



Localization of the Elko Field on the Bloch Branes [†]

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[†] Presented at the 3rd International Online Conference on Universe, 4–6 March 2026; Available online: <https://sciforum.net/event/IOCU2026>.

Abstract

In this work, we investigate the localization properties of five-dimensional Elko spinor fields on the Bloch brane, a thick brane generated by two interacting scalar fields and characterized by a rich internal structure. We begin by studying the dynamics of an Elko field in the five-dimensional Bloch brane background and derive the corresponding Schrödinger-like equation governing the Kaluza–Klein modes. Our analysis shows that, due to the specific geometric features and the non-trivial warp factor of the Bloch brane, the zero mode of a free Elko field is not normalizable and therefore cannot be localized on the brane. To resolve this issue, we introduce a non-minimal coupling term into the five-dimensional Elko spinor field action. By considering a different form of coupling between the Elko field and the background scalar fields, we demonstrate that the localization of the Elko zero mode can be achieved. We identify the explicit conditions under which a normalizable and physically acceptable zero mode can be trapped on the brane. Our results reveal that the brane's internal layers play a crucial role in shaping the effective potential felt by the Elko field, thus affecting both the localization behavior and the profile of the zero mode.

Keywords: field theories in higher dimensions; localization of fields; zero modes

1. Introduction

In recent years, theories with extra dimensions have received considerable attention as they offer possible solutions to some long-standing problems of the Standard Model, such as the hierarchy problem and the cosmological constant problem [1–3]. Among these frameworks, braneworld models are particularly important because they describe our four-dimensional universe as a brane embedded in a higher-dimensional space–time. The well-known Randall and Sundrum (R-S) braneworld models with thin structures do not provide the smallest scale in a fundamental theory [4,5]. Therefore, realistic branes need to have thickness and inner structure. Thick braneworld models have been suggested as smooth generalizations of the R-S models to eliminate singularities that arise from thin branes [6,7]. The Bloch brane, introduced by Bazeia and Gomes [8], is a thick brane scenario generated by two real scalar fields with a specific potential that depends on a tunable parameter. This setup produces a stable warped geometry with an internal structure that allows the localization of gravity and other matter fields of the Standard Model of particle physics on the brane.

The localization of matter fields is a crucial issue in braneworld scenarios because it helps us to recover the observed four-dimensional physics. Only fields with normalizable



Academic Editor: Gerald B. Cleaver

Published: 15 July 2026

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zero modes can be interpreted as four-dimensional particles. New spin-1/2 quantum fields beyond the Standard Model have been recently proposed by the authors of Ref. [9] that are called Elko spinors with mass dimension one. They are neutral under charge conjugation and have been applied as possible candidates for dark matter particles [10]. Unlike the usual Dirac spinors, Elko fields possess different Lorentz transformation properties, which make their localization mechanism in warped geometries more complicated. Previous studies have shown that free Elko spinors cannot be localized on most thick branes. Liu et al. [11] investigated the localization of 5D Elko spinors on Minkowski branes and found that the zero mode exists only on thin Randall–Sundrum branes, while it fails to localize on thick branes. Similar difficulties were reported in tachyonic de Sitter braneworlds [12–14] and in $f(R)$ gravity models [15]. To solve this problem, different coupling mechanisms have been proposed. Jardim et al. [16] presented two consistent solutions that satisfy the correct boundary conditions: one with a delta-function coupling and another with a Yukawa-type coupling to the Ricci scalar. More recently, non-minimal couplings with the background scalar field have also been explored [17]. However, the localization of Elko spinors on the Bloch brane generated by two scalar fields has not yet been studied in detail.

In the present work, we first analyze the free Elko field on the Bloch brane and demonstrate that it cannot be localized. We then introduce a Yukawa-like coupling term in the five-dimensional action and derive the conditions under which a normalizable zero mode appears.

2. The Bloch Brane Model

In this section, we review a Bloch thick brane setup constructed by gravity and two bulk scalar fields, ϕ and χ , in a curved five-dimensional space–time. The action is given by

$$S_c = \int d^4x dy \sqrt{-g} \left[\frac{1}{4} R - \frac{1}{2} \partial_a \phi \partial^a \phi - \frac{1}{2} \partial_a \chi \partial^a \chi - V(\phi, \chi) \right], \quad (1)$$

where $g = \det(g_{ab})$ with $a, b = 0, 1, 2, 3, 5$ and the metric is

$$ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (2)$$

where $\eta_{\mu\nu}$ is the usual Minkowski metric with $\text{diag}(-, +, +, +, +)$ and $e^{2A(y)}$ represents the warp factor. The parameter y denotes the extra-dimension coordinate. We suppose that the scalar fields ϕ, χ and the warp factor $A(y)$ only depend on the extra coordinate y . In this case, the equations of motion arising from the action (1) with respect to the metric (2) are given by

$$\phi'' + 4A'\phi' = \frac{\partial V(\phi, \chi)}{\partial \phi}, \quad (3)$$

$$\chi'' + 4A'\chi' = \frac{\partial V(\phi, \chi)}{\partial \chi}, \quad (4)$$

$$A'' = -\frac{2}{3}(\phi'^2 + \chi'^2), \quad (5)$$

$$A'^2 = \frac{1}{6}(\phi'^2 + \chi'^2) - \frac{1}{3}V(\phi, \chi), \quad (6)$$

where the prime represents a derivative with respect to y . To solve Equations (3)–(6), we can consider an approach that includes the first-order formalism in the braneworld scenario

initially introduced in Ref. [18]. The method consists of a superpotential function $W(\phi, \chi)$ related to the scalar field potential as

$$V(\phi, \chi) = \frac{1}{8} \left[\left(\frac{\partial W}{\partial \phi} \right)^2 + \left(\frac{\partial W}{\partial \chi} \right)^2 \right] - \frac{1}{3} W^2, \quad (7)$$

leading to the following differential equations

$$\phi' = \frac{1}{2} \frac{\partial W}{\partial \phi}, \quad (8)$$

$$\chi' = \frac{1}{2} \frac{\partial W}{\partial \chi}. \quad (9)$$

$$A' = -\frac{1}{3} W. \quad (10)$$

In this paper, we consider the following form of the superpotential $W(\phi, \chi)$ given by Ref. [8]

$$W(\phi, \chi) = 2\phi - \frac{2}{3}\phi^3 - 2r\phi\chi^2. \quad (11)$$

The solutions of Equations (8)–(10) can be easily found by using the superpotential above as

$$\phi(y) = \tanh(2ry), \quad (12)$$

$$\chi(y) = \sqrt{\frac{1}{r} - 2} \operatorname{sech}(2ry), \quad (13)$$

$$A(y) = \frac{1}{9r} \left[(1 - 3r) \tanh^2(2ry) - 2 \ln \cosh(2ry) \right]. \quad (14)$$

We note that r is a real parameter that controls the thickness of the brane. In other words, the width of these solutions (12) and (13) is related to $\frac{1}{r}$, so it increases with a decreasing r . For the scalar field $\chi(y)$ in Equation (13) to be real-valued, we require $0 < r \leq \frac{1}{2}$. This range ensures the solutions remain physically meaningful and is consistent with the original Bloch brane literature. We also notice that the two-field solutions change to the one-field solution when $r \rightarrow \frac{1}{2}$. The warp factor (14) shapes the potential that governs the zero-mode wave function profile and a sufficiently rapid decay of the warp factor can trap massless or light modes near the brane.

3. Localization of Five-Dimensional Elko Spinor

Elko is a novel matter field useful for exploring cosmological issues like the horizon and dark energy problems. Researchers have investigated the localization of a 5D Elko spinor field's zero mode on Minkowski branes using a Yukawa-type coupling [11]. A recent study introduced a non-minimal coupling mechanism that also enables localization on these types of branes [17]. In this section, we focus on the localization of Elko spinors on the Bloch branes via a Yukawa-type coupling. We consider the action of the Elko spinor field λ coupled to gravity as

$$S = \int d^5x \sqrt{-g} \left[-\frac{1}{4} g^{MN} (D_M \lambda D^M \bar{\lambda} + D_M \bar{\lambda} D^M \lambda) - F(y) \lambda \bar{\lambda} \right], \quad (15)$$

where $F(y)$ denotes a general scalar function of the extra-dimension coordinate y . The covariant derivatives are $D_M \lambda = \partial_M \lambda + \Omega_M \lambda$ and $D_M \bar{\lambda} = \partial_M \bar{\lambda} - \bar{\lambda} \Omega_M$ where Ω_M repre-

sents the non-vanishing components of the spin connection. It can be useful to apply the following conformal metric

$$ds^2 = e^{2A(z)} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2), \quad (16)$$

where we have transformed the parameter y to z by $dz = e^{-A(y)} dy$. Regarding the action (15) with the metric (16), the equation of motion for the Elko spinor field can be expressed as

$$\frac{1}{\sqrt{-g}} D_M (\sqrt{-g} g^{MN} D_N \lambda) - 2F(z) \lambda = 0. \quad (17)$$

In order to solve the above equation, we use a KK decomposition of the five-dimensional Elko spinor field as $\lambda = \lambda_+ + \lambda_-$ with $\lambda_\pm = e^{-\frac{3A}{2}} \sum_n [\alpha_n(w) \zeta_\pm^n(x) + \alpha_n(w) \tau_\pm^n(x)]$ where $\zeta_\pm^n(x)$ and $\tau_\pm^n(x)$ are two linearly independent four-dimensional (4D) Elko spinor fields which satisfy four-dimensional Klein–Gordon equations. The KK modes $\alpha_n(w)$ satisfy the following equation of motion,

$$\ddot{\alpha}_n - [V_0(z) - m_n^2 + im_n \dot{A}] \alpha_n = 0, \quad (18)$$

where m_n is the mass of the Elko spinor field on the brane and the dot denotes a derivative with respect to the extra-dimensional coordinate y . The effective potential of Elko is given by

$$V_0(z) = \frac{3}{2} \ddot{A} + \frac{13}{4} \dot{A}^2 + 2e^{2A} F(z). \quad (19)$$

Furthermore, in order to obtain the effective action on the brane for the four-dimensional massless Elko spinor field, we need the following orthonormality condition for the KK modes $\alpha_n(z)$:

$$\int_{-\infty}^{+\infty} \alpha_n \alpha_m dz = \delta_{mn}. \quad (20)$$

Next, we consider the localization of the zero mode of a 5D Elko spinor, $\alpha_0(z)$ on the Bloch brane by setting $m_n = 0$ in Equation (18), leading to a Schrödinger-like equation $[-\partial_z^2 + V_0(z)] \alpha_0(z) = 0$. It is clear that the potential $V_0(z)$ depends on the warp factor exponent $A(z)$ and the scalar coupling function $F(z)$. In this work, we first investigate free Elko fields by considering $F(z) = 0$ and examine whether the effective potential is able to trap the zero mode or not. Then, we study massive Elko fields by using $F(z) \neq 0$ and obtain a localized zero mode of Elko on the Bloch brane.

3.1. Case I: $F(z) = 0$

Here, we first suppose a simple function $F(z) = 0$ in the effective potential relation (19) and then put $m_n = 0$ in the Equation (18) to obtain a 5D free massless Elko spinor field zero mode. In this case, the effective potential (19) can be rewritten as follows by using the solution (14)

$$V_0(y) = e^{\frac{2(1-3r)}{9} \tanh^2(2ry)} \text{sech}^{\frac{2}{9r}}(2ry) \left[\beta_1 \text{sech}^6(2ry) + \beta_2 \text{sech}^4(2ry) + \beta_3 \text{sech}^2(2ry) + \frac{76}{81} \right], \quad (21)$$

where

$$\beta_1 = -\frac{76}{81} (1 - 3r)^2, \quad (22)$$

$$\beta_2 = \frac{4}{27} (8r + 19) (1 - 3r), \quad (23)$$

$$\beta_3 = -\frac{4}{27} (27r + 19) (1 - 2r). \quad (24)$$

Information about the presence of a gap in the continuum spectrum of Kaluza–Klein (KK) massive modes can be achieved by studying the asymptotical behavior of the potential (21). In order to get a localized zero mode of an Elko field, the value of the potential at its minimum should be negative. Furthermore, the region between two maxima of the effective potential, i.e., in the area where our four-dimensional universe is localized, informs us about the massive modes of the Elko field. We are interested in studying the effect of the internal structure-related parameter, r , on the profile of effective potential and the zero mode as shown in Figure 1.

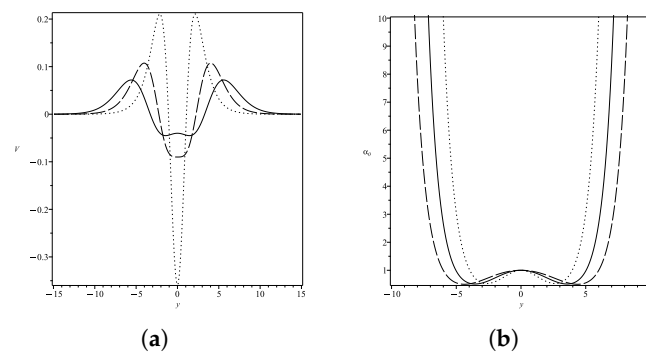


Figure 1. The profile of (a) the potential $V_0(z)$ and (b) the zero mode $\alpha_0(y)$ for $r = 0.1$ (solid line), $r = 0.15$ (dashed line) and $r = 0.3$ (dotted line).

It can be seen that the values of the effective potentials V_0 at infinity in the extra dimension tend to vanish, $V(y \rightarrow \pm\infty) \rightarrow 0$, and the shape of the potential is like a volcano. This means that the mass spectrum of the Elko field is continuous and there is no mass gap between the zero and massive modes. Also, the depth of the well increases with r while the distance between maxima decreases. Although the effective potential always has a negative minimum, i.e., $-2r^2$, as Figure 1b shows the zero mode of the Elko field cannot be trapped on the Bloch brane and diverges in the region around the center of the brane. The dominant behavior of the warp factor and potential leads to a solution that grows rather than decays, preventing normalizability. This can be seen by analyzing the asymptotic form or by direct numerical integration of the zero-mode equation, which confirms the divergence as shown in Figure 1b.

3.2. Case I: $F(z) \neq 0$

In this section, we show that the Elko field can be localized on the Bloch brane if we consider a non-zero Yukawa-like coupling as

$$F(y) = d_1 \text{sech}^6(2ry) + d_2 \text{sech}^4(2ry) + d_3 \text{sech}^2(2ry) + d_4, \quad (25)$$

where

$$d_1 = -\frac{2}{81}(1-3r)(18r\zeta + 57r - 19), \quad (26)$$

$$d_2 = -\frac{2}{27}(8r + 19 + 16r\zeta + 6r^2\zeta^2)(1-3r), \quad (27)$$

$$d_3 = \frac{10}{3}r^2\zeta + 2r^2\zeta^2 - 4r^2 - \frac{22}{27}r + \frac{38}{27}, \quad (28)$$

$$d_4 = -\frac{38}{81} - 2r^2\zeta, \quad (29)$$

where ζ is a real constant parameter. We note that the coupling function $F(y)$ in Equation (25) is engineered by substituting the assumed zero-mode profile $\alpha_0(y) = \text{sech}^\zeta(2ry)$ into

the Schrodinger-like equation for $m_n = 0$ and solving for the required $F(y)$ so that it exactly produces this normalizable zero-mode solution while preserving the desired asymptotic behavior of the effective potential. The explicit coefficients $d_1 - d_4$ are obtained by matching powers of $\text{sech}(2ry)$ after this substitution. The profile of the effective potential $V_0(y)$ is plotted in Figure 2 by substituting the above relations in Equation (19).

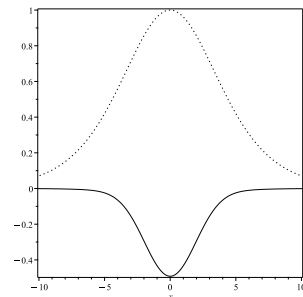


Figure 2. The profile of the effective potential (solid line) and the zero mode (dotted line) for $r = 0.1$ and $\xi = 2$.

It should be mentioned that using the zero-mode solution $\text{sech}^\xi(2ry)$ in condition (3) leads to

$$I = \int_{-\infty}^{+\infty} e^{\frac{(3r-1)}{9} \tanh^2(2ry)} \text{sech}^{2(\xi - \frac{1}{9r})}(2ry) dy. \quad (30)$$

It is well known that if the integrand in Equation (30) has a Gaussian profile, the normalization condition is satisfied, which requires $\xi > \frac{1}{9r}$. The behavior of the integrand in the above equation is shown in Figure 3. It is clear that for $\xi < \frac{1}{9r}$, the normalization condition is not satisfied.

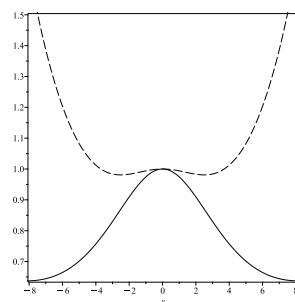


Figure 3. The profile of the integrand of I for $r = 0.1$, $\xi = 2$ (solid line) and $\xi = 1$ (dashed line).

4. Conclusions

In this paper, we have investigated the localization of five-dimensional Elko spinor fields on the Bloch brane. It was found that free (massless) Elko fields fail to localize, as the effective Schrödinger-like potential does not support a normalizable zero mode. By introducing a non-minimal Yukawa-like coupling to the background scalars, we modified the effective potential, creating a sufficiently deep attractive well that traps a normalizable zero mode under the condition $\xi > \frac{2}{9r}$ for the brane structure parameter r . From a broader perspective, this result suggests that Elko fields can be consistently trapped on thick branes and may therefore serve as viable dark matter candidates in higher-dimensional scenarios. The mechanism presented here may also be extended to other multi-scalar thick brane constructions, potentially leading to richer localization patterns and new features in the corresponding Kaluza–Klein spectrum.

Author Contributions: Conceptualization, M.M.S.; methodology, M.M.S.; formal analysis, M.M.S.; investigation, M.M.S.; writing—original draft preparation, M.M.S.; visualization, Z.G.; software, Z.G.;

writing—review and editing, M.M.S. and Z.G.; supervision, M.M.S. All authors have read and agreed to the published version of the manuscript.

Funding: The research of the corresponding author was in part supported by a grant from Kosar University of Bojnord (No. 0409241320).

Institutional Review Board Statement: Not applicable. This study does not involve human participants or animals.

Informed Consent Statement: Not applicable.

Data Availability Statement: No new data were created in this study. The results are based on analytical calculations presented within the article.

Conflicts of Interest: The authors declare no conflicts of interest.

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