

## Article

# Optimization of a Four-Bar Mechanism for Knee Prosthesis Using a Genetic Algorithm Based on Freudenstein's Equation

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## Abstract

**Background:** The natural motion of the human knee involves a combination of rotation and translation, resulting in a variable Instantaneous Center of Rotation (ICR) throughout the gait cycle. Traditional prosthetic knee designs often fail to reproduce this complex kinematic behavior. **Objectives:** This study aims to propose a customized, biomimetic knee mechanism through the synthesis of a four-bar linkage capable of approximating the physiological ICR trajectory with high precision. **Methods:** A Genetic Algorithm (GA) was implemented to optimize the geometric parameters of the four-bar mechanism, specifically its link lengths and inter-link angles. The optimization process is based on Freudenstein's equation, which analytically relates the input and output angles of the linkage to the lengths of its links. The desired ICR trajectory was derived from experimental data, and the objective function minimized the Euclidean error between the generated and target trajectories. **Results:** The proposed method yielded customized mechanisms that closely approximate the target ICR curves, achieving an overall mean Euclidean tracking error of 1.95% ( $\pm 1.68\%$ ) across diverse patient profiles, with a best-case optimization error as low as 0.355%. Furthermore, the GA demonstrated high computational efficiency, converging on optimal geometric configurations in an average execution time of just 3.98 min. **Conclusions:** These numerical results validate the robustness of the GA in navigating the design space while strictly adhering to kinematic constraints, Grashof's condition, and anatomical motion limits. The integration of Freudenstein's equation with GA-based optimization techniques enables the customized synthesis of four-bar linkages with a high capacity to reproduce the physiological kinematics of the knee. This computational approach could be highly beneficial for the design of polycentric knee prostheses, as it reduces design and manufacturing time by providing the initial parameters for the development of the four-bar mechanism, ultimately ensuring a better biomechanical fit between prosthetic and natural human movement.



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**Keywords:** four-bar mechanism; genetic algorithm; Freudenstein equation; instantaneous center of rotation; knee prosthesis

## 1. Introduction

The kinematics of the knee joint constitutes a fundamental role in human gait, as it is one of the most important and complex joints in the musculoskeletal system due to its displacement and rotation during flexion–extension [1–5]. Unlike simple joints, which are characterized by a fixed axis of rotation, the knee joint exhibits polycentric kinematics, where the axis of rotation between the femur and the tibia is not fixed. This means that the

location of the Instantaneous Centers of Rotation (ICR) varies continuously throughout the motion cycle as a result of the geometric interaction between the articular surfaces and tibiofemoral displacement patterns [6]. The polycentric configuration of the knee optimizes the relationship between stability and freedom of movement, achieving a functional balance between both properties [7]. The knee executes its natural movements thanks to its complex structure; however, in individuals with transfemoral amputation, the insertions of the adductor muscles are lost while those of the abductors are preserved, creating a muscular imbalance. This condition can lead to flexion and abduction contractures, hindering prosthesis fitting and functional mobility [8].

Currently, the development of devices such as knee orthoses and prostheses is primarily based on standardized models [9], which limits their adaptability to individual needs [10]. When customized solutions are sought, the associated costs are usually significantly higher [9]. With the continued increase in amputation rates, especially in the lower limbs, there is a greater need to develop customized prostheses that meet the specific requirements of each patient [11]. In the specific case of transfemoral prostheses, these can be classified into three main categories: monocentric, polycentric, and robotic [11]. Among them, polycentric prostheses are the most widely used due to their greater similarity to the kinematics of the tibiofemoral joint and their lower cost compared to robotic alternatives [10,12,13]. This design typically incorporates a four-bar linkage mechanism. Unlike the monocentric system, polycentric prostheses offer greater stability, as the location of the ICR is defined by the combined movement of the anterior and posterior joints during gait [14].

Knee prostheses possess complex mechanisms that establish direct relationships between prosthetic structures and human biomechanics [10]. Several authors highlight that the main function of a knee prosthesis is to guarantee stability during the stance phase, so polycentric mechanisms—mainly four-bar configurations—are usually incorporated to meet this requirement and avoid abrupt or unnatural movements [11,13,15]. Freudenstein's equation forms the basis of modern analytical methods for function synthesis, establishing a direct relationship between the angles of the input and output links of four-bar mechanisms. This formulation allows the development of the equations necessary for generating functions with five points of precision, which is especially useful in the design and optimization of these types of mechanisms [10,16,17].

The research conducted by Kinga Grodzka (2023) is based on the kinematic analysis of the trajectories of the ICR in 11 commercial knee prostheses that employ four-bar mechanisms [18]. The resulting trajectories were compared in terms of their ability to change position and shape, revealing that the location of the ICR is directly related to the configuration of the mechanism's axes. It was found that each prosthesis analyzed exhibits different levels of stability and energy demand and that both the position and shape of the ICR trajectory vary depending on the specific type of mechanism. Moreover, the author suggests that, through proper design of the link lengths within the mechanism, it is possible to manipulate the position of the ICR to achieve kinematic characteristics that align with the real needs of patients. However, a limitation of this study is that it is based solely on the kinematics of commercial prostheses, without considering the individual kinematics of the patient. Therefore, it is recommended to develop personalized knee prostheses based on the specific joint kinematics of each user, or on the kinematics of the healthy contralateral leg, to achieve a more accurate emulation of natural movement [18].

Likewise, through a two-dimensional optical gait analysis involving 17 individuals with unilateral transfemoral amputation, Andrysek (2022) determined that gait quality significantly influences a patient's ambulation capacity and overall mobility, making it a key component of the prosthetic rehabilitation process [19]. Consequently, the design of knee

prostheses can have a direct impact on gait quality. Furthermore, the study identified that friction-based swing-phase control mechanisms in the knee represent a viable and low-cost alternative for the development of functional and accessible prosthetic solutions [19].

On the other hand, Shay R. Piney (2022) establishes that during human locomotion, each limb contributes step by step to the work performed on the body's center of mass, enabling forward progression in gait [20]. This energy exchange relies on physiological mechanisms that are altered or impaired in individuals with transfemoral amputation. The study explores how changes in energy transfer at each step, particularly at higher walking speeds, reveal characteristic impairments associated with this condition, thereby providing valuable insights to improve gait efficiency in this patient population. The findings indicate that the observed inefficiency may stem from an almost complete loss of energy transfer capacity from the prosthetic limb, forcing the contralateral limb to bear a greater load in propulsion and movement modulation. This dependence highlights the importance of considering the long-term effects on the intact limb's health and suggests that microprocessor-controlled knee joints could offer an effective solution to compensate for these biomechanical deficiencies [20].

In developing countries, many individuals require prosthetic devices such as orthoses, prostheses, or rehabilitation aids, whose kinematics should align with the natural movement of the residual limb [21]. For this reason, the development of knee prostheses is aimed at individuals who have undergone transfemoral amputation and who adapt to the natural kinematic movements of their healthy contralateral leg [22].

In biomechanical applications, GAs have been used as optimization tools for the design of mechanisms, particularly in humanoid knee joints, allowing the adjustment of structural parameters to minimize the tracking error between the kinematics of the mechanism and the physiological trajectory of the human knee generated during gait cycles [23,24].

In the present study, baseline ICR trajectories are utilized as a reference. By leveraging the GA, the configuration parameters of the four-bar mechanism are defined to achieve the closest possible approximation between the anatomical ICR and the ICR generated by the proposed mechanism.

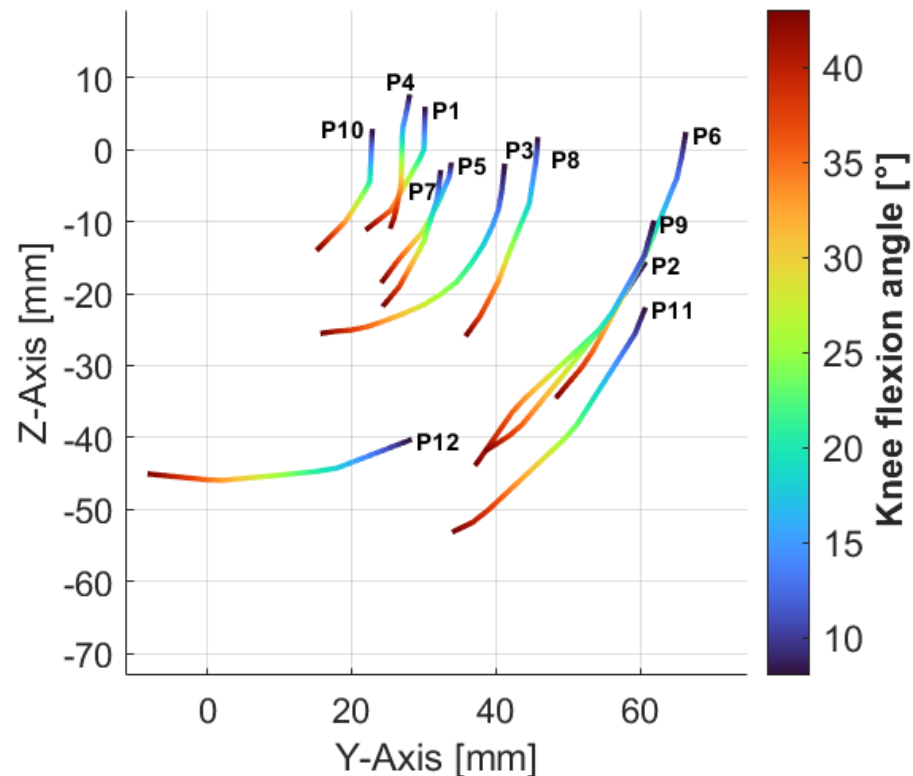
## 2. Materials and Methods

This section presents the general framework of the proposed methodology for optimizing a four-bar mechanism. The primary objective is to accurately emulate the anatomical ICR, seeking the highest possible congruence between the physiological joint and the optimal configuration of the linkage. The workflow begins with the biomechanical conceptualization of tibiofemoral motion. Based on this analysis, the target trajectory is defined by the ICR in the sagittal plane, derived from experimental gait data, as shown in Figure 1. Using this reference kinematics, the four-bar mechanism is mathematically modeled utilizing Freudenstein's equation. Subsequently, a GA is implemented to optimize the link lengths and angular configurations of the mechanism, aiming to minimize the Euclidean error between the synthesized trajectory and the target curve. This comprehensive computational approach facilitates the development of a personalized, biomimetic foundation applicable to the robust design of polycentric knee prostheses [25].

### 2.1. Knee Biomechanical Conceptualization

From a biomechanical perspective, the knee can be modeled as a system where the body weight load line acts at a specific distance from the articular center, generating a flexion moment. To counteract this tendency toward instability, the system requires a balance between passive and active forces. Passive stability is achieved when the gravity vector is positioned slightly behind the ICR, allowing tension in the dorsal capsuloligamentous

structures to lock the joint. Conversely, if the load shifts further behind this center, an extension moment is induced that compromises stability, potentially leading to undesirable flexion (recurvatum or functional instability). Meanwhile, active stabilization is primarily provided by the quadriceps femoris, whose action counteracts flexion forces and ensures the functional integrity of movement.



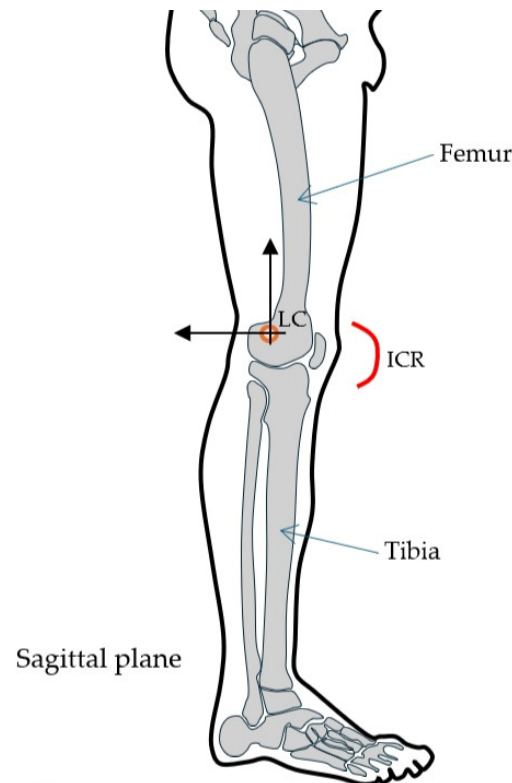
**Figure 1.** Twelve representative CIR sagittal plane trajectories, presented as target paths.

In the framework of this study, the analysis of the ICR trajectory forms the basis for the kinematic description of the joint. Given the high inter-individual variability in joint kinematics, the technical challenge lies in identifying the geometric parameters that allow for the synthesis of a four-bar mechanism capable of precisely emulating the target curve. Determining these optimized parameters is essential for the design of personalized prosthetic devices, facilitating greater efficiency in rehabilitation processes and superior biomechanical integration with the specific needs of each patient.

## 2.2. Target Curve

The reference ICR trajectory used in this study corresponds to the representative kinematic pattern of the knee, obtained in the experimental study reported by [25,26]. In that work, the kinematics of the tibiofemoral joint are assumed to be the evolution of the instantaneous center of rotation during the knee flexion movement and are represented in the sagittal plane; this was determined through a gait analysis performed on healthy subjects under controlled laboratory conditions [25–27]. The knee movement during several gait cycles was recorded and subsequently processed to obtain the corresponding ICR trajectories in the sagittal plane. After analyzing the behavior of these trajectories across the cycles, the representative trajectories for each subject were selected based on their continuity. In this study, each ICR trajectory is adopted as the target curve for the synthesis of the four-bar mechanism. The discrete coordinates of the reference trajectory, presented in Figure 1, correspond to sampled points of the ICR trajectory obtained from the experimental study, which were linearly interpolated according to their knee flexion angle and for a common

flexion range. The reference frame used to obtain the coordinates of the ICR is schematically illustrated in Figure 2. The coordinate system is defined with its origin located on the lateral surface of the lateral condyle (LC). The Y-axis represents the anteroposterior direction, while the Z-axis corresponds to the proximal–distal direction of the knee joint, considering the analysis in the sagittal plane.



**Figure 2.** Representation of the anatomical reference system and the target ICR; plot based on [28].

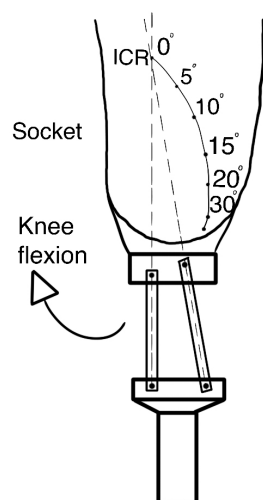
The ICR trajectory is parameterized as a function of the knee flexion angle, within an approximate range of  $10^{\circ}$  to  $40^{\circ}$ . The reported discrete coordinates serve as reference points for the optimization process, allowing the algorithm to evaluate the similarity between the trajectory generated by the synthesized four-bar mechanism and the anatomical ICR trajectory. During each evaluation of the GA, the ICR trajectory generated by the candidate four-bar mechanism is calculated at the same flexion angles and compared against the reference points.

### 2.3. Four-Bar Mechanism

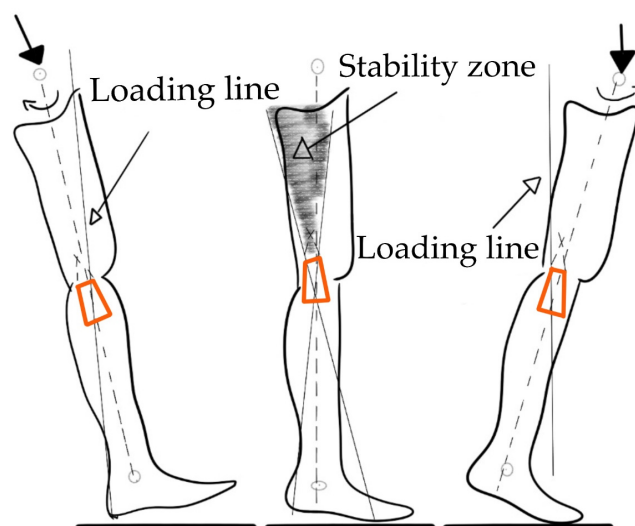
The four-bar mechanism is an alternative for the development of knee prostheses due to the variability of the ICR in the sagittal plane during the flexion of the tibiofemoral joint [26]. The ICR is defined as the intersection between the extensions of the posterior and anterior links, as shown in Figure 3.

The ICR trajectory in the sagittal plane is configured based on the parameters of the four-bar mechanism; that is, its position and evolution in space depend directly on the lengths of its links and the angles between them. Furthermore, by achieving a better fit with respect to the anatomical kinematics, its stability and control could improve significantly.

Figure 4 shows the stability diagram of the mechanism: the ground reaction force (GRF), represented by the load line, is positioned in front of the knee's ICR during heel strike, ensuring knee stability, and behind the ICR during toe-off, allowing voluntary flexion.

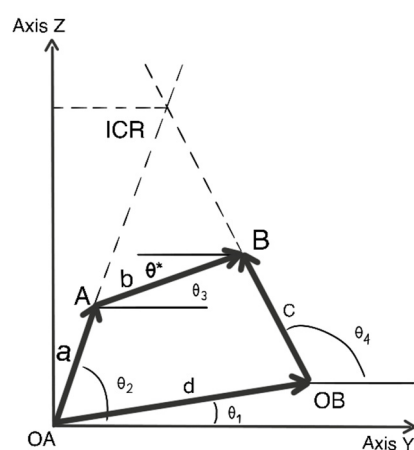


**Figure 3.** Four-bar mechanism and representation of the ICR during its angular displacement [29].



**Figure 4.** Stability diagram of the voluntary control four-bar polycentric mechanism [28].

To develop the four-bar mechanism with its corresponding ICR, represented in the sagittal plane during flexion, it is necessary to illustrate its configuration. The mechanism consists of four links: “a”, “b”, “c” and “d”, connected as follows: link “a” to “b” at joint A, “b” to “c” at joint B, “c” to “d” at joint OB, and “d” to “a” at pin A, as shown in Figure 5.



**Figure 5.** Positioning and General Configuration of the Four-Bar Mechanism.

The angle  $\theta^*$  is a design angle of link “b” and is measured from the joint between links “c” and “d” (joint B). The angle  $\theta_1$  corresponds to the design angle of link “d”, measured from the joint between links “a” and “d” (joint OA). The ICR varies as a function of the knee’s flexion–extension angle. Likewise, angle  $\theta_3$  is identified as an independent variable, as it is associated with link “b” (fixed to the thigh) and serves as the input for movement. Given the known lengths of the four links “a”, “b”, “c” and “d” along with the input angle  $\theta_3$  and the design angles  $\theta_1$  and  $\theta^*$ .

### 2.3.1. Anatomical–Prosthetic Reference System for the Alignment Between the Prosthesis and the Patient

- The feature that best describes the mechanical function of the knee is the so-called ICR. This center can be located by extending lines through the anterior and posterior axes of the knee; the point where these lines intersect is the instantaneous center. Mechanical control of the knee is determined at this center. It is at this point that the amputee perceives control of the fully extended prosthesis. It is important to understand that as the knee joint flexes, the position of the instantaneous center changes.
- Manufacturers provide guidelines to assist prosthetists in the static alignment of four-bar knee joints in the sagittal plane. Typically, a reference weight line is provided, preferably intersecting the anterosuperior pivot point of the four-bar knee unit. The Center of Balance Reference (CBR) line is considered a more appropriate reference than an arbitrary line established by the manufacturer. This line accounts for a reference extending from the center of pressure in the foot proximally through the individual’s center of mass.

### 2.3.2. There Are Three Main Prosthetic Functions That Are Determined by the Centroid Pattern of the Joint

- The distance at which the ICR is located posterior to the amputee’s CBR line determines the stability of the knee. The farther back this point is, the more stable the knee unit will be at heel strike.
- The better the ICR, the less muscular effort is required to control knee stability at heel strike. Since the control point of the knee unit is closer to the hip joint, the amputee gains a mechanical advantage for the residual limb, thereby increasing efficiency.
- An additional feature to consider is the CBR line, as it determines voluntary knee control. When the ICR is in front of the CBR line, greater muscular effort is required from the amputee. In contrast, when the ICR is positioned behind the CBR line, better voluntary control is achieved.

These factors can affect the amputee’s control over the prosthesis. Professionals rely on clinical experience and manufacturers’ recommendations. When considering a knee prosthesis, the prosthetist selects from the available knee centroids and then applies alignment principles to achieve the most efficient and balanced gait for the amputee.

The goal is to align the knee with the patient, rather than aligning the socket and the patient to the knee unit.

Proper alignment between the prosthetic knee mechanism and the patient’s anatomical reference system is essential to ensure stability, efficient load transfer, and natural gait dynamics. Therefore, patients place considerable importance on comparing their bodies to others, as well as on the perceived positive or negative reactions of those around them. It is essential to integrate the amputee’s center of balance between the prosthesis and the sound limb to determine optimal balance and reduce compensatory equilibrium.

### 2.3.3. Unification of Reference Systems for the Alignment of the Anatomical ICR and the Mechanism

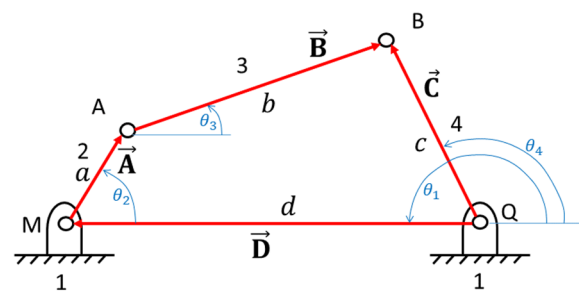
As shown in Figure 2, the original points are in different positions. The reference system for the anatomical ICR is located at the LC (lateral condyle), whereas the reference system for the mechanism's ICR is located at the joint OA. Additionally, the orientation of the Y-axis is offset by  $180^\circ$ . Therefore, it is necessary to reference a single coordinate system, specifically OA, to align the four-bar mechanism with the obtained anatomical ICR.

To align the two reference systems, the difference between the Y and Z axes has been determined. These offsets have been estimated as 50 mm along the Y-axis and 62 mm along the Z-axis and are incorporated into the algorithm.

### 2.4. Freudenstein's Equation

The kinematic behavior of the four-bar mechanism can be analytically described using the Freudenstein equation, which establishes the relationship between the input and output angles of the mechanism as a function of the link lengths. This equation is widely used in the synthesis and analysis of planar four-bar mechanisms due to its compact formulation and suitability for computational implementation.

Considering the four-bar mechanism shown in Figure 6, the link lengths are defined as  $a, b, c$ , and  $d$ , where  $d$  represents the ground link. The angular variables  $\theta_2, \theta_3$  and  $\theta_4$  correspond to the input, coupler, and output link angles, respectively.



**Figure 6.** Physical-mathematical model for obtaining the Freudenstein equation.

The loop-closure equation of the mechanism can be expressed in the classical Freudenstein form as:

$$K_1 \cos(\theta_4) - K_2 \cos(\theta_2) + K_3 = \cos(\theta_2 - \theta_4), \quad (1)$$

where the constants  $K_1, K_2$  and  $K_3$  are defined as:

$$K_1 = \frac{d}{a} \quad (2)$$

$$K_2 = \frac{d}{c} \quad (3)$$

$$K_3 = \frac{a^2 - b^2 + c^2 + d^2}{2ac} \quad (4)$$

This formulation allows the determination of the output angle  $\theta_4$  for a given input  $\theta_2$ . In the present study, the equation is solved using the half-angle tangent substitution method, which enables the analytical computation of the angular configuration of the mechanism.

By applying this transformation, the output angle can be obtained using the mathematical formulation in Equation (5):

$$\theta_4 = 2 \arctan\left(\frac{-B + \sqrt{B^2 - 4AC}}{2A}\right), \quad (5)$$

where  $a$ ,  $b$ ,  $c$  and  $d$ , represent the lengths of the corresponding links in the four-bar mechanism, as illustrated in Figure 6, and the coefficients  $A$ ,  $B$ , and  $C$  are functions of the link lengths and the input angle  $\theta_2$ , defined explicitly as:

- $A = \cos(\theta_2) - K_1 - K_2 \cos(\theta_2) + K_3$
- $B = -2 \sin(\theta_2)$
- $C = k_1 - (k_1 + 1) \cos(\theta_2) + K_3$

The formulation enables efficient numerical evaluation of the mechanism configuration during the optimization process performed by the GA. For each candidate solution, the angular configuration of the mechanism is computed using Freudenstein's equation, allowing the ICR to be determined and compared with the target trajectory.

#### Kinematic Derivation of the ICR Coordinates

During the optimization process with the GA, it is essential to define how the ICR coordinates are mathematically derived from the link lengths and angular positions at each iteration.

According to the Kennedy–Aronhold theorem, the ICR of the coupler link, link “b”, representing the knee joint, relative to the ground link (link “d”) is located at the intersection of the extended lines of the two cranks (links “a” and “c”).

Establishing a local Cartesian coordinate system ( $Y$ ,  $Z$ ) with its origin at the joint OA (0,0) and the  $Y$ -axis aligned with the ground link “d”, the coordinates of the main joint can be defined as follows:

- Joint OA: (0, 0)
- Joint A: ( $a \cdot \cos \theta_1$ ,  $a \cdot \sin \theta_1$ )
- Joint OB: ( $d$ , 0)
- Joint B: ( $d + c \cdot \cos \theta_3$ ,  $c \cdot \sin \theta_3$ )

The line passing through link a (from OA to A) is defined by the equation:

$$Z = \tan(\theta_1) \cdot Y, \quad (6)$$

The line passing through link c, from OB to B, is defined by the equation:

$$Z = \tan(\theta_3) \cdot (Y - d), \quad (7)$$

During each GA evaluation, the output angle  $\theta_3$  is calculated for a given input angle  $\theta_1$  using Freudensteins's equation. To find the intersection point representing the ICR coordinates ( $Y_{ICR}$ ,  $Z_{ICR}$ ) in the mechanism's local frame, we equate Equations (6) and (7):

$$\tan \theta_1 \cdot Y_{ICR} = \tan \theta_3 \cdot (Y_{ICR} - d)$$

Solving for  $Y_{ICR}$

$$Y_{ICR} = \frac{d \cdot \tan \theta_3}{\tan \theta_3 - \tan \theta_1}$$

By applying trigonometric identities (multiplying by  $\cos \theta_1 \cdot \cos \theta_3$ , the local coordinate  $Y_{ICR}$  is simplified to:

$$Y_{ICR} = \frac{d \cdot \sin(\theta_3) \cos(\theta_1)}{\sin(\theta_3 - \theta_1)}, \quad (8)$$

Substituting  $Y_{ICR}$  back into Equation (6), we obtain the local vertical coordinate  $Z_{ICR}$ :

$$Z_{ICR} = \frac{d \cdot \sin(\theta_1) \sin(\theta_3)}{\sin(\theta_3 - \theta_1)}, \quad (9)$$

Finally, to compute the Euclidean error between the generated mechanism and the patient's anatomical data during the optimization process, these local coordinates must be

transferred to the anatomical reference using the translational offsets defined in Section 2.3.3, 50 mm along the Y-axis and 62 mm along the Z-axis:

$$Y_{anat} = Y_{ICR} + 50 \quad (10)$$

$$Z_{anat} = Z_{ICR} + 62 \quad (11)$$

In each iteration of the GA, Equations (8)–(11) generate the simulated ICR trajectory across the flexion angle range, allowing the objective function to evaluate the Euclidean distance against the discrete target coordinates, as shown in Figure 1.

## 2.5. Genetic Algorithm

A GA was implemented to determine the optimal parameters of the four-bar mechanism that best reproduces the target ICR trajectory. Genetic algorithms are stochastic optimization techniques inspired by the principles of natural evolution, where populations of candidate solutions evolve through processes of selection, mutation, and recombination [29–31].

In the present study, everyone in the population represents a possible configuration of the four-bar mechanism defined by the parameter vector:

$$X = [a, b, c, d, \theta_1, \theta_3],$$

where  $a$ ,  $b$ ,  $c$  and  $d$  correspond to the link lengths of the mechanism, and  $\theta_1$  and  $\theta_3$  represent the design angles that define the kinematic configuration.  $\theta_3 = \theta^*$  (see Figure 5).

The optimization process aims to minimize the difference between the target ICR trajectory obtained from experimental gait analysis and the trajectory generated by the synthesized four-bar mechanism.

The objective function is defined using the Euclidean distance between corresponding points of the target and generated trajectories:

$$J = \frac{1}{N} \sum_{i=1}^N \sqrt{(Y_i^{\text{sim}} - Y_i^{\text{target}})^2 + (Z_i^{\text{sim}} - Z_i^{\text{target}})^2} \quad (12)$$

where:

$N$  is the number of sampled trajectory points;  
 $Y_i^{\text{target}}, Z_i^{\text{target}}$  are the coordinates of the reference ICR trajectory;  $Y_i^{\text{sim}}, Z_i^{\text{sim}}$  are the coordinates obtained from the four-bar mechanism.

The GA begins by generating a random initial population of candidate solutions. At each generation, the objective function is evaluated for all individuals, and the population is sorted according to their fitness.

The evolutionary process includes three main operators:

- **Elitism:** The best individuals of the population are preserved in the next generation to ensure that the best solutions are not lost during evolution.
- **Mutation:** A subset of individuals undergoes mutation by randomly modifying one parameter within its predefined bounds. This operation introduces diversity into the population and helps the algorithm explore the search space.
- **Recombination (Crossover):** New individuals are generated by combining parameters from randomly selected parents. Each parameter of the offspring may be inherited from a different parent, producing new candidate configurations of the four-bar mechanism.

The algorithm iteratively updates the population until a stopping criterion is satisfied, defined by the maximum number of generations.

### 2.5.1. Genetic Algorithm Pseudocode

In this study, the GA was defined as follows: population size  $NP = 3000$ , the maximum number of generations  $G_{max} = 100$ , elitism size of 50 individuals, mutation applied to 250 individuals per generation, and crossover applied to 700 individuals per generation. Figure 7 presents a genetic-algorithm-based methodology used that best approximates the kinematics of the test subject.



**Figure 7.** GA methodology to obtain the four-bar mechanism parameters.

### GA Sensitivity Analysis

The graph demonstrates that the error decreases drastically in the first generations and stabilizes before generation number 100, evidencing that 100 generations are robust enough to avoid unnecessary computational cost.

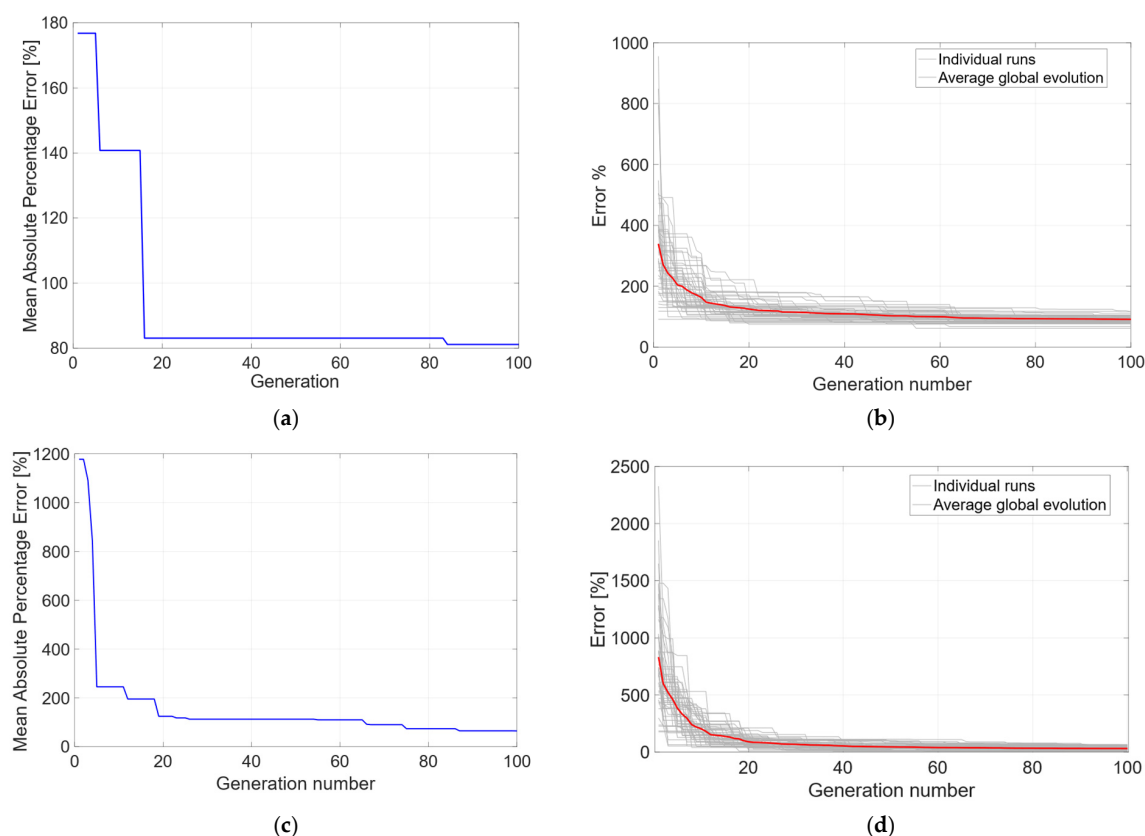
A conservative elitism rate of 1.6% is configured, which ensures that the best configurations of the four-bar mechanism are not lost. Additionally, a mutation rate of 8.3% and a crossover rate of 23.3% are applied to balance the exploitation of the best solutions and the exploration of the design space.

The rest of the population was randomly generated at each iteration to guarantee high genetic diversity against the geometric complexity imposed by the Freudenstein equation.

As shown in Figure 8 (a) and (c) present the convergence curves of the Mean Absolute Percentage Error (MAPE) for run #45, corresponding to subject P4, and run #4 for subject P10, respectively. These single-run curves display pronounced stepped decreases initially, followed by stabilization from generation 20 onwards. The low variance at the end of the evolutionary process, observable in all plots, demonstrates high repeatability, indicating that the algorithm is not prone to chronic stagnation in local optima.

Similarly, Figure 8 (b) and (d) display the mean of the global evolution (red line) for each subject (P4 and P10, respectively), superimposed on individual runs (gray lines), confirming the convergence trend observed in single runs.

Due to the stochastic nature of GAs, a statistical analysis was performed to evaluate the repeatability and robustness of the proposed configuration. The algorithm was executed across independent trials under identical initial conditions. Regarding the computational cost, the average execution time per independent trial was 3.98 min (with a standard deviation of 0.5044 s), operating in a computational environment equipped with a 2.8 GHz Intel Core i7 processor and 16 GB of RAM. Furthermore, the algorithm demonstrated a convergence variance of 0.1439 and a 95% confidence interval for the total error of [1.5786%, 1.7942%]. This processing time indicates that the method is computationally efficient and viable for the iterative synthesis of customized prosthetic mechanisms in clinical settings.



**Figure 8.** Subject P4. Convergence curve for the solution obtained using the GA method, the red line in subplots (b,d) represents the global average evolution. (a) Convergence of Mean Absolute Percentage Error (MAPE) for Subject P4 (Single Run #45), (b) Global Evolution of Percentage Error for Subject P4 (Average of Multiple Runs), (c) Convergence of Mean Absolute Percentage Error (MAPE) for Subject P10 (Single Run #4), (d) Global Evolution of Percentage Error for Subject P10 (Average of Multiple Runs).

### 3. Results

To evaluate the performance of the proposed optimization approach, the target ICR trajectories previously presented were used. The algorithm was executed 10 times independently to analyze the stability of the optimization process.

The parameters corresponding to the configuration with the lowest error for each subject are reported in Table 1. The results demonstrate that the GA can generate a four-bar mechanism whose ICR trajectory closely approximates the anatomical reference trajectory.

**Table 1.** Dimensions and angles of the optimal mechanism obtained.

| Parameter        | P1    | P2    | P3    | P4          | P5    | P6    | P7    | P8    | P9    | P10   | P11   | P12    | Mean  | SD    |
|------------------|-------|-------|-------|-------------|-------|-------|-------|-------|-------|-------|-------|--------|-------|-------|
| a                | 74.00 | 74.73 | 64.48 | 72.28       | 73.74 | 79.95 | 65.70 | 71.72 | 75.98 | 69.66 | 77.30 | 49.64  | 70.77 | 8.01  |
| b                | 22.73 | 21.85 | 22.37 | 21.06       | 24.17 | 24.18 | 21.48 | 21.76 | 21.47 | 21.07 | 23.43 | 23.10  | 22.39 | 1.13  |
| c                | 59.06 | 45.26 | 30.23 | 54.75       | 55.98 | 51.80 | 38.21 | 41.84 | 47.75 | 55.01 | 46.10 | 44.01  | 47.50 | 8.36  |
| d                | 57.18 | 55.43 | 58.68 | 57.77       | 56.66 | 58.50 | 59.59 | 58.72 | 54.00 | 49.56 | 57.22 | 31.25  | 54.55 | 7.82  |
| $\theta_1$ [rad] | 0.03  | 0     | 0.30  | 0.04        | 0.06  | 0.008 | 0.11  | 0.098 | 0     | 0.114 | 0.02  | 0.0004 | 0.067 | 0.088 |
| $\theta_3$ [rad] | 0.43  | 0     | 0.24  | 0.31        | 0.45  | 0.005 | 0.15  | 0.06  | 0     | 0.50  | 0.05  | 0.24   | 0.205 | 0.188 |
| Error [mm]       | 1.06  | 1.754 | 3.09  | <b>0.35</b> | 1.39  | 1.74  | 1.11  | 1.03  | 1.518 | 1.16  | 2.38  | 6.82   | 1.955 | 1.686 |

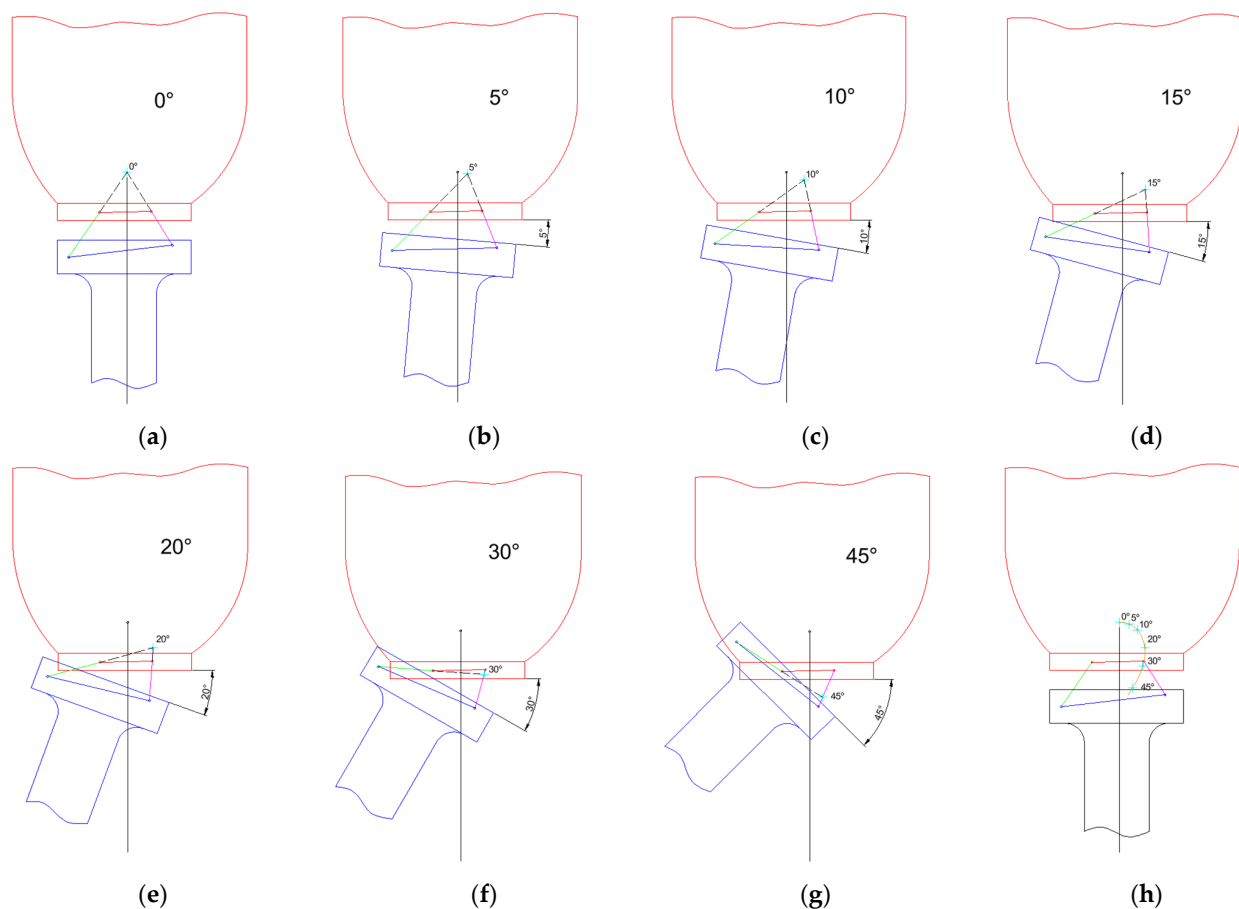
To quantitatively demonstrate the reproducibility and robustness of the optimization algorithm across different inter-subject trials, a statistical analysis of the synthesized parameters was conducted. The tracking error (Euclidean distance) across the tested configurations yielded a mean value of 1.95 mm with a standard deviation of 1.68 mm. As

observed in the table, the algorithm achieved an outstanding fit, fluctuating from a minimum error of 0.355 mm (for subject P4) to a maximum error of 6.821 mm (for subject P12). Regarding computational cost, the GA proved highly efficient, registering an average execution time of 3.98 min ( $\pm 0.50$  s) per independent run.

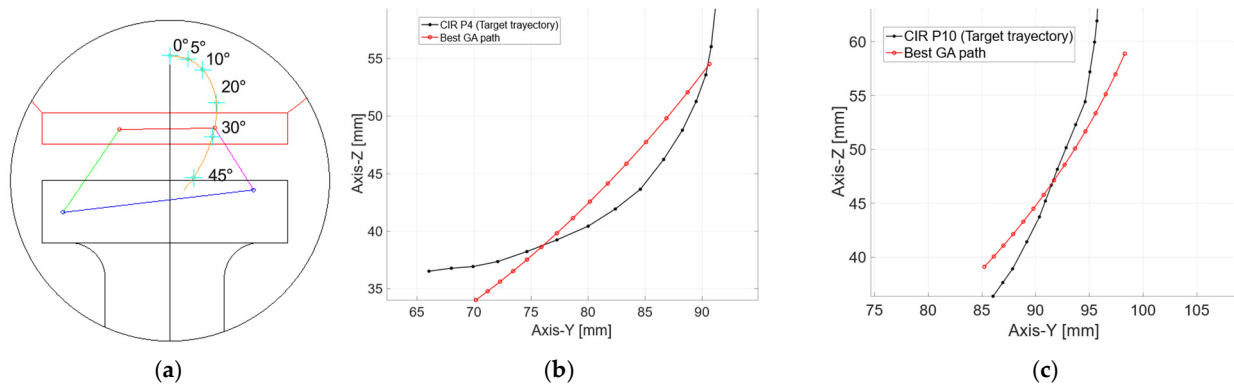
Furthermore, the standard deviations observed in the optimized link dimensions—specifically 8.0 mm for link a and 1.1 mm for link b—reveal an important biomechanical insight: the dimension of the coupler link (link b) remains highly constrained and stable across different patients. In contrast, the other links provide the necessary parametric flexibility to adapt to the unique anatomical ICR variations of everyone.

### Validation

To validate the optimized solutions, the four-bar mechanisms corresponding to the synthesized parameters were modeled computationally. Figure 9 illustrates the kinematic evolution of the ICR from  $0^\circ$  to  $45^\circ$  of knee flexion. Specifically, Figure 9a–g demonstrate the instantaneous positioning of the links at discrete flexion angles, while Figure 9h displays the complete consolidated trajectory of the mechanism across the entire  $0^\circ$  to  $45^\circ$  range. Furthermore, Figure 10a presents a detailed schematic of the synthesized four-bar linkage and its resulting ICR curve. To verify the accuracy of the GA optimization, Figure 10b,c compare the trajectory generated by the synthesized mechanisms against the target anatomical trajectories derived from previous experimental gait analysis [26] for subjects P4 and P10, respectively.



**Figure 9.** Kinematic evolution of the ICR trajectory for the optimized four-bar mechanism during knee flexion: (a)  $0^\circ$  flexion, (b)  $5^\circ$  flexion, (c)  $10^\circ$  flexion, (d)  $15^\circ$  flexion, (e)  $20^\circ$  flexion, (f)  $30^\circ$  flexion, (g)  $45^\circ$  flexion, and (h) complete consolidated trajectory from  $0^\circ$  to  $45^\circ$ .



**Figure 10.** Validation of the synthesized models: (a) Schematic of the optimized four-bar mechanism and its continuous ICR curve; (b) Comparison between the target anatomical trajectory and the GA-generated trajectory for subject P4; (c) Comparison between the target anatomical trajectory and the GA-generated trajectory for subject P10.

#### 4. Discussion

The development of various medical devices, such as external knee prostheses, exoskeletons, and rehabilitation equipment, is fundamentally based on the kinematics of the tibiofemoral joint. Conventionally, authors seek to adapt an analogous mechanism that can reproduce these kinematics—a process where dimensional synthesis and parameter determination require a considerable investment of time. Aiming to optimize this critical step, the present study proposes a GA capable of determining the link lengths and configuration angles of a four-bar mechanism in an average computational time of just 3.98 min. This approach drastically reduces computational effort, optimizing the time and resources necessary for designing these devices.

Anatomical variability represents a major challenge in prosthetic design. Recent studies on “Continual Human-in-the-loop optimization” address this by dynamically adjusting control parameters. Although generative strategies open the door to personalized systems, they face a physical limitation: Software can only execute efficient corrections if the underlying mechanical base is rigorously designed. If the hardware’s kinematic model is deficient, even the most advanced control algorithm will yield suboptimal results. In contrast, the GA proposed here solves the problem from its biomechanical root. By utilizing Freudenstein’s equation, it ensures that the mechanism provides a highly optimized and kinematically valid physical foundation, which is essential for the safe operation of modern control algorithms.

Furthermore, current literature has explored assistive design via reinforcement learning (RL). However, these simulated models often assume simplified joints with a fixed center of rotation, leading to unrealistic biomechanical scenarios. The tibiofemoral joint is a complex system characterized by a variable ICR; its dynamics are difficult to simulate precisely, particularly in pathological gaits. The GA-based methodology proposed herein represents a significant advantage by deriving parameters directly from the kinematics of the individual’s healthy or contralateral leg. This ensures a mechanical fit faithful to the patient’s natural ICR.

For this study, ICR trajectories corresponding to 10 healthy subjects were utilized. These trajectories describe regular, continuous curves devoid of abrupt variations—a critical factor for successful optimization. Previous literature reports trajectories with sudden directional changes where kinematic synthesis becomes unfeasible, preventing the GA from converging. An in-depth analysis of the four-bar mechanism reveals that, due to geometric constraints, it tends to generate relatively uniform coupler curves. Consequently,

future work should explore five-bar configurations or gear-integrated systems, as suggested by Ayala, to offer greater design versatility for replicating complex ICR trajectories.

Although the optimization process yielded a mean Euclidean error of 1.95%, it is essential to contextualize this magnitude within the biomechanical framework of the human knee. The natural joint is not a perfect rigid-body system; soft tissue deformations—such as cartilage compression and meniscal displacement—introduce physiological variations in the joint's center of rotation. While this study focuses on computational synthesis, the clinical viability of the GA-generated parameters was validated through physical prototyping. Hardware implementations built upon this GA geometric framework achieved highly accurate ICR emulation, providing the necessary dynamic stability and comfort during actual gait scenarios.

Biomechanical studies indicate that natural soft-tissue laxity typically ranges between 2 mm and 5 mm. Consequently, a tracking error of 1.95% falls within acceptable physiological boundaries, suggesting the synthesized mechanism can emulate natural kinematics without inducing abnormal joint stresses. Moreover, this represents a significant improvement over commercial polycentric prostheses, which typically exhibit ICR deviations ranging from 5 mm to over 12 mm. By reducing trajectory deviation, the GA-optimized mechanism ensures a precise replication of the patient's J-shaped physiological curve, expected to reduce compensatory movements and lower energy expenditure in transfemoral amputees.

## 5. Conclusions

The integration of the Freudenstein equation with a GA has proven to be an effective strategy for the synthesis of four-bar mechanisms capable of accurately replicating the physiological motion of the human knee joint. The proposed approach enabled the optimization of both link dimensions and angular configurations to closely match the target ICR trajectory.

Although this study presents a purely computational optimization framework, the clinical viability of the resulting geometries has been corroborated by recent physical prototyping efforts. The GA-derived parameters successfully guided the manufacturing of a physical prosthesis, which, when complemented with gear mechanisms, demonstrated excellent stability and gait emulation in experimental hardware tests.

The mechanisms generated through this process achieved an average Euclidean error of 1.95 mm relative to the reference trajectory, validating the effectiveness of the GA and its ability to efficiently explore complex search spaces while adhering to anatomical and kinematic constraints.

The developed methodology establishes a foundation for designing personalized polycentric knee prostheses that more closely mimic natural tibiofemoral kinematics. This approach has the potential to improve gait efficiency, dynamic stability, and long-term biomechanical compatibility in individuals with transfemoral amputation.

Furthermore, the methodology constitutes a robust approach for the rapid prototyping of functional prosthetic devices. By reducing trajectory deviation to an average of 1.95, the GA-optimized mechanism ensures a highly precise replication of the patient's specific J-shaped physiological curve. Compared to generic commercial prostheses—which can exhibit deviations exceeding 12 mm—this customized biomechanical congruence significantly improves the potential for reducing compensatory movements and lowering energy expenditure in patients with transfemoral amputations.

Finally, the proposed optimization strategy, supported by experimental data and computational modeling, offers a robust tool for the development of functional knee prostheses. The resulting mechanism geometry is well-suited for additive manufacturing technologies such as 3D printing, facilitating the efficient production of customized physical

prototypes. Future work will include experimental validation of the optimized mechanism using motion capture systems and mechanical prototyping to evaluate its biomechanical performance in real gait conditions.

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## Abbreviations

The following abbreviations are used in this manuscript:

|     |                                  |
|-----|----------------------------------|
| ICR | Instantaneous Center of Rotation |
| GA  | Genetic Algorithm                |
| CBR | Center of Balance Reference      |
| LC  | lateral condyle                  |

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