physical sciences forum

Proceeding Paper

The Status of Geometry and Matter in the Reinterpreted WdW Equation †

Avadhut V. Purohit

S. P. Mandel's Kankavli College, Kankavli 416602, Maharashtra, India; avdhoot.purohit@gmail.com

† Presented at the 2nd Electronic Conference on Universe, 16 February–2 March 2023; Available online: https://ecu2023.sciforum.net/.

Abstract: This paper shows that the field defined by the Wheeler–DeWitt equation for *pure gravity* is neither a standard gravitational field nor the field representing a particular universe. The theory offers a unified description of geometry and matter, with geometry being fundamental. The quantum theory possesses gravitational decoherence when the signature of $R^{(3)}$ changes. The quantum theory resolves singularities dynamically. Application to the FLRW $\kappa = 0$ shows the creation of local geometries during quantum evolution. The 3-metric is modified near the classical singularity in the case of the Schwarzschild geometry.

Keywords: reinterpretation; Wheeler–DeWitt equation; geometric matter; gravitized quantum theory

check for updates

Citation: Purohit, A.V. The Status of Geometry and Matter in the Reinterpreted WdW Equation. *Phys. Sci. Forum* **2023**, *7*, 37. https://doi.org/10.3390/ECU2023-14097

Academic Editor: Douglas Singleton

Published: 2 March 2023

1. Introduction

In search for the most fundamental theory, two schools of thought are prevalent. One school of thought states that quantum theory is fundamental because small objects together make a large object. Therefore, gravity is quantized at a small scale. Often, the existence of singularities in the classical theory indicates the limitations of this theory. Theories such as loop quantum gravity and the Wheeler–DeWitt take this approach. The quantization of a relativistic particle shows that the quantization by raising $E \to i\hbar\partial_t$ and $\vec{p} \to -i\hbar\vec{\nabla}$ to operators on the Hilbert space faces several mathematical and conceptual issues. Working from quantum field theories, Ref. [1] and others tried to reinterpret the Wheeler–DeWitt equation. However, this approach also has severe issues. Although the gravitational and matter fields are fields defined over a 4-dimensional manifold, the matter fields involve reinterpretation. However, the gravitational field is a generalization of the special theory of relativity. It does not involve reinterpretation. When reinterpreting the full Wheeler–DeWitt equation, the gravitational field and matter fields are not on equal footing. Therefore, such quantization is dubious (refer to [2,3] for more details).

Another school of thought states that gravity, as a dynamical theory of spacetime itself, is more fundamental. Quantum theory would be modified in the case of a gravitational scenario. Recently, there have been new developments [4] in this approach. Everything else ceases to exist in the absence of background spacetime. Apart from the search for a unified particle theory that included gravity as a natural ingredient, theoreticians pursued a separate line of investigation based primarily in general relativity and topology. Clifford presented his paper "On the Space Theory of Matter" [5], and Einstein gave substance to this line of inquiry. He wrote, "The material particle has no place as a fundamental concept in field theory. Even electrodynamics is not complete for this reason. Gravity as a field theory must also deny a preferred status to matter." John Wheeler also attempted to build such a theory. He wrote, "What else is there out of which to build a particle except for geometry (spacetime) itself?" Canonical theories such as the Wheeler–DeWitt theory quantize gravity. However, they do not describe geometric matter ([6], page 8). Unfortunately, even with the philosophical background, there has been no success in this direction so far.

In this paper, I analyze the Wheeler–DeWitt equation for *pure gravity* in the light of standard quantum field theories. Because the field is defined only over the space of 3-metric, it can be reinterpreted without the issue discussed above. The defined field satisfies the ADM constraints for pure gravity. Therefore, one would interpret the field Φ to be a pure gravitational field. However, I observe that even gauge fields obey ADM constraints for pure gravity. I also observe that these fields have nontrivial stress tensors, whereas the stress tensor for a pure gravitational field is $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0$. The quadratic coupling always remains nonnegative regardless of the signature of $R^{(3)}$. I also observe that the higher-order couplings with $\Phi \sim e^{iq_{ab}P^{ab}}$ allow us to interpret it as a matter-like term. The other interpretation is that the field Φ describes a particular Universe. I observe that such an interpretation faces problems due to the interaction between different fields. It shows that neither interpretation is true. The field Φ is a unified description of the gravity and scalar matter.

The reinterpretation partly modifies the quantum theory and the classical gravity theory. On the quantization of the field, we obtain the geometric quantum corresponding to the field. There is no quantum of gravity or graviton. With the curvature's signature-dependent decoherence, the vector-valued creation and annihilation operators show gravitational effects on the nature of quantum theory, whereas the dynamical singularity resolution shows a modification of the classical gravity theory. The quantum with a particular 3-metric has a definite energy. However, it does not have well-defined momentum. The theory applied to the FLRW $\kappa = 0$ Universe creates nontrivial local geometric quanta. The theory applied to the Schwarzschild geometry shows the existence of the Planck scale black hole. The sense of distance arises from the interaction between fields.

The reinterpretation connects the requirements of the correct quantum theory and the expectations of a general relativistic school of thought.

2. The Scalar Field

The Wheeler–DeWitt equation written in DeWitt's coordinates

$$\left(\frac{1}{\sqrt{-G}}\partial_\mu\sqrt{-G}G^{\mu\nu}\partial_\nu + \mu^2\right)\Phi = 0 \tag{1}$$

is interpreted as a classical field equation. The field Φ is defined over the superspace $\zeta^\mu := (\zeta, \zeta^A)$ with $A = 1,2,3,4,5$. The DeWitt supermetric is borrowed from [7].

$$G_{\mu\nu} := \begin{pmatrix} 1 & 0 \\ 0 & -\frac{3\zeta^2}{32}\bar{G}_{AB} \end{pmatrix} \quad \bar{G}_{AB} := \mathrm{Tr}\left(\mathbf{q}^{-1}\frac{\partial\mathbf{q}}{\partial\zeta^A}\mathbf{q}^{-1}\frac{\partial\mathbf{q}}{\partial\zeta^B}\right) \tag{2}$$
$$\zeta := \left(\tfrac{32}{3}\right)^{\frac{1}{2}}(\det q_{ab})^{\frac{1}{4}} \qquad \sqrt{-G} := \sqrt{-\det G_{\mu\nu}}$$

Here, $\mathbf{q} := q_{ab}$. $\bar{G}_{AB}$ is the symmetric supermetric on the 5D manifold M identified with $SL(3,\mathbb{R})/SO(3,\mathbb{R})$ (refer to [7] and [8] for more discussion on the geometry of superspace). As discussed by DeWitt [7] in appendix A, ζ^A are new orthogonal coordinates chosen from components of the 3-metric as they act as "good" coordinates. The coupling function is defined as

$$\mu^2\left(\zeta,\zeta^A\right) := -\frac{3\zeta^2}{32}R^{(3)}. \tag{3}$$

The field Φ is functional over the space of 3-metric only. Hence, $\frac{\partial\Phi}{\partial q_{ab}}$ is also functional over 3-metric. Trivially, it satisfies the diffeomorphism constraints

$$D_aP^{ab} \Rightarrow D_a\frac{\partial\Phi}{\partial q_{ab}} \approx 0. \tag{4}$$

The action functional for the geometric scalar field that gives field Equation (1) is assumed to have the following form.

$$\mathcal{A}_\Phi := \int \mathcal{D}\zeta \frac{\sqrt{-G}}{2}\left(\partial_\mu \Phi G^{\mu\nu}\partial_\nu \Phi - \mu^2\Phi^2\right) \tag{5}$$

$\mathcal{D}\zeta$ is suitable measure over the 6D manifold. The ADM theory does not have an issue with operator ordering. However, for the classical geometric scalar field, different combinations of P^{ab} and q_{ab} give non-equivalent field equations. I took the combination of field variables with a consistent self-adjoint extension. In other words, the combination allows the Hamiltonian operator to be self-adjoint. On a single spacetime-like interpretation $\Phi \sim e^{iq_{\mu\nu}P^{\mu\nu}}$, the ADM Hamiltonian constraints for pure gravity in the DeWitt coordinates (5.20, [7])

$$P_0^2 - \frac{32}{3\zeta^2}\bar{G}^{AB}P_A P_B + \frac{3\zeta^2}{32}R^{(3)} \approx 0 \tag{6}$$

are recovered.

The invariance of an action under variation $\zeta^\mu \to \zeta^\mu + \delta\zeta^\mu$ gives the stress tensor.

$$T_\mu^\nu := \frac{\partial \mathcal{L}}{\partial\left(\frac{\partial\Phi}{\partial\zeta^\nu}\right)}\frac{\partial\Phi}{\partial\zeta^\mu} - \mathcal{L}\delta_\mu^\nu \tag{7}$$

For now, I assume ζ as time and perform the Legendre transformation to obtain the Hamiltonian

$$\Pi_\Phi := \frac{\partial(\sqrt{-G}\mathcal{L})}{\partial\frac{\partial\Phi}{\partial\zeta}} = \sqrt{-G}\frac{\partial\Phi}{\partial\zeta} \tag{8}$$

$$\mathbf{H}_\Phi = \int \mathcal{D}\zeta^A \, \frac{1}{2}\left(\frac{\Pi_\Phi^2}{\sqrt{-G}} + \frac{32}{3\zeta^2}\sqrt{-G}\frac{\partial\Phi}{\partial\zeta^A}\bar{G}^{AB}\frac{\partial\Phi}{\partial\zeta^B} + \sqrt{-G}\mu^2\Phi^2\right) \tag{9}$$

For $\mu^2 < 0$, the field is self-coupled, i.e., it has $h\Phi^4$ with some $h > 0$. The Hamiltonian shows that the quadratic coupling will be nonnegative regardless of the signature of the 3-Ricci curvature scalar and justifies the use of ζ as the time for the geometric scalar field. On a single-geometric interpretation, we obtain

$$P_0^2 - \frac{32}{3\zeta^2}\bar{G}^{AB}P_A P_B + \frac{3\zeta^2}{32}R^{(3)} \approx h \tag{10}$$

The coupling parameter h is free and does not necessarily depend on the geometry of spacetime. From the ADM theoretical viewpoint, $h \neq 0$ represents the matter field. For $\mu^2 < 0$, the minima lies at $\Phi_0 = \pm\sqrt{\frac{-2\mu^2}{\lambda}}$. The field rolls down to obtain positive quadratic coupling. h appears due to field theoretical reason, and gravity guides us to interpret the coupling term h as a matter-like term.

If we look at the Green's function for the Wheeler–DeWitt operator 1, in the limit $\zeta \to \infty$, the middle spatial terms become negligible, and we effectively obtain

$$\left(\frac{\partial^2}{\partial\zeta^2} + \frac{1}{\zeta}\frac{\partial}{\partial\zeta} + \mu^2\right)Q(\zeta,\zeta') = \delta(\zeta - \zeta'). \tag{11}$$

The Green's function for the non-zero constant μ^2 is

$$Q = \frac{\pi}{2}\theta(\zeta - \zeta')\zeta'(Y_0(\mu\zeta)J_0(\mu\zeta') - J_0(\mu\zeta)Y_0(\mu\zeta')), \tag{12}$$

and for $\mu^2 = 0$, it is

$$Q = \theta(\zeta - \zeta')\ln\left(\frac{\zeta}{\zeta'}\right). \tag{13}$$

The Heaviside function $\theta(\zeta - \zeta')$ is zero for $\zeta < \zeta'$ and 1 for $\zeta > \zeta'$. Hence, the signal propagates forward in ζ. It shows that ζ acts as time for the field $\Phi(\zeta^\mu)$.

Note: *The Ricci curvature scalar $R^{(3)}$ and cosmological constant Λ are not on equal footing. $R^{(3)}$ appears in quadratic coupling, whereas Λ contributes to the vacuum.*

2.1. Features

The field $\Phi(q_{ab})$ is identified with an intrinsic property μ and defined over q_{ab}, which is the solution to the intrinsic curvature $R^{(3)}$. The existence of dynamical background shows that the field is gravitational.

Even when the 3-geometry has a positive curvature, the quadratic coupling remains positive. This shows the resemblance of the field to the standard matter fields.

The field Φ can also have charge.

$$\mathcal{A}_{\text{complex}} := \int \mathcal{D}\zeta \frac{\sqrt{-G}}{2} \left(\partial_\mu \Phi^\star G^{\mu\nu} \partial_\nu \Phi - \mu^2 \Phi^\star \Phi \right) \tag{14}$$

Pure gravity, on the other hand, does not have opposite charges. This is the property that resembles charged matter.

The existence of $\mu^2 = 0$ or $R^{(3)} = 0$ does not necessarily mean the line element is zero.

$$ds^2 = d\zeta^2 - \frac{3\zeta^2}{32} \bar{G}_{ij} dx^i dx^j \qquad \bar{G}_{ij} := \bar{G}_{AB} \frac{\partial \zeta^A}{\partial x^i} \frac{\partial \zeta^B}{\partial x^j} \tag{15}$$

For the 3-metric $q_{ab} := q(t)\,\text{diag}\big(f(r), r^2, r^2 \sin^2\theta\big)$, the line element is given as follows.

$$ds^2 = G^{abcd} dq_{ab} dq_{cd} = \begin{pmatrix} dq & df \end{pmatrix} \begin{pmatrix} \frac{-6}{f^{\frac{1}{2}} q^{\frac{1}{2}}} & -2\frac{q^{\frac{1}{2}}}{f^{\frac{3}{2}}} \\ -2\frac{q^{\frac{1}{2}}}{f^{\frac{3}{2}}} & 0 \end{pmatrix} \begin{pmatrix} dq \\ df \end{pmatrix} \tag{16}$$

For the *distance* in the superspace, in the large q limit, $ds^2 = -4\frac{q^{\frac{1}{2}} dq\, df}{f^{\frac{3}{2}}}$ increases with q. It shows cosmological expansion. The distance decreases with an increase in f, showing attraction between two objects. This distance is actually the distance between two 3-metrics (refer to [7]).

The energy of the field Φ is always well-defined. A gravitational field may not always have a time-like Killing vector field. Therefore, defining energy in general relativity is not straightforward.

3. Reinterpretation and Gauge Invariance

A famous experiment performed in 1975 by Colella, Overhauser, and Werner did confirm that quantum mechanics respects the principle of equivalence (page 11, [4]). In the general relativization of the quantum theory, wave functions obtain nontrivial phase differences. In the ADM theory, the set of lapse functions N and shift vector N^a identify the frame of reference.

The field Φ contains information about the reference frame, which can be easily seen by the single-geometric interpretation $\Phi \sim e^{\pm i P^{ab} q_{ab}}$ leading to (6). Here, P^{ab} contains information about the lapse function and shift-vector. The complex scalar field Φ is not invariant under transformation $\Phi \to e^{i\alpha(\zeta^\mu)}\Phi$. It requires the field A_μ that makes (14) gauge invariant. This theory fully respects the equivalence principle.

$$\mathcal{L}_{\text{complex}} = \tfrac{1}{2}(D_\mu \Phi)^\star G^{\mu\nu} (D_\nu \Phi) - \tfrac{\mu^2}{2} \Phi^\star \Phi \tag{17}$$

$$D_\mu := \partial_\mu - i\alpha A_\mu \tag{18}$$

The third quantized theories interpret Φ as a description of a particular universe. However, we can see that the Lagrangian describes the interaction between a charged field and its gauge field. Here, different universes interacting with each other destroys the very definition of a universe. Since the third quantized theories include matter fields in the superspace, the field Φ cannot be interpreted as a matter field as well. For the interpretation presented in this paper, there is no such problem, because the superspace is defined only over the space of the 3-metric.

The action functional for field A_μ is assumed to have the following form.

$$\begin{aligned} \mathcal{A}_{\text{vector}} &:= -\tfrac{1}{4}\int \mathcal{D}\zeta\sqrt{-G}F_{\mu\nu}F^{\mu\nu} \\ F^{\mu\nu} := G^{\mu\rho}G^{\nu\sigma}F_{\rho\sigma} &\ \text{ and }\ F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu \end{aligned} \tag{19}$$

$F_{\mu\nu}$ is a completely antisymmetric tensor and therefore satisfies the Bianchi identity. The field equations in the presence of source J^μ and in the gauge selected above are obtained as

$$\partial_\mu F^{\mu\nu} + \frac{F^{\mu\nu}}{\sqrt{-G}}\partial_\mu\left(\sqrt{-G}\right) = J^\nu. \tag{20}$$

In addition to other sources J^ν, the second geometric term also acts as a source for the gauge field. In the absence of a source, fields $B_A := \epsilon_{ABC}F_{BC}$ as well as $E_A := F_{0A}$ satisfy the Wheeler–DeWitt equation.

$$\frac{1}{\sqrt{-G}}\partial_\mu\left(\sqrt{-G}G^{\mu\nu}\partial_\nu\right)\begin{pmatrix}E_C\\ B_C\end{pmatrix} = 0 \tag{21}$$

In an asymptotic flat limit (i.e., $\sqrt{-G}\approx 1$), the field equations become $\partial_\mu F^{\mu\nu} = J^\nu$.

The field A_μ also satisfies the ADM constraints for pure gravity and has a nontrivial stress tensor. If Φ is interpreted as a pure gravitational field, then the field A_μ would also have to be interpreted as a pure gravitational field. However, a vector-valued gravitational field A_μ is a disaster. This shows that fields Φ and A_μ are not pure gravitational fields, at least not standard gravitational fields.

4. Quantization

I define the vector-valued annihilation and creation operators

$$\begin{aligned} a_A &:= \frac{(-G)^{\frac{1}{4}}}{\sqrt{2}}\left(\frac{\Pi}{(-G)^{\frac{1}{2}}}n_A + i\sqrt{\frac{32}{3\zeta^2}}\frac{\partial\Phi}{\partial\zeta^A} + i\omega\Phi n_A\right), \\ a_B^\dagger &:= \frac{(-G)^{\frac{1}{4}}}{\sqrt{2}}\left(\frac{\Pi}{(-G)^{\frac{1}{2}}}n_B - i\sqrt{\frac{32}{3\zeta^2}}\frac{\partial\Phi}{\partial\zeta^B} - i\omega\Phi n_B\right), \end{aligned} \tag{22}$$

$n^A := \frac{\zeta^A}{\sqrt{\bar{G}_{AB}\zeta^A\zeta^B}}$, $(-G)^{\frac{1}{4}} := \left(-\det G_{\mu\nu}\right)^{\frac{1}{4}}$, and $\omega\in\mathbb{R}$ is chosen as the solution to following Riccati equation

$$\sqrt{-G}\omega^2 - \frac{\partial}{\partial\zeta^C}\left(\omega\sqrt{-G}\sqrt{\frac{32}{3\zeta^2}}n_A\bar{G}^{AC}\right) = \sqrt{-G}\mu^2. \tag{23}$$

The equation should be solved using the 'correct' boundary conditions. Such a solution is unique. Examples of such boundary conditions are

- FLRW $\kappa = 0$ model: ω that makes the spectrum of the Hamiltonian operator continuous in the limit $q(t)\to\infty$.
- Schwarzschild spacetime: ω that makes the spectrum of the Hamiltonian operator continuous in the limit $q_{ab}\to\eta_{ab}$ with η_{ab} being flat 3-metric.

Computing the nontrivial commutator using the property of the Dirac delta function $f(x)\delta'(x) = -f'(x)\delta(x)$ for $f(x)\neq$ constant, we obtain

$$[a,a^\dagger] = \frac{\varepsilon_{Pl}}{2}\left(\sqrt{\frac{32}{3\zeta^2}}\partial_C n^C - \omega\right)\delta(\vec{\zeta},\vec{\zeta}') = \beta(\zeta,\zeta^A)\delta(\vec{\zeta},\vec{\zeta}'). \tag{24}$$

There is a conserved quantity corresponding to ζ that I call energy. I introduce ε_{Pl} instead of $\hbar$ to maintain ζ as dimensionless, because here, I interpret the theory relative to the ADM theory. If we interpret ζ as time in seconds without refering to the ADM theory, we can replace $\varepsilon_{Pl} \to \hbar$. The above commutator was possible because the inverse metric $\bar{G}^{AB}$ is symmetric. These identities allow us to write the Hamiltonian operator in the discrete space. (i.e., $\int \mathcal{D}\zeta^A \to \sum_{\zeta^A}$)

$$\mathbf{H}_\Phi = \sum_{\zeta^A} a^\dagger_A \bar{G}^{AB} a_B + \frac{\delta(0)}{2} \varepsilon_{Pl} \sum_{\zeta^A} \left(\sqrt{\frac{32}{3\zeta^2}} \partial_C n^C - \omega \right) \tag{25}$$

The second term is the vacuum term. The quantum vacuum is a sea of constantly creating and annihilating geometries. We discard this term and write the Hamiltonian operator in terms of the number operator $\hat{\mathbf{n}} = \sum_A \hat{\mathbf{n}}_A$ with $a^{\dagger A} a_A := \left(\sqrt{\frac{32}{3\zeta^2}} \partial_C n^C - \omega \right) \hat{\mathbf{n}}_A$

$$\hat{\mathbf{H}}_\Phi = \varepsilon_{Pl} \sum_{\zeta^A} \left| \sqrt{\frac{32}{3\zeta^2}} \partial_C n^C - \omega \right| \hat{\mathbf{n}} \tag{26}$$

The appearance of the differential Equation (23) is not surprising. It is a consequence of using coordinate-space for quantization. The scalar quantum of the Klein–Gordon field satisfies $\omega^2 = k^2 + m^2$. Similarly, the quantum of the geometric scalar field follows $frequency = \sqrt{\frac{32}{3\zeta^2}} \partial_C n^C - \omega$ with ω being the solution to (23). Φ has a single degree of freedom. Therefore, the quantum is scalar. Π is a collection of creation and destruction operators. However, Φ depends nonlinearly on creation and annihilation operators.

The momentum operator defined using the stress tensor

$$\hat{\mathbf{P}}^C = -\frac{32}{3\zeta^2} \sum_{\zeta^A} \left(\bar{G}^{CB} \frac{\partial \Phi}{\partial \zeta^B} \right) \Pi \tag{27}$$

does not share eigenstates with the Hamiltonian operator. This is because Φ depends nonlinearly on creation and annihilation operators. Therefore, the quantum with a particular 3-metric does not have well-defined momentum at the quantum level.

4.1. Application

If the spacetime is spherically symmetric with the 3-metric $q_{ab} := q(t)\,\mathrm{diag}(f(r), 1, 1)$ with ADM coordinates $dr, rd\theta$, and $r \sin\theta\, d\phi$ chosen keeping dimensionality in mind, then the DeWitt supermetric becomes $G_{\mu\nu} := \mathbf{diag}\left(1, -\frac{3\zeta^2}{32 f^2}\right)$, with $\zeta^A := f(r) = f$ being one-dimensional. The determinant of the supermetric $-\det G_{\mu\nu} = \frac{3\zeta^2}{32 f^2}$, $\bar{G}_{AB} = \frac{1}{f^2}$, and $n^A = f$. That implies $\partial_C n^C = 1$. Then, the Hamiltonian becomes

$$\mathbf{H}_\Phi = \varepsilon_{Pl} \sum_f \left| \sqrt{\frac{32}{3}} \frac{1}{\zeta} - \omega \right| \hat{\mathbf{n}}. \tag{28}$$

The ω is a solution to the following Riccati equation.

$$\omega^2 - \sqrt{\frac{32}{3}} \frac{f}{\zeta} \frac{\partial \omega}{\partial f} = \mu^2 \tag{29}$$

4.1.1. Spatially Flat FLRW Universe

For the spatially flat spacetime $\mu^2 = 0$, I chose a trivial solution $\omega = 0$. The Hamiltonian in such case becomes

$$\hat{\mathbf{H}}_\Phi = \sqrt{\frac{32}{3}} \frac{\varepsilon_{Pl}}{\zeta} \hat{\mathbf{n}} = \frac{\varepsilon_{Pl}}{a^{\frac{3}{2}}} \hat{\mathbf{n}}. \tag{30}$$

Here, a represents the scale factor. The Hamiltonian spectrum becomes continuous in the limit $\zeta \to \infty$ that justifies the selected trivial solution $\omega = 0$. The frequency (or energy) of the quantum is redshifted (proportional to $\frac{1}{\zeta}$) with time. There exists a conserved quantity corresponding to ζ. I call it *energy*. If we assume the Universe as a collection of n identical quanta, then the total energy of the Universe is $E_U \approx \varepsilon_{Pl} \frac{n}{\zeta_{min}}$. The finiteness of E_U results in the existence of finite nonzero $\zeta = \zeta_{min}$. The energy of a quantum in a particular state decreases with time ζ. We need to create more quanta to conserve the total energy. However, at a given time, every quantum has the same energy. That means we cannot conserve the total energy in this way. One of the possible ways to conserve the energy is by creating local geometries, that is, by introducing $f \neq 1$ with $R^{(3)} = 0$. An individual quantum radiates energy.

$$\frac{\varepsilon_{Pl}}{\zeta_{min}} \xrightarrow{\zeta \text{ evolution}} \frac{\varepsilon_{Pl}}{\zeta_{min} + \Delta\zeta} + \varepsilon(\zeta, f) \tag{31}$$

Here, $\varepsilon(\zeta, f)$ indicate created quanta in the evolution. This is locally allowed by geometries such as the Schwarzschild geometry, where $R^{(3)} = 0$ but $f \neq 1$. It shows that even though the metric in the beginning is ζ dependent only, the geometric quantum theory naturally introduces local variations of the 3-metric to conserve the total energy of the Universe.

4.1.2. Schwarzschild Geometry

The Schwarzschild geometry is written in the isotropic radial coordinates as

$$q_{ab} := f \,\mathrm{diag}\Big(1, r^2, r^2 \sin^2\theta\Big), \qquad f = \left(1 + \frac{M}{2r}\right)^4 \tag{32}$$

Here, I set $\zeta = 1$ for the static geometry. The Hamiltonian spectrum in this situation is obtained by taking the trivial solution to the Riccati equation, as it gives the correct quantum theory.

$$\hat{\mathbf{H}}_\Phi \approx 3.266\, \varepsilon_{Pl}\, \hat{\mathbf{n}} \sum_f 1 \tag{33}$$

As $f(r) \in (1, \infty)$, integrating f from 1 to f_{max}, we obtain

$$\hat{\mathbf{H}}_\Phi \approx 3.266\, \varepsilon_{Pl} (f_{max} - 1)\hat{\mathbf{n}}. \tag{34}$$

The energy of a quantum state increases with f. The energy spectrum shows area quantization, as $\sqrt{f}$ is a dimensionless length. The black hole with energy E_{bh} has $f_{max} \approx \frac{E_{bh}}{3.266\varepsilon_{Pl}} + 1$. Clearly, $f_{max} \geq 16$, because $f(r)$ at the event horizon is 16. $f_{max} < 16$ would mean the inner radius was greater than the event horizon. The minimum energy that a black hole can have is

$$E_{bh,min} \approx 52.25\, \varepsilon_{Pl}. \tag{35}$$

This is the Planck energy Schwarzschild black hole (PESBH), where $f_{max} = f_0$.

5. Interaction

The free field geometric quanta represent isolated geometries. I turned on their interaction to ascertain what happens when fields interacted with other fields.

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{H}_{int} \tag{36}$$

Here, the noninteracting Hamiltonian $\mathbf{H}_0 = \mathbf{H}_\Phi$ has explicit ζ-dependence, and $\mathbf{H}_{int}$ has interaction. The unitary operator is obtained using the following formula.

$$i \frac{d}{d\zeta} U(\zeta, \zeta') = \mathbf{H}_0 U(\zeta, \zeta'); \qquad U(\zeta, \zeta') = \left(\frac{\zeta}{\zeta'}\right)^{-i\sqrt{\frac{32}{3}}\mathbf{n}}$$

The evolution of the quantum state in interaction picture is given by

$$i\frac{d}{d\zeta}|\psi_{\mathbf{n}}(\zeta)\rangle_I = \mathbf{H}_I|\psi_{\mathbf{n}}(\zeta)\rangle_I$$
$$\mathbf{H}_I = \left(\frac{\zeta}{\zeta_0}\right)^{i\sqrt{\frac{32}{3}}\mathbf{n}}\mathbf{H}_{int}\left(\frac{\zeta}{\zeta_0}\right)^{-i\sqrt{\frac{32}{3}}\mathbf{n}}.$$

I obtained the S-matrix from the unitary operator in the interaction picture as,

$$S := U_I(\zeta_0, -\infty) = T\exp\left[-i\int_{\zeta_0}^{\infty} d\zeta\,\mathbf{H}_I\right]. \quad (37)$$

$|\langle 0|\hat{S}|0\rangle|^2 \neq 1$ implies the production of the geometric quantum from a vacuum due to the presence of a source. Let us consider two free fields interacting in the following way.

$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2 + \frac{\lambda}{2}(\Phi_1 - \Phi_2)^2 \quad (38)$$

In the large ζ limit, we can write the total Hamiltonian as a sum of two normal mode fields (Φ_+, Φ_-). These normal mode fields are obtained by rotating (Φ_1, Φ_2) by an angle $\alpha = \pm\arctan\sqrt{\frac{2\lambda+\omega_1^2-\omega_2^2}{\lambda}}$. Then, normal mode frequencies are obtained as

$$\omega_+ = \sqrt{\frac{\omega_1^2+\omega_2^2}{2}} \qquad \omega_- = \sqrt{\frac{4\lambda+\omega_1^2+\omega_2^2}{2}}.$$

The antisymmetric state has higher energy than the symmetric state. Without coupling, quanta are as good as individual Universes. If we take the weak gravity limit of the Schwarzschild metric, i.e., $f \approx 1 + \frac{\phi_{grav}}{r}$, the coupling effectively increases the value of $f(r)$. In other words, the radius is less than the sum of two noninteracting geometric quanta. The coupling gives the measure of the distance between quanta. The stronger the coupling, the less the distance between them.

6. Results

I showed that the field satisfied by the Wheeler–DeWitt Equation (1) cannot be a pure gravitational field. It cannot be a theory of multiverse or a scalar theory of gravity. In addition to having geometric properties, fields satisfying the Wheeler–DeWitt equation have properties similar to the corresponding matter fields.

The field with a positive intrinsic spatial curvature scalar $R^{(3)}$ is necessarily self-interactive. However, the field with a negative intrinsic spatial curvature scalar $R^{(3)}$ is not necessarily self-interactive. Fields with $R^{(3)} < 0$ do not have interaction other than the geometric one as discussed in Section 2.1. The self-interaction of fields depends upon the sign of $R^{(3)}$.

I observed that the gravitational field variables (q_{ab}, P^{cd}) and matter fields are not on equal footing. The gravitational field variables are first quantized. However, the matter fields are second quantized. Any theory that treats gravity and matter fields on equal footing concerning the quantum level is dubious.

The reinterpretation geometrizes the quantum theory itself. The field Φ has the geometric quantum. There is no quantum corresponding to the pure gravitational field.

The creation and annihilation operators follow deformed algebra. β is a function over superspace and not a constant. Since the role of creation and annihilation operators change depending upon the sign of β, the coherent state loses its coherence in the transition $\beta > 0 \leftrightarrow \beta < 0$.

The theory dynamically resolves the big bang singularity and the black hole singularity. The Universe begins at finite minimum time ζ_{min}. The ADM interpretation is that the

Universe has a nonzero initial volume. The big bang resolution is distinct from the loop quantum cosmology [9], where there is a quantum big bounce. Initially, the 3-metric is exclusively time-dependent, and the nontrivial local geometries arise dynamically.

In the case of the Schwarzschild black hole, there exists an upper limit on the value of the 3-metric. The upper limit depends on the total energy of a given black hole. Since $\sqrt{f}$ is a dimensionless length of a black hole, the area of the black hole is quantized.

7. Conclusions

The field $\Phi(\zeta, \zeta^A)$ describes the geometric matter. It satisfies the ADM constraints for *pure gravity*. It also shows the properties that are close to the standard matter fields. Unlike pure gravity, the field has a nontrivial stress-energy tensor, and unlike matter fields that live in a spacetime, Φ itself is geometric. The field has a consistent asymptotically flat limit. Its single-geometric interpretation recovers the ADM theory.

The single geometric interpretation $\Phi \sim e^{\pm i P^{ab} q_{ab}}$ indicates that the higher-order couplings correspond to the matter fields. In the Higgs field as well, ϕ^4-coupling gives mass. In the Klein–Gordon theory, the negative quadratic coupling constant is a mathematical possibility only. The geometric field, on the contrary, has purely geometric intrinsic quadratic coupling. The h-coupling is not necessarily geometric. It is responsible for interpretation as geometric matter. This interpretation does not contradict either standard field theory or gravity.

The quantum theory does not give the quantum of gravity. Instead, it gives the geometric quantum corresponding to a particular field. The nonlinearity of the theory makes the creation and annihilation operators vector-valued in the spatial part of the supermetric. There exists a domain, where the role of creation operator changes depending on the signature of (24). In such case, the coherent state loses its coherence.

The theory resolves classical singularities dynamically by modifying the 3-metric near a singularity. In the case of the Schwarzschild geometry, an object falling inside cannot reach the center. It can approach only f_{max}. In the case of FLRW geometry, the Universe began at $\zeta_0 \neq 0$ time. Even if we start with 3-metric $q_{ab}(t)$, quantum dynamics inevitably introduces $q_{ab}(t, r)$.

The free-field quanta are isolated geometries. There is no measure of the distance between noninteracting geometric quanta. In reality, fields interact with each other and give a sense of closeness. The geometric coupling λ has dimensions of μ^2-coupling. Closer geometric quanta have stronger geometric coupling between them. h-coupling has different units, and therefore, it has a different interpretation.

During the Planck epoch of the very early Universe, the energy of the geometric quanta is of the order of the Planck energy. Therefore, during this time, PESBHs are created. Beyond the Planck domain, PESBHs cannot form. These PESBHs undergo mergers and Hawking evaporation. The observation of primordial black holes would mean the existence of PESBH. In this way, the theory expects primordial black holes.

The resolution of classical singularities are quantum gravity effects, and situations such as the existence of gravitational decoherence show that the gravitational principles affect quantum theory. In this sense, quantum theory and gravity are both modified.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The author declares no conflict of interest.

References

1. McGuigan, M. Third quantization and wheeler-dewitt equation. *Phys. Rev. D* **1988**, *38*, 1988. [CrossRef] [PubMed]
2. Kucher, K.V. Time and interpretations of quantum gravity. *Int. J. Mod. Phys. D* **2011**, *20* (Suppl. S1), 3–86. [CrossRef]
3. Kiefer, C. *Quantum Gravity*, 3rd ed.; Oxford Press: Oxford, UK, 2012.
4. Penrose, R. On the gravitization of quantum mechanics 1: Quantum state reduction. *Found. Phys.* **2014**, *44*, 557–575. [CrossRef]
5. Clifford, W.K. On the space theory of matter. In *General Theory of Relativity*; Pergamon Press Ltd.: Oxford, UK, 1973. [CrossRef]
6. Odenwald, Z.S.F. A modern look at the origin of the universe. *J. Sci. Relig.* **1990**, *25*, 25–45. [CrossRef]
7. DeWitt, B.S. Quantum theory of gravity. I. The canonical theory. *Phys. Rev. (Series I)* **1967**, *160*, 1113. [CrossRef]
8. Giulini, D. What is the geometry of superspace? *Gen. Relativ. Gravit.* **1995**, *51*, 5630. [CrossRef] [PubMed]
9. Banerjee, K. Introduction to loop quantum cosmology. *Symmetry Integr. Geom. Methods Appl.* **2012**, *8*, 73. [CrossRef]