

Article

Theoretical Modeling of Multiphysics for CO₂ Phase-Change Ejection Power Unit

Jun Xia ^{1,2,*} and Mao Tang ^{1,2,*}¹ College of Aerospace Science and Engineering, National University of Defense Technology, Changsha 410073, China² State Key Laboratory of High-Efficiency Reusable Aerospace Transportation Technology, Beijing 102206, China

* Correspondence: putian9988@163.com (J.X.); maotsang@163.com (M.T.)

Abstract

A multiphysics theoretical framework is presented for CO₂ phase-change ejection power units, integrating the GERG-2008 equation of state for multiphase CO₂ thermophysical properties with the Jones–Wilkins–Lee (JWL) combustion model through customized Abaqus VUEOS and VHETVAL subroutines. The Coupled Eulerian–Lagrangian (CEL) method resolves the strongly coupled interaction between high-pressure compressible CO₂ and solid structures across supercritical, liquid–vapor coexistence, and gaseous phases. Validation against experimental pressure–time measurements demonstrates a maximum relative error below 10% throughout the pressurization trajectory, with peak pressure prediction accuracy of 4.1%. This validated framework offers quantitative design guidance for next-generation cold launch systems, enabling systematic optimization of high-pressure chamber configurations, release pressures, and operational parameters for enhanced energy efficiency and reliable performance in variable deployment environments.

Keywords: CO₂ phase-change; power unit; ejection; CEL

1. Introduction

Against the backdrop of rapid advancements in modern defense systems and kinetic energy actuation systems, ejection technology—widely employed for the launch and deployment of missiles, unmanned aerial vehicles (UAVs), pyrotechnic launching systems, and actuation devices in automotive crash tests—has garnered extensive academic attention, and its ongoing development has become increasingly critical [1,2]. Traditional launch approaches, encompassing hot launching via direct ignition and cold launching using compressed air or gas generators, each present inherent limitations that constrain their operational flexibility and battlefield adaptability. Hot launching systems generate exhaust gases with temperatures typically exceeding 1500 °C, containing harmful species such as CO, H₂S, and SO_x [3–5] that pose significant threats to missile instrumentation and launch facilities while causing severe water pollution in underwater applications [6]. Conversely, conventional compressed-air ejection systems are constrained by bulky infrastructure, complex pipeline networks, and substantial maintenance requirements, making them unsuitable for rapid deployment scenarios and field operations [7].

In response to these challenges, phase-transition ejection technologies utilizing cryogenic or liquefied gases have emerged as promising alternatives, offering unique advantages, including clean, low-temperature exhaust, high-density energy storage, and simplified logistical support. Among various working fluids, carbon dioxide (CO₂) has attracted



Academic Editor: Hassan El Fadil

Received: 3 June 2026

Revised: 2 July 2026

Accepted: 23 July 2026

Published: 29 July 2026

Copyright: © 2026 by the authors.

Licensee MDPI, Basel, Switzerland.

This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

considerable research attention owing to its favorable thermodynamic properties, including a critical temperature of 31.1 °C and a critical pressure of 7.38 MPa, which enable high-density liquid storage under moderate pressures and rapid phase transition upon thermal excitation [8,9]. The liquid CO₂ phase-transition ejection technology exploits the substantial volume expansion ratio (approximately 450:1) during liquid-to-gas transition to generate high-pressure working fluid for propulsive applications; this approach achieves clean, low-temperature launch conditions while eliminating combustion byproducts inherent to conventional propellants.

The thermodynamic behavior of CO₂ during rapid depressurization and phase transition has been extensively investigated through both experimental and computational approaches. Ke et al. [1] established comprehensive thermodynamic models for analyzing explosion energy in CO₂ blasting systems, demonstrating the complex interplay between metastable liquid expansion, nucleation dynamics, and shock wave formation. Zhang, F. et al. [10] developed accurate look-up table equations of state for two-phase compressible CO₂ flow simulations, highlighting the computational challenges associated with capturing phase boundaries and property discontinuities. These foundational studies underscore the necessity of sophisticated multiphysics modeling frameworks that can simultaneously resolve fluid dynamics, heat transfer, and thermodynamic phase equilibrium in transient, high-gradient environments [2,9–14].

Accurate prediction of CO₂ thermodynamic behavior under launch conditions requires careful consideration of real gas effects, particularly within supercritical and two-phase regions where ideal-gas assumptions break down. Numerous equations of state have been proposed for CO₂ property calculations, ranging from cubic formulations such as the Peng-Robinson and Redlich-Kwong equations to more complex multiparameter approaches including the Benedict-Webb-Rubin-Starling (BWRs) and Span-Wagner formulations [15–19]. The GERG-2008 wide-range equation of state [19] and NIST REFPROP database [20–23] currently represent the most accurate standards for industrial thermophysical property calculations; however, their high computational overhead poses significant challenges for coupled multiphysics simulations. Recent work by Liu et al. [24] has addressed this limitation through phase-diagram-formulated equations of state specifically designed for eco-friendly blasting applications, highlighting the potential of application-specific thermodynamic models for phase-transition ejection systems.

From a computational mechanics perspective, the simulation of CO₂ phase-transition ejection presents substantial challenges associated with large deformation, fluid–structure interaction, and extreme pressure gradients. The Coupled Eulerian-Lagrangian (CEL) approach implemented in Abaqus/Explicit has proven effective for similar applications involving blast loading and high-velocity impact [23–35], offering advantages in handling severe mesh distortion and multi-material interfaces. Alternative numerical strategies including Smoothed Particle Hydrodynamics (SPH) and Arbitrary Lagrangian-Eulerian (ALE) formulations have also been applied to explosive and impact problems [30–47], though comparative studies suggest that CEL methods may provide superior accuracy for fluid-dominated phenomena with complex free surfaces [27].

Despite these advances, critical knowledge gaps remain in the integrated multiphysics modeling of CO₂ phase-transition ejection systems, particularly regarding the complex interplay between real-gas thermodynamics, non-equilibrium phase transition kinetics, and combustion-driven excitation device mechanisms. Existing studies have predominantly focused on isolated thermodynamic property calculations [23,24] or simplified single-phase ballistic analyses [1,25], without establishing a comprehensive coupling between the high-pressure chamber discharge dynamics, the low-pressure chamber multiphase filling processes, and projectile acceleration mechanics. Furthermore, the influence of

supercritical CO₂ real-gas behavior, metastable phase-transition dynamics, and conjugate heat-transfer effects on overall system performance remains inadequately characterized. The combustion model of the excitation device [6,8] has not been systematically integrated with the multiphase CO₂ flow equations, leading to significant uncertainties in predicting chamber pressurization rates and peak pressure attainment.

The aforementioned studies have laid a solid methodological foundation and provided valuable references for the application of the CEL method to strongly coupled problems involving high-pressure compressible fluids and solid structures investigated in this work [48–52]. Nevertheless, further refinement is required for the integrated coupling of the GERG-2008 multiphase equation of state, the ignition charge combustion model, the dynamic failure model for metallic structures, and the CEL fluid–structure interaction scheme to enable full-process numerical simulation of supercritical CO₂ phase-transition-driven launch systems.

This study investigates the complex thermodynamic and combustion processes in CO₂ phase-change ejection power units. By integrating the GERG-2008 wide-range equation of state for CO₂ multiphase thermophysical properties with the JWL equation of state for excitation charge combustion, and employing the Coupled Eulerian-Lagrangian (CEL) methodology with customized VUEOS and VHETVAL subroutines, a comprehensive multiphysics theoretical framework has been established. The specific research content is as follows:

- (1) Based on the first law of thermodynamics and mass conservation principles, this analysis examines the phase transition dynamics of CO₂ under combustion-driven transient loading, spanning supercritical, liquid-vapor coexistence, and gaseous phases.
- (2) The CO₂ phase-change ejection sequence is decomposed into fundamental multiphase flow modules. By incorporating the GERG-2008 equation of state and the Tait equation for solid-phase CO₂, a multiphase physical property model capable of accurate phase-state determination has been constructed.
- (3) The performance of the excitation device is calculated using a JWL-based combustion modeling approach, employing the HETVAL subroutine to compensate for temperature evolution and heat flux calculation under full working conditions of the combustion chamber.
- (4) Encapsulate the thermophysical property calculation of CO₂ and the combustion dynamics of the excitation device as integrated functional modules, construct validation protocols comparing simulation predictions against experimental pressure-time curves, and analyze their predictive accuracy and error margins.

The main innovations and contributions of this paper can be summarized as follows:

- (1) This study establishes the first integrated multiphysics framework that couples the GERG-2008 real-gas equation of state with JWL combustion modeling within a Coupled Eulerian–Lagrangian (CEL) formulation, enabling full-process simulation of CO₂ phase-transition ejection from chemical energy release to mechanical work extraction.
- (2) A unified phase-state discrimination methodology is developed to enable seamless transitions among gaseous, liquid, supercritical, and solid CO₂ states within a single solver environment.

2. Theoretical Modeling of Interior Ballistic Multiphysics

2.1. Construction of Fundamental Physical Property Models

2.1.1. Multiphase Equation of State

Equations of state (EOS) describe the pressure-volume-temperature (PVT) relationships of fluids and enable the calculation of thermodynamic properties including enthalpy,

entropy, speed of sound, specific heat capacity, and Joule-Thomson coefficients. They represent the most widely employed methodology for predicting and calculating fluid thermodynamic properties. A large number of EoS formulations have been published to date, and refined variants are continuously being proposed in the literature [1,6–8]. Many of these equations are derived from universal formulations with supplementary corrections, while others are specifically designed for precise calculation of particular thermophysical properties.

The phase diagram of CO₂ (Changsha Chang Gang Gas Co. Ltd., Changsha, China), the working fluid investigated in this study, exhibits the following key characteristics [1]: its triple point is defined by a temperature of 216.7 K and a pressure of 518 kPa. Since the triple point temperature is below ambient temperature while the pressure substantially exceeds atmospheric pressure, CO₂ can only exist in gaseous and solid (dry ice) forms under ambient conditions. The vapor-liquid equilibrium curve extending from the triple point to the critical point describes the saturation vapor pressure of CO₂ as a function of temperature. In the supercritical region, when both temperature and pressure exceed the critical values (Table 1), CO₂ enters the supercritical state where the gas-liquid interface disappears, and the fluid properties are intermediate between those of gas and liquid.

Table 1. Critical point parameters of CO₂ [6].

Property	Parameter
Critical temperature T _c	304.2 K (31.04 °C)
Critical pressure P _c	7.38 MPa
Critical volume V _c	94 cm ³ /mol
Critical density ρ _c	0.468 g/cm ³
Compressibility factor Z	0.274

As mathematical models describing fluid thermodynamic properties, multiphase equations of state must accurately predict the density, phase equilibrium, and thermodynamic properties of CO₂ in gaseous, liquid, and supercritical phases. Due to intermolecular interactions and non-ideal behavior in the critical region of CO₂ molecules, conventional ideal-gas equations of state fail to meet accuracy requirements. Consequently, more sophisticated equations of state are required to characterize the multiphase physical properties of CO₂.

Under varying pressure and temperature conditions, CO₂ can exist in four distinct thermodynamic phases: solid, liquid, gas, and supercritical fluid. Therefore, when investigating the actual state of CO₂ under combustion shock loading, appropriate equations of state and thermophysical parameters must be selected according to different loading conditions. The energy during state transitions can be expressed as [6]:

$$\rho \frac{\partial E_m}{\partial t} = (p - p_{bv}) \frac{1}{\rho} \frac{\partial \rho}{\partial t} + S : \dot{\epsilon} + \rho \dot{Q} \quad (1)$$

where p is the pressure, p_{bv} is the pressure due to volume viscosity, S and $\dot{\epsilon}$ represent deviatoric stress and strain, respectively, and $\dot{Q}$ is the heat rate per unit mass. Under high-temperature and high-pressure conditions, the contribution of deviatoric stress is relatively small and can be neglected. While the contribution of deviatoric stress to the energy equation may indeed influence energy partitioning via viscous dissipation under high-pressure transient loading conditions, its magnitude is comparatively minor under high-temperature and high-pressure regimes and can therefore be safely neglected:

- (1) The stress state of a fluid element subjected to instantaneous isotropic pressure loading closely approximates a hydrostatic condition, wherein the deviatoric stress tensor

S identically vanishes by definition ($S = \sigma + pI$). Under such conditions, the viscous dissipation term contributes nothing to the energy budget regardless of the loading rate.

- (2) Even in the presence of transient shear deformation, the magnitude of viscous dissipation is governed by the Newtonian constitutive relation $S = 2\mu\dot{\epsilon}'$, where μ denotes the dynamic viscosity and $\dot{\epsilon}'$ the deviatoric strain rate tensor. For supercritical CO_2 under elevated temperatures and pressures, the dynamic viscosity has been widely reported to decrease markedly, approaching values typical of low-density gases [53]. Consequently, the ratio of viscous dissipation to pressure work, defined as $\eta = \frac{|S:\dot{\epsilon}|}{|p-p_{bv}|\frac{1}{\rho}\frac{\partial \rho}{\partial t}} \sim \frac{\mu|\Delta v|^2}{p|\Delta \cdot v|}$ is expected to remain small under the characteristic flow conditions of interest, since the dominant energy transfer mechanism is compression work rather than shear dissipation.
- (3) In the specific context of high-pressure gas launch systems, comparative studies have demonstrated that one-dimensional inviscid simulations yield predictions in close agreement with higher-fidelity axisymmetric viscous analyses for the core performance metrics of interest, including projectile velocity and breech pressure histories [54–56].

The equation indicates that in investigating CO_2 state transitions and their effects on surrounding media, primary consideration must be given to the influence of pressure, density, and temperature. According to research results documented in reference [19], among CO_2 multiphase equations of state, the model based on the GERG-2008 equation exhibits the highest computational accuracy and adequately covers the engineering applications of this study. Therefore, considering the prediction accuracy and applicability range of various models, the GERG-2008 model is selected for this research.

2.1.2. Construction of Multiphase Gaseous Property Equations

The specific implementation is illustrated in Figure 1. Through the VUEOS subroutine framework construction in Abaqus [8], the calling procedure receives inputs including temperature, density, and component concentration. The density solution module performs inverse calculation of density (if pressure is input), computes thermodynamic properties including pressure, enthalpy, entropy, and specific heat, and returns the calculated results. The VUEOS is periodically invoked by the finite element solver (typically during the material state update phase at each integration point), receiving current state information transmitted from the finite element solver, such as temperature T , pressure P , and strain state. The GERG-2008 subroutine is called to calculate thermodynamic properties to satisfy the requirements of the material constitutive relationship. The material state, stress, and stiffness matrix are updated according to the calculation results and returned to the finite element solver.

2.1.3. Construction of Solid-State Gas Property Equations

The construction of solid CO_2 property equations typically requires independent treatment due to fundamental differences in physical behavior compared to fluid (gas–liquid) phases. This study primarily focuses on the compressibility of solids under conditions of limited temperature variation. Under this specific premise, a simplified yet practical approach employs a modified form of the Tait equation to describe the pressure–volume (P - V) relationship of solid CO_2 [7]

$$\frac{V}{V_0} = 1 - C \ln \left(1 + \frac{P - P_0}{B} \right) \quad (2)$$

where P is the pressure, V is the volume, V_0 is the volume at reference pressure P_0 (which can be taken as the volume at zero pressure or atmospheric pressure). C and B are empirical parameters; parameter B approximates the bulk modulus, while C reflects the nonlinear compressive behavior of the material under pressure. In the calculation process, B and C are assigned values of 5.2 GPa and 1.2, respectively. The advantage of this equation lies in its simple form with fewer parameters, and it can adequately fit experimental data within a certain pressure range.

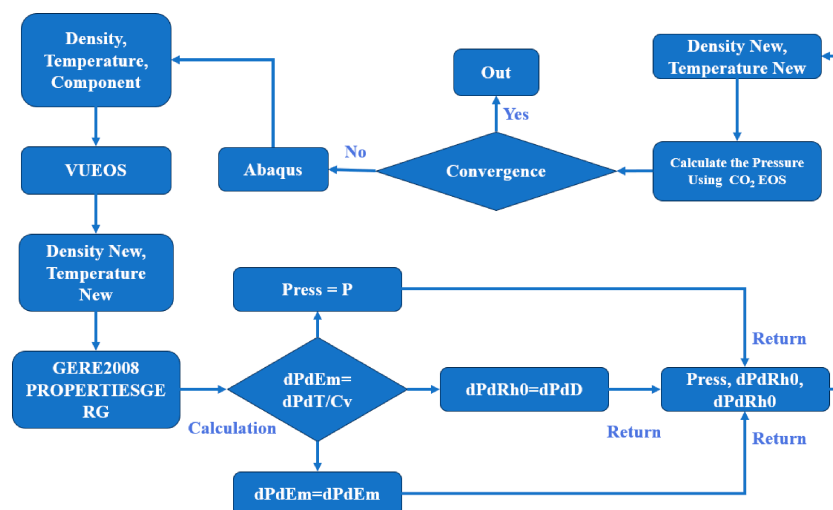


Figure 1. Flowchart of Abaqus calling GERG-2008 through VUEOS subroutine.

2.1.4. Phase State Determination Equations

Under different conditions, CO_2 exhibits various phase states, including gaseous, liquid, supercritical fluid, and solid (dry ice) states (Figure 2). As illustrated, phase determination can be performed based on Curve A, Curve B, the triple point, and the critical point. Therefore, the critical condition segmented curves A and B are first fitted to obtain relational expressions between pressure and temperature, with the general form [25]:

$$p_t = p_0 + p_1 \cdot \exp(T/T_0) \quad (3)$$

where p_0 , p_1 , and T_0 are constants, and p_t represents the pressure corresponding to Curve A (white) or Curve B (black).

Subsequently, based on the triple point and critical point temperature-pressure coordinates in Figure 2 and the aforementioned equations, different phase states are determined as follows:

(1) Solid region: Located in the “low-temperature high-pressure” region on the left side of Figure 2, enclosed by the front section of Curve A (up to the triple point) and Curve B. In the solid state, only the pressure condition needs to be satisfied:

$$p_A < p, \quad p_B < p \quad (4)$$

(2) Liquid region: Distributed in the “high-pressure” central region of Figure 2, enclosed by the rear section of Curve A (from triple point to critical point) and Curve B. Determination requires both temperature and pressure conditions:

$$\begin{cases} p_A < p, & p < p_B \\ p < p_{crt}, & T < T_{crt} \end{cases} \quad (5)$$

where p_{crt} and T_{crt} represent the critical point pressure and temperature, with values of 7.3 MPa and 303.4 K, respectively.

(3) Gaseous region: Distributed in the “low-pressure high-temperature” lower region of Figure 2, enclosed by Curve A and the critical point (pressure line). Only the pressure condition needs to be satisfied:

$$p < p_A, \quad p < p_{\text{crt}} \quad (6)$$

(4) Supercritical fluid region: Defined as the region outside the above three cases.

Based on the CO₂ phase state determination method reported in Refs. [56–59], the fitting results for Curves A and B are presented in Figure 3, confirming that the adopted equation of state exhibits satisfactory agreement with the experimental data (Figure 4). Accordingly, the thermodynamic phase state at any given time and spatial location can be determined once the local pressure and temperature are computed via the VUEOS. The corresponding fitting parameters for Curves A and B are summarized in Table 2.

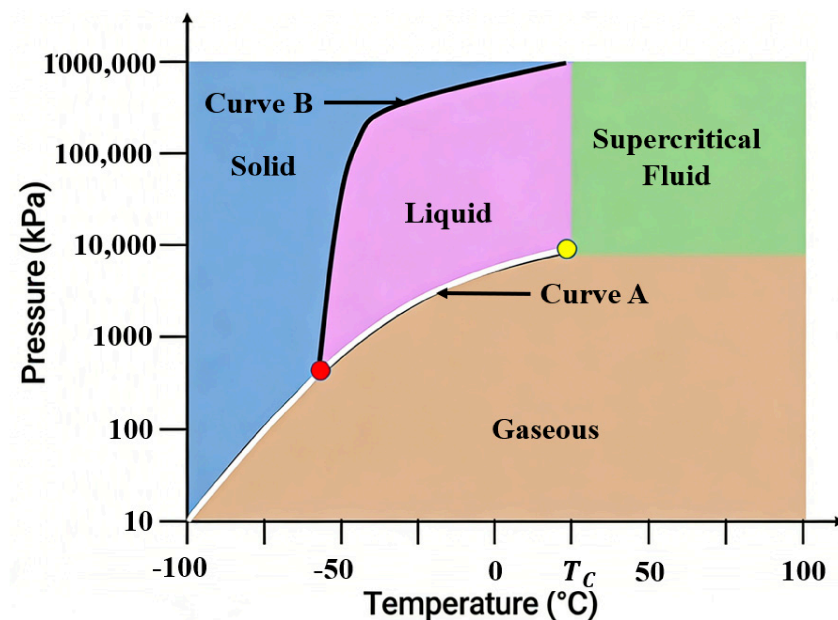


Figure 2. Schematic diagram of phase state determination.

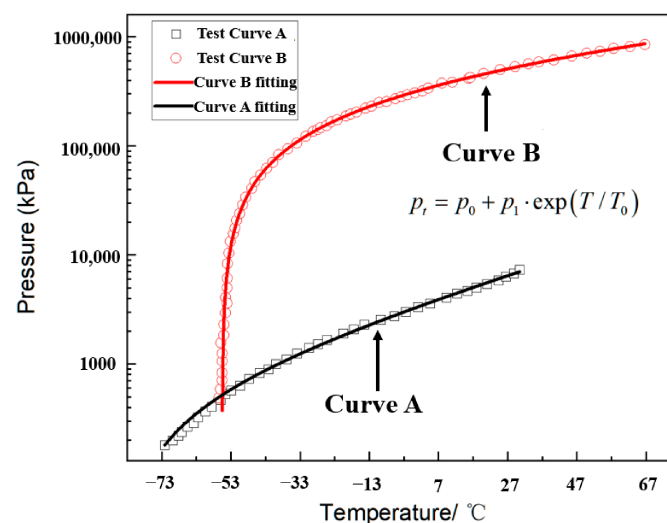


Figure 3. Fitting results of phase state determination.

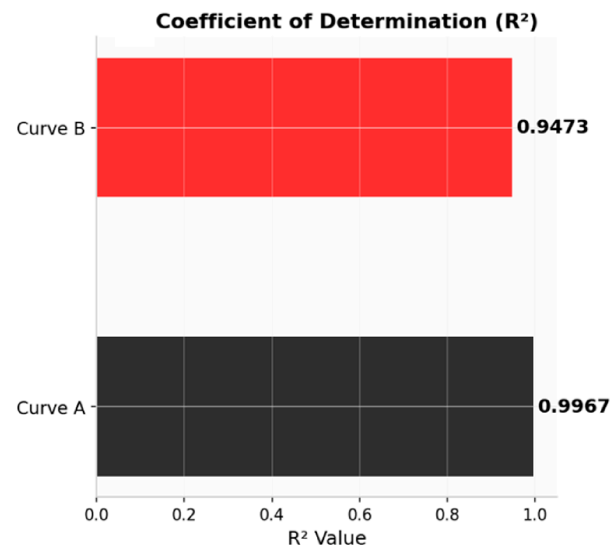


Figure 4. Results of coefficient of determination.

Table 2. CO₂ phase state determination conditions.

Curve	p_0 (Pa)	p_1 (Pa)	T_0 (K)
A	1.9×10^8	-7.47×10^8	159.1
B	7.9×10^3	-5.65×10^5	44.2

Within the Abaqus finite element simulation environment, the pressure–volume (P–V) constitutive relation is defined by specifying material compressibility and deploying a user-defined equation-of-state subroutine, designated VUEOS. As depicted in Figure 5, the determination of CO₂ thermophysical properties across fluid (gaseous, liquid, supercritical) and solid (dry ice) phases follows a structured two-branch workflow. At each iteration, a phase state discriminator first identifies whether CO₂ is present in a fluid phase (gaseous, liquid, supercritical, or near-critical) or a solid phase (dry ice), and routes the computational procedure to the corresponding property calculation branch.

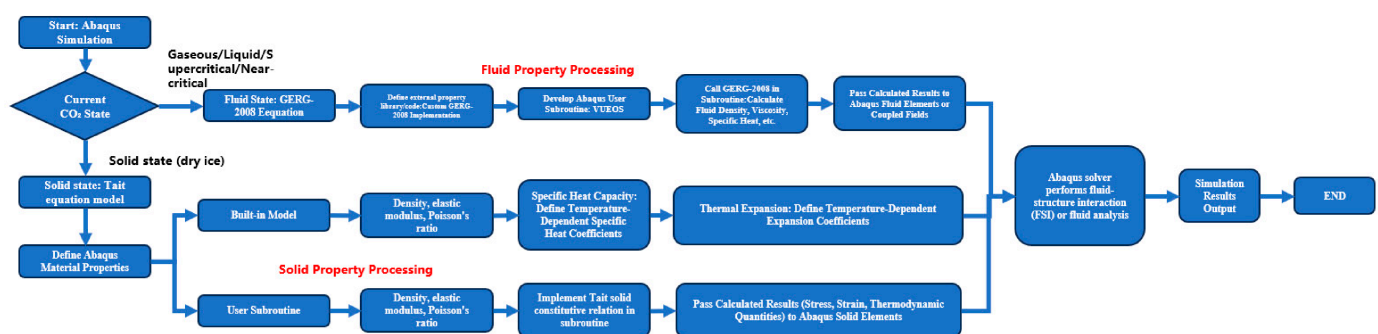


Figure 5. Implementation of CO₂ thermophysical properties across phase states.

For fluid phases exhibiting complex thermophysical behavior, Abaqus relies on the integration of external thermophysical property libraries (e.g., the GERG-2008 formulation). The computed density, viscosity, and associated thermophysical parameters are passed to the fluid elements through the VUEOS subroutine, which supports high-fidelity fluid–dynamic simulation and coupled fluid–structure interaction (FSI) analysis. By contrast, for solid-phase CO₂, the native material models built into Abaqus are sufficient to characterize its basic thermomechanical properties, including elastic behavior, compressibility, and thermal expansion. For more refined modeling accuracy, custom constitutive relations can

be implemented via user-defined material subroutines (e.g., UMAT), with the calculated stress tensors and material state variables returned to the solid elements. The full multi-phase thermophysical property model is ultimately executed by the Abaqus solver, and the corresponding simulation outputs are generated accordingly.

2.2. Construction of Excitation Device Combustion Model

2.2.1. Introduction to Excitation Device Combustion Model

Traditional combustion models often struggle to capture the complex behavior of energetic materials under extreme conditions. The JWL (Jones-Wilkins-Lee) equation of state [32,33], originally developed primarily for describing the thermodynamic behavior of explosive detonation products, possesses formal flexibility that enables effective application to describing rapid combustion processes of an excitation device when appropriately modified and parameterized, particularly when combustion is accompanied by significant pressure and temperature elevation [34]. The JWL equation after introduction and combustion modeling [33]:

$$P = A \left(1 \frac{\omega}{R_1 V} \right) e^{-R_1 V} + B \left(1 \frac{\omega}{R_2 V} \right) e^{-R_2 V} + \frac{\omega E}{V} \quad (7)$$

where P is the product pressure, V is the relative volume. E is the internal energy per unit initial volume, and A , B , R , and ω are characteristic parameters of the JWL equation—these are empirical constants representing material properties. When describing excitation device combustion, the process is not directly treated as detonation but rather as a rapid exothermic process generating substantial gaseous products. The introduction of the JWL equation enables correlation of the state of gaseous products generated during combustion with combustion progress, treating combustion as a transformation process from solid charge to gaseous products, with the JWL equation describing the pressure-volume-energy relationship of these gaseous products.

During the combustion process, a mixture of unburned and burned products exists within the container. The JWL equation describes the state of burned products at this time. Therefore, the actual pressure contribution is the pressure of burned products at a given volume and energy. The chemical energy released by combustion is converted into the internal energy of products. Thus, E can be correlated with combustion heat release and combustion progress λ :

$$E = \lambda \cdot Q_v \cdot \rho_0 \quad (8)$$

where Q_v is the combustion heat per unit mass.

As illustrated in Figure 6, through this approach, the JWL equation can describe the pressure response of combustion products at different combustion progress and volumes, thereby constructing a macroscopic combustion kinetics model. Combined with a combustion rate law driving the evolution of λ , the pressure within the container at the current combustion progress can be calculated using the JWL equation.

2.2.2. Combustion Heat Flow Model

In finite element software such as Abaqus, the JWL equation of state directly reflects only the pressure-volume-internal energy relationship and cannot explicitly simulate temperature evolution processes.

As combustion progresses, the chemical energy Q_v released per unit mass of the excitation device during combustion is:

$$Q_v = \frac{E_0 - E_{\text{product}}}{\rho_0} = \frac{1}{\lambda \rho_0} \cdot E_0 \quad (9)$$

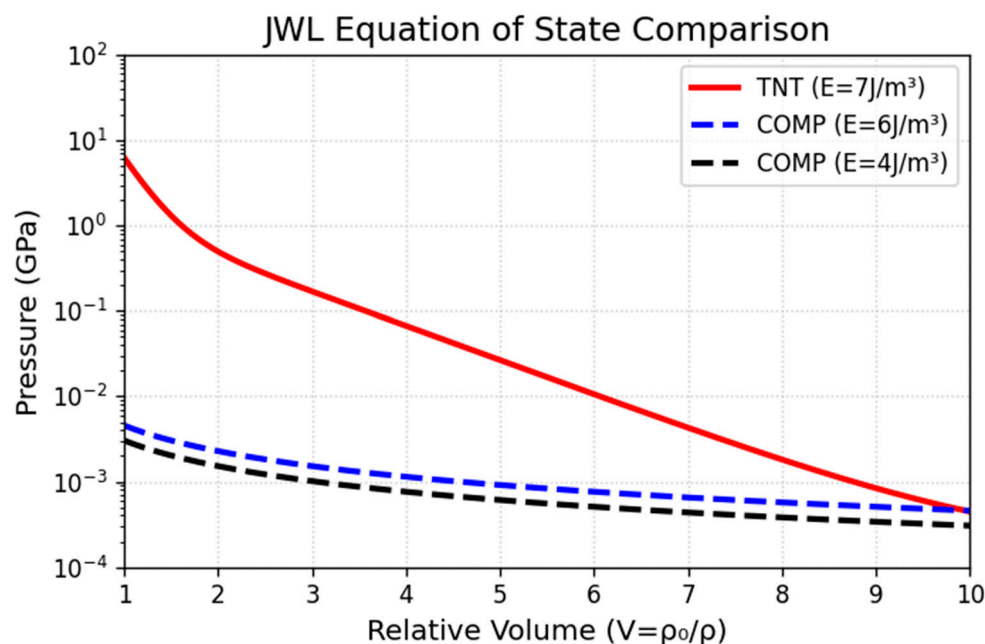


Figure 6. P-V curves of combustion products at different specific internal energies.

Representing the initial energy of the excitation minus the energy of detonation products. The excitation device rapidly releases energy Q_v instantaneously upon combustion, subsequently utilized for expansion pressure work and temperature elevation. For constant-volume combustion, all released energy is converted into increased system internal energy. Using ideal thermodynamic relationships, the internal energy-temperature relationship is:

$$Q_v = C_v(T - T_0) \Rightarrow T - T_0 = \frac{Q_v}{C_v} \quad (10)$$

where C_v is the specific heat at constant volume. Therefore, for the combustion of a unit mass charge, the released energy causes an internal energy increase in its products, thereby inducing temperature elevation.

The burning velocity D of the combustion front relative to unburned charge is defined as a function of pressure:

$$D = c + a \cdot P^n \quad (11)$$

where P is the CO_2 environmental pressure, and c , a , and n are burning rate coefficients and pressure exponents, which can be calibrated through experiments (such as closed-bomb tests).

Defining the combustion progress variable λ as the ratio of charge mass burned at time t to total charge mass, then:

$$\lambda(t) = \frac{x(t)}{L_0} = D(P)dt \quad (12)$$

where L_0 is the characteristic length. The total energy release rate (or power) can be defined as the energy released per unit initial volume of charge per unit time, i.e., heat flux H :

$$H = \rho_0 Q_v \frac{D(P)}{L_0} \quad (13)$$

The energy release rate H describes the chemical energy released per unit volume per unit time into the system, which is subsequently converted into internal energy, leading to pressure and temperature elevation.

The Abaqus user subroutine VHETVAL (Heat Generation Value) is adopted to encode a custom volumetric heat-source formulation. The workflow for defining the heat flux, as specified in Equation (13), is implemented within the VHETVAL subroutine via three sequential steps. As illustrated in Figure 7, first, the pressure P at the current integration point is extracted from user-defined material properties. Subsequently, the burning rate is evaluated using the predefined function $D(P)$. Finally, the initial density, volumetric heat of combustion Q_v , the evaluated burning rate, and the characteristic dimension are substituted into the heat flux expression to compute and return the magnitude of the volumetric heat source at the respective integration point. This numerical framework accurately replicates the energy release process of the initiating agent, which in turn drives the subsequent temporal evolution of the temperature and pressure fields within the system.

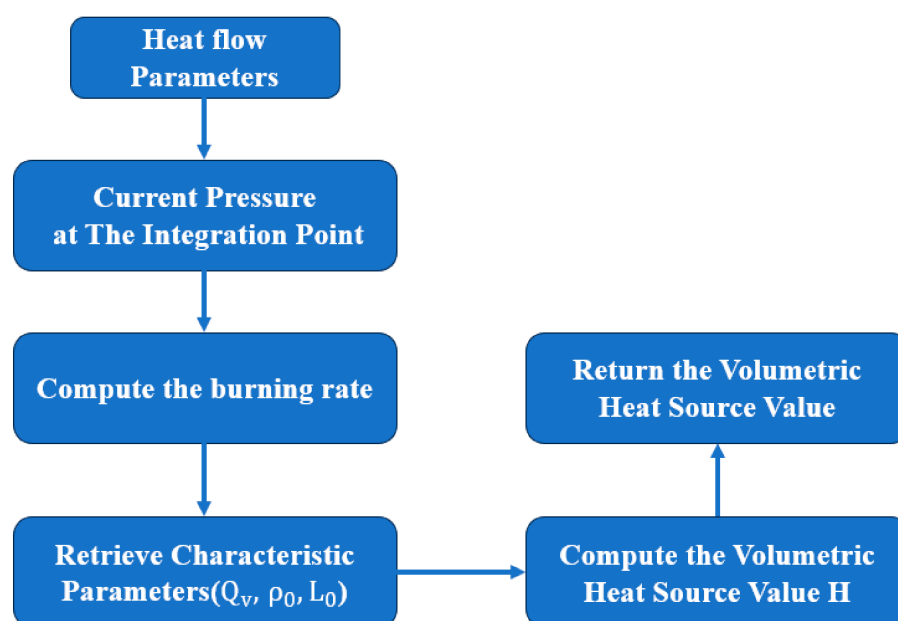


Figure 7. The workflow for defining the heat flux.

3. Construction of Multiphase Flow Coupling Methodology

3.1. Coupled Eulerian-Lagrangian Method

The present study investigates the thermodynamic phase transitions of carbon dioxide across various thermodynamic states and the combustion-driven expansion process of the ejection device; accordingly, the Coupled Eulerian–Lagrangian (CEL) method, as implemented in the finite element framework, was selected for numerical analysis. All numerical simulations were conducted using the commercial finite element code Abaqus 2021, in accordance with the methodology detailed in Refs. [25,59]. Carbon dioxide was modeled using the Eulerian formulation. The ignition device in its pre-combustion state was treated as a Lagrangian body, whereas combustion products were modeled using the same Eulerian approach for numerical consistency. The overall computational workflow of the present study is illustrated in Figure 8.

3.2. Construction of Multiphase Flow Coupling Simulation Model

Based on actual experimental specimens, the simulation model shown in Figure 9 is established. Considering specimen symmetry and improving computational efficiency, a 1/4 model is constructed. The model includes a CO_2 region, excitation device, and wall.

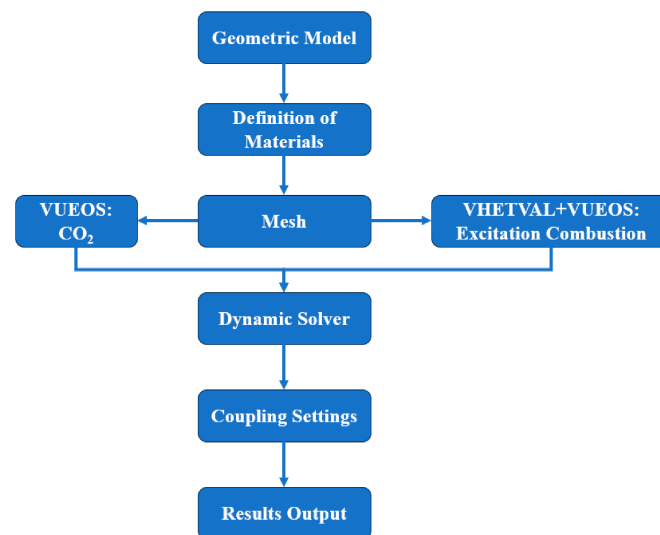


Figure 8. Flowchart of multiphase flow coupling calculation.

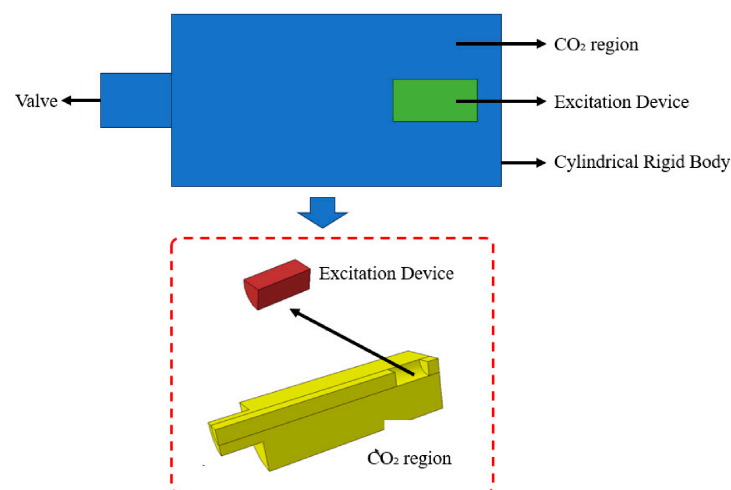


Figure 9. Geometric modeling of the flow field model.

To accurately simulate supercritical CO₂ phase-transition behavior driven by solid excitation device combustion and its dynamic interaction with the blasting energy-release device, this project employs the Coupled Eulerian-Lagrangian (CEL) method on the Abaqus/Explicit platform for explicit dynamic analysis. The excitation device ignition scheme strictly replicates experimental conditions: an instantaneous ignition source is established at the bottom-center position of the model, with a high-temperature ignition pulse applied at time $t = 0$ s. This design ensures combustion wave propagation path consistency with flame spread patterns in physical experiments.

(1) Geometric model: As shown in Figure 8, it is composed of an excitation device, CO₂, a valve, and a cylindrical rigid body.

(2) Boundary conditions: Considering the sensitivity of supercritical fluid phase transition processes to mesh quality, the entire Eulerian domain employs structured hexahedral mesh discretization, significantly improving computational efficiency while ensuring material interface capture precision. The model applies axisymmetric boundary conditions based on physical symmetry, implementing circumferential displacement constraints in cylindrical coordinates:

$$u_{\theta} = 0, \partial u_r / \partial \theta = 0, \partial u_z / \partial \theta = 0 \quad (14)$$

(3) Initial state: Strictly following experimental conditions, non-void regions (i.e., CO₂-filled cavity and combustion chamber) are assigned an initial pressure field of 7.2 MPa and initial temperature field of 298 K. To ensure accurate capture of dynamic processes, adaptive time stepping control is employed in calculations (stable time increment: $\Delta t \approx 1 \times 10^{-8}$ s).

(4) The initial conditions for the simulation are summarized in Table 3, where the density is 780 kg/m³, the temperature is 298 K, the pressure is 7.2×10^6 Pa, and the initial specific internal energy is 60,000 J/kg.

Table 3. Initial conditions.

Density (kg/m ³)	Temperature (K)	Pressure (Pa)	Initial Specific Internal Energy (J/kg)
780	298	7.2×10^6	60,000

4. Result and Discussion

4.1. Mesh Quality Criteria and Independence Study

To thoroughly analyze the influence of mesh size on simulation results, this study systematically establishes five different mesh size schemes for the Eulerian region: 15 mm, 10 mm, 5 mm, 2 mm, and 1 mm, with corresponding computational models constructed. As Eulerian region mesh sizes are progressively refined, the total mesh quantity of the entire model exhibits exponential growth. Under the aforementioned five mesh-size schemes, the corresponding total mesh quantities reach approximately 10,000, 35,000, 150,000, 2.26 million, and 18.7 million elements (Table 4). This sharp increase in mesh quantity directly leads to a significant elevation in computational model scale and complexity. To quantitatively evaluate the influence of different mesh sizes on simulation accuracy, pressure time-history curves at a specific monitoring point in the model are examined as the focus, with comparison results clearly presented in Figure 10. Analysis of these curves yields the following key conclusions:

Table 4. Mesh parameters of the model.

Mesh Size (mm)	Cells
15	10 thousand
10	35 thousand
5	150 thousand
2	226 thousand
1	1870 thousand

First, when relatively coarse 15-mm meshes are adopted, the simulation results exhibit obvious deficiencies. The calculation fails to precisely capture peak pressure response values, with both showing substantial deviations from the results under finer meshes. This indicates that excessively large mesh sizes cannot adequately resolve details of key physical processes including pressure wave propagation, reflection, and superposition, leading to degraded predictive capability for transient high-intensity physical fields and reduced credibility of computational results.

Second, as mesh size decreases, simulation accuracy is progressively improved. When mesh size decreases from 15 mm to 10 mm, and further to 5 mm, calculated pressure curves, particularly peak pressures, gradually converge toward results from finer meshes (2 mm and 1 mm). A critical finding is that once the mesh size is refined to 5 mm or smaller, pressure responses at this monitoring point, including peak pressure, pressure

pulse morphology, and subsequent oscillation characteristics, exhibit virtually no significant differences. This demonstrates that for the physical quantity of interest (point pressure here), the 5 mm mesh configuration for the Eulerian domain already achieves a mesh-independent solution; further mesh refinement yields negligible accuracy gains, which is unwarranted from an engineering application standpoint.

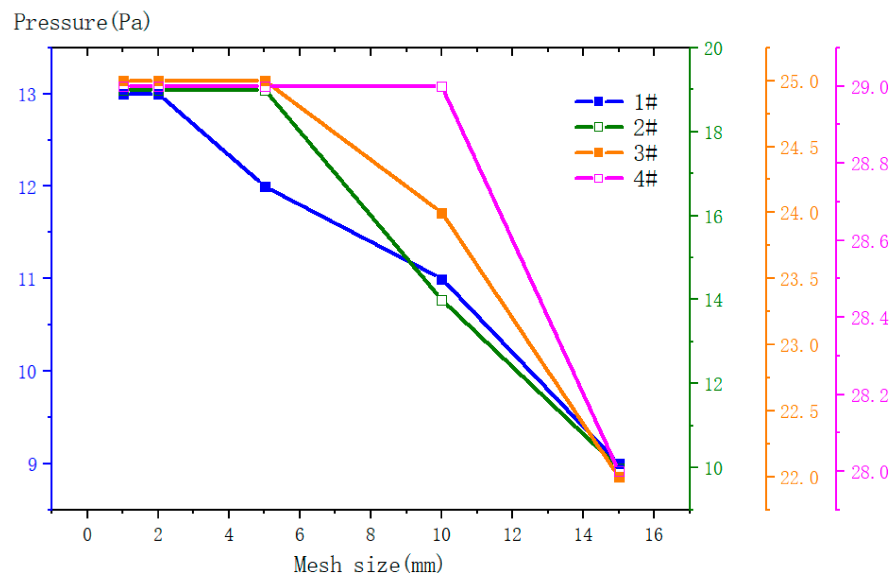


Figure 10. Pressure comparison at key points under different mesh sizes.

However, the total number of elements increases from 10,000 for the 15 mm mesh to 150,000 for the 5 mm mesh, with a corresponding rise in computation time, which remains within an acceptable range. When the mesh size is further refined from 5 mm to 1 mm, the mesh quantity surges to 18.7 million. This sharp increase in mesh quantity directly leads to rapid growth in computational memory requirements, disk read/write operations, and single computation cycle duration. The increase in computational time may reach several orders of magnitude, rendering computational costs exceptionally high for engineering practices requiring extensive parametric studies or optimization design. Therefore, a 5 mm mesh is recommended for the Eulerian region.

4.2. Model Validation

To ensure the reliability of computational results, the numerical model was developed with rigorous consideration of the physical geometry and material constitutive behavior. As depicted in Figure 11, the computational domain consists of four core components: the excitation device, the cylindrical rigid body, the CO₂ fluid domain, and the valve assembly. As the energy input unit of the system, the excitation device is described by the Jones–Wilkins–Lee (JWL) equation of state, with its key parameters summarized in Table 5.

Table 5. Parameters of the JWL state equation.

$\rho_0/(\text{kg/m}^3)$	A/MPa	B/MPa	ω	R_1	R_2	e/kJ	$c_0/(\text{mm/s})$
1725	0	0	1	1	1	6000	14.5

Guided by this numerical framework, a prototype of the CO₂ phase-transition ejection power unit was designed and manufactured. A pressure transducer was installed at the front end of the power unit to capture the temporal evolution of internal pressure following ignition of the excitation device. The valve was set to open at $t = 10$ post-ignition.

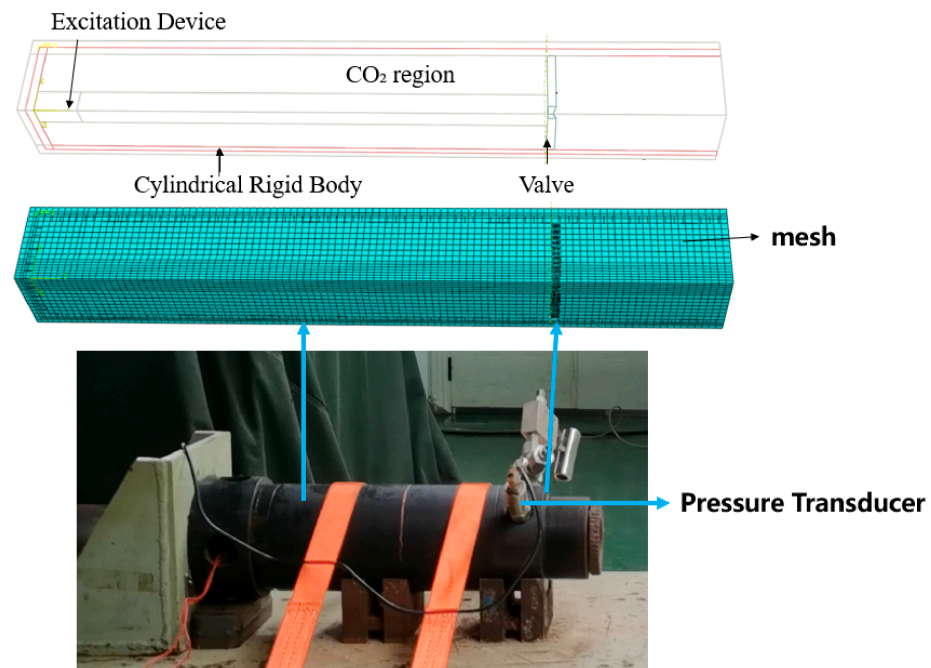


Figure 11. Mesh of the model and experimental setup.

The pressure–time history predicted by the multiphysics-coupled fluid–structure interaction (FSI) model is compared with experimental measurements in Figure 12a, and the corresponding relative errors are plotted in Figure 12b. The maximum relative error remains below 10% over the entire pressure evolution process, except for a transient deviation peak during the initial ignition stage. This favorable agreement verifies that the developed multiphysics FSI model achieves satisfactory predictive accuracy for pressure–transient processes, thus validating the reliability of the simulation results.

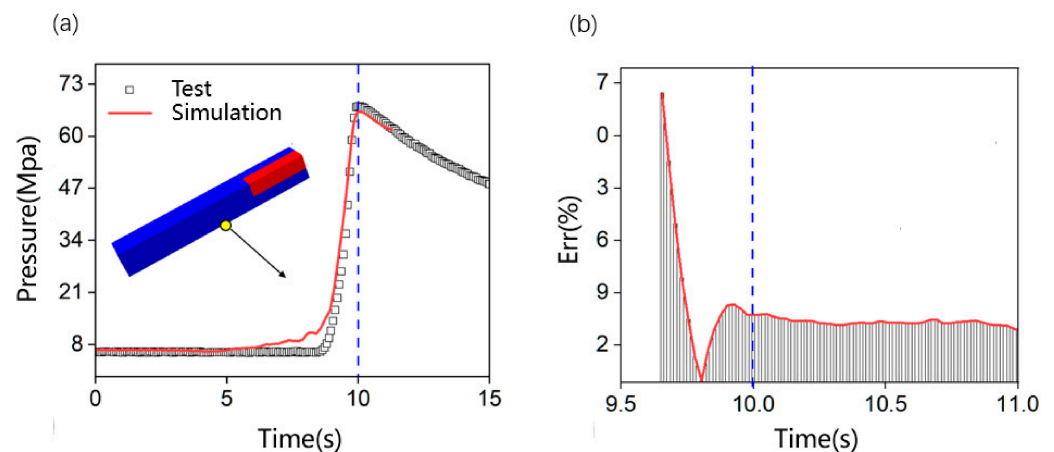


Figure 12. (a) Comparison of test results with simulation predictions. (b) Analysis of error between test results and simulation predictions.

Further analysis demonstrates that combustion of the excitation device charge initiates a coupled multiphysics process governed by the interplay of chemical energy release, compressible gas dynamics, and fluid–structure interaction. Upon ignition at the coordinate origin (0, 0, 0), the charge undergoes deflagration at a prescribed burning rate of 15 mm/ms, converting solid propellant into high-temperature, high-pressure combustion products. The thermodynamic behavior of these products is described by the JWL equation of state, with parameters tabulated in Table 5. A specially simplified set of coefficients ($A = B = 0$, $\omega = 1$)

effectively reduces the formulation to an ideal-gas-like thermodynamic response, in which pressure varies linearly with internal energy and density. This simplification is physically justified under the present study conditions, as the dominant energy transfer mechanism is the mechanical expansion work exerted on the surrounding CO₂ medium, rather than the complex detonation chemistry captured by complete JWL parameter sets. The combustion front propagates in accordance with the geometric burn law, consuming the charge and generating gaseous products that expand into the available void space. The expansion process is non-isentropic; instead, it is characterized by strong compressive heating of the adjacent CO₂ and irreversible entropy generation across the contact discontinuity separating the combustion products from the compressed working fluid. The numerical model reproduces this process via the Eulerian formulation, which inherently conserves mass, momentum, and energy while accommodating large-strain, large-deformation flow without mesh entanglement.

4.3. Combustion Shock Pressure Response Curves

Figures 13 and 14 Comparison between experimental results and simulation predictions. Systematic validation of the multiphysics theoretical framework for CO₂ phase-transition ejection power units confirms satisfactory predictive accuracy, as demonstrated by rigorous comparison between numerical simulations and experimental measurements throughout the critical pressurization stage. The pressure-time curve analysis reveals that the integrated modeling approach, combining the GERG-2008 equation of state for CO₂ thermophysical properties with the JWL combustion model for excitation-charge behavior, faithfully reproduces the experimental trajectory from initial ignition through peak pressure attainment. The error quantification methodology employed in this study utilizes the relative error formulation expressed:

$$\text{err} = \left| \frac{\chi_{\text{sim}} - \chi_{\text{test}}}{\chi_{\text{test}}} \right| \times 100\% \quad (15)$$

where χ_{sim} and χ_{test} represent the simulated and experimental values, respectively, enabling precise assessment of model fidelity at each temporal increment. Application of this metric to the pressure evolution data demonstrates that throughout the entire pressurization process—from the initial quiescent phase through the rapid acceleration period to the maximum pressure peak—the calculated errors remain consistently within the prescribed 10% tolerance band, with the critical peak pressure prediction exhibiting merely 4.1% deviation from experimental observations. This remarkable agreement validates the coupled treatment of supercritical CO₂ phase transition dynamics, combustion-driven energy release mechanisms, and multiphase flow interactions, confirming that the pressure-based Eulerian multifluid approach with relaxation-based phase equilibrium treatment adequately captures the complex physics governing system behavior. The systematic validation across all operational phases, including the sensitive near-critical region where CO₂ compressibility exhibits extreme sensitivity to pressure variations, establishes that the modeling framework satisfies the stringent accuracy requirements specified for engineering design applications. Consequently, the demonstrated predictive capability, with errors bounded well below the 10% threshold throughout the entire pressurization trajectory and particularly at the decisive peak-pressure moment, provides definitive evidence that the theoretical model is sufficiently robust and reliable for practical implementation in the design optimization of next-generation cold launch systems.

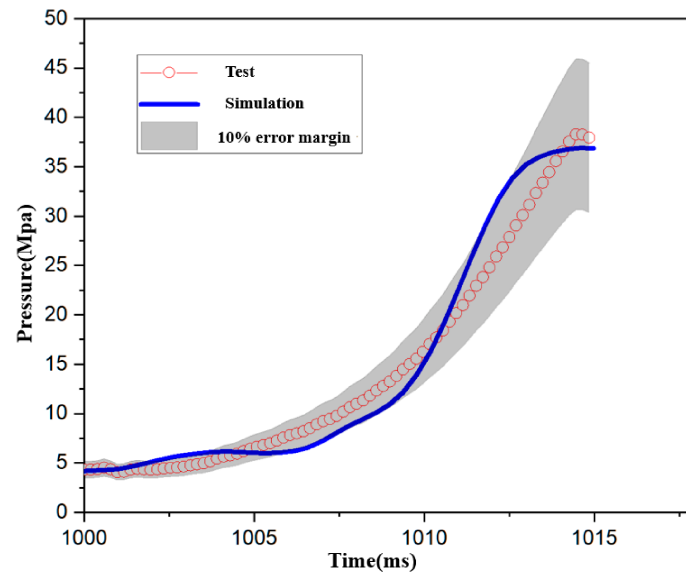


Figure 13. Comparison between experimental results and simulation predictions.

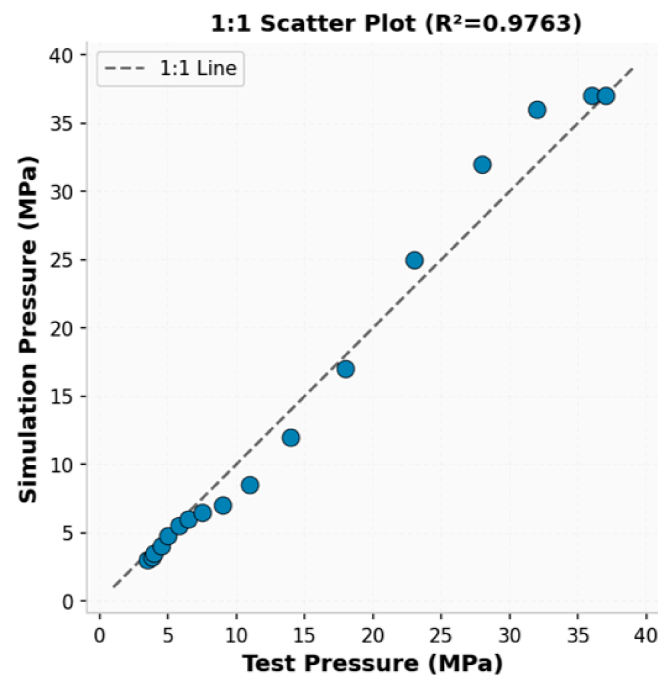


Figure 14. The error between the experimental results and the simulation predictions.

4.4. Excitation Device Combustion Shock Analysis

4.4.1. Shock Wave Formation and Pressure Transient Dynamics

As the combustion products expand, they function as a piston-like driving medium, generating a compression wave that steepens into a shock front as it propagates through the CO₂ medium. The shock formation distance can be estimated based on the pressure gradient and speed of sound in the undisturbed CO₂. Prior to shock formation, the pressure at the sensor location increases gradually, reflecting the acoustic precursor ahead of the expanding gas front. Once the shock is fully established, the pressure exhibits a nearly discontinuous jump, followed by either a pressure plateau or a sustained rise governed by the continuous energy input from the burning propellant charge.

The pressure–time history recorded at the central sensor location (Figures 15 and 16) presents three distinct regimes:

- (1) An initial acoustic precursor stage characterized by a mild pressure rise;

- (2) A rapid compression phase coinciding with the arrival of the shock front;
- (3) A quasi-steady high-pressure plateau stage maintained by the continuous expansion of the combustion products.

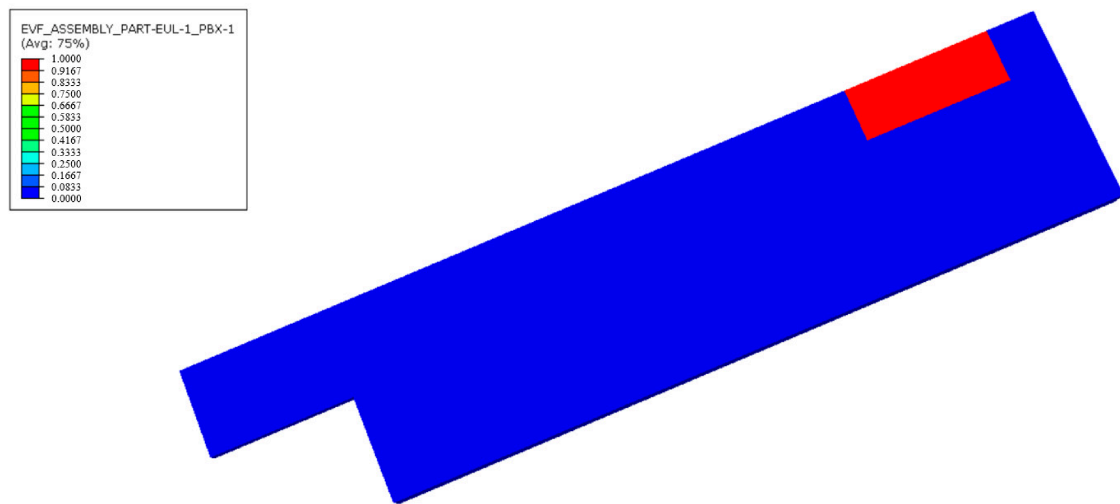


Figure 15. Spatial distribution of materials at initial moment.

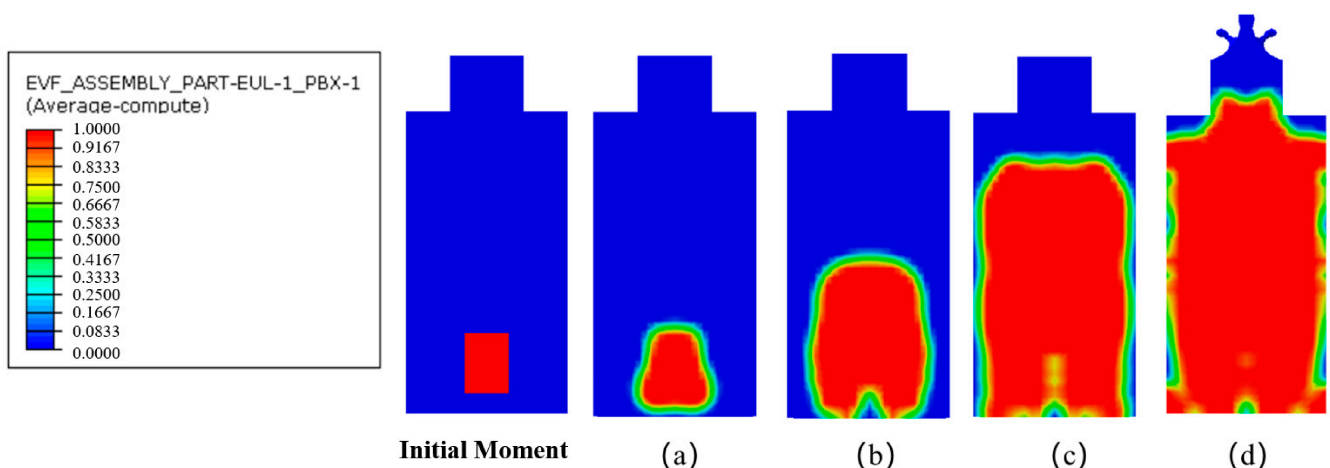


Figure 16. Diffusion process of excitation device at different moments. (a) 1.5 ms, (b) 5 ms, (c) 8 ms, (d) 15 ms.

The magnitude of the peak pressure is determined by the balance between the energy deposition rate from charge combustion and the pressure relief rate via CO₂ compression and eventual structural failure of the energy release device. Notably, the pressure does not exhibit an abrupt cliff-like drop immediately after the peak, as the energy release device remains structurally intact at this stage, confining the high-pressure combustion products and preserving the pressure boundary condition.

4.4.2. Temperature Field Evolution and Thermodynamic Coupling

The temperature distribution of the combustion products (Figure 17) is not directly prescribed by the JWL equation of state; instead, it is reconstructed a posteriori via the user-defined subroutine VHETVAL, which computes temperature from the product pressure and density using the caloric equation of state. This indirect approach is necessary because the standard JWL formulation adopts internal energy as the primary thermodynamic state variable, with no explicit temperature dependence.

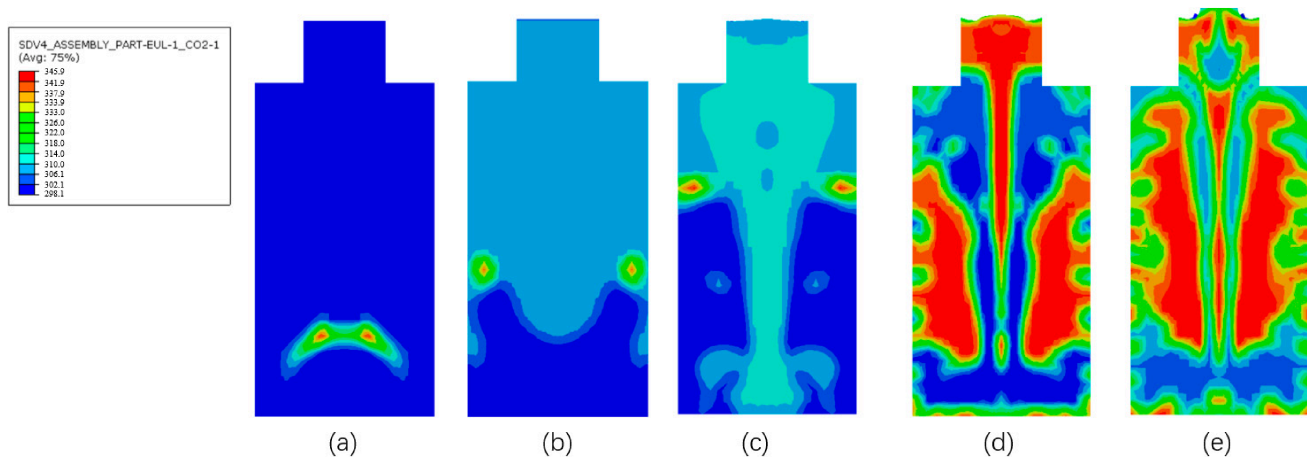


Figure 17. Temperature variations in charge at different moments. (a) 0.5 ms, (b) 1.5 ms, (c) 5 ms, (d) 8 ms, (e) 15 ms.

The spatial correlation between the volume fraction of combustion products (EVF-PBX) and the temperature field originates from the adiabatic nature of the expansion process in the initial stage. As the products expand, they undergo approximately isentropic cooling; however, this cooling effect is partially offset by compression heating of the CO₂ medium at the contact interface. The resulting temperature field therefore reflects a competition among three competing mechanisms:

- (1) Adiabatic expansion of the product gases, which acts to decrease the temperature;
- (2) Shock compression of the CO₂ medium, which acts to increase the temperature;
- (3) Conductive and convective heat transfer between the combustion products and CO₂, which drives thermal equilibration at the interface.

Numerical results indicate that the latter two mechanisms dominate in the near-field region, yielding a temperature distribution that closely mirrors the spatial pattern of product diffusion.

4.4.3. Validation and Predictive Capability

The fidelity of the developed numerical model is evaluated via comparison with experimental pressure measurements. The maximum relative error between simulation predictions and experimental data is approximately 10%, with larger transient deviations observed during the ignition phase. This level of agreement confirms that the multiphysics coupling framework adequately captures the dominant energy transfer mechanisms. The remaining discrepancies can be attributed to three primary sources:

- (1) Idealization of the JWL equation of state parameters, which neglects the effects of secondary chemical reactions;
- (2) Numerical diffusion inherent to the Eulerian formulation at the combustion product–CO₂ interface;
- (3) Uncertainties in the propellant charge burning rate and ignition transient behavior, which are challenging to quantify experimentally.

Nevertheless, the model serves as a robust predictive tool for parametric investigations into the effects of charge mass, geometric configuration, and confinement strength on overall ejection performance.

5. Conclusions

This study establishes a comprehensive multiphysics theoretical framework that successfully integrates real-gas thermodynamics, combustion dynamics, and multiphase flow

mechanics for CO₂ phase-change ejection power units. By coupling the GERG-2008 equation of state with the JWL combustion model within a Coupled Eulerian-Lagrangian (CEL) methodology, the proposed approach resolves the complex interplay between supercritical CO₂ phase transitions and excitation device combustion, achieving high-fidelity prediction of pressure evolution from ignition through peak pressure attainment.

(1) Multiphase Thermophysical Property Modeling

The phase state discrimination framework, which adopts fitted critical-condition curves (Curves A and B) with determination coefficients R^2 of 0.9967 and 0.9473, respectively, enables reliable real-time identification of CO₂ phase states (gaseous, liquid, supercritical, and solid) under transient combustion-driven loading. The GERG-2008 equation of state, implemented via the VUEOS user subroutine, accurately describes the strongly nonlinear density behavior characteristic of supercritical and two-phase CO₂.

(2) Combustion-Driven Pressure Transient Dynamics

The JWL-based combustion model, coupled with the VHETVAL heat-generation subroutine, reproduces the experimentally observed three-stage pressure evolution process. Numerical results demonstrate that the magnitude of the peak pressure is not governed solely by the total chemical energy release, but by the energy deposition rate relative to the pressure-relief rate via CO₂ compression and structural deformation. The validated model yields peak pressure predictions with a maximum relative deviation of 4.1% from experimental measurements, and all relative errors remain below 10% over the entire pressurization process.

(3) Engineering Applicability

The validated model provides quantitative design guidance for next-generation ejection systems, offering a robust foundation for optimizing high-pressure chamber configurations, release pressures, and nozzle geometries in variable operational environments.

In summary, the integrated GERG-2008/JWL/CEL framework successfully bridges the gap between real-gas thermodynamics, combustion kinetics, and structural mechanics in a unified computational model. The numerical findings confirm that supercritical CO₂ phase-transition ejection is a viable and controllable alternative to conventional hot launch systems with predictable performance characteristics. Future work will extend the present model to include non-equilibrium phase transition kinetics and three-dimensional asymmetric configurations to further enhance predictive fidelity under off-design conditions.

Author Contributions: Methodology, J.X.; Software, M.T.; Validation, M.T.; Formal analysis, J.X.; Investigation, J.X.; Writing—original draft, M.T. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author(s).

Conflicts of Interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Nomenclature

P	Pressure
P_c	Critical pressure
p_t	Phase transition pressure
p_0	Reference pressure
p_1	Fitting parameter

p_A	Curve A pressure
p_B	Curve B pressure
p_{bv}	Bulk viscosity pressure
p_{crt}	Critical point pressure
S	Resent deviatoric stress
$\dot{\epsilon}$	strain
Q	Heat rate per unit mass
Q_v	Volumetric heat of combustion
P	Density
ρ_c	Critical density
V	Volume
V_c	Critical volume
Z	Compressibility factor
ω	JWL equation parameter
λ	Combustion progress variable
μ	Dynamic viscosity
η	Ratio of viscous dissipation to pressure work
χ	Variable for error calculation
T_c	Critical temperature
T_0	Reference temperature
E_m	Mechanical energy
E_0	Initial energy

References

1. Ke, B.; Zhou, K.; Xu, C.; Ren, G.; Jiang, K. Thermodynamic properties and explosion energy analysis of carbon dioxide blasting systems. *Min. Technol.* **2019**, *128*, 39–50.
2. Tang, M.; Li, G.; Zhang, M.; Zhou, X.H.; Cai, K.; Hu, S.C.; Wu, L.; Cao, W.X. Comprehensive analysis and optimization of waste heat recovery and utilization in paper drying process. *Energy* **2025**, *341*, 134015. [[CrossRef](#)]
3. Li, J.T.; E, J.Q.; Ding, J.J.; Cai, L.; Luo, B. Investigation on thermophotovoltaic performance enhancement of the hydrogen fueled micro combustors with the center-cleared twisted tapes for micro-thermophotovoltaic system. *Appl. Therm. Eng.* **2025**, *264*, 125483. [[CrossRef](#)]
4. Li, J.T.; E, J.Q.; Ding, J.J.; Cai, L.; Luo, B. Effects of deformed twisted tapes on thermal performance enhancement of hydrogen fueled micro combustor for micro-thermophotovoltaic system. *Appl. Therm. Eng.* **2025**, *264*, 125413. [[CrossRef](#)]
5. Kou, C.; Zeng, S.; Chen, X.; E, J.; Ma, Y. Effects of air–fuel ratio on combustion performance enhancement and emission reduction of the methanol–diesel dual-fuel engine. *Can. J. Chem. Eng.* **2026**, online version. [[CrossRef](#)]
6. Yu, F.; Marco, D.; Philippe, L.; Sébastien, P.; Yann, B. An Accurate and Efficient Look-up Table Equation of State for Two-Phase Compressible Flow Simulations of Carbon Dioxide. *Ind. Eng. Chem. Res.* **2018**, *57*, 7676–7691. [[CrossRef](#)]
7. Keim, V.; Paredes, M.; Nonn, A.; Münstermann, S. FSI-simulation of ductile fracture propagation and arrest in pipelines: Comparison with existing data of full-scale burst tests. *Int. J. Press. Vessel. Pip.* **2020**, *182*, 104067. [[CrossRef](#)]
8. Yang, L.J.; Xia, J.; Shen, Z.Q.; Zhang, F.Z.; Xi, Y.Z. Sc-CO₂ phase transition evolution migration and heat–mass transfer characteristics. *Results Eng.* **2026**, *30*, 1230–2590. [[CrossRef](#)]
9. He, Z.; Tian, S.; Li, G.; Wang, H.; Shen, Z.; Xu, Z. The pressurization effect of jet fracturing using supercritical carbon dioxide. *J. Nat. Gas Sci. Eng.* **2015**, *27*, 842–851. [[CrossRef](#)]
10. Dai, H.; Zhang, F.; Liao, G.; Jiaqiang, E. Performance analysis and-improvement of the battery thermal management system based on topology-optimized cold plates using sCO₂ as coolant. *Renew. Energy* **2026**, *266*, 125731. [[CrossRef](#)]
11. Ren, L.; Wen, R.; Zheng, D.; Sun, J. The Influence of Detached Rib Locations on the Channel Thermal Performance. *Heat Transf. Eng.* **2026**, 1–18. [[CrossRef](#)]
12. Zhang, B.L.; Xia, J.; Zhang, F.Z.; Shen, Z.Q.; Zhang, F. Investigation on the Effect of Recuperator Design on the Supercritical Carbon Dioxide Power Cycle for Hypersonic Vehicles. *Aerosp. Sci. Technol.* **2026**, *177*, 112314. [[CrossRef](#)]
13. Zhang, B.L.; Xia, J.; Tao, W.; Shen, Z.Q.; Zhang, F.Z.; Zhang, F. Film Cooling Characteristics of a Pintle in a Thrust-Controlled Solid Rocket Motor. *Aerosp. Sci. Technol.* **2026**, *168*, 111219. [[CrossRef](#)]
14. Zhou, J.; Cao, L.; Liao, G.; Zhang, F. Performance analysis of a He-Xe Brayton cycle based on recuperator configuration for nuclear space power systems. *Appl. Therm. Eng.* **2026**, *292*, 130292. [[CrossRef](#)]

15. Zhang, B.L.; Xia, J. Film Cooling Performance of the Moving Pintle in a Thrust-Controlled Solid Rocket Motor. *Aerospace* **2026**, *13*, 651. [\[CrossRef\]](#)
16. Yang, G.; Li, X. Modified Peng-Robinson equation of state for CO₂/hydrocarbon systems within nanopores. *J. Nat. Gas Sci. Eng.* **2020**, *84*, 103700. [\[CrossRef\]](#)
17. Rostamian, H.; Lotfollahi, M.N. New functionality for energy parameter of Redlich-Kwong equation of state for density calculation of pure carbon dioxide and ethane in liquid, vapor and supercritical phases. *Period. Polytech. Chem. Eng.* **2016**, *60*, 93–97. [\[CrossRef\]](#)
18. Starling, K.E.; Powers, J.E. Enthalpy of mixtures by modified BWR equation. *Ind. Eng. Chem. Fundam.* **1970**, *9*, 531–537. [\[CrossRef\]](#)
19. Ashton, G.J.; Haselden, G.G. Measurements of enthalpy and phase equilibrium for simulated natural gas mixtures and correlation of the results by a modified Starling equation. *Cryogenics* **1980**, *20*, 41–47. [\[CrossRef\]](#)
20. Nishiumi, H.; Saito, S. An improved generalized BWR equation of state applicable to low reduced temperatures. *J. Chem. Eng. Jpn.* **1975**, *8*, 356–360. [\[CrossRef\]](#)
21. You, M.; Feng, M.; Zhang, F.; Liao, G. Numerical investigation on the unsteady flow and coupled heat transfer characteristics between supercritical carbon dioxide and pulsating flow of water within a PCHE under offshore conditions. *Int. Commun. Heat Mass Transf.* **2026**, *172*, 110249. [\[CrossRef\]](#)
22. Song, H.; Yan, Y.; Zhang, F.; Liao, G.; E, J. Numerical investigation on the unsteady flow and heat transfer characteristics of supercritical carbon dioxide within a vertical tube under offshore conditions. *Appl. Therm. Eng.* **2025**, *274*, 126796. [\[CrossRef\]](#)
23. Lian, L.; Qin, J.; Liu, T.; Li, S.; Ji, Z.; Peng, X. Modified mixing rule for BWR-type equation of state and its application in phase behavior calculation of CO₂-crude oil system. *J. China Univ. Pet.* **2013**, *37*, 81–88.
24. Liu, K.; Yao, S.; Wang, Z.; Lu, Z. A phase-diagram-formulated EOS and fluid-solid coupling analysis for eco-friendly blasting of carbon dioxide phase transition. *Energy* **2024**, *313*, 134015. [\[CrossRef\]](#)
25. Kunz, O.; Wagner, W. The GERG-2008 wide-range equation of state for natural gases and other mixtures: An expansion of GERG-2004. *J. Chem. Eng. Data* **2012**, *57*, 3032–3091. [\[CrossRef\]](#)
26. Huber, M.L.; Lemmon, E.W.; Bell, I.H.; McLinden, O. The NIST REFPROP database for highly accurate properties of industrially important fluids. *Ind. Eng. Chem. Res.* **2022**, *61*, 15449–15472. [\[CrossRef\]](#) [\[PubMed\]](#)
27. Lemmon, E.W.; Bell, I.H.; Huber, M.L.; McLinden, M.O. *REFPROP Documentation, Release 10*; National Institute of Standards and Technology: Boulder, CO, USA, 2018.
28. Feng, Q.; Zhou, L.; Chen, Z.; Liu, J. Binary gas-solid coupling numerical simulation of CO₂ disposal in coal seams. *Geol. J. China Univ.* **2009**, *15*, 63–68.
29. Farag, G.; Chinnayya, A. On the Jones-Wilkins-Lee equation of state for high explosive products. *Propellants Explos. Pyrotech.* **2024**, *49*, e202300223. [\[CrossRef\]](#)
30. Castedo, R.; Natale, M.; López, L.M.; Sanchidrián, J.A.; Santos, A.P.; Navarro, J.; Segarra, P. Estimation of Jones-Wilkins-Lee parameters of emulsion explosives using cylinder tests and their numerical validation. *Int. J. Rock Mech. Min. Sci.* **2018**, *112*, 290–301. [\[CrossRef\]](#)
31. Baudin, G.; Serradeill, R. Review of Jones-Wilkins-Lee equation of state. *EPJ Web Conf.* **2010**, *10*, 00021. [\[CrossRef\]](#)
32. Kavvadias, D.; El Khatib, O.; Patil, S.A.; Kiefer, B.; Baxevanis, T. A phase-field regularization of porous-ductile fracture. *Eng. Fract. Mech.* **2026**, *332*, 111783. [\[CrossRef\]](#)
33. Su, S.; Wu, Y.; Wang, G.; Miao, Z.; Xiong, Y.P.; Guo, F.X.; Liu, H.B. Optimal calibration of Q235B steel Johnson-Cook model parameters based on global response surface algorithm. *J. Mar. Sci. Appl.* **2024**, *23*, 470–478. [\[CrossRef\]](#)
34. Lin, Z.; Zihao, W.; Heming, W. On the accuracy of the Johnson-Cook constitutive model for metals. *Chin. J. High Press. Phys.* **2019**, *33*, 040101. (In Chinese) [\[CrossRef\]](#)
35. Zhu, H.; Zhang, C.; Chen, S.; Wu, J. A modified Johnson-Cook constitutive model for structural steel after cooling from high temperature. *Constr. Build. Mater.* **2022**, *340*, 127746. [\[CrossRef\]](#)
36. Wen, B.; Huang, Y. *Handbook of Metal Materials*; Publishing House of Electronics Industry: Beijing, China, 2009.
37. Zhang, C.; Lei, N.; Cheng, J.; Shu, C.W. Interface preserving mesh optimization method for multi-material simulations. *J. Comput. Phys.* **2025**, *543*, 114389. [\[CrossRef\]](#)
38. Staubach, P. Hydro-mechanically coupled CEL analyses with effective contact stresses. *Int. J. Numer. Anal. Methods Geomech.* **2024**, *48*, 2207–2215. [\[CrossRef\]](#)
39. Zeng, X.; Zhang, D. Numerical simulation of metal pipe explosion based on ABAQUS. *Int. J. Mech. Res.* **2024**, *13*, 126.
40. Zhang, X.; Zhang, Q.; Huang, X.; Hu, H.; Cao, Y.; Pan, G. Numerical simulation of steel plate under close-range underwater explosion based on CEL method. *Ship Sci. Technol.* **2022**, *44*, 8–11.
41. Mougeotte, C.; Carlucci, P.; Recchia, S.; Ji, H. *Novel Approach to Conducting Blast Load Analyses Using Abaqus/Explicit-CEL*; Technical Report ADA558175; Defense Technical Information Center: Fort Belvoir, VA, USA, 2010.

42. Ben Belgacem, I.; Khochtali, H.; Cheikh, L.; Barhoumi, E.M.; Ben Salem, W. Comparison between two numerical methods SPH/FEM and CEL by numerical simulation of an impacting water jet. In *Advances in Materials, Mechanics and Manufacturing*; Chaari, F., Barkallah, M., Bouguecha, A., Stephen, R., Eds.; Springer International Publishing: Cham, Switzerland, 2020; pp. 50–60. [\[CrossRef\]](#)
43. Ismail, A.F. Validation study for flow over a sphere using CEL method in Abaqus. In Proceedings of the SIMULIA Community Conference, Vienna, Austria, 21–24 May 2013.
44. Ramírez, E.I.; Ruiz, O.; Reyes-Ruiz, C.; Ortiz, A. Preform geometry determination for a connecting rod forging by CEL model in Abaqus™. *Int. J. Adv. Manuf. Technol.* **2023**, *127*, 5569–5576. [\[CrossRef\]](#)
45. Bakroon, M.; Daryaei, R.; Aubram, D.; Rackwitz, F. Arbitrary Lagrangian-Eulerian finite element formulations applied to geotechnical problems. *Numer. Methods Geotech.* **2017**, *41*, 33–44.
46. Zheng, J.; Hossain, M.S.; Wang, D. 3D large deformation FE analysis of spudcan foundations on layered clays using CEL approach. In *Constitutive Modeling of Geomaterials*; Yang, Q., Zhang, J.M., Zheng, H., Zhang, Z., Eds.; Springer: Berlin/Heidelberg, Germany, 2013; pp. 803–810. [\[CrossRef\]](#)
47. Chen, Y.; Ji, S.; Wu, X.; An, H.Y.; Zhu, H.T.; Shen, D.G.; Lin, W.L. Simulation of brain mass effect with an arbitrary Lagrangian and Eulerian FEM. In *Medical Image Computing and Computer-Assisted Intervention—MICCAI 2010*; Lecture Notes in Computer Science; Jiang, T., Navab, N., Pluim, J.P.W., Robert, N., Eds.; Springer: Berlin/Heidelberg, Germany, 2010; Volume 6362, pp. 274–281. [\[CrossRef\]](#) [\[PubMed\]](#)
48. Karlsson, T.; Olsson, L.G.; Gustavsson, J.; Arborelius, U.P.; Risling, M.; Günther, M. An Experimental Cold Gas Cannon for the Study of Porcine Lung Contusion and Behind Armor Blunt Trauma. *Ann. Biomed. Eng.* **2023**, *51*, 2762–2771. [\[CrossRef\]](#) [\[PubMed\]](#)
49. Zheng, W.; Zhang, X.B. Modeling and optimization of the entire process of pneumatic catapult for small folding tandem-wing unmanned aerial vehicles. *Aerosp. Sci. Technol.* **2026**, *176*, 112118. [\[CrossRef\]](#)
50. Yang, G.; Cheng, W.; Yuanbo, S. Design, fabrication, and experimental study of a full-scale compressed air ejection system based on missile acceleration limitation. *Aerosp. Sci. Technol.* **2023**, *141*, 108519. [\[CrossRef\]](#)
51. Lee, C.F.; Tsai, J.C.; Kuo, C.H. Multi-physical modeling and analysis on UAV launching system. *Adv. Mech. Eng.* **2024**, *16*, 1–18. [\[CrossRef\]](#)
52. Liu, X.; Jian, X.; Xiao, Y.; Liu, H.; Zhang, X.; Xu, Y.; Chen, H. Research progress on pneumatic catapult systems. *Acta Aeronaut. Astronaut. Sin.* **2024**, *45*, 30123. [\[CrossRef\]](#)
53. Fenghour, A.; Wakeham, W.; Wa, W. The viscosity of carbon dioxide. *J. Phys. Chem. Ref. Data* **1998**, *27*, 31–44. [\[CrossRef\]](#)
54. Moradi, A.; Ahmadikia, H. One-dimensional and axisymmetric numerical simulation of a single-stage gas gun. *Adv. Theor. Appl. Mech.* **2011**, *4*, 101–111.
55. Muller, J.; Rulière, R.; Ruyer, P.; Clausse, M. Pressure transient and vaporization process following the rapid heating of a liquid Experiments and modelling. *Exp. Therm. Fluid Sci.* **2022**, *137*, 110674. [\[CrossRef\]](#)
56. Raman, S.K.; Kim, H.D. Solutions of supercritical CO₂ flow through a convergent-divergent nozzle with real gas effects. *Int. J. Heat Mass Transf.* **2018**, *116*, 127–135. [\[CrossRef\]](#)
57. Xue, Y.; Wang, L.; Cai, Y.; Liu, J.; Ranjith, P.G.; Gao, F.; Xie, H.; Liu, Y. Effects of heating–cooling on the mechanical properties and damage evolution of granite for underground cryogenic storage. *Acta Geotech.* **2026**. [\[CrossRef\]](#)
58. Teng, T.; Wang, Y.; Ren, C.; Chen, Y. Modelling and simulation on anisotropic non-Darcy seepage characteristics of gas in naturally fractured coal. *Geomech. Energy Environ.* **2026**, *46*, 100828. [\[CrossRef\]](#)
59. Aursand, E.; Aursand, P.; Berstad, T.; Dørum, C.; Hammer, M.; Munkejord, S.T.; Nordhagen, H.O. CO₂ pipeline integrity: A coupled fluid-structure model using a reference equation of state for CO₂. *Energy Procedia* **2013**, *37*, 3113–3122. [\[CrossRef\]](#)

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.