evolution of injection pressure under varying injection rates. An increase in injection rate leads to an accelerated fluid pressurization rate within the fracture. Consequently, both the breakdown pressure and propagation pressure gradually increase with the injection rate. The recorded breakdown pressures are 58.57, 61.05, 62.71, and 63.35 MPa, respectively. Figure 15b shows the corresponding evolution of fracture width at the injection point under different injection rates. With an increasing injection rate, the growth rate of the fracture width accelerates significantly. Under the same injected volume, the fracture widths at the injection point are 4.28, 4.79, 5.07, and 5.17 mm, respectively.

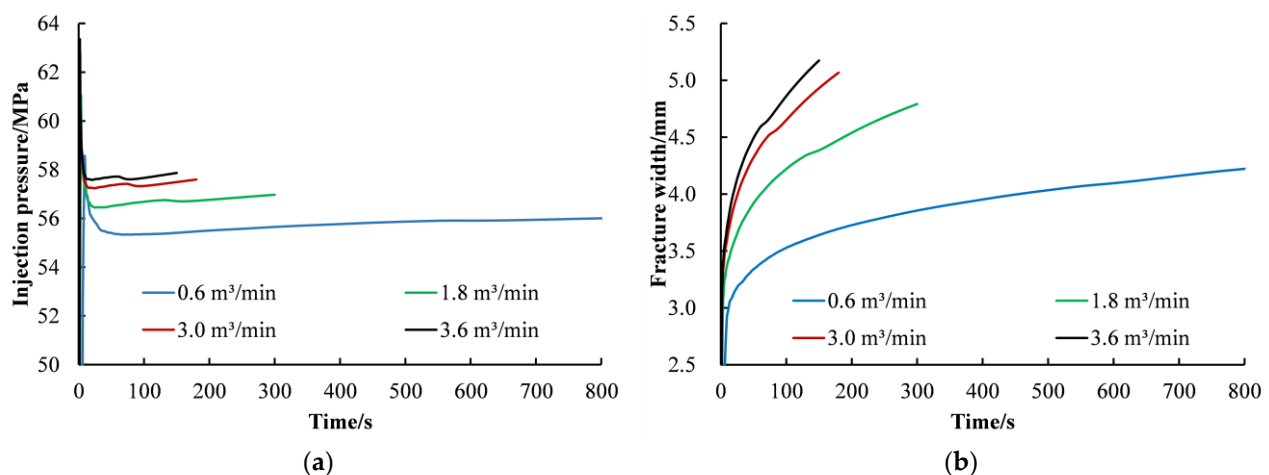


Figure 15. The evolution curves of (a) injection pressure and (b) fracture width under different injection rates.

6. Discussion

6.1. The Cross-Layer Propagation Pattern in the Lianggaoshan Interbed-Type Sandstone

To quantitatively characterize the vertical cross-layer propagation capability of hydraulic fractures and the effectiveness of reservoir stimulation, this study establishes a novel comprehensive evaluation index—the Effective Vertical Propagation Index (EVI). Introduced to overcome the limitations associated with single geometric parameters, such as fracture height or length, this index is defined as the product of the normalized fracture area and the fracture aspect ratio (see Equation (10)), thereby comprehensively reflecting both the vertical penetration depth and the overall scale of volumetric stimulation.

$$S = \frac{A}{L \times H}; \quad (13)$$

$$AR = \frac{H}{L}; \quad (14)$$

$$EVI = S \times AR = \frac{A}{L \times H} \times \frac{H}{L}, \quad (15)$$

where A is the vertical fracture area, L is the fracture length, and H is the fracture height.

Based on a comprehensive statistical analysis of simulation results considering various combinations of geological and engineering parameters, this study identifies three distinct vertical propagation modes for hydraulic fractures in the Lianggaoshan sandwich-type sandstone. These modes fundamentally reflect the competitive mechanism between the intra-fracture net pressure and the interlayer resistance:

- (1) Mode I ($EVI \leq 0.2$): Restricted Vertical Propagation. As illustrated in Figure 16a, the hydraulic fracture is strictly confined within the sandstone reservoir. In this scenario, strong stress barriers or high interface strength at the interlayer boundaries constitute an insurmountable vertical resistance. Since the net pressure is insufficient to breach this critical threshold, hydraulic energy is predominantly dissipated during lateral propagation. Consequently, the fracture geometry exhibits a distinct “long and narrow” characteristic (extensive length but limited height), failing to communicate with the overlying and underlying mudstone layers, resulting in a minimal vertical stimulated reservoir volume (SRV).
- (2) Mode II ($EVI \geq 0.4$): Effective Vertical Propagation. Depicted in Figure 16b, this mode represents the ideal scenario for achieving vertical breakthrough. Here, the accumulated net pressure is sufficient to overcome the interlayer stress contrast and interface toughness, facilitating simultaneous propagation (“co-development”) within both the sandstone reservoir and the mudstone barriers. This “cross-layer” propagation mode maximizes the vertical contact area between the fracture and the formation, ensuring effective mobilization and volumetric stimulation of the vertically stacked layer system.
- (3) Mode III ($0.2 < EVI < 0.4$): Partially Effective Cross-Layer Propagation. As shown in Figure 16c, the fracture morphology exhibits significant spatial non-uniformity. In the near-wellbore region, the high concentration of hydraulic energy enables the fracture to successfully penetrate the interlayer interfaces. However, as the fracture propagates into the far-field region, the fluid pressure within the fracture gradually attenuates. When the net pressure at the fracture tip drops below the vertical breakthrough threshold, the fracture morphology transitions, reverting to a contained propagation within the layer. This ultimately results in a hybrid morphology characterized by “cross-layer propagation near the wellbore and contained propagation in the far field”.

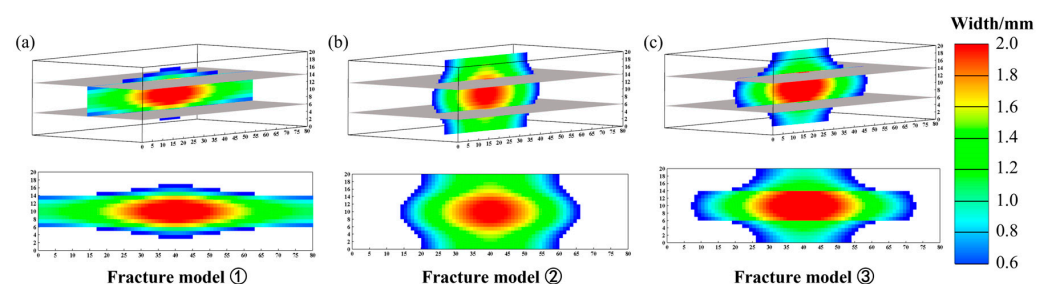


Figure 16. The cross-layer propagation pattern in the Lianggaoshan interbed-type sandstone.

6.2. Key Controlling Factors of Vertical Propagation in the Lianggaoshan Interbed-Type Sandstone

Taking the EVI as the reference sequence, Gray Relational Analysis (GRA) was employed to rank the sensitivity of various influencing factors, as shown in Figure 17. The quantitative results reveal that the interlayer stress contrast is the primary controlling factor restricting cross-layer propagation, followed by engineering parameters—specifically injection rate and fluid viscosity—which facilitate propagation by elevating the intra-fracture

net pressure to overcome formation resistance; conversely, the vertical stress contrast exhibits a minimal impact. Based on this sensitivity analysis, the study further delineates the operational optimization window for achieving effective cross-layer fracturing. Simulation results indicate that for a typical formation characterized by a strong 4 MPa interlayer stress barrier, ensuring sufficient fluid energy input as a prerequisite for achieving effective vertical breakthrough; specifically, when the fracturing fluid viscosity exceeds 25 mPa·s or the injection rate surpasses 3.6 m³/min, the generated net pressure becomes sufficient to overcome the interface resistance, thereby realizing the anticipated vertical propagation and volumetric stimulation of the reservoir.

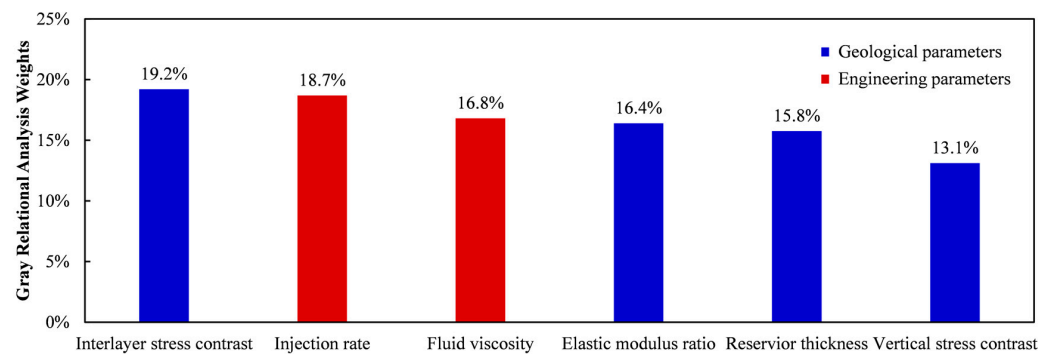


Figure 17. Gray relational weights of geological and engineering factors affecting vertical fracture propagation.

6.3. Model Limitations and Applicability

It should be noted that sandstone and mudstone are treated as equivalent linear elastic materials in the present model, while their lithological differences are represented by different elastic moduli, Poisson's ratios, strengths, and in situ stress states. This simplification allows the study to focus on the dominant effects of interlayer stress contrast, elastic modulus contrast, reservoir thickness, fluid viscosity, and injection rate on vertical fracture propagation. The model is mainly applicable to small-strain, elastic-dominated deformation before large-scale plastic yielding occurs. Under this condition, fracture propagation is primarily controlled by stress redistribution around the fracture tip and tensile or shear failure of contact elements. Therefore, the model can be used to evaluate the general tendency of fracture initiation, vertical extension, and interface crossing under an equivalent elastic framework.

However, mudstone plasticity, intrinsic anisotropy, and pre-existing weakened bedding planes are not explicitly considered. Ignoring these factors may underestimate plastic energy dissipation and bedding-parallel deflection, and may therefore overestimate the fracture crossing capability in formations with strong mudstone plasticity or well-developed weak planes. For such formations, elastoplastic and anisotropic constitutive models with explicitly weakened bedding planes should be incorporated in future work to further improve the prediction of cross-layer fracture propagation.

7. Conclusions

In this study, a 3D hydraulic fracturing model reflecting the “sandstone–mudstone interbedded” characteristics of the Lianggaoshan Formation was established using the continuum–discontinuum element method (CDem). The influence of geological and engineering parameters on vertical cross-layer propagation was systematically investigated. The main conclusions are drawn as follows:

- (1) The interlayer stress contrast is the primary geological factor governing vertical fracture propagation. Simulation results indicate that the interlayer stress contrast exerts

a significant inhibitory effect on cross-layer behavior, whereas the vertical stress contrast has a negligible impact on fracture morphology within the investigated range. Additionally, a higher barrier-to-reservoir modulus ratio (indicating a more brittle mudstone barrier) helps mitigate the interface hindrance effect, facilitating vertical extension. Conversely, an increase in reservoir thickness leads to the dissipation of hydraulic energy during lateral propagation, thereby reducing the vertical penetration capability.

- (2) Three distinct vertical evolution modes for hydraulic fractures in sandwich-type sandstone were identified. These are: (I) Restricted Vertical Propagation, where the fracture is strictly confined within the sandstone reservoir; (II) Effective Vertical Propagation, where the fracture propagates simultaneously within both sandstone and mudstone layers; and (III) Partially Effective Cross-Layer Propagation, characterized by a hybrid morphology of near-wellbore breakthrough and far-field containment.
- (3) GRA indicates that the sensitivity ranking is as follows: interlayer stress contrast > injection rate > fluid viscosity > elastic modulus ratio > reservoir thickness > vertical stress contrast. Under the baseline 4 MPa interlayer stress barrier, increasing the fluid viscosity to 25 mPa·s or the injection rate to 3.6 m³/min can promote effective vertical breakthrough.

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