

Article

Process Modeling and Load Prediction for Pear-Shaped Expander Restoration of Deformed Casing in Horizontal Oil and Gas Wells

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Abstract

Casing deformation in horizontal wells reduces the effective drift diameter of the wellbore and may interrupt workover and hydraulic fracturing operations. Accurate prediction of reshaping load is therefore important for deformed-casing repair and wellbore integrity management. In this study, the mechanical response of C110-grade elliptically deformed casing restored by a pear-shaped expander was investigated without cement-sheath constraint. Two analytical reshaping-force models based on curved beam theory and thick-walled cylinder theory were established, and a three-dimensional finite element model was developed to simulate nonlinear tool–casing contact and elastoplastic deformation. Five expander outer diameters of 122–130 mm were analyzed, and a 1:8 similarity-scaled experiment was conducted for validation. The results show that the reshaping force increased from 50.2 t to 141.7 t as the expander outer diameter increased from 122 mm to 130 mm, whereas the improvement in minor-axis expansion was relatively limited. The finite element results agreed well with the experimental results, with average errors of 8.542% for reshaping force and 8.822% for minor-axis expansion. The thick-walled cylinder model showed better prediction accuracy than the curved beam model, with an average error of 4.11%. The proposed analytical–numerical–experimental framework provides a basis for reshaping-load estimation, expander selection, and stepwise restoration process design in deformed-casing repair.

Keywords: casing deformation; casing reshaping; pear-shaped expander; C110 casing; reshaping force prediction; restoration process design; finite element simulation; similarity-scaled experiment; wellbore integrity



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1. Introduction

With the continuous development of unconventional oil and gas resources, horizontal wells combined with multistage hydraulic fracturing have become an important approach for improving reservoir stimulation efficiency and hydrocarbon recovery from shale gas, tight oil, and other low-permeability reservoirs [1]. In such wells, the casing system plays a critical role in maintaining wellbore integrity, providing mechanical support, isolating formations, and ensuring the safe passage of fracturing and workover tools [2]. During hydraulic fracturing and long-term production, casing strings in horizontal sections are subjected to complex coupled loads, including non-uniform in situ stress, fracturing-induced pressure fluctuations, thermal effects, formation slip, and changes in cement-sheath support [3,4]. Under the combined effects of geological conditions and engineering

disturbances, casing deformation may occur in the form of ovalization, diameter reduction, bending, local indentation, or shear offset [5]. Among these failure modes, elliptical deformation directly reduces the minimum drift diameter of the wellbore and may obstruct the passage of bridge plugs, perforating guns, milling tools, and other downhole assemblies. Severe casing deformation can interrupt fracturing operations, delay production, increase workover costs, and threaten the long-term service life of oil and gas wells. Therefore, understanding the mechanical response of deformed casing during repair and accurately predicting the required reshaping load are important for maintaining wellbore integrity and improving the operational reliability of unconventional oil and gas development [6,7].

To restore the passability of deformed casing, several mechanical repair methods have been developed and applied in field operations, including pear-shaped expander reshaping, spinning casing swages, hydraulic rolling reshapers, expansion-cone reshaping, and other mechanical restoration methods [8–11]. Among these methods, the pear-shaped expander has attracted considerable attention because of its simple structure, low manufacturing cost, convenient operation, and adaptability to local casing deformation. During restoration, the axial load transmitted by the drill string drives the expander into the deformed interval. The tapered working surface subsequently contacts the casing inner wall and converts part of the axial load into radial expansion force. This radial force induces elastoplastic deformation of the elliptically deformed casing and gradually restores its effective inner diameter [12]. From a process-engineering perspective, pear-shaped expander restoration can be regarded as a nonlinear tool–casing interaction process involving axial-to-radial load transfer, contact-pressure evolution, frictional resistance, material yielding, and geometric recovery [13]. The reshaping load and restoration effect are jointly affected by casing geometry, material properties, initial ovality, expander outer diameter, cone angle, and friction coefficient. If the expander diameter is too small, the restoration effect may be insufficient; if it is too large, the required reshaping load may increase sharply, thereby increasing the risks of local stress concentration, tool sticking, or secondary casing damage. Therefore, establishing a reliable load-prediction method and clarifying the effect of expander diameter on casing recovery are essential for the safe design and optimization of deformed-casing restoration processes [14].

Previous studies have investigated casing deformation mechanisms, repair-tool structures, reshaping-force calculation methods, and numerical simulation approaches [15]. In terms of casing deformation and wellbore integrity, researchers have analyzed the effects of in situ stress, cement-sheath behavior, hydraulic-fracturing disturbance, thermal loading, and formation movement on casing damage and failure [16,17]. In theoretical analyses, elastoplastic mechanics, contact mechanics, curved-beam theory, and thick-walled-cylinder theory have been used to describe casing deformation, collapse resistance, radial expansion, and repair load under different assumptions [18,19]. In numerical studies, the finite element method has been widely adopted to analyze stress distribution, deformation recovery, contact behavior, and load evolution during tool–casing interaction [20–22]. In experimental research, laboratory loading tests, similarity-scaled model tests, and field tests have been conducted to evaluate the restoration performance of casing repair tools and the integrity of casing–cement sheath systems [23,24]. These studies have provided an important foundation for the development of casing-repair technologies and wellbore-integrity management.

Under actual wellbore conditions, however, the casing does not behave as an isolated tubular component but interacts mechanically with the surrounding cement sheath and formation. In addition to providing zonal isolation and annular sealing, an intact and well-bonded cement sheath can transmit radial and shear stresses through the casing–cement interface, thereby restraining radial deformation and ovalization of the casing [25]. As a result, the cement sheath contributes to the effective radial stiffness and resistance to

ovalization of the casing–cement system, while also modifying the local bending response and stress redistribution of the casing under non-uniform loading [26]. The magnitude of this mechanical restraint depends on the elastic properties and thickness of the cement sheath, the bonding condition at the casing–cement and cement–formation interfaces, the stiffness of the surrounding formation, and the loading history [27]. Previous studies have shown that hydraulic-fracturing-induced pressure cycling, temperature variation, formation deformation, and non-uniform in situ stress may lead to cement cracking, casing–cement interfacial debonding, or cement–formation interfacial failure. Once the cement sheath or its interfaces are damaged, the external confinement provided to the casing may be partially or substantially reduced, and the local mechanical response of the casing may gradually approach that of a partially unsupported tubular structure [28].

The mechanical role of the cement sheath should therefore be considered when evaluating casing deformation and repair behavior under actual downhole conditions. Nevertheless, directly incorporating cement–sheath restraint, interfacial damage, formation confinement, and non-uniform in situ stress into a pear-shaped expander restoration analysis would introduce multiple coupled mechanisms, including tool–casing contact, casing plastic deformation, cement damage, interface degradation, and formation interaction [29,30]. These coupled effects would make it difficult to distinguish the fundamental load-transfer behavior between the pear-shaped expander and the deformed casing. Accordingly, the absence of cement–sheath constraint is adopted in the present study as a deliberately defined baseline condition rather than as a representation of all field conditions. This simplification enables the primary tool–casing interaction to be isolated and establishes a reference framework that can subsequently be extended to casing–cement sheath–formation coupled conditions. The simplified condition may also be relevant to local intervals where cement support has been significantly weakened by cracking or interfacial debonding [31].

Despite the above progress, important limitations remain in the available studies. Previous investigations have examined the effects of pear-shaped swage parameters on reshaping force, springback, and restoration performance, while other tubular-expansion studies have developed force-prediction and numerical models for conical- or mandrel-driven expansion processes under specific contact conditions [32,33]. However, these models are generally tool-specific and are derived from their respective expansion geometries, loading modes, and contact assumptions. The progressive tapered contact associated with a pear-shaped expander differs from the axisymmetric or mandrel-driven expansion conditions considered in many tubular-expansion studies. During pear-shaped expander restoration, the axial thrust is transformed into a non-uniform radial expansion force, while the tool–casing contact region evolves continuously as the expander advances. Consequently, the direct applicability of existing expansion-force models to pear-shaped expander restoration requires further assessment [34].

The restoration of an elliptically deformed casing also involves the coupled effects of bending-dominated deformation of the elliptical cross-section and contact-pressure-induced elastoplastic expansion of the casing wall. Previous studies have shown that the expansion force and deformation response are sensitive to tool geometry, friction, expansion ratio, material behavior, and boundary conditions [35–37]. A single analytical description may therefore be insufficient to capture the complete evolution of reshaping load, stress redistribution, and geometric recovery. In particular, the relative applicability of bending-based and contact-pressure-based reshaping-force models has not been systematically evaluated for the same pear-shaped expander–casing configuration under identical geometric, material, contact, and boundary conditions.

Although experimental and numerical investigations have provided valuable insights into tubular expansion and casing-repair behavior, comprehensive validation remains demanding because it requires controlled specimen geometry, sufficient loading capacity, dedicated fixtures, and well-defined boundary conditions [38,39]. Differences among theoretical assumptions, numerical contact conditions, and experimental loading arrangements may further affect the comparison of reshaping force and residual deformation. Therefore, the quantitative relationship among analytical prediction, nonlinear finite element simulation, and physical testing still requires further clarification for pear-shaped expander restoration of elliptically deformed casing.

Based on these considerations, this study investigates the mechanical response and reshaping-force prediction of C110-grade elliptically deformed casing during pear-shaped expander restoration under a baseline condition that excludes cement-sheath restraint and external confining pressure. The central contribution of the present work is the establishment of a unified comparison framework for two reshaping-force prediction approaches with different physical bases under the same pear-shaped expander–casing configuration, material properties, and boundary conditions.

Within this framework, a curved-beam model is employed to characterize the bending-dominated response of the elliptical cross-section, including local stress concentration and the onset of yielding under radial loading. A thick-walled-cylinder model is used to describe the elastoplastic expansion of the casing wall induced by the progressive development of contact pressure and frictional resistance during expander advancement. By evaluating these two analytical approaches under identical casing geometry, material parameters, expander configuration, and loading conditions, their prediction performance, associated load-transfer mechanisms, and applicability under the investigated baseline condition can be systematically assessed.

A three-dimensional nonlinear finite element model is further established to simulate the geometric interference, finite-sliding contact, frictional behavior, and elastoplastic deformation involved in the pear-shaped expander–casing interaction. In the present study, the numerical model serves as a common benchmark for evaluating the two analytical reshaping-force models under consistent conditions. Five expander outer diameters ranging from 122 to 130 mm are then considered to quantify the effects of tool size on reshaping force, stress distribution, and minor-axis recovery. This analysis clarifies the trade-off between increasing load demand and the corresponding improvement in geometric recovery, thereby providing a quantitative basis for preliminary stepwise expander-size selection.

Finally, a 1:8 geometrically scaled physical test is conducted to corroborate the variation trends in reshaping load and minor-axis recovery under different expander-diameter conditions. The experimental observations are used to support the comparative trends identified in the analysis rather than to establish direct equivalence with full-scale field performance. The resulting framework provides a reference for rapid reshaping-load estimation, analytical-model selection, and preliminary expander-size design for elliptically deformed C110 casing. Further investigation is required to extend the proposed framework to field conditions involving intact cement support, formation confinement, non-uniform in situ stress, and complex downhole friction.

2. Computational Method

To clarify the mechanical behavior involved in restoring an elliptically deformed casing using a pear-shaped expander, the structural configuration and shaping mechanism of the tool are first examined. On this basis, two shaping-force prediction models are developed using curved beam theory and the elastoplastic theory of thick-walled cylinders. The relationships among axial thrust, radial expansion force, casing geometry, tool cone

angle, and friction coefficient are then analyzed, providing a theoretical foundation for the subsequent finite element simulations and scaled experimental validation.

2.1. Structure and Working Principle of the Pear-Shaped Expander

The pear-shaped expander is a typical mechanical tool used for casing reshaping. During operation, the tool is driven into the deformed section of the casing by the axial load transmitted through the drill string. As the tapered working surface comes into contact with the inner wall of the casing, the resulting compressive interaction converts part of the axial load into a radial expansion force. This force induces elastoplastic deformation in the deformed casing and gradually restores its effective inner diameter. The structural configuration of the pear-shaped expander is shown in Figure 1a, and the corresponding reshaping process is illustrated in Figure 1b.

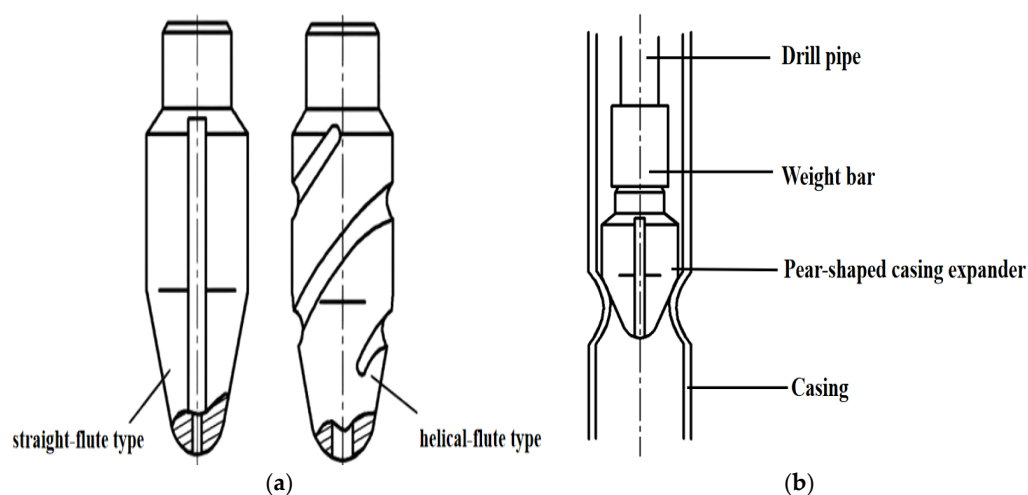


Figure 1. Pear-shaped casing expander: (a) structural configurations of the straight-flute and helical-flute types; (b) schematic of the casing reshaping process.

As illustrated in Figure 1, when the tapered surface of the expander contacts the deformed casing section under the axial load transmitted by the drill string, a radial expansion force is generated. Through sustained compressive interaction, the elliptically deformed casing is progressively re-rounded. During operation, the tool is first positioned at the deformed interval, after which the drill string is reciprocated repeatedly. Driven by the gravitational load and kinetic energy of the drill string, the tapered head gradually penetrates the deformed section, enabling the expansion and restoration of the casing.

The pear-shaped expander consists of three functional sections: a tapered section, a working section, and a connection-control section. The tapered section is 300 mm in length with a cone angle of 14° , and is designed to guide the tool smoothly into and through the deformed casing interval. The working section has a length of 400 mm and serves as the primary region for casing expansion and restoration. The connection-control section is 320 mm long and is equipped with a drill-string interface, allowing the tool to be connected to the surface power system for load transmission and operational control during downhole reshaping. The detailed structural configuration of the pear-shaped expander is shown in Figure 2.

Under axial loading, concentrated stresses are generated when the tapered section of the expander comes into contact with the deformed casing interval. These stresses promote circumferential plastic deformation of the casing wall, thereby enabling casing restoration. In a typical operation, the tool is first positioned at the deformed section, and the tapered head is then driven to expand the deformed region radially through repeated lifting and lowering of the drill string. The main advantage of this tool lies in its ability to convert

axial loading into radial expansion while maintaining a relatively simple and convenient operating procedure.

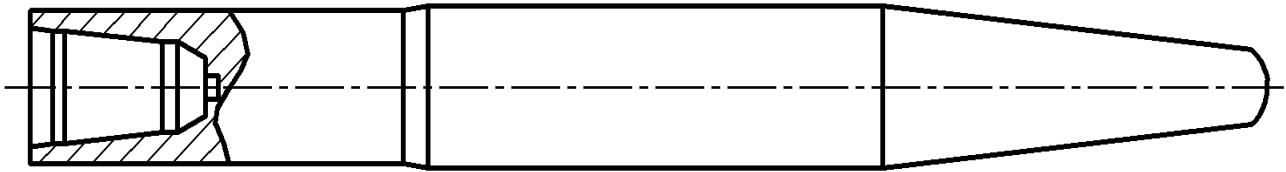


Figure 2. Basic structure of the pear-shaped expander.

2.2. Restoration Model of the Pear-Shaped Expander Based on Curved Beam Theory

To establish the shaping-force model of the pear-shaped expander, the elliptically deformed casing is first idealized as an equal-thickness elliptical ring. The casing material is assumed to be homogeneous and isotropic, and the deformation of the cross-section is considered to be mainly governed by bending under an asymmetric radial load. The schematic cross-section of the elliptically deformed casing is shown in Figure 3.

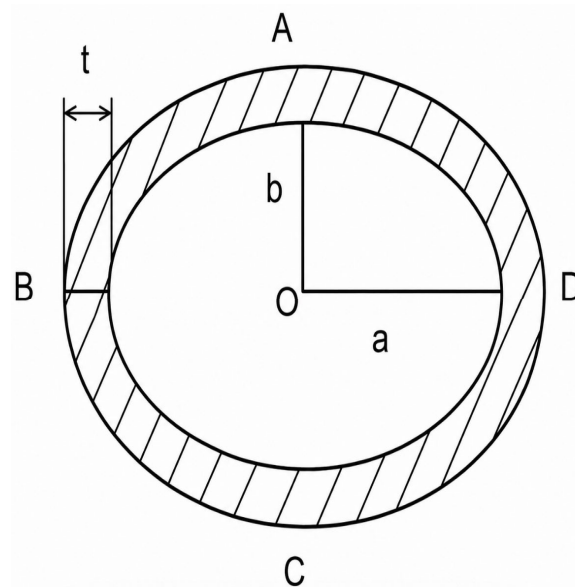


Figure 3. Cross-section of the elliptically deformed casing, where A–D denote the topmost, leftmost, bottommost, and rightmost points on the outer boundary, respectively.

The force response during restoration of the deformed casing using the pear-shaped expander is shown in Figure 4a, while the simplified mechanical model of the elliptically deformed casing subjected to radial concentrated loads is presented in Figure 4b. To facilitate the formulation of the shaping-force model, the contact compression between the tapered working surface of the expander and the deformed casing section is idealized as a radial concentrated load F_2 acting along the minor-axis direction of the casing. By neglecting the elastic deformation of the tool itself and secondary frictional losses, the axial load F_p transmitted from the drill string to the expander can be converted into the radial shaping force F_2 acting on the inner wall of the casing through the tapered working surface. Based on the force equilibrium of the tapered surface, the relationship between these two forces can be expressed as follows:

To establish the curved beam model, the pear-shaped expander was assumed to be rigid, and the interaction between the tapered surface of the expander and the casing inner wall was simplified as an idealized normal contact. In this initial axial–radial force–conversion relationship, tangential frictional resistance along the tapered surface was not

explicitly considered; therefore, the friction coefficient does not appear in Equation (1). Under this assumption, the axial load F_p transmitted from the drill string to the expander can be converted into the radial shaping force F_2 acting on the inner wall of the casing according to the cone angle of the tapered surface. Based on the force equilibrium of the frictionless tapered surface, the relationship between these two forces can be expressed as follows:

$$F_p = 2F_2 \tan \alpha \quad (1)$$

where F_p denotes the axial force transmitted from the drill string to the expander, N; F_2 is the radial shaping force acting on the casing wall, N; and α represents the half cone angle of the pear-shaped expander, rad.

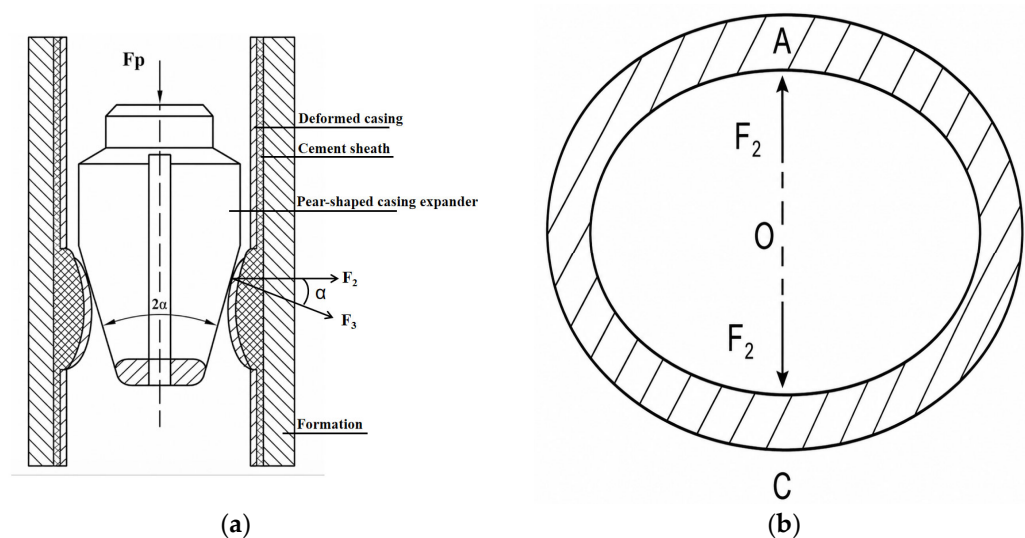


Figure 4. Force analysis and simplified model of deformed casing restoration using the pear-shaped expander: (a) force diagram of the deformed casing restoration process; (b) mechanical model of the elliptically deformed casing under radial concentrated loads.

It should be noted that Equation (1) represents an idealized frictionless force-conversion relationship used in the curved beam model. This simplification enables the bending-dominated response of the elliptical casing cross-section to be analyzed, but it may underestimate the total axial thrust required during actual expander advancement. The effect of frictional resistance is further considered in the thick-walled cylinder model and in the finite element contact formulation.

Figure 5 presents the loading distribution of the deformed casing. In this mechanical model, the redundant bending moment X_1 is taken as the primary unknown to be solved.

Based on the force analysis, the loading on the elliptical casing ring is symmetric, and the axial force has a negligible influence on the cross-sectional restoration. Therefore, the model can be simplified to a quarter of the elliptical ring for calculation. During the restoration process, the $B - D$ section is assumed to remain constrained, meaning that the total angular displacement of this section is zero. According to the displacement theory of curved beams, the compatibility equation can be written as follows:

$$\delta_{11} X_1 + \Delta_{1F} = 0 \quad (2)$$

where δ_{11} denotes the angular displacement caused by a unit moment, rad; X_1 is the redundant bending moment, N·mm; and Δ_{1F} represents the angular displacement induced by the external load alone, rad.

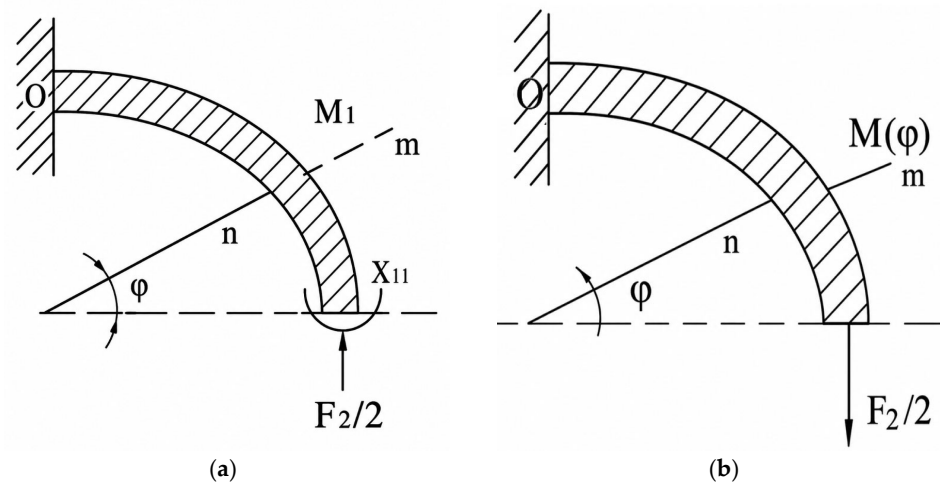


Figure 5. Force analysis of the quarter elliptical casing ring: (a) unit redundant moment X_{11} acting alone; (b) Force analysis of external loads acting alone.

Owing to the symmetry of the loading and geometry, a quarter model is selected for the mechanical analysis, as shown in Figure 5. The moment equilibrium equation for an arbitrary section of the casing can then be established as follows:

$$M_1 = X_{11} = 1 \quad (3)$$

where M_1 denotes the bending moment generated at an arbitrary cross-section of the casing under the unit moment loading condition, N·m.

Based on the displacement theory of curved beams, the deformation compatibility problem of the quarter elliptical ring can be formulated as the solution of the redundant moment X_1 . The angular displacement caused by a unit moment characterizes the flexibility of the elliptical ring structure and is expressed as:

$$\delta_{11} = \frac{b}{EI} \int_0^{\pi/2} \sqrt{1 + k(\sin \phi)^2} d\phi \quad (4)$$

where $k = \frac{a^2 - b^2}{b^2}$, k denotes the ovality of the deformed casing.

To further determine the internal force distribution of the elliptically deformed casing under the radial concentrated load, an arbitrary cross-section of the quarter elliptical ring is selected for force analysis, as shown in Figure 6.

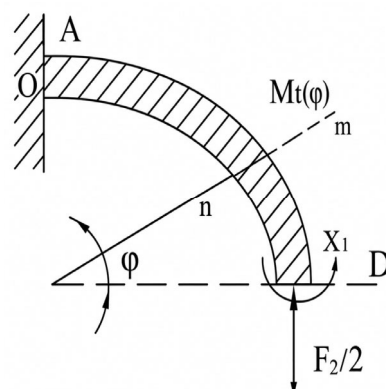


Figure 6. Force analysis of an arbitrary cross-section of the elliptically deformed casing.

Here, φ denotes the position angle of the cross-section, $Mt(\varphi)$ is the total bending moment at this section, X_1 is the redundant moment determined from the displacement

compatibility condition, and $F_2/2$ represents the equivalent radial load acting on the boundary of the quarter model. Based on the moment equilibrium of the arbitrary cross-section, the total bending moment of the casing section can be expressed as:

$$M(\phi) = \frac{Fa}{2}(1 - \cos \phi) \quad (5)$$

Based on Mohr's theorem in structural mechanics, the angular displacement caused by the external load acting alone can be expressed as:

$$\Delta_{1F} = \frac{1}{EI} \int M_1 \cdot M(\phi) \cdot R d\phi \quad (6)$$

Combining the bending moment expression of the quarter elliptical ring with the differential arc-length relationship, the angular displacement induced by the external load can be obtained as:

$$\Delta_{1F} = \frac{Fab}{2EI} \int_0^{\frac{\pi}{2}} (\cos \phi - 1) \sqrt{1 + k(\sin \phi)^2} d\phi \quad (7)$$

The redundant bending moment X_1 can then be determined from the displacement compatibility condition:

$$X_1 = \frac{Fa \int_0^{\frac{\pi}{2}} \sqrt{1 + k(\sin \phi)^2} (1 - \cos \phi) d\phi}{2 \int_0^{\frac{\pi}{2}} \sqrt{1 + k(\sin \phi)^2} d\phi} \quad (8)$$

Equation (8) indicates that the redundant bending moment X_1 is governed by the radial concentrated load F_2 , the geometric dimensions of the elliptical cross-section, and the casing ovality (k). This moment reflects the bending constraint of the elliptically deformed casing under the radial concentrated load and serves as a key parameter for determining the total bending moment and stress distribution of the casing section.

Based on the analysis shown in Figure 6, the total bending moment $M_t(\phi)$ and the total stress $\sigma_t(\phi)$ at each casing section can be determined as follows:

$$M_t(\phi) = \frac{Fa}{2}(1 - \cos \phi) - X_1 \quad (9)$$

$$\sigma_t(\phi) = \frac{M_t(\phi)}{W} + \sigma_N(\phi) \quad (10)$$

where $\sigma_N(\phi)$ denotes the axial stress at each cross-section of the deformed casing, MPa; and W represents the bending section modulus of the casing wall per unit thickness, m^3 .

To determine whether the deformed casing enters an effective plastic reshaping stage under the action of the pear-shaped expander, the stress state at the critical section of the deformed region must be further examined. Since the radial concentrated load mainly acts along the minor-axis direction of the casing, pronounced bending deformation and stress concentration are generally expected near section D. Therefore, based on the expression for the total stress at an arbitrary section, the stress distribution near section D can be obtained as follows:

$$\sigma_{tD} = \frac{6X_1}{t^2} + \frac{F_2}{2t} \quad (11)$$

When the maximum stress near section D, σ_{tD} reaches the yield strength of the casing material, σ_y , the casing wall begins to enter the plastic deformation stage. At this point, the radial load F_2 applied by the expander can be regarded as the minimum radial force

required to produce effective casing reshaping. By combining the axial–radial force conversion relationship, the solution for the redundant bending moment X_1 , and the yielding condition at section D, the shaping-force model of the pear-shaped expander based on curved beam theory can be established as follows:

$$F_2 = \frac{4t^2\sigma_s \tan \alpha}{\frac{\pi}{12\int_0^2 (1 - \cos \phi) \sqrt{1 + k(\sin \phi)^2} d\phi} + t} \quad (12)$$

$$\frac{\pi}{2\int_0^2 \sqrt{1 + k(\sin \phi)^2} d\phi}$$

The curved beam model described above uses the initial yielding of the critical casing section as the criterion for reshaping, and can be used to estimate the minimum radial load required for the pear-shaped expander to initiate effective casing restoration. In the actual expansion process, however, the casing does not undergo only local elastic yielding. Instead, under continuous radial compression, its deformation evolves from elastic bending to local yielding and finally to full-wall plastic deformation. Therefore, an elastoplastic bending analysis is required to further characterize the plastic load-bearing capacity of the casing wall during reshaping.

In the theoretical model, the casing material is simplified as an ideal elastoplastic material. As the radial shaping force increases, the stress distribution across the casing wall can be divided into three stages, as shown in Figure 7. The first stage is the elastic stage, in which the stress is approximately linearly distributed through the wall thickness and no plastic deformation occurs. The second stage is the local yielding stage, where the edge region of the section first reaches the yield strength, leading to a combined elastic–plastic stress state. The third stage is the full-wall yielding stage, in which the plastic zone extends through the entire wall thickness and the casing wall enters a fully plastic deformation state.

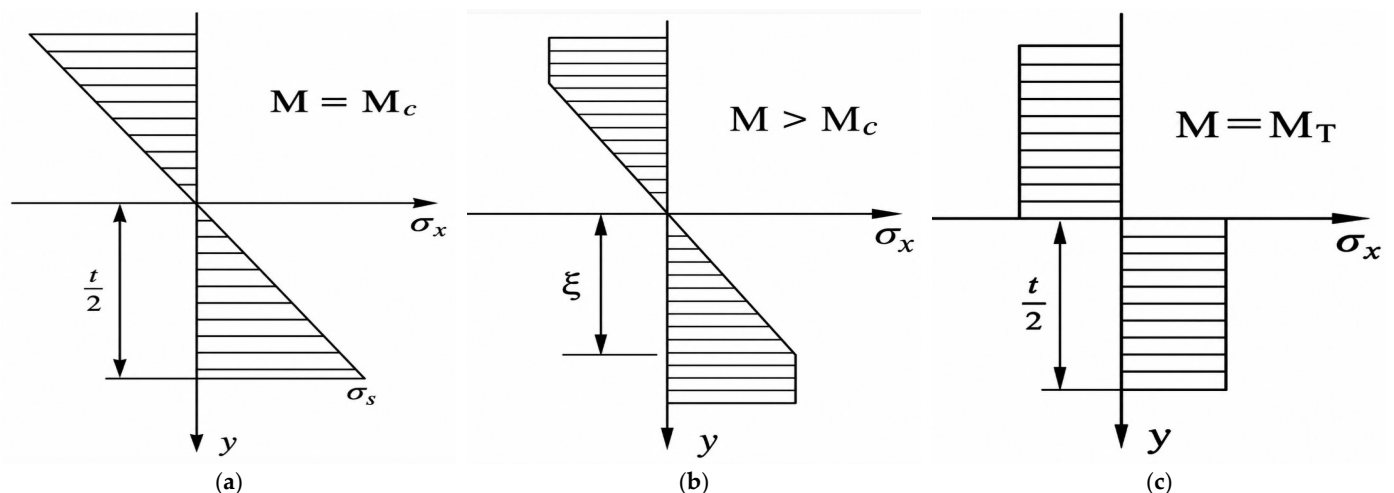


Figure 7. Elastoplastic bending deformation process of the casing wall: (a) elastic state; (b) local yielding state; (c) full-wall yielding state.

In Figure 7, ξ denotes the thickness of the elastic region, M_e is the elastic-limit bending moment, M_T is the fully plastic bending moment, t represents the original casing wall thickness, and h is the axial length of the casing segment considered in the analysis. The

stress distribution across the section at different deformation stages can be expressed as follows:

$$\sigma_x = \begin{cases} \sigma_s \frac{y}{\zeta}, & |y| \leq \zeta \\ \pm \sigma_s, & |y| \leq \zeta \end{cases} \quad (13)$$

Under elastoplastic bending conditions, the cross-sectional bending moment is jointly borne by the elastic region and the plastic region. According to the stress distribution on the cross section, the expression for the bending moment M of the casing cross section can be derived as follows:

$$M = 2 \int_0^{\zeta} \sigma_s \frac{y}{\zeta} h y d y + 2 \int_{\zeta}^{\frac{t}{2}} \sigma_s h y d y \quad (14)$$

Further integrating the cross-sectional stress over the wall thickness gives the relationship between the bending moment and the elastic region thickness ζ

$$M = \frac{\sigma_s h t^2}{4} \left[1 - \frac{1}{3} \left(\frac{\zeta}{\frac{t}{2}} \right)^2 \right] \quad (15)$$

When the casing is at the critical elastic state, the thickness of the elastic region ζ equals $(t/2)$ and the elastic limit bending moment M_e can be obtained accordingly. When the casing enters the full-wall plastic yielding state, ζ approaches zero, and the corresponding bending moment is the fully plastic bending moment M_T . The ratio of the fully plastic bending moment to the elastic limit bending moment is thus derived.

$$\mu = \frac{M_T}{M_e} = 1.5 \quad (16)$$

Based on the elastoplastic bending analysis, the casing wall undergoes full plastic deformation from initial yielding to full-wall yielding. The corresponding radial force can be taken as the criterion for the shaping force required for stable diameter expansion and repair using the pear-shaped expander. According to the elastoplastic bending theory of beam structures, the bearing capacity in the full-wall yielding stage is higher than that at the elastic limit. Accordingly, a model for the radial shaping force required for full-wall yielding of the casing is established based on the initial yield load calculated by the curved beam theory.

$$F_{Pmin} = \frac{6t^2 \sigma_y \tan \alpha}{\frac{\pi}{12a \int_0^{\frac{\pi}{2}} \left(1 - \cos \phi \sqrt{1 + k(\sin \phi)^2} \right) \sqrt{1 + k(\sin \phi)^2} d\phi} + t} \quad (17)$$

An analysis of Equation (17) indicates that the minimum radial force F_{pmin} for overall plastic deformation of the casing wall depends on key parameters including casing geometry, initial ellipticity k , material yield strength and the cone angle of the shaping tool.

In actual repair operations, pear-shaped expanders with different outer diameters are selected according to the casing deformation degree. A larger outer diameter of the tool corresponds to a greater shaping capacity, and the required axial thrust varies accordingly. Let the target shaping capacity be λ the axial shaping force required to achieve this value is expressed as:

$$F_{PA} = \lambda \frac{EI \tan \alpha}{2a^3 J_F} \quad (18)$$

$$J_F = \int_0^{\pi} \frac{\pi}{2} \left(0.5 - R_{X1} - 0.5 \times \sqrt{1 + k(\sin \phi)^2} \cos \phi \right)^2 \left(1 + k(\sin \phi)^2 \right) d\phi \quad (19)$$

Taking both the full-wall yielding condition of the casing and the requirement for target shaping capacity into account, the actual axial shaping force required for the pear-shaped expander shall be the larger of the two values, namely:

$$F_P = \max\{F_{P\lambda}, F_{\min}\} \quad (20)$$

Based on the calculation model of Equation (20), the shaping force of expanders with different specifications for various deformed casings can be predicted, which provides a theoretical basis for the design of on-site construction schemes. This method can effectively reduce additional damage to the casing and cement sheath during repair operations, lower the risk of downhole sticking, and improve the overall efficiency of the shaping process.

2.3. Shaping Force Model of Pear-Shaped Expander Based on Thick-Walled Cylinder Theory

Curved beam theory mainly analyzes the casing rounding process from the perspective of bending deformation of elliptical cross-sections. However, continuous contact and extrusion occur between the cone surface of the pear-shaped expander and the inner wall of the casing during actual shaping. To further describe the elastoplastic diameter expansion behavior of the casing under contact pressure, this paper establishes a shaping force model for the pear-shaped expander based on the thick-walled cylinder theory.

The structural parameters of the pear-shaped expander and the elastoplastic mechanical model of the casing under contact pressure are shown in Figure 8.

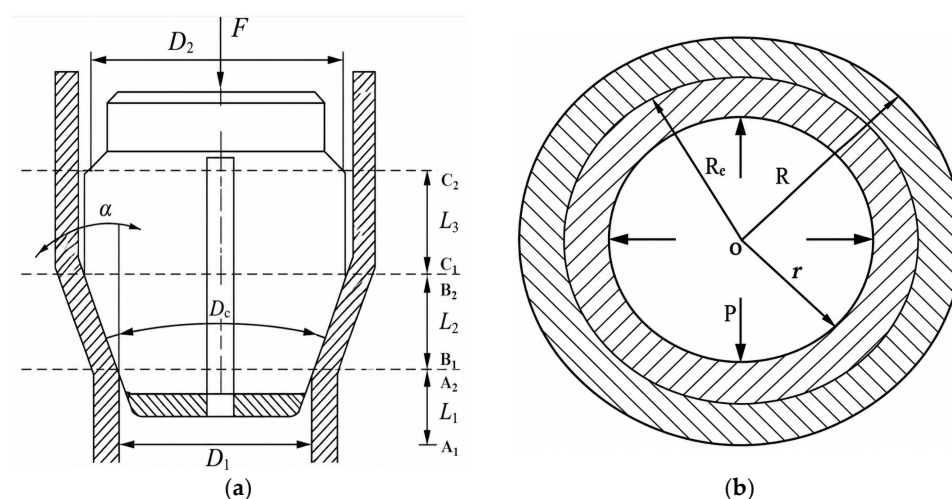


Figure 8. Structural configuration of the pear-shaped expander: (a) overall structural schematic; (b) geometric definition of the expansion mechanism and key dimensional parameters.

During the expansion process, the casing is idealized as a thick-walled cylindrical structure, and the interaction between the tool and the inner wall of the casing is assumed to be a continuous contact condition. Under this assumption, the compressive action exerted by the tool on the casing can be equivalently represented as an internal contact pressure P_P . As the contact pressure increases, yielding first occurs at the inner wall of the casing, and the plastic zone subsequently propagates toward the outer wall. When the entire wall reaches full plastic yielding, the corresponding plastic limit pressure can be expressed as:

$$P_P = \frac{4}{3} \sigma_f \ln\left(\frac{R}{r}\right) \quad (21)$$

Considering the effect of external confining pressure P_0 , the plastic limit pressure can be further written as:

$$P_p = \frac{4}{3}\sigma_f \ln\left(\frac{R}{r}\right) + P_0 \quad (22)$$

where P_p is the plastic limit pressure of the casing (MPa); σ_f is the flow stress of the material (MPa); R and r denote the outer and inner radii of the casing (mm), respectively; and P_0 represents the external confining pressure (MPa). The flow σ_f can be determined based on the yield strength and tensile strength of the casing material, representing the equivalent stress in the plastic flow regime.

The reshaping function of the pear-shaped expander is mainly achieved through the combined action of the tapered section and the sizing section. The tapered section is responsible for the initial expansion of the casing, while the sizing section maintains the required inner diameter and completes secondary calibration. The length of the tapered transition section can be expressed as:

$$L_2 = \frac{D_2 - D_1}{2 \tan \alpha} \quad (23)$$

The force characteristics of the tapered and sizing sections are illustrated in Figure 9.

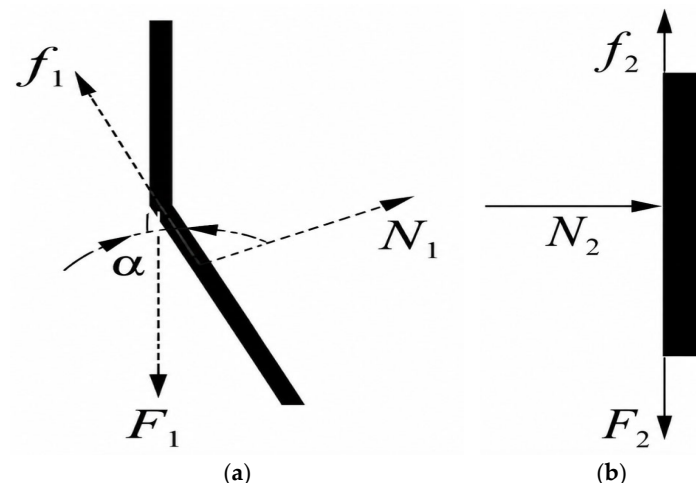


Figure 9. Mechanical force analysis of the pear-shaped expander during casing reshaping: (a) tapered section; (b) sizing section.

For the tapered section, the normal reaction force at the casing–tool interface must reach the critical condition required for sufficient plastic deformation of the casing. The contact area of the tapered section is given by:

$$S_1 = \frac{\pi}{4 \sin \alpha} (D_2^2 - D_1^2) \quad (24)$$

For the sizing section, after the tapered section passes through the deformed zone, continuous contact is maintained between the sizing section and the casing wall, and the corresponding contact area is:

$$S_2 = \pi D_2 L_3 \quad (25)$$

Considering a friction coefficient μ , the axial resistance in the sizing section is primarily governed by interfacial friction. By combining the thrust contribution from the tapered

section and the frictional resistance from the sizing section, the total axial shaping force required for casing repair can be obtained as:

$$F = \left[\frac{1}{4} (D_2^2 - D_1^2) (\mu \cot \alpha + 1) + \mu D_2 L_3 \right] \pi P_p \quad (26)$$

where F is the axial shaping force of the pear-shaped expander (N); μ is the friction coefficient between the tool and the casing; D_1 and D_2 are the inner diameter of the deformed casing and the outer diameter of the sizing section, respectively (mm); and L_3 is the length of the sizing section (mm).

Equation (25) indicates that the shaping force is mainly governed by the deformation magnitude $D_2 - D_1$, the friction coefficient μ , the taper angle α , and the length of the sizing section L_3 . A larger deformation requires greater plastic expansion capacity, leading to an increase in shaping force. A higher friction coefficient results in greater axial resistance during tool advancement. The taper angle influences the conversion efficiency between axial load and radial expansion, while an increase in sizing length improves stability but also increases frictional resistance. Overall, the thick-walled cylinder model provides a contact-pressure-based description of the shaping mechanism of the pear-shaped expander and serves as a complementary framework to the curved beam model, offering theoretical support for subsequent numerical simulations and experimental validation.

3. Finite Element Model and Numerical Simulation Analysis

3.1. Geometric Model Construction

To characterize the mechanical response of an elliptically deformed casing during restoration under the condition without a cement sheath, a three-dimensional finite element model was developed, consisting of a pear-shaped expander and a C110 deformed casing. Owing to the much higher stiffness of the pear-shaped expander compared with that of the casing, and considering that the present study focuses primarily on the elastoplastic deformation behavior of the casing during the reshaping process, the expander was treated as a rigid body in the numerical model. Its elastic deformation was therefore neglected, while its geometric profile and contact interaction with the inner wall of the casing were retained. The simplified geometric model of the pear-shaped expander and the corresponding three-dimensional finite element model for restoring the deformed casing without a cement sheath are shown in Figure 10.

Under the applied axial displacement load, the pear-shaped expander gradually entered the deformed section of the casing and contacted the inner wall, thereby producing a radial expansion effect along the minor-axis direction. During the reshaping process, the Von-Mises equivalent stress, displacements along the major- and minor-axis directions, and reshaping reaction force of the C110 casing were extracted to characterize the degree of plastic deformation, geometric recovery, and variation in reshaping load. Considering the symmetry of the elliptical casing geometry and the loading boundary conditions, a quarter model was adopted in the finite element calculation to improve computational efficiency.

To analyze the influence of the outer diameter of the pear-shaped expander on the casing reshaping process, five finite element simulation cases with different outer diameters were established. In all cases, the cone angle of the pear-shaped expander was kept constant at 14° , and only its outer diameter was varied to simulate the stepwise expansion restoration process of the deformed casing. The pear-shaped expander mainly consisted of a tapered guiding section and a reshaping working section. The geometric parameters, material properties, contact settings, and simulation cases of the model are listed in Table 1.

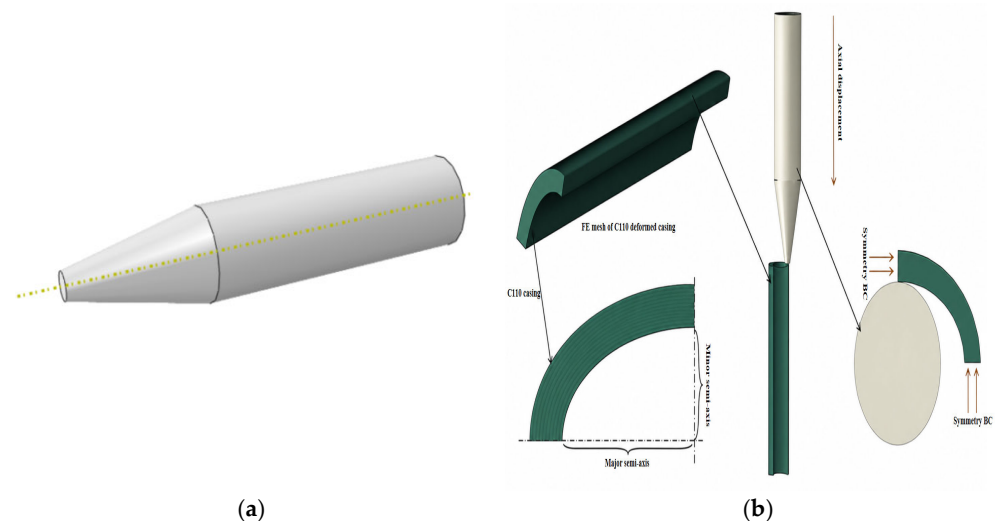


Figure 10. Finite element model of the pear-shaped expander and the deformed casing without cement sheath: (a) simplified geometric model of the pear-shaped expander; (b) 3D finite element model of C110 deformed casing restoration without cement sheath using the pear-shaped expander.

Table 1. Main parameters and simulation cases of the finite element model.

Category	Parameter	Symbol	Unit	Value
Deformed casing	Major axis of elliptical section	A	mm	186.6
Deformed casing	Minor axis of elliptical section	B	mm	120
Material property	Elastic modulus of C110 casing	E	GPa	210
Material property	Yield strength of C110 casing	σ_y	MPa	850
Material property	Poisson's ratio	ν	—	0.30
Pear-shaped expander	Cone angle	α	°	14
Pear-shaped expander	Length of tapered guiding section	L1	mm	200
Pear-shaped expander	Length of reshaping working section	L2	mm	400
Simulation case	Outer diameter of expander	D	mm	122, 124, 126, 128, 130
Simulation case	Outer-diameter increment	ΔD	mm	2
Contact setting	Friction coefficient	μ	—	0.08
Model setting	Fraction of symmetric model	—	—	1/4

3.2. Mesh Generation

After the finite element geometry was established, mesh generation was carried out for the C110 deformed casing and the pear-shaped expander. The C110 casing was discretized using eight-node linear brick elements with reduced integration (C3D8R). This element type is suitable for elastoplastic deformation analysis and helps improve computational efficiency. A structured hexahedral mesh was adopted for the casing to ensure mesh quality and numerical stability during the reshaping process.

Since the main contact interaction occurred between the pear-shaped expander and the inner wall of the deformed casing, local mesh refinement was applied to the casing inner wall and the deformed section. This refinement improved the accuracy of contact stress calculation, displacement response, and reshaping force extraction. In regions far from the contact area, relatively coarser meshes were used to reduce computational cost.

The pear-shaped expander was defined as a rigid body in the finite element model. Therefore, its mesh was mainly used to describe the geometric profile and contact surface, rather than to calculate its internal stress or deformation. The mesh density of the expander was selected to ensure smooth contact interaction with the casing inner wall. In the quarter finite element model, approximately 50,000 elements were generated for the C110

deformed casing, which provided sufficient accuracy for analyzing the stress distribution, deformation response, and reshaping force during the restoration process.

3.3. Contact and Boundary Conditions

A surface-to-surface contact pair was defined between the pear-shaped expander and the inner wall of the C110 deformed casing. The normal contact behavior was described using hard contact, allowing pressure transfer between the tool and the casing during contact while permitting separation after unloading. The tangential contact behavior was described using the penalty friction formulation, and the friction coefficient was set as $\mu = 0.08$. A finite-sliding contact algorithm was adopted to capture the large relative motion between the expander and the casing during the reshaping process.

In this study, the friction coefficient of $\mu = 0.08$ was adopted as a baseline contact parameter to maintain consistent contact conditions among different expander-diameter cases and to focus on the effect of tool outer diameter on reshaping force and casing geometric recovery. It should be noted that, under actual downhole conditions, the tool–casing frictional state may be affected by drilling mud, corrosion products, surface wear, and lubrication conditions, and the actual friction coefficient may differ from the baseline value used in this study. According to the axial reshaping-force expression in the thick-walled cylinder model, an increase in the friction coefficient would increase the interfacial frictional resistance and consequently raise the axial thrust required for expander advancement. Therefore, the adopted value of $\mu = 0.08$ is mainly used for model calculation and parameter comparison under the present baseline condition. The variation in friction parameters under the coupled effects of drilling mud, corrosion, and wear, as well as its quantitative influence on the total thrust force, should be further investigated through interface-friction tests and calibrated finite element simulations in future work.

A prescribed downward axial displacement was applied to the pear-shaped expander to drive it through the deformed section of the casing. The bottom end of the casing was constrained in the axial direction to prevent rigid-body motion, while symmetric boundary conditions were applied to the two cut planes of the quarter model. Since this study focuses on the casing restoration process without a cement sheath and without external confining pressure, no external pressure was applied to the outer wall of the casing.

To accurately obtain the reshaping force, a reference point was established at the bottom end of the casing and coupled with the corresponding cross-section using a multi-point constraint. The axial reaction force extracted from the reference point was taken as the reshaping force during the restoration process.

4. Results and Discussion

4.1. Stress and Deformation Response During the Reshaping Process of the Pear-Shaped Expander

To analyze the reshaping process of the C110 elliptically deformed casing using the pear-shaped expander under the condition without cement sheath constraint, a pear-shaped expander with a cone angle of 14° and an outer diameter of 122 mm was selected as a representative case for finite element analysis. By extracting the Von-Mises equivalent stress, major-axis displacement, and minor-axis displacement during the reshaping process, the contact loading, plastic deformation, and geometric recovery characteristics of the casing during the advancement of the expander were investigated.

Figure 11 shows the Von-Mises equivalent stress contours of the C110 deformed casing during restoration using the pear-shaped expander. As shown in the figure, before the expander contacted the inner wall of the casing, the overall stress level of the casing remained low, and no obvious stress concentration was observed. As the expander gradually entered the deformed section and contacted the inner wall of the casing, the equivalent stress in

the contact region increased rapidly, accompanied by distinct local stress concentration. With the further advancement of the expander, the casing inner wall was subjected to radial compression and gradually entered the plastic deformation stage. The local equivalent stress exceeded the yield strength of the C110 casing, indicating that irreversible plastic deformation occurred in this region. After completion of the reshaping process, a certain level of residual stress remained near the minor-axis region of the casing, suggesting that the restoration process was not merely elastic recovery but involved significant elastoplastic deformation.

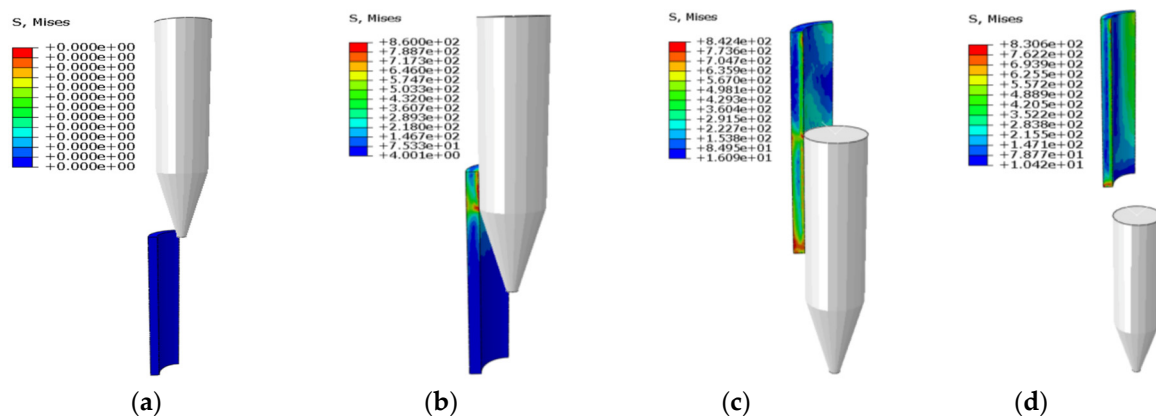


Figure 11. Evolution of Von-Mises equivalent stress in the C110 deformed casing during the restoration process using the pear-shaped expander: (a) non-contact stage; (b) initial contact stage; (c) plastic deformation stage; (d) completion stage.

Figure 12 shows the displacement contours of the C110 casing along the major-axis direction during the reshaping process. As shown in the figure, with the advancement of the pear-shaped expander, a certain displacement response occurs along the major-axis direction of the casing. This indicates that, while the pear-shaped expander produces radial expansion along the minor-axis direction, it also induces coordinated deformation of the entire elliptical cross-section. Although the major-axis displacement is not the primary restoration direction, its variation reflects the transverse deformation characteristics of the casing cross-section as it gradually changes from an elliptical shape toward a more circular profile.

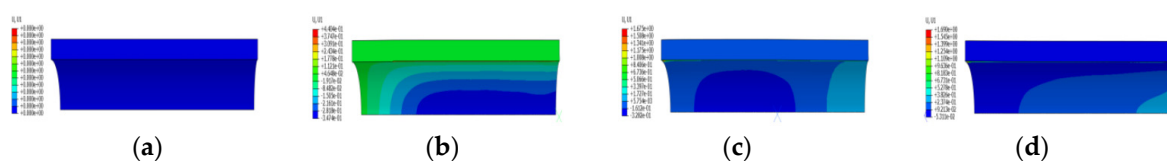


Figure 12. Evolution of displacement along the major-axis direction (U1) in the C110 deformed casing during the restoration process using the pear-shaped expander: (a) non-contact stage; (b) initial contact stage; (c) plastic deformation stage; (d) completion stage.

Figure 13 shows the displacement contours of the C110 casing along the minor-axis direction during the reshaping process. The minor-axis direction is the primary restoration direction of the elliptically deformed casing; therefore, its displacement variation directly reflects the reshaping effect. As shown in the figure, with the increasing contact between the expander and the casing inner wall, the displacement along the minor-axis direction gradually increases, indicating that the pear-shaped expander can effectively push the minor-axis region of the casing outward. After completion of the reshaping process, a certain amount of plastic expansion remains along the minor-axis direction. However, due

to elastic recovery of the material, the final restored amount is smaller than the nominal expansion of the expander.

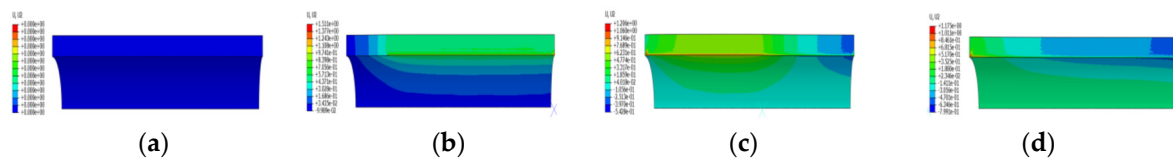


Figure 13. Evolution of displacement along the minor-axis direction (U2) in the C110 deformed casing during the restoration process using the pear-shaped expander: (a) non-contact stage; (b) initial contact stage; (c) plastic deformation stage; (d) restoration completion stage.

The evolution of stress and displacement during the reshaping process indicates that the restoration of the C110 elliptically deformed casing using the pear-shaped expander exhibits distinct stage-dependent characteristics. At the initial stage, the expander has not yet contacted the casing inner wall, and the casing shows a relatively low stress level with negligible displacement. As the expander gradually enters the deformed section and contacts the casing inner wall, the equivalent stress in the contact region increases rapidly, accompanied by an increase in the reshaping reaction force, indicating that the casing begins to experience radial compression. With the further advancement of the expander, obvious expansion occurs in the minor-axis region, and the local material enters an elastoplastic deformation state. As a result, the casing cross-section gradually changes from an elliptical shape toward a more circular profile. After completion of the reshaping process, residual plastic deformation remains in the casing, accompanied by elastic springback, indicating that the final restoration effect is jointly determined by plastic expansion and elastic recovery.

Therefore, the restoration process of the C110 elliptically deformed casing using the pear-shaped expander is essentially a nonlinear contact expansion process between the tool and the casing inner wall. At the early reshaping stage, the contact area is relatively small, and the local stress and reshaping reaction force increase rapidly. After entering the main plastic deformation stage, the casing deformation gradually becomes stable, but the reshaping reaction force still fluctuates with the contact state and the degree of cross-sectional recovery. This indicates that the casing deformation, contact state, and reshaping resistance do not follow a simple linear relationship, but are jointly affected by the tool outer diameter, expansion of the contact region, and elastoplastic response of the casing material.

4.2. Effect of Pear-Shaped Expander Outer Diameter on Casing Reshaping Performance

To further analyze the influence of the outer diameter of the pear-shaped expander on the reshaping effect of the deformed casing without a cement sheath, five pear-shaped expanders with outer diameters of 122, 124, 126, 128, and 130 mm were selected for finite element simulations of the C110 elliptically deformed casing. In all simulation cases, the cone angle of the pear-shaped expander was kept constant at 14° , and only the outer diameter was varied to investigate its effect on the reshaping load and deformation response of the casing.

The reshaping force and the displacements of the casing along the major- and minor-axis directions under different outer diameter conditions are listed in Table 2.

To more clearly illustrate the variation patterns of the parameters, the relationship between the reshaping force and the outer diameter of the expander, as well as the relationships between the major- and minor-axis displacements and the outer diameter of the expander, were plotted, as shown in Figure 14.

Table 2. Simulation results of repairing casing with pear-shaped expander at different outer diameters.

Outer Diameter (mm)	Reshaping Force RF3 (MPa)	Semi-Major Axis U1 (mm)	Semi-Minor Axis U2 (mm)
122	50.2	1.766	1.690
124	67.9	1.767	1.699
126	87.7	1.779	1.725
128	114.8	1.804	1.738
130	141.7	1.9443	1.743

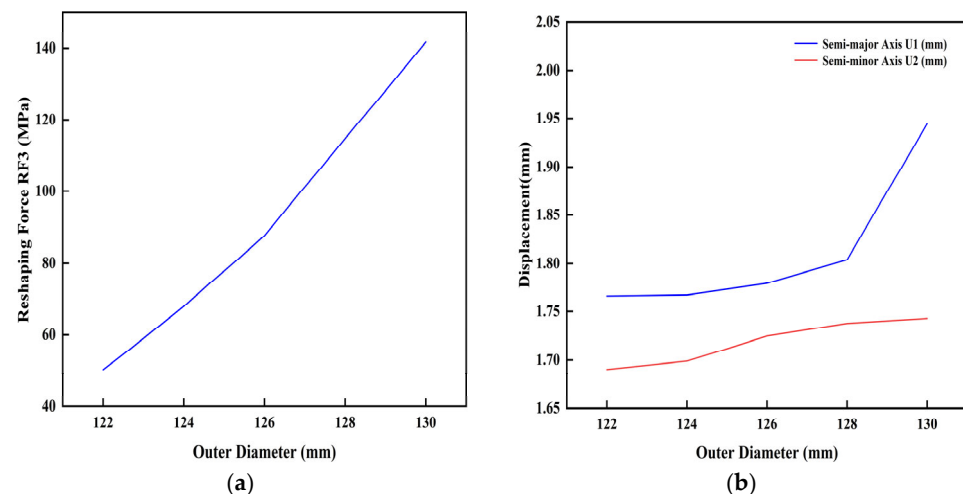
**Figure 14.** Effects of pear-shaped expander outer diameter on reshaping force and casing displacement: (a) variation of reshaping force with expander outer diameter; (b) variations of major-axis and minor-axis displacements with expander outer diameter.

Figure 14a shows that the reshaping force of the casing increases continuously with the increase in the outer diameter of the pear-shaped expander. When the expander outer diameter increases from 122 mm to 130 mm, the reshaping force increases from 50.2 t to 141.7 t, showing a significant increase. This is mainly because a larger expander outer diameter leads to greater geometric interference between the expander and the minor-axis region of the casing. As a result, the deformation resistance that must be overcome during plastic expansion of the casing increases, and a higher axial driving load is required. This result indicates that the outer diameter of the pear-shaped expander is an important factor affecting the reshaping load.

Figure 14b shows that the displacement along the minor-axis direction gradually increases with the increase in expander outer diameter, indicating that a larger expander can produce a stronger radial expansion effect and improve the geometric recovery of the casing along the minor-axis direction. Meanwhile, the displacement along the major-axis direction also changes to some extent, suggesting that the casing reshaping process is not merely a local expansion along the minor-axis direction, but rather a coordinated deformation of the entire elliptical cross-section under contact extrusion. The radial expansion along the minor-axis direction induces corresponding adjustment along the major-axis direction, causing the casing cross-section to gradually recover from an elliptical shape toward a more circular profile.

Further analysis indicates that although the displacement along the minor-axis direction increases with the outer diameter of the expander, its growth rate is much lower than that of the reshaping force. This suggests that simply increasing the outer diameter of the expander does not lead to a proportional improvement in the reshaping effect. From an engineering application perspective, an excessively large expander diameter can enhance

the radial expansion effect, but it also significantly increases the reshaping load and may intensify local stress concentration, thereby increasing the risk of secondary casing damage or tool sticking. Therefore, the selection of the pear-shaped expander diameter should comprehensively consider the casing recovery degree, reshaping load, and operational safety.

In summary, the outer diameter of the pear-shaped expander has a significant influence on the reshaping load and restoration effect of the C110 elliptically deformed casing under the condition without a cement sheath. When the cone angle was kept constant at 14° , the reshaping force increased from 50.2 t to 141.7 t as the expander outer diameter increased from 122 mm to 130 mm, indicating that a larger outer diameter significantly increases the load required for plastic expansion of the casing. Meanwhile, the displacement along the minor-axis direction also increased with the expander outer diameter, suggesting that a larger expander is beneficial for improving the geometric recovery of the casing along the minor-axis direction. However, compared with the increase in reshaping force, the improvement in displacement was relatively limited, indicating that simply increasing the expander outer diameter does not lead to a proportional improvement in the restoration effect. Therefore, the selection of expander outer diameter should comprehensively consider the casing recovery degree, reshaping load, and operational safety. In engineering applications, a stepwise expansion strategy from smaller to larger expander diameters can be adopted to gradually complete the restoration process.

5. Similarity-Scaled Experimental Design

5.1. Similarity Model Design

To verify the predictive reliability of the finite element model for the reshaping process of the deformed casing without a cement sheath, a scaled experiment for restoring the deformed casing using the pear-shaped expander was carried out based on similarity theory. Since the full-scale casing–pear-shaped expander system is large in size, direct prototype testing involves difficulties in specimen fabrication, loading equipment, and experimental operation. Therefore, a scaled model was used to physically simulate the casing reshaping process.

During the design of the scaled model, geometric similarity, mechanical response similarity, and boundary condition similarity between the model and the prototype were considered. By maintaining consistency in the casing cross-sectional shape, expander structural form, loading direction, and boundary constraints, the scaled experiment can reasonably reproduce the main reshaping process of the prototype casing under the action of the pear-shaped expander.

The experimental model was designed according to a geometric scaling ratio of 1:8. The experimental system consisted of an elliptically deformed casing model and five pear-shaped expander models with different outer diameters. The cross-sectional dimensions, length, and wall thickness of the casing model, as well as the outer diameter, working-section length, tapered-section length, and cone angle of the pear-shaped expander, were converted according to the prototype dimensions. The main dimensional correspondence between the prototype and the scaled model is listed in Table 3.

To clarify the similarity criteria used in the scaled experiment, the main scaling relationships between the scaled model and the prototype are described as follows. The subscripts m and p denote the scaled model and the full-scale prototype, respectively. The geometric scale factor is defined as

$$C_L = \frac{L_m}{L_p} = \frac{1}{8} \quad (27)$$

According to geometric similarity, the corresponding linear dimensions of the model and prototype remain proportional. Therefore, the area scale factor and volume scale factor can be expressed as

$$C_A = C_L^2 \quad (28)$$

$$C_V = C_L^3 \quad (29)$$

For the deformation response, the displacement scale factor is taken to be consistent with the geometric scale factor:

$$C_u = \frac{u_m}{u_p} = C_L \quad (30)$$

Table 3. Prototype and model dimensions.

	Physical Quantity	Prototype	Model	Scale (Prototype:Model)
Elliptically deformed casing	Major axis length	186.000 mm	23.325 mm	1:8
	Minor axis length	120.000 mm	15.000 mm	1:8
	Overall length	500.000 mm	62.500 mm	1:8
	Wall thickness	23.400 mm	2.925 mm	1:8
Pear-shaped expander	Outer diameter	122.000 mm	15.25 mm	1:8
		124.000 mm	15.50 mm	1:8
		126.000 mm	15.75 mm	1:8
		128.000 mm	16 mm	1:8
		130.000 mm	16.25 mm	1:8
	Working section length	400.000 mm	50.000 mm	1:8
	Tapered section length	200.000 mm	25.000 mm	1:8
	Cone angle	14°	14°	1:1

Since the reshaping experiment was conducted under quasi-static loading, inertial effects were not taken as the primary similarity criterion. Therefore, velocity and acceleration similarity were not emphasized in the present experiment. The strain scale factor can be obtained as

$$C_\epsilon = \frac{\epsilon_m}{\epsilon_p} = \frac{C_u}{C_L} = 1 \quad (31)$$

For the load response, the stress scale factor is related to the force scale factor and the area scale factor. Since stress is defined as force divided by area,

$$C_\sigma = \frac{\sigma_m}{\sigma_p} = \frac{\frac{F_m}{A_m}}{\frac{F_p}{A_p}} = \frac{C_F}{C_L^2} \quad (32)$$

Thus, the force scale factor is

$$C_F = \frac{F_m}{F_p} = C_\sigma C_L^2 \quad (33)$$

Accordingly, the prototype-scale reshaping force can be converted from the model-scale measured force as

$$F_P = \frac{F_m}{C_\sigma C_L^2} \quad (34)$$

and the prototype-scale displacement can be converted from the model-scale displacement as

$$u_P = \frac{u_m}{C_L} \quad (35)$$

The cone angle of the pear-shaped expander was kept unchanged because it is a dimensionless geometric parameter:

$$C_\alpha = \frac{\alpha_m}{\alpha_p} = 1 \quad (36)$$

Through the above similarity relationships, the scaled experimental results can be converted to the prototype scale and compared with the finite element simulation results.

5.2. Specimen Preparation and Experimental Setup

The similarity-scaled experiment mainly consisted of two stages: specimen preparation and reshaping loading. First, according to the similarity model design, the elliptically deformed casing model and five pear-shaped expander models with different outer diameters were fabricated using selective laser melting. Then, axial loading tests were carried out on the deformed casing using an electronic universal testing machine to obtain the load and deformation responses during the casing restoration process. The fabricated deformed casing model and pear-shaped expander models are shown in Figure 15.

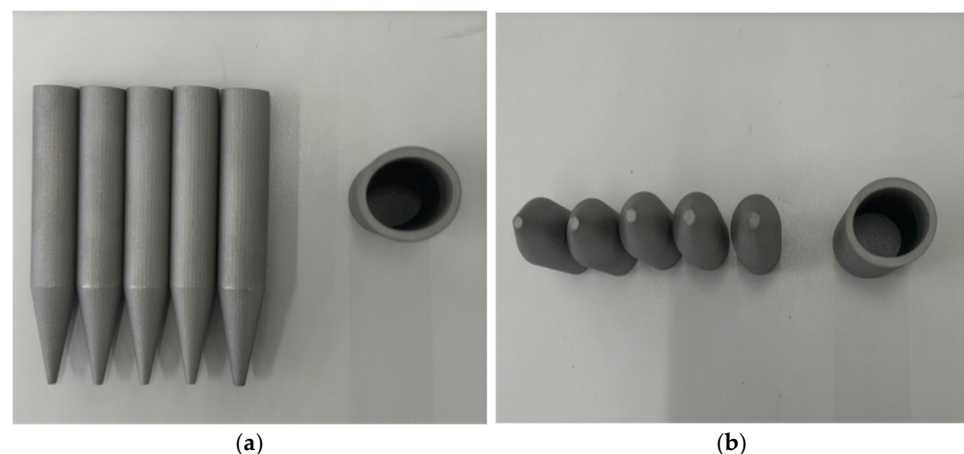


Figure 15. Similarity-scaled specimens of the deformed casing and pear-shaped expanders: (a) vertical arrangement of the pear-shaped expanders and deformed casing model; (b) horizontal arrangement of the pear-shaped expanders and deformed casing model.

Based on the similarity criteria described above, the scaled experiment was designed primarily to reproduce the geometric configuration, loading mode, boundary conditions, and deformation trend of the pear-shaped expander restoration process. The purpose of the scaled experiment was to provide structural-level verification for the finite element model, rather than to establish complete constitutive equivalence between the model material and the prototype C110 casing steel.

The scaled casing and pear-shaped expander specimens were fabricated using a TiC-reinforced stainless steel matrix composite, FeCrNi-TiC, by selective laser melting. In the scaled experimental design, the TiC particle content was 20 wt.%. The use of FeCrNi-TiC was mainly determined by the requirements of additive manufacturing, small-scale specimen forming accuracy, load-bearing capacity, structural stability, and dimensional stability during repeated reshaping tests. As a ceramic reinforcement phase, TiC can improve the hardness, wear resistance, and structural stability of the stainless steel matrix, which is beneficial for maintaining the integrity and geometric accuracy of the scaled casing and expander specimens during the reshaping process.

It should be emphasized that FeCrNi-TiC was not used as a direct constitutive substitute for C110 casing steel. In the finite element simulation of the prototype casing, the mechanical properties of C110 steel were still adopted. The scaled experiment was mainly

used to corroborate the variation trends of reshaping force, pull-up force, minor-axis expansion, and springback under different expander diameters. Therefore, the comparison between the scaled experiment and the numerical simulation should be interpreted as a validation of the structural response trend under controlled similarity conditions, rather than as a complete material-equivalence verification.

The use of FeCrNi-TiC may introduce differences from the response of C110 casing steel. Since TiC is a ceramic reinforcement phase, the post-yield response of FeCrNi-TiC may not be identical to that of ductile C110 casing steel. Such material differences may influence local tool–casing contact, unloading springback, residual minor-axis expansion, and the measured deformation after repeated reshaping passes. For this reason, the experimental results in this study were used mainly to support the overall trend of reshaping load and casing recovery, while the detailed elastoplastic response of the full-scale C110 casing was evaluated using the finite element model.

During specimen preparation, three-dimensional models of the elliptically deformed casing and pear-shaped expanders were first established using SOLIDWORKS 2024. The model files were then imported into the additive manufacturing system for slicing and printing. After fabrication, the casing cross-sectional dimensions, expander outer diameters, and key geometric parameters were measured and checked to ensure that the specimen dimensions satisfied the similarity model design requirements.

The experimental loading system used an electronic universal testing machine, as shown in Figure 16. The testing machine was used to provide the axial load during the advancement of the pear-shaped expander, while the dynamic/static strain testing system and sensors were used to collect the load, displacement, and local strain responses during the reshaping process. During the experiment, the casing model and the pear-shaped expander were fixed using a special fixture, and axial alignment was performed to reduce the influence of eccentric loading on the experimental results.



Figure 16. Electronic universal testing machine in kind and three-dimensional model drawing.

The installation and reshaping process of the pear-shaped expander is shown in Figure 17. Before the experiment, the initial major- and minor-axis dimensions of the casing

were measured using a caliper, and the specimen positioning and axial alignment of the expander were completed. During the experiment, the pear-shaped expander gradually entered the deformed section along the casing axis under the action of the electronic universal testing machine, producing radial expansion against the casing inner wall. After each test, the major- and minor-axis dimensions of the casing were measured again, and key parameters, including reshaping force, pull-up force, minor-axis expansion, springback, and reshaping amount, were recorded to provide experimental evidence for the subsequent validation of the finite element simulation results.

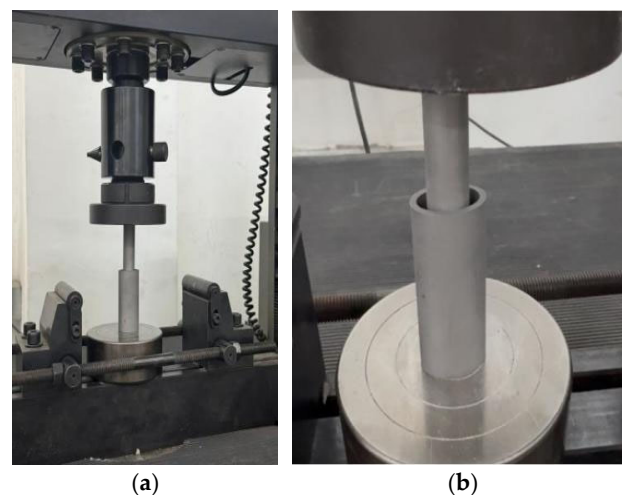


Figure 17. Experimental setup and reshaping process using the pear-shaped expander: (a) installation of the pear-shaped expander in the electronic universal testing machine; (b) reshaping process of the deformed casing using the pear-shaped expander.

5.3. Similarity Model Experimental Results

After the preparation of the similarity-scaled specimens and the setup of the experimental system were completed, the C110 elliptically deformed casing model was reshaped using a stepwise expansion procedure from smaller to larger expander diameters. During the experiment, pear-shaped expanders with outer diameters of 15.25, 15.50, 15.75, 16.00, and 16.25 mm were successively used to reshape the deformed casing. After each expander specification completed the reshaping process, the changes in the major- and minor-axis dimensions of the casing were measured, and key parameters, including reshaping force, pull-up force, minor-axis expansion, springback, and the number of round-trip cycles, were recorded. To improve the reliability of the experimental data, each test condition was repeated three times, and the average value was taken as the final experimental result. The model-scale experimental results are listed in Table 4.

Table 4. Model-scale experimental results for C110 deformed casing restoration using pear-shaped expanders.

Outer Diameter of Tool (mm)	Shortening of Major Axis (mm)	Expansion of Minor Axis (mm)	Springback Amount (mm)	Reshaping Deformation (mm)	Reshaping Force (t)	Pull-Up Force (t)	Number of Reciprocating Passes
15.25	0.05	0.2	0.05	0.2	5.862	3.15	7
15.50	0.071	0.225	0.075	0.225	7.763	3.675	7
15.75	0.094	0.213	0.113	0.213	10.332	5.312	7
16	0.11	1.163	0.2	1.163	13.575	8.05	8
16.25	0.116	0.138	0.313	0.138	15.225	9.688	8

As shown in Table 4, with the increase in the outer diameter of the pear-shaped expander, the reshaping force and pull-up force at the model scale generally show an increasing trend. This indicates that a larger expander diameter enhances the radial expansion effect on the casing, but also significantly increases the loading demand during the reshaping process. Meanwhile, the minor-axis expansion and springback vary with the expander diameter, suggesting that the final restoration effect of the casing is affected not only by the tool diameter but also by the elastoplastic response of the material, the tool–casing contact state, and the elastic springback after reshaping.

To facilitate comparison with the full-scale finite element simulation results, the model-scale experimental results were further converted to the prototype scale according to the similarity criteria. After conversion, the corresponding prototype outer diameters of the pear-shaped expanders were 122, 124, 126, 128, and 130 mm. The prototype-scale converted results are listed in Table 5.

Table 5. Prototype-scale converted results for C110 deformed casing restoration using pear-shaped expanders.

Outer Diameter of Tool (mm)	Shortening of Major Axis (mm)	Expansion of Minor Axis (mm)	Springback Amount (mm)	Reshaping Deformation (mm)	Reshaping Force (t)	Pull-Up Force (t)	Number of Reciprocating Passes
122	0.4	1.6	0.4	1.6	46.9	25.2	7
124	0.57	1.8	0.6	1.8	62.1	29.4	7
126	0.75	1.7	0.9	1.7	82.66	42.5	7
128	0.88	1.3	1.6	1.3	108.6	64.4	8
130	0.93	1.1	2	1.6	121.8	77.5	8

As shown in Table 5, after conversion to the prototype scale, both the reshaping force and pull-up force increase significantly with the outer diameter of the pear-shaped expander. When the tool outer diameter increases from 122 mm to 130 mm, the reshaping force increases from 46.90 t to 121.80 t, and the pull-up force increases from 25.20 t to 77.50 t. This indicates that the outer diameter of the pear-shaped expander is an important parameter affecting the reshaping load of the casing. A larger tool diameter can provide a stronger radial expansion effect, but it also significantly increases the axial loading demand during the reshaping process.

From the perspective of geometric recovery, the pear-shaped expander can effectively improve the cross-sectional shape of the elliptically deformed casing by producing a certain amount of expansion along the minor-axis direction. However, the minor-axis expansion does not increase linearly with the tool outer diameter, indicating that increasing the expander diameter does not lead to a proportional improvement in the final restoration effect. This phenomenon may be related to material springback, local contact state variation, and stress redistribution during the reshaping process. Therefore, in the subsequent comparison between the finite element simulation and experimental results, attention should be paid to the variation trends of reshaping force, minor-axis expansion, and springback characteristics to comprehensively evaluate the predictive reliability of the finite element model for the pear-shaped expander reshaping process.

6. Comparison of Finite Element, Experimental, and Theoretical Results

6.1. Comparison Between Simulation and Similarity-Scaled Experimental Results

To verify the predictive reliability of the finite element model for restoring the C110 deformed casing without a cement sheath using the pear-shaped expander, the finite element simulation results were compared with the prototype-scale results converted from the similarity model experiment. It should be noted that the experimental data were

not obtained directly from full-scale prototype tests, but were converted from the scaled experimental results according to the similarity criteria. The comparison indicators mainly included the reshaping force of the pear-shaped expander and the minor-axis expansion of the casing, as listed in Table 6. The variation trends of the reshaping force and minor-axis expansion are shown in Figure 18 respectively.

Table 6. Comparison of simulated and tested values of shaping force of pear-shaped expander.

Outer Diameter of Tool (mm)	Reshaping Force (t)			Expansion of Minor Axis (mm)			
	Simulated Value	Experimental Value	Error (%)	Simulated Value	Experimental Value	Error (%)	
122	50.2	46.9	6.57	1.690	1.6	5.32	
124	67.9	62.1	8.54	1.699	1.8	5.94	
126	87.7	82.66	5.75	1.725	1.7	1.45	
128	114.8	108.6	7.84	1.738	1.3	23.20	
130	141.7	121.8	14.00	1.743	1.6	8.20	
Average Error (%)			8.542	Average Error (%)			8.822

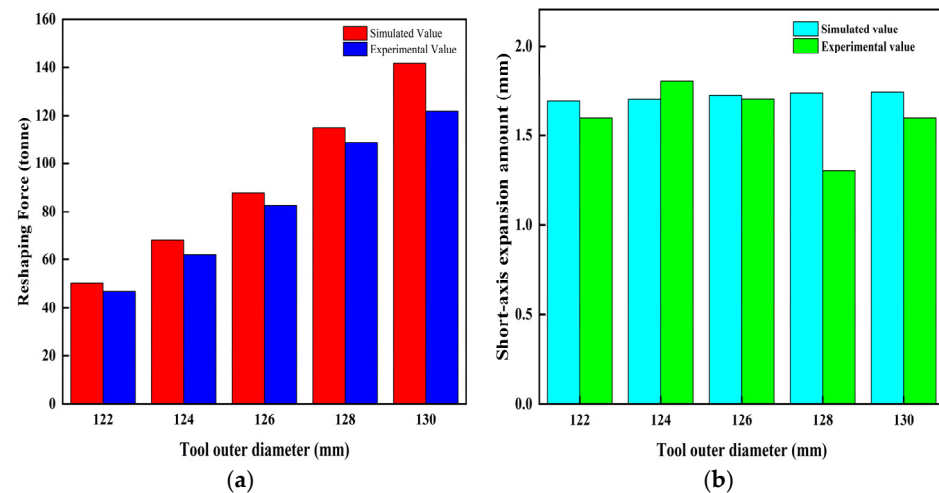


Figure 18. Comparison between finite element simulation and experimental results for pear-shaped expander restoration: (a) reshaping force; (b) minor-axis expansion.

The comparison of minor-axis expansion shows that the finite element simulation results and experimental results exhibit generally consistent variation trends, with an average error of 8.822%. Good agreement was observed for most tool-diameter cases, indicating that the finite element model can effectively capture the geometric recovery tendency of the casing along the minor-axis direction. For the 128 mm case, the simulated minor-axis expansion was 1.738 mm, whereas the prototype-scale converted experimental value was 1.30 mm, corresponding to a relatively higher individual error of 23.20%. This result indicates that the residual minor-axis expansion is more sensitive to local deformation recovery than the reshaping force.

This relatively higher discrepancy can be reasonably associated with the combined effects of elastic springback after unloading, local variation in the tool–casing contact state, repeated loading history, and the difference between the scaled specimen material and the prototype C110 casing. In particular, the residual minor-axis expansion is a small deformation quantity after unloading; therefore, slight changes in springback, specimen alignment, or dimensional measurement may lead to a more pronounced percentage difference after similarity conversion. Nevertheless, the finite element and experimental results still show a consistent overall trend, suggesting that the model can reasonably

reproduce the main geometric recovery behavior of the casing under the investigated baseline conditions.

Overall, the finite element simulation results and similarity-scaled experimental results show good consistency in terms of reshaping force and minor-axis expansion, thereby supporting the reliability of the finite element model. The model can reasonably describe the load variation and geometric recovery characteristics of the C110 elliptically deformed casing during restoration using the pear-shaped expander under the condition without cement-sheath constraint, and can be used to analyze casing reshaping performance under different tool parameters.

6.2. Comparison Between Simulation and Theoretical Calculation Results

To further evaluate the predictive capability of the theoretical models for the reshaping force of the pear-shaped expander, the finite element simulation results were compared with the calculation results obtained from the curved beam theory and the thick-walled cylinder theory, respectively. Five pear-shaped expanders with outer diameters of 122, 124, 126, 128, and 130 mm were selected for comparison, and the corresponding calculation results are listed in Table 7.

Table 7. Comparison between finite element simulation results and theoretical calculations of pear-shaped pipe expander.

Casing Grade	Tool Outer Diameter (mm)	Wall Thickness (mm)	Minimum Inner Diameter (mm)	Yield Stress (MPa)	Reshaping Force (t)		Simulated Value (t)	Error (%)	
					Curved Beam Theory	Thick-Walled Cylinder Theory			
C110	122	12.65	120	850	54.2	51.6	50.2	7.97	2.89
	124	12.65	120	850	74.4	70.0	67.9	9.57	3.09
	126	12.65	120	850	99.2	92.8	87.7	13.11	5.82
	128	12.65	120	850	129.2	120.2	114.8	12.54	4.71
	130	12.65	120	850	158.6	147.4	141.7	11.92	4.02
Average Error (%)								11.02	4.11

As shown in Table 7, the calculation results obtained from both the curved beam theory and the thick-walled cylinder theory can reflect the increasing trend of the reshaping force of the pear-shaped expander with the increase in tool outer diameter. This indicates that both theoretical models have a certain predictive capability for the variation trend of the reshaping force. However, according to the error analysis, the calculated values based on the thick-walled cylinder theory show better agreement with the finite element simulation results. The average error between the curved beam theory and the finite element simulation results is 11.02%, whereas that of the thick-walled cylinder theory is 4.11%, indicating that the thick-walled cylinder theory provides higher calculation accuracy in predicting the reshaping force of the pear-shaped expander.

Compared with the curved beam theory, the thick-walled cylinder theory can better describe the stress state of the casing under radial expansion, and its calculation results are closer to the finite element simulation results. This is mainly because the curved beam model simplifies the tool–casing interaction as an equivalent radial concentrated force and primarily characterizes the bending-dominated deformation of the elliptical casing cross-section, whereas the thick-walled cylinder model describes the reshaping process from the perspective of contact-pressure-induced elastoplastic expansion and further incorporates the effect of frictional resistance. Therefore, the reshaping-force calculation method based on the thick-walled cylinder theory can be used for rapid estimation of the reshaping force of C110 deformed casing under the condition without cement-sheath constraint, providing

a reference for the selection of pear-shaped expander specifications and the determination of field restoration parameters.

It should be noted that the above comparison was conducted within the scope of the single quasi-static reshaping process considered in this study. The two theoretical models are mainly intended to describe the reshaping load induced by instantaneous tool–casing contact and radial expansion during expander advancement, while the progressive change in interfacial conditions during repeated operations is not included. In practical downhole repair operations, repeated contact and relative sliding between the expander and the casing may alter the local surface condition and frictional behavior, thereby affecting the subsequent reshaping resistance. Since the present study focuses on establishing a reshaping-load prediction framework and validating the geometric recovery behavior of deformed casing through finite element simulation and similarity-scaled experiments, the evolution of the contact interface with repeated operating cycles was not incorporated into the current models. This effect will be further considered in future work when extending the proposed method to repeated reshaping or long-duration downhole operation conditions.

Based on the comparison results of the similarity-scaled experiment, theoretical calculations, and finite element simulation, the finite element model established in this study can reasonably predict the reshaping force and minor-axis expansion of the C110 elliptically deformed casing during restoration using the pear-shaped expander under the condition without cement-sheath constraint. Meanwhile, the calculation results based on the thick-walled cylinder theory show good agreement with the finite element simulation results, suggesting that the thick-walled-cylinder-theory-based method can serve as an effective approach for rapid prediction of reshaping force in engineering calculations.

7. Conclusions

This study combined analytical modeling, finite element simulation, and similarity-scaled experiments to investigate the reshaping behavior of C110-grade elliptically deformed casing using a pear-shaped expander under a baseline condition without cement-sheath constraint. The effects of expander outer diameter on reshaping force, stress evolution, and geometric recovery were analyzed, and the applicability of curved beam theory and thick-walled cylinder theory for reshaping-force prediction was compared. The main conclusions are as follows:

- (1) The restoration of elliptically deformed casing using a pear-shaped expander is characterized by nonlinear tool–casing contact and elastoplastic deformation. As the expander advances into the deformed section, the contact state between the tool and the casing inner wall evolves from initial contact to plastic expansion and final diameter restoration. The minor-axis region undergoes the most significant radial expansion, while stress concentration mainly occurs in the local contact and transition regions. These results indicate that casing restoration is governed by the coupled effects of contact loading, plastic deformation, stress redistribution, and elastic recovery.
- (2) The outer diameter of the pear-shaped expander has a significant influence on both reshaping load and casing recovery. With the cone angle fixed at 14° , increasing the expander outer diameter from 122 mm to 130 mm increased the reshaping force from 50.2 t to 141.7 t. However, the improvement in minor-axis expansion was much smaller than the increase in reshaping force, indicating that simply increasing the expander diameter does not lead to a proportional improvement in restoration performance. In the similarity-scaled experimental results, the minor-axis expansion did not show a strictly monotonic increase with tool diameter. This behavior may be attributed to elastic springback after unloading, changes in the local tool–casing contact state, differences between the scaled specimen material and the prototype C110 casing, and

cumulative deviations associated with repeated loading and measurement. Therefore, expander selection should consider both geometric recovery and load safety, rather than relying only on increasing tool diameter.

- (3) The finite element model was evaluated using a 1:8 similarity-scaled experiment. After conversion according to the similarity criteria, the converted experimental reshaping forces and minor-axis expansions showed reasonable agreement with the numerical results. The average errors were 8.542% for reshaping force and 8.822% for minor-axis expansion. These results suggest that the established finite element model can reasonably reproduce the load-response trend and geometric recovery behavior of the casing reshaping process under the investigated baseline conditions. The similarity-scaled experiment provides useful physical corroboration for the numerical model, while future calibration using full-scale experiments and field operation data will further improve its prediction reliability and engineering applicability.
- (4) Both the curved beam model and the thick-walled cylinder model can reflect the increasing trend of reshaping force with increasing expander outer diameter. The thick-walled cylinder model showed better agreement with the finite element results, with an average error of 4.11%, compared with 11.02% for the curved beam model. This indicates that the thick-walled cylinder model is more suitable for preliminary estimation of reshaping force during radial contact expansion. The difference in prediction accuracy is mainly related to the different treatment of tool–casing contact and friction: the curved beam model simplifies the contact as an equivalent radial concentrated force and does not explicitly include friction, whereas the thick-walled cylinder model describes contact-pressure-induced elastoplastic expansion and incorporates frictional resistance in an equivalent manner.
- (5) The integrated analytical–numerical–experimental framework proposed in this study provides a useful preliminary reference for pear-shaped expander specification selection, reshaping-load estimation, restoration-parameter optimization, and stepwise process design for C110 deformed casing under the simplified baseline condition without cement-sheath constraint. In practical casing repair operations, a stepwise expansion strategy using smaller to larger expander diameters may help balance restoration efficiency, reshaping load, and operational safety. Considering that the present validation is mainly based on similarity-scaled experiments, the proposed framework is more appropriately regarded at this stage as a reference method for repair-parameter design and reshaping-load estimation. Further verification using full-scale experiments and field operation data will help enhance its applicability to field-scale engineering applications.

The present study focuses on the fundamental reshaping behavior of C110 elliptically deformed casing under quasi-static loading and without cement-sheath constraint. Cement-sheath constraint, formation pressure, non-uniform in situ stress, complex downhole friction, repeated reshaping, and long-term tool–casing contact evolution were not included in the current baseline model. Future work will further incorporate these factors together with full-scale experimental validation and field operation data, which will help improve the proposed method and broaden its applicability to more realistic downhole casing-repair conditions.

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Abbreviations

The following abbreviations are used in this manuscript:

BC	Boundary Condition
FE	Finite Element Mesh
t	metric ton-force

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