

Article

PSO-Based Accuracy Optimization of Parallel Mass Flow Controllers in Multi-Component Gas Mixing Systems

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Abstract

Accurate gas mass flow allocation is important for multi-component gas mixing systems and fuel-cell test benches, especially when a wide total-flow range is covered by several parallel mass flow controllers (MFCs). This study evaluates the coordinated allocation of three parallel MFCs using traceable third-party calibration data and a reproducible numerical simulation workflow. The calibration curves define a constrained flow-allocation problem in which several MFC setpoint combinations can satisfy the same total target flow, while producing different full-scale error levels. A modified particle swarm optimization (PSO) strategy is compared with a rule-based allocation strategy and standard PSO. The numerical evaluation was extended with 500-run reliability statistics, ablation analysis, parameter sensitivity analysis, expanded switching-region evaluation, numerical perturbation checks, and benchmark optimizer comparisons. Across the main 12 operating points, the probability of obtaining a solution within 0.10 %FS (percentage of full scale (%FS)) increased from 0.207 with standard PSO to 0.310 with the improved PSO, and the probability within 0.05 %FS increased from 0.135 to 0.193. The positive-improvement rate relative to the rule baseline increased from 0.384 to 0.547. The results support the use of calibration-curve-based optimization to reduce switching-region error peaks and to improve repeated-run low-error solution probability in parallel-MFC allocation.

Keywords: mass flow controller; accuracy optimization; particle swarm optimization; repeated-run reliability; numerical simulation; fuel-cell test bench



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1. Introduction

Hydrogen energy and proton exchange membrane fuel cells are important technologies for low-carbon transport and power systems [1–5]. Fuel-cell stack testing requires accurate hydrogen and air delivery so that operating conditions can be reproduced under standard test procedures.

In high-flow fuel-cell test benches, a single mass flow controller (MFC) cannot always cover the required flow range with suitable accuracy. Parallel-MFC configurations are therefore used, but each controller has a flow-dependent calibration curve, and sequential rule-based switching can create local error peaks near controller activation thresholds. Particle swarm optimization (PSO) is considered below as a numerical allocation method for this constrained flow-allocation problem.

A proton exchange membrane fuel cell (PEMFC) converts hydrogen into electrical energy efficiently [6–8] and is widely used in transport and energy applications. Performance testing is a critical step in fuel-cell development, and the stack test bench is the main platform for this work. As shown in Figure 1, the fuel-cell stack test bench includes the gas-circuit subsystem, which regulates hydrogen and air mass flow; the humidification subsystem, which conditions the reactant gases; the thermal-management subsystem, which controls operating temperature; and the electrical-energy feedback subsystem, which absorbs the generated electrical energy.

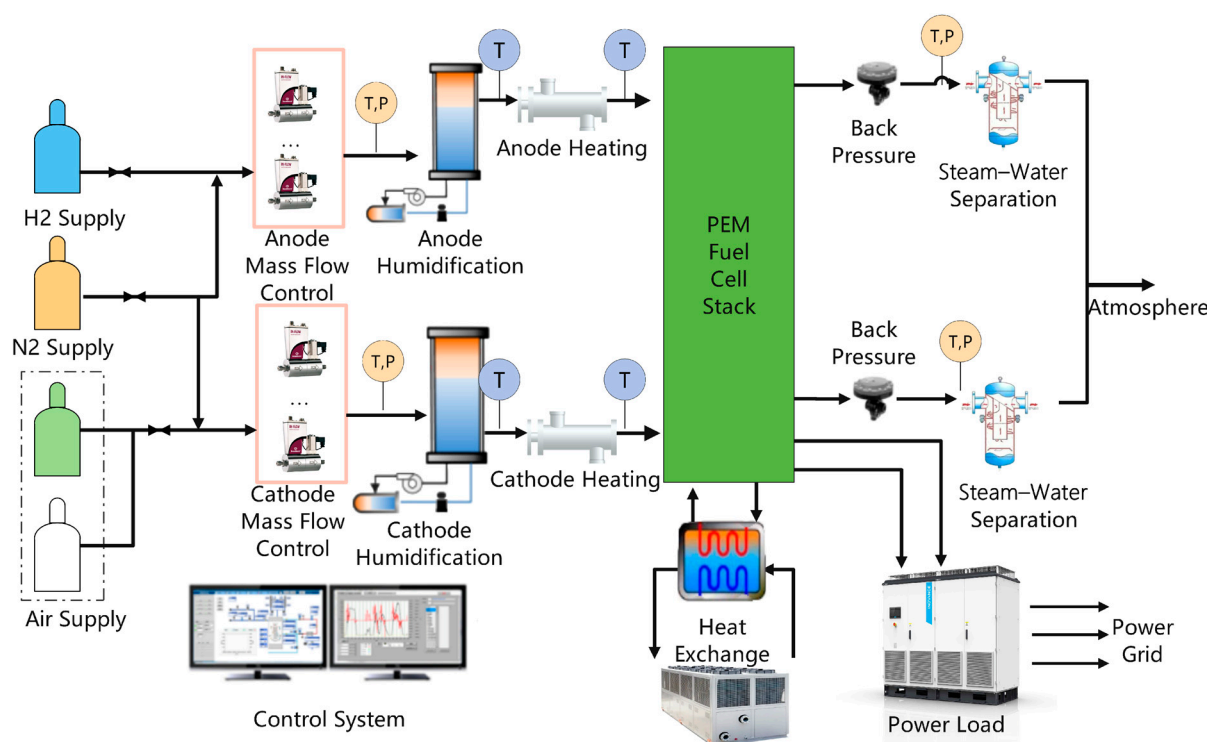


Figure 1. Structure of fuel-cell stack test bench.

Previous studies have addressed fuel-cell modeling, thermal management, control design, electrocatalysts, mass flow measurement, and general optimization methods [9–15]. Less attention has been given to the allocation of one total gas-flow demand among several parallel MFCs using traceable calibration-curve information. Compared with prior PSO-based turbine-system controller-tuning studies [16], this work addresses a parallel-MFC allocation problem in which several calibrated controllers must be coordinated to meet one total flow demand while minimizing full-scale error. This paper focuses on the gas-circuit allocation layer of the test bench.

The practical challenge is that the same total target flow can be obtained from many combinations of individual MFC setpoints. Because the full-scale error of each MFC varies with flow, the lowest-error allocation is not necessarily the one selected by a simple sequential switching rule.

To address this problem, this study formulates parallel-MFC allocation as a constrained numerical optimization problem and evaluates a modified PSO strategy. The contribution is the calibration-curve-driven allocation model for parallel MFCs, together with repeated-run reliability statistics, switching-region evaluation, ablation analysis, numerical perturbation checks, and benchmark optimizer comparison within one reproducible simulation-based workflow. The manuscript uses ‘modified’ or ‘adapted’ PSO wording because dynamic inertia, penalty handling, and random perturbation are established PSO or constrained-

optimization mechanisms applied to this engineering allocation problem rather than a new PSO theory.

2. Materials and Methods

2.1. Fuel-Cell Stack Model Development

A fuel-cell stack model was used to calculate the hydrogen and air requirements for a specified output power. The stack is primarily composed of the flow field plate (FFP), gas diffusion layer (GDL), catalyst layer (CL), and proton exchange membrane (PEM). Hydrogen is introduced at the anode, where it undergoes oxidation, releasing protons and electrons. Concurrently, oxygen or air is supplied to the cathode, where it undergoes a reduction reaction, combining with the protons and electrons to form water. Protons traverse the stack from the anode to the cathode via the membrane, while electrons travel through an external circuit, completing the electrical circuit and generating power [17]. The anode-side reaction in the fuel-cell stack is given as follows:



The cathode-side reaction is as follows:



Combining the reactions on the anode and cathode sides forms the general reaction of a fuel cell. The following is the overall reaction equation of the fuel cell, with a schematic diagram of the fuel-cell reaction process shown in Figure 2 [18,19]:

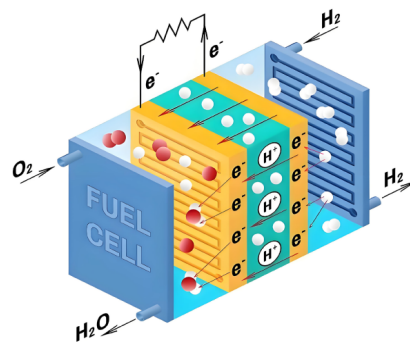
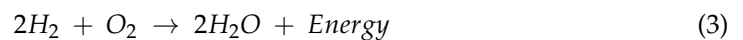


Figure 2. Reaction of a fuel cell [19]. Arrows indicate the transport directions of electrons, protons, reactants, and products.

The hydrogen consumption rate can be calculated from the current using Faraday's law:

$$m_{H_2} = \frac{M_{H_2} I}{n_{H_2} F} \quad (4)$$

Similarly, the oxygen consumption rate can be calculated as

$$m_{O_2} = \frac{M_{O_2} I}{n_{O_2} F} \quad (5)$$

where m_{H_2} and m_{O_2} denote the mass flow rates of hydrogen and oxygen consumption (g/s), respectively; M_{H_2} and M_{O_2} represent the molar masses of hydrogen (2 g/mol) and oxygen (32 g/mol), respectively; I is the operating current of the fuel cell (A); n_{H_2} and

n_{O_2} are the numbers of electrons transferred per mole of hydrogen (2) and oxygen (4), respectively; and F represents Faraday's constant (approximately 96,485 C/mol).

The volume of hydrogen consumed per minute, V_{H_2} (L/min), is calculated as follows:

$$V_{H_2} = \frac{N I_{cell} sto_H}{F \rho_{H_2}} \times 60 \quad (6)$$

The volume of air consumed per minute, V_{Air} (L/min), is calculated as follows:

$$V_{Air} = \frac{N I_{cell} sto_A}{F \rho_{Air}} \times 60 \quad (7)$$

In these equations, F is Faraday's constant (96,485 C/mol); ρ_{H_2} and ρ_{Air} are the densities of hydrogen and air, respectively; sto_A and sto_H denote the stoichiometric ratios of air and hydrogen; I_{cell} is the operating current of the fuel cell (A); and N is the number of single cells in the fuel-cell stack.

Following Refs. [20–22], the output power of the fuel-cell stack is calculated as

$$P_{stackout} = V_{stackout} \times I_{stackout} \quad (8)$$

where $P_{stackout}$ represents the output power of the fuel-cell stack; $V_{stackout}$ is the output voltage of the fuel-cell stack; and $I_{stackout}$ is the output current of the fuel-cell stack.

The Nernst open-circuit voltage is the electromotive force when the reversible electrochemical reaction in the fuel cell reaches equilibrium. It represents the maximum theoretical voltage of a single cell under ideal conditions [23].

$$E_{Nernst} = 1.229 - 0.85 \times 10^{-4}(T_{fc} - 298.15) + 4.3085 \times 10^{-5} T_{fc} [\ln(P_{H_2}) + \frac{1}{2} \ln(P_{O_2})] \quad (9)$$

where E_{Nernst} represents the Nernst open-circuit voltage; T_{fc} indicates the fuel-cell temperature (K); P_{H_2} is the hydrogen partial pressure; and P_{O_2} is the oxygen partial pressure.

The activation loss voltage is caused by activation polarization during the electrochemical reactions and can be expressed as

$$V_{act} = X_1 + X_2 T_{fc} + X_3 T_{fc} \ln(C_{O_2}) + X_4 T_{fc} \ln(I_{st}) \quad (10)$$

where I_{st} represents the fuel-cell current; T_{fc} is the temperature (K); C_{O_2} indicates the dissolved oxygen concentration at the gas–liquid interface; and X_1 , X_2 , X_3 , and X_4 are empirical constants related to operating temperature and excess air coefficient [24]. In this study, $X_1 = -1.0457$, $X_2 = 3.8 \times 10^{-3}$, $X_3 = 9.8 \times 10^{-5}$, and $X_4 = -9.54 \times 10^{-5}$.

The ohmic loss voltage is mainly caused by internal resistance in the electrolyte, electrodes, and current collectors. Because ion transport is slower than electron transport, catalyst and diffusion-layer losses are neglected in this model. It mainly depends on the proton exchange membrane's impedance and is given by

$$V_{ohm} = i R_{ohm} = i R_m = \frac{i \delta_m}{\sigma_m} = \frac{i \delta_m}{(0.514 \lambda_m - 0.033) e^{1268(\frac{1}{303} - \frac{1}{T})}} \quad (11)$$

where i is the fuel-cell current, R_{ohm} is the ohmic resistance, R_m indicates the ionic transport impedance, δ_m is the proton exchange membrane thickness, σ_m is electrical conductivity, λ_m indicates the membrane water content, and T represents the fuel-cell temperature.

Concentration polarization overvoltage results from concentration differences caused by reactant mass-transfer limitations within the fuel cell. The concentration-overvoltage expression is written in the commonly used single-logarithm form:

$$V_{conc} = -a \ln\left(1 - \frac{i}{i_{max}}\right) \quad (12)$$

where a is related to temperature and excess air coefficient. In this article, a is 0.0129 on the anode side and 0.0064 on the cathode side. i_{max} is the maximum current density, and i is the current density [24].

The electrochemical stack model estimates the hydrogen and air demand associated with the selected fuel-cell operating points. The allocation optimization is then performed at the gas-circuit layer using the MFC calibration curves and the flow-sum constraint. This separation keeps the evidence chain reproducible: the stack model supplies operating-point demand context, while the PSO objective function evaluates MFC allocation error from the traceable calibration data. The stack model was implemented in MATLAB/Simulink R2021a (MathWorks, Natick, MA, USA). The resulting model structure and parameter settings are shown in Figure 3 and Table 1, respectively.

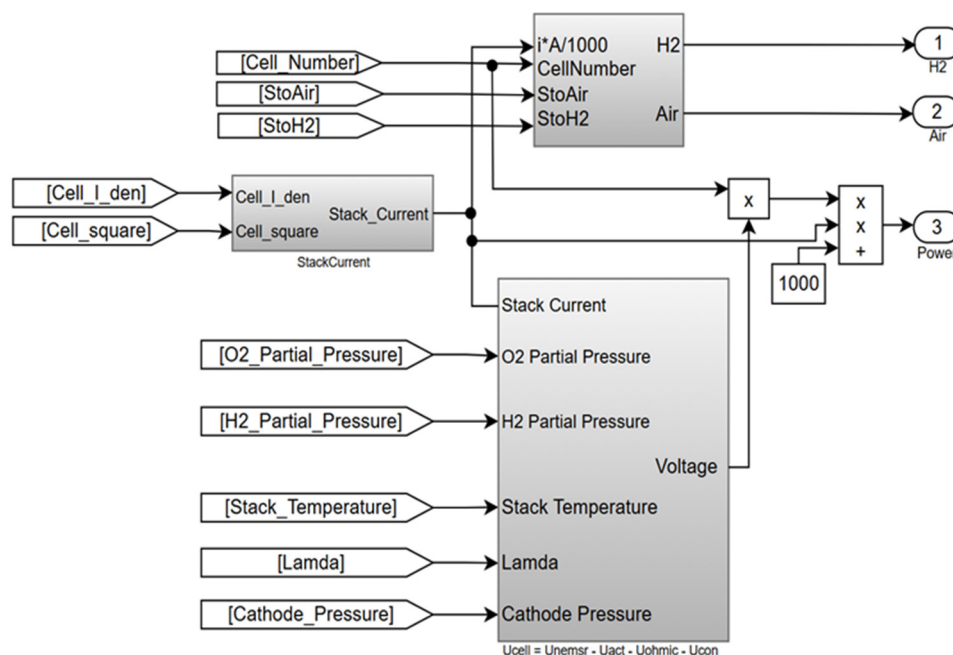


Figure 3. Fuel-cell stack model implemented in MATLAB/Simulink R2021a. The asterisk denotes multiplication.

Table 1. Fuel-cell stack model parameters.

Parameter Name	Value	Unit
Cell Number	316	-
Catalyst Surface Area	260	cm ²
Anode Stoichiometric Ratio	1.5	-
Cathode Stoichiometric Ratio	1.8	-
Temperature	353	K
Membrane Water Content (λ_m)	23	-
Anode Partial Pressure	1.5	bar
Cathode Partial Pressure	1.8	bar

The fuel-cell stack parameters were compiled from commercial stack technical-manual information, engineering estimates, and literature-supported PEMFC model parameters. After defining the calibrated MFC data source and the fuel-cell operating points, the model outputs were compared with the available calibrated/datasheet gas-flow values shown in Tables 2 and 3. The comparison is used to confirm the operating-point demand calculation, while the subsequent MFC allocation analysis uses the traceable calibration curves as the numerical basis for optimization.

Table 2. Anode mass flow data: datasheet values compared with model values.

Current Density (mA/cm ²)	Datasheet of Anode (L/min)	Model Anode (L/min)	Error (%)
1900	1643	1636	0.43
1800	1556	1550	0.39
1700	1469	1464	0.34
1600	1382	1378	0.29
1500	1295	1292	0.23
1400	1208	1206	0.17
1300	1122	1119	0.27
1200	1036	1033	0.29

Table 3. Cathode mass flow data: datasheet values compared with model values.

Current Density (mA/cm ²)	Datasheet Cathode (L/min)	Model Cathode (L/min)	Error (%)
1900	4683	4664	0.41
1800	4440	4418	0.50
1700	4193	4173	0.48
1600	3946	3927	0.48
1500	3699	3682	0.46
1400	3452	3436	0.46
1300	3206	3191	0.47
1200	2959	2945	0.47

2.2. Mass Flow Controller Calibration

In the fuel-cell test bench, thermal or Coriolis-type gas mass flow controllers are used to regulate gas flow. Institutions such as Ford Motor Company in the United States, the Japan Automobile Research Institute (JARI), and Tsinghua University in China have conducted relevant research on the measurement accuracy of gas flowmeters [25]. These studies indicate variations in the accuracy of flowmeters under different operating environments and flow conditions.

Gas mass flowmeter calibration methods include the mass-time, bell prover, and Pressure–Volume–Temperature–Time (pVtT) methods, which are primary standards but expensive and require complex setups. In contrast, secondary standard methods offer good repeatability and stability at lower costs, leading to their widespread use. Since the 1960s, institutions such as NIST, NIM, and NMII/AIST have extensively researched the sonic nozzle method for calibration [26–30], which is now widely adopted, as shown in Figure 4.

The calibration of gas mass flowmeters utilizing sonic nozzles is fundamentally governed by the critical flow property of these nozzles. Specifically, in the critical flow state

of the gas, its instantaneous mass flow becomes exclusively reliant on the pressure and temperature conditions at the nozzle inlet, remaining invariant to any alterations in downstream parameters. When the gas reaches a critical flow state, the flow rate through the sonic nozzle can be expressed by the following formula:

$$q_m = A^* C C^* P_0 \sqrt{\frac{M}{R T_0}} \quad (13)$$

where q_m is the gas mass flow rate (kg/s); A^* is the internal cross-sectional area of the nozzle throat (m^2); C is the outflow coefficient, normally close to 1; C^* denotes the critical-flow function under the current temperature, humidity, and pressure; P_0 indicates the stagnation pressure at the nozzle inlet (Pa); M is the molar mass of the measured gas (kg/kmol); and T_0 is the absolute stagnation temperature of the measured gas (K).

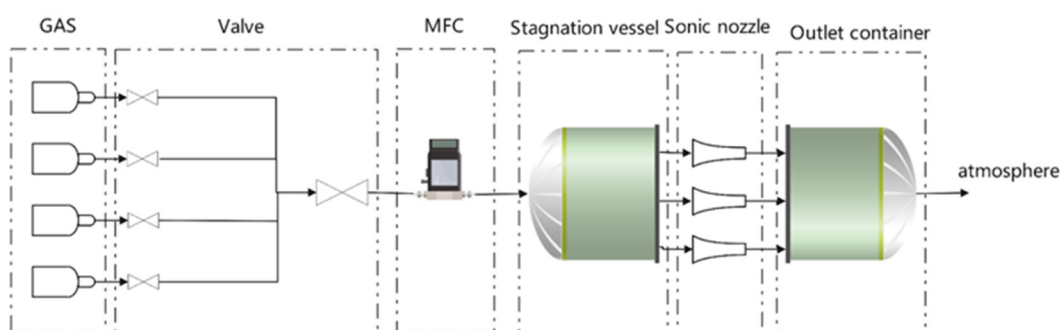


Figure 4. Sonic nozzle calibration system.

Three gas mass flow controllers with different ranges from a specific brand were selected and calibrated at a CNAS-certified institution. CNAS certification is recognized internationally through agreements with organizations like IAF, ILAC, and APAC. The calibration data include 13 entries for each controller, including the zero-flow and low-flow boundary entries, to characterize signed full-scale error across flow rates. Neighborhood linear interpolation was used to examine the relationship between commanded flow rate and signed calibration error, shown in Figure 5.

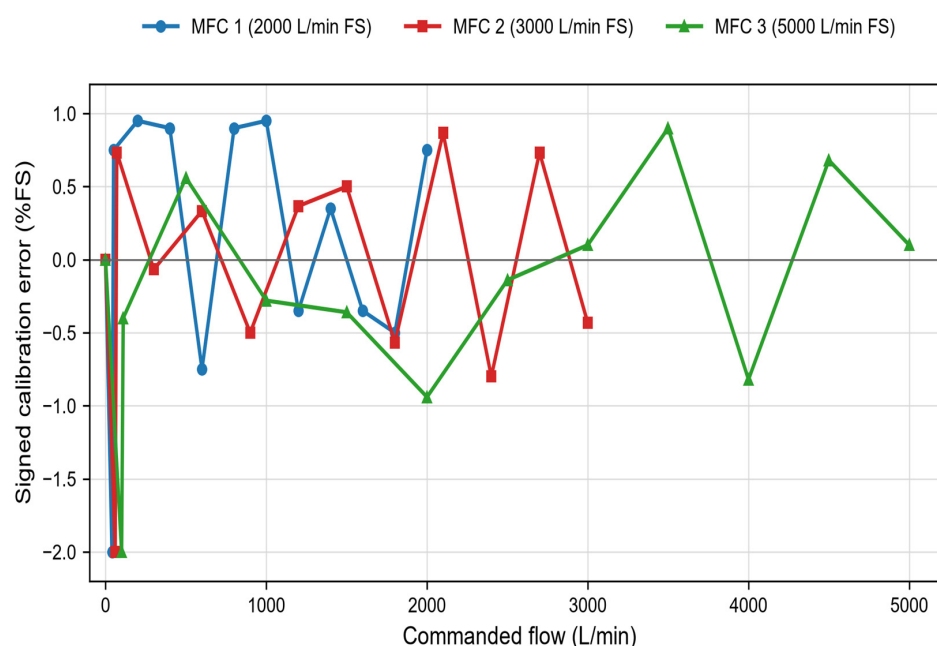


Figure 5. Calibration-error profiles for the three mass flow controllers (MFCs).

Figure 5 shows that the three MFCs have distinct flow-dependent calibration-error profiles. The local variations in the calibration curves explain why parallel allocation can improve system-level accuracy: for the same target total flow, the optimizer can select an allocation that avoids high-error regions on individual MFC curves.

2.3. Multi-MFC Switching Control Strategy

The current gas mass flow control strategy for the fuel-cell test bench is basic and relies on sequential activation of multiple MFCs based on their individual flow ranges. When the set flow is below the minimum range, only the MFC with the smallest range is activated. Once the flow exceeds this range, the next MFC is engaged. For instance, in a test bench with a 0–10,000 SLPM flow range and the three MFCs (2000, 3000, and 5000 L/min) defined above, Figure 6 shows the rule-based allocation error near the two main controller-activation regions.

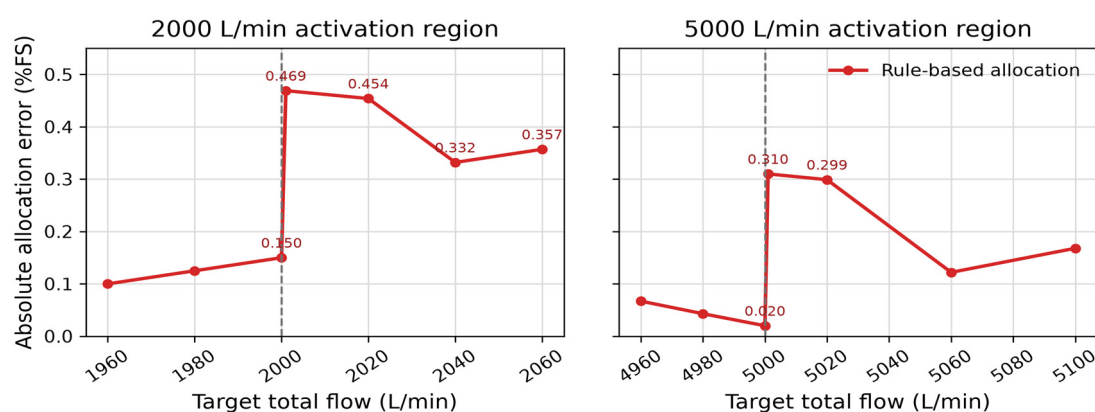


Figure 6. Rule-based allocation error near MFC activation regions.

Because the MFCs have a 50:1 effective turndown ratio, the lowest 2% of each range is a low-accuracy boundary region. In the rule-based switching strategy, local deviations appear near activation thresholds, especially around the transition from one active controller to multiple active controllers. These local switching-region peaks motivate the use of calibration-curve-based optimization rather than fixed sequential activation alone.

For the parallel-MFC allocation task, each PSO particle represents a candidate vector of MFC setpoints. The objective is to minimize the system flow-error metric derived from the calibrated MFC curves while satisfying the equality constraint that the sum of allocated flows equals the target total flow and the individual MFC operating limits.

The core of the PSO algorithm is the speed and position update formula, as shown below.

$$V_{m,b}^{h+1} = V_{m,b}^h + c_1 r_1 (P_{m,b}^h - X_{m,b}^h) + c_2 r_2 (P_{g,b}^h - X_{m,b}^h) \quad (14)$$

$$X_{m,b}^{h+1} = X_{m,b}^h + V_{m,b}^{h+1} \quad (15)$$

where c_1 and c_2 are the cognitive and social learning factors; r_1 and r_2 are random numbers between 0 and 1; m denotes the particle index and b denotes the spatial dimension; $V_{m,b}^h$ and $X_{m,b}^h$ are the velocity and position of particle m in dimension b at iteration h ; $P_{m,b}^h$ is the particle best position; and $P_{g,b}^h$ is the global best position [31].

The PSO algorithm flowchart is presented in Figure 7, illustrating the iterative search for an allocation solution.

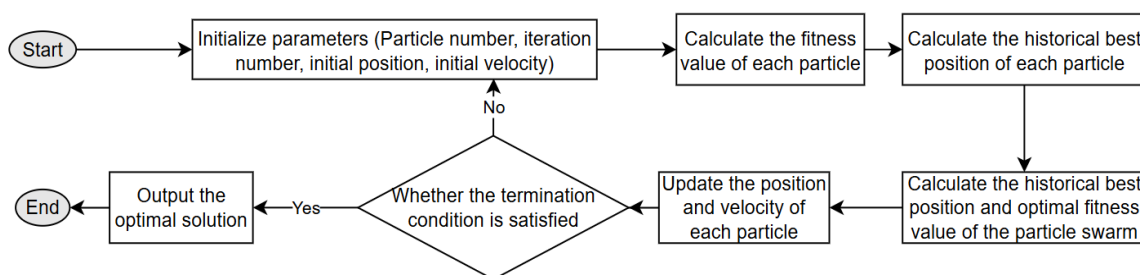


Figure 7. Particle swarm optimization (PSO) algorithm flow.

In optimization algorithms, the most crucial elements are the objective and constraint functions. This study focuses on coordinated allocation of multiple gas mass flow controllers to reduce full-scale error over the target-flow range while maintaining continuous total-flow coverage. The constraint function ensures that the sum of the flow values of multiple gas mass flowmeters equals the set flow, while the objective function aims to minimize the aggregate full-scale error corresponding to the current flow of each MFC.

This study uses three MFCs with full-scale ranges of 2000, 3000, and 5000 SLPM. Neighborhood linear interpolation was used to construct the flow–accuracy relationship for each controller from the calibration data. The resulting three-dimensional allocation problem minimizes the full-scale error metric while satisfying the target-flow equality constraint and individual controller operating limits.

The relationship function between gas mass flow and full-scale error for the 2000 SLPM MFC is denoted as $f_1(Q_1)$, and the corresponding functions for the 3000 SLPM and 5000 SLPM MFCs are denoted as $f_2(Q_2)$ and $f_3(Q_3)$, respectively. Here, Q_1 , Q_2 , and Q_3 represent the gas flow assigned to the first, second, and third MFCs, respectively, and each function output represents the signed calibrated full-scale error. In this study, the allocation metric is expressed as absolute full-scale error (%FS), i.e., the absolute flow deviation normalized by the full-scale range of the corresponding MFC. A smaller %FS value indicates a lower calibrated allocation error. PSO therefore minimizes the absolute aggregate %FS metric while satisfying the target-flow equality constraint, so that the selected MFC setpoints remain close to the required total flow and operate in lower-error calibration regions. The signed aggregate full-scale error metric is denoted as shown below:

$$E_{sys} = \frac{f_1(Q_1) \times 2000 + f_2(Q_2) \times 3000 + f_3(Q_3) \times 5000}{2000 + 3000 + 5000} \quad (16)$$

In the formula, E_{sys} is the signed overall full-scale error metric of the gas subsystem. The optimization objective uses its absolute value. The objective and constraint functions are presented as follows:

$$\begin{aligned} \min |E_{sys}| \\ Q_{target} &= Q_1 + Q_2 + Q_3 \\ Q_1 &\in \{0\} \cup [MFC1Range_{min}, MFC1Range_{max}] \\ Q_2 &\in \{0\} \cup [MFC2Range_{min}, MFC2Range_{max}] \\ Q_3 &\in \{0\} \cup [MFC3Range_{min}, MFC3Range_{max}] \end{aligned} \quad (17)$$

where $\min |E_{sys}|$ is the objective function. Q_{target} denotes the target setpoint flow, which equals the sum of the current mass flow rates assigned to the three MFCs. Q_1 , Q_2 , and Q_3 are constrained by the respective off-state and operating ranges of MFC1, MFC2, and MFC3. The zero-flow/off-state entries are included in the calibration data used for the allocation workflow.

PSO was selected as the main implementation route because each particle can directly represent the MFC flow-allocation vector, the method does not require gradient information from the piecewise calibration-curve objective, and the same formulation can be extended by adding allocation variables and operating-range constraints for additional MFCs. Standard PSO was adapted to this constrained allocation problem through three established mechanisms: dynamic inertia balances early global exploration with later local refinement, dynamic penalty handling guides the equality and operating-range constraints, and random perturbation reduces premature trapping in local low-quality regions of the discretized feasible space [32]. These mechanisms are used as engineering adaptations for parallel-MFC allocation and are evaluated by ablation analysis.

The algorithmic settings are reported transparently. The revised numerical evaluation uses 100 particles and 200 iterations as the reference setting, while the sensitivity analysis covers 50, 100, and 150 particles and 100, 200, and 300 iterations to show how this choice affects accuracy and runtime. The ablation study further separates the effect of dynamic inertia, dynamic penalty, and random perturbation; it shows that random perturbation is the primary source of the repeated-run reliability gain.

3. Results

3.1. Testing Conditions

Fuel-cell testing conditions vary widely. This study uses the Chinese standard GB/T 24554-2022, Fuel Cell Engine Performance Test Methods, as the operating-context reference for assessing gas-flow allocation accuracy. The standard covers power performance, efficiency, environmental adaptability, and reliability tests, including start-up characteristics, rated power, peak power, dynamic response, and steady-state characteristics.

The 150 kW-rated fuel-cell stack was selected because it represents a standard industrial grade, with many fuel-cell systems operating at or above this level. At 150 kW and higher, gas mass flow rates are significant, making multiple MFCs essential for achieving precise control. Based on the simulation model in Section 2 and the GB/T 24554-2022 standard, Figure 8 shows the relationship between current density and cathode gas mass flow rate, where both increase simultaneously, while Table 4 lists the test conditions and parameters used.

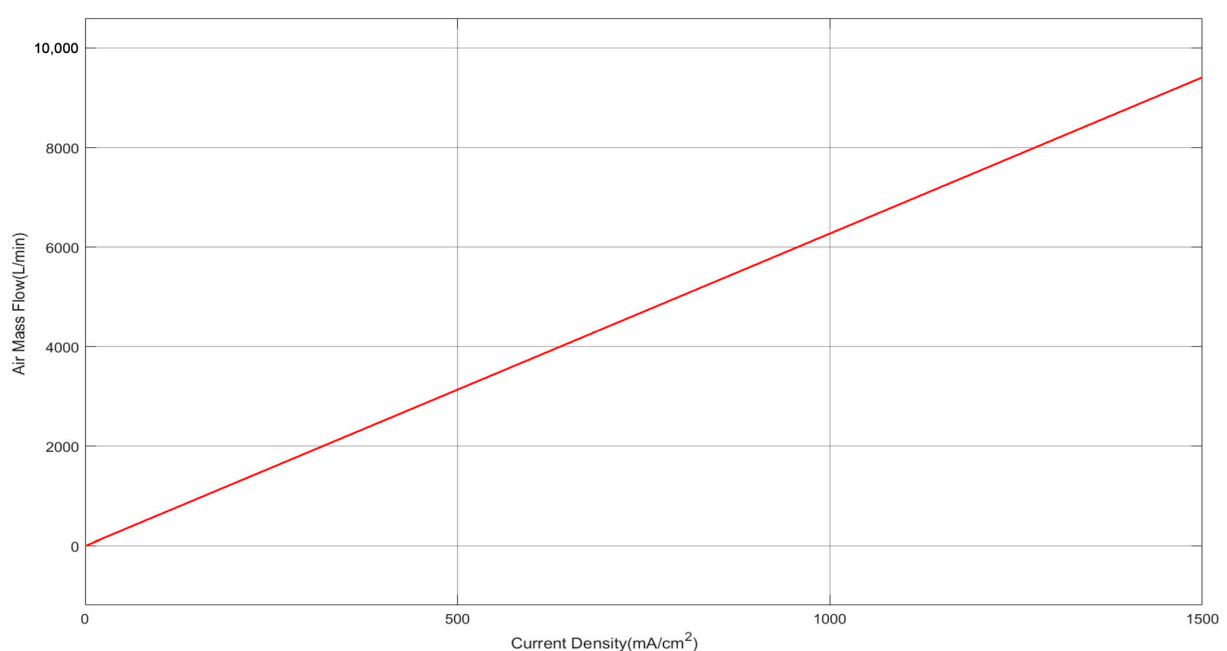


Figure 8. Relationship between current density and cathode air mass flow.

Table 4. Operating points used for the numerical evaluation.

Condition Name	Current Density (mA/cm ²)	Cathode Air Mass Flow (L/min)	Power (kW)
Idle (PI)	29	182	7.5
Rated power (PE)	1420	8905	150
Peak power (Pp)	1520	9535	160
10%PE	125	785	15
20%PE	270	1695	30
30%PE	410	2572	45
40%PE	555	3481	60
50%PE	695	4360	75
60%PE	835	5238	90
70%PE	980	6147	105
80%PE	1130	7088	120
90%PE	1275	7998	135

3.2. Simulation-Based Flow-Allocation Results

The operating points were evaluated with the rule-based allocation strategy and with optimization-based allocation. The analysis uses two clearly separated numerical protocols. The revised repeated-run numerical evaluation protocol supports reliability, ablation, and parameter sensitivity analysis under stochastic runs, whereas the constraint-handled numerical comparison projects the allocation search onto the feasible domain and supports switching-region expansion, numerical perturbation checks, and benchmark optimizer comparisons.

Figure 9 is presented as a representative single-run comparison under the revised repeated-run numerical evaluation protocol. The repeated-run analysis indicates that the main advantage of the improved PSO is higher low-error solution probability under stochastic runs. Across the main 12 operating points, 500 independent runs per operating point and algorithm show that the probability of reaching an error not exceeding 0.10 %FS increases from 0.207 for standard PSO to 0.310 for improved PSO. At the stricter 0.05 %FS threshold, the probability increases from 0.135 to 0.193. The positive-improvement rate relative to the rule baseline increases from 0.384 to 0.547. Confidence intervals, per-condition probabilities, paired Wilcoxon tests under matched run indices, and restart-probability interpretation are reported in Supplementary Tables S1–S5 and Supplementary Figures S1–S3.

The effective turndown ratio of the selected MFCs is 50:1, so local accuracy degradation is expected near the low-flow boundary of each activated controller. The switching-region comparison therefore focuses on the 2000–2060 SLPM and 5000–5100 SLPM regions, where the rule-based strategy is most likely to create local error peaks.

The switching-region evaluation summarized in Table 5 was expanded beyond the two original switching points. In the constraint-handled numerical comparison, 14 switching-region points in total, including the two original switching points, were evaluated around the approximately 2000 SLPM and 5000 SLPM activation regions. The rule baseline shows local error peaks around the switching thresholds, including about 0.469/0.454 %FS at 2001/2020 SLPM and about 0.310/0.299 %FS at 5001/5020 SLPM, while optimization-based allocation suppresses these peaks to near-zero numerical error under the constraint-handled comparison. The complete switching-region results are provided in Supplementary Table S9 and Supplementary Figure S6. In this comparison, the table reports improved-PSO results as a representative optimization-based allocation result; Section 3.4 further notes

that all tested optimization algorithms converge to similar near-zero errors under the same constraint-handled protocol.

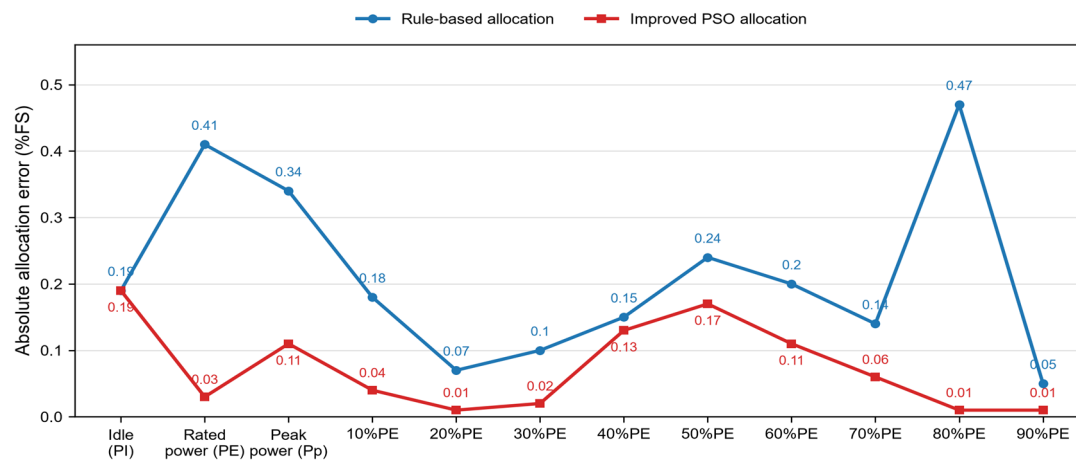


Figure 9. Representative single-run allocation-error comparison between rule-based allocation and improved PSO allocation under the revised repeated-run numerical evaluation protocol.

Table 5. Switching-region error comparison under the constraint-handled numerical comparison protocol.

Cathode Air Flow (L/min)	Rule-Based Error (%FS)	Improved PSO Median Error (%FS)
1960	0.100	<0.001
1980	0.125	<0.001
2000	0.150	<0.001
2001	0.469	<0.001
2020	0.454	<0.001
2040	0.332	<0.001
2060	0.357	<0.001
4960	0.067	<0.001
4980	0.043	<0.001
5000	0.020	<0.001
5001	0.310	<0.001
5020	0.299	<0.001
5060	0.122	<0.001
5100	0.168	<0.001

3.3. Implementation and Runtime Observations

The parameter sensitivity analysis evaluates 50, 100, and 150 particles combined with 100, 200, and 300 iterations. The setting of 100 particles and 200 iterations is used as the reference setting for the revised numerical evaluation because it stays within the tested sensitivity range and keeps the revised runs comparable with the updated baseline implementation. The previous computation-time statement has been clarified as a numerical runtime observation rather than a measured hardware control response.

Figure 10 summarizes the numerical runtime across 20 repeated simulation runs. The reported runtime characterizes computational demand in the numerical workflow. It is not used as evidence of closed-loop hardware response time. In the parameter sensitivity

analysis, the selected setting of 100 particles and 200 iterations is used as a fixed reference setting for comparability within the revised numerical workflow.

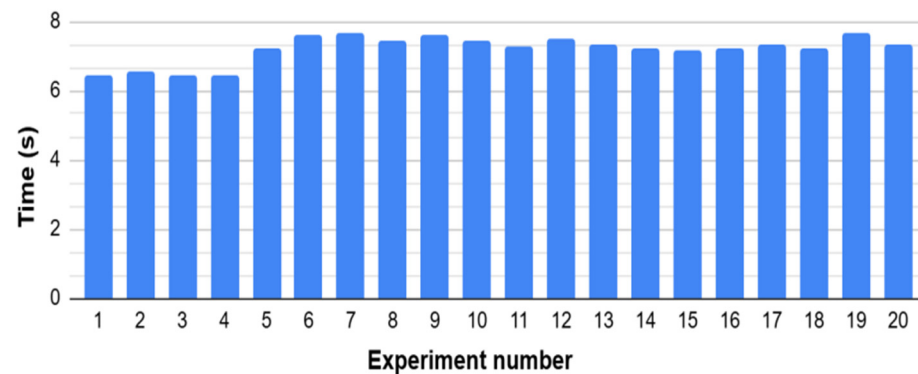


Figure 10. Numerical runtime of the PSO algorithm under the tested simulation settings.

For engineering deployment, optimized allocation results can be precomputed and stored in a lookup table so that a supervisory control program can retrieve selected MFC setpoints rapidly. The LabVIEW interface is therefore presented as an implementation concept for table retrieval and supervisory control integration. The supervisory interface was developed in LabVIEW Community Edition 2026 Q1 (National Instruments, Austin, TX, USA) and is shown in Figure 11.

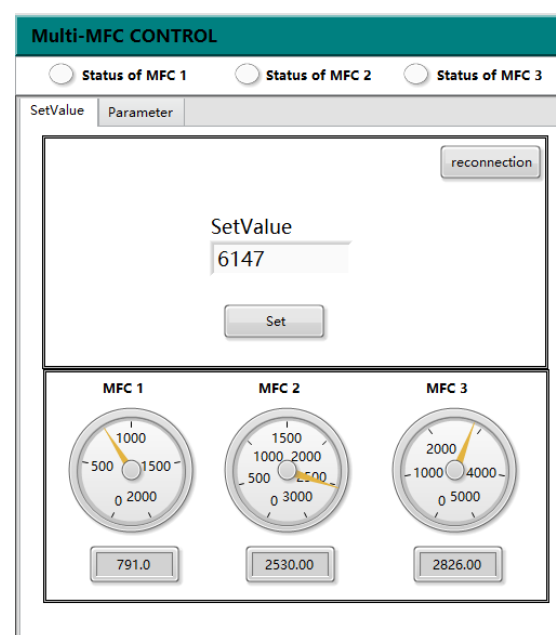


Figure 11. Multi-MFC supervisory control-program interface.

The LabVIEW retrieval demonstration indicates that lookup-table access can be fast once the optimized allocation table is available. Full end-to-end validation with real MFC dynamics, communication delay, sensor noise, thermal coupling, and controller drift is an important future bench-test step.

3.4. Ablation, Convergence, and Robustness Checks

The ablation study compares standard PSO, dynamic inertia, dynamic inertia with dynamic penalty, and the full improved PSO with random perturbation. For the main 12 operating points, the mean of the per-condition median absolute error decreases from

0.337 %FS for standard PSO to 0.251 %FS for the full improved PSO, and the mean positive-improvement rate increases from 0.392 to 0.533. These values are per-condition means in the ablation analysis, whereas the 0.384-to-0.547 values in Section 3.2 are pooled repeated-run rates across the main operating points. The ablation results identify random perturbation as the dominant contributor and dynamic inertia as a secondary contributor, while the dynamic penalty term has limited marginal benefit under the current discretized evaluation protocol. The full ablation, sensitivity, and fitness-convergence results are provided in Supplementary Tables S6–S8 and Supplementary Figures S4, S5 and S9.

Numerical perturbation checks evaluate whether bounded synthetic disturbances amplify the optimized allocation error. Synthetic uniform predicted-flow perturbations with amplitude $L = 0, 0.05, 0.10$, and 0.20 %FS were applied on the evaluation side, corresponding to perturbation ranges of $\pm L$ %FS. The main operating points and switching points show nearly overlapping mean noisy errors that increase approximately linearly with the imposed perturbation, indicating no additional switching-region amplification. The idle point is slightly higher, which reflects the low-flow boundary condition under the same evaluation-side perturbation. This is reported as a numerical robustness check, not as a hardware-noise experiment. The detailed perturbation results are provided in Supplementary Table S10 and Supplementary Figure S7.

A benchmark optimizer comparison was also performed in the constraint-handled numerical comparison procedure. Standard PSO, improved PSO, differential evolution, genetic algorithm, and grid search converge to similar near-zero numerical errors under this comparison setting, as reported in Supplementary Tables S11 and S12 and Supplementary Figure S8. This benchmark positions the supported contribution as improved repeated-run low-error probability relative to standard PSO in the revised repeated-run numerical evaluation protocol, together with calibration-curve-based mitigation of rule-baseline switching-region peaks. The constraint-handled comparison is therefore used to show feasible-domain allocation behavior across optimizers, while the repeated-run protocol is used to evaluate single-run stochastic reliability and the practical need for fewer restarts in offline table generation.

4. Discussion

The results show that optimization-based allocation is valuable for parallel-MFC systems because it uses calibration-curve information to select among multiple feasible flow allocations. This is especially relevant near switching regions, where a rule-based strategy can produce local error peaks. In fuel-cell engine test procedures such as GB/T 24554-2022 [33], stable low-error gas-flow allocation is useful because repeated test conditions require consistent reactant delivery over a wide flow range. For the 0–10,000 SLPM parallel-MFC range used in this study, the 0.10 %FS low-error threshold corresponds to about 10 L/min instantaneous metering difference and about 14.4 m^3 over a 24 h steady segment, while the stricter 0.05 %FS threshold corresponds to about 5 L/min and about 7.2 m^3 over 24 h. The rule-baseline switching-region peaks of 0.469/0.454 %FS and 0.310/0.299 %FS correspond to tens of L/min at this full-scale range. These conversions illustrate the engineering scale of small %FS differences in large-flow benches and do not represent measured 24 h hydrogen or air consumption experiments.

The fuel-cell stack model and the MFC optimization have distinct roles in the workflow. The stack model defines gas-demand context for the selected operating points. The optimization problem then allocates that demand among parallel MFCs based on calibrated MFC curves. Temperature, pressure, and electrochemical state variables are therefore outside the current PSO decision vector, which keeps the allocation study focused on the gas-circuit control layer.

The current evidence base is a simulation-based evaluation using traceable calibration data. The numerical perturbation test evaluates bounded synthetic disturbances and supports numerical robustness screening. Physical validation under real sensor noise, MFC dynamics, communication delay, thermal coupling, and long-term drift remains the next experimental step. The response-time discussion is therefore framed as numerical runtime and lookup-table retrieval context.

The method can be extended from three MFCs to N parallel MFCs by increasing the dimension of the allocation vector and adding the corresponding range constraint and calibration-error function for each controller. As the number of controllers grows, the feasible search space and offline computation cost also grow. Grid search is useful as a small-scale numerical reference but becomes increasingly expensive with additional MFCs and finer resolution; GA and DE remain valid comparison methods but also require more evaluations as dimensionality increases. The improved PSO route is therefore used as the main practical implementation path for offline lookup-table generation. Calibration-curve quality, interpolation uncertainty, and periodic recalibration become increasingly important when MFC characteristics drift or age.

5. Conclusions

This paper presents a simulation-based accuracy optimization method for parallel-MFC allocation in fuel-cell test-bench gas circuits. Using traceable MFC calibration data and a reproducible revised numerical evaluation workflow, this study shows that optimization-based allocation can reduce rule-baseline switching-region error peaks and that the improved PSO increases the probability of obtaining low-error solutions under repeated stochastic runs. Across the main 12 operating points, the improved PSO increases the 0.10 %FS good-solution probability from 0.207 to 0.310 and the 0.05 %FS good-solution probability from 0.135 to 0.193. The ablation analysis identifies random perturbation as the dominant source of the reliability improvement. Future work will focus on physical bench validation, online calibration updates, uncertainty-aware interpolation, and extension to larger MFC arrays.

The benchmark optimizer comparison also motivates future studies on optimizer selection under unified hardware constraints, especially when real-time implementation, calibration drift, and controller communication delay are included.

Supplementary Materials: The following supporting information can be downloaded at <https://www.mdpi.com/article/10.3390/pr14142344/s1>: Supplementary Tables S1–S12 and Supplementary Figures S1–S9, including repeated-run reliability statistics, ablation and sensitivity analyses, switching-region evaluation, numerical perturbation checks, and benchmark optimizer comparisons.

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Conflicts of Interest: Authors Hong Chang, Yi Ma, Maojun Tian, Hao Yu and Dejun Huang were employed by the company China Automotive Engineering Research Institute Co., Ltd. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as potential conflicts of interest.

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