

Article

Artificial-Lift System Control: A Reduced-Order Transient Model for Real-Time Predictive Control of Electrical Submersible Pump Wells

Mikhail Petrushin ^{1,*}, Efim Kherson ², Nikita Smirnov ¹, Evgeniy Yudin ², Shadfar Davoodi ^{3,4,*}
and Viktoriia Gorbacheva ⁵

¹ FSBIS Institute of Problems of Mechanical Engineering, Saint-Petersburg 199178, Russia; smirnov.nandr@gmail.com

² Independent Researcher, Saint-Petersburg 191181, Russia; kherson.fima@yandex.ru (E.K.)

³ School of Earth Sciences & Engineering, Tomsk Polytechnic University, Lenin Avenue, Tomsk 634050, Russia

⁴ Applied AI Institute, Moscow, Russia

⁵ Center for Technological Development of the Fuel and Energy Complex, Ufa State Petroleum Technological University, Ufa 450078, Republic of Bashkortostan, Russia; gorbacevaviktor4@gmail.com

* Correspondence: petrushinmikhail@yandex.com (M.P.); davoodis@hw.tpu.ru (S.D.)

Abstract

Unstable well operations, driven by reservoir depletion, high gas–oil ratios, unstable inflow, and surface network interactions, have become a major challenge in modern production — especially in Western Siberian fields — causing flow instabilities and production losses. While numerous studies have advanced transient modeling for field optimization, their practical application remains largely limited to recommendation systems running on hourly or daily cycles, making recommendations irrelevant by the time they are applied. At the opposite end, PLCs (programmable logic controllers) relying on PID (proportional–integral–derivative) control cannot solve the main problem: determining the structure of the intermittent cycle. This work proposes a reduced-order transient model of the coupled “reservoir–tubing–annulus” system that captures the essential behavior of transient multiphase flow while remaining compact enough for on-edge real-time model predictive control. Constrained optimization algorithms built on this model provide autonomous closed-loop well control, ensuring equipment and reservoir limits are respected and enabling safe, smooth mode transitions. The solution was validated against high-fidelity simulations and real operating data from Western Siberian fields, demonstrating reliable intermittent ESP (electrical submersible pump) well operation, robust response to rapid operational changes, and effective disturbance rejection.

Keywords: model predictive control; edge-of-stability operation; modeling and simulation; unstable well stock; intermittent well operations; production optimization



Academic Editor: Qingbang Meng

Received: 26 June 2026

Revised: 29 July 2026

Accepted: 3 August 2026

Published: 5 August 2026

Copyright: © 2026 by the authors.

Licensee MDPI, Basel, Switzerland.

This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

1. Introduction

The growing share of unstable well operations has become a central challenge in modern well stock management. In West Siberian oil fields, reservoir depletion, elevated gas–oil ratios, unstable inflow, and complex surface network interactions combine to produce frequent flow instabilities and production losses [1]. As oil and gas production increasingly moves toward unconventional and hard-to-recover resources [2,3], poor reservoir properties are becoming more common. Such properties cause a fast inflow decline after

start-up, while a high free-gas fraction at the pump intake degrades the head–capacity performance and can trigger “gas lock” [4,5]. Together, these factors destabilize ESP operation, producing repeated loss of pump delivery, intra-shift downtime, and associated production losses [4,5].

Three conventional responses to this problem are available: replacing the pump with a smaller unit or redesigning the installation [6–10]; adjusting the operating point of the installed pump by reducing shaft frequency and/or applying wellhead choking [7,11,12]; or *converting the well to an intermittent production mode* [13]. The last option is our focus. It allows the existing higher-capacity ESP to stay in service, now operated “periodically” with alternating “work/idle” intervals. Modern VSD/VFD control stations support such cycles as a standard scheduled on/off function [14,15]. Physically, the cycle acts in two ways: under insufficient inflow, production draws additional fluid from the annulus, which is recovered during the accumulation period; under elevated gas content, a higher liquid column is maintained above the intake, effectively “compressing” the produced fluid. This also tends to be economically favorable [13], since higher-capacity pumps are more efficient than low-rate units and no pump replacement is required.

In artificial-lift practice, the simple on/off cycle is known as “automatic reclosure” (AR) or “periodic short-term start” (PSS). It is a particular case of the more general variable-frequency periodic mode [16], in which different nonzero frequency setpoints are prescribed over different segments of the cycle; in AR/PSS, the lower setpoint always equals zero. Variable-frequency cycling is used as an alternative to AR when full shut-in is not feasible — for example, with compromised check valves — and the pump instead runs at a reduced non-zero frequency during accumulation. From a control-automation standpoint, intermittent production is therefore not merely a relay-type switch between “on” and “off” states but a piecewise-defined rotational-speed control law, in which separate tuning parameters (including separate PID loops) may apply to the production and accumulation segments [17]. During production, phase, controller is tuned for stable delivery, required head, and acceptable motor load; during accumulation, it is tuned for a safe low-flow state with constraints on overheating, oscillations, and power consumption. Intermittent operation thus constitutes a control tool for the coupled “reservoir–well–surface” system under low inflow and high gas content.

The problem addressed in this work sits strictly at that operational level: given an already-installed pump in an already-drilled well, how should the cyclic program be chosen and continuously revised so that the unit produces as much liquid as possible without violating equipment and reservoir limits. The reservoir-engineering question of what to do with these wells in the longer run is outside our scope. A key feature of the resulting regime is that intermittent operation runs near the “ESP stall” limit — a quasi-stationary edge-of-instability regime [18] in which the pump head curve and the system resistance curve approach tangency, formally expressed in the flow-rate existence condition derived later in (30); useful production is obtained precisely from the excursions that a conventional stabilizing controller would suppress.

Challenges of this kind are conventionally addressed with detailed numerical and computational models of the underlying subsurface and downhole processes [19,20], and the management of intermittent wells is no exception; the transient modeling of ESP wells accordingly has a substantial lineage. Quasi-steady nodal-analysis approaches [21,22], rooted in Brown and Lea [23] and Burakov et al. [24], remain the backbone of engineering practice. Dynamic formulations targeted specifically at the intermittent regime were proposed in Yudin et al. [25] and further refined for group optimization [26] and long-horizon asymptotic analysis [27]. This body of work has established that sufficiently accurate

transient simulation of a periodic ESP well is achievable; the remaining question is how to use such simulation *within the real-time closed-feedback control loop*.

This is where existing solutions fall short. Recommendation systems and automated optimization pipelines built on top of high-fidelity transient models [25,26,28] typically run on hourly to daily cycles, produce a set-point suggestion, and rely on manual or semi-manual dispatch to the field. The phenomena they are meant to mitigate—ESP stall, slug arrival, a sudden change in inflow composition—evolve on the order of seconds to minutes; by the time the recommended cycle parameters reach the drive, they are no longer relevant. In contrast, the programmable logic controllers (PLCs) wired to the drive, and the proportional–integral–derivative (PID) loops they host, operate on the right time scale but carry no representation of the underlying multiphase physics. They can protect equipment against an instantaneous violation, but they cannot decide whether a cycle as a whole will converge to a stable regime, stall within a few cycles, or drift into a sub-optimal steady state. Neither tier alone is adequate for the edge-of-instability operation that intermittent wells require.

In parallel, international practice has developed automatic control approaches for ESP-lifted wells, predominantly based on Model Predictive Control (MPC) [29,30]. Specifically, Sharma and Glemmestad [31] developed a nonlinear steady-state optimizer coupled with PI controllers for a multi-well ESP field, focusing on power minimization and separator capacity allocation, while Pavlov et al. [32] derived a simple two-control-volume hydraulic model of continuous ESP production and demonstrated linear MPC for intake-pressure setpoint tracking in a full-scale test facility; Krishnamoorthy et al. [33] subsequently extended the robustness analysis of the same control strategy to a high-fidelity simulator incorporating heavy-oil PVT properties and emulsion viscosity effects, showing adequate performance under watercut variations and choke nonlinearities. More recently, Santana et al. [34] embedded both nonlinear and robust infinite-horizon MPC formulations on a microcontroller with zone control that directly enforces the ESP operational envelope, demonstrating real-time feasibility on edge-class hardware. All three lines of work, however, address *continuous* production regimes with intake-pressure or power-consumption objectives and rely on linearized or empirical models identified around a steady operating point. The present work differs in both the model and the control law: it derives a reduced-order transient model of the coupled reservoir–wellbore system whose closed-form solution admits an analytical per-cycle stability criterion, and embeds this model inside a constrained predictive controller designed specifically for *intermittent* (cyclic on/off) ESP operation on a standard PLC. These distinctions are summarized in Table 1.

Table 1. Comparison of representative automatic-control approaches for ESP-lifted wells against the present work.

Study	Model (Type/Order)	Control Scope	Target Regime	Execution Target
Sharma and Glemmestad [31]	Nonlinear steady-state (algebraic) field model; no transient dynamics	Steady-state optimizer with PI loops; field-wide power minimization and separator-capacity allocation (multi-well)	Continuous	Supervisory real-time optimization layer
Pavlov et al. [32]	Low-order (two-control-volume) linearized hydraulic model	Linear MPC; intake-pressure setpoint tracking (single well)	Continuous	Full-scale test facility
Krishnamoorthy et al. [33]	Same low-order model; robustness study on a high-fidelity simulator	Robust linear MPC; intake-pressure tracking under watercut and choke variation	Continuous	High-fidelity simulator

Table 1. Cont.

Study	Model (Type/Order)	Control Scope	Target Regime	Execution Target
Santana et al. [34]	Nonlinear/robust model identified around a fixed operating point	Nonlinear and robust infinite-horizon MPC with zone control enforcing the ESP envelope	Continuous	Microcontroller (edge, hardware-in-the-loop)
This work	Reduced-order <i>transient</i> coupled reservoir–wellbore model; closed-form per-cycle solution with an analytical stability criterion	Constrained per-cycle predictive control that optimizes the cyclic on/off structure (F_{work} , T_{work} , T_{idle}) for a cycle-averaged rate target under an intake-pressure constraint	Intermittent (cyclic on/off, AR/PSS)	Standard PLC (edge, no cloud)

Consequently, two complementary limitations remain in the existing literature. Cloud-level simulation pipelines and recommendation systems built on high-fidelity transient models [25,26,28] accurately capture the multiphase physics of intermittent ESP operation. However, their hourly-to-daily update cycles cannot respond to the second-to-minute dynamics that determine whether a given on/off program will converge, stall, or drift. In contrast, edge-deployable MPC formulations [31–34] provide the closed-loop response and PLC-class computational footprint required for field deployment. However, existing formulations are primarily designed for continuous, steady-state production regime. Their underlying models assume a fixed operating point, the objective functions penalize time-averaged pressure or power deviations, and they do not explicitly evaluate cycle-by-cycle stability or optimize the discrete on/off structure that characterizes intermittent operation.

Together, these limitations motivate the development of a real-time, physics-informed predictive controller specifically designed for cyclic on/off ESP operation and capable of running on standard field hardware without cloud connectivity. To address this need, the present study proposes an on-site, edge-level model predictive controller (MPC) that bridges these two research directions by combining physics-based predictive capability with real-time, field-deployable control for cyclic on/off ESP wells.

2. Process Description and Control Problem Formulation

2.1. Periodic Well Operation as a Control Object

From a control-object perspective, the intermittent (periodic) operation of an oil well is defined as a repeating two-phase cyclic process consisting of an active “production” (“work”) phase, during which the ESP pumps fluid to the surface, followed by an “accumulation” (“idle”) phase, during which the equipment operates at reduced intensity, allowing reservoir inflow to restore the fluid column in the wellbore (Figure 1).

Let the k -th cycle start at t_k (1b) and consist of two consecutive phases of durations $T_{1,k}$ and $T_{2,k}$, with piecewise-constant control intensities $F_{1,k}$ and $F_{2,k}$ applied on the respective phases. Denoting the total cycle length (1a) and the local (in-cycle) time (1c):

$$\begin{cases} T_{\text{total},k} = T_{1,k} + T_{2,k}, & (1a) \\ t_{k+1} = t_k + T_{\text{total},k}, & (1b) \\ \tau = t - t_k, \quad \tau \in [0, T_{\text{total},k}), & (1c) \end{cases}$$

the phase-switching signal is

$$\sigma_k(\tau) = \begin{cases} 1 \text{ (work)}, & \tau \in [0, T_{1,k}), \\ 2 \text{ (idle)}, & \tau \in [T_{1,k}, T_{\text{total},k}), \end{cases} \quad (2)$$

and the in-cycle control takes the piecewise-constant form (3).

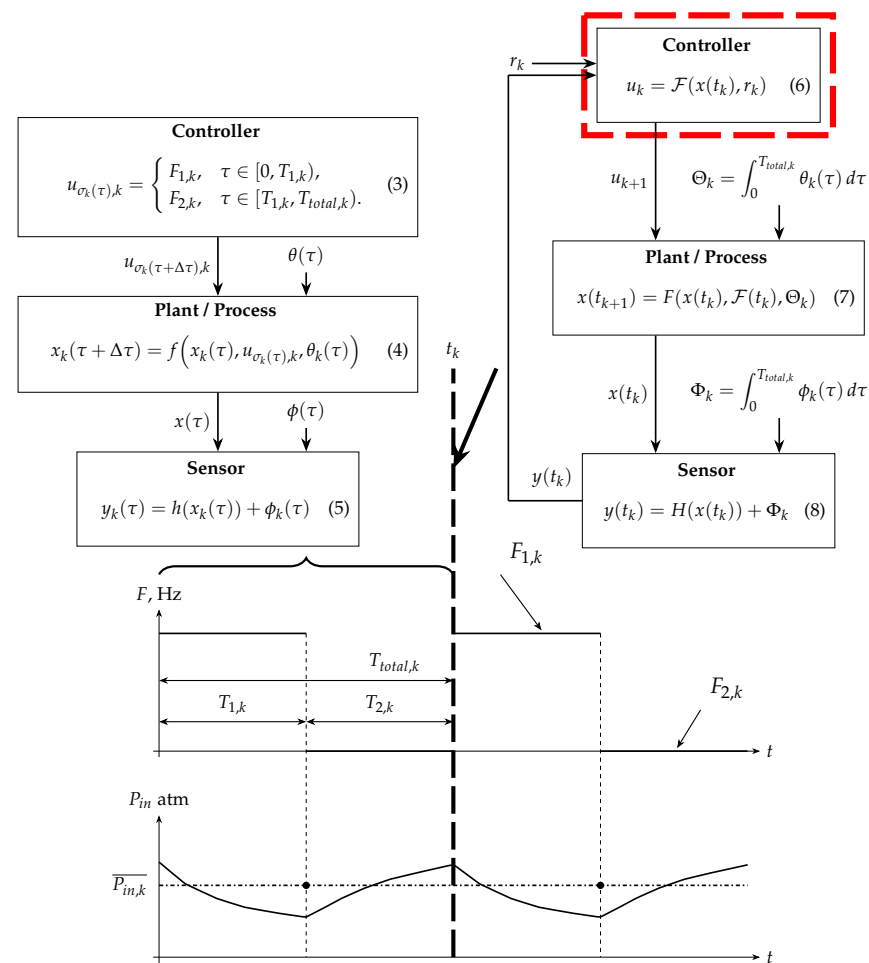


Figure 1. Intermittent well control scheme. (Left): intra-cycle open-loop dynamics — Controller, Plant/Process, and Sensor blocks with disturbance and noise inputs. (Right): per-cycle closed-loop structure (Controller block highlighted in red as the object of this study) receiving the reference r_k and cycle-boundary measurement $y(t_k)$ and issuing the updated control vector u_{k+1} . The vertical dashed line marks the cycle boundary t_k at which the per-cycle law is applied. (Bottom panels): typical time profiles over two consecutive cycles — piecewise-constant ESP shaft frequency F (work phase $T_{1,k}$ at $F_{1,k}$, idle phase $T_{2,k}$ at $F_{2,k}$) and the resulting intake-pressure response $P_{in}(t)$ with its cycle-averaged value $\overline{P_{in,k}}$.

The cycle parameters ($F_{1,k}$, $F_{2,k}$, $T_{1,k}$, $T_{2,k}$) are frozen within a cycle and may be updated only at cycle boundaries t_k . This defines the cycle-wise control vector:

$$u_k = \begin{bmatrix} F_{1,k} & F_{2,k} & T_{1,k} & T_{2,k} \end{bmatrix}^\top. \quad (9)$$

The resulting control object is therefore inherently multistage, acting on two separate time horizons: an intra-cycle horizon with open-loop phase-dependent control, and a “per-cycle” horizon with a discrete parameter update at the boundary.

By definition, intermittent well operation is applied when reservoir inflow is insufficient to sustain continuous operation: each cycle deliberately follows a non-stationary trajectory toward the “ESP stall” boundary, and the pump is shut down precisely before stall occurs, accumulating inflow during the idle phase and delivering it in the next work phase. **The regime is therefore characterized as *quasistationary edge-of-instability operation*** — a mode that a conventional stabilizing controller avoids entirely, seeking instead a

stable continuous-operation point at a low frequency where the deliverable liquid rate is negligibly small.

Let $x(t) \in \mathbb{R}^n$ denote the internal state (pressures, temperatures, PVT properties, flow rates, and electrical parameters), and let $y(t) \in \mathbb{R}^m$ denote the vector of available measurements (a subset of $x(t)$ — pressures, temperatures, and motor electrical signals). The process evolves under uncontrolled disturbances $\theta(t) \in \mathbb{R}^q$ (e.g., water cut, gas content, variations in reservoir productivity) and measurement noise $\phi(t) \in \mathbb{R}^p$. Within a cycle, the dynamics are described by (4) and (5), while the per-cycle functions (7) and (8) capture the net effect of an entire cycle at its boundary and thereby define the “per-cycle” dynamics on which u_k acts.

2.2. Control Objective

Because the process is intrinsically non-stationary inside a cycle, the control objective is not formulated as stabilization of an instantaneous output. Instead, control quality is defined as a “cycle-averaged” (“cycle-mean”) performance index computed from $y(t)$. Selecting a controlled scalar output $z(t) = c^\top y(t)$, the natural target is the cycle mean

$$\bar{z}_k = \frac{1}{T_{\text{total},k}} \int_0^{T_{\text{total},k}} z(\tau) d\tau, \quad (10)$$

which is required to track a setpoint $z_k^* = c^\top r_k$ within a prescribed tolerance ε :

$$|\bar{z}_k - z_k^*| < \varepsilon. \quad (11)$$

In some formulations of this control problem, “phase-averaged” (“phase-mean”) values are used instead:

$$\begin{cases} \bar{z}_{1,k} = \frac{1}{T_{1,k}} \int_0^{T_{1,k}} z(\tau) d\tau, \\ \bar{z}_{2,k} = \frac{1}{T_{2,k}} \int_{T_{1,k}}^{T_{1,k}+T_{2,k}} z(\tau) d\tau. \end{cases} \quad (12)$$

The same criterion extends naturally to a multivariable setting through a weighted norm of the cycle-boundary error $e(t_k) = r_k - y(t_k)$:

$$\|e(t_k)\|_Q = \sqrt{\sum_{i=1}^m q_i (y_i(t_k) - r_{i,k})^2} < \epsilon, \quad (13)$$

where ϵ is the allowed deviation in a multidimensional state space and q_i is the weight of the i -th component.

2.3. Constraints

Cycle constraints are defined as phase-dependent trajectory constraints that reflect the different safety and technological limits of the “work” and “idle” phases:

$$\begin{cases} g_1(y(\tau)) \leq 0, & \tau \in [0, T_{1,k}), \\ g_2(y(\tau)) \leq 0, & \tau \in [T_{1,k}, T_{\text{total},k}), \end{cases} \quad (14)$$

$$\begin{cases} y(\tau) \in \mathcal{Y}_1, & \tau \in [0, T_{1,k}), \\ y(\tau) \in \mathcal{Y}_2, & \tau \in [T_{1,k}, T_{\text{total},k}). \end{cases} \quad (15)$$

Additionally, the cycle parameters themselves are bounded by equipment capability and by the maximum admissible “per-cycle” variation:

$$\begin{cases} u_{\min} \leq u_k \leq u_{\max}, \\ \Delta u_{\min} \leq u_{k+1} - u_k \leq \Delta u_{\max}. \end{cases} \quad (16a)$$

$$(16b)$$

2.4. Control Problem Statement

In the context of the “multi-horizon” process described above, the control decision at the cycle boundary t_k consists of selecting the next cycle parameter vector u_k based on the currently available state estimate $x(t_k)$ and the reference r_k . Formally, we seek a per-cycle control law (6) such that the closed-loop system (4) and (5) and (7) and (8) satisfies the tracking condition (13) while respecting the phase-dependent trajectory constraints (14) and (15) and the input bounds (16) — *determine the function $\mathcal{F}$* .

2.5. Scope and Simplifying Assumptions

Treating (6) in full form — with four decision variables per cycle and arbitrary disturbances — would lead to a scope too broad for a single study and would obscure the objective of formulating the fundamental “per-cycle” principles of intermittent well control. We therefore narrow the scope, for the sake of practicality, to the most relevant and widely used forms of intermittent operation with a reduced set of controlled variables, namely “periodic short-term start” (PSS) and “automatic reclosure” (AR):

Assumption 1. *The actuation during the accumulation phase vanishes,*

$$F_{2,k} = \text{const.} = 0, \quad (17)$$

so that phase 2 is a purely passive “buildup” phase.

Assumption 2. *No phase-local PID loop is active; cycle parameters are the sole control degrees of freedom.*

Assumption 3. *Disturbances $\theta(t)$ and measurement noise $\phi(t)$ are small relative to the characteristic scale of the dominant technological variables and therefore do not govern the qualitative per-cycle behavior.*

Under Assumptions 1–3 the cycle-wise control vector (9) reduces to

$$u_k = \begin{bmatrix} F_{1,k} & T_{1,k} & T_{2,k} \end{bmatrix}^T. \quad (18)$$

From now on, we use the following notation interchangeably: $F_{1,k} \equiv F_{\text{work},k} \equiv F_{\text{work}}$; $T_{1,k} \equiv T_{\text{work},k} \equiv T_{\text{work}}$; $T_{2,k} \equiv T_{\text{idle},k} \equiv T_{\text{idle}}$.

Scope of the Small-Disturbance Assumption 3

The assumption is understood to apply to intra-cycle fluctuations about the current operating conditions, not to the slow evolution of those conditions themselves. The technological quantities that drive that evolution (GOR, water cut, and the associated PVT properties) change over characteristic times of weeks to months, whereas the per-cycle control decision and the three-cycle identification window operate on timescales of seconds to minutes (hours — maximum). Over the horizon relevant to a single decision they are effectively constant, and their slow drift across successive cycles is absorbed at every cycle boundary through re-identification of the model coefficients from fresh measurements.

Rapid, discrete events that genuinely violate 3 — slug arrivals, water and gas breakthrough — fall outside the present scope; their treatment, together with the robustness of the controller to larger disturbances and to measurement noise, is discussed in Section 7.2.

3. Reduced-Order Transient Model for an ESP Well

We begin with the definition of the intake-pressure derivative obtained in Yudin et al. [25] (Appendix A, Equation (19)) based on the balance of fluid flows (Figure 2) under conditions at the pump intake (which serves as the connecting node of the “reservoir–tubing–gathering network” system). It can be written in compact form as

$$\frac{\partial P_{in}}{\partial t} = \left[Q_{f \text{ tubing}}(P_{in}, T_{in}) - Q_{f \text{ casing}}(P_{in}, T_{in}) \right] \cdot A, \quad (19)$$

where A is a compound coefficient that translates the net flow imbalance ($Q_{f \text{ tubing}} - Q_{f \text{ casing}}$) into the rate of intake-pressure change — in effect, the pressure sensitivity of the annular volume — encapsulating the effects of annular geometry, fluid properties, and thermodynamic (PVT) corrections. It is determined by the wellbore geometry, the average phase densities in the annulus, and a PVT rescaling factor between downhole and reference conditions, and is expressed as

$$A = \frac{g(\bar{\rho}_{gas}^{ann} - \bar{\rho}_{liquid}^{ann})}{S_{ann}} \cdot \frac{D_{ann}(\bar{P}_{ann}, \bar{T}_{ann})}{D_{ann}(P_{in}, T_{in})}, \quad (20)$$

where the D coefficient is described in Yudin et al. [25] (Appendix A, Equations (3)–(5)).

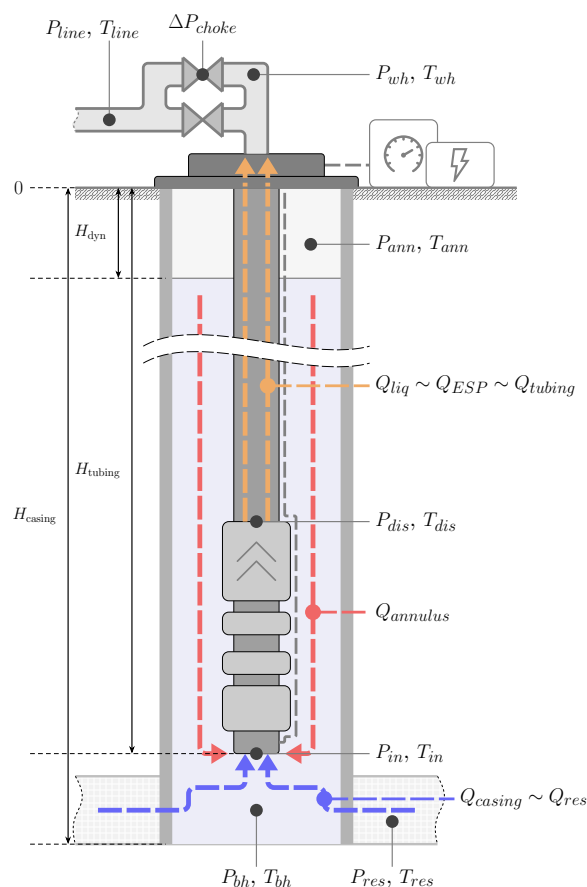


Figure 2. ESP Well Scheme.

3.1. Inflow Approximation

The well inflow function $Q_{f \text{ casing}}$ in (19) is commonly modeled using the equations of steady-state planar-radial flow in the near-wellbore zone [35] (Equation (43)). While this provides a sufficiently detailed description for the problem at hand, it also prevents the explicit derivation of the functional form itself. We therefore begin by simplifying this term. We replace it with a linear inflow relationship (function of “drawdown”) [36] (Equation (1.31)):

$$Q_{ipr} = K \cdot (P_{res} - P_{bh}), \quad (21)$$

where K is the well productivity index, P_{res} the reservoir boundary pressure, and P_{bh} the bottomhole pressure. The justification for this linear approximation is established in Golan and Whitson [36] (pp. 29–33), where the linear IPR is shown to provide the accuracy typically required for engineering calculations of ESP-lifted wells.

To eliminate P_{bh} in favor of the intake pressure P_{in} , we use the quasi-steady hydrostatic balance in the casing section between the pump intake and the bottomhole:

$$\begin{cases} P_{bh} = P_{in} + \rho gh + \Delta P_{fric}^{casing} \approx P_{in} + \rho gh, & (22a) \\ \Delta P_{fric}^{casing} \ll P_{in} + \rho \cdot g \cdot h, & (22b) \end{cases}$$

which neglects the frictional and kinetic components (ΔP_{fric}^{casing}) of the pressure gradient along this interval, see (22b). This simplification reflects a *key trade-off between speed and accuracy*: keeping these terms would require evaluating PVT properties and flow regimes along the entire intake-to-perforations interval, which is impractical for real-time use. For real-time control of intermittent operation, where the delay must stay below the cycle length, it is acceptable to keep only the correct qualitative behavior of the process at a low computational cost, even if this introduces a predictable bias in the absolute values.

Since, in a fully shut-in well, the bottomhole pressure tends to the reservoir pressure, P_{res} may be represented through a “maximum” intake pressure P_{in}^{max} corresponding to complete pressure build-up:

$$P_{res} = P_{in}^{max} + \rho gh, \quad (23)$$

so that, for a given reservoir pressure and given (P, T) conditions on the interval, $P_{in}^{max} = P_{res} - \bar{\rho}_{P,T} \cdot gh$ is uniquely determined, where $\bar{\rho}_{P,T}$ is the average mixture density along the interval and h the corresponding true vertical depth.

Substituting (22a) and (23) into (21) cancels the gravitational terms and yields an explicit linear expression for the casing-side flow in terms of the sought state variable:

$$Q_{f \text{ casing}} \equiv Q_{ipr} = K \cdot (P_{in}^{max} - P_{in}). \quad (24)$$

3.2. ESP Characteristic Approximation

In the quasi-stationary mass balance, the pressure increase delivered by the pump can be written (see Appendix A.1, Equation (A8)) as

$$\Delta P_{esp} = \rho_{mix,i} \cdot g \cdot k_{degr} \cdot N \cdot H_{esp}(Q_{mix}, F), \quad (25)$$

where k_{degr} is the composite ESP degradation factor (see Equation (A7)) accounting for wear, free-gas influence [37] and viscosity corrections [38], N is the number of pump stages, and H_{esp} is the known pump characteristic curve rescaled to the actual shaft frequency via the affinity laws [39] (pp. 27–30, pp. 34–36).

Equation (25) can be recast as the tubing-column pressure balance (see Appendix A.1, Equation (A17)):

$$\begin{cases} k_{\text{degr}} \cdot N \cdot H_{\text{esp}}(Q_{\text{mix}}, F) = \frac{P_{\text{wh}} - P_{\text{in}}}{\rho_{\text{mix}} \cdot g} + h_{\text{tubing}}, & (26a) \\ (P_{\text{wh}} - P_{\text{in}}) + \rho_{\text{mix}} \cdot g \cdot h_{\text{tubing}} \geq 0. & (26b) \end{cases}$$

Using interpolation to calculate H_{esp} requires a numerical search for the operating point at every integration step, and this is the main obstacle to a direct analytical solution of (26).

To eliminate the numerical interpolation dependency, we approximate the relevant portion of the head–rate characteristic by a quadratic polynomial,

$$H_{\text{esp}}(Q_{\text{liq}}) = a \cdot Q_{\text{liq}}^2 + b \cdot Q_{\text{liq}} + c, \quad (27)$$

whose coefficients a , b , c are identified from manufacturer data for each specific pump characteristic curve by the least-squares method. The justification for the quadratic fit over the operating range of centrifugal pumps is established in Ulanicki et al. [40] (p. 89), where this functional form is shown to reproduce both head and power characteristics of centrifugal units with an accuracy sufficient for modeling and regime optimization.

Applying the affinity laws for frequency rescaling and the standard corrections for free-gas influence, we rewrite (27) as a quadratic equation for the flow rate at the current shaft frequency F_i , with coefficients explicitly depending on F_i , F_{nom} and k_{degr} (see Appendix A.2). Substituting this expression into the tubing pressure balance and neglecting tubing friction losses — under the same speed/accuracy trade-off argument as in (22b) — yields a quadratic equation for the liquid flow rate (A25) that is pumped through the ESP and tubing at time t_i :

$$\mathcal{A} \cdot Q_{F_i}^2 + \mathcal{B} \cdot Q_{F_i} + \mathcal{C} = 0, \quad (28a)$$

$$\mathcal{A} = a \cdot k_{\text{degr}}, \quad (28b)$$

$$\mathcal{B} = b \cdot k_{\text{degr}} \cdot \frac{F_{\text{nom}}}{F_i}, \quad (28c)$$

$$\mathcal{C} = c \cdot \left(\frac{F_i}{F_{\text{nom}}} \right)^2 \cdot k_{\text{degr}} - \frac{P_{\text{wh}} - P_{\text{in}}}{\rho_{\text{mix}} \cdot g \cdot N} - \frac{h_{\text{tubing}}}{N}. \quad (28d)$$

The coefficients $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ depend on the approximation parameters of (27), the shaft frequency, the tubing-side mixture density, the pump setting depth, the number of stages, and the current values of P_{in} and P_{wh} .

Solving (28a) gives:

$$Q_{F_i} = \frac{-\mathcal{B} \pm \sqrt{\mathcal{B}^2 - 4\mathcal{A}\mathcal{C}}}{2\mathcal{A}}. \quad (29)$$

Out of the two roots, the one corresponding to the stable branch of the pump operating curve must be selected; this is the larger of the two roots (see Figure 3), as discussed in Yudin et al. [25] (p. 37, Appendix A, Equation (20), Figure 3): the first intersection lies on the increasing part of the head curve and represents an extreme, unstable operating mode, while the second lies on the decreasing part and represents the actual well operating mode. This gives a unique answer for the tubing-side flow Q_{vlp} :

$$\begin{cases} Q_{f \text{ tubing}} \equiv Q_{vlp} = \max \left(\frac{-B + \sqrt{B^2 - 4AC}}{2A}; \frac{-B - \sqrt{B^2 - 4AC}}{2A} \right), & (30a) \\ B^2 - 4AC \geq 0. & (30b) \end{cases}$$

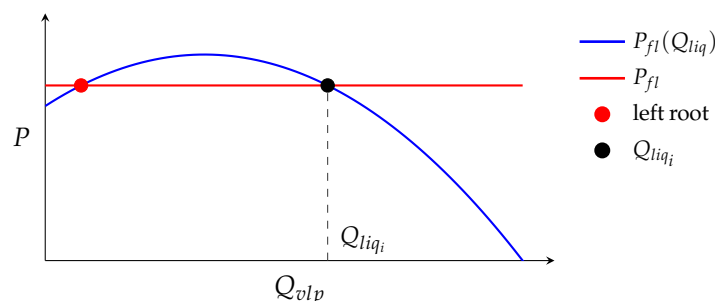


Figure 3. Visualization of the physically correct root for the tubing flow.

The non-negativity of the discriminant plays the role of an existence condition for a pump operating regime and can be interpreted as an analytical form of the “ESP stall” condition. Physically, a negative discriminant ($B^2 - 4AC < 0$) means that the pump characteristic curve and the system resistance curve do not intersect: the pump is unable to generate sufficient head to overcome the combined hydrostatic and friction load at any flow rate, and no steady delivery is possible. In a real well this manifests as pump stall — the unit continues to rotate but produces no net upward flow. A related, more complex phenomenon is reverse flow (the “pump-as-turbine” regime), in which the pressure differential across the pump drives fluid downward through the impellers. This regime requires a fundamentally different modeling framework and is outside the scope of the present work; a detailed treatment is given in Petrushin et al. [16]. Within the adopted model, both the stall case and the reverse-flow case are therefore captured by the single condition $B^2 - 4AC < 0$, which flags the absence of a physically admissible operating point on the stable branch of the pump curve. During the accumulation phase, when $F = 0$ Hz, the absence of free-flow through the idle ESP implies $Q_{f \text{ tubing}} = 0$.

3.3. Closed-Form Expression for $P_{in}(t)$

Substituting the explicit expressions (24) and (30) into the intake-pressure balance (19) transforms the original implicit nodal system into a first-order ordinary differential equation with constant coefficients in $P_{in}(t)$:

$$\frac{\partial P_{in}}{\partial t} = [Q_{vlp} - K \cdot (P_{in}^{max} - P_{in})] \cdot A. \quad (31)$$

Integration of (31) yields the general solution

$$P_{in}(t) = C \cdot e^{A \cdot K \cdot t} - \frac{Q_{vlp}}{K} + P_{in}^{max}, \quad (32)$$

with integration constant C determined from the initial condition $P_{in}(0) = P_{in_0}$, i.e., the intake pressure at the start of the considered half-cycle.

Evaluating the integration constant (see Appendix A.3, Equation (A27)) from (32) at $t = 0$ and substituting back yields the closed-form intake-pressure law during the “work” (“drawdown”) phase, when $F > 0$ Hz and $Q_{vlp} > 0$:

$$P_{in}(t) = \left(P_{in_0} + \left(\frac{Q_{vlp}}{K} - P_{in}^{max} \right) \right) \cdot e^{A \cdot K \cdot t} - \left(\frac{Q_{vlp}}{K} - P_{in}^{max} \right). \quad (33)$$

During the “idle” (“buildup”) phase, with $F = 0$ Hz, the no-free-flow assumption gives $Q_{vlp} = 0$ and (33) reduces to an exponential recovery of the intake pressure toward its maximum value:

$$P_{in}(t) = (P_{in_0} - P_{in}^{max}) \cdot e^{A \cdot K \cdot t} + P_{in}^{max}. \quad (34)$$

The pair of expressions (33) and (34) constitutes the reduced-order model of the intake-pressure dynamics under intermittent well operation, referred to in the remainder of this paper as

$$P_{in}(t) = \begin{cases} \left(P_{in_0} + \left(\frac{Q_{vlp}}{K} - P_{in}^{max} \right) \right) e^{AKt} - \left(\frac{Q_{vlp}}{K} - P_{in}^{max} \right), & F > 0 \text{ Hz}, \\ (P_{in_0} - P_{in}^{max}) e^{AKt} + P_{in}^{max}, & F = 0 \text{ Hz}. \end{cases} \quad (35a)$$

$$(35b)$$

Unlike the original equation obtained in Yudin et al. [25] (Equations (1), and (2)), the model (35) requires neither numerical integration of distributed equations nor iterative nodal coupling: the predicted value of $P_{in}(t)$ at an arbitrary instant is obtained from a single exponential evaluation and a finite number of arithmetic operations.

4. Intermittent Mode Stability Criterion

4.1. Cycle-Averaged Intake Pressure

Let $\overline{P}_{in}(N)$ denote the cycle-averaged intake pressure over the N -th “work–idle” cycle:

$$\overline{P}_{in}(N) = \frac{\overline{P}_{in}^{dd}(N) + \overline{P}_{in}^{bd}(N)}{T_{work} + T_{std}}, \quad (36a)$$

$$\overline{P}_{in}^{dd}(N) = \int_{N \cdot T_{total}}^{T_{work} + N \cdot T_{total}} P_{in}^{dd}(t) dt, \quad (36b)$$

$$\overline{P}_{in}^{bd}(N) = \int_{T_{work} + N \cdot T_{total}}^{T_{total} + N \cdot T_{total}} P_{in}^{bd}(t) dt. \quad (36c)$$

where $P_{in}^{dd}(t)$ and $P_{in}^{bd}(t)$ are, respectively, the “drawdown” (35a) and “buildup” (35b) branches of the solution.

The primitive functions of the integrands are obtained by directly integrating the solution (32).

Strictly speaking, the coefficients C , A , Q_{vlp} and P_{in}^{max} vary over time; however, since we are interested in the global trend $\overline{P}_{in}(N)$, they are treated as constant over the N cycles (Figure 4).

The drawdown integral then reads

$$\int P_{in}^{dd}(t) dt = \frac{C}{A \cdot K} \cdot e^{A \cdot K \cdot t} + \left(-\frac{Q_{vlp}}{K} + P_{in}^{max} \right) \cdot t + C_1, \quad (37)$$

or, in compact form,

$$\int P_{in}^{dd}(t) dt = \frac{C}{b_1} \cdot e^{b_1 \cdot t} + b_2 \cdot t + C_1, \quad (38)$$

where C_1 is the integration constant of the drawdown branch, $b_1 = A \cdot K$ and $b_2 = \left(-\frac{Q_{vlp}}{K} + P_{in}^{max} \right)$.

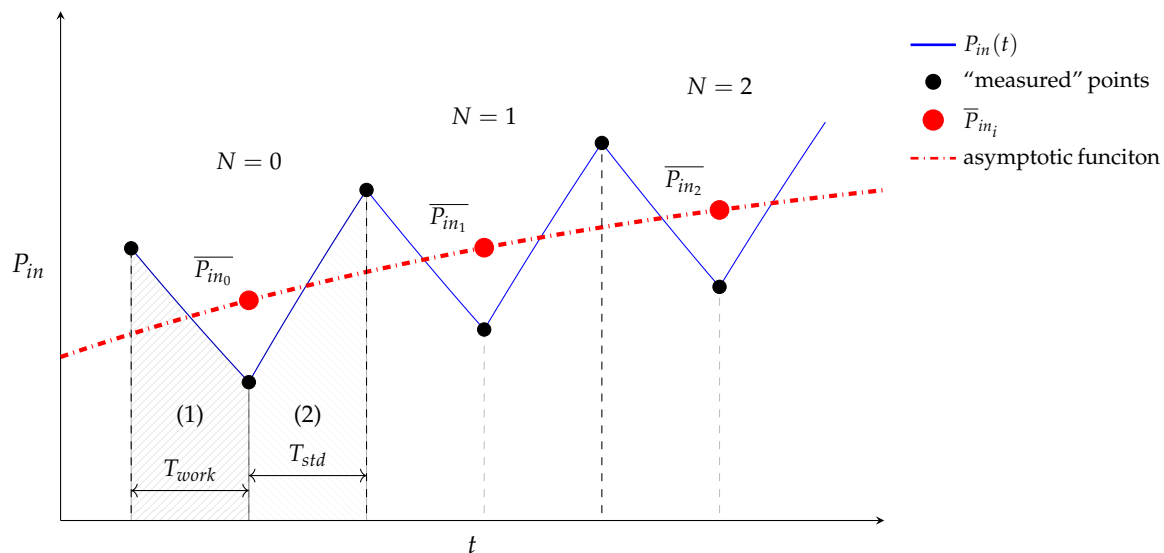


Figure 4. Trend of cycle-averaged intake pressure across “work (1)–idle (2)” cycles.

During the accumulation phase $Q_{vlp} = 0$, hence

$$\int P_{in}^{bd}(t) dt = \frac{C}{b_1} \cdot e^{b_1 \cdot t} + P_{in}^{max} \cdot t + C_2, \quad (39)$$

or, equivalently,

$$\int P_{in}^{bd}(t) dt = \frac{C}{b_1} \cdot e^{b_1 \cdot t} + b_3 \cdot t + C_2, \quad (40)$$

with C_2 the integration constant of the build-up branch and $b_3 = P_{in}^{max}$.

Following the operations described in Appendix B.1, the cycle-averaged intake pressure (the cycle-averaged P_{in} across cycles, not within a cycle) admits the closed form

$$\begin{cases} \overline{P_{in}}(N) = a \cdot e^{b \cdot N} + c, & N \in \{0, 1, 2, 3, \dots\}, \end{cases} \quad (41a)$$

$$\begin{cases} a = \frac{C}{A \cdot K} \cdot \frac{e^{A \cdot K \cdot (T_w + T_s)} - 1}{T_{work} + T_{std}}, \end{cases} \quad (41b)$$

$$\begin{cases} b = A \cdot K \cdot (T_w + T_s), \end{cases} \quad (41c)$$

$$\begin{cases} c = P_{in}^{max} - \frac{Q_{vlp}}{K} \cdot \frac{T_w}{T_{work} + T_{std}}. \end{cases} \quad (41d)$$

It is essential to note that the parameters C , A , K , P_{in}^{max} and Q_{vlp} defined in (41) cannot be evaluated directly, since they represent effective “averaged” characteristics of $P_{in}(t)$ over the N cycles considered. Nevertheless, the relation (41) proves that $\overline{P_{in}}(N)$ has the structural form $a e^{bN} + c$, so the remaining task is only to identify its three coefficients a , b , c .

Based on the specific physics of the process described in Yudin et al. [25] (pp. 15–19), a stable (quasi-stationary in the limit) intermittent regime corresponds to a bounded, monotone trajectory of $\overline{P_{in}}(N)$ converging to an asymptote. This places the following structural constraints on the coefficients (Figure 5):

- $b < 0$: convergence condition, expressing the decay of the transient;
- $a < 0, b < 0$: increasing trajectory of $\overline{P_{in}}$ approaching the steady value c from below;
- $a > 0, b < 0$: decreasing trajectory of $\overline{P_{in}}$ approaching the steady value c from above.

Cases $b \geq 0$ correspond to unbounded growth or decay of $\overline{P_{in}}(N)$ and are therefore incompatible with convergence to a quasi-stationary regime.

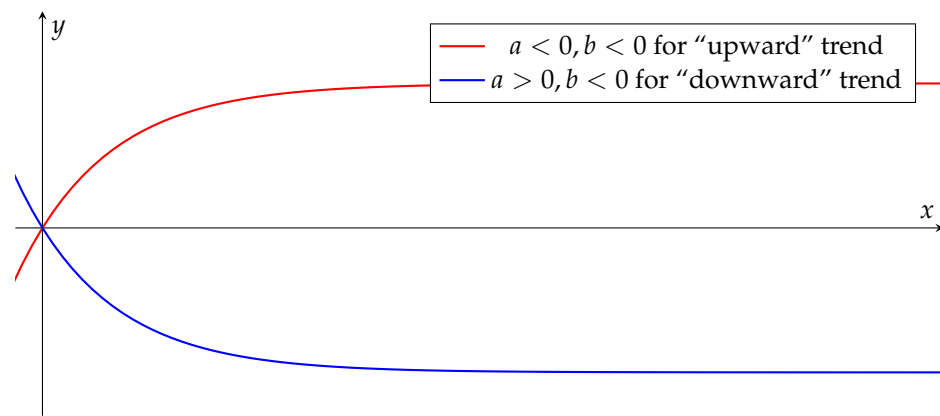


Figure 5. Expected shapes of the cycle-averaged intake pressure $\overline{P}_{in}(N)$.

4.2. Analytical Identification of a , b , c from Three Cycles

Knowing how $\overline{P}_{in}$ evolves over three full “work–idle” cycles at fixed regime parameters ($F_{work} = \text{const}$, $T_{work} = \text{const}$, $T_{std} = \text{const}$) lets us determine the coefficients a , b , c analytically.

Taking the first of the three cycles as $N = 0$, we form the system

$$\begin{cases} \overline{P}_{in0} = a \cdot e^{0 \cdot b} + c, \\ \overline{P}_{in1} = a \cdot e^{1 \cdot b} + c, \\ \overline{P}_{in2} = a \cdot e^{2 \cdot b} + c. \end{cases} \quad (42)$$

Following the manipulations detailed in Appendix B.2, this yields the closed-form solution (A49) for the three coefficients:

$$a = \frac{(\overline{P}_{in1} - \overline{P}_{in0})^2}{\overline{P}_{in2} - 2\overline{P}_{in1} + \overline{P}_{in0}}, \quad (43a)$$

$$b = \ln\left(\frac{\overline{P}_{in2} - \overline{P}_{in0}}{\overline{P}_{in1} - \overline{P}_{in0}} - 1\right), \quad (43b)$$

$$c = \overline{P}_{in0} - a, \quad (43c)$$

$$\overline{P}_{in0} \neq \overline{P}_{in1}, \quad (43d)$$

$$\overline{P}_{in2} - 2\overline{P}_{in1} + \overline{P}_{in0} \neq 0, \quad (43e)$$

$$\frac{\overline{P}_{in2} - \overline{P}_{in0}}{\overline{P}_{in1} - \overline{P}_{in0}} - 1 > 0. \quad (43f)$$

A sign analysis (see Appendix B.3) shows that the physically admissible shapes of $\overline{P}_{in}(N)$ correspond to:

- Increasing trend ($a < 0$, $b < 0$): $\overline{P}_{in0} < \overline{P}_{in1} < \overline{P}_{in2} < 2\overline{P}_{in1} - \overline{P}_{in0}$;
- Decreasing trend ($a > 0$, $b < 0$): $2\overline{P}_{in1} - \overline{P}_{in0} < \overline{P}_{in2} < \overline{P}_{in1} < \overline{P}_{in0}$.

Hence, the analytical identification of the coefficients of (41) requires, and is satisfied by, the observation of at least three consecutive cycles at fixed regime parameters:

$$\begin{cases} N_{\min} &= 3, \\ F_{work} &= \text{const.}, \\ T_{work} &= \text{const.}, \\ T_{std} &= \text{const.} \end{cases} \quad (44)$$

In practice, the triplet $(\overline{P_{in_0}}, \overline{P_{in_1}}, \overline{P_{in_2}})$ is taken either from measurements over three consecutive cycles at fixed $(F_{work}, T_{work}, T_{idle})$, or from model-calculated cycle averages at the candidate parameters.

4.3. Asymptotic Behavior as $N \rightarrow +\infty$

For regimes satisfying $b < 0$, the limit of (41a) as $N \rightarrow +\infty$ is

$$\overline{P_{in}}(+\infty) = \lim_{N \rightarrow +\infty} (a \cdot e^{b \cdot N} + c) = c. \quad (45)$$

The coefficient c therefore coincides with the steady-state value of the cycle-averaged intake pressure under “absolute” quasi-stationary operation. Substituting this value into the expression for c in (41d) yields the corresponding steady-state pump rate

$$\overline{Q_{vlp}}(+\infty) = \frac{T_{work} + T_{std}}{T_{work}} \cdot K \cdot (P_{in}^{max} - \overline{P_{in}}(+\infty)), \quad (46)$$

and the associated daily-averaged liquid rate (using the conversion (A59) of Appendix B.4):

$$\overline{Q_{liq}}(+\infty) = K \cdot (P_{in}^{max} - \overline{P_{in}}(+\infty)). \quad (47)$$

4.4. Stability Criterion

Following the approach of Petrushin et al. [27] (Equation (7)), we adopt the “per-cycle” drop of the cycle-averaged intake pressure as a measure of proximity to the quasi-stationary regime,

$$\Delta \overline{P_{in}}(N) = \overline{P_{in}}(N) - \overline{P_{in}}(N+1). \quad (48)$$

Substituting (41), this drop reduces to the closed form

$$|\Delta \overline{P_{in}}(N)| = \left| a \cdot e^{b \cdot N} \cdot (1 - e^b) \right| \leq \Delta \overline{P_{in}}^{\text{const}}, \quad (49)$$

where $\Delta \overline{P_{in}}^{\text{const}} \geq 0$ is a user-specified threshold that encodes the technological tolerance on regime repeatability; in practice it takes values on the order of 10^1 atm.

Combining the proximity condition (49) on a defined horizon N , the asymptotic value $\overline{P_{in}}(+\infty) = c$ together with the corresponding $\overline{Q_{vlp}}(+\infty)$ from (45), and the technological constraints on intake pressure and production rate introduced in Section 2.3, the stability criterion of an intermittent regime is formulated as

$$\left\{ \begin{array}{l} |a e^{bN} (1 - e^b)| \leq \Delta \overline{P}_{in}^{const}, \end{array} \right. \quad (50a)$$

$$b < 0, \quad (50b)$$

$$\overline{P}_{in}(N) = a e^{bN} + c \geq P_{in_{min}}, \quad (50c)$$

$$\overline{Q}_{liq}(N) = K(P_{in}^{max} - \overline{P}_{in}(N)) \in [Q_{liq}^{min}, Q_{liq}^{max}], \quad (50d)$$

$$\text{as } N \rightarrow +\infty : c \geq P_{in_{min}}, K(P_{in}^{max} - c) \in [Q_{liq}^{min}, Q_{liq}^{max}]. \quad (50e)$$

The criterion (50) is stated as a conditional requirement on a defined horizon N , chosen according to the planning needs of the operator (typical N corresponds to a daily or shift planning horizon). The limit $N \rightarrow +\infty$ is then a consequence: substituting $\overline{P}_{in}(+\infty) = c$ and the corresponding $\overline{Q}_{liq}(+\infty)$ into the technological constraints yields the existence condition for an “absolute” quasi-stationary regime, for which the first inequality is automatically satisfied since its left-hand side vanishes as $N \rightarrow +\infty$ provided $b < 0$.

In contrast with the empirical criterion of Petrushin et al. [27] (Equation (7)), the criterion (50) is fully analytical, needs only three control cycles for its evaluation (condition (44)), and admits analysis as a closed-form function of the regime parameters ($F_{work}, T_{work}, T_{std}$). This makes it possible to embed the criterion in the cycle-to-cycle control law (6).

5. Control Algorithm

The general structure of the adopted MPC loop is shown in Figure 6.

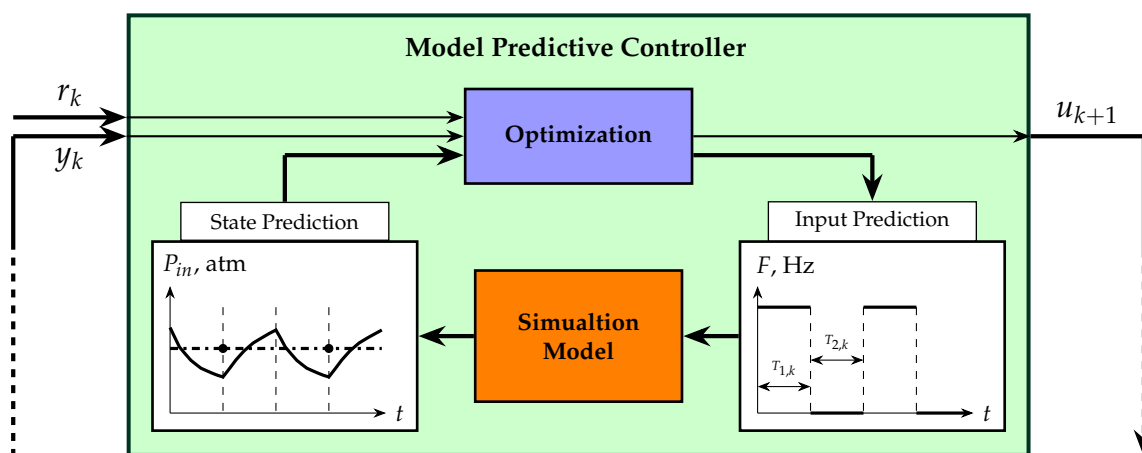


Figure 6. Predictive control loop scheme.

At each cycle boundary t_k the controller receives two external inputs: the production target r_k and the measured cycle-averaged output y_k (intake pressure and liquid rate). The *Optimization* block iterates over the discrete admissible set of candidate control vectors $u \in \mathcal{A}_k$ (54): for every candidate it first constructs the predicted piecewise-constant frequency program (*Input Prediction*, right panel of Figure 6), passes it to the *Simulation Model*, and obtains the corresponding predicted trajectory over the N -cycle horizon (*State Prediction*, left panel of Figure 6). The candidate that minimizes the cost functional (53) subject to the feasibility predicate (55) is selected and its first element is issued as the updated control u_{k+1} , which is applied to the well for the next cycle. The outer feedback loop (dashed arrows) returns the resulting measurements y_{k+1} at the following boundary, and the procedure repeats — realizing the standard receding-horizon principle.

5.1. Prediction Horizon

A specific feature of the present problem is the elementary unit of prediction: the natural “discretization step” is not an arbitrary time interval Δt , but one full cycle of duration $T_{\text{total},k}$ (see Equation (1a)). The prediction horizon N is therefore defined as a number of consecutive cycles, over which the model reconstructs both the in-cycle state trajectory at every local time τ (see Equation (1c)) and the cycle-boundary output.

Evaluation of the stability criterion (50) requires the simulation of three full “work-idle” cycles (condition (44)), which defines the minimum required prediction horizon length:

$$T_{\text{horizon},k} = \sum_{i=1}^N T_{\text{total},k+i}, \quad N = 3. \quad (51)$$

This minimum is assumed as the baseline horizon in what follows and denoted simply by N ; the extension to $N > 3$ is straightforward and does not affect any of the constructions below (it merely increases the number of terms in the sum and the weight matrices).

A by-product of the stability criterion evaluation is the asymptotic value of the cycle-averaged variables (Equation (45)) as $k \rightarrow \infty$, which formally corresponds to an infinite prediction horizon. This makes it possible to define the cost functional in a mixed form, with its main part evaluated on the finite horizon N and the asymptotic indicators incorporated through constraints on the admissible output and the regime stability condition.

5.2. Cost Function

In Section 2.2, the control objective was stated as keeping the cycle-averaged controlled variable $\bar{z}_k$ — or, in the multivariable case, the tracked output vector $y(t_k)$ — close to the reference r_k within a prescribed tolerance. In the predictive setting this requirement extends naturally over the full prediction horizon: the quality of a regime is assessed across the N predicted cycles rather than on a single one.

Let

$$\hat{e}_{k+i|k} = r_{k+i} - \hat{y}_{k+i|k}, \quad i = 1, \dots, N, \quad (52)$$

denote the predicted tracking error at cycle $k + i$, where $\hat{y}_{k+i|k}$ is the prediction of the output vector at the boundary of cycle $k + i$, computed at t_k using the model obtained in the Section 3 for the candidate control vector (18). The notation $(\dots)_{k+i|k}$ reads “prediction for cycle $k + i$ made at time t_k ”; the hat $(\hat{\dots})$ throughout this section indicates a calculated value.

The cost function is defined as a weighted sum of the tracking-error norms over the finite horizon, augmented by an asymptotic penalty:

$$J_k(u_k) = \sum_{i=1}^N \left\| \hat{e}_{k+i|k} \right\|_{Q_i} + \left\| \hat{e}_{\infty|k} \right\|_{Q_{\infty}}, \quad (53)$$

where Q_i and Q_{∞} are weight matrices assigned separately to each cycle i and to the asymptotic term, reflecting the common field practice of discretely tuning the importance of each “sub-horizon” (e.g., preferring an early arrival at the setpoint at $i = 1$ versus a slower but smoother approach at $i = 2, 3$, or trading off asymptotic optimality against mid-term regime quality). $\|\dots\|_Q$ is the weighted norm defined in (13), and $\hat{e}_{\infty|k}$ is the tracking error on the asymptotic regime.

5.3. Constraints

The constraints adopted in the predictive control problem are inherited directly from Section 2.3 and naturally split into two classes: constraints on the control vector and constraints on the state.

The control-vector constraints on the absolute regime parameters and on their “per-cycle” increments are given by (16). They jointly define, at each cycle k , the admissible control set

$$\mathcal{U}_k = \left\{ u \in \mathbb{R}^3 \mid u_{\min} \leq u \leq u_{\max}, u - u_{k-1} \in [\Delta u_{\min}, \Delta u_{\max}] \right\}. \quad (54)$$

State constraints are phase-dependent (see Equations (14) and (15)). In the predictive formulation, this requirement is propagated over the entire prediction horizon and the asymptotic steady regime. We collect these conditions in an aggregate feasibility predicate

$$g(\hat{x}_{k+1|k}, \hat{x}_{k+2|k}, \hat{x}_{k+3|k}, \hat{x}_{\infty|k}) \in \{0, 1\}, \quad (55)$$

which equals 1 if the predicted state trajectories over the $N = 3$ cycles, the corresponding outputs, and the asymptotic value $\hat{x}_{\infty|k}$ all lie in the admissible region; and 0 otherwise.

It is essential that the two classes of constraints play different algorithmic roles. The control-vector constraints, encoded in $\mathcal{U}_k$, allow the elimination of inadmissible candidates *before* any prediction is computed. The state constraints, encoded in $g(\dots)$, can only be checked *after* the predicted trajectory has been computed. This distinction is exploited explicitly in the algorithm described next.

5.4. The Control Law

In the predictive framework the sought-after solution for $\mathcal{F}(\dots)$ becomes the constrained-optimization problem

$$u_k^* = \arg \min_{u_k \in \mathcal{U}_k} J_k(u_k) \quad \text{subject to} \quad g(\hat{x}_{k+1|k}, \hat{x}_{k+2|k}, \hat{x}_{k+3|k}, \hat{x}_{\infty|k}) = 1, \quad (56)$$

whose solution defines the target mapping

$$\mathcal{F} : (x(t_k), r_k) \mapsto u_k^*, \quad (57)$$

turning the abstract control law (6) into an explicit computational procedure.

Since the prediction model (35) is a *recurrent analytical function that does not admit further analytical inversion*, problem (56) belongs to the class of nonlinear mixed-constraint optimization problems.

However, the structure of the control vector (18) exhibits several useful features: the order is small (three scalar parameters); each parameter $(F_{1,k}, T_{1,k}, T_{2,k})$ is bounded above and below by physical and technological limits; and the “per-cycle” increments are themselves bounded, allowing the search at each step to remain in a localized neighborhood of the previous regime.

These properties make it possible to solve (56) by *exhaustive search on a grid of admissible controls*, without resorting to any gradient-based procedure (Section 7.6). At each cycle k , define uniform discretizations of the admissible ranges of the control parameters,

$$\mathcal{G}_{F_1,k} = \left\{ F_1^{(i)} \right\}_{i=1}^{N_{F_1}}, \quad \mathcal{G}_{T_1,k} = \left\{ T_1^{(j)} \right\}_{j=1}^{N_{T_1}}, \quad \mathcal{G}_{T_2,k} = \left\{ T_2^{(l)} \right\}_{l=1}^{N_{T_2}}, \quad (58)$$

with steps $\Delta F_1, \Delta T_1, \Delta T_2$ inside the bounds defined by $\mathcal{U}_k$ (54).

The Cartesian product of the three grids forms the finite candidate set

$$\mathcal{G}_{u,k} = \mathcal{G}_{F_1,k} \times \mathcal{G}_{T_1,k} \times \mathcal{G}_{T_2,k} \subset \mathcal{U}_k. \quad (59)$$

By construction, $\mathcal{G}_{u,k} \subset \mathcal{U}_k$, so every grid element automatically satisfies both the absolute bounds and the increment bounds (the latter being enforced through the grid lower/upper limits relative to the previous control u_{k-1}).

The control-vector constraints (16) are thus “baked into” the grid structure and need not be checked again inside the algorithm.

The state constraints, encoded in the predicate (55), cannot be enforced in advance and are therefore checked individually for every grid element after the prediction step. Putting these elements together, the control law (57) is realized by the following procedure:

- (1) Given the current state estimate $x(t_k)$, the previous control u_{k-1} and the bounds $u_{\min}$, $u_{\max}$, $\Delta u_{\min}$, $\Delta u_{\max}$, build the admissible control grid $\mathcal{G}_{u,k}$ (59).
- (2) Initialize the empty set of admissible candidates $\mathcal{A}_k = \emptyset$.
- (3) For each candidate $u^{(i,j,l)} = (F_1^{(i)}, T_1^{(j)}, T_2^{(l)}) \in \mathcal{G}_{u,k}$, perform:
 - (a) compute the state predictions $\hat{x}_{k+1|k}$, $\hat{x}_{k+2|k}$, $\hat{x}_{k+3|k}$ and output predictions $\hat{y}_{k+1|k}$, $\hat{y}_{k+2|k}$, $\hat{y}_{k+3|k}$ from (35);
 - (b) compute the asymptotic value $\hat{x}_{\infty|k}$ (Equation (45)) under the stability conditions of (50);
 - (c) evaluate the feasibility predicate (55); if $g(\dots) = 0$, discard the candidate and proceed to the next grid element;
 - (d) otherwise compute $J_k(u^{(i,j,l)})$ from (53) and add the candidate to the admissible set $\mathcal{A}_k \leftarrow \mathcal{A}_k \cup \{u^{(i,j,l)}\}$.
- (4) Once the enumeration is complete, the control applied at cycle k is taken as

$$u_k^* = \arg \min_{u \in \mathcal{A}_k} J_k(u). \quad (60)$$

- (5) If $\mathcal{A}_k = \emptyset$ — no grid element satisfies the trajectory constraints — a fall-back control $u_k^* = u_{k-1}$ is applied, i.e., the previous regime is held unchanged. This is treated as a safety action and accompanied by an alarm to the operator indicating that no admissible prediction could be constructed.

This branch is included primarily to ensure logical completeness of the algorithm and, in the present work, acts as a stub. In a production implementation; however, this branch would encapsulate non-trivial logic grounded in fuzzy-rule systems and engineering field-practice heuristics, providing a structured and safe recovery from states that lie outside the algorithm’s applicability. A full treatment of such a fallback controller is beyond the scope of this study.

The resulting control law admits the compact form

$$u_k^* = \mathcal{F}(x(t_k), r_k) = \arg \min_{u \in \mathcal{G}_{u,k} : g(\hat{x}(u)) = 1} J_k(u), \quad (61)$$

where $\hat{x}(u) = (\hat{x}_{k+1|k}, \hat{x}_{k+2|k}, \hat{x}_{k+3|k}, \hat{x}_{\infty|k})$ is the full collection of state predictions generated by candidate u .

Compatibility of the Fixed-Parameter Assumption with Receding-Horizon Control

The derivation of the cycle-averaged intake pressure (41) and the identification of the coefficients a , b , c from three consecutive cycles (43) both rely on the assumption that the regime parameters (F_{work} , T_{work} , T_{idle}) remain constant over the prediction horizon

(condition (44)). At first glance, this may appear to conflict with the receding-horizon structure of the control law (61), which is free to update u_k at every cycle boundary.

No inconsistency arises, however, because the two mechanisms operate at different conceptual levels. The fixed-parameter assumption is invoked *inside* the evaluation of a single candidate control vector: when the optimizer considers a particular $u_k \in \mathcal{G}_{u,k}$, it propagates three cycles with that candidate held constant and extracts the corresponding asymptotic forecast $\hat{x}_{\infty|k}$ via (45). This “compressed” long-horizon prediction serves as an inexpensive proxy for the steady-state behavior that the candidate would eventually produce, supplying the stability and feasibility information encoded in (50) and (55). Once the best candidate u_k^* has been selected and applied, the controller discards the previous forecast, acquires fresh measurements at t_{k+1} , re-identifies the model coefficients, and solves (56) anew — exactly the receding-horizon principle.

The asymptotic term therefore acts as a *quality indicator* attached to each candidate, not as a commitment to hold the control constant indefinitely. It enriches the cost functional (53) and the feasibility check (55) with long-term trend information that would otherwise be unavailable within the short $N = 3$ horizon, while the receding-horizon re-optimization ensures that the controller adapts to new measurements and disturbances at every cycle. In the limit, when the process reaches a quasi-stationary state, the successive optimal controls converge, $u_k^* \approx u_{k+1}^* \approx \dots$, and the fixed-parameter assumption becomes exactly satisfied — so the asymptotic forecast coincides with the true steady-state outcome. This self-consistency property is characteristic of MPC schemes that embed a terminal cost or terminal constraint derived from a stationary model [41,42].

6. Results

6.1. Benchmark Against Reference Solution

The reduced-order model (35) was benchmarked against two independent references: the in-house high-fidelity transient simulator from which it was derived [25], referred to below as the “original model”, and the industry-standard multiphase flow simulator OLGA 2022.1.0 [28]. Both reference solutions were generated for the same well configuration, reservoir parameters, and operating schedule, so that the comparison isolates the effect of the modeling simplifications introduced in Section 3. Two characteristic intervals were examined: the initial drawdown transient and the subsequent convergence to a periodic (quasi-stationary) regime. The well configuration and experiment parameters are identical to those provided in Yudin et al. [25] (Section “Comparison of periodic well model with existing solutions”, pp. 4–13).

6.1.1. Drawdown Phase

Figure 7 compares the three solutions during the first ≈ 5 h of continuous pumping. Figure 7a shows the intake pressure $P_{in}(t)$: all three curves follow the same monotonically decreasing exponential envelope, from the initial reservoir-equilibrium value down to the vicinity of the first pump shutoff. The reduced-order model tracks the original simulator closely (RMSE ≈ 1 atm, MAPE $\approx 1.3\%$) and remains within the spread between the two high-fidelity references. The OLGA solution itself deviates from the original model by a comparable amount, confirming that the discrepancy introduced by the reduced-order formulation does not exceed the inherent uncertainty between two independent numerical solvers.

Figure 7b shows the corresponding liquid rate $Q_{liq}(t)$. Here the offset is more pronounced: the reduced-order curve is shifted by roughly $20 \text{ m}^3/\text{day}$ relative to the original model (MAPE $\approx 11\%$), while the OLGA solution lies between the two. This systematic bias in the absolute flow-rate level is a direct consequence of the omitted tubing friction

(A16) and casing-side losses (22b), both of which affect the nodal balance and hence the computed Q_{vlp} . Despite the offset, the temporal shape of the liquid-rate transient — the rate of decline, the curvature, and the approach to a quasi-steady plateau — is reproduced faithfully.

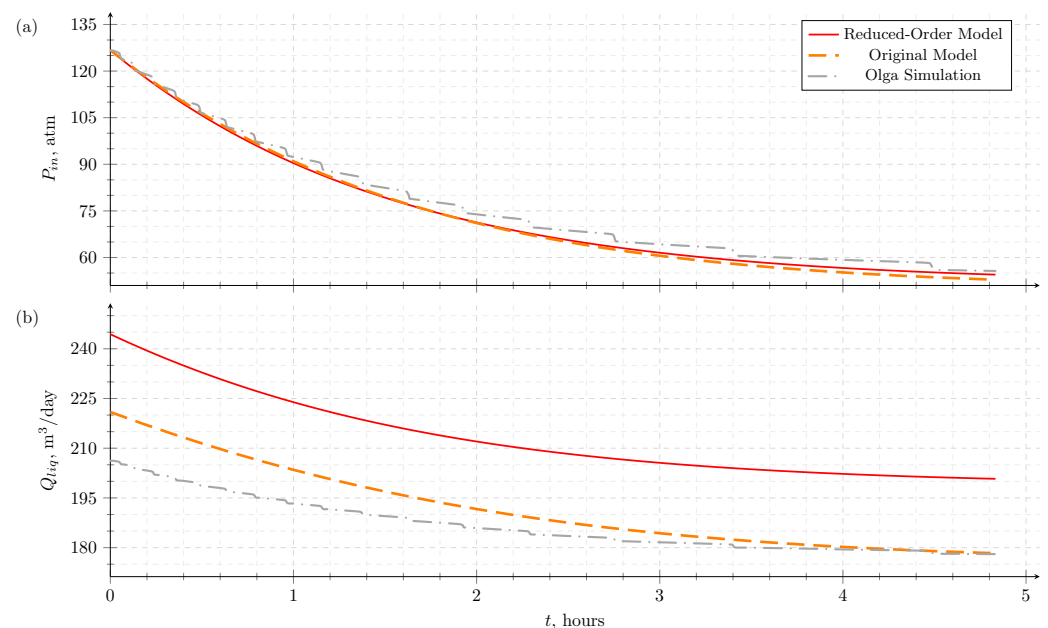


Figure 7. Drawdown-phase comparison of intake pressure (a) and liquid rate (b) among the reduced-order model (red), original numerical simulator (orange, dashed) and OLGA simulation (gray, dash-dotted).

6.1.2. Asymptotic Phase

Figure 8 extends the comparison to the periodic regime that develops after several “work/idle” cycles with $T_{work} = T_{idle} = 10$ min ($t \in [4.85, 10.85]$ h). Panels (a, b) show the full intake-pressure traces, while panels (c, d) display the liquid rate.

The pressure signal (Figure 8a) reveals that the reduced-order model reproduces the period, the amplitude, and the progressive drift of the oscillation envelope with fidelity sufficient for control purposes (MAPE $\approx 4.3\%$ against the original model and $\approx 2.7\%$ against OLGA). Notably, the zoomed view (Figure 8b) shows the cycle-averaged pressure points $\bar{P}_{in}(N)$ and the fitted exponential $a e^{bN} + c$ converging toward the predicted asymptote $P_{in}(+\infty)$, confirming that the analytical forecast (45) captures the long-term trend of the high-fidelity solution.

The liquid-rate comparison (Figure 8c,d) again shows a larger absolute spread: the pump-on RMSE reaches ≈ 35 m³/day against the original model and ≈ 53 m³/day against OLGA. However, all three solutions share the same qualitative structure—the same work-cycle pattern, the same on/off switching instants, and the same tendency toward a repeatable periodic waveform. A large fraction of the numerical discrepancy stems from the sensitivity of instantaneous flow rate to the exact position of the ESP operating point on its characteristic curve, which amplifies even small pressure offsets into significant rate differences.

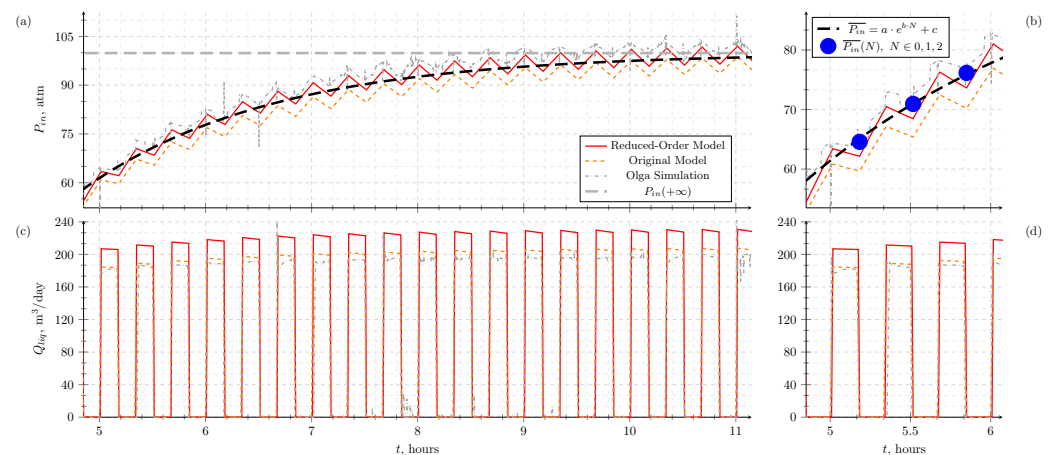


Figure 8. Asymptotic-phase comparison of intake pressure (a,b) and liquid rate (c,d). Panels (b,d) are zoomed views of the first cycles. Blue dots in (b): cycle-averaged $\bar{P}_{in}(N)$; black dashed: fitted $a e^{bN} + c$; gray dashed: predicted asymptote $P_{in}(+\infty)$.

6.1.3. Interpretation

The benchmark confirms the central design premise of the reduced-order model: the aggressive simplifications adopted in Section 3 — elimination of distributed PVT calculations, tubing and casing friction, and iterative nodal coupling — introduce a predictable bias in the absolute values of pressure and, especially, of liquid rate, yet preserve the qualitative dynamics of the system. The intake pressure is reproduced with good absolute accuracy, whereas the liquid rate exhibits a systematic offset whose magnitude is consistent with the omitted frictional terms. Crucially, the temporal behavior of both variables — the exponential drawdown, the cyclic oscillation pattern, and the convergence toward a quasi-stationary regime — is virtually identical across all three solutions.

This trade-off is deliberate. Since the model is intended for real-time predictive control rather than for standalone simulation, the relevant performance metric is not the pointwise accuracy of a single forward run but the ability to detect trends, identify regime transitions, and predict the asymptotic state from a small number of observed cycles. Within the control loop, the residual bias is compensated at every cycle boundary by re-identifying the model coefficients from fresh measurements and re-evaluating the stability criterion (50) — a strategy analogous to the integral action of a classical PID controller. The analytical closed-form nature of (35), which requires only a single exponential evaluation and a finite number of arithmetic operations per prediction step, makes such repeated re-identification computationally trivial and ensures that the controller operates well within the real-time budget of a standard PLC cycle.

6.2. Closed-Loop Feedback Control Simulation

The preceding benchmark established that the reduced-order model (35) reproduces the qualitative dynamics of the high-fidelity reference with sufficient accuracy for predictive control purposes. The present subsection addresses the next logical question: whether the complete control law (61), when coupled to the process (Figure 1) through a closed feedback loop, is capable of (i) driving the system toward a prescribed production target, (ii) maintaining the target in a stable manner over multiple consecutive cycles, and (iii) rejecting external disturbances entering through the measurable boundary conditions.

6.2.1. Motivation for Virtual Testing

Closed-loop validation of a model predictive controller for intermittent ESP wells faces a fundamental practical barrier. At the present stage of MPC deployment in Western

Siberian oilfields, the process safety division prohibits direct autonomous actuation of the ESP drive; the controller may issue recommendations, but the final switching decision remains with the operator. Consequently, a fully closed-loop field trial is not yet feasible. An alternative laboratory route is equally constrained: intermittent operation is, by definition, a regime that arises when the well operates near or beyond the edge of stability, and the transient dynamics of the coupled reservoir–wellbore–pump system are known to depend on the absolute geometric scale of the installation. A laboratory rig capable of faithfully reproducing the cycle-averaged pressure and flow-rate transients would therefore require a full-scale well, which is impractical. For these reasons, the closed-loop experiments reported below were conducted in a virtual environment [43] in which the original high-fidelity transient simulator [25], equipped with the complete surface network model and boundary conditions (A2), served as the controlled plant, and the proposed controller (61) was connected to this plant through a standard feedback loop.

6.2.2. Test-Case Configuration

A synthetic well whose reservoir and completion parameters are representative of a typical Western Siberian AR/PSS candidate was used as the control object (see Table A1 for its parameters). The well was initialized on a baseline intermittent regime with $T_{\text{work}} = T_{\text{idle}} = 20$ min and $F_{\text{work}} = 40.0$ Hz, under a constant wellhead boundary condition $P_{\text{wh}} = P_{\text{fl}} = 15$ atm (A2).

The cost functional (53) was configured with the weight matrices Q_2 , Q_3 and Q_∞ set to zero, so that the optimization penalizes only the tracking error on the first predicted cycle. The sole state constraint was a lower bound on the intake pressure, $P_{\text{in}}^{\text{min}} = 40.0$ atm, and the control target was set to a cycle-averaged liquid rate of $\bar{Q}_{\text{liq}} = 95.0$ m³/day with a tolerance band of $\pm 5\%$ (approximately ± 5 m³/day). The admissible control ranges were $F_{\text{work}} \in [40.0, 60.0]$ Hz and $T_{\text{work}} = T_{\text{idle}} \in [5.0, 30.0]$ min, which correspond to the typical operational envelope of an AR/PSS well. The optimization grid (58) comprised $N_{F_1} = 8$ nodes in frequency and $N_{T_1} = N_{T_2} = 7$ nodes in cycle time; within each candidate element, the in-cycle trajectory was discretized with 20 points per work phase and 20 points per idle phase. These settings (Table 2) were chosen so that the control decisions remain easy for an expert to inspect.

Table 2. Controller configuration for the setpoint-tracking experiment.

Category	Parameter	Value
Cost functional	Active weight	$Q_1 \neq 0$
	Suppressed weights	$Q_2 = Q_3 = Q_\infty = 0$
	Target $\bar{Q}_{\text{liq}}$	95.0 m ³ /day
	Tolerance band	$\pm 5\%$ ($\approx \pm 5$ m ³ /day)
State constraint	$P_{\text{in}}^{\text{min}}$	40.0 atm
Admissible controls	F_{work}	[40.0, 60.0] Hz
	T_{work}	[5.0, 30.0] min
	T_{idle}	[5.0, 30.0] min
Optimization grid	N_{F_1} (frequency nodes)	8
	$N_{T_1} = N_{T_2}$ (time nodes)	7
	In-cycle points (work / idle)	20 / 20

6.2.3. Setpoint Tracking (Figure 9)

Figure 9 shows the first ≈ 7.4 h of the experiment. Panels (a–c) display, respectively, the ESP shaft frequency $F(t)$, the intake pressure $P_{\text{in}}(t)$ and the liquid rate $Q_{\text{vlp}}(t)$; red curves represent instantaneous values recorded by the virtual plant, black markers denote cycle-averaged quantities, and blue vertical lines mark control events at which the controller

updated the operating parameters. The orange dashed line in panel (b) indicates the hard lower bound $P_{in}^{\min}$, while the hatched band and the orange dashed line in panel (c) show the $\pm 5\%$ tolerance zone and the target value of $95.0 \text{ m}^3/\text{day}$, respectively.

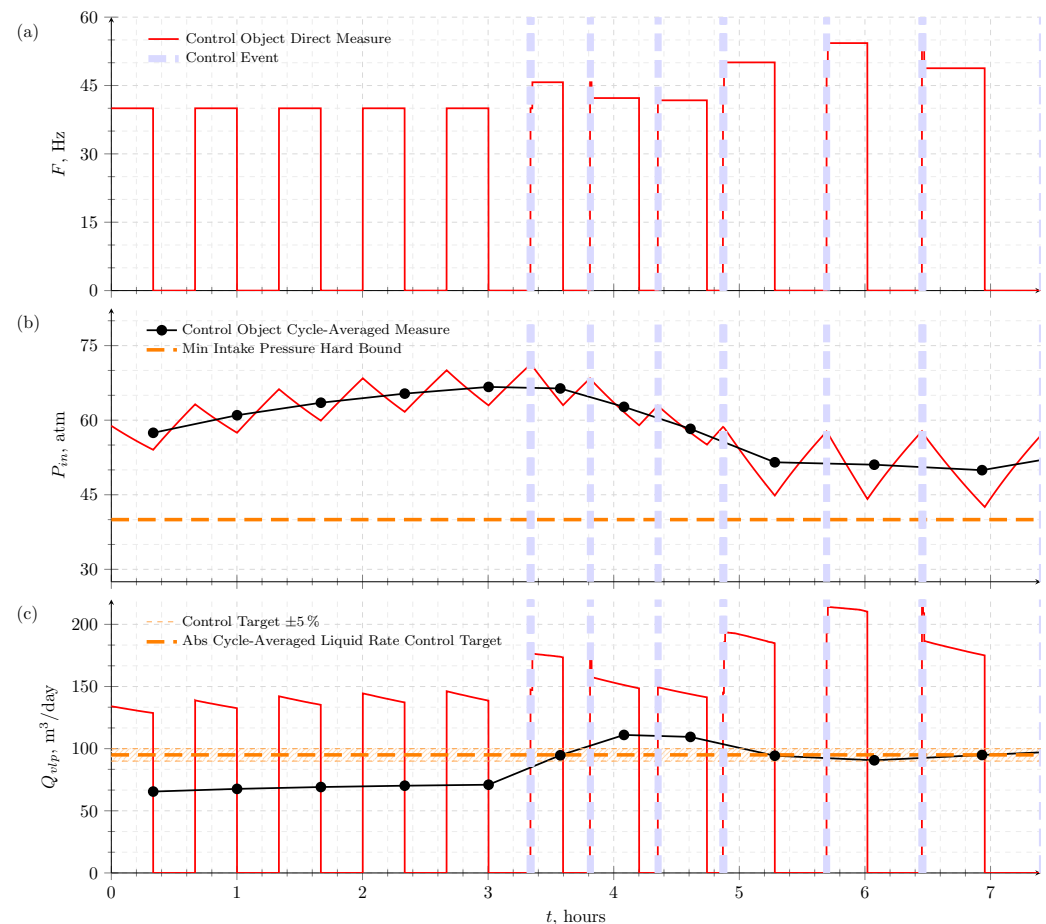


Figure 9. Closed-loop setpoint-tracking experiment: (a) ESP shaft frequency, (b) intake pressure with the hard lower bound (orange dashed), and (c) cycle-averaged liquid rate with the control target (orange dashed) and $\pm 5\%$ tolerance band (hatched). Blue vertical lines mark control update events; black markers denote cycle-averaged values.

During the first five cycles (0–3.3 h) the well operated in open loop at the baseline parameters, reaching a quasi-stabilized cycle-averaged rate of approximately $70 \text{ m}^3/\text{day}$ —well below the target. At $t \approx 3.3$ h the controller was engaged and immediately began adjusting the regime. Over the first three control iterations the algorithm explored the admissible space: frequency rose from the baseline 40.0 Hz to 45.7 Hz and then settled near 42 Hz, while the cycle proportions were redistributed toward longer work phases and shorter idle phases. By the fourth iteration ($t \approx 4.9$ h) the cycle-averaged liquid rate entered the prescribed tolerance band for the first time, and the subsequent three iterations maintained it there: frequency stabilized in the range 49–54 Hz and the cycle durations converged to values near $T_{\text{work}} \approx 20\text{--}30$ min, $T_{\text{idle}} \approx 26\text{--}27$ min. Throughout this transient, the intake pressure remained above the hard bound of 40 atm, confirming that the feasibility predicate (55) was satisfied at every control step.

The complete regime-change schedule is provided in Table 3 below:

Table 3. Setpoint -tracking control-iteration schedule.

Control Iteration	Time (hr)	F_{work} (Hz)	T_{work} (min)	T_{idle} (min)
№1	3.3	45.7	15.5	12.9
№2	3.8	42.2	23.4	9.0
№3	4.4	41.8	23.4	7.6
№4	4.9	50.1	24.7	24.7
№5	5.7	54.3	19.5	26.1
№6	6.5	48.8	30.0	27.4

6.2.4. Disturbance Rejection (Figure 10)

Starting from the stabilized regime reached at the end of the previous test, the wellhead boundary condition was deliberately perturbed to emulate the most common operational disturbance encountered in Western Siberian fields: a change in the surface gathering-network pressure. Two step changes were imposed. At $t \approx 11.0$ h the wellhead pressure was increased from $P_{wh} = 15.0$ atm to 30.0 atm, and at $t \approx 13.5$ h it was partially released to 25.0 atm. Both perturbations enter the measurement vector $y(t)$ and simultaneously act as boundary conditions of the predictive model (35), so that they are *measurable* disturbances available to the controller at each cycle boundary.

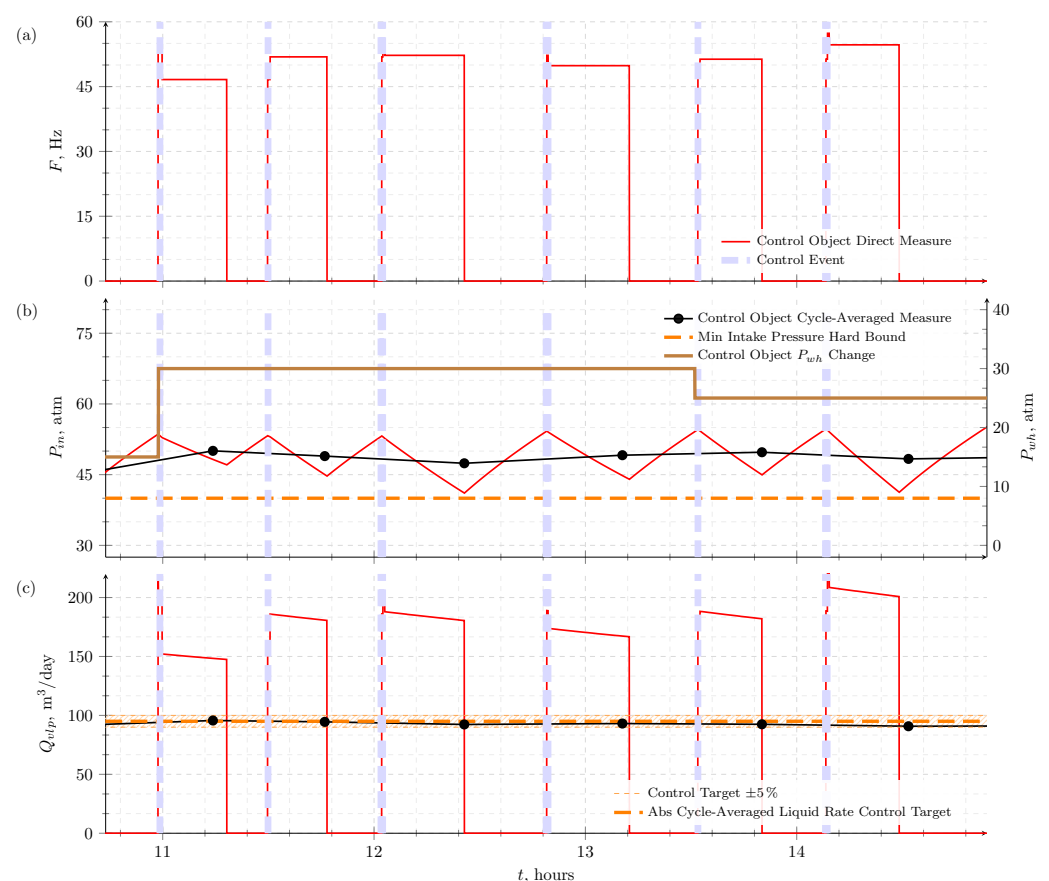


Figure 10. Closed-loop disturbance-rejection experiment: (a) ESP shaft frequency, (b) intake pressure (left axis) and imposed wellhead pressure perturbation $P_{wh}(t)$ (right axis, brown), and (c) cycle-averaged liquid rate with the control target and tolerance band. Notation as in Figure 9.

Figure 10 presents the system response. The brown curve on the secondary axis of panel (b) shows the imposed $P_{wh}(t)$ profile. The first pressure step (15 $\rightarrow$ 30 atm) substantially altered the operating point: the intake pressure surged and the instantaneous liquid rate decreased. The controller responded by increasing the frequency to approximately

52 Hz and adjusting the cycle proportions, bringing the cycle-averaged production rate back into the tolerance band within two to three iterations. The second step (30 → 25 atm), being smaller in magnitude, required a more modest correction: the algorithm increased the frequency further to approximately 57 Hz and rebalanced the cycle times, again recovering the target rate. Across all seven control iterations following the disturbances, the cycle-averaged liquid rate remained within or very close to the prescribed $\pm 5\%$ band, and the intake pressure never violated its lower bound.

The complete regime-change schedule is provided in Table 4 below:

Table 4. Disturbance-rejection control-iteration schedule.

Control Iteration	Time (hr)	F_{work} (Hz)	T_{work} (min)	T_{idle} (min)
№1	11.0	46.6	19.5	11.6
№2	11.5	51.9	16.8	15.5
№3	12.1	52.2	23.4	23.4
№4	12.8	49.9	23.4	19.5
№5	13.5	51.3	18.2	18.2
№6	14.1	54.7	20.8	26.1

6.2.5. Summary

The two experiments jointly confirm that the control law (61), operating on the reduced-order model (35), is capable of driving a closed-loop intermittent ESP system to the prescribed production setpoint, maintaining stable operation over an extended sequence of cycles, and recovering from significant step disturbances in the wellhead boundary conditions—all while respecting the imposed intake-pressure constraint. The controller exhibited no divergence, limit-cycle instability, or constraint violation throughout either test, which provides confidence that the proposed MPC framework is suitable for deployment in autonomous control.

6.3. Open-Loop Validation Against Field Operator Decisions

The benchmark of Section 6.1 confirmed that the reduced-order model (35) reproduces the qualitative dynamics of the high-fidelity reference, while the closed-loop experiments of Section 6.2 demonstrated that the control law (61) can autonomously drive the process to a prescribed setpoint and reject boundary-condition disturbances in a virtual environment. Two questions, however, remain open: first, whether the model and the controller retain their effectiveness when applied to telemetry from a real production well, as opposed to the synthetic test cases used so far; and second, how the recommendations produced by the algorithm compare with the decisions actually taken by the field operator, who at present implements the control law (6) manually.

6.3.1. Rationale for the Open-Loop Back-Test Methodology

As discussed in Section 6.2 (Section 6.2.1), a fully closed-loop field trial is not yet feasible: the process safety division currently prohibits autonomous actuation of the ESP drive, and the operator retains the final switching authority. To bridge this gap, the validation reported below adopts a *retrospective open-loop back-test* methodology [43–45]. A historical interval was identified on a producing well in a Western Siberian field (see Table A2 for its parameters) in which three conditions were simultaneously satisfied: (i) the well was operating in a stable intermittent regime; (ii) a measurable disturbance event occurred during this interval; and (iii) the operator responded to the event, and the well was subsequently returned to stable operation. The recorded telemetry was then fed to the proposed controller as input data. At each cycle boundary, the algorithm computed its recommended control actions using the model (35), but these actions were not applied to

the plant (open loop). The generated recommendations were instead compared with the historical operator decisions.

6.3.2. Test-Case Configuration

The well was initially operating in a stable AR/PSS regime with the following cycle parameters: $F_{\text{work}} = 54.3 \text{ Hz}$, $T_{\text{work}} = 1.6 \text{ min}$, and $T_{\text{idle}} = 27.8 \text{ min}$. The control objective was defined as maintaining the cycle-averaged liquid production rate at the target $\bar{Q}_{\text{liq}} = 7.47 \text{ m}^3/\text{day}$ within the tolerance band $[7.10, 7.84] \text{ m}^3/\text{day}$. Intake-pressure constraints were not activated in this configuration (Table 5), since the operating point was far from the lower P_{in} bound. The cost functional (53) was configured with $Q_1 = 0$ and nonzero weights Q_2, Q_3 , so that the optimization penalized the tracking error on the second and third predicted cycles. The first-cycle horizon was suppressed because the cycle duration of this particular well (approximately 29 min) is too short for a meaningful “first-cycle” response to develop. The asymptotic term Q_∞ was also set to zero; comparing the asymptotic forecast with the operator’s response would not have been meaningful, since field operators do not account for the long-term convergence behavior of the regime when making manual adjustments, and such a comparison would have been *a priori* inconclusive.

Table 5. Controller configuration for the retrospective open-loop back-test.

Category	Parameter	Value
Baseline regime	F_{work}	54.3 Hz
	T_{work}	1.6 min
	T_{idle}	27.8 min
Cost functional	Active weights	$Q_2, Q_3 \neq 0$
	Suppressed weights	$Q_1 = Q_\infty = 0$
	Rationale for $Q_1 = 0$	cycle too short ($\approx 29 \text{ min}$)
State constraint	$p_{\text{in}}^{\text{min}}$	not activated
	(operating point far from bound)	—
Control target	$\bar{Q}_{\text{liq}}$	$7.47 \text{ m}^3/\text{day}$
	Tolerance band	$[7.10, 7.84] \text{ m}^3/\text{day}$
	Tolerance band ($\pm\%$)	$\approx \pm 5 \%$

6.3.3. Disturbance Event and Initial Forecast (Figure 11)

The selected test interval is illustrated in Figure 11. The three panels display the ESP shaft frequency (a), the intake and flowline pressures (b), and the instantaneous and cycle-averaged liquid rate (c). A vertical marker near $t \approx 18 \text{ min}$ marks the moment when a step-like decrease in the wellhead (flowline) pressure P_{fl} from about 54 atm to about 50 atm was recorded (brown curve on the secondary axis of panel b). This type of disturbance — a sudden drop in the downstream gathering-network pressure — is the most common operational perturbation encountered in intermittent wells and is directly analogous to the synthetic disturbance studied in Section 6.2.

Two model-based forecasts are shown, constructed from (35) at the cycle boundary immediately preceding and immediately following the event, respectively: the “pre-event” forecast (black dashed line, corresponding to $P_{\text{fl}} = 54 \text{ atm}$) and the “post-event” forecast (orange dash-dotted line, corresponding to $P_{\text{fl}} = 50 \text{ atm}$). Panel (c) reveals the key consequence of the pressure drop: the cycle-averaged liquid rate under the unchanged control parameters rises from approximately $7.47 \text{ m}^3/\text{day}$ at the pre-event baseline to $8.02 \text{ m}^3/\text{day}$ on the second predicted cycle (red markers), exceeding the upper tolerance bound of $7.84 \text{ m}^3/\text{day}$. By comparison, the pre-event forecast (green markers) remains within the prescribed corridor over the entire prediction horizon. The model thus identifies the need for a corrective control action immediately at the cycle boundary following the disturbance.

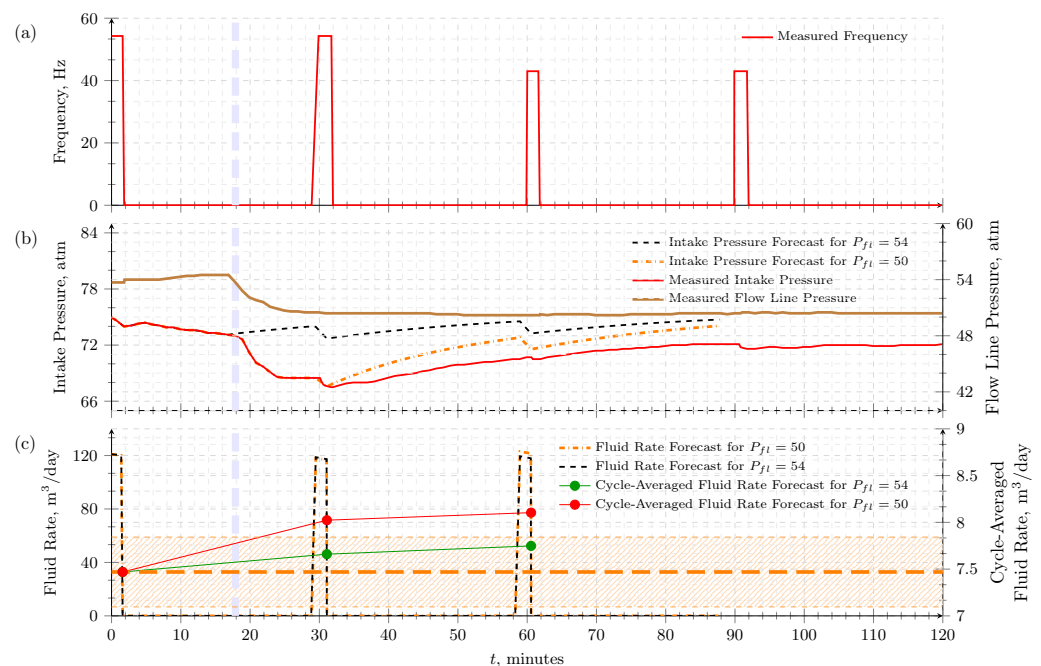


Figure 11. Initial forecast at the disturbance event: (a) measured ESP shaft frequency, (b) intake pressure (left axis) and measured flowline pressure P_{fl} (right axis, brown) with model forecasts for $P_{fl} = 54$ atm (black dashed) and $P_{fl} = 50$ atm (orange dash-dotted), and (c) instantaneous liquid rate (left axis) and cycle-averaged rate (right axis) with the control target (orange dashed) and tolerance band (hatched). Green markers: baseline forecast; red markers: post-disturbance forecast.

6.3.4. Decomposition of the Control Problem

The full control vector (18) comprises three parameters (F_{work} , T_{work} , T_{idle}), and the corrective recommendation produced by the optimizer is, in general, a simultaneous adjustment of all three. Evaluating the “correctness” of such a recommendation in the four-dimensional space (F_{work} , T_{work} , T_{idle} , $\bar{Q}_{liq}$) and, moreover, comparing it with the operator’s reference trajectory is a task of considerable complexity that merits a dedicated study. To make the present analysis transparent, the optimization was therefore decomposed into two independent one- and two-dimensional scenarios by restricting the admissible grid (58). In the first scenario, only the ESP shaft frequency F_{work} was varied while the cycle-time parameters were held fixed at their baseline values ($T_{work} = 1.6$ min, $T_{idle} = 27.8$ min). In the second scenario, the frequency was frozen at its baseline $F_{work} = 54.3$ Hz and the optimizer was free to adjust T_{work} and T_{idle} .

Scenario 1: Frequency Adjustment at Fixed Cycle Proportions (Figures 12 and 13)

The results of the frequency-only optimization are presented in Figure 12, with the corresponding cross-section of the cost-function landscape shown in Figure 13. With T_{work} and T_{idle} held constant, the algorithm recommends a reduction in the shaft frequency from the baseline 54.3 Hz to 52.2 Hz. This correction reduces the predicted cycle-averaged liquid rate on the second cycle from 8.02 m³/day (which violates the upper tolerance bound) to 7.26 m³/day, which lies within the prescribed corridor. The control trajectory in the (F_{work} , $\bar{Q}_{liq}$) plane (Figure 13) provides a geometrical interpretation: the initial operating point (red marker at 54.3 Hz, 7.66 m³/day) is displaced upward by the pressure disturbance to the post-event point (green marker at 54.3 Hz, 8.02 m³/day), and the algorithm then slides along the $P_{fl} = 50$ atm response curve to the new admissible operating point (blue marker at 52.2 Hz, 7.26 m³/day).

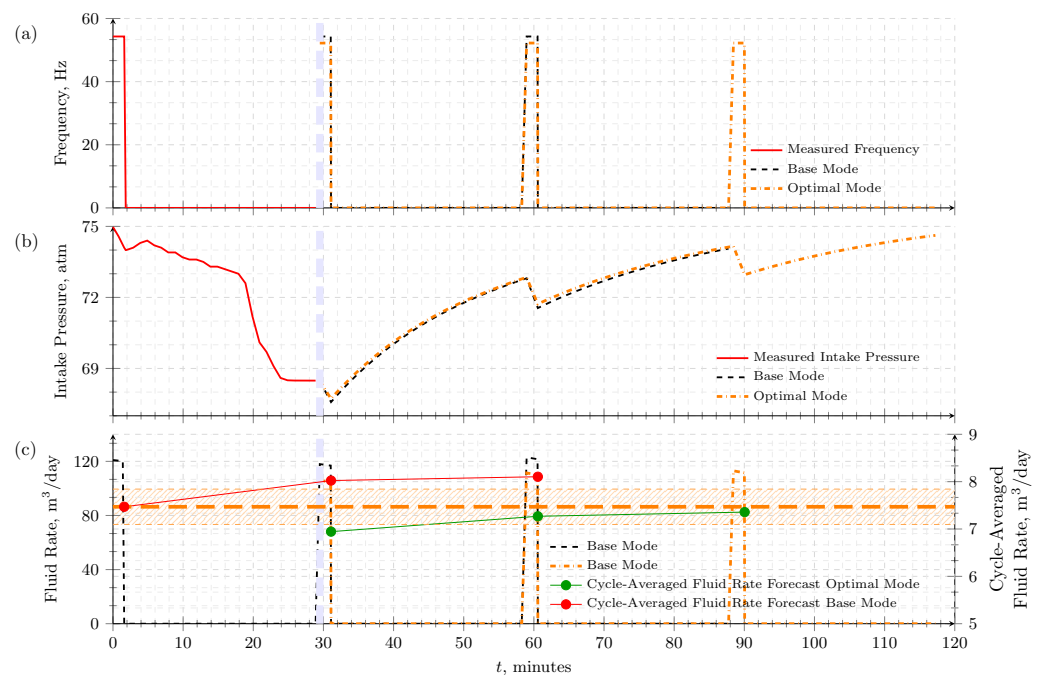


Figure 12. Frequency-only control scenario (T_{work} , T_{idle} fixed): (a) ESP shaft frequency, (b) intake pressure, and (c) liquid rate with cycle-averaged values. Red curves and markers: base (unchanged) mode after the disturbance; green curves and markers: optimized mode. Notation and tolerance band as in Figure 11.

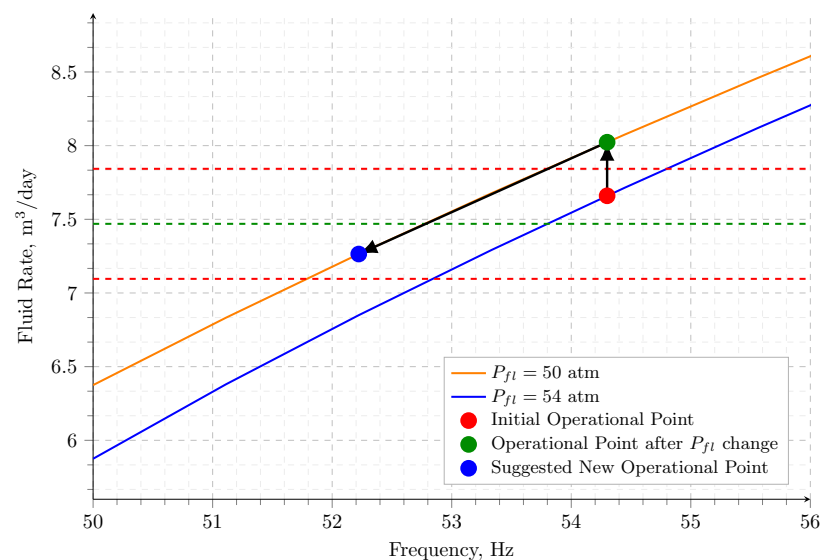


Figure 13. Cross-section of the predicted liquid rate as a function of F_{work} at fixed $T_{\text{work}} = 1.6$ min and $T_{\text{idle}} = 27.8$ min. Orange and blue curves: model response for $P_{fl} = 50$ and 54 atm, respectively. Horizontal dashed lines: tolerance band (red) and target (green). Markers: initial operating point (red), post-event point (green), and algorithm-recommended point (blue). Arrows indicate the control trajectory.

Scenario 2: Cycle-Time Adjustment at Fixed Frequency (Figures 14 and 15)

The results of the cycle-time optimization are presented in Figure 14, with the corresponding control-surface visualization shown in Figure 15. With $F_{\text{work}} = 54.3$ Hz held constant, the algorithm recommends adjusting the cycle proportions from $(T_{\text{work}}, T_{\text{idle}}) = (1.6, 27.8)$ min to $(1.5, 29.5)$ min: a slight reduction in the work phase and a moderate extension of the idle (accumulation) phase. This correction reduces the predicted cycle-

averaged liquid rate on the second cycle from $8.02 \text{ m}^3/\text{day}$ to $7.34 \text{ m}^3/\text{day}$, again returning it to within the tolerance band. The three-dimensional surface plot (Figure 15) illustrates the mechanism: the two surfaces correspond to the model response at $P_{fl} = 54 \text{ atm}$ (blue) and $P_{fl} = 50 \text{ atm}$ (orange); the horizontal planes delineate the tolerance band (red) and the target (green). The disturbance lifts the operating point from the red marker on the lower surface to the green marker on the upper surface, and the optimizer identifies a new admissible point (blue marker) on the upper surface by shifting toward shorter work times and longer accumulation periods.

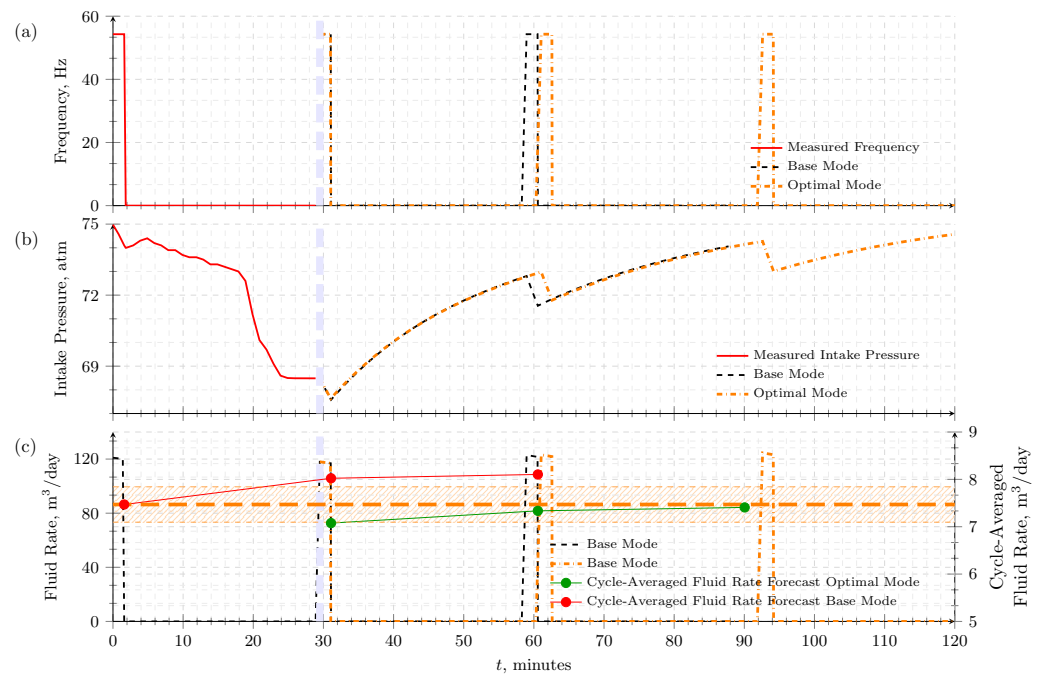


Figure 14. Cycle-time control scenario (F_{work} fixed): (a) ESP shaft frequency, (b) intake pressure, and (c) liquid rate with cycle-averaged values. Notation as in Figure 12.

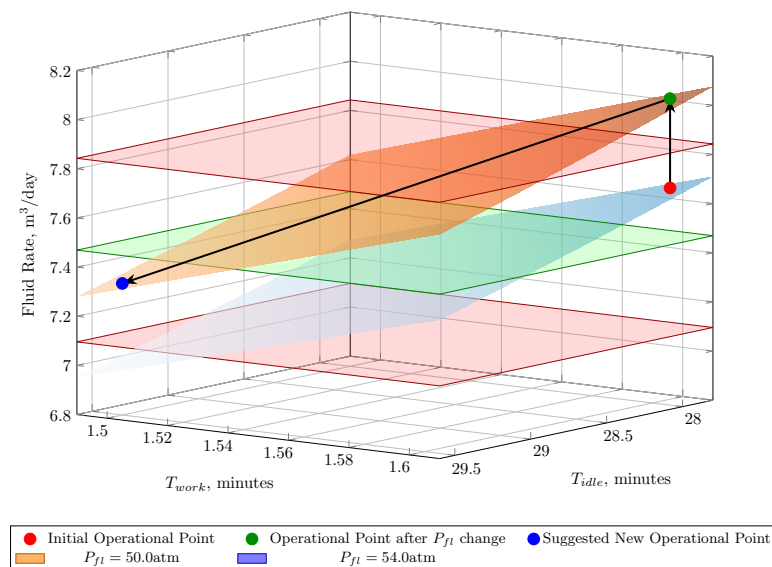


Figure 15. Control-surface visualization for the fixed-frequency scenario: predicted cycle-averaged liquid rate as a function of T_{work} and T_{idle} at $F_{work} = 54.3 \text{ Hz}$. Orange and blue surfaces: $P_{fl} = 50$ and 54 atm , respectively. Horizontal planes: tolerance band boundaries (red) and target (green). Markers as in Figure 13.

6.3.5. Comparison with the Operator's Historical Response (Figure 16)

The full chronology of the operator's actual decisions over the approximately five-hour interval following the disturbance is shown in Figure 16. Four distinct operating regimes can be identified. The baseline regime ($F_{\text{work}} = 54.3 \text{ Hz}$, $T_{\text{work}} = 1.6 \text{ min}$, $T_{\text{idle}} = 27.8 \text{ min}$; orange-shaded region) was followed by an initial conservative correction ($F_{\text{work}} = 45.0 \text{ Hz}$, $T_{\text{work}} = 1.7 \text{ min}$, $T_{\text{idle}} = 28.0 \text{ min}$; green-shaded region), a refinement ($F_{\text{work}} = 44.5 \text{ Hz}$, $T_{\text{work}} = 1.7 \text{ min}$, $T_{\text{idle}} = 27.5 \text{ min}$; purple-shaded region), and finally a near-restoration of the original frequency ($F_{\text{work}} = 53.0 \text{ Hz}$, $T_{\text{work}} = 1.6 \text{ min}$, $T_{\text{idle}} = 28.0 \text{ min}$; blue-shaded region). It should also be noted that the flowline pressure continued to evolve slightly over this four-hour interval (panel c), introducing additional variability that was not present at the initial decision horizon.

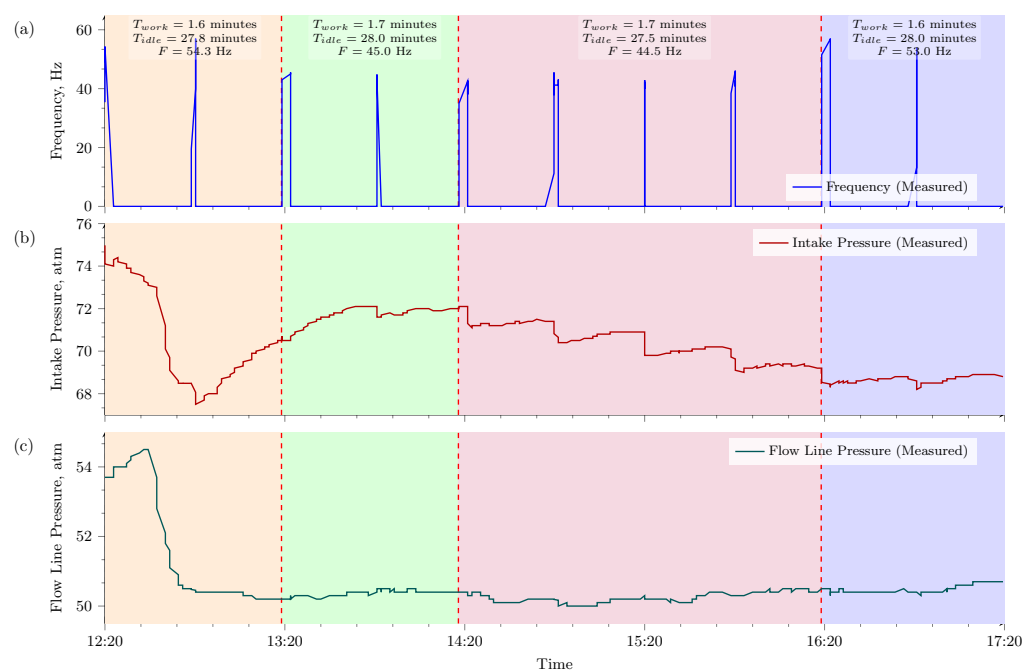


Figure 16. Chronology of the operator's historical regime changes following the wellhead pressure disturbance: (a) ESP shaft frequency with annotated regime parameters, (b) intake pressure, and (c) measured flowline pressure. Colored regions delimit distinct operating regimes; red dashed vertical lines mark the transition instants.

The operator's final settled regime can now be compared with the two algorithmic recommendations. In terms of frequency, the operator arrived at 53.0 Hz — a smaller reduction from the baseline than the 52.2 Hz recommended by Scenario 1, but in the same direction. The work time $T_{\text{work}} = 1.6 \text{ min}$ was left unchanged relative to the baseline (consistent with Scenario 1 and intermediate between the baseline and the 1.5 min of Scenario 2), while the idle time was increased to 28.0 min, a change whose direction matches Scenario 2 ($27.8 \rightarrow 29.5 \text{ min}$) but whose magnitude is smaller. The final operator solution therefore occupies an intermediate position between the two decomposed algorithmic scenarios: a moderate frequency reduction combined with a slight extension of the accumulation phase. Given that the disturbance boundary conditions continued to evolve over the four-hour adjustment period and that the operator worked iteratively from incomplete information, the agreement is considered satisfactory.

6.3.6. Summary

Two principal observations emerge from the comparison. First, the algorithm's recommendation is model-grounded and immediate: it is computed at the first cycle boundary after the disturbance event and is accompanied by a quantitative forecast of the resulting production trajectory. By contrast, the operator required approximately three hours and three intermediate trial regimes — including a conservatively large initial frequency reduction to 45.0 Hz followed by a gradual recovery — before converging to a stable operating point. This iterative trial-and-error pattern is consistent with the practical difficulty noted in Section 1: when three control parameters are changed simultaneously, the outcome of a regime transition at the cycle-averaged level is extremely difficult to predict intuitively, which encourages operators to overshoot in the conservative direction and then correct incrementally. Second, the final regime reached by the operator lies within the envelope bounded by the two decomposed scenarios, supporting the conclusion that the controller's recommendations are at least as accurate as the current manual practice. Taken together, these results provide evidence that the proposed MPC framework, built on the reduced-order model (35) and the control law (61), is capable of producing timely and physically meaningful control recommendations under conditions representative of real intermittent ESP well operation.

7. Discussion

The model, the analytical stability criterion, and the predictive control law proposed in this work were constructed to preserve the structural generality of the intermittent-mode framework: extensions to variable-frequency cycles ($F_2 \neq 0$) or to additional physical effects are admissible, provided they can be written in a form that fits naturally into the reduced-order structure without sacrificing the closed-form analytical solution — the property that makes real-time performance on edge hardware possible.

Nevertheless, several deliberate compromises were made in order to obtain a formulation that is both analytically tractable and fast enough to run inside a real-time control loop on edge hardware. These compromises define the present scope of applicability and, at the same time, set a clear agenda for further research.

7.1. Restriction to AR/PSS-Type Intermittent Regimes

The first group of limitations originates from the simplifying Assumptions 1–3 introduced in the problem statement. Setting $F_{2,k} = 0$ collapses the accumulation phase to a purely passive “buildup” and reduces the per-cycle control vector. This is fully consistent with the dominant industrial use cases (AR/PSS) for which the present results were derived and validated. It does, however, exclude the more general variable-frequency periodic regime in which $F_{2,k} > 0$. Extending the model and the optimization procedure to such cases is therefore the most immediate direction for further research.

Such an extension does not require revisiting the reduced-order ODE itself, since (35a) already describes the $F > 0$ Hz branch; rather, it requires re-deriving the cycle-averaged form (41a) for the case where neither phase is passive, and revisiting the sign analysis underlying the stability criterion (50).

Removing the no-PID Assumption 2 and treating disturbances $\theta(t)$ explicitly, rather than as small fluctuations under Assumption 3, would also allow the framework to accommodate slug arrivals, sudden composition changes, and other intra-cycle events whose mean effect over the cycle is currently absorbed into the identified coefficients of (41).

7.2. Robustness to Disturbances and Measurement Noise

The small-disturbance Assumption 3 and its consequences for robustness deserve separate discussion, particularly because the target well stock is characterized by fluctuating gas–oil ratio, water cut and associated PVT properties. It is useful to distinguish two qualitatively different classes of disturbance.

Slow drifts. Gradual changes in GOR, water cut, and fluid properties evolve on characteristic timescales of weeks to months — orders of magnitude slower than the per-cycle control horizon and the three-cycle identification window. Within the framework, these are handled by construction: because the model coefficients are re-identified from fresh measurements at every cycle boundary, the cycle-mean effect of a slow drift is continuously absorbed into the identified parameters of (41), in direct analogy with the integral action of a PID controller. Where required, such properties may additionally be injected as externally measured boundary inputs, keeping them outside the predictive responsibility of the reduced-order model itself.

Fast, discrete events. Slug arrivals, water breakthrough, and gas breakthrough — all common in this well stock — produce abrupt, large-amplitude excursions that genuinely violate Assumption 3. The present formulation does not claim to represent these events: removing the no-PID Assumption 2 and treating the disturbance term $\theta(t)$ explicitly, rather than as a small fluctuation, is the extension required to accommodate them, and constitutes a natural and substantial subject for dedicated future research within the same reduced-order framework.

Measurement noise. We regard the rejection of measurement noise $\phi(t)$ as primarily a signal-conditioning and state-estimation problem — a concern of metrology and data processing [46–48] — best addressed in the acquisition and pre-processing layer rather than inside the physical model. The controller further mitigates high-frequency noise structurally, since it acts on cycle-integrated quantities (the cycle-integrated disturbance Θ_k and noise Φ_k), whose averaging over the cycle attenuates zero-mean fluctuations.

A first, if limited, indication of closed-loop robustness is already provided by the synthetic experiments of Section 6.2. We nonetheless emphasize that this does not constitute a systematic robustness study against large composition swings or discrete flow events; such a study, spanning the amplitude, rate, and type of disturbance, is an important direction for future work, deliberately left outside the present scope to keep the manuscript focused on the fundamental per-cycle control principles.

7.3. Modeling Compromises

A second group of limitations stems from the modeling choices made inside the reduced-order description itself. The neglect of the casing-side frictional and kinetic pressure losses in (22b), the assumption of a fixed wellhead boundary condition in (A2), the omission of tubing friction in (A16), and the existence condition imposed on the ESP operating point in (A15) were each justified by the same speed/accuracy trade-off: each term, if retained, would force a distributed PVT and flow-regime calculation along the corresponding interval at every integration step, which is incompatible with on-edge real-time use.

The benchmark in Section 6 (Figures 7 and 8) shows that, despite these omissions, the reduced-order model reproduces the qualitative dynamics of the high-fidelity simulator and of the industry-standard reference with an accuracy that is sufficient to drive a robust control law and to identify a quasi-stationary regime.

It also makes clear, however, that a predictable bias is introduced in the absolute values of intake pressure and liquid rate. Reintroducing each of the omitted terms would substantially improve absolute accuracy and broaden the range of physical phenomena that

can be represented inside the predictive loop. An alternative route, noted for completeness, is to replace the omitted physics with a data-driven surrogate — for example, a proper-orthogonal-decomposition (POD) reduced-order model or a neural-network-based flow emulator, approaches that have recently been shown to approximate distributed multiphase-flow calculations at low online cost [49–51]. Such surrogates, however, would substitute a trained approximation for the closed-form analytical structure of (35), which is the primary enabler of on-edge real-time re-identification; the interpretability and robustness of that substitution under distribution shift fall outside the scope of the present work.

7.4. Extensions of the Control Law

Improvements are also possible on the control side, independently of the modeling refinements above. The present controller is dedicated to the intermittent regime and assumes that the well is and remains in periodic operation. Real wells, however, transition between steady-state and intermittent operation as inflow conditions evolve, and the moments of conversion are themselves critical in terms of production losses and equipment integrity. Embedding the steady-state operating point as a specific case of (50) — formally as the $T_{\text{idle}} \rightarrow 0$ limit — and equipping the optimization problem (56) with explicit handover logic between the two modes would allow the same controller to manage the full operational lifecycle of an unstable well.

A second extension concerns the lower control tier. The PLC-hosted PID loops that protect the equipment within a cycle are currently treated as fixed; once the per-cycle controller has identified the asymptotic regime through (45), it has, in principle, all the information needed to retune those PID parameters on the fly so that the intra-cycle behavior matches the predicted optimal trajectory. This would close the gap between the two control tiers identified in Section 1 and turn the proposed MPC into a supervisor of the existing low-level loops rather than a parallel decision layer.

7.5. Analytical Structure as a Research Opportunity

Finally, we believe that the analytical structure obtained in Section 4 has value beyond its immediate use inside the controller. The closed form $\overline{P}_{in}(N) = a e^{bN} + c$, the exponent $b = AK(T_w + T_s)$, and the explicit asymptote c together constitute a small set of “stability coefficients” that summarize the per-cycle dynamics of an arbitrary AR/PSS regime in three numbers identifiable from only three consecutive cycles.

We expect that a systematic study of these coefficients — their sensitivity to PVT properties, to the productivity index K , and to the pump degradation factor k_{degr} — could yield closed-form classifiers of stable, marginal, and unstable operating envelopes, and ultimately a deeper understanding of non-stationary (intermittent and transient) well operation than is currently available from numerical experimentation alone.

7.6. Real-Time Performance

The reduced-order model (35) replaces the iterative nodal-analysis solves of the high-fidelity reference with a single closed-form exponential evaluation per candidate regime. By construction, this makes the exhaustive grid search over the admissible set (54) orders of magnitude faster than any numerical predecessor. The complete solution surface shown in Section 6 is computed in under 1.0s on a commodity processor (Intel Core i3, 14th generation), well within the characteristic time interval between two measurements $\Delta\tau \approx 10\text{--}30\text{s}$ inside the cycle of a typical intermittent well.

7.7. Scope of the Experimental Validation

The experiments reported in Section 6 address the three questions that are essential for a first deployment: model fidelity against high-fidelity and industry-standard refer-

ences (Figures 7 and 8), closed-loop stability in a synthetic sandbox (Figures 9 and 10), and consistency of the open-loop controller output with the decisions actually taken by experienced production engineers on historical telemetry (Figures 11–16). To make the comparisons meaningful and the results interpretable, the weights in the cost functional (53) were deliberately simplified: in particular, the asymptotic penalty term was suppressed, so that the optimizer selects cycles on the basis of the immediate predicted flow rate and constraint satisfaction alone, mirroring the information set available to a human operator adjusting the regime in the field. This choice was instrumental for validation — it allowed a direct, cycle-by-cycle comparison between the controller’s suggestions and the engineer’s actual decisions — but it leaves the full multi-objective landscape of (53) unexplored. A more comprehensive study is therefore a necessary next step before the framework can be recommended for continuous closed-loop deployment: it should systematically vary the weight vector, reintroduce the asymptotic convergence term, and evaluate the controller inside a modified virtual environment capable of replaying multi-day operational scenarios with evolving reservoir and surface-network conditions.

8. Conclusions

This work was motivated by a structural gap in the control of unstable ESP wells: recommendation systems built on high-fidelity transient models operate on hourly to daily cycles and deliver setpoints that are obsolete by the time they reach the drive, while PLC-hosted PID loops act on the right time scale but carry no representation of the underlying multiphase physics. Neither tier alone is adequate for the edge-of-instability regime that intermittent operation requires.

To bridge this gap, we formulated the per-cycle control of an intermittent ESP well as a multi-horizon problem and derived a reduced-order transient model of the coupled “reservoir–tubing–annulus” system whose closed-form solution predicts the intake pressure through a single exponential and a finite number of arithmetic operations. Building on this model, we obtained an analytical stability criterion for the cycle-averaged intake pressure, identifiable from only three consecutive cycles, and embedded it inside a per-cycle predictive control law solved by exhaustive search over a discretized admissible set. The entire solution surface is computed in under 1.0 s on a commodity processor, well within the characteristic measurement interval of a typical intermittent well.

The proposed framework was validated in three stages. In the model benchmark, the reduced-order formulation reproduced the intake pressure of both the in-house high-fidelity simulator and OLGA 2022.1.0 with $\text{RMSE} \approx 1 \text{ atm}$ and $\text{MAPE} \approx 1.3 \%$; the liquid-rate offset ($\text{MAPE} \approx 11 \%$) was consistent with the omitted frictional terms and did not affect the qualitative dynamics. In the closed-loop sandbox experiments, the controller drove the cycle-averaged liquid rate to the prescribed setpoint and held it within the $\pm 5 \%$ tolerance band across all control iterations, including recovery from two imposed step disturbances in wellhead pressure, without violating the intake-pressure lower bound at any cycle boundary. In the retrospective open-loop back-test on field telemetry, the algorithm issued a physically grounded recommendation at the first cycle boundary following a disturbance event; the operator required approximately three hours and three intermediate trial regimes to converge to a final settled point whose cycle parameters lay within the envelope bounded by the two algorithmic scenarios, confirming consistency between the controller output and experienced field practice.

The present formulation is restricted to AR/PSS-type intermittent regimes and relies on several modeling simplifications that trade absolute accuracy for real-time tractability. Principal directions for further research include extension to variable-frequency periodic operation, relaxation of the modeling compromises, integration with steady-state and

PLC-level control tiers, a systematic study of the analytical stability coefficients, a more comprehensive exploration of the cost functional weight space within an extended virtual testing environment, and closed-loop field validation, which is currently precluded by process safety requirements that prohibit autonomous actuation of the ESP drive at the present stage of deployment.

Author Contributions: Conceptualization, M.P., E.K., N.S. and E.Y.; methodology, M.P., E.K. and N.S.; software, M.P. and E.K.; validation, M.P., E.K. and N.S.; investigation, V.G.; data curation, S.D. and V.G.; writing—original draft preparation, M.P. and E.K.; writing—review and editing, M.P. and S.D.; visualization, M.P. and E.K.; supervision, E.Y. and S.D.; project administration, N.S. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the Strategic Academic Leadership Program “Priority-2030” of the Ministry of Science and Higher Education of the Russian Federation, project “Technological Engineering and Autonomous Production Processes in the Energy Sector”.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding authors.

Conflicts of Interest: Authors Mikhail Petrushin and Nikita Smirnov were studying at the FSBIS Institute of Problems of Mechanical Engineering. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Nomenclature

Latin symbols

A	Compound pressure-sensitivity coefficient; translates net flow imbalance into rate of intake-pressure change (annular geometry and PVT corrections)
a, b, c	Coefficients of the cycle-averaged intake-pressure model $\overline{P_{in}}(N) = a e^{bN} + c$; also used (context-dependent) as quadratic head approximation coefficients
A, B, C	Coefficients of the quadratic equation for the tubing-side flow rate Q_{vlp}
A_k	Admissible candidate set at cycle k
D_{ann}	PVT rescaling coefficient for the annulus (downhole-to-reference conditions)
$e(t_k)$	Cycle-boundary tracking error vector
$\hat{e}_{k+i k}$	Predicted tracking error at cycle $k + i$, computed at t_k
F	ESP shaft rotational frequency [Hz]
$F_{1,k} \equiv F_{work}$	Work-phase shaft frequency [Hz]
$F_{2,k}$	Idle-phase shaft frequency (= 0 in AR/PSS) [Hz]
F_{nom}	Nominal shaft frequency supplied by the vendor [Hz]
g	Gravitational acceleration [$m\ s^{-2}$]
$\mathcal{G}_{u,k}$	Discrete admissible control grid at cycle k
h	True vertical depth [m]
h_{tubing}	Pump setting depth (TVD) [m]
hf	Head-degradation correction factor (wear)
H_{esp}	ESP head–rate characteristic curve [m]
$J_k(u)$	MPC cost functional at cycle k
k	Cycle index
k_{degr}	Composite ESP degradation factor ($= hf \cdot k_{gas} \cdot k_{\mu}$)
k_{gas}	Gas-related head-degradation correction
k_{μ}	Viscosity-related head-degradation correction
K	Well productivity index [$m^3\ day^{-1}\ atm^{-1}$]

m	Dimension of the measurement vector
n	Dimension of the state vector
N	Number of ESP stages; also prediction horizon (number of cycles) — context-dependent
P_{bh}	Bottomhole pressure [atm]
P_{fl}	Flowline (gathering-network) pressure [atm]
P_{in}	Pump intake pressure [atm]
p_{in}^{max}	Maximum intake pressure at complete pressure build-up [atm]
p_{in}^{min}	Lower bound on intake pressure [atm]
$\overline{P}_{in}(N)$	Cycle-averaged intake pressure at cycle N [atm]
P_{res}	Reservoir boundary pressure [atm]
P_{wh}	Wellhead pressure [atm]
q_i	Weight of the i -th component in the tracking norm
Q_i, Q_{∞}	Weight matrices for the i -th horizon step and the asymptotic term in J_k
Q_{ipr}	Inflow (IPR) volumetric flow rate [$m^3 day^{-1}$]
Q_{liq}	Daily-averaged liquid production rate [$m^3 day^{-1}$]
Q_{mix}	Gas–liquid mixture flow rate at pump intake [$m^3 s^{-1}$]
$Q_{f casing}$	Casing-side (reservoir inflow) volumetric flow rate [$m^3 day^{-1}$]
$Q_{f tubing}, Q_{vlp}$	Tubing-side (VLP) volumetric flow rate [$m^3 day^{-1}$]
r_k	Reference (setpoint) vector at cycle k
R_p	Gas–oil ratio [$m^3 m^{-3}$]
S_{ann}	Annulus cross-sectional area [m^2]
t	Absolute time [h or min]
t_k	Start time of cycle k
$T_{1,k} \equiv T_{work}$	Work-phase duration [min]
$T_{2,k} \equiv T_{idle}$	Idle-phase duration [min]
$T_{horizon,k}$	Prediction horizon length [min]
$T_{total,k}$	Total cycle duration [min]
u_k	Per-cycle control vector at cycle k
$\mathcal{U}_k$	Admissible control set at cycle k
$x(t)$	Internal process state vector
$y(t)$	Measured output vector
$z(t)$	Scalar controlled output ($= c^{\top} y(t)$)
$\bar{z}_k$	Cycle-averaged controlled output
<i>Greek symbols</i>	
$\Delta \overline{p}_{in}^{const}$	Stability threshold on cycle-to-cycle pressure drop [atm]
ΔP_{esp}	Pressure increase across the ESP [atm]
ΔP_{tubing}	Frictional pressure loss along the tubing [atm]
ΔP_{choke}	Pressure drop across the wellhead choke [atm]
ϵ, ϵ	Tracking tolerance (scalar and multivariable)
τ	Local (in-cycle) time, $\tau = t - t_k$ [min]
$\theta(t)$	Uncontrolled process disturbances
Θ_k	Cycle-integrated disturbances
$\phi(t)$	Measurement noise
Φ_k	Cycle-integrated measurement noise
ρ_{mix}	Average gas–liquid mixture density [$kg m^{-3}$]
$\overline{\rho}_{gas}^{ann}$	Average gas density in the annulus [$kg m^{-3}$]
$\overline{\rho}_{liq}^{ann}$	Average liquid density in the annulus [$kg m^{-3}$]
$\sigma_k(\tau)$	Phase-switching signal (1 = work, 2 = idle)
$\mathcal{F}$	Per-cycle control law $u_k = \mathcal{F}(x(t_k), r_k)$
<i>Abbreviations</i>	
AR	Automatic reclosure
ESP	Electrical submersible pump
IPR	Inflow performance relationship

MPC	Model predictive control
PID	Proportional–integral–derivative (controller)
PLC	Programmable logic controller
PSS	Periodic short-term start
PVT	Pressure–volume–temperature (fluid properties)
TVD	True vertical depth
VLP	Vertical lift performance
VSD/VFD	Variable-speed / variable-frequency drive

Appendix A. Auxiliary Derivations for the Reduced-Order ESP Model

Appendix A.1. Derivation of the ESP Pressure-Drop Function

We start from the pressure balance along the tubing:

$$P_{fl} = (P_{in} + \Delta P_{esp}) - \rho_{mix} \cdot g \cdot h_{tubing} - \Delta P_{tubing} - \Delta P_{choke}, \quad (A1)$$

where P_{fl} is the flowline pressure; ΔP_{esp} is the pressure increase across the ESP; ρ_{mix} is the average mixture density in the tubing; h_{tubing} is the tubing setting depth (TVD); ΔP_{tubing} is the tubing pressure loss; ΔP_{choke} is the pressure drop across the wellhead choke; and $(P_{in} + \Delta P_{esp})$ is the pump-discharge pressure.

The pressure drop across the choke, ΔP_{choke} , is typically determined by the external surface network infrastructure, which, in the general case, may be represented as a function $P_{wh,i} = f_{network}(Q_{liquid}^{i \neq j})$, where i and j index the wells, i.e., the nodes of the system.

Within the scope of the present formulation, the effect of the network is neglected based on Assumption 3. This assumption is justified because the mutual influence of the wells on one another during the optimization of their operating conditions is sufficiently small and does not materially affect the final solution.

Furthermore, the time scale of the problem (Section 2.4) is short enough that flow redistribution within the field network, and the associated variations in wellhead pressures, may be regarded as negligible.

Accordingly, the wellhead pressure for each well is assumed to be a known boundary condition and is treated as constant at the wellhead:

$$P_{wh,i} - \Delta P_{choke,i} = P_{fl,i} = \text{const.} \quad (A2)$$

Using Equation (A2), the choke pressure drop is written as

$$\Delta P_{choke} = P_{wh} - P_{fl}. \quad (A3)$$

The pressure increase across the i -th stage of the ESP is computed via interpolation of the known pump characteristic curves [39] (pp. 27–30):

$$\Delta P_{esp,i} = \rho_{mix,i} \cdot g \cdot H_{esp}(Q_{mix,i}, F, hf, k_{gas,i}, k_{mu,i}), \quad (A4)$$

where H_{esp} is the head–rate characteristic of the ESP; $Q_{mix,i}$ is the gas–liquid mixture flow rate at the inlet of the i -th stage; F is the shaft rotational frequency; $\rho_{mix,i}$ is the fluid density evaluated at the i -th stage inlet conditions; hf is the head-degradation correction, assumed known; $k_{gas,i}$ is the gas-related head-degradation correction at the i -th stage [37]; and $k_{mu,i}$ is the viscosity-related head-degradation correction at the i -th stage [38].

The total pressure increase across an N -stage pump is then

$$\Delta P_{esp} = \sum_{i=1}^N \rho_{mix,i} \cdot g \cdot H_{esp}(Q_{mix,i}, F, hf, k_{gas,i}, k_{mu,i}). \quad (A5)$$

In most cases, the stage-by-stage evaluation (A5) is regarded as excessive and is reserved for cases that require a detailed analysis of stage-by-stage degradation, or a particularly high model accuracy and sensitivity.

Assuming that the PVT properties of the mixture remain unchanged across stages, equation (A5) can be rewritten as

$$\Delta P_{esp} = \rho_{mix,i} \cdot g \cdot hf \cdot k_{gas} \cdot k_{mu} \cdot \sum_{i=1}^N H_{esp}(Q_{mix}, F). \quad (A6)$$

Introducing the composite degradation coefficient

$$k_{degr} = hf \cdot k_{gas} \cdot k_{mu}, \quad (A7)$$

the total ESP pressure-drop function takes the compact form

$$\Delta P_{esp} = \rho_{mix,i} \cdot g \cdot k_{degr} \cdot N \cdot H_{esp}(Q_{mix}, F). \quad (A8)$$

Substituting (A3) and (A8) into the tubing-balance equation (A1) yields

$$P_{fl} = \left(P_{in} + \rho_{mix} \cdot g \cdot k_{degr} \cdot N \cdot H_{esp}(Q_{mix}, F) \right) - \rho_{mix} \cdot g \cdot h_{tubing} - \Delta P_{tubing} - (P_{wh} - P_{fl}). \quad (A9)$$

The flowline pressure P_{fl} cancels on both sides:

$$\cancel{P_{fl}} = P_{in} + \rho_{mix} \cdot g \cdot k_{degr} \cdot N \cdot H_{esp}(Q_{mix}, F) - \rho_{mix} \cdot g \cdot h_{tubing} - \Delta P_{tubing} - P_{wh} + \cancel{P_{fl}}, \quad (A10)$$

$$0 = P_{in} + \rho_{mix} \cdot g \cdot k_{degr} \cdot N \cdot H_{esp}(Q_{mix}, F) - \rho_{mix} \cdot g \cdot h_{tubing} - \Delta P_{tubing} - P_{wh}, \quad (A11)$$

and rearranging the remaining terms gives

$$\rho_{mix} \cdot g \cdot k_{degr} \cdot N \cdot H_{esp}(Q_{mix}, F) = (P_{wh} - P_{in}) + \rho_{mix} \cdot g \cdot h_{tubing} + \Delta P_{tubing}, \quad (A12)$$

$$k_{degr} \cdot N \cdot H_{esp}(Q_{mix}, F) = \frac{P_{wh} - P_{in}}{\rho_{mix} \cdot g} + h_{tubing} + \frac{\Delta P_{tubing}}{\rho_{mix} \cdot g}. \quad (A13)$$

Equation (A13) also makes it possible to formulate an existence condition for the solution. In the present setting, we exclude “pump-as-turbine” operating modes [16], in which the ESP behaves as a hydraulic resistance, and therefore impose constraints on the left-hand side. The right-hand side must then satisfy

$$\frac{P_{wh} - P_{in}}{\rho_{mix} \cdot g} + h_{tubing} + \frac{\Delta P_{tubing}}{\rho_{mix} \cdot g} \geq 0, \quad (A14)$$

$$(P_{wh} - P_{in}) + \rho_{mix} \cdot g \cdot h_{tubing} + \Delta P_{tubing} \geq 0. \quad (A15)$$

As in (22b), we further assume that the hydraulic losses along the tubing have a negligible effect on the final pressure balance:

$$\Delta P_{tubing} \ll (P_{wh} - P_{in}) + \rho_{mix} \cdot g \cdot h_{tubing}. \quad (A16)$$

The final ESP system is then written as

$$\begin{cases} k_{\text{degr}} \cdot N \cdot H_{\text{esp}}(Q_{\text{mix}}, F) = \frac{P_{\text{wh}} - P_{\text{in}}}{\rho_{\text{mix}} \cdot g} + h_{\text{tubing}}, \\ (P_{\text{wh}} - P_{\text{in}}) + \rho_{\text{mix}} \cdot g \cdot h_{\text{tubing}} \geq 0. \end{cases} \quad (\text{A17})$$

Appendix A.2. Derivation of the ESP Flow-Rate Function

Adopt the quadratic head approximation (see Equation (27))

$$H_{\text{esp}}(Q_{\text{liq}}) = a \cdot Q_{\text{liq}}^2 + b \cdot Q_{\text{liq}} + c, \quad (\text{A18})$$

where, for any given pump characteristic head–rate curve, the coefficients a , b and c can be pre-fitted by the least-squares method.

The affinity laws [39] (pp. 34–36) provide

$$Q_{F_i} = Q_{F_{\text{nom}}} \cdot \frac{F_i}{F_{\text{nom}}}, \quad (\text{A19})$$

$$H_{F_i} = H_{F_{\text{nom}}} \cdot \left(\frac{F_i}{F_{\text{nom}}} \right)^2, \quad (\text{A20})$$

where $(...)_{F_{\text{nom}}}$ denotes the “nominal” values at the nominal shaft frequency provided by the vendor with the equipment, and $(...)_{F_i}$ denotes the values at the actual frequency F_i .

From (A19), in order to **rescale a flow rate from F_i to the F_{nom} scale**, we use the transformation:

$$Q_{\text{nom } F_i} = Q_{F_i} \cdot \frac{F_{\text{nom}}}{F_i}. \quad (\text{A21})$$

Combining (A20) and (A21) with the quadratic form (A18) gives

$$\begin{aligned} H_{\text{esp}}(Q_{\text{liq}}) = & \left(a \cdot \left(Q_{F_i} \cdot \frac{F_{\text{nom}}}{F_i} \right)^2 + b \cdot \left(Q_{F_i} \cdot \frac{F_{\text{nom}}}{F_i} \right) + c \right) \times \\ & \times \left(\left(\frac{F_i}{F_{\text{nom}}} \right)^2 \cdot k_{\text{degr}} \right). \end{aligned} \quad (\text{A22})$$

Expanding the products and collecting terms in (A22) yields

$$\begin{aligned} H_{\text{esp}}(Q_{F_i}) = & a \cdot k_{\text{degr}} \cdot Q_{F_i}^2 + b \cdot k_{\text{degr}} \cdot \frac{F_{\text{nom}}}{F_i} \cdot Q_{F_i} + \\ & + c \cdot \left(\frac{F_i}{F_{\text{nom}}} \right)^2 \cdot k_{\text{degr}}. \end{aligned} \quad (\text{A23})$$

Substituting (A23) into the pressure balance (26a) and dividing both sides by the number of stages N gives

$$\begin{aligned} a \cdot k_{\text{degr}} \cdot Q_{F_i}^2 + b \cdot k_{\text{degr}} \cdot \frac{F_{\text{nom}}}{F_i} \cdot Q_{F_i} + c \cdot \left(\frac{F_i}{F_{\text{nom}}} \right)^2 \cdot k_{\text{degr}} = \\ = \frac{P_{\text{wh}} - P_{\text{in}}}{\rho_{\text{mix}} \cdot g \cdot N} + \frac{h_{\text{tubing}}}{N}. \end{aligned} \quad (\text{A24})$$

Equation (A24) can be recast in standard quadratic form as

$$\begin{cases} \mathcal{A} \cdot Q_{Fi}^2 + \mathcal{B} \cdot Q_{Fi} + \mathcal{C} = 0, \\ \mathcal{A} = a \cdot k_{\text{degr}}, \\ \mathcal{B} = b \cdot k_{\text{degr}} \cdot \frac{F_{nom}}{F_i}, \\ \mathcal{C} = c \cdot \left(\frac{F_i}{F_{nom}} \right)^2 \cdot k_{\text{degr}} - \frac{P_{wh} - P_{in}}{\rho_{mix} \cdot g \cdot N} - \frac{h_{tubing}}{N}. \end{cases} \quad (\text{A25})$$

Appendix A.3. Derivation of the Integration Constant

Taking the boundary conditions at $t = 0$ as known, the general solution (32) yields the system

$$\begin{cases} P_{in_0} = C \cdot e^{A \cdot K \cdot 0} - \frac{Q_{vlp}}{K} + P_{in_{max}}, \\ P_{in_i} = C \cdot e^{A \cdot K \cdot t_i} - \frac{Q_{vlp}}{K} + P_{in_{max}}. \end{cases} \quad (\text{A26})$$

Solving the first equation for the integration constant gives

$$C = P_{in_0} + \left(\frac{Q_{vlp}}{K} - P_{in_{max}} \right). \quad (\text{A27})$$

Appendix B. Auxiliary Derivations for the Stability Criterion

Appendix B.1. Derivation of the Function $\overline{P_{in}}(N)$

To keep the manipulations compact, we denote the numerator of (36) by "...", and — since the integrals are definite — we drop the integration constants C_1 and C_2 from (38) and (40). The numerator of (36) can then be written as

$$\begin{aligned} \dots &= \frac{C}{b_1} \cdot e^{b_1 \cdot (T_w + N \cdot (T_w + T_s))} + b_2 \cdot (T_w + N \cdot (T_w + T_s)) \\ &\quad - \frac{C}{b_1} \cdot e^{b_1 \cdot N \cdot (T_w + T_s)} - b_2 \cdot N \cdot (T_w + T_s) \\ &\quad + \frac{C}{b_1} \cdot e^{b_1 \cdot ((T_w + T_s) + N \cdot (T_w + T_s))} + b_3 \cdot ((T_w + T_s) + N \cdot (T_w + T_s)) \\ &\quad - \frac{C}{b_1} \cdot e^{b_1 \cdot (T_w + N \cdot (T_w + T_s))} - b_3 \cdot (T_w + N \cdot (T_w + T_s)) \end{aligned} \quad (\text{A28})$$

$$\begin{aligned} \dots &= b_2 \cdot (T_w + N \cdot (T_w + T_s)) - N \cdot (T_w + T_s) - \frac{C}{b_1} \cdot e^{b_1 \cdot N \cdot (T_w + T_s)} \\ &\quad + \frac{C}{b_1} \cdot e^{b_1 \cdot (T_w + T_s)} \cdot e^{b_1 \cdot N \cdot (T_w + T_s)} \end{aligned} \quad (\text{A29})$$

$$\begin{aligned} &+ b_3 \cdot (T_w + T_s + N \cdot (T_w + T_s)) - T_w - N \cdot (T_w + T_s) \\ \dots &= \frac{C}{b_1} \cdot e^{b_1 \cdot N \cdot (T_w + T_s)} \cdot (e^{b_1 \cdot (T_w + T_s)} - 1) + b_2 \cdot T_w + b_3 \cdot T_s. \end{aligned} \quad (\text{A30})$$

Expanding all coefficients in equation (A30) and combining like terms,

$$\dots = \frac{C}{A \cdot K} \cdot e^{A \cdot K \cdot N \cdot (T_w + T_s)} \cdot (e^{A \cdot K \cdot (T_w + T_s)} - 1) - \frac{Q_{vlp}}{K} \cdot T_w + P_{in}^{max} \cdot T_w + P_{in}^{max} \cdot T_s, \quad (\text{A31})$$

$$\dots = \frac{C}{A \cdot K} \cdot e^{A \cdot K \cdot N \cdot (T_w + T_s)} \cdot (e^{A \cdot K \cdot (T_w + T_s)} - 1) - \frac{Q_{vlp}}{K} \cdot T_w + (T_w + T_s) \cdot P_{in}^{max}. \quad (\text{A32})$$

Substituting (A32) into the original Equation (36) and simplifying gives

$$\overline{P}_{in}(N) = \frac{\frac{C}{A \cdot K} \cdot e^{A \cdot K \cdot N \cdot (T_w + T_s)} \cdot (e^{A \cdot K \cdot (T_w + T_s)} - 1) - \frac{Q_{vlp}}{K} \cdot T_w + (T_w + T_s) \cdot P_{in}^{max}}{T_{work} + T_{std}}, \quad (A33)$$

$$\overline{P}_{in}(N) = \frac{C}{A \cdot K} \cdot \frac{e^{A \cdot K \cdot (T_w + T_s)} - 1}{T_{work} + T_{std}} \cdot e^{A \cdot K \cdot N \cdot (T_w + T_s)} - \frac{Q_{vlp}}{K} \cdot \frac{T_w}{T_{work} + T_{std}} + P_{in}^{max}. \quad (A34)$$

Equation (A34) is then equivalent to

$$\begin{cases} \overline{P}_{in}(N) = a \cdot e^{b \cdot N} + c, \quad N \in \{0, 1, 2, 3, \dots\} \\ a = \frac{C}{A \cdot K} \cdot \frac{e^{A \cdot K \cdot (T_w + T_s)} - 1}{T_{work} + T_{std}} \\ b = A \cdot K \cdot (T_w + T_s) \\ c = P_{in}^{max} - \frac{Q_{vlp}}{K} \cdot \frac{T_w}{T_{work} + T_{std}} \end{cases} \quad (A35)$$

Appendix B.2. Identification of the Coefficients of $\overline{P}_{in}(N)$

We solve the system (42). Eliminating c through $c = \overline{P}_{in_0} - a$ and substituting into the two remaining equations gives

$$\begin{cases} \overline{P}_{in_1} = a \cdot e^b + \overline{P}_{in_0} - a \\ \overline{P}_{in_2} = a \cdot e^{2 \cdot b} + \overline{P}_{in_0} - a \end{cases} \quad (A36)$$

which, after rearrangement, reads

$$\begin{cases} c = \overline{P}_{in_0} - a \\ \overline{P}_{in_1} - \overline{P}_{in_0} = a \cdot (e^b - 1) \\ \overline{P}_{in_2} - \overline{P}_{in_0} = a \cdot (e^{2 \cdot b} - 1) \end{cases} \quad (A37)$$

and, solving the last two equations for a ,

$$\begin{cases} c = \overline{P}_{in_0} - a \\ a = \frac{\overline{P}_{in_1} - \overline{P}_{in_0}}{e^b - 1} \\ a = \frac{\overline{P}_{in_2} - \overline{P}_{in_0}}{e^{2 \cdot b} - 1} \end{cases} \quad (A38)$$

Equating the two expressions for a yields

$$\frac{\overline{P}_{in_1} - \overline{P}_{in_0}}{e^b - 1} = \frac{\overline{P}_{in_2} - \overline{P}_{in_0}}{e^{2 \cdot b} - 1}. \quad (A39)$$

Setting $x = e^b$, so that $e^{2b} = x^2$,

$$\frac{\overline{P}_{in_1} - \overline{P}_{in_0}}{x - 1} = \frac{\overline{P}_{in_2} - \overline{P}_{in_0}}{x^2 - 1}. \quad (A40)$$

Recalling that $x^2 - 1 = (x - 1)(x + 1)$, we cancel the factor $(x - 1)$ under the constraint $x \neq 1$:

$$\frac{\overline{P_{in_1}} - \overline{P_{in_0}}}{1} = \frac{\overline{P_{in_2}} - \overline{P_{in_0}}}{x + 1}. \quad (\text{A41})$$

Hence

$$x + 1 = \frac{\overline{P_{in_2}} - \overline{P_{in_0}}}{\overline{P_{in_1}} - \overline{P_{in_0}}}, \quad (\text{A42})$$

$$x = \frac{\overline{P_{in_2}} - \overline{P_{in_0}}}{\overline{P_{in_1}} - \overline{P_{in_0}}} - 1. \quad (\text{A43})$$

Returning to $x = e^b$,

$$e^b = \frac{\overline{P_{in_2}} - \overline{P_{in_0}}}{\overline{P_{in_1}} - \overline{P_{in_0}}} - 1 \quad (\text{A44})$$

$$b = \ln \left(\frac{\overline{P_{in_2}} - \overline{P_{in_0}}}{\overline{P_{in_1}} - \overline{P_{in_0}}} - 1 \right), \quad \frac{\overline{P_{in_2}} - \overline{P_{in_0}}}{\overline{P_{in_1}} - \overline{P_{in_0}}} - 1 > 0. \quad (\text{A45})$$

Substituting (A44) into the expression for a in system (A38) yields

$$a = \frac{\overline{P_{in_1}} - \overline{P_{in_0}}}{\left(\frac{\overline{P_{in_2}} - \overline{P_{in_0}}}{\overline{P_{in_1}} - \overline{P_{in_0}}} - 1 \right) - 1}, \quad (\text{A46})$$

$$a = \frac{\overline{P_{in_1}} - \overline{P_{in_0}}}{\frac{\overline{P_{in_2}} - \overline{P_{in_0}}}{\overline{P_{in_1}} - \overline{P_{in_0}}} - 2}. \quad (\text{A47})$$

Multiplying the numerator and denominator by $(\overline{P_{in_1}} - \overline{P_{in_0}})$ gives

$$a = \frac{(\overline{P_{in_1}} - \overline{P_{in_0}})^2}{\overline{P_{in_2}} - 2 \cdot \overline{P_{in_1}} + \overline{P_{in_0}}}. \quad (\text{A48})$$

The coefficients a , b and c are therefore determined by the following relations (subject to the conditions illustrated in Figure 5):

$$\left\{ \begin{array}{l} a = \frac{(\overline{P_{in_1}} - \overline{P_{in_0}})^2}{\overline{P_{in_2}} - 2 \cdot \overline{P_{in_1}} + \overline{P_{in_0}}} \\ b = \ln \left(\frac{\overline{P_{in_2}} - \overline{P_{in_0}}}{\overline{P_{in_1}} - \overline{P_{in_0}}} - 1 \right) \\ c = \overline{P_{in_0}} - a \\ \overline{P_{in_0}} \neq \overline{P_{in_1}} \\ \overline{P_{in_2}} - 2 \cdot \overline{P_{in_1}} + \overline{P_{in_0}} \neq 0 \\ \frac{\overline{P_{in_2}} - \overline{P_{in_0}}}{\overline{P_{in_1}} - \overline{P_{in_0}}} - 1 > 0 \end{array} \right. \quad (\text{A49})$$

Appendix B.3. Analysis of the Conditions Imposed on the Coefficients of $\overline{P_{in}}(N)$

The conditions illustrated in Figure 5 can be derived directly from the analysis of Equation (43).

For brevity, let us denote

$$P_0 = \overline{P_{in_0}}, \quad P_1 = \overline{P_{in_1}}, \quad P_2 = \overline{P_{in_2}}. \quad (\text{A50})$$

System (43) then gives

$$a = \frac{(P_1 - P_0)^2}{P_2 - 2P_1 + P_0}, \quad b = \ln\left(\frac{P_2 - P_0}{P_1 - P_0} - 1\right), \quad (\text{A51})$$

subject to $P_1 \neq P_0$, $P_2 - 2P_1 + P_0 \neq 0$, and $\frac{P_2 - P_0}{P_1 - P_0} - 1 > 0$.

The numerator of a is always positive ($(P_1 - P_0)^2 > 0$ whenever $P_1 \neq P_0$), so the sign of a is determined by the sign of its denominator:

- if $P_2 - 2P_1 + P_0 < 0$, then $a < 0$;
- if $P_2 - 2P_1 + P_0 > 0$, then $a > 0$.

Furthermore, $b < 0$ requires the argument of the logarithm to be smaller than unity (and positive):

$$0 < \frac{P_2 - P_0}{P_1 - P_0} - 1 < 1. \quad (\text{A52})$$

Adding 1 to all sides yields

$$1 < \frac{P_2 - P_0}{P_1 - P_0} < 2. \quad (\text{A53})$$

Two cases are considered, distinguished by the sign of $P_1 - P_0$.

1. Case $P_1 - P_0 > 0$ (i.e., $P_1 > P_0$).

Multiplying $\left(1 < \frac{P_2 - P_0}{P_1 - P_0} < 2\right)$ by the positive quantity $P_1 - P_0$ yields:

- $P_2 - P_0 > P_1 - P_0$, i.e., $P_2 > P_1$;
- $P_2 - P_0 < 2(P_1 - P_0)$, i.e., $P_2 < 2P_1 - P_0$.

The condition for a becomes $a < 0 \iff P_2 - 2P_1 + P_0 < 0$, equivalent to $P_2 < 2P_1 - P_0$.

Hence, for $P_1 > P_0$, the condition $b < 0$ requires

$$P_1 < P_2 < 2P_1 - P_0, \quad (\text{A54})$$

and $a < 0$ is automatically equivalent to $P_2 < 2P_1 - P_0$.

2. Case $P_1 - P_0 < 0$ (i.e., $P_1 < P_0$).

From $\left(1 < \frac{P_2 - P_0}{P_1 - P_0}\right)$: $P_2 - P_0 < P_1 - P_0 \implies P_2 < P_1$.

From $\frac{P_2 - P_0}{P_1 - P_0} < 2$: $P_2 - P_0 > 2(P_1 - P_0) \implies P_2 > 2P_1 - P_0$.

Hence, for $b < 0$ with $P_1 < P_0$, we must have $2P_1 - P_0 < P_2 < P_1$.

Meanwhile, the sign of a obeys $a > 0 \iff P_2 - 2P_1 + P_0 > 0$, which here means $P_2 > 2P_1 - P_0$.

Therefore, for $P_1 < P_0$, the joint condition $a > 0$ and $b < 0$ is equivalent to

$$2P_1 - P_0 < P_2 < P_1. \quad (\text{A55})$$

Note that for $P_1 < P_0$ the conditions $a < 0$ and $b < 0$ are mutually inconsistent (since $a < 0$ would require $P_2 < 2P_1 - P_0$, while $b < 0$ requires $P_2 > 2P_1 - P_0$). This is consistent with intuition: an increasing trend in P_{in} cannot be expected if the cycle-averaged pressure over the observed three pumping–accumulation cycles is in fact decreasing, and vice versa.

In summary:

- For $a < 0$ and $b < 0$ to hold (the *increasing* -trend case):
 - it must be that $P_1 > P_0$;
 - and $P_1 < P_2 < 2P_1 - P_0$.
- For $a > 0$ and $b < 0$ to hold (the *decreasing*-trend case):
 - it must be that $P_1 < P_0$;
 - and $2P_1 - P_0 < P_2 < P_1$.

Appendix B.4. Conversion from Per-Cycle to Daily-Averaged Flow Rate

Note that Q_{vlp} is the flow rate during the “work” phase, i.e., the rate delivered at frequency F over the interval T_{work} .

To convert it to the daily-averaged liquid rate Q_{liq} , we apply the following transformation:

$$V_{work} = Q_{vlp} \cdot T_{work} \quad (A56)$$

$$V_{total} = V_{work} \cdot n = V_{work} \cdot \frac{T_{day}}{T_{work} + T_{std}} = Q_{vlp} \cdot T_{work} \cdot \frac{T_{day}}{T_{work} + T_{std}} \quad (A57)$$

$$Q_{liq} = \frac{V_{total}}{T_{day}} = \frac{Q_{vlp} \cdot T_{work} \cdot \frac{T_{day}}{T_{work} + T_{std}}}{T_{day}} \quad (A58)$$

$$Q_{liq} = Q_{vlp} \cdot \frac{T_{work}}{T_{work} + T_{std}} \quad (A59)$$

Appendix C. Well Configuration for Experiments

Appendix C.1. Closed-Loop Feedback Control Simulation

Table A1. Well Parameters #1.

Group	Parameter	Value	Unit
General	p_{wh}	1,519,875.0	Pa
General	t_{fl}	293.15	°K
General	t_{res}	303.15	°K
General	f_{work}	40.0	Hz
General	t_{work}	1200.0	s
General	t_{std}	1200.0	s
General	k_{prod}	7.996×10^{-11}	(m ³ /s)/Pa
General	p_{res}	20,872,950.0	Pa
General	k_{sep}	0.75	—
Fluid	WCT	0.6	—
Fluid	γ_{wat}	1.0	—
Fluid	γ_{gas}	0.6	—
Fluid	γ_{oil}	0.8	—
Fluid	R_p	60.0	m ³ /m ³
ESP	N_{stages}	350	—
ESP	$q_{mix,min}$	0.0	m ³ /s
ESP	$q_{mix,max}$	2.872×10^{-3}	m ³ /s
ESP	a_{head}	-1.058×10^6	m
ESP	b_{head}	1,287.382	m
ESP	c_{head}	5.390	m
ESP	f_{nom}	50.0	Hz
ESP	H_{nom}	5.46	m

Table A1. *Cont.*

Group	Parameter	Value	Unit
Construction	h_{mes}	1400.0	m
Construction	h_{bot}	1800.0	m
Construction	d_{casing}	0.146	m
Construction	d_{tubing}	0.062	m
Construction	s_{wall}	0.0055	m
Casing PVT	R_{sb}	80.0	m^3/m^3
Casing PVT	$R_{sb,p}$	10,000,000.0	Pa
Casing PVT	$R_{sb,t}$	303.15	$^{\circ}K$
Casing PVT	B_{ob}	1.5	m^3/m^3
Casing PVT	$B_{ob,p}$	10,000,000.0	Pa
Casing PVT	$B_{ob,t}$	303.15	$^{\circ}K$
Casing PVT	μ_{ob}	0.5	cP
Casing PVT	$\mu_{ob,p}$	10,000,000.0	Pa
Casing PVT	$\mu_{ob,t}$	303.15	$^{\circ}K$

*Appendix C.2. Real-Life Well Open-Loop Feedback Back-Test Case***Table A2.** Well Parameters #2.

Group	Parameter	Value	Unit
General	t_{fl}	278.15	$^{\circ}K$
General	t_{res}	308.15	$^{\circ}K$
General	f_{work}	54.3	Hz
General	t_{work}	96.0	s
General	t_{std}	1668.0	s
General	k_{prod}	4.362×10^{-11}	$(m^3/s)/Pa$
General	p_{res}	13,131,720.0	Pa
General	k_{sep}	0.75	—
Fluid	WCT	0.002	—
Fluid	γ_{wat}	1.0	—
Fluid	γ_{gas}	0.7	—
Fluid	γ_{oil}	0.835	—
Fluid	R_p	74.40	m^3/m^3
ESP	N_{stages}	206	—
ESP	$q_{mix,min}$	0.0	m^3/s
ESP	$q_{mix,max}$	2.308×10^{-3}	m^3/s
ESP	a_{head}	-2.454×10^6	m
ESP	b_{head}	2,131.401	m
ESP	c_{head}	8.206	m
ESP	f_{nom}	58.3	Hz
ESP	H_{nom}	7.37	m
Construction	h_{mes}	1357.39	m
Construction	h_{bot}	2093.43	m
Construction	d_{casing}	0.1246	m
Construction	d_{tubing}	0.062	m
Construction	s_{wall}	0.0055	m
Casing PVT	R_{sb}	300.0	m^3/m^3
Casing PVT	$R_{sb,p}$	18,238,500.0	Pa
Casing PVT	$R_{sb,t}$	308.15	$^{\circ}K$
Casing PVT	B_{ob}	1.33	m^3/m^3
Casing PVT	$B_{ob,p}$	18,238,500.0	Pa
Casing PVT	$B_{ob,t}$	308.15	$^{\circ}K$
Casing PVT	μ_{ob}	0.99	cP
Casing PVT	$\mu_{ob,p}$	18,238,500.0	Pa
Casing PVT	$\mu_{ob,t}$	308.15	$^{\circ}K$

References

1. Bedrin, V.G.; Khasanov, M.M.; Khabibullin, R.A.; Krasnov, V.A.; Pashali, A.A.; Litvinenko, K.V.; Elichev, V.A.; Prado, M. High GLR ESP Technologies Comparison, Field Test Results. In Proceedings of the SPE Russian Oil and Gas Technical Conference and Exhibition, Moscow, Russia, 28–30 October 2008; Paper SPE-117414-MS. <https://doi.org/10.2118/117414-MS>.
2. Li, Q.; You, D.; Li, Q.; Wang, F.; Wang, Y.; Yang, Y. Analysis of Sedimentation Behavior and Influencing Factors of Solid Particles in CO₂ Fracturing Fluid. *Processes* **2025**, *13*, 4049. <https://doi.org/10.3390/pr13124049>.
3. Li, Q.; Li, Q.; Wu, J.; He, K.; Xia, Y.; Liu, J.; Wang, F.; Cheng, Y. Wellhead Stability During Development Process of Hydrate Reservoir in the Northern South China Sea: Sensitivity Analysis. *Processes* **2025**, *13*, 1630. <https://doi.org/10.3390/pr13061630>.
4. Suleimanov, R.I.; Khabibullin, M.Y.; Suleimanov, R.I. Influence of Complicating Factors on the Operation of an Electric Centrifugal Pump Installation. *IOP Conf. Ser. Mater. Sci. Eng.* **2020**, *905*, 012091. <https://doi.org/10.1088/1757-899X/905/1/012091>.
5. Mamchistova, E.I.; Levitina, E.E.; Nazarova, N.V. Impact Assessment of Various Factors on Operational Efficiency of ESCP. *IOP Conf. Ser. Earth Environ. Sci.* **2019**, *378*, 012093. <https://doi.org/10.1088/1755-1315/378/1/012093>.
6. Bagci, A.S.; Kece, M.; Nava, J. Challenges of Using Electrical Submersible Pump (ESP) in High Free Gas Applications. In Proceedings of the CPS/SPE International Oil & Gas Conference and Exhibition in China, Beijing, China, 8–10 June 2010; Paper SPE-131760-MS. <https://doi.org/10.2118/131760-MS>.
7. Powers, M.L. Special Considerations for Electric Submersible Pump Applications in Underpressured Reservoirs. *SPE Prod. Eng.* **1992**, *7*, 301–306. Paper SPE-22786-PA, <https://doi.org/10.2118/22786-PA>.
8. Mokhov, M.; Tsunevskiy, A. Research of Gas Separator with Disperser Influence of ESP Work and Increase of Well Operation Efficiency by the Use of ESP. In Proceedings of the SPE Annual Technical Conference and Exhibition, New Orleans, LA, USA, 4–7 October 2009; Paper SPE-124775-MS, <https://doi.org/10.2118/124775-MS>.
9. Pedrotti, M.M.; Biazussi, J.L.; Bannwart, A.C.; Sassim, N.A. A Time-Frequency Study of the Behavior of an Electrical Submersible Pump Operating Nearly the Gas-Locking Condition. In Proceedings of the SPE Artificial Lift Conference – Latin America and Caribbean, Salvador, Brazil, 27–28 May 2015; pp. 300–309; Paper SPE-173951-MS. <https://doi.org/10.2118/173951-MS>.
10. Vachon, G.; Furui, K. Production Optimization in ESP Completions with Intelligent Well Technology by Using Downhole Chokes to Optimize ESP Performance. In Proceedings of the SPE Middle East Oil & Gas Show and Conference, Manama, Bahrain, 12–15 March 2005; Paper SPE-93621-MS. <https://doi.org/10.2118/93621-MS>.
11. Camilleri, L. Free Gas and ESP; Case Studies Illustrating the Difference Between Flowrate Oscillations, Gas Locking and Instability. In Proceedings of the SPE Annual Technical Conference and Exhibition, Virtual, 26–29 October 2020; Paper SPE-201476-MS. <https://doi.org/10.2118/201476-MS>.
12. Brinkhorst, J.W. Successful Application of High GOR ESPs in the Lekhwair Field. In Proceedings of the Abu Dhabi International Petroleum Exhibition and Conference, Abu Dhabi, United Arab Emirates, 11–14 October 1998; Paper SPE-49466-MS. <https://doi.org/10.2118/49466-MS>.
13. Khabibullin, R.; Sarapulov, N. ESP Energy Efficiency Analysis on Western Siberia Fields. In Proceedings of the SPE Russian Petroleum Technology Conference, Moscow, Russia, 15–17 October 2018; Paper SPE-191538-18RPTC-MS. <https://doi.org/10.2118/191538-MS>.
14. Lu, X.; Han, G.; Wang, L.; Zhang, Z.; Liang, X. Fault Diagnosis of Electric Submersible Pump System Based on Motor Current Signal Analysis and Deep Learning Method. *SPE J.* **2025**, *30*, 2238–2255. <https://doi.org/10.2118/225426-PA>.
15. Camilleri, L.A.; Gong, H.; Al-Maqseed, N.H.; Al-Jazzaf, A.M. Tuning VSDs in ESP Wells to Optimize Oil Production—Case Studies. In Proceedings of the SPE Artificial Lift Conference and Exhibition – Americas, The Woodlands, TX, USA, 28–30 August 2018; Paper SPE-190940-MS. <https://doi.org/10.2118/190940-MS>.
16. Petrushin, M.A.; Kherson, E.Y.; Liubimov, V.S.; Popravko, K.A.; Petyaev, V.A.; Shpakov, S.O. Modeling of Mechanized Wells Operating in Alternating Frequency Mode Considering Check Valve Leakage and Practical Application for Efficient Well Management. In Proceedings of the SPE Annual Technical Conference and Exhibition, Houston, TX, USA, 20–22 October 2025; Paper SPE-227858-MS. <https://doi.org/10.2118/227858-MS>.
17. Kurchukov, M.O.; Dudnik, P.D.; Kherson, E.Y.; Petrushin, M.A.; Smirnov, N.A.; Yudin, E.V.; Isaeva, S.M. Slug Flow Regime Modeling Using Machine Learning and Practical Application for ESP Well Optimization. In Proceedings of the SPE Annual Caspian Technical Conference and Exhibition, Baku, Azerbaijan, 25–27 November 2025; Paper SPE-230456-MS. <https://doi.org/10.2118/230456-MS>.
18. Shostak, V.F.; Shostak, O.V. The Control of Technological Complexes in Emergency (Extreme) Regimes. *IFAC Proc. Vol.* **1995**, *28*, 137–139. [https://doi.org/10.1016/S1474-6670\(17\)51506-2](https://doi.org/10.1016/S1474-6670(17)51506-2).
19. Li, Q.; Wang, F.; Wu, J.; Li, Q.; Zhang, G. Multivariate Coupling Model and Reservoir Characteristics of Enhanced Geothermal Reservoirs. *Energies* **2026**, *19*, 3180. <https://doi.org/10.3390/en19133180>.
20. Liu, Y.; Li, W.; Xiao, Z.; Ji, S.; Liu, Q.; Tang, Y.; Zhang, Y.; Wang, J. Recent Progress in Organic Inhibitors for Anticorrosion in Complex Acid Environments. *Coatings* **2026**, *16*, 150. <https://doi.org/10.3390/coatings16020150>.

21. Pashali, A.A.; Khalfin, R.S.; Silnov, D.V.; Topolnikov, A.S.; Latypov, B.M. On the Optimization of the Periodic Mode of Well Production Operated by Submersible Electric Pumps in Rosneft Oil Company. *Neft. Khozyaystvo Oil Ind.* **2021**, 2021, 92–96. (In Russian) <https://doi.org/10.24887/0028-2448-2021-4-92-96>.
22. Topolnikov, A.S. Argumentation of Application if Quasi-Stationary Model to Describe the Periodic Regime of Oil Well. *Proc. Mavlyutov Inst. Mech.* **2017**, 12, 15–26. (In Russian) <https://doi.org/10.21662/uim2017.1.003>.
23. Brown, K.E.; Lea, J.F. Nodal Systems Analysis of Oil and Gas Wells. *J. Pet. Technol.* **1985**, 37, 1751–1763. <https://doi.org/10.2118/14714-PA>.
24. Burakov, I.M.; Garipov, T.T.; Sharipov, T.R.; Zaydullin, R.I. Integrated Hydrodynamic Modelling of the Well–Reservoir System. *Nauchno-Tekhnicheskij Vestnik OAO “NK Rosneft”* **2009**, 15–17. (In Russian) <https://doi.org/10.13140/RG.2.2.36319.01445>.
25. Yudin, E.V.; Piotrovskiy, G.A.; Smirnov, N.A.; Bayrachnyi, D.; Petrushin, M.A.; Isaeva, S.M.; Yudin, P. Modeling and Optimization of ESP Wells Operating in Intermittent Mode. In Proceedings of the SPE Annual Caspian Technical Conference, Nur-Sultan, Kazakhstan, 15–17 November 2022; Paper SPE-212116-MS. <https://doi.org/10.2118/212116-MS>.
26. Yudin, E.V.; Piotrovskiy, G.A.; Smirnov, N.A.; Petrushin, M.A.; Isaeva, S.M.; Zamakhov, S.V.; Popravko, K.A.; Kherson, E.Y. Group Optimization and Modeling of Mechanized Wells Operating in Intermittent Mode. In Proceedings of the Abu Dhabi International Petroleum Exhibition and Conference (ADIPEC), Abu Dhabi, United Arab Emirates, 4–7 November 2024; Paper SPE-222942-MS. <https://doi.org/10.2118/222942-MS>.
27. Petrushin, M.A.; Kherson, E.Y.; Smirnov, N.A.; Yudin, E.V. Performance Optimization of ESP Wells Under Intermittent Operation Using Long-Term Asymptotic Model Predictions. In Proceedings of the SPE Annual Caspian Technical Conference and Exhibition, Baku, Azerbaijan, 25–27 November 2025; Paper SPE-230454-MS. <https://doi.org/10.2118/230454-MS>.
28. Schlumberger. *OLGA User Manual: Version OLGA 2016.2*; User Manual; Schlumberger: Abingdon, UK, 2017;
29. García, C.E.; Prett, D.M.; Morari, M. Model Predictive Control: Theory and Practice—A Survey. *Automatica* **1989**, 25, 335–348. [https://doi.org/10.1016/0005-1098\(89\)90002-2](https://doi.org/10.1016/0005-1098(89)90002-2).
30. de Keyser, R.M.C.; van de Velde, P.G.A.; Dumortier, F.A.G. A Comparative Study of Self-adaptive Long-range Predictive Control Methods. *Automatica* **1988**, 24, 149–163. [https://doi.org/10.1016/0005-1098\(88\)90024-6](https://doi.org/10.1016/0005-1098(88)90024-6).
31. Sharma, R.; Glemmestad, B. Optimal Control Strategies with Nonlinear Optimization for an Electric Submersible Pump Lifted Oil Field. *Model. Identif. Control* **2013**, 34, 55–67. <https://doi.org/10.4173/mic.2013.2.2>.
32. Poland, J.; Stadler, K.S. Stochastic optimal planning of solar thermal power. In Proceedings of the 2014 IEEE Conference on Control Applications (CCA), Juan Les Antibes, France, 8–10 October 2014; pp. 586–592; Part of 2014 IEEE Multi-conference on Systems and Control (MSC), 8–10 October 2014. <https://doi.org/10.1109/CCA.2014.6981404>.
33. Krishnamoorthy, D.; Bergheim, E.M.; Pavlov, A.; Fredriksen, M.; Fjalestad, K. Modelling and Robustness Analysis of Model Predictive Control for Electrical Submersible Pump Lifted Heavy Oil Wells. *IFAC-PapersOnLine* **2016**, 49, 544–549. <https://doi.org/10.1016/j.ifacol.2016.07.399>.
34. Santana, B.A.; Matos, V.S.; Santana, D.D.; Martins, M.A.F. Embedded MPC Strategies for ESP-Lifted Oil Wells: Hardware-in-the-Loop Performance Analysis of Nonlinear and Robust Techniques. *Processes* **2023**, 11, 1354. <https://doi.org/10.3390/pr11051354>.
35. Al-Hussainy, R.; Ramey, H.J.; Crawford, P.B. The Flow of Real Gases Through Porous Media. *J. Pet. Technol.* **1966**, 18, 624–636. Paper SPE-1243-A-PA, <https://doi.org/10.2118/1243-A-PA>.
36. Golan, M.; Whitson, C.H. *Well Performance*, 2nd ed.; Prentice Hall: Englewood Cliffs, NJ, USA, 1991.
37. Zhu, H.; Zhu, J.; Lin, Z.; Zhao, Q.; Rutter, R.; Zhang, H.Q. Performance Degradation and Wearing of Electrical Submersible Pump (ESP) with Gas-Liquid-Solid Flow: Experiments and Mechanistic Modeling. *J. Pet. Sci. Eng.* **2021**, 200, 108399. <https://doi.org/10.1016/j.petrol.2021.108399>.
38. Vieira, T.S.; Siqueira, J.R.; Bueno, A.D.; Morales, R.E.M.; Estevam, V. Analytical Study of Pressure Losses and Fluid Viscosity Effects on Pump Performance during Monophase Flow inside an ESP Stage. *J. Pet. Sci. Eng.* **2015**, 127, 245–258. <https://doi.org/10.1016/j.petrol.2015.01.014>.
39. Takács, G. *Electrical Submersible Pump Manual: Design, Operations, and Maintenance*; Gulf Professional Publishing: Burlington, MA, USA, 2009.
40. Ulanicki, B.; Kahler, J.; Coulbeck, B. Modeling the Efficiency and Power Characteristics of a Pump Group. *J. Water Resour. Plan. Manag.* **2008**, 134, 88–93. [https://doi.org/10.1061/\(ASCE\)0733-9496\(2008\)134:1\(88\)](https://doi.org/10.1061/(ASCE)0733-9496(2008)134:1(88)).
41. Mayne, D.Q.; Rawlings, J.B.; Rao, C.V.; Sokaert, P.O.M. Constrained Model Predictive Control: Stability and Optimality. *Automatica* **2000**, 36, 789–814. [https://doi.org/10.1016/S0005-1098\(99\)00214-9](https://doi.org/10.1016/S0005-1098(99)00214-9).
42. Rawlings, J.B.; Mayne, D.Q.; Diehl, M.M. *Model Predictive Control: Theory, Computation, and Design*, 2nd ed.; Nob Hill Publishing: Madison, WI, USA, 2017.
43. Seidel, D.; Winkelmann, M.; Hartrumpf, M.; Heppner, N. Data-driven Validation for the Safety of Automated Driving. *ATZelectronics Worldw.* **2023**, 18, 38–41. <https://doi.org/10.1007/s38314-023-1463-7>.
44. Kang, Y.; Yin, H.; Berger, C. Test Your Self-Driving Algorithm: An Overview of Publicly Available Driving Datasets and Virtual Testing Environments. *IEEE Trans. Intell. Veh.* **2019**, 4, 171–185. <https://doi.org/10.1109/TIV.2018.2886678>.

45. Zhang, P.; Zhu, B.; Zhao, J.; Fan, T.; Sun, Y. Safety Evaluation Method in Multi-Logical Scenarios for Automated Vehicles Based on Naturalistic Driving Trajectory. *Accid. Anal. Prev.* **2023**, *180*, 106926. <https://doi.org/10.1016/j.aap.2022.106926>.
46. Yudin, E.; Kovaleva, M.; Shevchenko, V.; Bekh, G.; Gudilov, M.; Isaev, D.; Zaytsev, A. Maintaining ESP Operational Efficiency Through Machine Learning-Based Anomaly Detection. *Geoenergy Sci. Eng.* **2025**, *251*, 213864. <https://doi.org/10.1016/j.geoen.2025.213864>.
47. Yudin, E.; Khabibullin, R.; Kobzar, O.; Usikov, D.; Chernyshov, V.; Suleymanov, M.; Grigorev, I.; Ryzhikov, A.; Vrazhevsky, S.; Shabunin, M. Innovative Monitoring Technologies for Well Control Through Sensor Integration and Edge Computing. In Proceedings of the Middle East Oil, Gas and Geosciences Show (MEOS GEO), Manama, Bahrain, 16–18 September 2025; Paper SPE-226940-MS. <https://doi.org/10.2118/226940-MS>.
48. Yudin, E.; Galeev, R.; Isaev, D.; Zamakhov, S.; Kobzar, O. An Integrated Approach to Virtual Flow Metering of Wells with ESP Based on a Combined Hydraulic and Electric Model. In Proceedings of the Abu Dhabi International Petroleum Exhibition and Conference (ADIPEC), Abu Dhabi, United Arab Emirates, 3–6 November 2025; Paper SPE-229250-MS. <https://doi.org/10.2118/229250-MS>.
49. Fraces, C.G.; Tchelepi, H.A. Physics Informed Deep Learning for Flow and Transport in Porous Media. In *Proceedings of the SPE Reservoir Simulation Conference*; Society of Petroleum Engineers (SPE): Richardson, TX, USA, 2021; p. D011S006R002. <https://doi.org/10.2118/203934-MS>.
50. Zhao, M.; Wang, Y.; Gerritsma, M.; Hajibeygi, H. Efficient simulation of CO₂ migration dynamics in deep saline aquifers using a multi-task deep learning technique with consistency. *Adv. Water Resour.* **2023**, *178*, 104494. <https://doi.org/10.1016/j.advwatres.2023.104494>.
51. Brunton, S.L.; Noack, B.R.; Koumoutsakos, P. Machine Learning for Fluid Mechanics. *Annu. Rev. Fluid Mech.* **2020**, *52*, 477–508. <https://doi.org/10.1146/annurev-fluid-010719-060214>.

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.