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LRS Bianchi I Cosmological Model with Strange Quark Matter in $f(R,T)$ Gravity †

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Abstract: A locally rotationally symmetric Bianchi-I model filled with strange quark matter is explored in $f(R,T) = R + 2f(T)$ gravity, where R is the Ricci scalar, T is the trace of the energy-momentum tensor and λ is an arbitrary constant. Exact solutions are obtained by assuming that the expansion scalar is proportional to the shear scalar. The model is found to be physically viable for $\lambda < -\frac{1}{4}$. Strange quark matter at early times mimics ultra-relativistic radiation whereas at late times it behaves as dust, quintessence, or even the cosmological constant for some specified values of λ. The effective matter acts as stiff matter irrespective of the matter content and of $f(R,T)$ gravity. The model is shear-free at late times but remains anisotropic throughout the evolution.

Keywords: strange quark matter; $f(R,T)$ gravity; Bianchi-I model; dark energy

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1. Introduction

Observational data [1–3] suggest that the universe is currently in an accelerating phase. A plethora of attempts have been made to explain this phenomenon, but neither of them is compelling. The first attempt is dark energy (DE), which is the hypothesis of exotic matter with the unique feature of anti-gravity due to highly negative pressure, thus accelerating the expansion of the universe [4]. In the standard ΛCDM model, the cosmological constant (CC) is the primary candidate for DE. Secondly, there are modified theories of gravity [5], which attempt to resolve the shortcomings of the ΛCDM model [6–10]. Harko et al. [11] proposed $f(R,T)$ gravity. A noticeable feature of this theory is the late-time acceleration due to the geometrical contribution and matter content [12]. Observations indicate that there could be some small anisotropy present [13–19], and so, in this work, we consider the Bianchi-I (BI) model.

In order to comprehend the early stages of the evolution of the universe, it is important to study quark-gluon plasma. During the early stages, two-phase transitions occurred as it cooled down, viz., the quark-gluon phase, when quark matter is thought to have been formed, and the quark hadron phase [20,21]. Some authors [22–24] came up with the theoretical possibility of strange quark matter (SQM) constituting the ground state of hadronic matter. This implies that neutron stars could become strange stars [25–27]. Although SQM has not yet been detected, there are several possibilities where this type of matter can be located [28–30].

The work is organized as follows. In the introduction, an LRS Bianchi-I (BI) space-time model with SQM in the presence of a bag constant and $f(R,T)$ gravity is presented. In Section 2, solutions for $f(R,T) = R + 2\lambda T$ gravity in the presence of quark matter(QM)

and SQM are calculated. In Section 3, the field equations are discussed, while the behavior of SQM is explored in Section 4. Conclusions are made in Section 5.

2. The Formalism of $f(R,T)$ Gravity

In 2011, Harko et al. [11], formulated $f(R,T)$ gravity, whose general action with units in which $8\pi G = 1 = c$ is given by

$$S = \frac{1}{2}\int [f(R,T) + 2L_m]\sqrt{-g}d^4x, \tag{1}$$

where the symbols have their usual meanings. We assume that $f(R,T)$ has the form

$$f(R,T) = R + 2f(T), \tag{2}$$

and hence, (4) becomes

$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} = T_{ij} - 2(T_{ij} + \Theta_{ij})f'(T) + f(T)g_{ij}, \tag{3}$$

where a prime represents the derivative of $f(T)$ with respect to T.

3. Model and Field Equations

The spatially homogeneous and anisotropic BI space-time metric is given by

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2dz^2, \tag{4}$$

where A and B are the scale factors, and are functions of the cosmic time t. The average scale factor is defined by

$$a = (A^2B)^{\frac{1}{3}}. \tag{5}$$

The rates of expansion along the x, y and z-axes are defined as

$$H_1 = \frac{\dot{A}}{A} = H_1, H_2 = \frac{\dot{B}}{B}, \tag{6}$$

where a dot represents a derivative with respect to time. The average expansion rate, which is the generalization of the Hubble parameter in an isotropic scenario, is given by

$$H = \frac{1}{3}\left(\frac{2\dot{A}}{A} + \frac{\dot{B}}{B}\right). \tag{7}$$

An expansion scalar, θ and shear scalar, σ^2, respectively, are defined as

$$\theta = u^i{}_{;i} = 3H, \tag{8}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2. \tag{9}$$

Since the quark gluon plasma behaves as a perfect fluid, the EMT of SQM is given by

$$T_{ij} = (\rho_{sq} + p_{sq})u_iu_j - p_{sq}g_{ij}, \tag{10}$$

where ρ_{sq} is the energy density and p_{sq} is the thermodynamic pressure of the SQM. The trace T of (15) yields

$$T = \rho_{sq} - 3p_{sq}. \tag{11}$$

In the bag model, the energy density and pressure of the SQM are given by, respectively,

$$\rho_{sq} = \rho_q + B_c, p_{sq} = p_q - B_c \tag{12}$$

With the assumption that quarks are non-interacting and massless particles, the pressure is approximated by an EoS of the form

$$p_q = \frac{\rho_q}{3}. \tag{13}$$

Then $p_{sq} = \frac{1}{3}(\rho_{sq} - \rho_0)$ is the linear equation of state of the SQM, with ρ_0 the density at zero pressure. In a bag model, $\rho_0 = 4B_c$, and hence the EoS yields

$$p_{sq} = \frac{\rho_{sq} - 4B_c}{3}. \tag{14}$$

The assumption of $L_m = -p_{sq}$ is used and its variation with respect to g_{ij} yields

$$\Theta_{ij} = -2T_{ij} - p_{sq}g_{ij}. \tag{15}$$

Using (21) in (8) yields:

$$G_{ij} = \left[1 + 2f'(T)\right]T_{ij} + \left[2p_{sq}f'(T) + f(T)\right]g_{ij}. \tag{16}$$

These are the field equations of $f(R,T) = R + 2f(T)$ gravity with SQM. In considering $f(T) = \lambda T$, with λ is an arbitrary constant, using (16)–(19), $T = 4B_c$ which is a constant. Then $f(T) = 4\lambda B_c$ implies $f'(T) = 0$ and so:

$$R_{ij} - \frac{1}{2}Rg_{ij} = T_{ij} + 4\lambda B_c g_{ij}. \tag{17}$$

If we put $\Lambda = 4\lambda B_c g_{ij}$, then the field equations are equivalent to Einstein's field equations with CC. Then $f(R,T) = R + 2\lambda T$ becomes $f(R,T) = R + 8\lambda B_c$. Hence, SQM is equivalent to the ΛCDM model with CC as a results of the coupling of the parameter λ with the bag constant. If $\lambda = B_c = 0$, (23) is the same as in GR. In our case, using (4) and (18), we obtain:

$$\left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} = (\rho_q + B_c) + 4\lambda B_c, \tag{18}$$

$$\left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\ddot{A}}{A} = -(p_q - B_c) + 4\lambda B_c, \tag{19}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -(p_q - B_c) + 4\lambda B_c. \tag{20}$$

These three independent equations consist of four unknowns, namely, A, B, p_q, ρ_q. Therefore, in order to find exact solutions, one supplementary constraint is required. Agrawal [31] considered the expansion scalar, $\theta(= 3H)$ to be proportional to the shear scalar ($\sigma^2 = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2$), σ, which leads to

$$B = A^n, \tag{21}$$

where n is an arbitrary constant. From (19)–(22), one obtains

$$\frac{\ddot{A}}{A} + (n+1)\left(\frac{\dot{A}}{A}\right)^2 = 0, \tag{22}$$

which gives

$$A = \beta[(n+2)t]^{\frac{1}{n+2}}. \tag{23}$$

Consequently

$$B = \alpha[(n+2)t]^{\frac{n}{n+2}}. \tag{24}$$

It is observed that from (8)–(9), and by the use of (24)–(25), the isotropy condition, ($\sigma^2/\theta \to 0$ as $t \to \infty$) is satisfied in this instance. Then the energy density and pressure for quark matter takes the form

$$\rho_q = \frac{1+2n}{(2+n)^2 t^2} - (1+4\lambda)B_c, p_q = \frac{1}{3}\left[\frac{1+2n}{(2+n)^2 t^2} - (1+4\lambda)B_c\right] \tag{25}$$

and the density and pressure of SQM, are, respectively, given by

$$\rho_{sq} = \frac{1+2n}{(2+n)^2 t^2} - 4\lambda B_c, p_{sq} = \frac{1}{3}\left[\frac{1+2n}{(2+n)^2 t^2} - 4B_c(1+\lambda)\right] \tag{26}$$

These are the corrected expressions for the energy density and pressure as opposed to those obtained by Agrawal [31]. In Section 5 of his paper [31], he calculated some geometrical parameters namely, the expansion, shear and volume scalar by the use of some of his equations. Again, we can see that all these parameters can be defined in terms of only the metric potentials A, B, which are different from those of the aforementioned paper. It is also important to mention that both the metric potential and geometrical parameters are independent of the additional terms of $f(R,T)$ gravity. In other words, we obtain the same results as in general relativity for the metric potential and geometrical parameters. For any physically realistic cosmological model, the energy density must be positive, meaning that the weak energy density condition (WEC) ought to be satisfied. Hence both ρ_q and ρ_{sq} remain positive under the constraint $\lambda < -1/4, n > -\frac{1}{2}$. It is clear from (28)–(29) that both the pressure and density depend on $f(R,T)$ gravity and the bag constant. Then $\rho_{sq} \to \infty$ as $t \to 0$, and $\rho_{sq} \to -4\lambda B_c$ as $t \to \infty$. Then again we notice that for $\lambda < -\frac{1}{4}$, the bag constant dominates at late times, and the energy density of the SQM becomes constant. Similarly $p_{sq} \to \infty$ as $t \to 0$, and $p_{sq} \to -\frac{4}{3}(1+\lambda)B_c$ as $t \to \infty$.

4. The Behavior of Strange Quark Matter

Since quarks are considered as a bag, the EoS parameter of SQM can be expressed by the following constraints: $n > -\frac{1}{2}, \lambda < -\frac{1}{4}, \omega_{sq} = p_{sq}/\rho_{sq}$. These yield:

$$\omega_{sq} = \frac{\frac{1}{3}\left[\frac{1+2n}{(2+n)^2 t^2} - 4B_c(1+\lambda)\right]}{\frac{1+2n}{(2+n)^2 t^2} - 4\lambda B_c}. \tag{27}$$

The above EoS indicates that ω_{sq} depends both on $f(R,T)$ gravity and the bag constant. However, if $B_c = 0$, the model neither depends on $f(R,T)$ nor the bag constant, i.e., $B_c = 0$. Then $\omega_{sq} = \frac{1}{3} = \omega_q$, where ω_q is the EoS of QM. Hence this exhibits ultra-relativistic radiation. Hence, for any values of $\lambda < -\frac{1}{4}, n > -\frac{1}{2}$, at the origin model started with $\omega_{sq} = \frac{1}{3}$ (ultra relativistic radiation). The future behavior of the model, i.e., $t \to \infty$ $\omega_{sq} = \frac{1}{3}\left[\frac{1+\lambda}{\lambda}\right]$, depends solely on $f(R,T)$ gravity. For particular values of λ, the model exhibits interesting behavior, i.e., for $\lambda = -1, \omega_{sq} = 0$ (dust), $\lambda = -\frac{1}{2}, \omega_{sq} = -\frac{1}{3}$ (quintessence), and $\lambda = -\frac{1}{4}, \omega_{sq} = -1$ (cosmological constant). Thus, the overall behavior of the model describes the evolution of the universe (ultra-relativistic matter, dust, quintessence, later mimics CC). If $\lambda = 0$, the model exhibits a smooth transition from $\omega_{sq} = \frac{1}{3}$ to $\omega_{sq} = -\infty$ (phantom matter). Therefore again we can see that SQM explains the transition from the early radiated epoch to the phantom phase.

It is mentioned in the introduction that due to the coupling of matter and geometry, some extra terms do appear in the field equations. These terms having λ in (19)–(21) can be associated with coupled matter. This can be distinguished as ρ_f and p_f, respectively, and then $\rho_f = 4\lambda B_c = -p_f$ Hence $\omega_f = -1$. Therefore these extra terms contribute as a cosmological constant.

Effective Matter

The energy density and the pressure of effective matter for $\rho_{eff} \geq 0$ for $n > -\frac{1}{2}$, is given by:

$$\rho_{eff} = \frac{1+2n}{(2+n)^2 t^2} = p_{eff}, \tag{28}$$

Then the effective matter acts as stiff matter in this model.

5. Discussion

In this paper, $f(R,T) = R + 2f(T)$, where $f(T) = \lambda T$, the model investigated in [31] was considered, where a BI model in $f(R,T)$ gravity with SQM was studied. To obtain solutions, the assumption of the expansion scalar proportional to the shear scalar was made [31]. The metric potentials A, B that were calculated are not correct as they can be obtained by means of his equations "(25)–(26)" and "(27)". The other setback of their model is that the LHS of their field equations is also not correct. Since the assumption $(\theta = 3H)$ has already been considered in [32,33], we can see that, surprisingly, the wrong signs do not affect the geometrical parameters. The comparisons of the outcomes in the presence of $f(R,T)$ gravity and the bag constant has been carried out by us to comprehend their roles. It is to be noted that the geometrical parameters of "$model : 1$" of [31] have been carried out by [16]. The physical viability constraints of the model ignored in [31] have been considered by us.

In this model $B = A^n$. In $f(R,T)$ gravity, we found that the model is physically viable for $\lambda < -1/4, n > -\frac{1}{2}$. It is also important to mention that when working with $f(R,T)$, there are some additional terms appearing on the right-hand side of the field equations. Due to the coupling of matter and geometry, those terms can be treated as some additional matter. If the coupling matter is treated as normal matter, they are physically viable for $\lambda > 0$ as they contribute as the CC.

The overall model depends both on $f(R,T)$ gravity and the bag constant B_c. Hence if $B_c = 0$, the model starts off with ultra-relativistic radiation, hence behaving the same as quark matter. We can also observe that B_c for future consideration of the model depends solely on $f(R,T)$ gravity. Then for some values of λ, the model describes a variety of matter including dust, quintessence and CC. We can conclude that $f(R,T)$ gravity enables a transition from ultra-relativistic radiation to the CC. In the absence of $f(R,T)$ gravity, i.e., $\lambda = 0$, the model of course relies on the bag constant only. Again, we can see clearly that the model starts off radiating, and then all the dynamical candidates including the phantom stage. Hence, in this case, we can see that the bag constant enables the transition from ultra-relativistic radiation to phantom matter.

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References

1. Riess, A.G.; Filippenko, A.V.; Challis, P.; Clocchiatti, A.; Diercks, A.; Garnavich, P.M.; Gilliland, R.L.; Hogan, C.J.; Jha, A.; Kirshner, R.P.; et al. Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. *Astron. J.* **1998**, *116*, 1009–1038. [CrossRef]
2. Perlmutter, S.; Aldering, G.; Goldhaber, G.; Knop, R.A.; Nugent, P.; Castro, P.G.; Deustua, S.; Fabbro, S.; Goobar, A.; Groom, D.E.; et al. Measurements of Ω and Λ from 42 High-Redshift Supernovae. *Astrophys. J.* **1999**, *517*, 565–586. [CrossRef]
3. Schmidt, B.P.; Suntzeff, N.B.; Phillips, M.M.; Schommer, R.A.; Clocchiatti, A.; Kirshner, R.P.; Garnavich, P.; Challis, P.; Leibundgut, B.; Spyromilio, J.; et al. The High-Z Supernova Search: Measuring Cosmic Deceleration and Global Curvature of the Universe Using Type Ia Supernovae. *Astrophys. J.* **1998**, *507*, 46–63. [CrossRef]
4. Bamba, K.; Capozziello, S.; Nojiri, S.; Odintsov, S.D. Dark energy cosmology: The equivalent description via different theoretical models and cosmography tests. *Astrophys. Space Sci.* **2012**, *342*, 155–228. [CrossRef]
5. Nojiri, S.; Odintsov, S.D. Unified cosmic history in modified gravity: From F(R) theory to Lorentz non-invariant models. *Phys. Rep.* **2011**, *505*, 59–114. [CrossRef]
6. Zlatev, I.; Wang, L.; Steinhardt, P.J. Quintessence, Cosmic Coincidence, and the Cosmological Constant. *Phys. Rev. Lett.* **1999**, *82*, 896. [CrossRef]
7. Peebles, J.E.; Ratra, B. The cosmological constant and dark energy. *Rev. Mod. Phys.* **2003**, *75*, 559. [CrossRef]
8. Linde, A.D. A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems. *Phys. Lett. B* **1982**, *108*, 389–393. [CrossRef]
9. Misner, C.W. The Isotropy of the Universe. *Astrophys. J.* **1968**, *151*, 431. [CrossRef]
10. Weinberg, S. *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, 1st ed.; John Wiley and Sons: New York, NY, USA, 1972.
11. Harko, T.; Lobo, F.S.N.; Nojiri, S.; Odintsov, S.D. $f(R,T)$ gravity. *Phys. Rev. D* **2011**, *84*, 024020. [CrossRef]
12. Singh, V.; Beesham, A. Plane symmetric model in $f(R,T)$ gravity. *Eur. Phys. J. Plus* **2020**, *135*, 319. [CrossRef]
13. Reddy, D.R.K.; Naidu, R.L.; Satyanarayana, B. Kaluza-Klein Cosmological Model in f(R,T) Gravity. *Int. J. Theor. Phys.* **2012**, *51*, 3222. [CrossRef]
14. Ram, S.; Priyanka. Some Kaluza-Klein cosmological models in $f(R,T)$ gravity theory. *Astrophys. Space Sci.* **2013**, *347*, 389–397. [CrossRef]
15. Sharif, M.F.; Zubair, M. Energy Conditions Constraints and Stability of Power Law Solutions in $f(R,T)$ Gravity. *J. Phys. Soc. Jpn.* **2013**, *82*, 014002. [CrossRef]
16. Shamir, M.F. Bianchi Type I Cosmology in $f(R,T)$ Gravity. *J. Exp. Theor. Phys.* **2014**, *119*, 242–250. [CrossRef]
17. Moraes, P.H.R.S.; Correa, R.A.C.; Ribeiro, G. Evading the non-continuity equation in the $f(R,T)$ cosmology. *Eur. Phys. J. C* **2018**, *78*, 192. [CrossRef]
18. Tiwari, R.K.; Beesham, A. Anisotropic model with decaying cosmological term. *Astrophys. Space Sci.* **2018**, *363*, 234. [CrossRef]
19. Esmaeili, F.M. Dynamics of Bianchi I Universe in Extended Gravity with Scale Factors. *J. High Energy Phys. Gravit. Cosmol.* **2018**, *4*, 716–730. [CrossRef]
20. Witten, E. Cosmic separation of phases. *Phys. Rev. D* **1984**, *30*, 272. [CrossRef]
21. Mak, M.K.; Harko, T. Quark stars admitting a one-parameter group of conformal motions. *Int. J. Mod. Phys. D* **2004**, *13*, 149–156. [CrossRef]
22. Itoh, N. Hydrostatic Equilibrium of Hypothetical Quark Stars. *Prog. Theor. Phys.* **1970**, *44*, 291–292. [CrossRef]
23. Bodmer, A.R. Collapsed Nuclei. *Phys. Rev. D* **1971**, **4**, 1601–1606. [CrossRef]
24. Farhi, E.; Jaffe, R.L. Strange matter. *Phys. Rev. D* **1984**, *30*, 2379. [CrossRef]
25. Alcock, C.; Farhi, E.; Olinto, A. Strange Stars. *Astrophys. J.* **1986**, *310*, 261–272. [CrossRef]
26. Haensel, P.; Zdunik, J.L.; Schaefer, R. Strange quark stars. *Astron Astrophys.* **1986**, *160*, 121–128.
27. Madsen, J. Physics and astrophysics of strange quark matter. In *Hadrons in Dense Matter and Hadrosynthesis, Lecture Notes in Physics*; Springer: Berlin/Heidelberg, Germany, 1999; Volume 516, pp. 162–203.
28. Drake, J.J.; Marshall, H.L.; Dreizler, S.; Freeman, P.E.; Fruscione, A.; Juda, M.; Kashyap, V.; Nicastro, F.; Pease, D.O.; Wargelin, B.J.; et al. Is RX J185635-375 a Quark Star? *Astrophys. J.* **2002**, *572*, 996–1001. [CrossRef]
29. Weber, F. Strange Quark Matter and Compact Stars. *Prog. Part. Nucl. Phys.* **2005**, *54*, 193–288. [CrossRef]
30. Aktas, C.; Yilmaz, I. Is the universe homogeneous and isotropic in the time when quark-gluon plasma exists? *Gen. Relativ. Grav.* **2011**, *43*, 1577–1591. [CrossRef]
31. Agrawal, P.K.; Pawar, D.D. Plane Symmetric Cosmological Model with Quark and Strange Quark Matter in f (R, T) Theory of Gravity. *J. Astrophys. Astron.* **2017**, *38*, 2. [CrossRef]
32. Mahanta, K.L. Bulk viscous cosmological models in f(R,T) theory of gravity. *Astrophys. Space Sci.* **2014**, *353*, 683. [CrossRef]
33. Shamir, M.F. Locally Rotationally Symmetric Bianchi Type I Cosmology in f(R,T) Gravity. *Eur. Phys. J. C* **2015**, *75*, 354. [CrossRef]