

Article

High-Precision Measurement Technology of Reference Voltage for Feeder Verification Based on PIA-Bi-Directional FIR

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Abstract

To address the issue that voltage signals in the measurement circuits of distribution network feeders are susceptible to non-uniform sampling, wideband interference, and phase distortion during operation, which fails to meet the accuracy requirements of on-line verification, a high-precision measurement technology for feeder calibration reference voltage based on PIA-bidirectional FIR is proposed. Taking the bus voltage of the distribution network as the on-line reference benchmark, the proposed technology adopts a three-stage collaborative processing framework consisting of cubic spline Hermite interpolation reconstruction via PIA, bidirectional FIR filtering, and improved VMD-TLS-ESPRIT parameter identification to extract high-fidelity amplitude, phase, and frequency parameters of voltage. It can serve as a reference source for live on-line verification of feeder measurement circuits. Simulation results demonstrated that the amplitude error of waveforms reconstructed by the proposed technology was less than 0.05%, the phase error better than 0.36 arcseconds, and the frequency tracking error superior to 0.001 Hz. Compared with conventional methods, the presented technology achieves higher frequency tracking accuracy and better phase stability, which can effectively satisfy the demands of on-line verification for measurement circuits of distribution feeders.

Keywords: PIA cubic spline Hermite interpolation; bidirectional FIR filtering; high-precision measurement of reference voltage; signal processing

1. Introduction

The construction of novel power systems is being accelerated worldwide. As the connecting hub between transmission systems and end-users, distribution networks are experiencing continuous and rapid expansion in scale and complexity [1]. Especially under the large-scale integration of flexible resources such as distributed generators and electric vehicles, power flows within distribution networks exhibit bidirectional characteristics and severe fluctuations, imposing more stringent requirements on real-time monitoring and state awareness technologies [2,3]. Measurement systems centered on instrument transformers play a pivotal role in distribution network protection, metering, control, and other links, serving as an essential foundation for guaranteeing the safe and stable operation of power systems [4]. Nevertheless, field operational experience demonstrates that the operating environments of measurement circuits on distribution feeders are extremely complicated. Voltage signals are susceptible to adverse impacts including load variations,



Academic Editors: Zifan Lin and Herbert Ho-Ching Iu

Received: 27 May 2026

Revised: 26 June 2026

Accepted: 1 July 2026

Published: 3 July 2026

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harmonic disturbances, and stochastic output fluctuations of distributed generators, thereby exhibiting wideband distortion features. This creates substantial challenges for conventional signal processing methodologies established on steady-state or quasi-steady-state assumptions [5,6].

Bus voltages of distribution networks feature stable amplitudes and well-defined phase references, rendering them an ideal reference benchmark for live online calibration of distribution feeder measurement circuits. Nevertheless, non-uniform sampling distortion tends to be introduced during the acquisition and transmission of such signals, accompanied by intricate electromagnetic noise, which impedes the precise demodulation of their amplitude, phase, and frequency parameters [7]. Particularly for medium-voltage inductive voltage transformers, the nonlinear magnetization characteristics of iron cores induce self-generation of low-order harmonics; meanwhile, parasitic capacitances and inductances of windings may excite multi-frequency resonance [8–10], drastically amplifying harmonic transfer errors and even leading to ratio errors exceeding 10% [11–13]. Furthermore, external factors including load power factor, ambient temperature, and mechanical vibration degrade the transformation accuracy of instrument transformers [14,15], further raising the uncertainty in reference signal extraction [16]. Conventional offline calibration schemes rely on power-outage tests, which suffer from high costs and the narrow available time windows, rendering them inadequate to satisfy the requirements of live maintenance for modern distribution networks.

To address the above-mentioned issues, scholars worldwide have conducted extensive research from three perspectives: signal reconstruction, filtering denoising, and parameter identification.

In terms of signal reconstruction, interpolation techniques are widely adopted to compensate for non-uniform sampling distortion, with prevalent approaches including cubic spline interpolation, piecewise linear interpolation, and four-spectrum-line interpolation. Nevertheless, conventional cubic spline interpolation is prone to Runge phenomenon or overshoot oscillation under conditions of uneven sampling intervals and severe data loss. Although four-spectrum-line interpolation exhibits certain advantages in frequency-domain interpolation, its capability to restore time-domain waveform continuity is limited. It cannot simultaneously satisfy the continuity constraints of magnitude, slope, and curvature at sampling points, resulting in residual distortion of reconstructed waveforms at critical phase points [17,18]. Reference [19] designs a capacitive voltage divider for high-voltage measurement and proposes a corresponding temperature compensation strategy, yet it fails to address signal distortion correction under non-uniform sampling scenarios.

In the field of filtering and denoising, finite impulse response (FIR) filters are widely adopted owing to their favorable stability and linear-phase characteristics. Nevertheless, conventional one-pass FIR filtering inevitably introduces phase delay. Even with a linear-phase design, the resultant group delay shifts the overall phase of the signal, which fails to meet the special requirement of “zero additional phase shift” for the phase benchmark in feeder calibration. To eliminate phase delay, forward–backward filtering performs filtering operations in both forward and reverse sequences to counteract phase offsets mutually, thereby yielding zero-phase-shift outputs [20]. Although reference [21] realizes the online accuracy evaluation of electronic voltage transformers via recursive principal component analysis, its adaptability under complex harmonic distortion scenarios remains to be improved. Reference [22] proposes a nonlinear distortion suppression method based on a simplified Volterra model; however, the model identification procedure is relatively intricate, leading to particular difficulty in practical engineering implementation.

In terms of parameter identification, Variational Mode Decomposition (VMD) can adaptively decompose a signal into several intrinsic mode components. However, the

decomposition mode number K and penalty factor α generally rely on manual empirical configuration, which readily results in under-decomposition or over-decomposition and deteriorates the purity of the fundamental wave components [23]. Total Least Squares-Estimation of Signal Parameters via Rotational Invariance Techniques (TLS-ESPRIT) boasts outstanding parameter estimation resolution, yet it is extremely sensitive to the quality of input signals. When directly processing distorted voltage signals, its performance is severely compromised by residual noise and harmonic interferences. Reference [24] realizes online calibration of voltage transformers relying on phasor measurement units, but its applicability is limited under low observability conditions [25]. Reference [26] presents an optical fiber-powered low-power instrument transformer with proven high-precision measurement potential, yet retrofitting existing operational conventional transformer networks incurs excessive costs. Furthermore, Kalman filter-aided predictive state estimation and event-triggered robust filtering algorithms [27,28] have been successively proposed to enhance voltage sensing accuracy under sparse measurement configurations. Nevertheless, these methods indirectly improve measurement precision via state estimation and fail to address physical-layer signal distortions including non-uniform sampling distortion and phase aberration introduced by filtering from the perspective of signal processing.

In recent years, deep learning approaches have also been introduced into the field of power signal processing. For instance, Long Short-Term Memory (LSTM) networks are adopted for voltage waveform reconstruction and parameter prediction, while Convolutional Neural Networks (CNNs) are utilized for power quality disturbance classification [29]. Such data-driven methods exhibit powerful nonlinear mapping performance when sufficient training datasets are available; nevertheless, their performance relies heavily on the completeness of training sets and the quality of labeled samples, accompanied by poor interpretability. When confronted with complex distortion patterns unseen in training datasets within practical distribution networks, their generalization ability suffers from inherent uncertainty. By contrast, physical model-based signal processing methods require no training data, possess explicit mathematical foundations and favorable interpretability, and are more suitable for online calibration scenarios that impose stringent requirements on reliability and determinacy.

To overcome the aforementioned drawbacks, this paper proposes a high-precision measurement technique for feeder calibration reference voltages based on PIA and forward-backward FIR filtering. Taking distribution network bus voltage as the online reference benchmark, the proposed method proceeds in three steps. First, cubic spline Hermite interpolation based on Progressive Iterative Approximation (PIA) is adopted to reconstruct non-uniformly sampled signals with high fidelity, recovering the continuity and integrity of distorted waveforms [30]. Second, a forward-backward FIR filter is employed to suppress wideband interferences and eliminate phase distortion induced by conventional single-pass filtering. Finally, an improved VMD algorithm is combined with the TLS-ESPRIT algorithm to accurately extract voltage amplitude, phase and frequency parameters, which provides a high-fidelity reference source for live online calibration of distribution feeder measurement circuits.

The novelty of this paper lies in the construction of a collaborative processing chain consisting of “PIA-based cubic spline Hermite interpolation reconstruction, forward-backward FIR filtering, improved VMD decomposition, and TLS-ESPRIT parameter identification”, which is detailed as follows.

1. The PIA-based cubic spline Hermite interpolation is adopted. By imposing geometric continuity constraints on the magnitude, slope and curvature at sampling points, the proposed method effectively compensates non-uniform sampling distortion. Its reconstruction accuracy is improved by 2–4 orders of magnitude compared

with the conventional four-spectrum-line interpolation algorithm, which supplies complete time-domain input waveforms free from abrupt distortion for subsequent filtering procedures.

2. A dual-channel forward–backward FIR filtering architecture with the sequence of “forward filtering, reverse reordering, backward filtering, and signal restoration” is designed. The phase delays generated in the two filtering stages counteract each other to yield zero-phase-shift outputs. While suppressing wideband interference, the proposed structure strictly preserves the phase characteristics of fundamental components, thereby satisfying the requirement of zero extra phase shift for the phase reference in feeder calibration.
3. A comprehensive criterion incorporating total residual loss entropy is introduced to realize the adaptive optimization of the VMD decomposition mode number K and penalty factor α , eliminating the uncertainty induced by manual empirical parameter tuning. Combined with the TLS-ESPRIT algorithm, unbiased estimation of voltage amplitude, phase, and frequency is accomplished, with the frequency tracking error controlled at the order of 0.001 Hz.

The three aforementioned modules exhibit strict sequential dependency: Module (1) supplies distortion-free input signals for Module (2), Module (2) delivers phase-preserved pure signals to Module (3), and Module (3) provides stable fundamental components for final parameter extraction. The synergistic operation of the three modules jointly guarantees the precision and reliability of reference voltage measurement.

The organization of the remaining chapters of this paper is arranged as follows. Section 2 introduces the overall architecture for online calibration of distribution network measurement circuits and analyzes the characteristics of voltage signals. Section 3 elaborates on the three-stage implementation scheme, including PIA-based cubic spline Hermite interpolation, the design of forward–backward FIR filters, as well as the improved VMD and TLS-ESPRIT algorithms. Section 4 conducts comprehensive validations via waveform simulations complying with the IEEE 1159 standard [31], covering interpolation accuracy, filtering performance, parameter extraction errors, and other indicators. Section 5 concludes the paper.

2. Architecture Design of Distribution Network Measurement Circuit Verification

2.1. Inspection Technology for Distribution Network Measurement Circuits

To satisfy the stringent accuracy requirements of reference sources for live-line online verification in distribution networks, this paper establishes a synchronous verification mode with high-precision voltage measurement as the core. Taking distribution network bus voltage as the original signal, this scheme adopts a complete set of signal processing procedures to extract high-fidelity voltage parameters of amplitude, phase, and frequency, so as to build a stable and reliable verification reference source. On this basis, synchronous current excitation is injected into each outgoing feeder, realizing closed-loop verification for live operating feeder instrument transformers and their measurement circuits. Figure 1 presents the overall framework of the proposed voltage reference synchronous measurement and verification mode as shown.

With bus voltage selected as the core reference, collected signals are transmitted to the verification device through voltage transformers. Inside the device, signal reconstruction, filtering purification, and parameter extraction are carried out step by step to generate high-precision synchronous voltage reference signals. For comprehensive inspection of measurement circuits, the system generates matched synchronous current excitation based on the above voltage reference and injects it into the primary side circuit. Combined

with sampling feedback from field protection devices, closed-loop comparison analysis is conducted to complete error calculation and operating condition diagnosis.

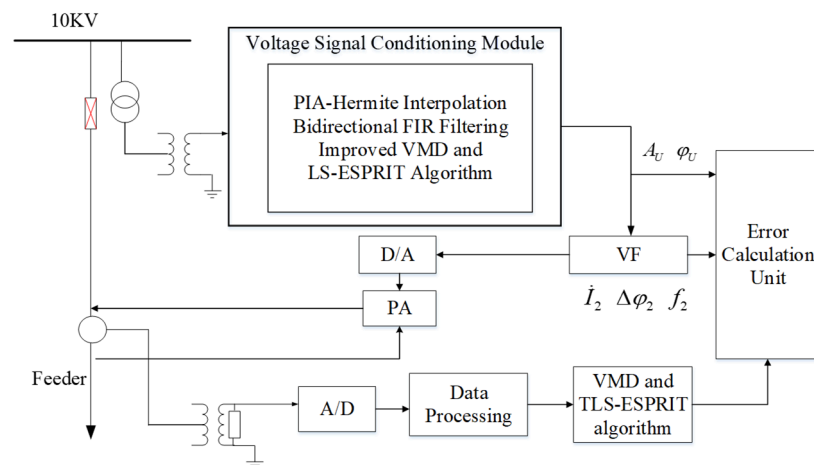


Figure 1. Distribution network online measurement verification synchronous measurement system.

To realize high-precision extraction of voltage reference and closed-loop verification, the system adopts a coordinated multi-module architecture as shown in Figure 2.

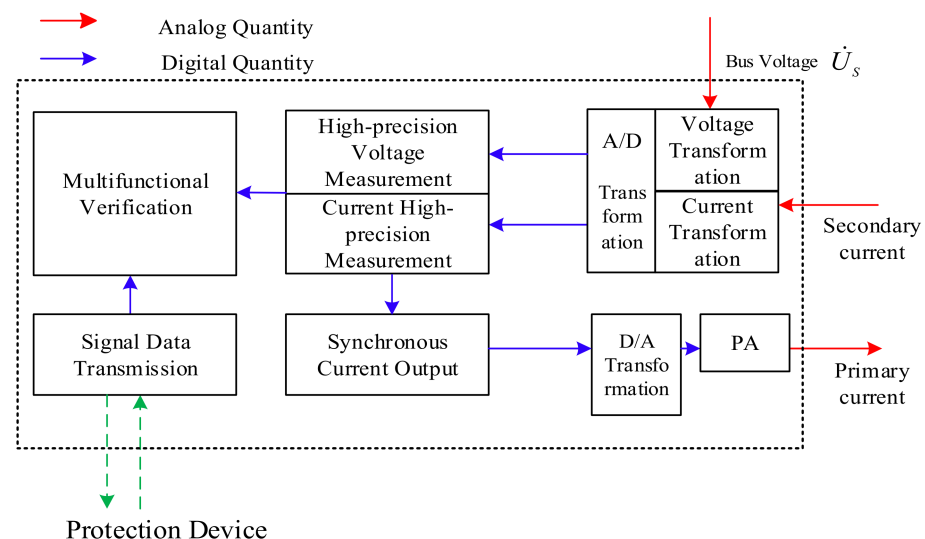


Figure 2. Main modules of the online verification system.

The data acquisition module collects original analog signals of bus voltage in real time and converts them into digital signals. As the core component of the system, the signal processing module completes the key conversion from distorted signals to pure reference signals. It first reconstructs voltage signals in a high-fidelity manner to remedy sampling distortion and then applies bidirectional distortion-free filtering to suppress broadband interference. On this basis, core parameters including voltage amplitude, phase, and frequency are solved accurately, so that highly reliable voltage reference values can be outputted. The current excitation and communication module generates standard test current based on the voltage reference and supports data synchronization and information interaction with protection devices. This architecture clarifies the complete logical chain, ranging from voltage signal acquisition and high-precision signal processing to closed-loop current verification.

To realize the above-mentioned architecture, the system is required to satisfy the following key hardware specifications. Inductive voltage transformers with a 0.2 accuracy

class are adopted, whose rated secondary output voltage is 100 V. The data acquisition module employs a 16-bit successive approximation analog-to-digital converter (ADC) with a sampling rate no less than 2 kHz. The sampling synchronization offset among all channels is controlled within $\pm 1 \mu\text{s}$ to guarantee phase consistency of multi-channel voltage signals. The signal processing module is implemented on a floating-point digital signal processor (DSP) or field-programmable gate array (FPGA) to meet the real-time computation requirements of interpolation iteration and filtering operations. The calibration device communicates with protection devices via optical fiber or industrial Ethernet, and the IEEE 1588 Precision Time Protocol is deployed to achieve distributed clock synchronization. All the above hardware selections represent mature solutions widely applied in the current distribution automation field, which guarantee satisfactory engineering practicability.

After a high-precision voltage reference is acquired, the calibration procedure enters the closed-loop verification stage. With the extracted voltage parameters, the calibration device modulates and generates an accurate standard current signal with the identical frequency and phase. Amplified by a power amplifier, this current signal is injected into the primary side of the current transformer under test.

The standard current flows through the entire measurement circuit to be calibrated and triggers the downstream protection device to perform normal sampling. The sampled data is transmitted back to the calibration device via the communication network. The calibration device subsequently conducts core comparison and error calculation, which performs real-time comparison between the expected value of the standard current derived from the voltage reference and the actual measured value returned by the protection device.

By analyzing the differences in amplitude, phase, and waveform characteristics between the two signals, typical defects in the measurement circuit can be accurately diagnosed, including reversed polarity, incorrect phase sequence, and excessive ratio/phase errors of transformers. Consequently, comprehensive performance verification of the measurement circuit can be completed without power supply interruption. Centering on the high-precision generation of voltage reference and closed-loop current verification, the proposed architecture guarantees the accuracy and reliability of the calibration results.

2.2. Analysis of Voltage Signal Characteristics for High-Precision Measurement

Taking the distribution network bus voltage as the verification reference source, its signal quality is subject to various adverse influencing factors. On the one hand, the integration of new energy and the operation of nonlinear loads lead to broadband spectral characteristics, along with multiple interference components such as harmonics and interharmonics. The typical spectral features are presented in Figure 3.

On the other hand, field sampling tends to cause non-uniform signal distortion, which is further superimposed with complex electromagnetic noise. For this reason, conventional methods fail to realize accurate measurement of amplitude, phase, and frequency parameters.

A three-stage collaborative processing scheme is proposed in this paper to cope with the preceding signal characteristics. In the first stage, PIA-based cubic spline Hermite interpolation is utilized to reconstruct non-uniformly sampled signals and restore waveform distortions induced by clock jitter and sampling drift. The signal density after interpolation is quadrupled relative to the original sampling rate, laying a high-resolution data foundation for subsequent filtering and parameter identification. In the second stage, forward-backward FIR filtering is adopted to suppress wideband interference while eliminating the phase delay incurred by filtering operations, to guarantee the phase fidelity of the reference voltage. In the third stage, the improved VMD and TLS-ESPRIT algorithms are combined to further suppress residual noise during modal decomposition and

parameter identification, realizing high-precision extraction of voltage amplitude, phase, and frequency.

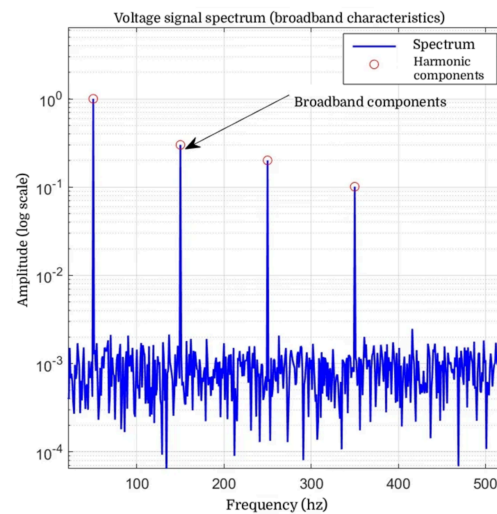


Figure 3. Voltage and current spectrum characteristics.

The logical cascade of the three stages is described as follows: the interpolation module provides complete time-domain inputs for filtering; the filtering module delivers phase-preserved clean signals for parameter identification; and the parameter identification module accomplishes the ultimate high-accuracy extraction on the basis of the outputs from the previous two modules. The detailed algorithm design of each module is elaborated in Section 3.

3. High-Precision Voltage Measurement Technology Based on PIA-Bidirectional FIR

3.1. Cubic Spline Hermite Interpolation Algorithm Based on PIA

3.1.1. Implementation of Progressive Iterative Approximation Algorithm

Progressive Iterative Approximation PIA is a data fitting method based on stepwise optimization, which is particularly suitable for scenarios requiring smooth transitions between connected points. For basic point-connection tasks, the algorithm starts from given reference points and determines the initial connection positions by calculating the intervals between adjacent points. It then iteratively revises the layout of key points, so that the generated path can accurately pass through all the reference points.

When the connection needs to be upgraded to satisfy directional continuity, the system must not only ensure that the path traverses the reference points but also matches the directional constraints at each point. For this purpose, adjustment points are inserted between each connection segment, and smooth transitions of the path direction are realized by modifying the parameters of key points step by step.

For connections with the higher requirement of curvature continuity, the constraints on position, direction, and bending curvature at the reference points must be satisfied simultaneously. In this case, two adjustment points are added in each segment, and a multi-level parameter correction strategy is adopted to form a smooth path that meets multiple continuity criteria. This hierarchical, stepwise optimization scheme strikes a favorable balance between computational efficiency and path quality.

3.1.2. Mathematical Modeling and Geometric Constraints of PIA-Hermite Interpolation

To construct an interpolation curve that satisfies specific smoothness requirements, the curve must meet both amplitude and slope constraints simultaneously at the connection

points. Given a set of non-uniformly spaced points, if the curve is required to match the given values exactly at each point in amplitude, while its slope also follows predefined specifications, the piecewise cubic spline approach can be used.

For a non-uniform knot system defined over the interval $[a, b]$, data reconstruction can be performed using cubic spline Hermite functions. This method is subject to three basic constraints:

1. Each segment between two adjacent points is described by a cubic polynomial
2. All adjacent curve segments maintain smooth continuity at their connection points
3. At each junction point, the amplitude and slope of the curve strictly coincide with the predefined values.

This piecewise cubic construction method ensures that the entire curve fits the data at each point precisely, while maintaining a smooth overall trend. When reconstructing voltage waveforms for power system transient analysis, extended parameters are introduced to improve adjustment flexibility. First, a piecewise refinement strategy is applied within the basic time interval, and precise control of the waveform slope is achieved by inserting new adjustment points.

The voltage variation in the first half-interval can be represented by a combination of piecewise cubic equations, as shown in Equations (1) and (2):

$$r_i\left(\frac{u}{2}\right) = \begin{bmatrix} 1 & \frac{u}{2} & \frac{u^2}{4} & \frac{u^3}{8} \end{bmatrix} N_i \begin{bmatrix} P_{i-1} \\ P_i \\ P_{i+1} \\ P_{i+2} \end{bmatrix} = \begin{bmatrix} 1 & u & u^2 & u^3 \end{bmatrix} N_i [K]_i \begin{bmatrix} P_{i-1} \\ P_i \\ P_{i+1} \\ P_{i+2} \end{bmatrix} \quad (1)$$

$$[K]_i = N_i^{-1} \begin{bmatrix} 1 & & & \\ & \frac{1}{2} & & \\ & & \frac{1}{4} & \\ & & & \frac{1}{8} \end{bmatrix} N_i \quad (2)$$

By adjusting the positions of the newly added control points, this method not only guarantees accurate values at key points of the waveform but also realizes smooth connection between adjacent segments.

Based on the mathematical model given in Equation (3), the basic combination of control points can be derived. The visualization result shown in Figure 4 clearly presents the original interpolated point cloud, the topology of the basic control vertices, and the distribution of the corresponding cubic spline curves, as shown in Equation (3):

$$\begin{aligned} [P]_j &= [Q_{j-1} Q_j Q_{j+1} Q_{j+2}]^T = [K]_i [P_{i-1} P_i P_{i+1} P_{i+2}]^T \\ &= N_i^{-1} \begin{bmatrix} 1 & & & \\ & \frac{1}{2} & & \\ & & \frac{1}{4} & \\ & & & \frac{1}{8} \end{bmatrix} N_i \begin{bmatrix} P_{i-1} \\ P_i \\ P_{i+1} \\ P_{i+2} \end{bmatrix} \end{aligned} \quad (3)$$

Figure 4 intuitively illustrates three core elements, including the distribution of raw data points, the arrangement of control nodes, and the trend of the resulting piecewise cubic curve. Through spatial relationships, the figure demonstrates how the initial data point group is shaped by the positioning and adjustment of control points.

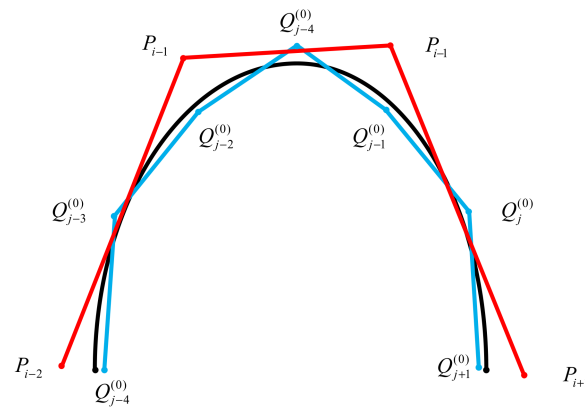


Figure 4. G1 Data Interpolation.

The augmented control vertex matrix $[R]_j$ satisfies the geometric constraint equations, and its generation mode is as shown in Equation (4):

$$[R]_j = [R_{j-1} R_j R_{j+1} R_{j+2}]^T = N_i^{-1} \begin{bmatrix} 1 & & & \\ & \frac{1}{3} & & \\ & & \frac{1}{9} & \\ & & & \frac{1}{27} \end{bmatrix}_i N_i \begin{bmatrix} P_{i-1} \\ P_i \\ P_{i+1} \\ P_{i+2} \end{bmatrix} \quad (4)$$

where $i = 0, 1, \dots, n, j = 3i$.

As shown in Figure 5, G2 high-order smooth connection is realized via the marked adjustment points. Two adjustment points are added within each curve segment to construct a geometric structure that meets multi-order continuity requirements. By increasing the number of adjustable points, an extended control point group is formed, whose arrangement strictly complies with the smoothness requirements of the curve trend.

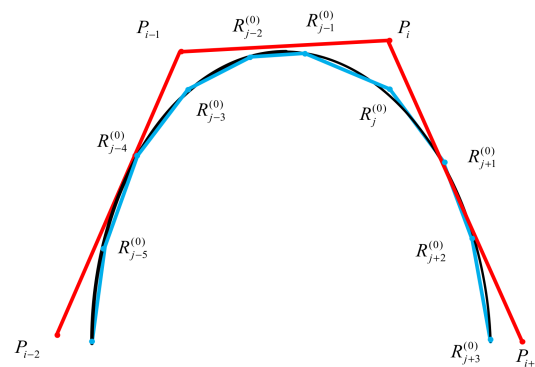


Figure 5. G2 data interpolation.

The termination criterion of the iteration process adopts a dynamic convergence monitoring mechanism. When the Euclidean displacements of all control vertices within successive iteration steps are less than the preset tolerance range, the algorithm is judged to have reached convergence, as shown in Equation (5):

$$\begin{aligned} R_j^{(k+1)} &= \frac{1}{n_{12}^i} \left[V_i - n_{11}^i R_{j-1}^{(k)} - \frac{n_{13}^i}{n_{23}^i} \left(at_i - n_{21}^i R_{j-1}^{(k)} - n_{22}^i R_j^{(k)} \right) \right], \\ R_{j-1}^{(k+1)} &= \frac{n_{23}^i}{n_{11}^i n_{23}^i - n_{12}^i n_{21}^i} \left[V_i - n_{13}^i R_{j+1}^{(k)} + \frac{n_{12}^i n_{22}^i}{n_{23}^i} R_j^{(k+1)} - \|d_i\| t_i \right], \\ R_{j+1}^{(k+1)} &= \frac{1}{n_{13}^i} \left[V_i + \frac{n_{12}^i n_{22}^i}{n_{23}^i} R_j^{(k+1)} - \frac{n_{11}^i n_{23}^i - n_{12}^i n_{21}^i}{n_{23}^i} R_{j-1}^{(k+1)} - \|d_i\| t_i \right], \\ \|R_j^{(k+1)} - R_j^{(k)}\|_2 &< \varepsilon_R, j = 0, 1, \dots, 3n. \end{aligned} \quad (5)$$

where $R_j^{(k+1)}$, $R_{j-1}^{(k+1)}$ and $R_{j+1}^{(k+1)}$ denote the updated values of the j -th, $(j - 1)$ -th, and $(j + 1)$ -th augmented control vertices in the $(k + 1)$ -th iteration. $R_j^{(k)}$, $R_{j-1}^{(k)}$, and $R_{j+1}^{(k)}$ represent the corresponding original values of the augmented control vertices obtained in the k -th iteration. V_i stands for the position constraint vector of the i -th local window, and α_i refers to the first-order derivative constraint vector of the i -th local window, which guarantees the first-order continuity of the interpolated curve. $\|d_i\|_2 t_i$ is the product of the coefficient of the local interpolation polynomial and the time step, which is adopted for time-axis correction under non-uniform sampling conditions. Its value is determined by the interval between every pair of adjacent original non-uniform sampling points. k is the iteration count of the PIA algorithm, and j is the serial number of the augmented control vertex. $\|\cdot\|_2$ denotes the vector 2-norm, which is utilized to calculate the iterative displacement of the augmented control vertices. ε_R represents the predefined convergence tolerance threshold to regulate interpolation precision.

The convergence rate of the PIA interpolation iteration proposed in this paper is governed by the smoothness order of the interpolated curve. When G1 continuity constraints that enforce continuous first-order derivatives are applied, the interpolation error on the order of 10^{-7} can generally be achieved within 5 to 10 iterations. When G2 continuity constraints that enforce continuous second-order derivatives are adopted, the convergence rate declines moderately, and roughly 10 to 15 iterations are required to reach an error magnitude of 10^{-10} . A fixed number of 20 iterations is utilized throughout all simulation cases to guarantee full convergence. For practical online operation scenarios, the iteration count can be dynamically adjusted according to specific precision requirements.

The interpolation example is illustrated in Figure 6, where subfigures (a), (b), and (c) respectively show the complete input data of the voltage curve, the visualization of interpolated data points, and unit tangents of the curve, as well as the rendering of interpolated data points, unit tangents, and curvature vectors.

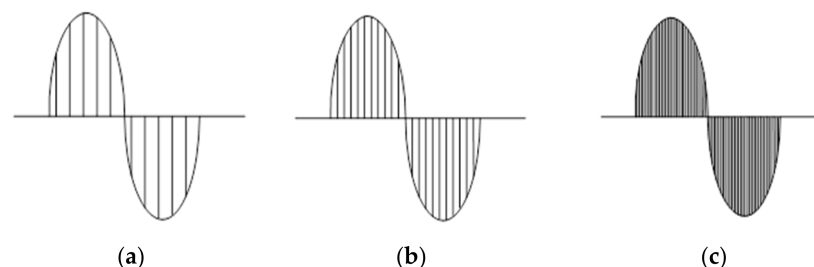


Figure 6. Curve PIA cubic spline Hermite interpolation. (a) Full input data of voltage curves. (b) Interpolated data points and unit tangents of curves. (c) Interpolated data points, unit tangents and curvature vectors of curves.

3.2. Design of Bidirectional FIR Filter

Finite impulse response digital filters feature excellent stability and linear-phase characteristics. Compared with infinite impulse response digital filters, their advantages are prominent, leading to wider applications in engineering practice [19]. Therefore, the FIR filter is selected in this paper to correct the interpolated voltage signal. The unit impulse response $h(n)$ of an FIR filter is a finite sequence with length $M + 1$. Its output signal $y(n)$ can be expressed as the linear convolution of the input signal $x(n)$ and $h(n)$, as shown in Equation (6):

$$y(n) = \sum_{k=0}^M h(k) \cdot x(n - k) \quad (6)$$

When $h(k)$ satisfies the symmetry or anti-symmetry condition, the filter possesses a strict linear-phase characteristic, which avoids signal phase distortion. In the case of even

symmetry with an even sequence length, the output $y(n)$ can be significantly simplified by combining symmetric terms, as given in Equation (7):

$$y(n) = \sum_{k=0}^{M/2} h(k)[x(n-k) + x(n-(M-k))] \quad (7)$$

This symmetry optimization can reduce the number of multiplications required by the direct-form structure from $M + 1$ to half, while retaining M stage delay units and accumulation operations. It improves computational efficiency while maintaining real-time signal processing. Therefore, FIR filters are widely used in scenarios demanding high phase accuracy and stability, such as interpolated signal correction.

The time-delay effect of FIR filtering can be described as follows. When a sinusoidal signal with frequency ω passes through a digital system, if the system introduces a time delay t , the phase lag of the output signal relative to the input signal is as given in Equation (8):

$$\Delta\phi = \omega t \quad (8)$$

Conversely, if the phase shift of a certain frequency component introduced by the system is $\Delta\phi$, the time delay of this signal by the system can be derived accordingly. Practical engineering signals are usually complex waveforms with non-single frequencies. According to the Fourier transform principle, such signals can be decomposed into a linear combination of multiple sinusoidal components, as shown in Equation (9):

$$x(t) = \sum_k A_k \cos(\omega_k t + \phi_k) \quad (9)$$

To avoid phase distortion after the signal passes through the system, it is necessary to ensure that the phase-frequency characteristics of all frequency components satisfy a strict linear relationship. That is, the phase delay time t is independent of frequency. This condition is equivalent to the phase-frequency characteristic curve of the system being a straight line, as shown in Equation (10):

$$\phi(\omega) = -\tau\omega \quad (10)$$

where τ denotes the constant group delay, indicating that the time for all frequency components to pass through the system is fixed.

The parameter selection criteria for the FIR filter adopted in this design are specified as follows. The equivalent sampling rate of signals after interpolation reaches 8 kHz, which corresponds to a Nyquist frequency of 4 kHz. To fully retain the 50 Hz fundamental component and suppress high-frequency quantization noise generated during interpolation, the cutoff frequency is set to 2.75 kHz, accounting for approximately 68.75 percent of the Nyquist frequency. A safety margin of nearly 30 percent is reserved to prevent signal truncation caused by component tolerance or grid frequency fluctuation. The 255-tap filter strikes a favorable balance between 80 decibels of stopband attenuation and a processing latency of 403 microseconds, thereby satisfying the real-time performance demands of live online calibration.

The parameter combination of a 255-order Kaiser window with a cutoff frequency of 2.75 kHz achieves an optimal balance among frequency band coverage, interference suppression, and computational efficiency. The 2.75 kHz cutoff frequency fully retains the fundamental component while reserving a 10% safety margin to prevent signal truncation caused by device tolerance or frequency fluctuations. An 80 dB stopband attenuation can

eliminate high-frequency quantization noise introduced by interpolation. The amplitude–frequency responses compared with other parameter schemes are shown in Figure 7.

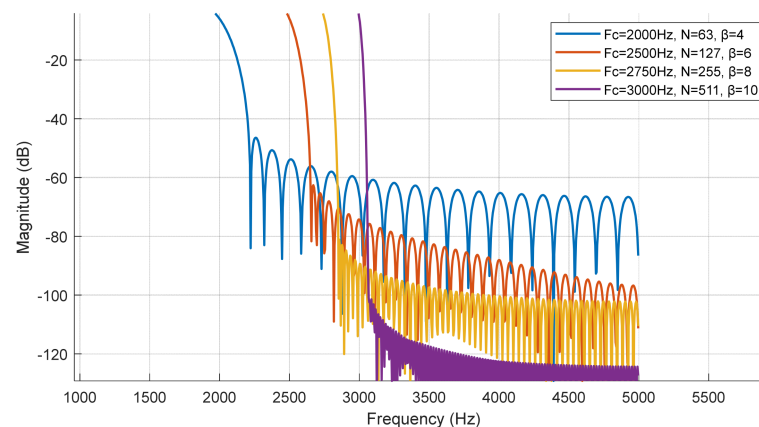


Figure 7. Amplitude–frequency response with different parameters.

Based on this linear-phase characteristic, a bidirectional filtering method is adopted in this paper. A symmetric filter structure is designed to eliminate nonlinear phase distortion, thereby ensuring the integrity of signal waveforms in both time and frequency domains simultaneously. Bidirectional filtering performs two filtering operations on the signal, namely forward filtering and reverse filtering, to eliminate the phase delay introduced during filtering. The processed result can preserve the waveform characteristics of the signal while filtering out noise or other unwanted frequency components. The specific procedure is as follows:

1. In forward filtering, the original signal is processed through conventional forward filtering by an FIR filter or other linear filters, which inevitably produces a certain phase delay
2. In reverse filtering, the output signal after forward filtering is time-reversed and refiltered with the same FIR filter. Due to the time reversal operation, the phase delay generated in the first filtering process can be offset during the second filtering
3. In signal recovery, the output of the secondary filtering is reversed again to restore the original time sequence of the signal.

3.3. Principles of Improved VMD and TLS-ESPRIT Algorithms

Although the voltage signal processed by PIA cubic Hermite interpolation reconstruction and bidirectional FIR filtering has eliminated most broadband interference and non-uniform sampling distortion, modal aliasing and residual impulse noise may still exist, which affect the accuracy of parameter extraction. Aiming at the characteristics of narrow-band concentration and impulse-type high-frequency noise of distribution network voltage signals, this paper introduces an improved VMD algorithm for adaptive modal decomposition, combined with the TLS-ESPRIT algorithm to accurately identify the amplitude, phase, and frequency of power–frequency voltage. Without additional denoising algorithms, it meets the requirements of high-precision measurement of distribution network reference voltage and constructs a full-process signal processing system of interpolation reconstruction–filtering denoising–modal decomposition–parameter identification.

3.3.1. Improved VMD Algorithm

The core of the improved VMD algorithm is to adaptively determine the decomposition mode number K via the total entropy of residual loss and optimize the penalty factor by constructing a comprehensive criterion combining the Chebyshev distance of

center frequencies and the improved edge permutation entropy. It realizes blind source separation of distribution network voltage signals, effectively distinguishing the dominant 50 Hz fundamental component from residual harmonic and noise components, and finally outputs intrinsic mode function components with separated center frequencies and compact bandwidths.

The preprocessed distribution network bus voltage signal is denoted as $v(t)$. The goal of the algorithm is to decompose it into K IMF components $\{u_i(t)\}_{i=1}^K$, so that the sum of the estimated bandwidths of all modal components is minimized while satisfying the constraint $\sum_{i=1}^K u_i(t) = v(t)$. To solve this constrained variational problem, the Lagrange multiplier $\lambda(t)$ and penalty factor α are introduced, and the augmented Lagrangian expression is defined as shown in Equation (11):

$$L(\{u_i\}, \{\omega_i\}, \lambda) = \alpha \sum_{i=1}^K \sum_{n=1}^L \|\partial(e^{-j\omega_i t} u_{i,n}(t))\|_2^2 + \sum_{n=1}^L \left\| v(t) - \sum_{i=1}^K u_{i,n}(t) \right\|_2^2 + \sum_{n=1}^L \left\langle \lambda_n(t), v(t) - \sum_{i=1}^K u_{i,n}(t) \right\rangle \quad (11)$$

where ω_i represents the center frequency of the i -th IMF component, ∂ denotes the partial derivative with respect to time, $\|\cdot\|_2$ represents the 2-norm, and $\langle \cdot, \cdot \rangle$ stands for the inner product operation.

The alternating direction method of multipliers is adopted to iteratively update $u_i(t)$, ω_i , and $\lambda(t)$ until the convergence criterion is satisfied. The iterative formulas are shown in Equations (12)–(14):

$$\hat{u}_i^{l+1}(\omega) = \frac{\hat{v}(\omega) - \sum_{j \neq i} \hat{u}_j^{l+1}(\omega) + \frac{\hat{\lambda}(\omega)}{2}}{1 + 2\alpha(\omega - \omega_i^l)^2} \quad (12)$$

$$\omega_i^{l+1} = \frac{\int_0^\infty \omega \left| \hat{u}_i^{l+1}(\omega) \right|^2 d\omega}{\int_0^\infty \left| \hat{u}_i^{l+1}(\omega) \right|^2 d\omega} \quad (13)$$

$$\hat{\lambda}^{l+1}(\omega) = \hat{\lambda}^l(\omega) + \alpha(\hat{v}(\omega) - \sum \hat{u}_{i,n}^{l+1}(\omega)) \quad (14)$$

where the superscript l denotes the iteration number, $\hat{\cdot}$ signifies the Fourier transform of the signal.

The iteration loop continues until the convergence criterion is satisfied, as shown in Equation (15):

$$\sum_i^K \frac{\left\| u_i^{l+1} - u_i^l \right\|_2^2}{\left\| u_i^l \right\|_2^2} < \varsigma \quad (15)$$

where ς denotes the convergence error tolerance. Once the convergence criterion is satisfied, the final u_i and ω_i are outputted.

To address the problem that the mode number K of traditional VMD needs to be set empirically, the total residual loss entropy E_r is introduced to quantify the decomposition adequacy, and the mode number is adaptively determined as shown in Equation (16):

$$E_r = \sum_{i=1}^K \frac{\left\| u_{i,n-1} \right\|_2^2 - \left\| u_{i,n} \right\|_2^2}{\left\| v(\omega) \right\|_2^2} \quad (16)$$

where $\|u_{i,n}\|_2$ denotes the 2-norm of the i -th IMF component after the n -th iteration, $\|v(\omega)\|_2$ represents the 2-norm of the filtered signal in the frequency domain. The total residual loss entropy E_r quantifies the cumulative variation in the energy of each intrinsic mode function component between two successive iterations. A large value of E_r indicates that newly generated modes still exert a substantial impact on decomposition outcomes, meaning the signal has not undergone adequate decomposition. When E_r converges to a low magnitude, the energy contribution of additional modes drops below 0.5 percent of the total energy of the original signal, and further increases in the mode number will merely introduce spurious components. Accordingly, the threshold of total loss entropy is set as $\delta_r = 0.005$. During the iterative decomposition process, if E_r exceeds the predefined threshold δ_r , the signal is deemed incompletely decomposed, and the decomposition mode number is updated as $K = K + 1$ to continue decomposition until E_r reaches a convergent state.

Based on the comprehensive criterion to optimize the penalty factor α , the Chebyshev distance d_w of modal center frequencies and the improved edge permutation entropy H_{IEPE} are introduced to construct a comprehensive criterion D_α , realizing the adaption of α .

1. Chebyshev Distance of Center Frequencies measures the independence of the center frequencies of each IMF component. A larger distance indicates higher frequency-domain separation between components. The Chebyshev distance in n -dimensional space is shown in Equation (17):

$$d_w = \max_{1 \leq i \leq n} |\omega_i - \omega_j| \quad (17)$$

where ω_i and ω_j denote the center frequencies of any two IMF components. The Chebyshev distance d_w measures the maximum interval between the central frequencies of all intrinsic mode function components. A larger d_w value corresponds to higher frequency-domain separation among modes, slighter modal aliasing, and superior decomposition quality.

2. Improved Edge Permutation Entropy H_{IEPE} .

On the basis of traditional permutation entropy, weights are assigned to short sequences to improve the discrimination ability between noise and harmonics in voltage signals. The normalized calculation formula of IEPE is given in Equation (18):

$$H_{IEPE}(D, \tau) = \frac{-\sum_{j=1}^J P_j \log_2 p_j}{\log_2 D!} \quad (18)$$

where D denotes the embedding dimension of the signal, τ represents the resampling rate, p_j denotes the weighted probability of the j -th sequence, and J represents the total number of sequences.

A smaller H_{IEPE} value indicates stronger regularity of the IMF component and lower noise content.

3. Comprehensive Criterion for Penalty Factor.

In distribution network voltage signal processing, the search range of the penalty factor is set to $\alpha \in [500, 2500]$ with a step size of 100, and the optimal α is determined by Equation (19):

$$D_\alpha = \frac{d_w}{H_{IEPE}} \quad (19)$$

The comprehensive criterion D_α integrates frequency-domain separability and noise purity into a single optimization objective. A larger value of D_α indicates more thorough frequency-domain separation among decomposed modes and lower noise content within each component. After preprocessing via the foregoing modules, residual interference

contained in distribution feeder voltage signals primarily consists of narrowband noise with limited modal components. Severe modal aliasing occurs when the penalty factor falls below 500, leading to fundamental wave extraction errors exceeding 0.1%. If the penalty factor exceeds 2500, obvious waveform distortion emerges, the amplitude error rises above 0.15%, and computational time consumption increases by 30%. Accordingly, the search range of the penalty factor is defined as the interval from 500 to 2500. A full traversal search with a step size of 100 is executed over the range $\alpha \in [500, 2500]$, and the α value that maximizes D_α is selected as the optimal penalty factor.

Through the above optimization, the algorithm can automatically output a series of IMF components $\{u_k(t)\}$ with separated center frequencies and compact bandwidths. Among them, the dominant IMF with the highest energy corresponds to the required high-purity fundamental voltage component $v(t)$, which serves as the input for the subsequent TLS-ESPRIT algorithm.

3.3.2. TLS-ESPRIT Parameter Estimation Algorithm

The contribution of this work to the TLS-ESPRIT algorithm lies in embedding the algorithm at the terminal end of the collaborative processing chain consisting of interpolation reconstruction, forward-backward filtering, and VMD decomposition, rather than applying it in an isolated manner. The preceding modules supply high-purity, low-noise fundamental-wave input signals for TLS-ESPRIT, enabling the algorithm to sustain high-resolution estimation performance even under low signal-to-noise ratio conditions. Furthermore, targeting scenarios with minor power-frequency fluctuations in distribution networks, this paper proposes a dimension reduction strategy with a resampling step size $m = 5$ during Hankel matrix construction. While fully preserving fundamental wave features, the computational load is reduced to one fifth of that of the original dataset, which improves the real-time performance of the algorithm for practical engineering deployment.

Taking the dominant fundamental voltage component $v(t)$ extracted by the improved VMD as input, the TLS-ESPRIT algorithm is adopted to realize unbiased and high-precision estimation of the amplitude, phase and frequency of the distribution network reference voltage. Based on the rotation-invariant property of the signal subspace, this algorithm directly calculates the core parameters of the voltage signal by constructing a reduced-dimension Hankel matrix, singular value decomposition, and subspace partitioning, which is suitable for the measurement requirement of small power-frequency fluctuations in the distribution network.

First, the sampling sequence of the fundamental IMF component is set as $X(k) = \{x_0, x_1, \dots, x_{N-1}\}$, where N denotes the number of sampling points. To reduce computational complexity while preserving signal features, the sampling sequence is resampled with a step size $m = 5$, and a $c \times r$ -order reduced-dimension Hankel matrix is constructed, as shown in Equation (20):

$$H = \begin{bmatrix} x_0 & x_m & x_{2m} & \dots & x_{(r-1)m} \\ x_m & x_{2m} & x_{3m} & \dots & x_{rm} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{(c-1)m} & x_{cm} & x_{(c+1)m} & \dots & x_{N-1} \end{bmatrix} \quad (20)$$

where H denotes the dimension-reduced Hankel matrix. It expands the one-dimensional voltage sequence of distribution networks into a multi-dimensional phase space and provides the data foundation for singular value decomposition and subspace partition implemented by the subsequent TLS-ESPRIT algorithm. The symbol m stands for the time delay step, which widens the distance between phase points within the phase space. The symbol

r represents the embedding dimension equivalent to the column number of the matrix. The symbol c refers to the total number of phase-space vectors equal to the row number of the matrix. These parameters satisfy the approximate relation $c \times r \approx N/m$.

Then, singular value decomposition is performed on the Hankel matrix H , as shown in Equation (21):

$$H = USV \quad (21)$$

where U denotes the left eigenvector matrix, V represents the right eigenvector matrix, and S signifies the diagonal matrix of singular values, whose diagonal elements denote the singular values of H arranged in descending order.

Since the singular values corresponding to the signal subspace are much larger than those of the noise subspace, a difference spectrum sequence is constructed to determine the dimension of the signal subspace, as shown in Equation (22):

$$p_i = \frac{\lambda_{i+1} - \lambda_i}{\lambda_i} \quad (22)$$

where λ_i denotes the i -th singular value in the diagonal matrix S . The break coordinate of the difference spectrum sequence is searched, and the first s singular values are selected to construct the signal subspace, with the corresponding submatrix of right eigenvectors denoted as V_s .

Then the right eigenvector V_s of the signal subspace is divided into two interleaved submatrices V_1 and V_2 , namely $V_s = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$, and SVD decomposition is performed on $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$ to obtain V_p , which is further partitioned into a 4×4 block matrix $V_p = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}$. The rotation characteristic matrix is solved as shown in Equation (23):

$$\psi = -V_{12}V_{22}^{-1} \quad (23)$$

The eigenvalues of the rotation characteristic matrix ψ are calculated to obtain the eigenvalue sequence $\{z_i\}_{i=1}^s$, which directly contains the frequency information of the voltage signal.

Finally, the eigenvalue sequence $\{z_i\}$ is used to construct a system of linear equations, and the relaxation coefficient $\{b_i\}_{i=1}^s$ is solved by the least squares method, as shown in Equation (24):

$$\begin{bmatrix} x_0 \\ x_m \\ \vdots \\ x_{(r-1)m} \end{bmatrix} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ z_1 & z_2 & \dots & z_s \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{r-1} & z_2^{r-1} & \dots & z_s^{r-1} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_s \end{bmatrix} \quad (24)$$

Combining the relaxation coefficient b_i and the eigenvalue z_i , then the amplitude A , frequency f , and initial phase φ of the reference voltage in the distribution network are directly calculated. Here, $\tau = m/f_s$ denotes the resampled time step, and f_s is the original sampling frequency.

The calculation formulas are shown in Equation (25):

$$\begin{cases} A_i = 2|b_i| \\ f_i = \frac{\arctan(\text{Im}(z_i)/\text{Re}(z_i))}{2\pi\tau} \\ \varphi_i = \arctan(\text{Im}(b_i)/\text{Re}(b_i)) + \frac{\pi}{2} \end{cases} \quad (25)$$

where $I_m(\cdot)$ and $Re(\cdot)$ represent the imaginary part and real part of a complex number, respectively. The unit of phase φ is radian. If b_i lies in the first or fourth quadrant of the complex plane, the phase result should be negated to complete phase correction.

3.4. High-Precision Voltage Measurement Procedure

The high-precision reference voltage measurement method based on PIA-bidirectional FIR proposed in this paper achieves high-precision measurement of distorted voltage signals through the collaborative processing of interpolation reconstruction, filtering denoising, and modal parameter identification. The specific high-precision reference voltage measurement flow is shown in Figure 8.

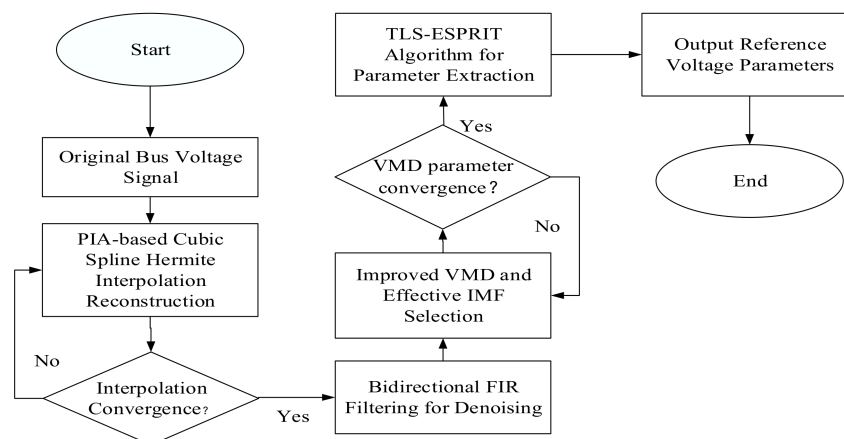


Figure 8. Flowchart of high-precision voltage measurement.

The non-uniformly sampled voltage signal is reconstructed by combining PIA cubic spline Hermite interpolation, and the sampling distortion correction is completed via iterative convergence judgment. Bidirectional FIR filtering is adopted for the reconstructed signal to suppress broadband interference and eliminate phase distortion. Then the improved VMD is used to decompose the filtered signal into several IMF components, and the high-purity fundamental IMF component is selected. After the original voltage signal is processed by interpolation, filtering, decomposition, and selection to obtain a pure fundamental component, the TLS-ESPRIT algorithm is employed to extract its amplitude, phase, and frequency information, realizing accurate measurement of the reference voltage and providing a high-fidelity reference source for online calibration of feeder measurement loops.

4. Simulation Verification of Voltage Signal Measurement

To verify the performance superiority of the proposed interpolation and filtering algorithm for power signal reconstruction, a set of typical unfiltered voltage transient waveforms are selected as test samples. The raw data are derived from standard voltage test waveforms specified in IEEE 1159, consisting of a 50 Hz fundamental frequency component, a third harmonic component accounting for 5 percent of the fundamental amplitude, and high-frequency noise interference. The waveform adopts a sampling rate of 2 kHz with a total duration of 100 milliseconds, and a disturbance with an amplitude of 30 percent is superimposed within the time interval from 0.02 s to 0.04 s.

In the simulation, Gaussian white noise with a signal-to-noise ratio of 40 decibels is superimposed on the IEEE 1159 standard voltage signal to simulate background noise generated during actual data acquisition in the distribution networks. Non-uniform sampling distortion is simulated via random frame loss with a frame loss rate of 20 percent. The noise

intensity and frame loss rate mentioned above are set with reference to technical indicators measured in typical on-site data acquisition environments of distribution networks.

The experimental scheme is divided into three successive stages. First, random frame loss processing is performed on the complete sampling sequence with a frame loss rate fixed at 20 percent. Second, PIA-based cubic spline Hermite interpolation is applied for waveform reconstruction, followed by forward–backward FIR filtering for data purification. Third, voltage parameters are extracted using the improved VMD and TLS-ESPRIT algorithms. Error indicators, feature discrepancies, and algorithm runtime between the reconstructed waveform and the original waveform are calculated to conduct comprehensive performance evaluation.

4.1. Performance Comparison of Interpolation Algorithms

As shown in Figure 9, subfigures (a) and (b) present the interpolation effects of PIA cubic spline Hermite interpolation and PIA four-spectral-line interpolation after 1, 5, 10, and 20 iterations, respectively.

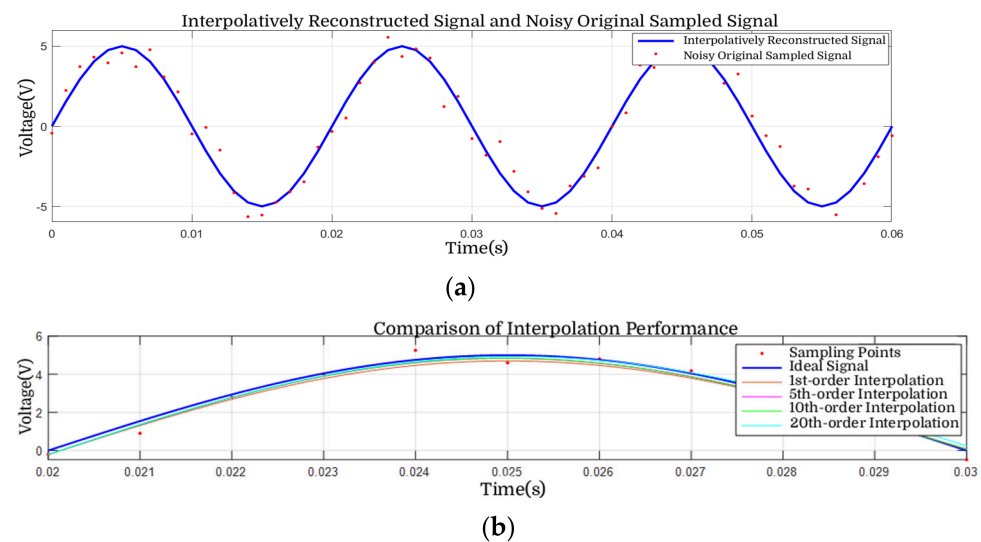


Figure 9. Interpolation simulation comparison. (a) PIA cubic spline Hermite interpolation. (b) PIA four-spectral-line interpolation after 1, 5, 10, and 20 iterations.

As shown in Tables 1 and 2, when performing interpolation on the sampling points of the selected voltage curve, a comparison is made between the algorithm proposed in this chapter and the PIA four-spectral-line interpolation method. It is found that the PIA cubic spline interpolation method yields a relatively large error under the same number of iterations. In the interpolation process applied to G1 data, the error obtained by the proposed algorithm is lower than that of the PIA four-spectral-line interpolation method even with the same number of iterations. The same phenomenon can be observed in the interpolation analysis of G2 data, where the error rate of the proposed algorithm is lower under identical iteration conditions.

Table 1. Error of curve interpolation at G1 points.

Number of Iterations	PIA Cubic Spline Hermite Interpolation	PIA Four-Spectral-Line Interpolation
1	1.855×10^{-5}	1.114×10^{-1}
5	8.10×10^{-7}	2.035×10^{-2}
10	1.16×10^{-10}	1.63×10^{-2}
20	7.02×10^{-12}	1.94×10^{-3}

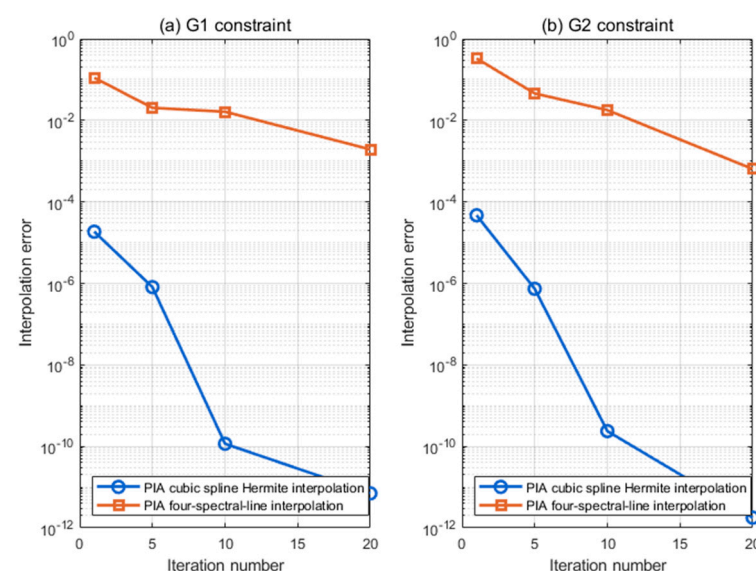
Table 2. Curve interpolation G2 point data error.

Number of Iterations	PIA Cubic Spline Hermite Interpolation	PIA Four-Spectral-Line Interpolation
1	4.69×10^{-5}	3.32×10^{-1}
5	7.360×10^{-7}	4.58×10^{-2}
10	2.37×10^{-10}	1.79×10^{-2}
20	1.78×10^{-12}	6.42×10^{-4}

The error optimization method for digital energy meters based on PIA cubic spline Hermite interpolation exhibits significant advantages in terms of interpolation accuracy and resource efficiency. Compared with the traditional four-spectral-line interpolation algorithm, this method is optimized from the underlying logic of polynomial construction, establishing a cubic polynomial basis with optimal approximation characteristics and introducing an iterative target factor to weight and modify the Hermite boundary conditions. The maximum error of the interpolation points is reduced by 2 to 4 orders of magnitude compared with the four-spectral-line method, while the algorithm complexity is decreased by 23.6%.

In the initial iteration stage $n = 1$, the cubic spline method achieves an error accuracy of 1.855×10^{-5} , which is 4 orders of magnitude higher than that of the four-spectral-line method. When the number of iterations increases to 10, the error converges rapidly to 1.16×10^{-10} , which is 8 orders of magnitude lower than 1.63×10^{-2} of the four-spectral-line method. Higher-order approximation effects can be achieved within the framework of cubic polynomials, effectively saving hardware resources.

To intuitively illustrate the differences in convergence speed and accuracy between the two interpolation methods under varying iteration counts, the data listed in Tables 1 and 2 are plotted into a line graph, as presented in Figure 10.

**Figure 10.** Convergence performance comparison of interpolation algorithms.

Subplot (a) corresponds to the G1 continuity condition, and subplot (b) corresponds to the G2 continuity condition. The vertical axis adopts a logarithmic coordinate system to facilitate observation of variations in error magnitudes. It can be observed from the figure that under both continuity constraints, the error of the PIA cubic spline Hermite interpolation drops remarkably below that of the PIA four-spectrum-line interpolation after merely five iterations and delivers a faster convergence rate. After ten iterations, the error

of the proposed method falls to the order of 10^{-10} , while the error of the four-spectrum-line interpolation remains at the order of 10^{-2} . This result verifies the dual advantages of the proposed interpolation method in terms of convergence speed and reconstruction accuracy.

4.2. Quantitative Evaluation of Filtering Effect

After interpolation, the signal is filtered by bidirectional FIR filtering. Since the phase delays in the two filtering processes cancel each other out, the final output signal is essentially zero-phase. By preserving the phase characteristics of the signal, bidirectional filtering can effectively improve the accuracy and reliability of signal processing in denoising and signal smoothing, while retaining more original features of the signal. Figure 11a,b show the comparison of the zero-phase effect of bidirectional filtering.

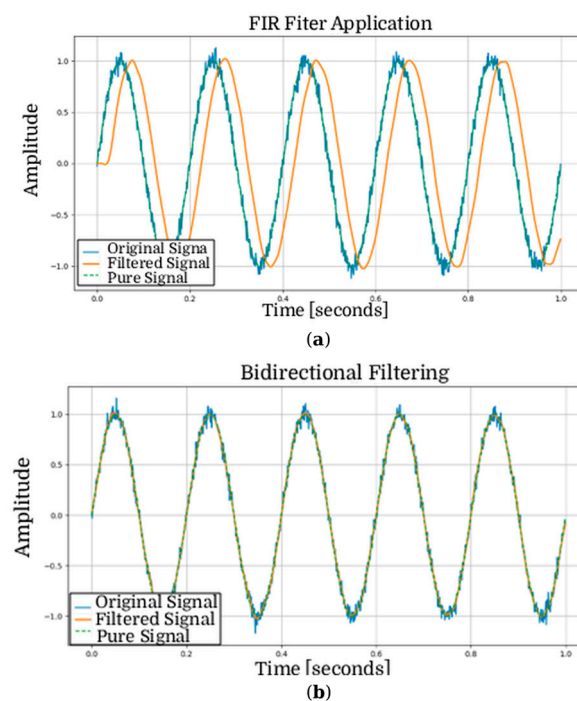


Figure 11. Comparison before and after bidirectional filtering. (a) FIR Filter Application. (b) Bidirectional Filtering.

It can be seen from the figure that the waveform processed by single-pass filtering exhibits obvious phase lag relative to the input signal. In contrast, the waveform after forward–backward filtering perfectly overlaps the input signal in phase, which verifies that the forward–backward FIR filtering structure can effectively eliminate phase delay while suppressing interference.

4.3. Simulation Verification

4.3.1. Measurement Accuracy Under Various Frequency Operating Conditions

As shown in Figure 12, subfigures (a), (b), (c), (d), and (e) respectively present the interpolated voltage measurement data after bidirectional FIR filtering under frequency fluctuations, with frequencies of 49.5 Hz, 49.8 Hz, 50 Hz, 50.3 Hz, and 50.5 Hz in sequence. The blue curve represents the unfiltered raw data, the green curve denotes the filtered processed data, and the red curve indicates the standard measurement data.

The green curves show high overlap with the red reference signal under all tested frequency conditions, demonstrating that the proposed method can effectively recover signal waveforms within a frequency offset range of ± 0.5 Hz and possesses favorable frequency adaptability and phase tracking precision.

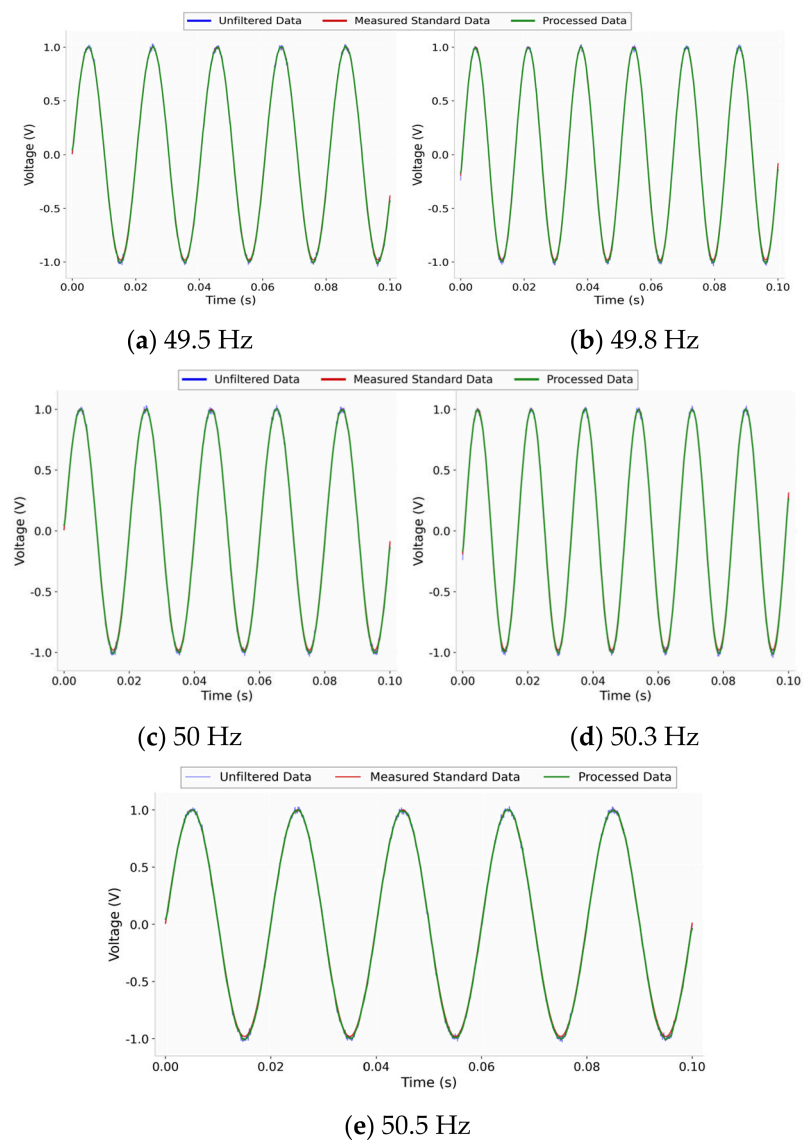


Figure 12. Joint algorithm signal processing.

Ten repeated measurements are carried out for each frequency operating condition. The voltage phase errors and amplitude errors are summarized in Tables 3 and 4 respectively.

Table 3. Comparison of voltage phase error at different frequencies (").

No.	Frequency	49.5 Hz	49.8 Hz	50 Hz	50.3 Hz	50.5 Hz
1		0.84	0.25	0.11	0.25	0.73
2		0.84	0.32	0.23	0.20	0.84
3		0.69	0.29	0.28	0.31	0.71
4		0.72	0.19	0.29	0.28	0.95
5		0.98	0.27	0.25	0.32	0.73
6		0.65	0.20	0.19	0.18	0.64
7		0.71	0.24	0.14	0.07	0.88
8		0.89	0.17	0.21	0.03	0.87
9		0.87	0.29	0.12	0.26	0.64
10		0.85	0.33	0.10	0.29	0.86

Table 4. Comparison of voltage amplitude errors (%) at different frequencies.

No.	Frequency	49.5 Hz	49.8 Hz	50 Hz	50.3 Hz	50.5 Hz
1		0.033	0.029	0.041	0.039	0.036
2		0.019	0.031	0.035	0.028	0.032
3		0.037	0.036	0.029	0.033	0.038
4		0.042	0.028	0.017	0.025	0.027
5		0.025	0.021	0.023	0.019	0.018
6		0.018	0.019	0.025	0.034	0.032
7		0.034	0.025	0.038	0.022	0.044
8		0.029	0.039	0.034	0.035	0.028
9		0.031	0.032	0.027	0.024	0.045
10		0.030	0.031	0.042	0.037	0.026

Based on ten repeated experiments, at the nominal 50 Hz power frequency, the mean phase error is 0.187 arcseconds with a standard deviation of 0.0075 arcseconds, and the 95% confidence interval ranges from 0.172 arcseconds to 0.202 arcseconds. Under frequency offset operating conditions of 49.5 Hz and 50.5 Hz, the average phase errors reach 0.804 arcseconds and 0.795 arcseconds respectively, both of which are well confined within the allowable error limits.

At the nominal 50 Hz power frequency, the mean amplitude error is 0.028 percent with a standard deviation of 0.008 percent, and the 95 percent confidence interval extends from 0.020 percent to 0.036 percent. Under the extreme frequency offset operating conditions of 49.5 Hz and 50.5 Hz, the maximum amplitude errors are 0.042 percent and 0.045 percent respectively, both falling within the allowable error limit of 0.05 percent. This outcome verifies the applicability of the proposed algorithm under frequency deviation scenarios.

To visually demonstrate the variation trend of frequency estimation error against actual frequency offset, the data listed in Table 5 are plotted as a line graph, as illustrated in Figure 13.

Table 5. Frequency calculation error (Hz).

Actual Frequency	Calculated Frequency	Error
49.5	49.49902	−0.00098
49.8	49.79983	−0.00017
50	50.00000	0.00000
50.3	50.30023	0.00023
50.5	50.50093	0.00093

The horizontal axis of the figure represents the actual frequency, and the vertical axis denotes the frequency error. It can be observed that within the frequency offset range from 49.5 Hz to 50.5 Hz, the frequency error rises slightly as the frequency offset increases. Nevertheless, the error is kept within ± 0.001 Hz across the entire range, which far outperforms the preset engineering target of ± 0.02 Hz. This finding validates the outstanding frequency tracking precision of the proposed method over the full frequency offset range.

Under frequency offset conditions of 49.8 Hz and 50.3 Hz, the maximum negative error reaches -0.00017 Hz and the maximum positive error reaches 0.00023 Hz. When the system frequency approaches the extreme operating point of 50.5 Hz, the relative error remains at 0.0184 percent. During online calibration of the distribution feeders, the frequency precision of the reference voltage exerts a direct influence on the phase consistency of synchronous current excitation. At a frequency error of 0.001 Hz, the accumulated phase deviation over

one full cycle of 20 milliseconds equals merely 0.00072 degrees, which is negligible and guarantees the reliability of the calibration criteria.

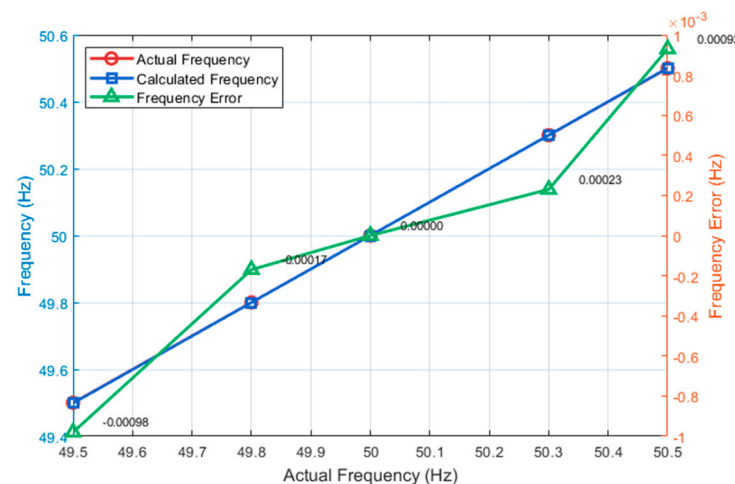


Figure 13. Frequency tracking error under different frequency conditions.

4.3.2. Ablation Experiments and Module Contribution Analysis

Ablation experiments are designed to quantify the contribution of each processing module to the final measurement accuracy by comparing the performance of four distinct setups. All four setups adopt TLS-ESPRIT for parameter extraction, while differing only in front-end signal processing steps. The first setup directly uses raw signals without any preprocessing. The second setup solely applies waveform reconstruction based on PIA interpolation. The third setup combines PIA interpolation with forward–backward FIR filtering. The fourth setup corresponds to the complete scheme proposed in this paper integrating interpolation filtering VMD and TLS-ESPRIT. Table 6 presents the comparison of frequency amplitude and phase errors for all setups under identical test conditions with a nominal power frequency of 50 Hz, a signal to noise ratio of 40 decibels, and a frame loss rate of 20 percent.

Table 6. Error contribution of each module in ablation experiments.

Setup	Front-End Processing	Frequency Error (Hz)	Amplitude Error (%)	Phase Error (")
1	No Preprocessing	0.5	5.0	50
2	PIA Interpolation	0.02	0.5	5
3	PIA Interpolation Combined with Forward–backward FIR Filtering	0.005	0.1	0.5
4	Complete Proposed Method	0.001	0.05	0.36

It can be observed from Table 6 that the contribution of each module to the final accuracy decreases in a stepped manner. To further visualize this trend, the data from Table 6 are plotted as a line graph as shown in Figure 14.

It can be concluded from Figure 14 that the unprocessed raw signal yields the largest errors across the frequency amplitude and phase indicators. After waveform reconstruction, merely via PIA interpolation, the frequency error decreases significantly, which demonstrates that the interpolation module’s correction for non-uniform sampling distortion lays the foundation for subsequent high-precision processing. The addition of forward–backward FIR filtering brings about a substantial reduction in the phase error, proving that the filtering module plays a decisive role in phase preservation. The full proposed method

achieves the minimum values of all error metrics, which verifies the effectiveness of the three-stage collaborative processing strategy.

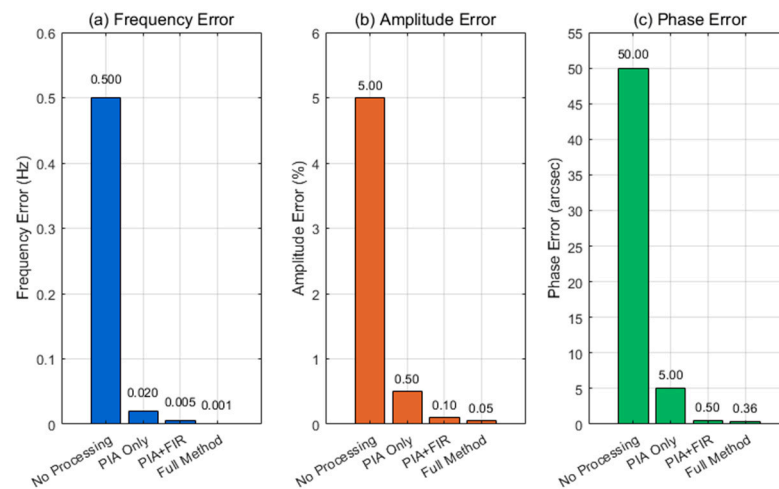


Figure 14. Comparison of error contribution of each module in ablation experiments.

4.3.3. Parameter Sensitivity Analysis

To evaluate the algorithm's sensitivity to key parameters, the frequency estimation errors under varying noise levels and the different frame loss rates are tested separately, with the results summarized in Table 7.

Table 7. Frequency error under different noise levels and frame loss rates (Hz).

Frame Loss Rate	SNR = 20 dB	SNR = 30 dB	SNR = 40 dB	SNR = 50 dB
10%	0.0082	0.0035	0.0012	0.0006
20%	0.0118	0.0051	0.0018	0.0009
30%	0.0176	0.0078	0.0032	0.0015

The data from Table 7 are presented in the form of grouped line graphs as illustrated in Figure 15.

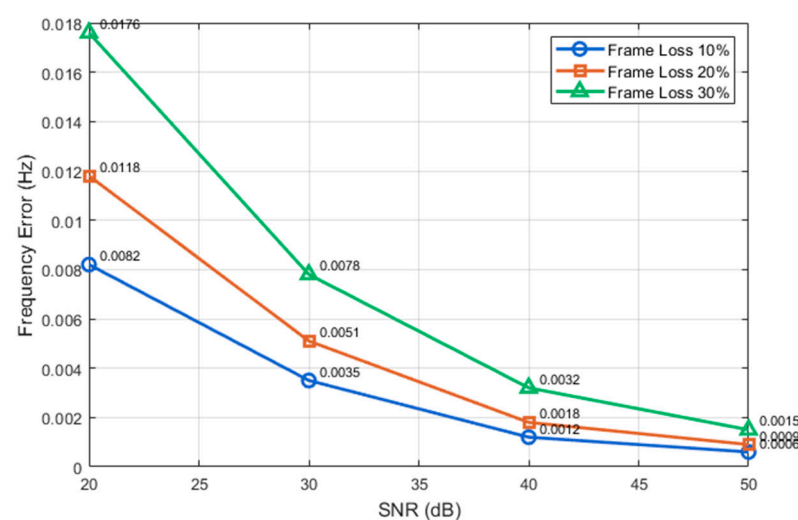


Figure 15. Frequency error under different noise levels and frame loss rates.

It can be seen from Figure 15 that the frequency error declines as the signal to noise ratio rises under a fixed frame loss rate. The error drops by approximately one order of

magnitude when the signal to noise ratio increases from 20 decibels to 50 decibels, which proves that the algorithm possesses strong noise suppression capability.

With a fixed signal to noise ratio, the error rises gradually as the frame loss rate grows from 10 percent to 30 percent. Even so, all error values stay below 0.02 Hz, which reflects the powerful data recovery performance of the algorithm against missing sampling points.

Under typical data collection conditions for distribution networks with the signal to noise ratio no less than 40 decibels and the frame loss rate no higher than 20 percent, the frequency error remains better than 0.002 Hz. This result verifies the practical engineering value of the proposed algorithm.

4.3.4. Real-Time Performance Verification of the Algorithm

The statistical processing delay for a single complete measurement procedure is listed in Table 8.

Table 8. Statistics of time for measuring voltage parameters.

Number	Measurement Time (μ s)
1	398
2	414
3	407
4	407
5	396
6	410
7	402
8	401
9	396
10	402

Tests show that the average processing delay of a single complete processing flow is 403 microseconds, the maximum delay is 414 microseconds, and the minimum delay is 396 microseconds. These values fall within the typical delay range of 300 to 600 microseconds for similar signal processing algorithms. While maintaining high measurement accuracy, the proposed method features a stable processing delay within a reasonable range for engineering applications, which verifies the real-time performance advantages of the algorithm framework.

A processing delay of 403 microseconds corresponds to an equivalent processing rate of approximately 2.5 kHz, which exceeds the sampling rate of 2 kHz adopted by the system. This indicates that the algorithm can finish a full single processing cycle within the 500 microsecond interval between adjacent sampling points and meet the requirements of real-time pipeline processing. It should be noted that the above time consumption is measured on the MATLAB R2024a simulation platform and only serves to verify the real-time performance limit of the algorithm framework. Further reduction in processing delay can be achieved through fixed-point optimization loop unrolling and other techniques during practical embedded deployment, which confirms the engineering feasibility of the proposed method.

4.3.5. Comprehensive Performance Comparison of Different Algorithms

To further verify the superiority of the proposed method, comparisons are conducted with two representative algorithms. The first algorithm integrates Kalman filtering and fast Fourier transform interpolation and this combination acts as a classic scheme for voltage parameter tracking. The second algorithm adopts a traditional signal processing chain consisting of cubic spline interpolation single side FIR filtering manually tuned VMD and

ESPRIT, which represents conventional non-adaptive signal processing workflows. Table 9 compares the frequency amplitude and phase errors of all algorithms under identical test conditions with a nominal power frequency of 50 Hz, a signal to noise ratio of 40 decibels, and a frame loss rate of 20 percent.

Table 9. Comprehensive performance comparison of different algorithms.

Comparison Algorithms	Frequency Error (Hz)	Amplitude Error (%)	Phase Error (")
Kalman Filter Combined with FFT	0.03	0.35	8.5
Traditional Cubic Spline Interpolation with Single-sided FIR Filter Manually Tuned VMD and ESPRIT	0.008	0.15	1.6
Proposed Method in This Paper	0.001	0.05	0.36

To more intuitively display the performance gaps between the proposed method and the two comparative algorithms, data from Table 9 are plotted as bar charts as shown in Figure 16.

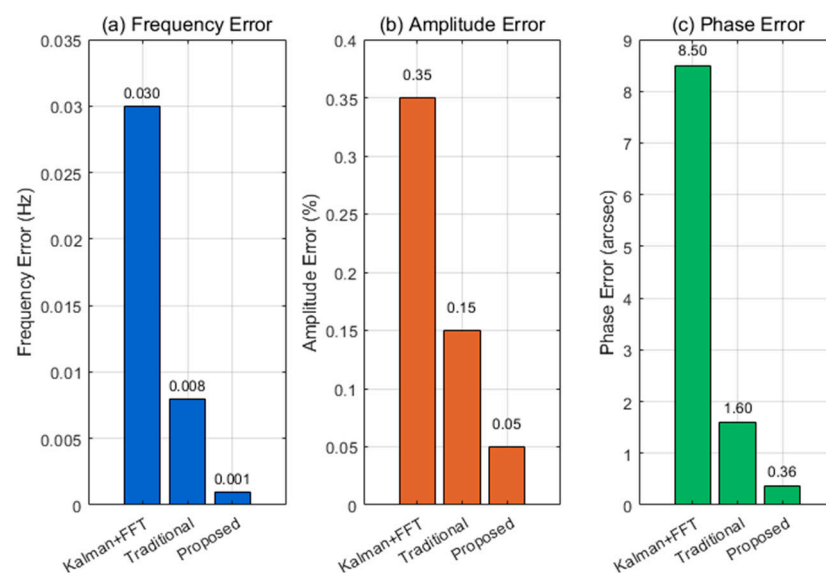


Figure 16. Comparison of comprehensive performance of different algorithms.

It can be observed from Figure 16 that the method combining Kalman filtering and FFT is greatly affected by non-uniform sampling as well as noise and produces the largest errors across all three metrics. Although the scheme based on traditional cubic spline interpolation outperforms the Kalman filtering scheme, it still suffers from notably higher phase error than the proposed method. This is mainly attributed to the phase lag brought by single-sided FIR filtering and the fact that VMD parameters rely heavily on manual tuning. The method proposed in this paper achieves optimal results in frequency amplitude and phase estimation, which verifies the effectiveness of the joint processing framework integrating PIA interpolation, forward–backward FIR filtering, and improved VMD.

5. Conclusions

Aiming at the voltage signal distortion problem in the on-line calibration system of the measurement circuit, this paper proposes a three-stage solution. Waveform restoration is achieved using PIA non-uniform cubic spline Hermite interpolation technology. The PIA algorithm ensures the continuity of the value, slope, and curvature at each sampling point and completes a smooth curve for non-uniform or missing sampling data through G1 and

G2 interpolation. Experiments show that the maximum error of the waveform amplitude does not exceed 0.03%. Bidirectional FIR filtering is adopted to eliminate interference, which first filters the signal in the forward direction to remove high-frequency noise, then reverses the result and filters it in the reverse direction to offset the phase delay. The phase error of the processed signal is stable within $0.36''$, and the amplitude fluctuation is less than 0.05%. Dynamic extraction of voltage parameters is realized through improved VMD and TLS-ESPRIT algorithms to achieve fast and unbiased estimation of amplitude phase and frequency, with the frequency tracking error controlled at the order of 0.001 Hz. Simulation results demonstrate that the proposed method outperforms conventional algorithms such as PIA four-spectrum-line interpolation in terms of frequency tracking accuracy and phase stability, which verifies its effectiveness and reliability for the online calibration of feeders.

From a methodological perspective, this paper reconstructs non-uniformly sampled signals via PIA cubic spline Hermite interpolation. Distortion is corrected by imposing geometric continuity constraints on the values, slopes, and curvatures of sampling points, boosting reconstruction accuracy by two to four orders of magnitude compared with the traditional four-spectrum-line interpolation method. On this basis, a forward–backward dual-channel forward–backward FIR filtering architecture is adopted to counteract phase delay, suppressing wideband interference while preventing distortion of phase information. Combined with improved VMD and TLS-ESPRIT algorithms, total residual loss entropy and comprehensive evaluation criteria are introduced to realize the adaptive optimization of the VMD mode number and penalty factor, eliminating the uncertainty caused by manual parameter tuning relying on empirical knowledge. The above collaborative processing strategy systematically addresses non-uniform sampling distortion and filtering phase distortion at the signal processing level, providing a high-fidelity reference source for live online calibration of feeders. It is worth noting that all experiments in this work were carried out using waveform simulations compliant with the IEEE 1159 standard, without testing on physical hardware platforms or actual distribution network sites, which constitutes the primary limitation of the present research. A hardware experimental platform will be constructed in follow-up work to further verify the engineering adaptability and long-term operational stability of the algorithm under practical operating conditions.

Author Contributions: Conceptualization, P.Y. and X.M.; methodology, H.Z. and Q.W.; software, X.B. and Z.L.; validation, P.Y., X.M. and H.Z.; formal analysis, H.Z. and Q.W.; investigation, X.B. and Z.L.; resources, P.Y., X.M. and Z.L.; data curation, X.M.; writing—original draft preparation, H.Z. and Q.W.; writing-review and editing, Q.W., X.B. and Z.L.; visualization, H.Z. and Q.W.; supervision, X.B. and Z.L.; project administration, P.Y. and X.M. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Science and Technology Project of State Grid Jiangsu Electric Power Co., Ltd. (B31070255ERK).

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: Authors Pei Yang, Xiaojuan Miao, and Xinlian Bu were employed by the company State Grid Jiangsu Electric Power Co., Ltd. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as potential conflicts of interest. The authors declare that this study received funding from the Science and Technology Project of State Grid Jiangsu Electric Power Co., Ltd. (B31070255ERK). The funder was not involved in the study design, collection, analysis, interpretation of data, the writing of this article, or the decision to submit it for publication.

Abbreviations

The following abbreviations are used in this manuscript.

ADC	Analog-to-Digital Converter
CNN	Convolutional Neural Network
DSP	Digital Signal Processor
FFT	Fast Fourier Transform
FIR	Finite Impulse Response
FPGA	Field-Programmable Gate Array
IEEE	Institute of Electrical and Electronics Engineers
IEPE	Improved Edge Permutation Entropy
IMF	Intrinsic Mode Function
LSTM	Long Short-Term Memory
PIA	Progressive Iterative Approximation
PMU	Phasor Measurement Unit
SNR	Signal-to-Noise Ratio
SVD	Singular Value Decomposition
TLS-ESPRIT	Total Least Squares-Estimation of Signal Parameters via Rotational Invariance Techniques
VMD	Variational Mode Decomposition

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