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Thermo–Fluid–Solid Coupled Prediction of Trapped Annular Pressure in Multi-Annulus Wells

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Abstract

Annular trapped pressure is an important factor affecting wellbore integrity during the production of high-temperature and high-pressure oil and gas wells. In wells with multi-layer casing structures, one or more enclosed annuli may form because of cement top, packer setting, wellbore structural constraints and wellhead sealing. When the temperature and pressure fields in the wellbore change during production, the annular fluid undergoes thermal expansion, compressive deformation and possible phase-state changes, causing trapped pressure to evolve continuously with time. Conventional annular pressure prediction methods are usually based on a single annulus, quasi-static assumptions or simplified fluid properties, and therefore cannot fully describe deformation transfer among annuli, thermal expansion of tubular strings and nonlinear gas–liquid compression. To address this problem, this study analyses the formation mechanism of annular trapped pressure from the perspective of thermal–fluid–solid coupling and develops a dynamic pressure evolution model that accounts for transient temperature variation, annular fluid thermal expansion and compression, elastic deformation of tubular strings and multi-annulus coupling. The analysis indicates that annular trapped pressure is essentially a pressure response produced by fluid thermal expansion under structural confinement. A multi-annulus system is not a set of independent annuli, but a pressure–deformation–volume feedback system coupled through shared casing walls. The proposed solution framework and pressure evolution analysis provide a theoretical basis for annular pressure prediction, casing safety assessment and wellbore-integrity management in high-temperature and high-pressure gas wells.

Keywords: annular trapped pressure; dynamic evolution; multi-annulus coupling; thermal–fluid–solid coupling; tubular deformation; wellbore integrity

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1. Introduction

As oil and gas exploration and development move towards deep, ultra-deep and high-temperature high-pressure reservoirs, the service environment of wellbores becomes substantially more complex than that of conventional wells [1–3]. Deep gas wells commonly have large well depths, high formation temperatures, high pressure gradients and multiple casing layers. During drilling, completion and subsequent production, several annular spaces are formed among the tubing, production casing, intermediate casing and surface casing [4,5]. Under the influence of cement top, packer setting, trapped

annular fluid columns and wellhead sealing, some annuli may become enclosed or semi-enclosed. When the wellbore changes from a static state to a production state, the temperature of the downhole fluid gradually increases. Heat is transferred radially and axially to the casing strings and annular fluids, causing fluid-volume expansion and thermal deformation of tubular strings [6,7]. Because the fluid in an enclosed annulus cannot be freely discharged, the volume increase caused by thermal expansion cannot be completely released and is ultimately manifested as an increase in annular pressure [8]. This phenomenon is commonly referred to as annular trapped pressure [9].

Sustained growth of annular trapped pressure can induce a series of wellbore-integrity risks [10,11]. On the one hand, excessive annular pressure changes the pressure difference across casing strings and imposes additional tensile, compressive and hoop stresses. In severe cases, it may lead to casing yielding, collapse or rupture. On the other hand, repeated annular pressure fluctuations can act on the cement sheath interface, inducing microcracks, interfacial debonding and isolation failure, which may further generate sustained annular pressure channels [12]. For high-temperature and high-pressure gas wells, the safety margin of the wellbore structure is relatively small. The evolution of annular pressure is often not instantaneous, but develops continuously with changes in production regime, wellbore heating and fluid state [13]. Therefore, calculating annular pressure at a single time point is insufficient for evaluating wellbore safety. It is necessary to understand the dynamic evolution mechanism of annular trapped pressure over time.

Existing annular pressure models can be broadly divided into single-annulus thermodynamic models, tubular-deformation models, and coupled multi-annulus models [14]. Single-annulus models generally determine pressure from fluid thermal expansion, compressibility, and an assumed annular-volume constraint [15]. These models explain the basic pressure-buildup mechanism but do not represent deformation transfer between adjacent annuli. Tubular-deformation models improve the geometric-volume calculation by considering casing expansion or contraction under thermal and pressure loads [16]. However, the surrounding annular pressures are often prescribed as boundary conditions rather than solved simultaneously. More recent coupled models have incorporated transient thermal loading, multi-annular temperature and pressure interactions, or casing-deformation effects [17]. These studies demonstrate the importance of coupled analysis. Nevertheless, the simultaneous relationship among annular pressure, shared casing displacement, and annular-volume compatibility remains insufficiently described in a unified reproducible formulation, particularly when axial temperature variation and state-dependent fluid properties are considered together.

The distinction of the present study is not the isolated inclusion of fluid thermal expansion, tubular deformation, or multiple annuli. Instead, these mechanisms are integrated into a synchronous pressure–deformation–volume solution framework. First, Annuli A–C are represented by individual volume-compatibility equations and solved simultaneously. Deformation of each shared casing wall is calculated from the pressures acting on both sides, and the resulting displacement enters the volume equations of the two adjacent annuli with opposite effects. Second, the enclosed annuli are axially segmented so that non-uniform temperature, temperature-dependent fluid properties, and local tubular deformation can be integrated over the enclosed length while maintaining one pressure unknown for each annulus. Third, state-dependent liquid expansion and compressibility, together with gas–liquid compressibility when applicable, are incorporated into the same iterative calculation. Annulus D is treated separately as an independently sealed packer annulus and is not included in the A–C coupling system.

The proposed formulation is evaluated against field pressure measurements and Wellcat (version 5000.17.2, Schlumberger) calculations under two prescribed temperature-rise conditions. Because time-resolved field measurements were unavailable, this

comparison evaluates the predicted endpoint pressure levels rather than the complete transient pressure trajectory. The contribution of this study therefore lies in the unified and reproducible treatment of deformation-mediated coupling among multiple enclosed annuli, rather than in claiming a general time-resolved validation.

2. Formation Mechanism of Annular Trapped Pressure

2.1. Physical Characteristics of an Enclosed Annular System

The formation of annular trapped pressure requires a certain degree of annular confinement. In actual wellbores, an enclosed annulus does not necessarily mean a perfectly leak-free space. Rather, it means that, within the time scale considered, the change in annular fluid mass is relatively small and the thermal expansion of the fluid cannot be sufficiently released by free discharge. The annulus can then be approximated as a constrained-volume system. This system consists of the inner and outer tubular strings or casing strings, the cement sheath and the annular fluid. The fluid is the direct carrier of pressure change, whereas the casing strings and cement sheath form the confining boundaries and are also the sources of deformation feedback.

From a thermodynamic perspective, the mass of fluid in an enclosed annulus is approximately conserved. When temperature increases, the fluid tends to expand. When volume is constrained, this expansion is resisted and converted into pressure increase. From a mechanical perspective, annular boundaries are not perfectly rigid. Casing strings and cement sheaths undergo small elastic deformation under pressure, which changes the annular volume to some extent [18]. Thus, annular trapped pressure is neither a purely fluid thermal expansion problem nor a purely casing-stress problem. It is the result of compatibility between fluid-volume change and structural deformation.

In high-temperature and high-pressure gas wells, the formation of annular trapped pressure generally involves three continuous stages [19]. The first is the initial heating stage during production, in which wellbore temperature changes rapidly, fluid thermal expansion is significant and annular pressure grows quickly. The second is the heat-transfer development stage, in which temperature continues to diffuse radially and axially along the wellbore, and the pressure-growth rate is gradually controlled by casing deformation and fluid compressibility. The third is the relatively stable stage, in which wellbore temperature tends to stabilize and pressure changes become slower. However, when the production regime is adjusted, or when shut-in, restart or production-rate changes occur, annular pressure may fluctuate again. Therefore, annular pressure exhibits clear time dependence and path dependence.

To describe the geometric constraint of an enclosed annulus, the initial volume of the i -th annulus can be expressed by Equation (1).

$$V_{a,i}^0 = \int_0^{L_i} \pi [r_{o,i}^2(z) - r_{in,i}^2(z)] dz \quad (1)$$

where $V_{a,i}^0$ denotes the initial geometric volume of the i -th enclosed annulus; the superscript 000 represents the initial undeformed state, and the subscript a,i represents the annulus domain and its corresponding index, respectively. L_i is the effective enclosed length of the i -th annulus. $r_{o,i}(z)$ and $r_{in,i}(z)$ represent the outer and inner boundary radii of the annulus at axial position z , respectively. Here, r_o corresponds to the outer casing boundary, while r_m represents the inner pipe-string boundary. z is the axial coordinate along the wellbore.

When the annulus geometry is assumed to be uniform along the axial direction, $r_{o,i}$ and $r_{m,i}$ can be treated as constants, and Equation (1) is reduced to Equation (2).

$$V_{a,i}^0 = \pi (r_{o,i}^2 - r_{m,i}^2) L_i \quad (2)$$

These relationships show that the effective volume of an enclosed annulus is determined jointly by casing hierarchy, packer position, cement top and wellhead sealing boundary. This volume provides the geometric basis for subsequent pressure response. For an annulus with a small enclosed volume or strong boundary constraint, a small fluid-volume change may be converted into a large pressure increment.

2.2. Thermal Expansion and Compression

The volume change in annular fluid is mainly affected by both temperature and pressure [20]. At constant pressure, an increase in temperature increases fluid volume. At constant temperature, an increase in pressure reduces fluid volume. For an enclosed annulus, the volume increment caused by temperature increase cannot be fully released. A new equilibrium can only be achieved through pressure-induced fluid compression and casing deformation.

When wellbore temperature increases, the annular fluid produces a thermal expansion volume increment. Considering the non-uniform axial temperature distribution in the wellbore, the thermal expansion volume of the fluid in the i -th annulus can be expressed by Equation (3).

$$\Delta V_{T,i} = \int_0^{L_i} \alpha_{f,i} V_i(z) \Delta T_i(z,t) dz \quad (3)$$

where $\Delta V_{T,i}$ represents the volume increment due to thermal expansion of the fluid, $\alpha_{f,i}$ is the thermal expansion coefficient of the fluid in the i -th annulus, $V_i(z)$ is the annular volume per well section, and $\Delta T_i(z,t)$ is the temperature increment at well depth z at time t .

Changes in the volume of a liquid can be described using the coefficient of thermal expansion and the isothermal compressibility coefficient. Let the volume of the fluid in the annulus be ΔV , the temperature change be ΔT , the pressure change be Δp , the coefficient of thermal expansion of the liquid be α_l , and the isothermal compressibility coefficient be K_T ; the fluid-volume change can be expressed by Equation (4).

$$\frac{\Delta V_l}{V_l} = \alpha_l \Delta T - K_T \Delta p \quad (4)$$

This equation indicates that increasing temperature tends to expand the liquid, whereas increasing pressure compresses it. In the limiting case of a fully enclosed annulus with a rigid boundary, almost all fluid thermal expansion is converted into pressure increase. With a deformable boundary, part of the thermal expansion volume is released by elastic casing deformation, reducing the magnitude of pressure growth.

Corresponding to thermal expansion, the increase in annular pressure causes fluid compression and therefore absorbs part of the thermal expansion volume. The pressure-induced volume reduction can be expressed by Equation (5).

$$\Delta V_{P,i} = \int_0^{L_i} c_{f,i} V_i(z) \Delta P_i(t) dz \quad (5)$$

where $\Delta V_{P,i}$ represents the decrease in fluid volume due to compression caused by the pressure increase, $C_{f,i}$ is the fluid compression coefficient, and $\Delta P_i(t)$ is the pressure increase in the i -th annulus at time t . The equivalent volume change in the annular fluid under the combined action of temperature and pressure can then be written as Equation (6).

$$\Delta V_{f,i} = \Delta V_{T,i} - \Delta V_{P,i} \quad (6)$$

This relationship shows that trapped pressure is governed by the competition between fluid thermal expansion volume and pressure-compression volume. If the thermal expansion volume cannot be fully absorbed by fluid compression, boundary deformation or volume release, annular pressure will continue to increase.

Under high-temperature and high-pressure conditions, however, the liquid thermal expansion coefficient and isothermal compressibility are not constants. They vary with temperature and pressure. If constant parameters are still used, pressure prediction errors may be introduced. A more reasonable treatment is to express the thermophysical properties of the fluid as functions of temperature and pressure, as shown in Equations (7) and (8).

$$\alpha_l = \alpha_l(T, p) \quad (7)$$

$$K_T = K_T(T, p) \quad (8)$$

This treatment better represents the volume response of fluids in deep wellbores under high-temperature and high-pressure conditions. In particular, when well depth is large, the axial difference in annular temperature is significant. Using an average temperature rise for the entire well can obscure the contribution of local high-temperature intervals to pressure growth. Therefore, the annulus should be discretized into finite segments along the well depth, and the temperature, pressure and volume changes in each segment should be calculated separately and summed to obtain the overall pressure response of the annulus.

2.3. Effects of Tubular Thermal Expansion and Elastic Deformation

The tubular strings are assumed to exhibit linear-elastic behavior within the operating range considered in this study. This assumption is applicable when the radial displacement is small relative to the tubular radius and the equivalent stress remains below the material yield strength, i.e.,

$$\frac{|u_r|}{r} \ll 1, \sigma_{\text{vM}} < \sigma_y \quad (9)$$

where u_r is the radial displacement, r is the tubular radius, σ_{vM} is the von Mises equivalent stress, and σ_y is the yield strength of the tubular material. Under these conditions, the pressure- and temperature-induced changes in tubular diameter can be described using the small-strain linear-elastic constitutive relationship. The cement sheath is represented as an equivalent continuous elastic constraint on the outer casing. Plastic yielding, casing buckling, cement cracking, interface debonding, and microannulus formation are not considered. Therefore, the present formulation is intended for trapped-pressure prediction before structural or interfacial failure occurs.

Annular volume is not fixed. It is determined jointly by the relative positions and deformation of the inner and outer tubular strings [21,22]. During production, tubular strings undergo thermal expansion as temperature increases and radial elastic deformation under internal and external pressure differences. For a given annulus, an increase in the outer diameter of the inner tubular string reduces annular volume, whereas an

increase in the inner diameter of the outer casing increases annular volume. These two effects together determine the direction and magnitude of annular geometric-space change.

The free radial thermal displacement of a tubular string was calculated from the isotropic linear thermal expansion relation:

$$u_T(r, z, t) = \alpha_s r \Delta T_s(z, t) \quad (10)$$

where u_T is the radial displacement due to free thermal expansion, α_s is the linear expansion coefficient of the casing, r is the radius at the calculation point, and $\Delta T_s(z, t)$ is the temperature rise in the casing at depth z and time t .

The geometric volume of the annulus changes with radial displacement of the inner and outer tubular boundaries. For the i -th annulus, the change in geometric volume can be characterized by the radial displacements of the inner and outer boundaries, as expressed by Equation (11).

$$\Delta V_{a,i} = \int_0^{L_i} 2\pi [r_{o,i} u_{o,i}(z, t) - r_{in,i} u_{in,i}(z, t)] dz \quad (11)$$

where $\Delta V_{a,i}$ represents the change in geometric volume of the annulus, $u_{o,i}(z, t)$ represents the radial displacement of the outer boundary of the annulus, and $u_{in,i}(z, t)$ represents the radial displacement of the inner boundary of the annulus. An outward displacement of the outer boundary increases the volume of the annulus, while an outward displacement of the inner boundary compresses the annulus space. Therefore, the deformation of the tubing string not only affects the stress state of the casing but also directly contributes to the volumetric compensation process of annulus pressure.

Pressure-induced tubular deformation was calculated using the elastic solution for a thick-walled cylinder. For the k -th tubular string, let a_k and b_k denote the inner and outer radii, respectively, and let $p_{i,k}$ and $p_{o,k}$ denote the pressures acting on its inner and outer surfaces. Positive pressure is taken as a compressive surface load, whereas positive stress denotes tension. The Lamé constants are given by Equations (12) and (13).

$$A_k = \frac{p_{i,k} a_k^2 - p_{o,k} b_k^2}{b_k^2 - a_k^2} \quad (12)$$

$$B_k = \frac{a_k^2 b_k^2 (p_{i,k} - p_{o,k})}{b_k^2 - a_k^2} \quad (13)$$

The radial and circumferential stresses at radius r given by Equation (14).

$$\sigma_{r,k}(r) = A_k - \frac{B_k}{r^2}, \sigma_{\theta,k}(r) = A_k + \frac{B_k}{r^2} \quad (14)$$

When the axial stress $\sigma_{z,k}$ is specified, the radial displacement follows from the three-dimensional thermoelastic constitutive relation:

$$u_k(r) = \frac{r}{E_k} [\sigma_{\theta,k}(r) - \nu_k (\sigma_{r,k}(r) + \sigma_{z,k})] + \alpha_k r \Delta T_k \quad (15)$$

The pressure-induced component of displacement is therefore

$$u_{p,k}(r) = \frac{1}{E_k} \left[(1 - \nu_k) A_k r + (1 + \nu_k) \frac{B_k}{r} - \nu_k \sigma_{z,k} r \right] \quad (16)$$

where E_k , ν_k , and α_k are the Young's modulus, Poisson's ratio, and linear thermal expansion coefficient of tubular string k , respectively. The pressures $P_{i,k}$ and $P_{o,k}$ are the local absolute pressures acting on the inner and outer surfaces of the tubular string.

The axial stress influences radial tubular displacement through the Poisson effect and must therefore be defined consistently with the adopted axial boundary condition. For the k -th tubular string, the axial strain is related to the three normal stresses and the temperature change by Equation (17).

$$\varepsilon_{z,k} = \frac{1}{E_k} \left[\sigma_{z,k} - \nu_k (\sigma_{r,k} + \sigma_{\theta,k}) \right] + \alpha_k \Delta T_k \quad (17)$$

Because $\sigma_{r,k} + \sigma_{\theta,k} = 2A_k$, the corresponding axial stress is shown in Equation (18).

$$\sigma_{z,k} = E_k (\varepsilon_{z,k}^c - \alpha_k \Delta T_k) + 2\nu_k A_k \quad (18)$$

where $\varepsilon_{z,k}^c$ is the axial strain permitted by the boundary constraints. For a freely expanding tubular section, the axial stress is set to zero and the axial strain is calculated from Equation (17). For a fully constrained section, $\varepsilon_{z,k}^c = 0$. For a section subjected to a prescribed axial force N_k , the axial stress is shown in Equation (19).

$$\sigma_{z,k} = \frac{N_k}{\pi(b_k^2 - a_k^2)} \quad (19)$$

In terms of deformation sources, the radial displacement of a tubular string can be further decomposed into thermal expansion displacement and pressure-induced elastic displacement, as shown in Equation (20).

$$u(z, t) = u_T(z, t) + u_P(z, t) \quad (20)$$

where $u(z, t)$ represents the thermal expansion displacement caused by temperature changes, and $u_P(z, t)$ represents the elastic deformation displacement caused by pressure differences and axial loads.

In actual wellbores, the effect of axial force on radial deformation should also be considered [23]. Temperature change, wellhead constraints, packer constraints and internal/external pressure differences generate axial stress in tubular strings. Through the Poisson effect, axial stress further induces radial strain and produces additional changes in annular volume. In the present model, cemented casing sections were treated as radially constrained boundaries. The mechanical deformation of the cement sheath itself was not calculated explicitly.

3. Dynamic Multi-Annulus Coupling Model

3.1. Description of the Multi-Annulus Structure

A typical high-temperature and high-pressure gas well generally consists of tubing, production casing, intermediate casing and surface casing (Figure 1). According to the tubular hierarchy, the annular spaces can be divided into Annulus A, Annulus B, Annulus C and other annuli. Annulus A is located between the tubing and the production casing, Annulus B is located between the production casing and the intermediate casing, and Annulus C is located between the intermediate casing and the surface casing. Different annuli are spatially isolated, but they are not mechanically independent because adjacent annuli share a casing string as their boundary [24,25].

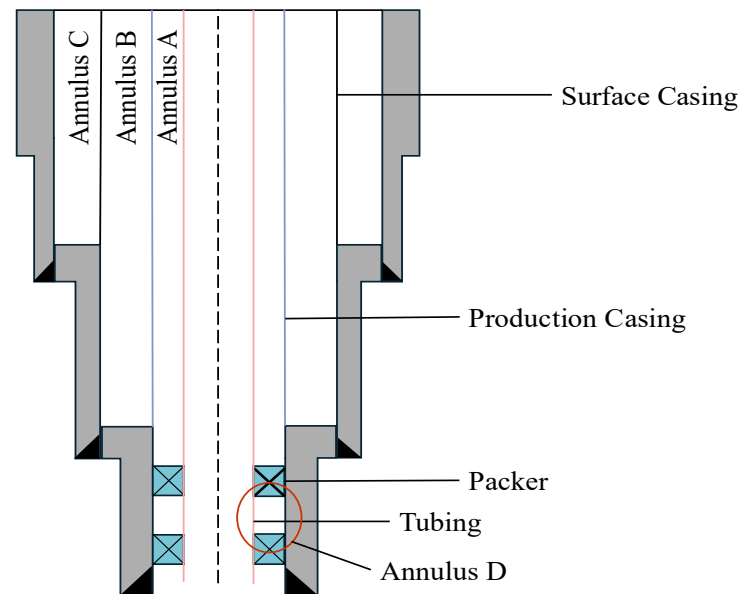


Figure 1. Schematic diagram of different annuli.

Taking Annuli A and B as an example, the production casing is both the outer boundary of Annulus A and the inner boundary of Annulus B. When the pressure in Annulus A increases, the production casing expands outward under internal pressure. This changes the volume of Annulus A and also compresses or affects the space of Annulus B. Conversely, when the pressure in Annulus B increases, the production casing is subjected to external pressure and may move inward, thereby changing the volume of Annulus A. Therefore, calculating the pressure of Annulus A alone without considering the pressure in Annulus B may underestimate or overestimate the actual pressure response. The coupled formulation described in Sections 3.2 and 3.3 applies to Annuli A–C, which interact through their shared casing walls. Annulus D in the field case has a different boundary configuration. It is a separate sealed interval bounded by two rigid and impermeable packers. Therefore, Annulus D is not pressure-coupled with Annuli A–C and is evaluated independently.

3.2. Pressure–Deformation Coupling Among Annuli

The fundamental reason for multi-annulus coupling is that adjacent annuli share the same casing wall. A pressure change in one annulus induces radial deformation of the shared casing wall. This deformation changes the geometric volume of the annulus itself and also changes the boundary position of the adjacent annulus. Thus, in a multi-annulus system, annular pressures are not independent scalar variables. Instead, they form an interacting pressure vector through casing deformation (Figure 2). Figure 2 illustrates how pressure is transferred between adjacent annuli through deformation of their shared casing wall rather than through direct fluid communication. Taking Annuli A and B as an example, when $P_A > P_B$, the net pressure difference causes the shared production casing to deform outward. This deformation increases the geometric volume of Annulus A but decreases that of Annulus B because the same casing constitutes the outer boundary of Annulus A and the inner boundary of Annulus B. Consequently, part of the pressure increase in Annulus A is relieved through boundary expansion, while the reduced volume of Annulus B produces a pressure response in that annulus. The opposite deformation occurs when $P_B > P_A$. Therefore, the pressure difference across each shared casing wall produces bidirectional pressure–deformation–volume feedback among adjacent annuli.

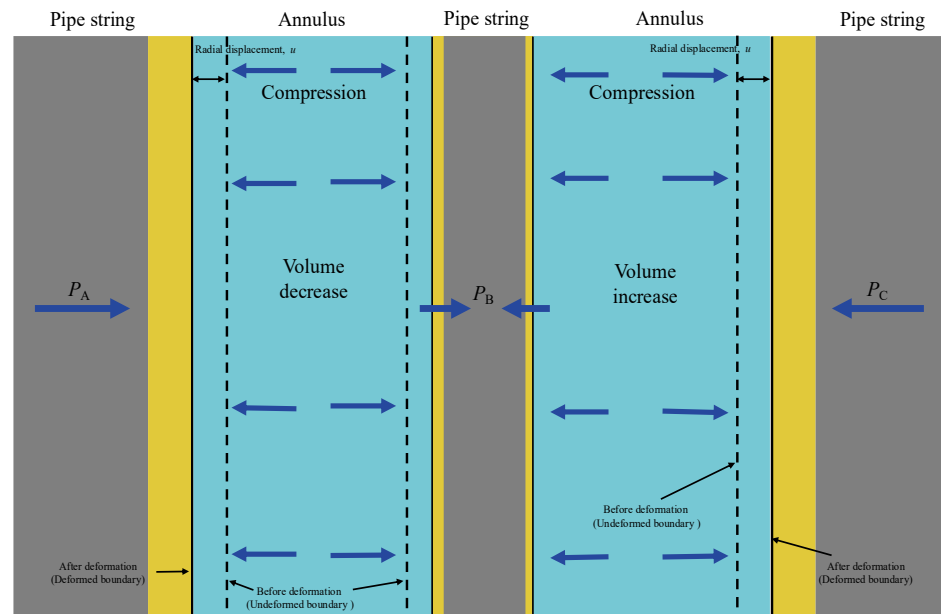


Figure 2. Compressive effects of different columns.

For Annulus A, its volume change can be written as a function of its own pressure, the pressure in Annulus B, the fluid temperature in Annulus A and the temperature of the production casing, as indicated in Equation (21).

$$\Delta V_A = \Delta V_A(P_A, P_B, T_A, T_{c1}) \quad (21)$$

where P_A represents the pressure in annulus A, P_B represents the pressure in annulus B, T_A represents the fluid temperature in annulus A, and T_{c1} represents the production casing temperature. This equation shows that the volume change in annulus A depends not only on its own pressure and temperature, but also on the pressure in the adjacent annulus B.

Similarly, the volume change in Annulus B depends on the pressures in Annuli A, B and C and on the temperatures of the relevant casing strings, as indicated in Equation (22).

$$\Delta V_B = \Delta V_B(P_A, P_B, P_C, T_B, T_{c1}, T_{c2}) \quad (22)$$

The volume change in Annulus C can be expressed in an analogous form by Equation (23).

$$\Delta V_C = \Delta V_C(P_B, P_C, T_C, T_{c2}, T_{c3}) \quad (23)$$

Therefore, a multi-annulus system actually constitutes a coupled set of equations. The pressure change in each annulus affects adjacent annuli through casing displacement, while the pressure changes in adjacent annuli feed back to the annulus itself. This coupling makes annular pressure evolution systemic rather than local.

Mechanistically, multi-annulus coupling includes three pathways [26]. The first is structural coupling, in which pressure is transferred between annuli through elastic deformation of casing strings. The second is volume coupling, in which the volume change in one annulus is affected by deformation of adjacent tubular strings. The third is thermal coupling, in which the temperature fields of different annuli interact through heat transfer across casing and cement. These three coupling pathways jointly determine the dynamic pressure response of a multi-annulus system.

3.3. Volume-Compatibility Governing Equation

The dynamic model of annular trapped pressure should satisfy the principle of volume compatibility [27,28]. For a single annulus at any time, the geometric volume of the annulus should be compatible with the fluid-volume change. This relationship is expressed by Equation (24).

$$V_{an}(t) = V_l(t) + V_g(t) \quad (24)$$

In this equation, the annular geometric volume is equal to the sum of the liquid volume and gas volume. If the annulus contains only liquid, the gas volume is zero. If gas is present, the change in gas volume with temperature and pressure must also be considered.

For a multi-annulus system, a set of governing equations can be established for all annuli simultaneously.

$$\begin{aligned} F_A(P_A, P_B, T_A, t) &= V_A(P_A, P_B, T_A, t) - V_{l,A}(P_A, T_A, t) - V_{g,A}(P_A, T_A, t) = 0 \\ F_B(P_A, P_B, P_C, T_B, t) &= V_B(P_A, P_B, P_C, T_B, t) - V_{l,B}(P_B, T_B, t) - V_{g,B}(P_B, T_B, t) = 0 \\ F_C(P_B, P_C, T_C, t) &= V_C(P_B, P_C, T_C, t) - V_{l,C}(P_C, T_C, t) - V_{g,C}(P_C, T_C, t) = 0 \end{aligned} \quad (25)$$

For the field case considered in Section 7, Annulus D is treated separately from the coupled A–C system. The packers bounding Annulus D are assumed to be rigid, impermeable, and non-deformable. Consequently, pressure changes in Annuli A–C do not change the volume of Annulus D. The geometric-volume change in Annulus D is caused only by thermal expansion of the tubular string.

To solve the coupled governing equations in Equation (26), the residual and pressure vectors are defined as

$$\mathbf{R} = [R_A \quad R_B \quad R_C]^T, \quad \mathbf{p} = [p_A \quad p_B \quad p_C]^T \quad (26)$$

where $R_i = \Delta V_{f,i} - \Delta V_{a,i}$ is the volume-compatibility residual of Annulus i . The Jacobian matrix is assembled as

$$J_{ij} = \frac{\partial R_i}{\partial p_j} \delta_{ij} \frac{\partial \Delta V_{f,i}}{\partial p_i} \frac{\partial \Delta V_{a,i}}{\partial p_j} \quad (27)$$

where δ_{ij} is the Kronecker delta. The diagonal terms account for the fluid compressibility and deformation of the two tubular boundaries of the corresponding annulus. The off-diagonal terms represent the change in one annular volume caused by the pressure-induced deformation of a casing shared with an adjacent annulus. Therefore, the Jacobian of the A–C coupled system is assembled as

$$\mathbf{J} = \begin{bmatrix} \frac{\partial R_A}{\partial p_A} & \frac{\partial R_A}{\partial p_B} & 0 \\ \frac{\partial R_B}{\partial p_A} & \frac{\partial R_B}{\partial p_B} & \frac{\partial R_B}{\partial p_C} \\ 0 & \frac{\partial R_C}{\partial p_B} & \frac{\partial R_C}{\partial p_C} \end{bmatrix} \quad (28)$$

At iteration m , the pressure correction is obtained from $\mathbf{J}^{(m)} \Delta \mathbf{p}^{(m)} = -\mathbf{R}^{(m)}$ and the pressure vector is updated using $\mathbf{p}^{(m+1)} = \mathbf{p}^{(m)} + \omega \Delta \mathbf{p}^{(m)}$, where ω is the damping factor. Annulus D is solved independently and is not included in the above Jacobian because the packer is assumed rigid and does not transmit pressure-induced deformation between Annulus D and Annuli A–C.

This system of equations reflects the fundamental constraints governing the dynamic evolution of pressure in multiple annuli. The pressure variables P_A , P_B and P_C must be solved simultaneously at each time step. If the annuli are solved independently one by one, the coupling effects caused by the deformation of the common casing cannot be accounted for. The simultaneous solution described above applies to Annuli A–C. Annulus D is excluded from this pressure vector and is solved separately using its independent volume-balance equation.

4. Dynamic Temperature-Field Evolution and Its Role

4.1. Control of Pressure Evolution by the Temperature Field

Temperature is the primary driving factor for the growth of annular trapped pressure [29]. Before production, the wellbore is usually in a relatively stable geothermal state. After production begins, high-temperature formation fluid flows upward along the tubing and exchanges heat with the surrounding casing strings, annular fluids and formation. As a result, the wellbore temperature field gradually changes. Because both heat conduction and convective heat transfer require time, annular temperature does not reach a stable state instantaneously, but exhibits clear dynamic evolution.

The wellbore temperature field can be expressed as a function of depth and production time, as shown in Equation (29).

$$\rho c \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(kr \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) \quad (29)$$

where z represents the coordinate in the well depth direction, and t represents the production time. This expression indicates that temperature varies not only with well depth, but also with time. For annular pressure calculation, replacing the entire wellbore temperature distribution with a single average temperature ignores the contribution of local high-temperature intervals. In practice, the temperature rise in deeper intervals is higher, fluid thermal expansion is stronger and the contribution to pressure growth is greater. Temperature and heat-flux continuity were imposed at the interfaces among the tubing, annular fluid, casing, cement sheath, and formation.

Annular pressure is related not only to the final temperature, but also to the temperature-rise path. If temperature increases rapidly, fluid thermal expansion occurs quickly and pressure may rise sharply within a short time. If temperature increases slowly, casing and cement have more time to deform compatibly, and the pressure-growth curve may become smoother. Therefore, even under the same final temperature condition, different heating paths may correspond to different pressure evolution processes.

This point is particularly important for production-regime changes. For example, during the early stage of well opening, wellbore temperature increases rapidly and annular pressure may grow significantly. During stable production, temperature variation slows and the pressure-growth rate decreases. After shut-in, the wellbore temperature redistributes, and the pressure in some annuli may decrease or increase again. Thus, annular pressure prediction should consider not only the final pressure, but also the pressure path during the entire production process.

4.2. Axial Segmentation Method

To account for the non-uniform axial temperature distribution, each enclosed annulus was uniformly discretized into 100 axial segments. The segment length was calculated as $\Delta z = L_i / 100$, where L_i is the effective enclosed length of the i -th annulus. The temperature and temperature-dependent fluid properties of each segment were evaluated at

its midpoint. The segment-level fluid-volume changes and tubular-displacement-induced geometric-volume changes were then summed over the complete enclosed length.

A single unknown pressure was assigned to each sealed annulus at each temperature-load step. Thus, annular pressure was assumed to be uniform within each enclosed annulus, while temperature and fluid properties were allowed to vary with depth. Annuli A–C were included in the coupled pressure system, whereas Annulus D was evaluated independently under the rigid-packer assumption. Therefore, the numerical formulation retains an incremental solution structure, whereas the field evaluation represents end-point calculations under prescribed thermal loads rather than a time-resolved transient simulation.

The length of the i -th segment is z_i , the temperature is $T_i(t)$, and the pressure is $P_i(t)$. Fluid-volume change and annular geometric-volume change are calculated for each segment, and then integrated or summed along the well depth, as indicated in Equations (30) and (31).

$$V_l(t) = \sum_{i=1}^n V_{l,i}(T_i(t), P_i(t)) \quad (30)$$

$$V_{an}(t) = \sum_{i=1}^n V_{an,i}(T_i(t), P_i(t)) \quad (31)$$

This segmentation method better represents the large temperature gradients in high-temperature and high-pressure wells. In deep gas wells, temperature variation near the bottomhole region contributes significantly to fluid thermal expansion, whereas temperature changes in shallow intervals are relatively small. Segment-by-segment calculation can avoid errors caused by using an average temperature.

The axial hydrostatic-pressure gradient and transient frictional pressure loss within each enclosed annulus are neglected. This assumption is consistent with the treatment of the annular fluid as a quiescent, sealed control volume. Consequently, the model predicts one representative trapped-pressure value for each annulus rather than a depth-dependent pressure profile.

5. Gas–Liquid Two-Phase Annular Pressure Evolution Model

5.1. Volume Compatibility in the Presence of Gas and Liquid

In some wellbore conditions, the annulus may contain gas [30]. For example, nitrogen may be injected into the annulus to mitigate pressure increase, or free gas may exist because of imperfect wellbore sealing. Once gas is present, the annular pressure response changes markedly. Compared with liquid, gas is more compressible, and its volume is more sensitive to changes in pressure and temperature. Therefore, gas may buffer pressure increase, but it may also introduce stronger nonlinearity under high-temperature and high-pressure conditions.

A gas–liquid two-phase annulus still satisfies the volume-compatibility relationship, as shown in Equation (32).

$$V_{an}(t) = V_l(t) + V_g(t) \quad (32)$$

The liquid volume is governed by liquid thermal expansion and compressibility, whereas the gas volume is governed by the gas equation of state. In high-temperature and high-pressure environments, the ideal-gas equation often cannot accurately describe gas-density changes. A real-gas equation of state is therefore required.

The gas state can be expressed as Equation (33), and the gas volume can be calculated by Equation (34).

$$\rho_g = \rho_g(P, T) \quad (33)$$

$$V_g(t) = \frac{m_g}{\rho_g(P, T)} \quad (34)$$

where m_g represents the mass of the gas, and ρ_g represents the density of the gas. For real gases such as nitrogen under high-temperature and high-pressure conditions, the Benedict–Webb–Rubin (BWR) equation of state can be used to calculate gas density. The basic idea of this equation is to describe intermolecular forces and volume-exclusion effects using higher-order terms, thereby improving the accuracy of density and compressibility calculations under high pressure.

A real-gas equation of state can be written in a unified implicit form, as shown in Equation (35).

$$f(\rho_g, P, T) = 0 \quad (35)$$

Under given pressure and temperature conditions, gas density can be solved iteratively. Gas volume is then calculated using gas mass conservation and substituted into the volume-compatibility equation. Because the relationship among gas density, pressure and temperature is nonlinear, pressure calculation for a gas–liquid two-phase annulus usually requires iterative solution.

5.2. Influence of Gas on Pressure Evolution

Gas has a dual influence on annular pressure evolution [11]. On the one hand, because gas is highly compressible, it can absorb part of the volume increment caused by liquid thermal expansion within a certain range, thereby slowing the rate of pressure increase. On the other hand, under high-temperature and high-pressure conditions, gas behavior deviates from ideal-gas behavior. Gas density changes nonlinearly with pressure, and improper treatment can lead to pressure prediction errors.

To describe the proportion of gas in the annular fluid, the gas volume fraction can be defined by Equation (36).

$$\phi_g(t) = \frac{V_g(t)}{V_g(t) + V_l(t)} \quad (36)$$

where $\phi_g(t)$ represents the volume fraction of the gas, and $V_g(t)$ and $V_l(t)$ are the volumes of the gas phase and liquid phase, respectively, in the annulus at time t . A larger gas volume fraction increases the overall compressibility of the system, and the annular pressure response to temperature change is usually more moderate.

For a gas–liquid two-phase annulus, an equivalent compressibility can be used to describe the overall response of the mixed fluid to pressure variation, as shown in Equation (37).

$$c_m(t) = \phi_g(t)c_g(t) + [1 - \phi_g(t)]c_l \quad (37)$$

where $c_m(t)$ is the equivalent compressibility coefficient of the gas–liquid mixture, $c_g(t)$ is the compressibility coefficient of the gas phase, and c_l is the compressibility coefficient of the liquid phase. Since $c_g(t)$ is typically much greater than c_l , an increase in the gas volume fraction significantly enhances the system's ability to absorb volumetric disturbances, thereby reducing the rate of annular pressure rise. However, when the gas volume fraction is small or the pressure is high, the compressible space in the gas phase

is limited, the system's equivalent compressibility decreases, and thermal expansion of the liquid phase may still dominate the pressure response.

Under high-temperature and high-pressure conditions, gas behavior should be described using a real-gas equation of state, as shown in Equation (38).

$$P_g V_g = Z n_g R T_g \quad (38)$$

where P_g is the vapor pressure, V_g is the vapor volume, Z is the gas deviation factor, n_g is the amount of gas, R is the gas constant, and T_g is the vapor temperature.

In an enclosed annulus, if gas leakage or dissolution loss is neglected, the gas mass should satisfy the conservation relationship in Equation (39).

$$m_g = \rho_g(P_g, T_g) V_g \quad (39)$$

where m_g represents the mass of the gas in the annulus, and $\rho_g(P_g, T_g)$ represents the density of the gas under the current pressure and temperature conditions. Therefore, when temperature increases or pressure changes, gas volume does not vary linearly. It is determined jointly by gas mass conservation and the real-gas equation of state.

The gas compressibility can be expressed by Equation (40).

$$c_g = -\frac{1}{V_g} \left(\frac{\partial V_g}{\partial P_g} \right)_{T_g} \quad (40)$$

This equation indicates that the gas-buffering capacity depends on the sensitivity of gas volume to pressure change. Under low pressure or at a high gas volume fraction, gas compressibility is large and gas can effectively absorb the volume increment caused by liquid thermal expansion. Under high pressure, gas density increases, the available compression space decreases and the buffering effect weakens.

Therefore, the influence of gas on annular pressure evolution should not be simply interpreted as a one-way reduction in pressure. It is a nonlinear regulatory process controlled jointly by gas volume fraction, initial gas-injection pressure, gas-injection amount, temperature level and real-gas behavior. When the gas volume fraction is high and the pressure is relatively low, the gas phase mainly acts as a pressure-buffering medium. When the gas volume fraction is low or when the system is under high-temperature and high-pressure conditions, the gas-buffering capacity may decline rapidly, and real-gas effects may become more important for prediction accuracy.

6. Model Solution Method

6.1. Time-Stepping Solution Strategy

The dynamic annular trapped-pressure model is strongly nonlinear and coupled, and is therefore suitable for solution using a time-stepping method [31]. The production process is divided into a series of time steps, as shown in Equation (41).

$$t_0, t_1, t_2, \dots, t_k \quad (41)$$

At each time step, the temperature field of the wellbore is updated. The fluid-volume changes, tubular deformation and annular geometric-volume changes in each annulus are calculated. The annular pressures satisfying the volume-compatibility conditions are then solved. The time-stepping procedure provides the model with the capability to calculate the transient evolution of temperature, fluid volume, tubular deformation, and annular pressure. However, because the available field dataset contains pressure records only for the two selected thermal conditions, the transient calculation capability and the field evaluation are distinguished in the following discussion. The former describes the numerical

formulation, whereas the latter assesses only the predicted pressure levels at the available field conditions.

The solution procedure is as follows. First, the initial wellbore structural parameters are input, including the inner and outer diameters, elastic modulus, Poisson ratio, linear expansion coefficient and cement sheath parameters of the tubing, production casing, intermediate casing and surface casing. Second, the initial annular-state parameters are input, including initial pressure, initial temperature, annular fluid type, liquid thermal expansion coefficient, compressibility and initial gas volume or gas mass. Third, the temperature distribution of each annulus at the current time step is obtained according to production time and the wellbore heat-transfer model. Fourth, the fluid-volume change under the current temperature and pressure conditions is calculated. Fifth, the radial deformation caused by tubular thermal expansion, internal and external pressure difference and axial force is calculated, and the annular geometric-volume change is then obtained. Sixth, the volume-compatibility residual equations are established and the annular pressures are solved iteratively. Seventh, convergence of the residual is checked. If the convergence criterion is satisfied, the calculation proceeds to the next time step. Otherwise, the pressure is updated and the iteration continues.

The convergence condition can be written as Equation (42), where the subscript j denotes the j -th annulus and the prescribed tolerance defines the acceptable residual level.

$$\eta^{(m)} = \max_{j \in \{A, B, C\}} \left[\frac{|V_{an,j}^{(m)} - V_{l,j}^{(m)} - V_{g,j}^{(m)}|}{V_{an,j}^{(m)}} \right] < \varepsilon_R, \varepsilon_R = 1.0 \times 10^{-6} \quad (42)$$

where $\eta^{(m)}$ is the maximum normalized volume-compatibility residual at the m -th Newton iteration; m is the Newton iteration index; j denotes Annulus A, B, or C; $V_{an,j}^{(m)}$ is the geometric volume of Annulus j after tubular deformation at the m -th iteration, m^3 ; $V_{l,j}^{(m)}$ and $V_{g,j}^{(m)}$ are the liquid and gas volumes in Annulus j , respectively, m^3 ; and ε_R is the prescribed dimensionless convergence tolerance. For a liquid-filled annulus, $V_{g,j}^{(m)} = 0$. Convergence is achieved only when the normalized volume residuals of all coupled annuli are smaller than ε_R .

Therefore, convergence is achieved only when the normalized volume residuals of all coupled annuli are simultaneously smaller than the prescribed tolerance.

6.2. Synchronous Iteration for Multiple Annuli

For a multilayer casing well, Annuli A, B and C are not independent. Taking the production casing between Annuli A and B as an example, a pressure change in Annulus A induces radial deformation of the production casing. This deformation simultaneously changes the outer boundary of Annulus A and the inner boundary of Annulus B. Similarly, a pressure change in Annulus B also affects the geometric volume of Annulus A. Therefore, any pressure increment in one annulus affects not only its own volume-compatibility relationship, but also adjacent annuli through the shared casing wall. Multi-annulus pressure evolution is essentially a coupled feedback problem involving pressure, deformation and volume. It cannot be solved by treating each annulus independently.

For a three-annulus system including Annuli A, B and C, the unknown pressures can be written in vector form, as shown in Equation (43).

$$\mathbf{P}^k = [P_A^k, P_B^k, P_C^k]^T \quad (43)$$

At the k -th time step, the pressure vector contains the pressures in Annuli A, B and C. Within this time step, each annulus must satisfy its own volume-compatibility condition. Thus, a residual equation system can be established, as shown in Equation (44).

$$\mathbf{F}(\mathbf{P}^k, \mathbf{T}^k, t_k) = [F_A, F_B, F_C]^T = 0 \quad (44)$$

where F_A , F_B and F_C represent the volumetric consistency residuals for the A, B, and C annuli, respectively. Accordingly, the number of pressure unknowns is determined by the number of enclosed annuli rather than by the number of axial segments. For the coupled A–C system, the pressure vector is $\mathbf{P} = [P_A, P_B, P_C]^T$. Annulus D also has a single spatially uniform pressure, p_D , but it is solved independently under the rigid-packer assumption. The residuals represent the volume-compatibility errors of Annuli A, B and C, respectively. Because the geometric-volume change in each annulus is affected by deformation of adjacent casing strings, each residual term depends not only on the pressure of the annulus itself, but also on the pressures of adjacent annuli, as shown in Equation (45).

$$F_i = F_i(P_A^k, P_B^k, P_C^k, \mathbf{T}^k, t_k), \quad i = A, B, C \quad (45)$$

This relationship indicates that the governing equations of the multi-annulus system are nonlinear and coupled. A Newton iterative method can be used to solve this equation system. At the m -th iteration, the residual equations are linearized according to Equation (46), where the pressure correction vector, current residual vector and Jacobian matrix are defined.

$$\mathbf{J}^m \Delta \mathbf{P}^m = -\mathbf{F}^m \quad (46)$$

where $\Delta \mathbf{P}^m$ is the pressure correction term for the m -th iteration, $\mathbf{F}^m$ is the residual vector based on the current pressure estimate, and $\mathbf{J}^m$ is the Jacobian matrix:

$$\mathbf{J}^m = \begin{bmatrix} \frac{\partial F_A}{\partial P_A} & \frac{\partial F_A}{\partial P_B} & \frac{\partial F_A}{\partial P_C} \\ \frac{\partial F_B}{\partial P_A} & \frac{\partial F_B}{\partial P_B} & \frac{\partial F_B}{\partial P_C} \\ \frac{\partial F_C}{\partial P_A} & \frac{\partial F_C}{\partial P_B} & \frac{\partial F_C}{\partial P_C} \end{bmatrix}^m \quad (47)$$

After the pressure correction is obtained, the multi-annulus pressure vector is updated by Equation (48).

$$\mathbf{P}^{m+1} = \mathbf{P}^m + \lambda \Delta \mathbf{P}^m \quad (48)$$

A fixed damping factor of $\lambda = 0.5$ was used to improve the stability of the nonlinear iteration. After solving the linearized residual equations, the pressure vector was updated according to

$$\mathbf{p}^{(k+1)} = \mathbf{p}^{(k)} + \lambda \Delta \mathbf{p}^{(k)} \quad (49)$$

where k denotes the nonlinear iteration number. The damped Newton iteration was applied to the coupled pressure vector of Annuli A–C. Annulus D was excluded from the A–C Jacobian matrix and was solved independently.

The iteration continues until the convergence conditions in Equation (50) are satisfied.

$$|\mathbf{F}^m| < \varepsilon_F \quad \text{or} \quad |\Delta \mathbf{P}^m| < \varepsilon_P \quad (50)$$

where ε_F and ε_P represent the convergence thresholds for residuals and pressure correction, respectively. When the convergence conditions are met, it is considered that the annular pressures, casing deformation, and annular volume changes at the current time step have reached a coordinated state, and the calculation proceeds to the next time step.

Through this synchronous iteration, pressure transfer among annuli is explicitly introduced into the solution framework. Compared with independent single-annulus calculation, the method can represent the mechanical coupling effect formed by adjacent annuli through shared casing walls and can therefore describe the dynamic evolution of trapped pressure in multilayer casing wells more accurately. The calculation process is shown in Figure 3.

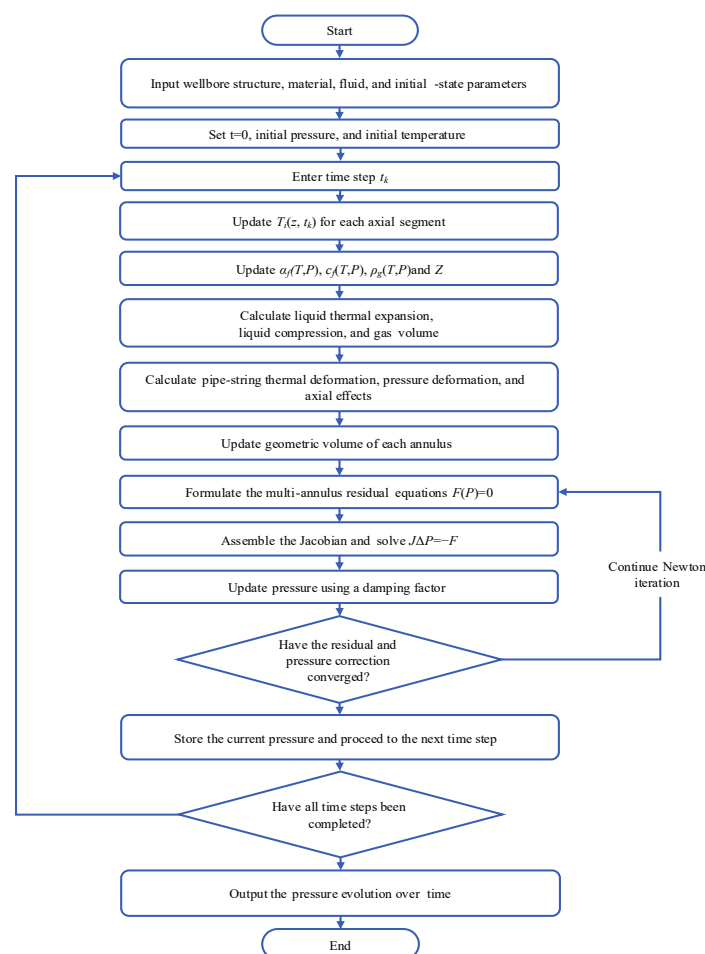


Figure 3. Solution process for transient pressure coupling in multi-ring cavities.

The synchronous Newton iteration is applied to the coupled pressure vector $P_{ABC} = [p_A, p_B, p_C]^T$. Annulus D is not included in the corresponding Jacobian matrix. After convergence of the A–C system, P_D is obtained separately from $F_D(p_D, T) = 0$. Thus, Annulus D is evaluated using the same temperature field and fluid-property model, but without pressure or deformation coupling with Annuli A–C.

The present model is intended for pre-failure pressure prediction in structurally intact and hydraulically sealed annuli. Conservation of the annular fluid mass is applicable only when the casing, packers, cement sheath, and sealing interfaces remain impermeable. Leakage, cement degradation, connected micro-annuli, or formation fractures would introduce fluid-mass exchange and may invalidate the constrained-volume assumption. The linear-elastic and homogeneous-material approximations are applicable when

tubular deformation remains small and the material stress remains below the yield strength; plastic deformation, buckling, cement cracking, interface debonding, and non-linear contact are outside the present scope. Although the brine properties vary with temperature and pressure, the mechanical properties of the tubular and cement materials are treated as constant equivalent values. Gas dissolution, interphase mass transfer, phase transition, and gas–liquid slip are also neglected. Therefore, the model should be applied to intact wellbore systems before significant leakage or structural damage occurs.

7. Field-Case Evaluation

7.1. Wellbore Structure

A typical offshore production well in a block of the Bohai Oilfield was selected as the study object to provide the wellbore-structure basis for the dynamic annular trapped-pressure model. The wellbore structure is shown in Figure 4, and the structural parameters are listed in Table 1. The well is located in a seawater environment and was completed with a multilayer casing structure consisting of surface casing, two production casing strings and tubing. Multiple annular spaces are formed between the tubular strings. Annulus A is formed between the tubing and the inner production casing, Annulus B is formed between the two production casing strings, and Annulus C is formed between the outer production casing and the surface casing. Because each annulus is affected by cement top, casing overlap, wellhead sealing and wellbore structural constraints, different degrees of enclosed or semi-enclosed spaces may form during production. These spaces provide the geometric conditions for annular trapped pressure.

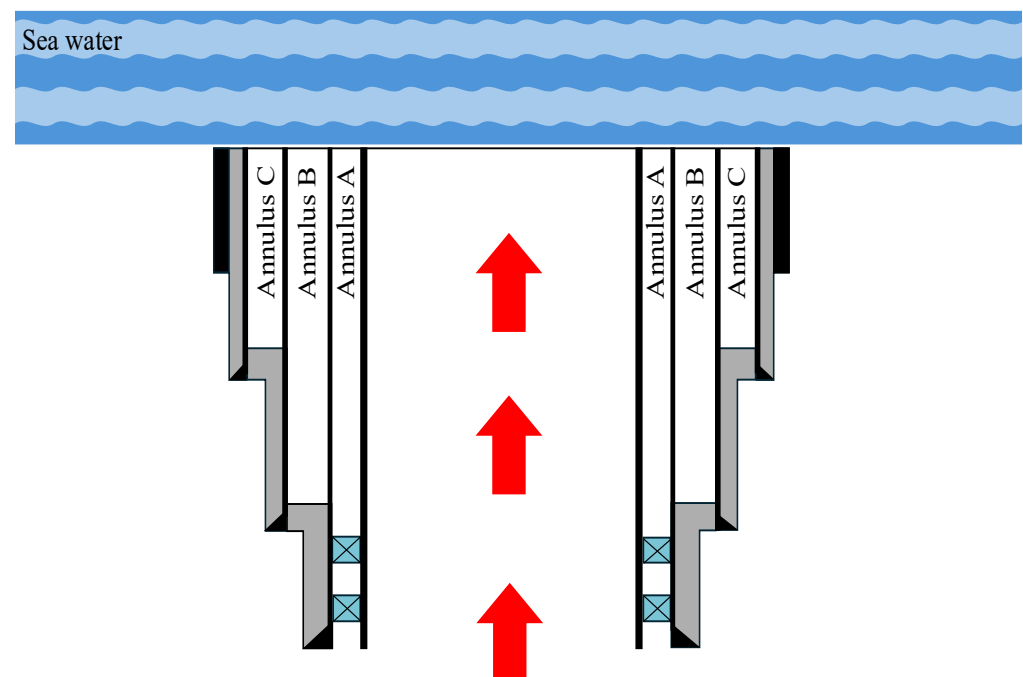


Figure 4. Wellbore structure.

In addition to Annuli A–C, the field well contains a sealed interval between two packers, denoted as Annulus D. The two packers are assumed to be rigid and impermeable. Therefore, Annulus D does not receive pressure-induced volume changes from Annuli A–C. Its pressure response is governed by the thermal expansion and compression of the enclosed fluid and by the reduction in available volume caused by thermal expansion of the tubular string.

Table 1. Well structure.

Column Name	Outer Diameter (mm)	Wire Weight (kg/m)	Depth (m)	Cement Overrun (m)
Surface Casing	762	460.12	182.11	162.18
Production Casing	508	139.89	728.13	701.22
Production Casing	244.47	76.62	2389.28	2376.11
Oil Pipe	88.9	18.92	3982.19	3962.19

7.2. Fluid Thermophysical Properties

Annular trapped pressure is strongly affected by the thermophysical properties of the annular fluid [32]. Therefore, obtaining the corresponding parameters is critical. In this study, the thermal expansion coefficient and compressibility of the fluid were obtained from fluid experimental data. The expansion coefficient describes the volume increase caused by temperature rise, whereas the compressibility describes the volume decrease caused by pressure increase. These parameters were used in the volume-compatibility equations to calculate the coupled response of fluid expansion, fluid compression and annular-volume variation.

The thermal expansion coefficient and compressibility must be evaluated along different thermodynamic paths. The isobaric thermal expansion coefficient was calculated from the temperature dependence of density at constant pressure:

$$\alpha_p = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p \quad (51)$$

The isothermal compressibility was calculated from the pressure dependence of density at constant temperature:

$$\kappa_T = \frac{1}{\rho} \left(\frac{\partial \rho}{\partial p} \right)_T \quad (52)$$

where $\rho(T, p)$ denotes the brine density at temperature T and pressure p . Thus, the density difference used to calculate α_p was taken between two temperatures at the same pressure, whereas the density difference used to calculate κ_T was taken between two pressures at the same temperature. This treatment separates the temperature and pressure effects and ensures positive values of both coefficients for the investigated brine conditions.

The NaCl-brine density data used in this study were not measured by the authors. They were obtained from the NIST ThermoML database record associated with the experimental study of Al Ghafri et al. The relevant information is shown in Table 2. The original study reported density measurements for aqueous NaCl solutions with molalities of 1.06, 3.16, and 6.00 mol kg⁻¹, over temperatures from 283 to 472 K and pressures up to 68.5 MPa.

In the original study, density was measured using a vibrating-tube densimeter calibrated under vacuum and with pure water. The reported relative uncertainty of the density measurements was 0.03–0.05%, and the reported average absolute relative deviation of the density correlation was approximately 0.02%.

The isobaric thermal expansion coefficient and isothermal compressibility were derived from the density data using $\alpha_p = -(1/\rho)(\partial \rho / \partial T)_p$ and $\kappa_T = (1/\rho)(\partial \rho / \partial p)_T$, respectively. The required values between the tabulated conditions were obtained using bilinear interpolation in temperature and pressure at the specified NaCl molality. No extrapolation beyond the source-data range was performed.

Because α_p and κ_T vary with temperature, pressure, and NaCl concentration, the constant-volume pressure–temperature response is state dependent. Therefore, the previous statement that the pressure increases universally by approximately 1.2 MPa per 1 °C has been removed.

Table 2. Coefficients of thermal expansion and compression of brine.

Temperature	Coefficient of Expansion/(10 ^{−4} °C ^{−1})			Compression Coefficient/(10 ^{−4} MPa ^{−1})		
	10 MPa	15 MPa	20 MPa	10 MPa	15 MPa	20 MPa
50	4.84	4.71	4.69	3.58	3.39	3.16
75	4.88	4.79	4.80	4.55	4.27	3.74
100	4.78	4.64	4.57	4.72	4.42	4.56
125	4.77	4.62	4.51	5.66	5.37	5.01
150	5.38	5.33	5.29	5.84	4.92	4.48

7.3. Calculation Results Under Different Conditions

According to the temperature variations in casing strings and annular fluids, annular pressure accumulation was solved iteratively. Simulations were conducted under three conditions. S1 considers the expansion and compression effects of the liquid and deformation of the pipe section. S2 considers liquid expansion and compression, casing-section deformation and multi-annulus coupling. S3 represents field measurements. The sealing area between the two packers is denoted as annulus D; the packer annulus is shown in Figure 5.

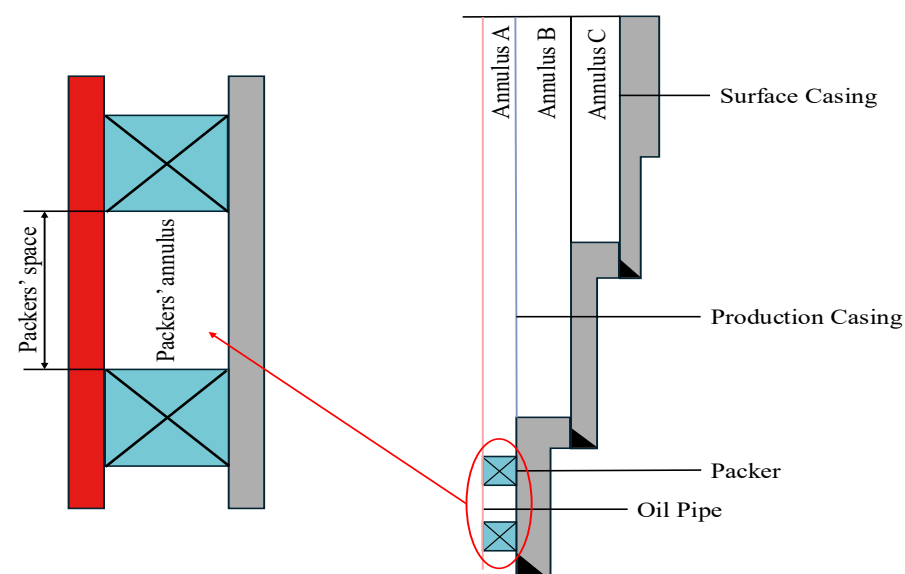


Figure 5. Annulus of the packer.

The comparison in Figure 6 shows that S1 was higher than the field-measured value S3 in all annuli. In Annuli A, B, C and D, S1 exceeded S3 by 2.78, 3.52, 3.53 and 1.87 MPa, respectively. The corresponding relative deviations were 11.08%, 11.84%, 13.02% and 61.31%. Although the absolute deviation in Annulus D was small, its relative deviation was more pronounced because of the low-pressure base value. These results indicate that, if multi-annulus coupling is ignored, the model tends to concentrate the constrained thermal expansion effect into pressure buildup in a single annulus, leading to pressure over-estimation.

By contrast, the S2 results after introducing multi-annulus coupling were closer to the field-measured S3 values. In Annuli A, B, C and D, the absolute deviations between S2 and S3 were reduced to 0.34, 0.45, 0.45 and 0.32 MPa, respectively. The corresponding relative deviations were 1.36%, 1.51%, 1.66% and 10.49%. The mean absolute deviation decreased from 2.93 MPa under S1 to 0.39 MPa under S2, giving an error reduction of approximately 86.67%. The reduction in prediction error is attributed to the consideration of deformation-mediated pressure interaction among adjacent annuli. In the uncoupled model (S1), the thermal expansion of fluid is converted directly into pressure buildup within each individual annulus, while the volume compensation caused by casing deformation is neglected. In the coupled model (S2), pressure variations in one annulus induce radial deformation of shared casing walls, which modifies the effective volumes of neighboring annuli and promotes pressure redistribution. Consequently, the calculated pressure response becomes closer to the field measurements.

The reduction in prediction error can be physically explained by deformation compatibility and pressure redistribution through the shared casing strings. When pressure increases in an inner annulus, the shared casing tends to deform outward, increasing the available volume of the inner annulus while simultaneously reducing the volume of the adjacent outer annulus. The same casing displacement therefore enters the volume equations of the two neighboring annuli with opposite signs, requiring their pressures to be solved simultaneously. The magnitude of this pressure transfer is governed by the relative radial stiffness of the casing strings: a more compliant casing provides greater volume compensation and stronger pressure redistribution, whereas a stiffer casing approaches the independent-annulus response. In addition, non-uniform radial heating causes the inner annuli to expand earlier than the outer annuli. The resulting pressure-induced deformation modifies the available volumes of the neighboring annuli and links their pressure responses to both the local temperature change and heat transfer through the multilayer wellbore.

Overall, the close agreement between S2 and S3 shows that, in multilayer casing structures, annular trapped pressure is not determined independently by thermal expansion of the fluid in a single annulus. Instead, it is jointly controlled by fluid thermal-pressure response, elastic casing deformation and pressure coupling among adjacent annuli. Therefore, incorporating multi-annulus coupling into annular pressure prediction can effectively reduce calculation bias and improve the interpretation of field pressure responses in deepwater or high-temperature high-pressure wellbore-integrity assessments.

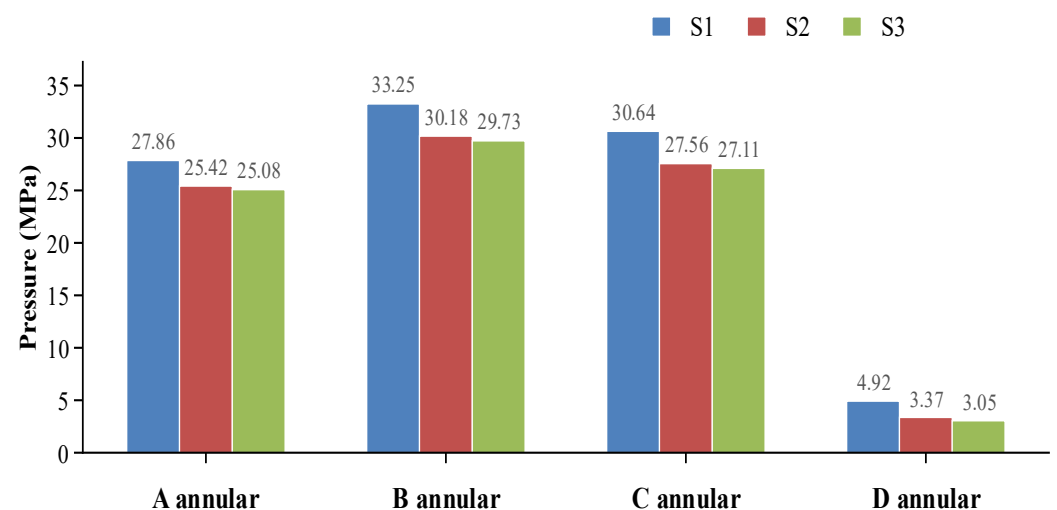


Figure 6. Comparison of annular trapped pressure under different conditions.

7.4. Endpoint Comparison Under Two Prescribed Temperature-Rise Conditions

The available field dataset did not contain time-resolved temperature and annular pressure measurements over the complete production period. Therefore, the field comparison was performed at two prescribed average temperature-rise conditions, namely 40 and 50 °C. These two conditions represent two thermal-loading levels rather than two points in a continuously measured transient history.

For each thermal condition, the prescribed temperature rise was applied to the wellbore thermal–mechanical model, and the resulting annular pressures were compared with the corresponding field pressure records and Wellcat calculations (version 5000.17.2, Schlumberger). This comparison evaluates the model’s ability to reproduce the annular pressure levels under the two investigated thermal conditions. It does not constitute a time-resolved validation of the transient pressure response, including the pressure-growth rate, response lag, or time to peak pressure.

Tables 3 and 4 show that Wellcat exhibited a clear overestimation trend under both temperature-rise conditions, whereas the proposed model was closer to the measured values. Under the 40 °C condition, the measured pressures in Annuli A, B, C and D were 17.52, 27.98, 24.22 and 4.23 MPa, respectively. The Wellcat calculated values were 22.18, 31.72, 29.91 and 5.12 MPa, corresponding to errors of 26.60%, 13.37%, 23.49% and 21.04%, with a mean relative error of 21.13%. By contrast, the calculated values from the proposed model were 17.85, 27.57, 23.06 and 4.39 MPa, with errors of only 1.88%, 1.47%, 4.79% and 3.78%. The mean relative error decreased to 2.98%. In terms of absolute deviation, the mean absolute deviation of Wellcat was 3.75 MPa, whereas that of the proposed model was only 0.52 MPa, corresponding to an error reduction of approximately 86.25%.

Under the 50 °C condition, the measured pressures in Annuli A, B and C were 18.86, 30.21 and 26.05 MPa, respectively. The Wellcat calculated values were 24.84, 34.18 and 30.81 MPa, with errors of 31.71%, 13.14% and 18.27%, giving a mean relative error of 21.04%. The proposed model produced calculated values of 18.12, 29.94 and 24.98 MPa, with errors of 3.92%, 0.89% and 4.11%, giving a mean relative error of 2.97%. Compared with Wellcat, the mean absolute deviation of the proposed model under the 50 °C condition decreased from 4.90 MPa to 0.69 MPa, corresponding to an error reduction of approximately 85.86%.

Under the 50 °C condition, the Wellcat predictions were higher than the field measurements in all four annuli, with relative errors of 31.71%, 13.14%, 18.27%, and 25.41% for Annuli A, B, C, and D, respectively. The corresponding errors of the proposed model were 3.92%, 0.89%, 4.11%, and 1.02%. When all four annuli, including Annulus D, were considered, the descriptive mean relative error decreased from 22.13% for Wellcat to 2.49% for the proposed model. For the coupled Annuli A–C alone, the corresponding mean relative errors were 21.04% and 2.97%, respectively. Annulus D was evaluated independently under the rigid-packer assumption. Therefore, the improved agreement for Annuli A–C supports the relevance of the casing-deformation and multi-annulus coupling treatment, whereas the result for Annulus D provides an additional assessment of the independent sealed-annulus calculation. For the investigated well and the two prescribed thermal-loading conditions, the coupled calculation produced smaller deviations from the field measurements than the corresponding Wellcat calculation. This result supports the relevance of casing-deformation-mediated volume interaction in the present case, but it does not establish general predictive superiority across different wells or operating conditions.

Table 3. Comparison between calculated and measured values under the 40 °C condition.

Annulus	Measuring Pressure (MPa)	Wellcat Calculated Value (MPa)	Error Value (%)	Measuring Pressure (MPa)	Calculated Values from the Model (MPa)	Error Value (%)
Annular A	17.52	22.18	26.60	17.52	17.85	1.88
Annular B	27.98	31.72	13.37	27.98	27.57	1.47
Annular C	24.22	29.91	23.49	24.22	23.06	4.79
Annular D	4.23	5.12	21.04	4.23	4.39	3.78

Table 4. Comparison between calculated and measured values under the 50 °C condition.

Annulus	Measuring Pressure (MPa)	Wellcat Calculated Value (MPa)	Error Value (%)	Measuring Pressure (MPa)	Calculated Values from the Model (MPa)	Error Value (%)
Annular A	18.86	24.84	31.71	18.86	18.12	3.92
Annular B	30.21	34.18	13.14	30.21	29.94	0.89
Annular C	26.05	30.81	18.27	26.05	24.98	4.11
Annular D	4.92	6.17	25.41	4.92	4.87	1.02

To provide a more objective quantitative assessment, the mean absolute error (MAE), root mean square error (RMSE), mean absolute percentage error (MAPE), and coefficient of determination R^2 were calculated as follows:

$$MAE = \frac{1}{n} \sum_{i=1}^n |P_i^{\text{cal}} - P_i^{\text{mea}}| \quad (53)$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (P_i^{\text{cal}} - P_i^{\text{mea}})^2} \quad (54)$$

$$MAPE = \frac{100\%}{n} \sum_{i=1}^n \left| \frac{P_i^{\text{cal}} - P_i^{\text{mea}}}{P_i^{\text{mea}}} \right| \quad (55)$$

$$R^2 = 1 - \frac{\sum_{i=1}^n (P_i^{\text{cal}} - P_i^{\text{mea}})^2}{\sum_{i=1}^n (P_i^{\text{mea}} - \bar{P}^{\text{mea}})^2} \quad (56)$$

where P_i^{cal} and P_i^{mea} are the calculated and measured annular pressures, respectively; $\bar{P}^{\text{mea}}$ is the mean of the measured pressures; and n is the number of comparison points.

Based on Table 5, the proposed model yielded an MAE of 0.524 MPa, an RMSE of 0.653 MPa, a MAPE of 2.733%, and an R^2 of 0.995. The corresponding descriptive values obtained using Wellcat were 3.868 MPa, 4.252 MPa, 21.628%, and 0.794, respectively. Thus, the proposed model showed closer within-case agreement with the field measurements for the investigated well and thermal-loading conditions.

Table 5. Descriptive error metrics for the eight available comparison points from one well.

Method	MAE (MPa)	RMSE (MPa)	MAPE (%)	R^2
Wellcat	3.868	4.252	21.628	0.794
Proposed model	0.524	0.653	2.733	0.995

These metrics should be interpreted cautiously because the eight points were obtained from four annuli in one well under only two prescribed thermal conditions and do not constitute eight independent validation cases. In addition, the relatively high R^2 may partly reflect the pressure differences among the annuli, whereas MAPE is sensitive to the relatively low pressure in Annulus D. Therefore, the metrics are used only as descriptive

measures of agreement and should not be interpreted as evidence of general model performance.

Owing to the confidentiality agreement with the well operator, the well identifier and certain commercially sensitive operating records cannot be disclosed. Nevertheless, the geometric, material, fluid, thermal, initial-state, and numerical parameters required to reproduce the reported calculations are provided in Tables 1 and 2, S1 and S2 in anonymized form. The field pressure measurements were used only for model evaluation, and no field-data-based parameter calibration was performed.

8. Conclusion and Future Work

Compared with independent single-annulus calculations, the principal contribution of the proposed framework is the synchronous enforcement of volume compatibility among Annuli A–C through the deformation of shared casing walls. The novelty lies in the unified pressure–deformation–volume solution architecture, rather than in the separate inclusion of thermal expansion or tubular deformation. The principal quantitative findings are summarized as follows.

(1) When multi-annulus coupling was neglected, the predicted pressures exceeded the field measurements by 2.78, 3.52, 3.53, and 1.87 MPa in Annuli A, B, C, and D, respectively. After casing-deformation-mediated multi-annulus coupling was incorporated, the corresponding absolute deviations decreased to 0.34, 0.45, 0.45, and 0.32 MPa. Consequently, the mean absolute deviation decreased from 2.93 to 0.39 MPa, representing an error reduction of approximately 86.67%.

(2) Under the 40 °C temperature-rise condition, the mean relative error of the proposed model was 2.98%, compared with 21.13% for Wellcat. The mean absolute deviation decreased from 3.75 MPa for Wellcat to 0.52 MPa for the proposed model, corresponding to a reduction of approximately 86.25%.

(3) Under the 50 °C temperature-rise condition, the mean relative error for the coupled Annuli A–C decreased from 21.04% for Wellcat to 2.97% for the proposed model. The corresponding mean absolute deviation decreased from 4.90 to 0.69 MPa, representing an error reduction of approximately 85.86%. When Annulus D was also included, the mean relative error decreased from 22.13% to 2.49%.

(4) For the investigated well and the two prescribed thermal-loading conditions, the reduction in prediction errors is consistent with volume compensation and pressure redistribution through the shared casing walls. In this case, treating the annuli independently resulted in pressure overestimation. Additional wells and independent operating conditions are required to determine whether this tendency can be generalized. Annulus D was evaluated separately under the rigid-packer assumption and therefore does not constitute part of the A–C pressure–deformation coupling system.

The field comparison was limited to two prescribed temperature-rise conditions because time-resolved temperature and pressure measurements were unavailable. Therefore, the reported results validate the predicted pressure levels under the investigated thermal-loading conditions but do not constitute time-resolved validation of pressure-growth rates, response delays, or peak times.

Our future work should focus on three areas. First, we need to conduct synchronized, time-resolved measurements of annular pressure and temperature to evaluate the predicted pressure-growth rate, response delay, and peak time. Second, we should further evaluate the model using additional wellbores, laboratory experiments, and independent benchmark cases covering different casing configurations, annular fluids, and operating conditions. Third, sensitivity and uncertainty analyses should be conducted to quantify the effects of temperature, fluid properties, casing elasticity, initial pressure, and gas volume fraction on the predicted annular pressure trap.

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