

Article

Theory and Application of Formation Pressure in Fractured-Vuggy Oil and Gas Reservoirs Based on Concentrated Source Theory

Dongmei Li ^{1,2} and Zhiwei Lu ^{3,*}¹ Engineering Technology Management Department, Sinopec Northwest Oilfield Company, Urumqi 830011, China² Sinopec Key Laboratory for Enhanced Recovery in Fractured-Caved Reservoirs, Urumqi 830011, China³ Anhui Inflywell Technology Co., Ltd., Hefei 230088, China

* Correspondence: wellfly_anhui@163.com

Abstract

In this study, a rapid analytical framework is developed for reconstructing transient formation-pressure distributions in sparsely connected fractured-vuggy carbonate reservoirs using well-test-derived fracture–cave geometry and source allocation. Pressure–flow relationships are established for caves, fractured bodies, and finite line-source fractures; unit-source solutions are combined via spatial superposition and Duhamel convolution for variable-rate production; and the framework is evaluated by comparing Well EX-1 against a PEBI-grid simulation. Cave response is derived from mass conservation and effective compressibility, while point- and line-source Green’s functions describe pressure diffusion. For the 30-day EX-1 case, analytical and PEBI pressures at four locations show close internal agreement, with a mean absolute error of 0.070 MPa, a root-mean-square error of 0.083 MPa, and a maximum absolute error of 0.13 MPa. This method is applicable mainly to single-phase, slightly compressible, linear-flow conditions with a constrained fracture–cave topology. It provides a rapid screening tool for deep carbonate reservoirs with sparse well control, while multiphase, strongly nonlinear, reactive, or geomechanically coupled cases require conventional numerical simulation.

Keywords: carbonate reservoirs; formation pressure; well-test analysis; concentrated source theory; convolution integral; petroleum exploration; fractured pore network



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1. Introduction

Production rate and formation pressure are fundamental parameters throughout the life cycle of an oil or gas well, from drilling to abandonment [1]. Development measures, such as acidizing, fracturing, water flooding, and polymer flooding, are designed and evaluated largely through their effects on these parameters. Early formation-pressure calculations assumed steady-state flow and yielded a single-well pressure distribution equation:

$$p(r) = p_w + \frac{QB\mu}{2\pi kh} \ln\left(\frac{r}{r_w}\right)$$

Based on this distribution, the concept of a formation-pressure funnel was proposed [2]. Using the single-well pressure distribution as a foundation, the superposition principle was applied to derive pressure distributions for well patterns, well rows, and well groups [3]. For complex regions, conformal mapping techniques from complex variable functions can

be used to determine pressure distribution, or alternatively, the series-parallel calculation method of hydroelectric analogies in porous media flow can be employed [4]. However, all these methods are based on steady-state flow theory. In fractured-vuggy reservoirs, the storage space mainly consists of vugs, fractures, and matrix pores, with the distribution of vugs and fractures being highly random [5]. These reservoirs are generally produced by natural flow, but pressure declines rapidly. When natural flow is no longer sustainable, energy is replenished by gas or water injection before further production [6–8]. This production mode cannot be analyzed using steady-state flow methods for pressure calculation.

Transient formation pressure is primarily calculated using reservoir numerical simulation, which involves gridding the computational domain and discretely solving the flow equations using finite-difference methods [9–11]. The simulation output provides the pressure at each grid block at different times. For multiphase flow, saturation distributions over time can also be obtained. Reservoir development involves geology, production technology, and reservoir fluids, and consequently, numerous numerical simulation methods have been proposed [12,13]. However, reservoir numerical simulation operates on large spatial and temporal scales, requiring extensive data preparation and long processing times. Meanwhile, pressure history matching requires continuous adjustments and iterations, consuming even more time. Fractured-vuggy reservoirs are mainly developed by drilling into fractures and caves, and wells are often unconnected to each other. The duration of natural flow production supported by reservoir pressure is also relatively short. For rapid pressure assessment in sparsely connected fracture–cave systems, conventional numerical simulation can be data- and time-intensive because it requires grid construction, property assignment, solver configuration, and iterative history matching. Therefore, this paper develops an analytical method for calculating formation-pressure distributions from well-test-derived fracture–cave information.

Well test analysis has a history of nearly a century in oil and gas field applications [14,15]. Due to differences in fluid properties, gas well testing requires the definition of pseudopressure and pseudotime to linearize the flow equations for analytical solutions of bottomhole pressure [16]. Recent studies have advanced pressure-transient interpretation, semi-analytical source solutions, connectivity characterization, geological modeling, material-balance analysis, and coupled numerical simulation for fractured-vuggy carbonate reservoirs [17–23]. These studies confirm that analytical or semi-analytical models are most useful when fracture–cave topology can be constrained from well tests and seismic interpretation, whereas full numerical simulation remains necessary for strongly nonlinear, multiphase, reactive, or geomechanically coupled processes. The present work addresses the intermediate need for rapid formation-pressure calculations after the principal caves and connecting fractures have been identified.

Well-test analysis provides a practical route for inferring reservoir parameters from transient pressure data. Gas-well analysis commonly uses pseudopressure and pseudotime to account for pressure-dependent fluid properties [24–26], whereas naturally fractured and fractured-vuggy systems have been represented by dual- and triple-porosity models [6,27]. These classical models identify storage ratios and interporosity flow coefficients but do not uniquely resolve the spatial arrangement and volumes of individual caves and connecting fractures. The coupled wave–flow model of Du et al. [24] improves geological interpretability by estimating fracture–cave locations, connectivity, and cave volumes from pressure transients. Building on that interpreted topology, the present study applies concentrated source theory, linear superposition, and Duhamel convolution to reconstruct formation-pressure distributions rather than only bottomhole pressure histories, thereby providing a rapid analytical tool when interwell data and full-field property grids are limited.

2. Theory and Method for Pressure Distribution Calculation

Deep carbonate fractured-vuggy reservoirs (at depths greater than 7000 m) are widely distributed in the Tarim Basin of China. For example, in Well EX-1 of the Shunbei area, when drilling reached a measured depth of 7694.57 m (7265 m vertical depth), severe mud loss of 435.32 m³ occurred during circulation and sand cleaning, indicating that the well had penetrated a cave or fault zone. Figure 1a shows the seismic time-migration section near this well, and Figure 1b shows the coherent energy volume and ant-tracking attribute maps. Both figures reveal the presence of multiple caves and ant-tracking lineaments near the wellbore. Coherence-based discontinuity attributes emphasize reflector discontinuities associated with faults, whereas ant-tracking enhances connected linear discontinuities and is widely used to delineate fractures and caves in heterogeneous carbonate reservoirs [28,29].

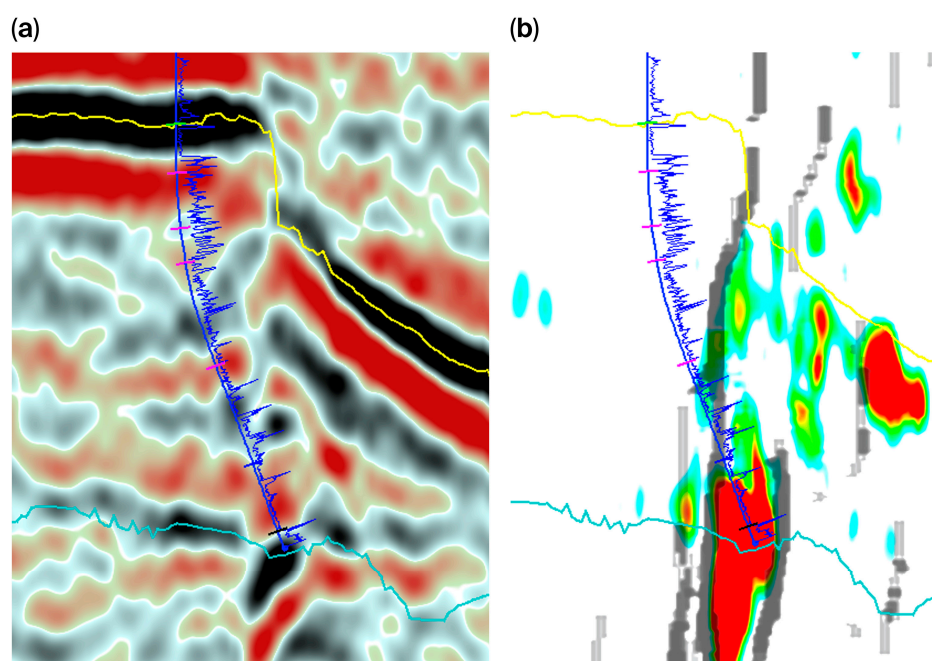


Figure 1. (a) Seismic time-migration section; (b) coherent energy gradient and ant-tracking attributes. Source: authors' processing and interpretation of proprietary field seismic data for Well EX-1.

Jiang et al. [30] classified such geological bodies into three types from the perspectives of hydrocarbon storage and flow: (1) the inter-breccia pore-type reservoir geological model, (2) the fault-cavity-type reservoir geological model, and (3) the tectonic fracture-pore-type reservoir geological model [30]. To obtain the formation pressure in such formations, the carbonate fractured-vuggy reservoir is simplified into components consisting of fractured bodies, fractures, and caves. The main fluid storage spaces are the fractured bodies and caves, while fractures serve as flow pathways connecting them. The pressure–flow relationships for these different storage units are discussed below.

2.1. Relationship Between Pressure Change and Inflow/Outflow in a Cave

Consider a 1500 m long pipe filled with water, with a pressure gauge installed at the left end and a piston at the right end. When a force is applied to the piston, the pressure disturbance reaches the gauge after the finite acoustic travel time L/C . For water, taking $C \approx 1450$ m/s gives $L/C \approx 1.0$ s, which is short relative to production and well-test time scales but not instantaneous. Du et al. [24] likewise modeled pressure changes in a cave as fluid compression (or rarefaction) waves.

Taking a small control volume of length δx within a cave (Figure 2b), the mass conservation equation gives:

$$\rho\Omega v - [\rho\Omega v + \frac{\partial}{\partial x}(\rho\Omega v)\delta x] = \frac{\partial}{\partial t}(\rho\Omega\delta x) \quad (1)$$

where $\Omega = A\delta x$ is the volume of the differential cave element (m^3), A is its effective cross-sectional area (m^2), δx is the elemental length (m), v is the fluid velocity (m/s), and ρ is the fluid density (kg/m^3). Here, Ω does not denote the total volume of the entire cave. The volume form of Equation (1) is obtained by multiplying every term of the conventional cross-sectional area form by the same δx .

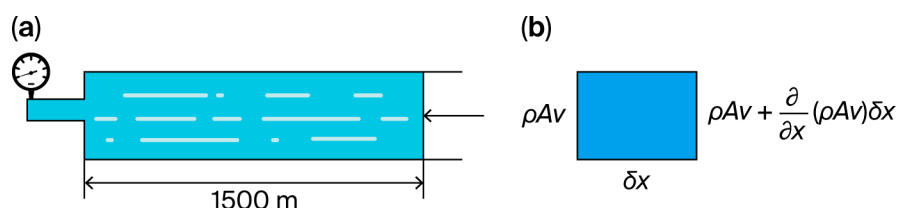


Figure 2. (a) A schematic of the fluid-pressure conduction experiment; (b) a schematic of fluid inflow/outflow in a differential control volume.

Expanding the equation yields:

$$\frac{\partial v}{\partial x} + \frac{v}{\Omega} \frac{\partial \Omega}{\partial x} + \frac{v}{\rho} \frac{\partial \rho}{\partial x} + \frac{1}{\Omega} \frac{\partial \Omega}{\partial t} + \frac{1}{\rho} \frac{\partial \rho}{\partial t} = 0 \quad (2)$$

Using the relationship between the total derivative and the partial derivative in fluid mechanics:

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + v \frac{\partial \rho}{\partial x}; \quad \frac{d\Omega}{dt} = \frac{\partial \Omega}{\partial t} + v \frac{\partial \Omega}{\partial x} \quad (3)$$

The equation can be rewritten as:

$$\frac{\partial v}{\partial x} + \frac{1}{\rho} \frac{d\rho}{dt} + \frac{1}{\Omega} \frac{d\Omega}{dt} = 0 \quad (4)$$

The density term can be expressed as a function of pressure:

$$\frac{1}{\rho} \frac{d\rho}{dt} = \frac{1}{K} \frac{dp}{dt} \quad (5)$$

where K is the fluid bulk modulus (Pa).

Since the fluid is surrounded by rock, and a well may be drilled through the cave, an equivalent bulk modulus K_{eff} is used in the equation, given by:

$$K_{eff}^{-1} = K^{-1} + G^{-1} \quad (6)$$

where K is the bulk modulus of the fluid (Pa), and G is the shear modulus of the surrounding formation (Pa).

The wave propagation velocity in the cave fluid can be expressed as:

$$C = \sqrt{\frac{K_{eff}}{\rho}} \quad (7)$$

According to the definition of wave speed, the equation can be rewritten as:

$$\frac{\partial v}{\partial x} + \frac{1}{\rho C^2} \frac{dp}{dt} + \frac{1}{\Omega} \frac{d\Omega}{dt} = 0 \quad (8)$$

The speed of sound differs among fluid media (e.g., approximately 340 m/s in air and 1450 m/s in pure water). The simplification below is not justified by $C \gg v$ alone. Equation (1) is interpreted using control volume balance: after volume averaging, the net boundary exchange is retained explicitly through the inflow/outflow term, whereas $\partial v/\partial x$ represents unresolved spatial variation in the internal velocity about its volume-averaged value. Let L_c be the characteristic cave length, U the mean internal velocity, C the acoustic velocity, T_p the shortest imposed production or pressure-variation time scale, and Δv the velocity variation across the cave. The lumped closure requires $Ma = U/C \ll 1$, $\varepsilon a = L_c/(CT_p) \ll 1$, and $\varepsilon v = (L_c/U) |\partial v/\partial x| \approx \Delta v/U \ll 1$. The first two conditions ensure that pressure equilibrates across the cave before the imposed operating condition changes, and the third states directly—rather than inferring from $C \gg v$ —that the internal velocity field is approximately uniform on the cave scale. Under these conditions, volume averaging reduces the distributed balance to the lumped storage relation used in Equations (9) and (10), while the net cave exchange remains. If εv is not small, or during the initial acoustic transient, the high-velocity flow or rapid rate changes, the distributed velocity-gradient term must be retained; thus, the lumped model is not applicable.

$$\frac{1}{\rho C^2} \frac{dp}{dt} = -\frac{1}{\Omega} \frac{d\Omega}{dt} \quad (9)$$

Here, $\frac{d\Omega}{dt}$ is the fluid exchange rate between the cave and surrounding media (such as fractured bodies, fractures, and matrix). Therefore, the relationship between the fluid inflow/outflow rate and the pressure change in the cave is:

$$Q(t) = -\frac{\Omega}{\rho C^2} \frac{dp}{dt} \quad (10)$$

2.2. Pressure Difference Under Unit-Source Intensity in a Fractured Body

In physics, there is a class of physical quantities that are concentrated in distributions (e.g., point masses, point charges, point heat sources, and concentrated forces). The delta function, δ , is a mathematical tool used to describe such concentrated distributions [31]. To mathematically represent these concentrated quantities using the δ function, it must satisfy two conditions:

$$\delta(M) = \begin{cases} 0 & M \in M_0 \\ \infty & M \notin M_0 \end{cases} \quad (11)$$

$$\iiint_v \delta(M) dv = \begin{cases} 1 & M_O \in v \\ 0 & M_O \notin v \end{cases} \quad (12)$$

For slightly compressible fluid flow in a fractured body, if a fluid mass of $\tilde{m}$ is injected (or produced) at point r' at time τ , the density ρ satisfies the following equation and boundary/initial conditions:

The governing diffusion equation can be obtained by combining Darcy's law, $u = -(k/\mu)\nabla p$, with mass conservation for a homogeneous, isotropic, slightly compressible, fractured continuum. For constant porosity ϕ , permeability k , viscosity μ , and total compressibility C_t , the pressure equation is $\partial p/\partial t = \chi \nabla^2 p$, where $\chi = k/(\phi\mu C_t)$ is the hydraulic

diffusivity (m^2/s). In a two-dimensional formulation, the Dirac delta function $\delta(x - x_0)\delta(y - y_0)$ has units m^{-2} and localizes an instantaneous source at (x_0, y_0) .

The Fourier transform converts the spatial diffusion operator into multiplication by $-(\xi^2 + \zeta^2)$, producing a first-order ordinary differential equation in time. Its exponential solution represents attenuation of high-wavenumber components. Inverse transformation yields the two-dimensional Green's function $G(\mathbf{r}, t - \tau) = [4\pi\chi(t - \tau)]^{-1} \exp[-r^2/(4\chi(t - \tau))]$, where $r^2 = (x - x_0)^2 + (y - y_0)^2$ and $t > \tau$. The Gaussian kernel conserves the injected quantity because its integral over the infinite plane equals unity.

The parameters for Equations (22)–(24) are defined as follows: p_i is the initial pressure (Pa); $p(x, y, t)$ is the pressure at the observation point (Pa); δv is the incremental source volume per unit reservoir thickness (m^2 in the two-dimensional representation); φ is dimensionless porosity; C_t is total compressibility (Pa^{-1}); k is permeability (m^2); μ is viscosity ($\text{Pa}\cdot\text{s}$); χ is hydraulic diffusivity (m^2/s); τ is the source time (s); and $t - \tau$ is the elapsed diffusion time (s). For a finite reservoir thickness h , a three-dimensional volumetric source must be converted consistently by dividing the source rate by h . Production and injection use opposite signs of source intensity.

$$\frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2} = \frac{1}{\chi} \frac{\partial \rho}{\partial t} \quad (13)$$

$$\rho(x \rightarrow \pm\infty, y \rightarrow \pm\infty, t) = \rho_i \quad (14)$$

$$\rho(x \rightarrow \pm\infty, y \rightarrow \pm\infty, t) = \rho_i \quad (15)$$

Let $\Delta\rho(x, y, t) = \rho_i - \rho(x, y, t)$, and apply a spatial Fourier transform to the above equations with respect to coordinates (x, y) .

$$\bar{\rho}(\xi, \zeta, t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \exp[(\xi x' + \zeta y')i] \Delta\rho(x', y', t) dx' dy' \quad (16)$$

Based on the boundary conditions and the properties of the Fourier transform, the equation becomes

$$\frac{d\bar{\rho}}{dt} = \chi \bar{\rho} [(-i\xi)^2 + (-i\zeta)^2] = -\chi (\xi^2 + \zeta^2) \bar{\rho} \quad (17)$$

$$\bar{\rho}|_{t=\tau} = \exp(\xi x_0 + \zeta y_0) \quad (18)$$

The solution to this ordinary differential equation and its initial condition is:

$$\bar{\rho}(\xi, \zeta, t) = \exp \left[-\chi (\xi^2 + \zeta^2) (t - \tau) + i(\xi x_0 + \zeta y_0) \right] \quad (19)$$

Applying the inverse Fourier transform, the solution $p(\mathbf{r}, t)$ in real space is expressed as:

$$\begin{aligned} \Delta\rho(x, y, t) &= \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \exp[-i(\xi x + \zeta y)] \cdot \Delta\bar{\rho} d\xi d\zeta \\ &= \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} \exp[-\chi \xi^2 - i\xi(x - x_0)] d\xi \cdot \int_{-\infty}^{+\infty} \exp[-\chi \zeta^2 - i\zeta(y - y_0)] d\zeta \end{aligned} \quad (20)$$

The integral term is evaluated as:

$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} \exp[-\chi \xi^2 - i\xi(x - x_0)] d\xi = \frac{1}{2\sqrt{\pi\chi(t - \tau)}} \exp \left[-\frac{(x - x_0)^2}{4\chi(t - \tau)} \right] \quad (21)$$

Substituting this result gives the explicit expression for $p(r,t)$:

$$\rho = \rho_i + \frac{A}{4\pi\chi(t-\tau)} \exp \left[-\frac{(x-x_0)^2 + (y-y_0)^2}{4\chi(t-\tau)} \right] \quad (22)$$

Using the definition of fluid compressibility $C_f = -\frac{1}{\rho} \frac{d\rho}{dp}$ and noting that this compressibility is small (e.g., the compressibility of pure water is $C_w = 4.75 \times 10^{-10} \text{Pa}^{-1}$), the relationship between density and pressure is $\rho = \rho_i [1 - C_f(p_i - p)]$. Considering that the injected or produced fluid mass flow rate $\delta_m = \rho_i \delta_v$ (where δ_v is the volumetric flow rate), the pressure distribution $p(x, y, t)$ in the formation is given by:

$$p(x, y, t) = p_i + \frac{\delta v}{4\pi\phi\chi C_t(t-\tau)} \exp \left[-\frac{(x-x_0)^2 + (y-y_0)^2}{4\chi(t-\tau)} \right] \quad (23)$$

The pressure difference under unit-source intensity is expressed as:

$$\frac{p_i - p(x, y, t)}{\delta v} = \frac{\Delta p(x, y, t)}{\delta v} = \frac{1}{4\pi\phi\chi C_t(t-\tau)} \exp \left[-\frac{(x-x_0)^2 + (y-y_0)^2}{4\chi(t-\tau)} \right] \quad (24)$$

where $\chi = k/(\phi\mu C_t)$ is hydraulic diffusivity (m^2/s), k is fractured-body permeability (m^2 or mD), μ is fluid viscosity ($\text{Pa}\cdot\text{s}$), ϕ is porosity (dimensionless), C_t is total compressibility (Pa^{-1}), (x_0, y_0) is the source location (m), τ is the source time (s), and t is the observation time (s). The solution assumes an infinite, homogeneous, isotropic, two-dimensional domain and constant fluid and rock properties.

2.3. Pressure Difference Under Unit-Source Intensity for a Fracture System (Line-Source)

A fracture is simplified as a line source, as shown in Figure 3. Points A, B, and C represent the two endpoints and the midpoint of the line source, respectively. The line source has a total length of $2L$ and forms an angle α with the horizontal axis. The center of the line source is located at a distance $OC = L_W$ from the origin. First, the xoy coordinate system is rotated by angle α to the $x'oy'$ coordinate system. A differential element ox' is taken along the AB line source at a distance $d\sigma$ from the origin. The pressure generated by this differential element in the formation is given by Equation (24). Integrating along ox' from OA to OB yields the pressure distribution caused by the line source.

Let the fracture midpoint be located at (x_c, y_c) , its half-length be L (m), and its orientation be α (rad). The rotated coordinates are $x' = (x - x_c)\cos \alpha + (y - y_c)\sin \alpha$ and $y' = -(x - x_c)\sin \alpha + (y - y_c)\cos \alpha$. A uniformly distributed line-source element $d\sigma$ carries the fraction $d\sigma/(2L)$ of the total source intensity. Substitution of the point-source Green's function followed by integration from $\sigma = -L$ to L gives Equation (26).

The exponential factor in Equation (26) controls pressure diffusion normal to the fracture, whereas the two error function terms account for the finite endpoints. As L approaches zero, the solution tends to the point-source kernel; when L is very large compared with the diffusion length $\sqrt{[\chi(t-\tau)]}$, the endpoint effects vanish, and the response approaches that of an infinite line source. This formulation treats the fracture as a zero-width source and therefore does not resolve aperture-scale velocity or pressure variation.

$$p(x', y', t) = p_i + \frac{\delta v}{4\pi\phi\chi C_t(t-\tau)} \int_{OA}^{OB} \exp \left[-\frac{(x-\sigma)^2}{4\chi(t-\tau)} \right] \frac{d\sigma}{2L} \quad (25)$$

After integration and applying the coordinate rotation transformation back to the (x,y) system, the pressure difference under unit-source intensity is:

$$\frac{\Delta p(x,y,t)}{\delta v} = \frac{1}{2\phi C_t \sqrt{2\pi\chi(t-\tau)}} \exp\left[-\frac{(-x \sin \alpha + y \cos \alpha)^2}{4\chi(t-\tau)}\right] \cdot \left\{ \operatorname{erf}\left[\frac{L+(x \cos \alpha + y \sin \alpha - L_w)}{\sqrt{4\chi(t-\tau)}}\right] + \operatorname{erf}\left[\frac{L-(x \cos \alpha + y \sin \alpha - L_w)}{\sqrt{4\chi(t-\tau)}}\right] \right\} \quad (26)$$

where $\operatorname{erf}(z) = (2/\sqrt{\pi}) \int_0^z \exp(-u^2) du$ is the error function, and $\operatorname{erfc}(z) = 1 - \operatorname{erf}(z)$ is the complementary error function.

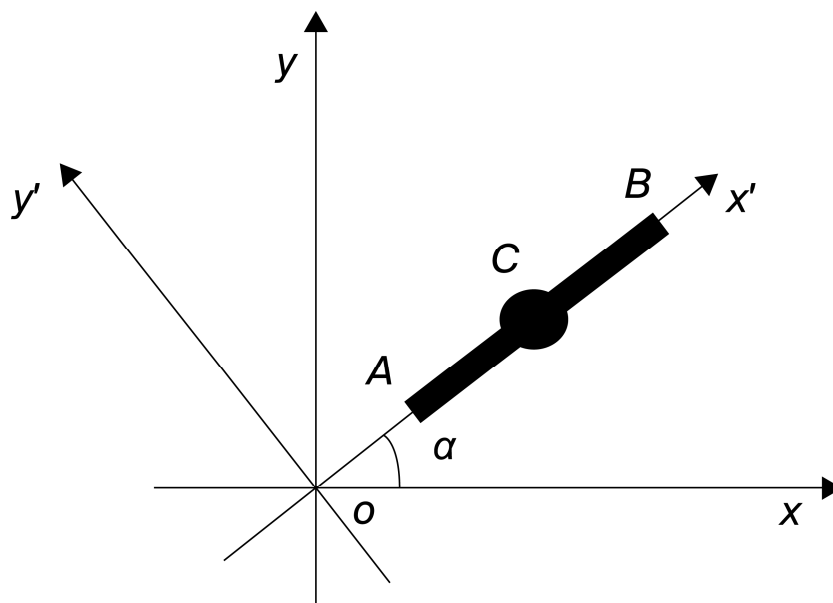


Figure 3. The position and length of the finite fracture line source.

2.4. Formation Pressure in Fractured-Vuggy Reservoirs via Convolution and Superposition Principles

A fractured-vuggy reservoir contains caves, fractures, and fractured bodies. Under the linear assumptions stated above, the unit-source pressure responses of a cave (Equation (10)), a fractured body (Equation (24)), and a finite fracture line source (Equation (26)) can be summed. For M caves, N fractures, and L fractured bodies, Equation (27) gives the composite unit-source pressure response at (x,y).

$$\Delta p'(x,y,t) = \sum_{m=1}^M \left[\frac{\Delta p_v}{\delta v_v} \right]_m + \sum_{n=1}^N \left[\frac{\Delta p_f}{\delta v_f} \right]_n + \sum_{l=1}^L \left[\frac{\Delta p_p}{\delta v_p} \right]_l \quad (27)$$

The development of a fractured-vuggy reservoir is a continuous production process, involving varying production rates over different periods and the allocation of production among caves and fractures at different locations. To address these issues, Duhamel's principle is applied to convolve the production rate with the unit-source solution, thereby obtaining the pressure at various locations in the formation:

$$p(x,y,t) = p_i - \int_0^t q(t-\tau) \Delta p'(x,y,\tau) d\tau \quad (28)$$

where $q(\tau)$ is the production rate supplied by caves and fractures. The allocation of these rates needs to be determined through well-test interpretation.

For a piecewise-constant production history, the convolution can be evaluated as a sum of rate increments as follows: $p(x,y,t_n) = p_i - \sum_s \sum_j \Delta q_{s,j} K_s[x,y,t_n - t_j]$, where s is the index of source units, j is the index of rate-change times, and K_s is the corresponding unit-source kernel. This form clarifies that Duhamel's principle superposes responses to individual rate changes rather than repeatedly solving a new spatial system. The source-rate fractions assigned to each cave or fracture must sum to the measured downhole rate after conversion from surface to reservoir conditions.

All quantities entering the convolution must use a consistent unit system: q in m^3/s at reservoir conditions, time in s , pressure in Pa during calculations, and source kernels in $\text{Pa}\cdot\text{s}/\text{m}^3$. Calculated pressures are converted to MPa only for reporting. If gas pseudopressure is used, the convolution is performed in pseudopressure space and subsequently transformed back to pressure using the PVT relation.

The superposition and Duhamel convolution formulation is reliable only while the governing pressure equation remains linear: geometry and source locations are fixed; permeability, porosity, viscosity, and compressibility are approximately constant or have been linearized through pseudopressure/pseudotime; source-rate allocation is prescribed; and external boundaries do not strongly affect the investigated time window. It is not expected to remain accurate after saturation fronts, strong gas property variation, aquifer influx, pressure-dependent fracture conductivity, non-Darcy flow, or geomechanical changes make the response nonlinear or history-dependent.

2.5. Model Assumptions, Applicability, and Limitations

The proposed method is applicable when the dominant fracture–cave topology is known, flow is single-phase or can be linearized via gas pseudopressure/pseudotime, Darcy flow is valid, and k , μ , ϕ , and C_t remain approximately constant over the investigated pressure range. The cave-lumped storage approximation additionally requires acoustic equilibration to be much faster than the production or shut-in time scale [$L_c/(C_t p) \ll 1$] and the internal velocity to be nearly uniform at the cave scale [$(L_c/U) |\partial v/\partial x| \ll 1$]. These are independent requirements; a low Mach number alone is not sufficient.

The main idealizations are as follows: (1) fractured bodies are homogeneous and isotropic continua; (2) fractures are represented as zero-width line sources with uniform source strength; (3) caves have spatially uniform pressure; (4) the Green's functions correspond to an infinite or sufficiently distant-boundary domain; and (5) pressure responses are linear and can therefore be superposed. These assumptions may become inaccurate near sealing faults, external aquifers, strong permeability contrasts, pressure-dependent fracture conductivity, or rapidly changing fluid properties.

The model does not explicitly resolve multiphase saturation, capillary pressure, gravity segregation, compositional phase behavior, non-Darcy flow, chemical reactions, thermal effects, or hydro-mechanical deformation. Such processes require a conventional numerical simulator or a coupled extension [13,21,23]. In addition, well-test inversion of cave volumes, locations, and rate allocation may be non-unique; sensitivity analysis or uncertainty propagation should therefore accompany forecasts when several structural interpretations fit the same pressure data.

Accordingly, the method is intended as a rapid screening and pressure-mapping tool for identified fracture–cave networks, not as a universal replacement for full reservoir simulation. It is most valuable for short-term pressure prediction, scenario ranking, and initialization or quality control of numerical models. Numerical simulation is preferred when the development decision depends on phase distribution, water/gas breakthrough, reaction, deformation, or complex boundary conditions.

3. Field Application and Model Evaluation

The model inputs for Well EX-1 in the Shunbei Block, Xinjiang, China, were constrained using bottomhole pressure-buildup tests after drilling and after two acid-fracturing treatments, static pressure and temperature surveys, multi-rate productivity tests, reservoir-fluid PVT analysis, and available drilling, mud-logging, wireline-logging, seismic, and geological information.

Figure 4 shows the wellbore temperature and pressure profiles obtained from static surveys. The static pressure gradient is $dp/dz = 0.0038$ MPa/m, and the temperature gradient is $dT/dz = 0.012$ °C/m. The flowing pressure gradient is 0.0037–0.0038 MPa/m, which is consistent with the static gradient. The approximately linear pressure–depth relationship indicates nearly uniform fluid properties over the surveyed interval and supports the inference that gas is the principal flowing phase.

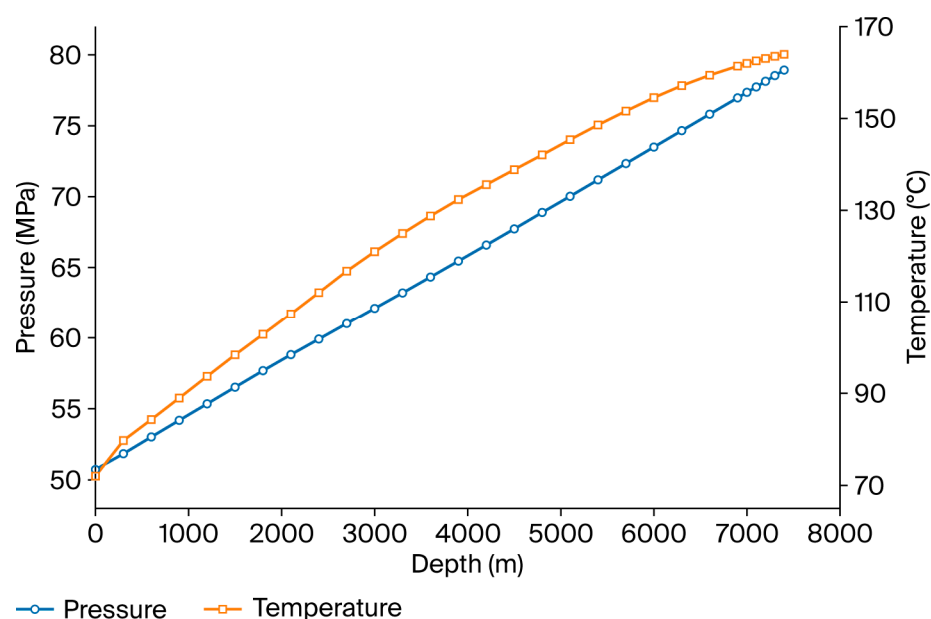


Figure 4. EX-1 wellbore pressure and temperature profiles with depth.

3.1. Obtaining Formation Parameters Through Well-Test Interpretation

After acidization, Well EX-1 was produced for 30 days through a 6 mm choke before the pressure-buildup test. The pressure gauge was lowered to a measured depth of 7680 m (7265 m true vertical depth approximately). The formation temperature was 164.63 °C, the effective reservoir thickness was 10 m, the porosity was 7.83%, and the wellbore radius was 0.08255 m. Gas mole fractions were CH₄ 76.47%, C₂H₆ 4.58%, C₃H₈ 2.52%, n-C₄H₁₀ 0.92%, i-C₄H₁₀ 1.52%, n-C₅H₁₂ 0.69%, i-C₅H₁₂ 0.87%, C₆+ 0.17%, CO₂ 10.48%, and N₂ 1.78%. The measured data were converted to pseudotime and pseudopressure [25], and type-curve fitting was performed using gas-well-test methods [26]. For the pressure-reconstruction calculations, the stabilized surface gas rate was $q_g = 3.0 \times 10^4$ Sm³/d at standard conditions $p_s = 0.101325$ MPa and $T_s = 293.15$ K. Using $q_r = q_g(p_s/p_r)(T_r/T_s)(Z_r/Z_s)$, with $p_r = 85.01$ MPa, $T_r = 437.78$ K, $Z_r = 1.415$ (estimated from the reported gas composition using the Peng–Robinson equation of state), and $Z_s \approx 1.0$, gives a reservoir-condition rate $q_r \approx 76$ m³/d.

Figure 5 shows the log–log type-curve match of pseudopressure and its derivative for the second post-acidization pressure-buildup test in Well EX-1. The fractured-vuggy well-test model of Du et al. [24] was used. The selected topology is well–Cave 1–fracture–Cave 2–fracture–Cave 3 (Figure 6). The interpreted parameters are permeability $k = 12.8$ mD $= 1.263 \times 10^{-14}$ m²; Cave 1 volume = 34,000 m³; Cave 2 volume = 102,300 m³ and 33%

of total flow; Cave 3 volume = 138,000 m³ and 41% of total flow; Cave 2 distance from the well = 196 m; Cave 3 distance from the well = 879 m; and fracture conductivity $k_{fw} = 4.79 \text{ D}\cdot\text{cm}$. The abscissa reports the measured elapsed time Δt for the field data and the dimensionless time T_D for the calculated type curve; the joint logarithmic display is used for type-curve shape matching rather than as a direct equality between dimensional and dimensionless time.

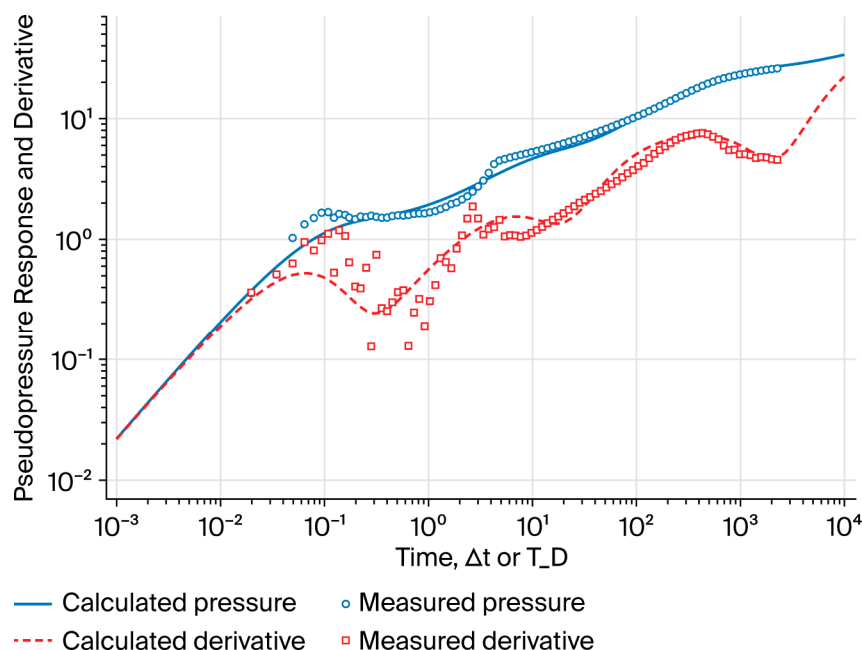


Figure 5. Log-log type-curve matching of pseudopressure and derivative for the second post-acidization pressure buildup test in Well EX-1.

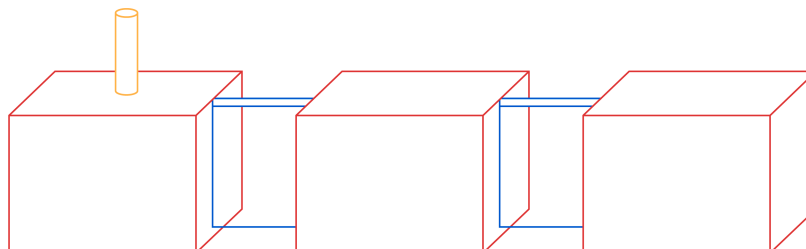


Figure 6. A schematic of the well-cave-fracture-cave-fracture-cave model for fractured-vuggy reservoirs.

3.2. Consistency Check of Well-Connected Cave Volumes

On 19 November 2023, Well EX-1 experienced mud loss at a measured depth of 7694 m (7265 m TVD). The drilling-mud density was 1650 kg/m³ (equivalent to 1.65 g/cm³), the cumulative mud loss was 435.32 m³, and the formation-pressure coefficient was (17).

Use the definition of fluid compressibility to estimate the cave volume:

$$C_f dp = -\frac{dV}{V} \quad (29)$$

Rearranging and integrating it gives:

$$C_f(p - p_i) = -\ln \frac{V}{V_i} \quad (30)$$

This yields the relationship between cave volume and the pressure drop:

$$V_i = \frac{\Delta V}{1 - \exp[-C_f(p - p_i)]} \quad (31)$$

where p_i is the initial formation pressure (Pa in calculations and MPa in reporting), ΔV is the cumulative mud-loss volume (m^3), V_c is the well-connected cave volume (m^3), and C_f is the fluid compressibility (Pa^{-1}).

For Well EX-1, the initial formation pressure at 7265 m TVD is 85.01 MPa. Using the stated mud density $\rho_m = 1650 \text{ kg/m}^3$, gravitational acceleration $g = 9.80665 \text{ m/s}^2$, and true vertical depth $h = 7265 \text{ m}$ gives a hydrostatic mud-column pressure $\rho_m g h = 117.55 \text{ MPa}$ and hence $\Delta p = 32.54 \text{ MPa} = 3.254 \times 10^7 \text{ Pa}$. Using water compressibility $C_f = 4.75 \times 10^{-10} \text{ Pa}^{-1}$ (equivalent to $4.75 \times 10^{-4} \text{ MPa}^{-1}$) and $\Delta V = 435.32 \text{ m}^3$ in Equation (31) gives $V_c \approx 2.8 \times 10^4 \text{ m}^3$. This is close to the $3.4 \times 10^4 \text{ m}^3$ well-test estimate for Cave 1, supporting the interpretation while also indicating uncertainty associated with mud properties, leak-off pathways, and the assumption of a closed cave.

3.3. Formation-Pressure Calculation

Well EX-1 penetrates Cave 1. The surface gas rate reported in Section 3.1 is converted to reservoir conditions, and the same reservoir-condition withdrawal is used in the analytical and PEBI calculations. A connected fracture is not assumed to have spatially uniform pressure. Instead, each connector is approximated as a quasi-steady one-dimensional Darcy conduit between cave-node pressures p_1 and p_2 , and Equation (32) represents the pressure drop along the fracture. This reduction is reliable when the fracture hydraulic-diffusion time $t_f = L^2/\chi_f$ is much shorter than T_p , flow remains in the Darcy regime, fracture storage is secondary, and aperture/conductivity do not vary materially with pressure. Long or tight fractures, rapid rate changes, high-velocity gas flow, or pressure-dependent aperture require a transient fracture-storage or non-Darcy numerical model.

$$p_2 = p_1 - \frac{q_f \mu L}{h w k_f} \quad (32)$$

where p_1 and p_2 are the pressures in Caves 1 and 2 (Pa in Equation (32)); L is fracture length (m); h is effective reservoir thickness (m); w is fracture aperture (m); k_f is fracture permeability (m^2); k_{fw} is fracture conductivity (m^3 when expressed in strict SI form, or conventionally D-cm in well-test reporting); μ is viscosity ($\text{Pa}\cdot\text{s}$); and q_f is fracture flow rate (m^3/s at reservoir conditions). Reported pressure results are converted from Pa to MPa. The downstream-cave flow fraction, such as 33% for Cave 2, is obtained via well-test interpretation and must be consistent with the total downhole rate.

Figure 6 shows the well-test model used for Well EX-1. Based on this model, the superposition principle (27) and convolution (28) were applied to calculate the pressures at key formation locations at the time of shut-in. The results are presented in Table 1. At $r = 0.1 \text{ m}$, the calculated pressure matches the measured bottomhole pressure at the shut-in time.

3.4. Comparison with Numerical Simulation Results

Based on the geological structure in the vicinity of Well EX-1 (see Figure 1a,b), a computational domain of $1000 \text{ m} \times 1000 \text{ m}$ was constructed. Using the well-test interpretation results for Well EX-1, three caves were placed in the domain, connected by fractures, as shown in Figure 7. Due to the irregular shapes of the caves, unstructured PEBI grids were used for grid generation (Figure 8). The interpreted parameters were assigned to each grid

cell, and reservoir numerical simulation was performed. The initial formation pressure was set to 85.01 MPa. The gas composition, formation temperature, and production rate were consistent with those used in the well-test interpretation. The simulation time was 30 days.

Table 1. Pressure at key formation points.

Distance (m)	Pressure (MPa)
0.1	83.84
69	84.02
196	84.18
985	84.44

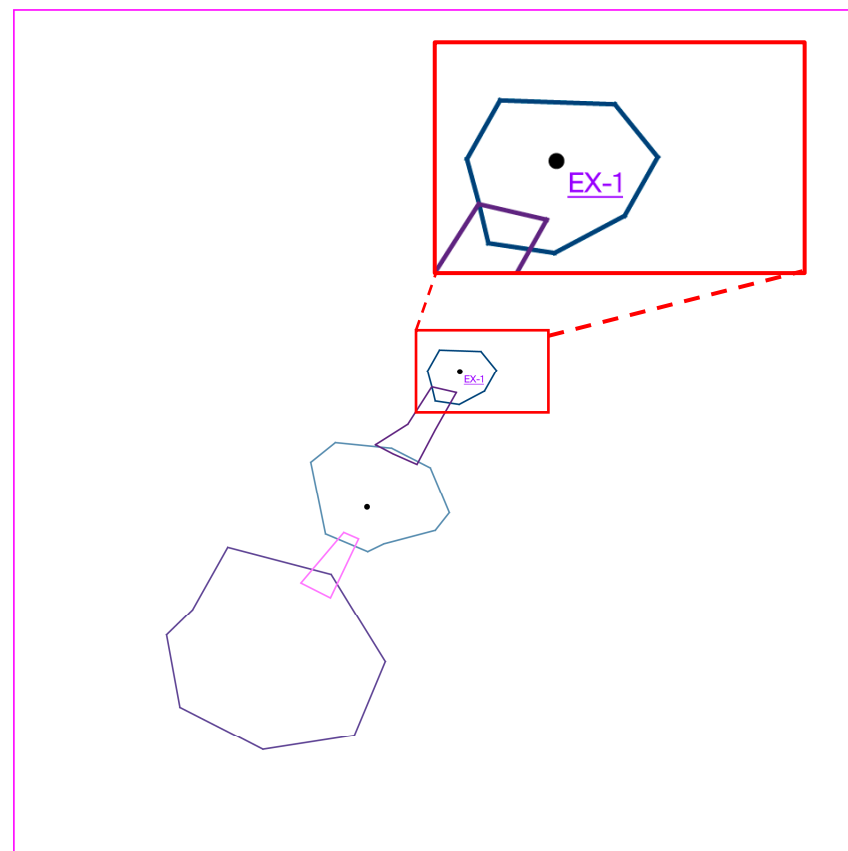


Figure 7. Well location and interpreted fracture–cave distribution for Well EX-1. The area around the well location is locally enlarged. The outlined irregular polygons denote the three interpreted caves, the narrow quadrilateral connectors between adjacent polygons denote the connecting fractures, and the black dot labeled EX-1 marks the well location within Cave 1.

Figure 9 shows the pressure distribution contours for Well EX-1 at different times. The colors represent formation pressure, with red indicating the highest initial pressure of 85.01 MPa and blue indicating the lowest pressure at the wellbore after 30 days of production (83.83 MPa). The simulated bottomhole pressure matches the measured shut-in pressure well. (a), (b), (c), and (d) in Figure 9 show the pressure distributions at production times of 1, 10, 20, and 30 days, respectively. The pressure drop is most rapid at the bottom of the well due to the well penetrating the cave; the pressure quickly diffuses into Cave 1 and then propagates along fractures toward Caves 2 and 3. Comparing the colors in panel (d), the pressures at distances of 69 m, 196 m, and 985 m along the fracture direction are approximately 83.97 MPa, 84.27 MPa, and 84.57 MPa, respectively. These values match the

calculated results in Table 2, supporting the internal consistency of the proposed analytical calculation with the calibrated PEBI model.

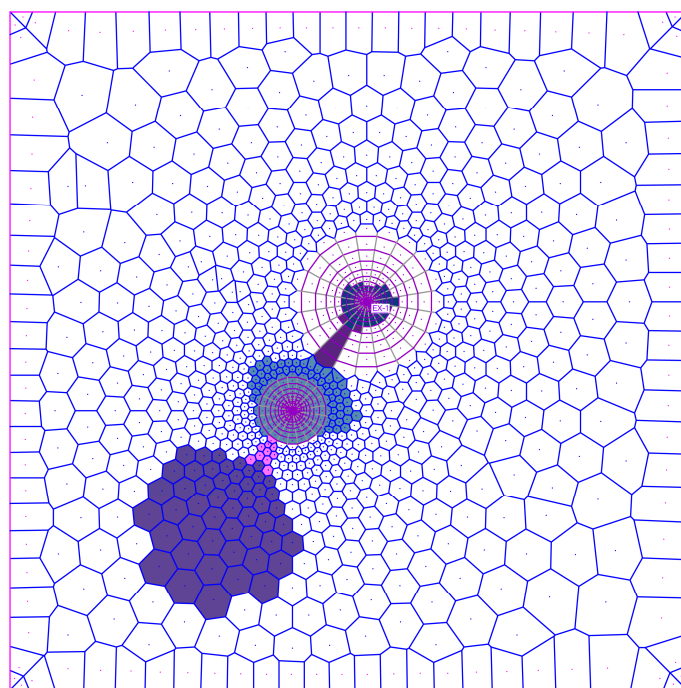


Figure 8. PEBI grid partitioning for numerical simulation of Well EX-1.

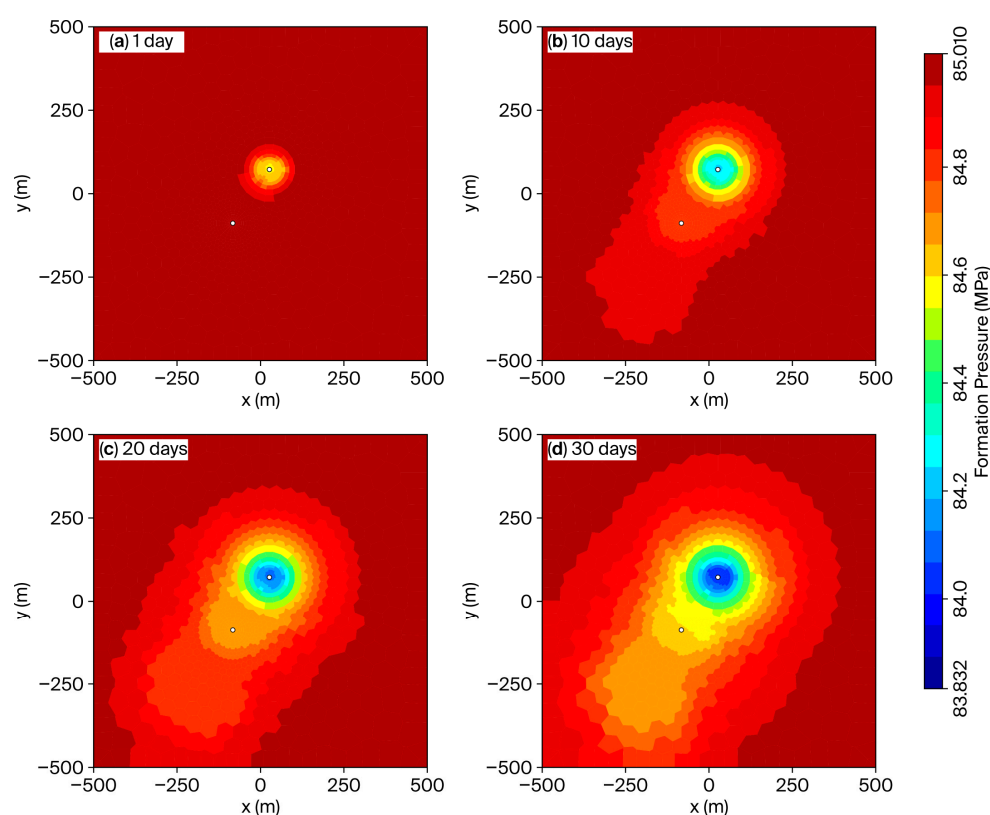


Figure 9. PEBI-simulated formation-pressure distribution for Well EX-1 after (a) 1 day, (b) 10 days, (c) 20 days, and (d) 30 days of production. The common color scale ranges from approximately 83.832 to 85.010 MPa; lower pressures first develop near the producing well and then propagate through Cave 1 and the connecting fractures toward Caves 2 and 3.

Table 2. Pointwise comparison between the proposed analytical method and the PEBI numerical simulation after 30 days of production.

Distance (m)	Analytical Pressure (MPa)	PEBI Pressure (MPa)	Absolute Error (MPa)	Relative Error (%)
0.1	83.84	83.83	0.01	0.012
69	84.02	83.97	0.05	0.060
196	84.18	84.27	0.09	0.107
985	84.44	84.57	0.13	0.154

3.5. Quantitative Agreement and Computational Requirements

The analytical values in Table 3 were compared with the PEBI numerical pressures obtained from the 30-day simulation. Because both models use the same interpreted topology, initial pressure, fluid description, and production history, this comparison measures numerical consistency rather than fully independent validation against new field data.

Across the four locations, the mean absolute error is 0.070 MPa, the root-mean-square error is 0.083 MPa, the mean absolute percentage error is 0.083%, and the maximum absolute error is 0.13 MPa. These small differences show that the analytical superposition reproduces the pressure levels of the calibrated PEBI model for this single-phase, short-term case.

Table 3. Comparison of computational requirements for the proposed method and conventional PEBI simulations.

Item	Proposed Analytical Method	Conventional PEBI Simulation
Spatial representation	M + N + L source units	Ng grid cells plus fracture/cave geometry
Main calculation	Direct evaluation and convolution of source kernels	Assembly and iterative solution of discretized flow equations at each time step
Approximate operation scaling	$O(N_t N_s N_q)$	$O(N_t N_g N_{it})$
Memory scaling	$O(N_s)$ to $O(N_t N_s)$	$O(N_g)$ plus sparse-matrix storage
Strength	Rapid pressure screening and scenario ranking	Detailed pressure and saturation fields; nonlinear physics
Limitation	Depends on linear assumptions and known topology	Requires gridding, property assignment, solver controls, and history matching

$N_s = M + N + L$ is the number of source units, N_t is the number of output times, N_q is the number of rate increments, N_g is the number of grid cells, and N_{it} is the average number of nonlinear/linear iterations per time step. The analytical method evaluates source kernels directly, with approximate operation scaling $O(N_t N_s N_q)$, whereas the PEBI method requires grid-based equation assembly and iterative solution, with approximate scaling $O(N_t N_g N_{it})$. Thus, when $N_s \ll N_g$, the source-based formulation has a lower computational burden, while PEBI simulation retains greater spatial and physical detail.

4. Discussion

4.1. Comparison with Published Literature

The EX-1 results extend prior pressure-transient studies by converting an interpreted fracture–cave topology into a spatial pressure solution. Du et al. [32], Pan et al. [17], Liu et al. [18], and Jia et al. [22] developed analytical or semi-analytical well-test models that identify cave volumes, fracture responses, or fault-controlled flow regimes primarily from bottomhole pressure behavior. In contrast, the present framework treats those interpreted storage units as concentrated, point, or finite line sources and evaluates pressure at arbitrary reservoir locations through superposition and convolution. The two approaches are therefore complementary: well testing constrains the topology and source allocation, whereas the proposed formulation reconstructs the associated pressure field.

The predicted pressure pathway from Cave 1 through the connecting fractures toward Caves 2 and 3 is consistent with the central role of connectivity reported in recent carbonate-

reservoir studies. Zhang et al. [20] showed that reliable connectivity assessment benefits from integrating static geological information with dynamic production indicators, while Chen et al. [19] demonstrated that multiscale fracture configurations produce distinct pressure-transient signatures. In the present case, the monotonic increase in pressure with distance from the producing well shows that both implementations represent the same dominant pressure-communication pathway over the 30-day period. Because the analytical and PEBI models share the interpreted geometry, initial pressure, fluid properties, and production history, their agreement establishes internal numerical consistency rather than independent field validation.

The pointwise errors (MAE = 0.070 MPa, RMSE = 0.083 MPa, MAPE = 0.083%, and maximum absolute error = 0.13 MPa) show that source superposition can reproduce the calibrated numerical pressure field for this single-phase, short-term case. Direct accuracy rankings against the cited analytical studies are not possible because those studies mainly report type-curve matching and field-interpretation performance rather than spatial pressure errors at the same observation points. Nevertheless, the current results support the broader conclusion of Pan et al. [17] and Jia et al. [22] that semi-analytical descriptions are efficient when a discrete fracture–cave structure can be represented explicitly.

4.2. Implications, Applicability, and Remaining Uncertainty

The proposed method should not be viewed as a replacement for high-fidelity reservoir simulation. Recent numerical and material-balance studies show that aquifer support, multiphase displacement, reactive transport, geomechanical deformation, and history-dependent distribution of remaining oil can materially alter fractured-vuggy reservoir behavior [8,21,23]. These processes violate the present assumptions of linear superposition, slightly compressible single-phase flow, fixed connectivity, and prescribed source allocation. Accordingly, the analytical model is best used for rapid screening, pressure-management diagnosis, sensitivity analysis, and initial reservoir evaluation and development planning; PEBI or other numerical simulators remain necessary when nonlinear physics, uncertain topology, complex boundaries, or detailed saturation forecasts control decisions.

The source-based method scales with the number of storage/flow units and rate changes, whereas PEBI simulation scales with the number of grid cells, time steps, and iterative solver operations. This structural difference makes the analytical method suitable for rapid pressure screening and repeated scenario evaluation, while numerical simulation remains preferable when detailed spatial discretization and nonlinear physics are required.

5. Conclusions

The principal quantitative findings and their validity limits are summarized as follows:

1. After 30 days, the analytical pressures at distances of 0.1, 69, 196, and 985 m were 83.84, 84.02, 84.18, and 84.44 MPa, respectively. Relative to the 85.01 MPa initial pressure, the corresponding pressure declines were 1.17, 0.99, 0.83, and 0.57 MPa, showing attenuation of drawdown away from the producing well along the interpreted fracture–cave pathway.
2. The corresponding PEBI pressures were 83.83, 83.97, 84.27, and 84.57 MPa. The analytical–PEBI differences yielded MAE = 0.070 MPa, RMSE = 0.083 MPa, MAPE = 0.083%, and a maximum absolute error of 0.13 MPa. These values demonstrate close internal agreement for the shared single-phase, 30-day model, but they do not constitute an independent field-validation error because both methods use the same interpreted topology and production history.
3. The mud-loss compressibility estimate provided a well-connected cave volume of approximately $2.8 \times 10^4 \text{ m}^3$, compared with $3.4 \times 10^4 \text{ m}^3$ from well-test interpretation.

The difference is approximately $0.6 \times 10^4 \text{ m}^3$, or 17.2% relative to the well-test estimate when calculated from the unrounded value, supporting the same order of magnitude while indicating uncertainties arising from mud properties, leak-off pathways, and the closed-cave assumption.

4. The analytical method evaluates a small number of caves, fractures, and fractured-body source units rather than solving a full grid system, giving it a lower computational burden for rapid pressure screening. Reliable application requires acoustic equilibration to be fast relative to operational time scales, linearized single-phase Darcy flow, fixed connectivity, and known source allocation. Multiphase flow, strong non-Darcy effects, rapid fracture transients, aquifer support, pressure-dependent conductivity, reactive transport, or geomechanical deformation require conventional numerical simulation or a coupled extension.

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Conflicts of Interest: Dongmei Li is employed by the Engineering Technology Management Department, Sinopec Northwest Oilfield Company. Zhiwei Lu is employed by Anhui Inflowwell Technology Co., Ltd. The authors declare that these employment relationships did not influence the design of the study; the collection, analysis, or interpretation of data; the writing of the manuscript; or the decision to publish the results.

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