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Contextuality in Neurobehavioural and Collective Intelligence Systems

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Abstract: Contextuality is often described as a unique feature of the quantum realm, which distinguishes it fundamentally from the classical realm. This is not strictly true, and stems from decades of the misapplication of Kolmogorov probability. Contextuality appears in Kolmogorov theory (observed in the inability to form joint distributions) and in non-Kolmogorov theory (observed in the violation of inequalities of correlations). Both forms of contextuality have been observed in psychological experiments, although the first form has been known for decades but mostly ignored. The complex dynamics of neural systems (neurobehavioural regulatory systems) and of collective intelligence systems (social insect colonies) are described. These systems are contextual in the first sense and possibly in the second as well. Process algebra, based on the Process Theory of Whitehead, describes systems that are generated, transient, open, interactive, and primarily information-driven, and seems ideally suited to modeling these systems. It is argued that these dynamical characteristics give rise to contextuality and non-Kolmogorov probability in spite of these being entirely classical systems.

Keywords: process; process algebra; contextuality; neurodynamics; collective intelligence; social insects; information dynamics; salience

1. Introduction

Human beings tend to think in terms of dichotomous categories—up/down, black/white/, good/evil, conscious/unconscious. Physics is no exception—linear/non-linear, discrete/continuous, classical/quantum, general relativity/quantum mechanics. Such dichotomies are frequently discussed in absolute terms as if they are statements about the nature of reality, the way things are, as opposed to how our minds perceive things to be. It took many centuries before the two-valued logic of Aristotle was supplemented by many-valued, modal, and fuzzy logics. Only in the past century has the objective Newtonian world view been supplemented by the contextual world view of quantum mechanics. Psychologists and neuroscientists understand that our perception of reality is, in part, an internal mental construction derived from theories and expectations which are projected onto the real world and mostly (sometimes) adjusted when the fit is sufficiently poor [1]. Physicists, too, view the world through the lens of theory—currently, reality is perceived through the lens of the Hilbert space. Mermin [2], however, has warned against the dangers of reification, the tendency to see our models of reality *as* reality.

In the past, these extensions of theoretical frameworks expanded and enhanced our view of reality. These days, we are told that reality itself does not exist. This seems excessive [3]. The loss of the traditional view of the object, and of the possibility of non-invasive, non-disturbing, objective measurement, does not imply the end of reality [4]. Indeed, biologists, psychologists, physicians, and anthropologists, among many others, have dealt successfully for a long time with the impossibility of pure objectivity. Whitehead advocated for the introduction of subjectivity into science in his Process Theory [5], which he referred to as a philosophy of organism. Had more attention been paid to biology [6],

ideas of subjectivity and contextuality might have been commonplace, and the only surprise from quantum mechanics might have been that these features apply even to the simplest of physical phenomena and not merely to the most complex. Biology is the domain of complexity and emergence [7], but in physics these concepts still play little role, if any, outside of condensed matter physics [8] True, there have been a few attempts to apply ideas of emergence to fundamental particles, but they have not received widespread acceptance.

Quantization, non-Kolmogorov probabilities, and contextuality are all said to distinguish the quantum realm from the classical realm. Quantum mechanics is held to be the fundamental theory; therefore, the question becomes why the classical world is best described as continuous, Kolmogorov, and non-contextual. A number of esoteric explanations have been proposed over the years to explain this discrepancy. Perhaps, however, a simpler explanation is at hand. Perhaps the problem lies not with quantum mechanics, but instead resides in our characterization of the classical realm. As with most other dichotomies, perhaps this one is false as well. Perhaps, at times, the classical realm can also be discrete, non-Kolmogorov, and contextual. Contextuality at the classical level was recognized even by Kolmogorov [9] when he formulated the axioms of probability theory. He stated explicitly that all probability measures needed to be linked to specific contexts, and the ability to combine such measures across distinct contexts was not a given; rather, there are precise mathematical conditions that must be met in order for that to be possible [10]. Unfortunately, this seems to have been forgotten during a century of usage. Dzhafarov [11] has described this as "contextuality by default" and has written extensively about this in the psychological literature.

Quantum mechanics has taken the notion of contextuality one step further through the introduction of the Bell inequality and its myriad variants [12–17]. These inequalities are formed from sets of correlations and are satisfied whenever the correlations are generated by systems following the laws of Kolmogorov probability. Quantum mechanics, being non-Kolmogorov, can violate these inequalities, at least some of the time. Much effort has gone to understanding the conditions under which such inequalities can be violated, the limits imposed upon quantum mechanics, and the possibility of supra-quantum correlations.

The conflation of classicality and Kolmogorov probability has led to endless speculation about non-realism, non-locality, multiverses, observer-dependent reality, and so on. However, Dzhafarov and colleagues have demonstrated that a condition on probabilities equivalent to these inequalities is violated in decision experiments involving groups [18] and individuals [19]. Aerts and colleagues have also demonstrated CHSH values in human cognition which appear to surpass the Cirel'son bound [20]. As Harris [21] points out, contextual effects are widespread; they have simply been either seen and ignored, or not even seen. Humans see what they believe, and if researchers do not believe in contextuality at the classical level, then they will not see it, but that does not mean that it does not exist. Paradoxically, many of our models of complex systems are unduly simplistic. Take, for example, physical models of neural networks, which describe them as akin to spin glasses where neurons simply fire (1) or not (0), and thus superficially resemble a binary computer [22]. Such models fit nicely into existing world views (two valued logic, binary computers, spin glasses) but they are, at best, cartoon versions of living neurons and quite misleading as to their dynamics. Biologically inspired models have been much more successful [23]. The same is true for physical models of collective intelligence systems [24], although some nuanced biologically relevant models have been remarkably successful [25–27]. The emerging field of quantum cognition offers new ways of understanding neural systems through the application of quantum formalisms (not quantum dynamics per se) to the description of cognition [28–30].

In the next two sections, there is first a concise review of our current knowledge about the dynamics of biological neural systems (in particular, neurobehavioural regulatory systems), and second, a review of biological collective intelligence systems (in particular, social insect colonies). The next two sections examine evidence for contextuality (at least contextuality by default) in these systems, and some of the dynamical aspects which might

underlie such contextuality. The first of these sections is more philosophical; the second presents a toy model to illustrate one form of contextuality. The Appendices A and B presents some details of the process algebra which is presented as a framework for describing and analyzing such systems. Process algebra is generative, contextual, transient, non-Kolmogorov, and interactive. Process algebra has been shown to reproduce the structure of non-relativistic quantum mechanics. Process algebra provides a high-level, formal language which addresses those features of these systems which potentially could give rise to contextuality, namely, their being processes, i.e., generated, transient, open, and dominated by interaction and information.

2. Neural and Collective Intelligence Systems

2.1. Neural Systems—Neurobehavioural Regulatory Systems

Prior to discussing the role of contextuality in neurodynamics, it is first necessary to describe what is currently known about the dynamics of collectives of neurons. Over the centuries, there have been different metaphors used to understand how the brain gives rise to objective behaviour and subjective experience. Nowadays, the brain is conceptualized as a biological computer [31], as a neural network [32], or as a spin glass [22]. Each conception takes one feature of neuronal architecture and elevates it to a central status: the computer metaphor interprets neuronal action potentials as binary values; neural networks focus on the mostly large-scale connectivity; and spin glass models interpret action potentials as spin states. The loss of detail involved in formulating each of these models is staggering, and the result in each case provides a caricature of neurodynamics at best.

In spite of more than a century of intense research, our knowledge of neurodynamics still barely scratches the surface. Nevertheless, it is still vast, and this brief overview cannot do it justice. It serves to catalogue the various players and their myriad contributions to the collective dynamics of neurons. This information will be important for understanding how contextuality arises in the dynamics of such systems.

It is generally agreed that the nervous system plays a central role in the determination of psychological phenomena, whether objective as in motor behaviour, or subjective as in thoughts, emotions, memories, perceptions, and so on [33]. It is further agreed that that the nervous system functions as a biological information processing system [34], although analogies with digital computers are far from the truth.

The main components of the nervous system are neurons, glial cells, and blood vessels. Neurons were long thought to be the principal information-processing component of the nervous system. Neurons possess a remarkably varied cell body which can be further subdivided into axonal and dendritic components. The axonal component usually consists of a single long process which terminates in an arborization, akin to the branches of a tree. At the ends of this arborization are specialized regions called axonal terminals, which provide the so-called pre-synaptic component of specialized structures called synapses which link this (upstream) neuron to other (downstream) neurons. The dendritic component also ends in arborizations which show remarkably diversity. At the ends of these arborizations are the terminal dendrites or dendritic spines, which possess a wide variety of structures called receptors which, in turn, form the post-synaptic component of synapses.

The cellular membrane of a neuron is electrically active, and the axons of many neurons are covered in a myelin sheath whose purpose is to facilitate the propagation of an electrical "action" potential along the length of the axon from a specialized region called the trigger zone to the axon terminals. The arrival of an action potential at the axon terminal often facilitates the release of neurotransmitters from vesicles located in the presynaptic structure. It is important to note that the release of neurotransmitters is stochastic and follows a Poisson distribution [35]. Once released, the neurotransmitter diffuses through an extracellular region, the synaptic cleft, before bonding with one of the post synaptic receptors. That results in a local alteration in the electrical properties of the membrane of the downstream neuron. If a large enough depolarization of the membrane occurs at the axon trigger zone which exceeds a specific threshold value, an action potential

is initiated. Once initiated, the action potential is an all-or-nothing phenomenon, having a fixed amplitude, duration, and form. Action potentials vary only in their frequency. There is, however, another means through which neurons can influence each other. Dendritic spines can extend back from the downstream to the upstream neuron, and thus directly assert an influence over the upstream neuron.

Cajal postulated two broad principles informing neural structure: (1) the principle of dynamic polarization, which states that the electrical activity of the neuron flows from dendrites to the trigger zone of the axon; and (2) the principle of connectional specificity, which states that neurons do not connect purely at random but rather only to specific populations of neurons.

Once formed, large-scale connectivity, which reflects how neurons interact with one-another through synaptic connections, is largely fixed. However, this belies the fact that local connectivity—the numbers of axon terminals and vesicles, the numbers and local geometries of dendritic spines and the numbers and types of receptors, are all dynamic. Every night, serotonin activity in the brain ceases, resulting in a contraction of dendritic spines which re-form again in differing geometries each morning [36]. Little attention in modeling is paid to the fact that these neuronal networks are profoundly dynamic.

Neurons are classified as afferent (meaning towards the central nervous system, usually from sensory receptors), efferent (meaning away from the central nervous system, usually to muscle receptors but many other organs as well), or as interneurons (meaning between neurons). They are classified as unipolar, bipolar, or multipolar, depending upon the number of processes that emerge from the cell body. They are classified morphologically by the shape and density of the dendritic arborization, and by the number of axons.

Synapses also come in several forms [33]. Gap junction synapses are electrical and provide direct bi-directional interactions between neurons. Chemical synapses involve a chemical intermediary between the upstream and downstream neurons which are separated by a gap junction. Chemical synapses come in two major forms. Ligand-gated receptors produce excitatory or inhibitory effects upon the local neuronal membrane when stimulated by an appropriate neurotransmitter. G-protein-coupled receptors, on the other hand, serve a modulatory role, altering the response of a neuron to ligand-gated receptor activation by modulating various membrane parameters such as resistance, length and time constants, the duration of action potentials, and so on. Ligand-gated receptors contribute to the production of an action potential, whereas G-protein-coupled receptors change the dynamics of a neuron. A third type of receptor, tyrosine kinase receptors, plays a role in gene transcription and metabolic processes, which play a role in disease induction as well as memory.

A fourth form of interaction which has been little recognized is called ephaptic (or ephatic) transmission. This has been described in the context of certain disease states such as those causing demyelinization. It occurs when two axons are sufficiently close together so that electrical activity in one axon influences the electrical activity in the other. This is a form of neuronal transmission that is extremely difficult to study, but recent theoretical work suggests that it might play a greater role in the nervous system than previously recognized, at least in the dynamics of unmyelinated fibres [37].

Finally, a fifth form of interaction has been recognized. This is volume transmission, originally associated with somatic influences on neural dynamics such as those arising from hormones, neuropeptides, cytokines, and interleukins, among many others. In recent years, however, it has been realized that neurons may also release large quantities of neurotransmitter directly into the extracellular space rather than delivering it to vesicles that participate in synaptic interactions. Volume transmission is thought to underlie phenomena such as mood, attention, arousal [38–40], and to regulate the dynamics of mesoscopic level cellular assemblies which operate over large spatio-temporal scales [41]. Somatic influences effected through the release of hormones, neuropeptides, and immune factors all influence neurodynamics through volume transmission. Even gut bacteria

influence neurodynamics through their production of neurotransmitter precursors as well as neurotransmitters themselves [42]. These, in turn, are all influenced by the environment.

Receptors frequently come in many different subtypes, and each subtype may result in a different effect on synaptic function. For example, there are at least 14 different receptor subtypes for serotonin, 5 for dopamine, 9 for norepinephrine, and so on. Some of these are ligand-coupled, but most are G-protein-coupled. They may be either excitatory or inhibitory. It is not possible to ascribe a specific neurophysiological action to any given neurotransmitter. It depends upon the receptor subtype and the target neuronal system. It has been conjectured that some large-scale features of behaviour, such as temperament traits, may be due to ensemble interactions among neurobehavioural regulatory systems [43,44] and reflect various dynamical phases among these systems [45–47]. Moreover, many neuronal systems act through volume transmission, the serotonin, norepinephrine, dopamine, acetylcholine, and glutamate systems being prominent examples [41]. These authors point out that commonly used experimental methods for studying neurophysiology at the cellular level, such as in vitro slices, destroy large-scale connectivity and long-range interactions, because these involve small numbers of cells which are isolated from one-another through restrictions of both vascular and extracellular spaces.

The connectivity among neurons is equally vast. Each neuron receives synaptic contributions from anywhere between 1000 and 1,000,000 presynaptic neurons. Every neuron serves as a presynaptic neuron for anywhere between 1000 and 10,000 postsynaptic neurons. The combinatorics is mind boggling. Furthermore, virtually every region of the brain receives inputs from the neurobehavioral regulatory systems (serotonin, dopamine, acetylcholine) and project back onto these systems, whereas among cortical neurons there are widespread connections between and within the cerebral hemispheres. Moreover, some neural systems play as important a role when inactive as when they are active. The serotonin system, for example, plays an important role in maintaining the physiological integrity of individual neurons and becomes inactive during sleep to allow change and repair processes to act [36]. During this time, dendritic bulbs can retract into the neuronal membrane, and new dendritic bulbs can form in the awake state, so that the ultrafine scale network structure is also variable. Activity from these neuroregulatory systems is closely regulated through multiple feedback processes, and departures from the "middle way" cause harm.

The brain comprises approximately 30 billion neurons and some 70 million supporting cells, or glia. Long thought to be merely a connective tissue scaffold for neurons, it is now apparent that glial cells play a major role in not only the maintenance of neurons, but in their processing of information, particularly learning and memory [34]. Glial cells respond to a wide variety of hormones and peptides and play a central role in the regulation of glutamate dynamics. It is not possible to fully understand the dynamics of neurons without taking into account the dynamics of the glial system, the body as a whole, and ultimately the environment which that body inhabits.

Physicists are fond of denying the existence of time, although neural systems treat time as fundamental and derive much of their functionality from the different time scales associated with different processes. For example, gap junction synapses are virtually instantaneous, ligand-gated synapses act over 0.3–5 milliseconds, G-protein-coupled synapses act over hundreds of milliseconds to minutes (as may volume transmission), whereas tyrosine kinase receptors have effects that can last up to weeks. These time scales all support behaviour at the organism level ranging from milliseconds to hours. Freeman [48,49] carried out extensive experiments, particularly involving the olfactory system, which demonstrated an important role for stochastic and chaotic processes, and for dynamical phase transitions between transient chaotic attractors [23]. Gerstein and Mandelbrot [35], and Shadlen and Newsome [50] have all provided striking evidence for stochasticity at all levels of the nervous system.

At the behavioural level, it has long been known that behaviour is not reproduced as a fixed pattern elicited by some stimulus; instead, behaviour is generated anew, "on the

fly", each and every time it is elicited. Different neurons, different muscle fibres, different sequencing, and different timing all distinguish one instance of a specific behaviour from another, even if superficially these behaviours all appear to be the same or express the same functionality [43,44,51]. Moreover, it is now apparent that internal "representations" of the external world such as internal maps and memories are using different neurons at different times, even though functionality at the macroscopic level appears to be preserved [52–54]. Neurons appear to be fungible. They can be recruited to carry out different tasks, to make different contributions at different times in different environmental contexts. There is no fixed relationship between individual neurons and individual acts of behaviour, any more than there is a fixed relationship between neurotransmitter or hormone and specific behaviours. There are correlations at the macro, but not at the micro level.

Even this simple overview should make it abundantly apparent that simplistic models such as feed-forward neural networks, spin glass neural network models, coupled oscillators, and so on, fail to come close to capturing the true complexity of neural dynamics.

2.2. Collective Intelligence Systems—Social Insects

Collective intelligence is often used as a euphemism for any form of collective behaviour such as that exhibited by crowds, mobs, swarms, or schools. It may also refer to various forms of multi-agent programming in computer science and engineering. Here, however, collective intelligence strictly refers to the appearance of ecologically salient, intelligent behaviour by a collective of individual agents in the absence of any central authority [55–57]. Collective intelligence is an emergent phenomenon. The collective behaves in ways that are not possible for the individual agents which form the collective. Its dynamics are not a simple scaling-up or extrapolation from the dynamics of the agents. The quintessential example of a collective intelligence is a social insect colony, particularly an ant colony, and that will serve as the focus of this section.

Social insects have been present on Earth for between 100 and 200 million years. They are highly successful species from an evolutionary perspective. Some species, such as termites, have constructed vast interconnected cities [58]. Ants appears everywhere except for the coldest climates.

Ants live in colonies with populations ranging from a few hundred to several million workers. The members of a colony are exclusively female—male workers (drones) appear only when the colony is about to reproduce. Each colony has a single queen whose task is to deposit the eggs that become the workers. The queen is not a central authority. Her entire life is spent within the nest laying eggs. The workers tend to the egg, larvae, and pupae, construct and maintain the nest, forage for food, and defend the nest, all without any central direction. The queen determines when the colony should reproduce, and at that time will produce one or more queen eggs and those for male drones.

The members of an ant colony are distinguished morphologically, forming distinct castes. In some species, morphology (caste) determines role [59]. In others, workers may perform different roles, although each is performed exclusively for an extended period of time. The main roles are queen, drone, midden worker, forager or scout, and soldier. In some species, transitions between roles are reversible, but in others, they are irreversible [60,61].

The brain of a typical worker consists of about one million neurons. The body of a worker possesses a wide variety of receptors for different kinds of sensory information. The antennae receive olfactory signals from other workers and their environment. These signals include trail-marking pheromones, alarm signals, affiliation markers (which distinguish members of the colony from others), food, and so on. They possess receptors for vibration and for touch. They possess glands which exude affiliation markers, trail-marking pheromones, and various signals such as alarm signals [60,61]. Larvae and pupae also secrete different signals at different stages of their development to inform workers as to their needs [61]. These different chemicals are definite signals, not merely generic metabolic products such as nutrients. Responses to alarm pheromones, for example, may be consis-

tent within a species, but vary among species. In response to an alarm pheromone, the workers of *Acanthomyops claviger* become aroused and converge, whereas workers of *Lasius alienus* scatter widely [60]. This is important, because it means that these various chemical agents have informational specificity which is linked to each individual species. There is no one-size-fits-all property such as energy or entropy which determines their interpretation. They are indeed signals, and they require a specific agent for their interpretation.

The body of a typical worker is asymmetric in the rostral–caudal direction. Roughly, the sensory apparatus is situated rostrally, whereas the secretory apparatus is situated caudally. Workers interact with one-another in a wide variety of ways. They may send chemical signals to each another, some of which are deposited on sensory receptors by direct touch (akin to synaptic transmission), whereas others are secreted into the environment and arrive through either diffusion (volume transmission) or direct contact with an object in the environment (akin to gap junctions). They may also interact through direct body contact, which often involves rubbing (akin to ephaptic transmission), and which stimulates different forms of behaviour such as tandem running, carrying (of themselves, pupae, corpses, or food pellets), defense, foraging, or changes in task allocation. These interactions are complex and depend not merely on the presence of contact, but on the orientation of the workers, their internal states, their current tasks, and the current state of the colony [59–61].

Colonies live in nests, and these come in a myriad of forms. Some colonies live underground, some live on the surface, and some live in trees [59]. Nests may be formed from materials in the environment such as topsoil, forming underground tunnels or surface mounds. The mounds of termites are spectacular. Some nests are formed from the bodies of the workers themselves. Some colonies form more or less permanent nests, whereas others are temporary, with emigration necessitated by environmental damage or foraging needs. The army ant, *Eciton burchelli*, has evolved a remarkable cyclical pattern in which a colony alternates between a statory phase, during which time the nest has a fixed location and each day the workers carry out radially directed raids into the neighbouring environment, with the angle between daily raids offset by 123 degrees, so that they sweep the area, and a [58] nomadic phase, in which the entire colony packs up and moves each day until a new statory site is reached.

The behaviour of a colony depends upon the number of workers contributing to it, an aspect termed mass action. Consider the army ant, *Lasius alienus*, for example: the foraging pattern of a single worker appears to be completely random, although in some species the return to the nest is remarkably direct [60]. As the number of workers increases, the colony may organize into milling behaviour, which, if vigorous, has been described as an ant tornado. With further increases in numbers, one may see the formation of organized columns of ants, which gives rise to the army descriptor, and which appear during migrations. Foraging is also highly organized into a shape somewhat resembling a mushroom, with a single column emanating from the nest which subsequently subdivides into numerous branches linked by transverse paths, thus forming a head. These foraging raids maintain their orientation to within a few degrees [58].

The cliff-dwelling ant, *Temnothorax albipennis*, is highly adapted to carrying out nest emigrations due to the high frequency with which its nest loses its structural integrity. The manner in which such colonies select sites for nest construction has been extensively studied by Nigel Franks and colleagues [62,63]. They have been able to demonstrate that colonies are able to distinguish between sites based on a number of characteristics such as entrance width, room height, and amount of light. Given a choice of sites with varying characteristics, a colony appears capable of making a choice using a weighted additive decision strategy, one which is difficult for humans to utilize [62]. Individual workers are unable to do so themselves. Pratt and colleagues [64,65] have shown that under some circumstances, colonies exhibit features of rational decision making which individual workers do not. In particular, they showed that colonies of *Temnothorax rugatulus* could avoid the decoy effect in making decisions even though individual workers appeared to fall prey to it. Prior experience, and thus perhaps a form of learning, has been shown

to influence decision making, sometimes leading to non-rational decisions at the colony level [66].

Colonies appear to adapt their decision-making strategy depending upon various parameters such as the ecological salience of the decision, the functional status of the colony, and environmental factors such as time, accuracy, resource use, and distance to target. For example, decision making changes depending upon the relative distances between targets [67]. Strategies change when there are trade-offs between different parameters. Different levels of ecological salience (food search under starvation versus plenty, mate selection, and nest selection) result in different decision-making strategies being employed, from weighted additive down to just "good enough" [62]. Thus, the strategies which are utilized by a collective intelligence should not be thought of as stored, represented, or embodied by the collective intelligence, but instead should be viewed as emergent phenomena, generated "on the fly" by the agents of the collective in interactions among themselves and with the environment in which they are embedded. This environmental dependency has been termed nonrepresentational contextual dependence [55].

At the colony level of decision making, it would appear as though the colony is able to compare different nest sites and choose based upon the differences in site characteristics using one of several different decision-making strategies. Robinson, Feinerman, and Franks [67] showed that this did not appear to be due to the individual ants comparing the different sites, but rather to the individual ants obeying a form of threshold rule. Individual ants make a judgment based upon whether or not the option exceeds some threshold, and the colony as a whole then adopts an option when the numbers of individual workers choosing that option surpasses a so-called quorum threshold. A low quorum threshold means that fewer scouts suffice; thus, more individualistic judgments dominate. A high quorum threshold requires more search and more consensus [63,68,69]. The individual thresholds appear to be highly heterogenous, which provides the colony with a decision-making advantage under conditions in which the individual choices are suboptimal [70]. These colonies are capable of using a wide variety of strategies in response to different environmental conditions [71–74].

This is the case for nest emigration in *Temnothorax albipennis*. In nest emigration, individual scouts explore the local environment. If a site exceeding their individual threshold is found, they return to the nest, laying a trail pheromone along the return trail. When they encounter other workers, they will touch the worker in a specific manner in order to elicit a response called tandem running, in which they both return to the chosen site. The new worker, assuming its threshold is exceeded, then returns to the nest as does the original worker and the process is repeated, each time resulting in more trail pheromone and more recruitment. If a quorum threshold is exceeded, then the colony as a whole shifts its activity, with movement shifting towards the new site and the initiation of carrying behaviour—of eggs, larvae, pupae, queen, other workers, and so on. It is important to note that this whole process unfolds only in a statistical sense. At any given time, workers are going back and forth between the nest and other sites. Workers may carry nest occupants to the new site, but they can just as easily carry them back to the original nest. This is very apparent in the migrations of *Eciton burchelli* colonies. In any given emigration, the workers that participate in the exploration, the workers that initiate the emigration, and the workers that participate in the emigration, are all chosen at random, and will be different each and every time. Each emigration is generated "on the fly", with new workers, new distributions of tasks, and new timings, although viewed at the colony levels, there are fairly consistent patterns. Rayleigh–Bénard convection cells provide a physical example where there appears to be stable structure at the macroscopic level with chaos ensuing at the microscopic level.

Stigmergy is another important feature of collective intelligence dynamics. It is defined as "the inducement to work by the products of work". It is most apparent in nest construction by termites and wasps. The nests of such social insect colonies are not entirely random, but possesses symmetries, structure, and form. Construction of these structures

occurs in the absence of an architect. There is no blueprint within the minds of individual workers, nor is there a foreman directing workers to different locations to perform different tasks. Again, these structures are generated "on the fly". The selection of a particular worker to carry out a particular task is entirely random. Individual workers are fungible. Each time a nest is constructed, it is constructed anew, with different workers, and the final structure will be different from any previous structure. Nevertheless, there will be common features and similarities in the form of the nests, and these features tend to be distinctive for each different species. Individual workers are induced to carry out particular tasks depending upon the state and form of the structure produced up to that point. As the workers act, the structure becomes modified in particular ways, and these modifications elicit further distinctive behaviours from the workers. Over time, the structure forms and transforms, until eventually a stable final structure is created. At that point, workers shift their activity to nest maintenance and repair.

2.3. Commonalities between Neural and Collective Intelligence Systems

By now, it should be apparent that neurobehavioural regulatory systems and collective intelligence systems possess a level of complexity in their dynamics which far exceeds that of simple Newtonian objects, or indeed that of nearly any physical model currently under investigation. It was suggested in 2009 [56] that nonrepresentational contextual dependence was a defining characteristic of collective intelligence systems. The question is whether contextuality, as manifested by collective intelligence and neural systems, has any relationship with contextuality as understood within the physical literature.

The neurobehavioural regulatory systems and social insect colonies were presented as examples precisely because they possess a remarkable number of dynamical features in common. First of all, in both cases, these systems consist of a collective of agents, neurons and glia in the case of the neurobehavioural regulatory systems, and workers in the case of social insect colonies. Their behaviour of interest is global in nature, which pertains to the system as a whole, and cannot be reduced to the actions of any single individual member of the collective. In both cases, there is no central authority to which one can refer to explain the dynamics of the collective. There is no CEO, no organizer, no planner. There are no codified rules, no scripts, blueprints, or instructions which shape the behaviour of the individual agents. The agents, whether neurons or ants, which participate in any collective behaviour are fungible—they are not necessarily the same in each instance; indeed, they may not even remain the same throughout the duration of the collective behaviour. These collective behaviours are not stored in some fashion within the structure of the system. Instead, they are generated "on the fly", de novo, each and every time they are elicited by their environment. Whether retrieving a long-term memory, or selecting a site for a new nest, the participants and actions which give rise to the collective behaviour are determined in the moment. The collective behaviour is generated, not elicited.

These systems are profoundly open systems. They exchange structural and energetic resources with their environments. More importantly, they exchange information. The environment provides information which is necessary to elicit, constrain and shape the activities of these systems, and the systems, in turn, act upon their environments, imparting information and altering their environments so as to create complex feedback relationships that then further shape future behaviour. This openness is not simply a complication—it is an essential contributor to the generation of behaviour by these systems. As a consequence, this open nature renders the environment a determining context in the generation of behaviour. It is not possible to treat the environment merely as an initial or boundary condition. The environment, its resources, and its geography all influence the possible behaviours that these systems can exhibit. However, these systems also act upon their environments in myriad ways, thus shaping their own contexts, altering the boundary conditions, and thus the possibilities for future behaviour.

There are deeper parallels between neurobehavioural regulatory systems and collective intelligence systems. The environment of the neurobehavioural regulatory system

consists of the matrix of glial cells, blood vessels, volume distribution of hormones, neuropeptides, and neurotransmitters, together with signals from sensory systems and other brain regions. This environment can be modified through central neuronal action on the regulation of hormones, the release of neuropeptides, the volume distribution of neurotransmitters, receptor and membrane properties through G-protein-coupled receptors, large-scale movements of the body which alter the sensory inputs, and fine-scale alterations in dendritic architecture, receptor distribution and types through morphological and genetic activities. The environment for a collective intelligence consists of the physical environment—the distribution of resources needed by the colony to function, the presence of competitors and predators, the volume distribution of emitted pheromones, and the spatial distribution and orientation of individual workers which impacts whether they interact, and how.

Neurons interact with one-another through five main channels: ligand-gated synapses, G-protein-coupled synapses, gap junctions, ephaptic transmission, and volume transmission. The workers in collective intelligence systems such as ant colonies also interact through five main channels: ligand-gated receptors (olfaction), various alarm signals (akin to G-protein-coupled systems) which alter the dynamics of the worker, touch (akin to gap junctions), patterned behaviour (akin to ephaptic transmission), and the volume transmission of pheromones. Both neurobehavioural regulatory systems and collective intelligence systems use a mix of local and global means for transmitting important information. In neurobehavioural regulatory systems, local information is transmitted via gap junctions, axo-dendritic synapses, and ephaptic connections. Global information is passed through the volume transmission of hormones, neuropeptides, and immunoregulatory peptides. Local information is passed within collective intelligence systems through somatic odorants and various forms of touch. Global information is passed through pheromones and diffusible signals such as alarm signals.

Both neurobehavioural regulatory systems and collective intelligence systems make global decisions based upon some form of collective action. In the case of social insect colonies, a shift in behaviour at the colony level, for example, the commencement of emigration to a new nest site, does not take place until a quorum of workers in favour of one site is formed. Each worker makes its own choice for a new nest site, and these choices may differ over a range of sites. Having made its choice, each worker returns to the nest to recruit other workers to their chosen site. These subsequent workers are free to choose that site or explore the space and choose a different site. However, over time, an increasing number of workers will be attracted to these sites. Once a quorum for one site is achieved, there is a shift in the internal dynamics of the nest and emigration to that site commences. Freeman argued for a similar phenomenon in neural systems, which he termed mass action [48]. It has long been known that individual neurons release neurotransmitters stochastically, and that the timing of spikes of a single neuron under repeated presentation of the same stimulus is also stochastic. However, a large number of neurons responding to the same stimulus will produce a noisy approximation to a more or less stable wave form. Moreover, an individual neuron initiates an action potential only after a sufficient number of neurons stimulate the receptors of the neuron within a very short time frame; in other words, they arrive more or less simultaneously. A pre-synaptic neuron may vote for (excitation) or against (inhibition), but if the balance sufficiently favours acceptance, then the post-synaptic neuron produces an action potential.

In the case of both the collective intelligence and the neurobehavioural regulatory system, there is a threshold which must be achieved (quorum threshold for social insects, firing threshold for neurons). If the stimulus is sub-threshold, the subsequent activity of the system is unaffected. If the threshold is met or exceeded, then the system undergoes a transition to a new dynamic. Another aspect of decision making is that once the decision has been made, it is irreversible. It is possible to disrupt the decision, but not reverse it. Thresholds provide these systems with a mixture of continuous and discrete dynamics.

One major difference between neurobehavioural regulatory systems and collective intelligence systems is that the agents of a collective intelligence are generally free to physically move through their environment, whereas the neurons in a neurobehavioural regulatory system are fixed in space and explore a virtual environment through local connectivity changes and alterations in their own local dynamics (somewhat akin to changes in task allocation). In many other respects, collective intelligence systems and neurobehavioural regulatory systems share a range of dynamical mechanisms. Collective intelligence systems are much smaller in terms of the number of agents and much more accessible to experimental study; therefore, it is suggested that the study of collective intelligence systems is worthwhile, not just in their own right, but also for the insights they might provide into the functionality and roles of various dynamical mechanisms in determining the behaviour of neurobehavioural regulatory systems.

The number of common features between neurobehavioural regulatory systems and collective intelligence systems is striking. Is this coincidental? Or could these characteristics provide the fundamental capabilities necessary for the expression of intelligence in a biological system (or any system)? Fundamentally, both neurobehavioural regulatory systems and collective intelligence systems are examples of process. Their dynamics are characterized by generativity, becoming, transience, openness, fungibility, context dependency, and information, all characteristics of process. This suggests framing the descriptions of both neurobehavioural regulatory systems and collective intelligence systems within the formal language of the process algebra, based upon Whitehead's Process Theory (see Appendix A).

3. Generativity, Interactions and Contextuality

Contextual effects have been recognized in biopsychosocial sciences for a long time. Rarely, however, are they explicitly denoted and considered in the analysis of experiments. Kolmogorov's caution about the necessity to link probability distributions to their contexts [9] has mostly been ignored. In psychology, it has taken decades to move beyond the almost dogmatic devotion to the Normal distribution, and the recognition that many situations possess a very different probability structure, particularly power law distributions [75]. In physics, the appearance of contextual effects took physicists by surprise, so entrenched were ideas of Newtonian objects and objectivity. The earliest awareness of contextuality arises from the non-commutativity, in general, of self-adjoint operators, which in the von Neumann formulation, represent measurements. It was recognized that it was impossible to simultaneously measure the eigenvalues corresponding to non-commuting operators. Thus, it becomes impossible to attribute a complete set of measurement eigenvalues to a quantum system. From von Neumann's initial proof of the non-existence of dispersion-free measures (discussed in [12]), there followed a series of non-commutativity results [76–78], and their later simplification by Mermin [79], showing the impossibility of finding a joint probability distribution for all possible observables. The wave function became contextually dependent, leading to endless discussions concerning the apparent non-reality of quantum measurements, or at least the observer dependence of such measurements. This is the first understanding of contextuality, namely, that the context under which a measurement is performed affects which measurements can be observed and what values can be obtained. From this, one is led to the Copenhagen interpretation of quantum mechanics and to Wheeler's famous dictum [80], that "no elementary phenomenon is a phenomenon until it is an observed phenomenon"; in other words, the system has no inherent properties until a measurement takes place.

The second form of contextuality again stems from EPR and involves the Bell inequalities [12–17]. Following experimental evidence of their violation in certain quantum mechanical situations, these inequalities and their violation have come to represent contextuality to many physicists today. Each of these inequalities is derived by assuming that the system can be described by a set of "hidden variables" whose dynamics obeys the structure of Kolmogorov probability. In particular, the hidden variables and component

systems are assumed to be statistically independent and that probabilities obey the classical Kolmogorov sum rule. From this, one can obtain an inequality involving certain expectation values of measurements which must be satisfied if the underlying assumptions are satisfied. Probabilities in quantum mechanics follow from the Born rule, and the sum rule in this case includes interference terms which do not appear in the Kolmogorov sum rule; therefore, by definition, they are non-Kolmogorov. Moreover, it should be no surprise that quantum mechanics violates the Bell inequalities because the usual situation studied, that of entanglement, by definition involves systems that are not statistically independent but are, in fact, highly correlated. It has always seemed odd that one would expect a model of independent systems to yield a full description of dependent or correlated systems.

The study of these inequalities has created something of an academic industry. So deeply ingrained is the conflation of classicality and Kolmogorov probability, that quantum mechanics is described as manifesting a supra-classical probability. More modestly, and more accurately, it should be described a manifesting a non-Kolmogorov probability. In fact, Dzhafarov and colleagues have studied contextuality within the framework of Kolmogorov probability in order to apply these ideas to psychological experiments. They formulated inequalities analogous to the Bell inequalities, but more importantly, introduced the idea of inconsistent couplings in the structure of the probability structure [11,81,82]. These ideas are presented in condensed form in Appendix B. Most importantly, they have demonstrated in at least two psychological experiments—one social and thus somewhat similar to collective intelligence [18], and one individual [19]—the presence of contextuality comparable to that observed in quantum mechanics. A complementary approach is that of Khrennikov, who developed an extension of Kolmogorov probability theory that he called contextual probability theory, which involves modifications of the sum rule for probabilities in a manner analogous to the development of non-Euclidean geometry [83–85].

This section focuses on contextuality by default, the first type of contextuality, in neurobehavioural regulatory systems and collective intelligence systems. The demonstration of a stronger form of contextuality by Dzhafarov in psychological experiments makes it likely that this strong contextuality will also be observed in the neurobehavioural regulatory systems (because psychological processes supervene upon such systems), and so likely in collective intelligence systems as well, although the current state of theory and experiment does not permit this to be explicitly demonstrated. Nevertheless, it is hoped that the discussion in this paper will motivate others to explore these questions further, both theoretically and experimentally.

Consider a simplified model of a collective intelligence systems: first of all, each system consists of a large collection of individual agents, neurons in the case of NBRS, ants in the case of CIS. These agents are described by processes, A_i, each of which comprises myriad subprocesses describing such entities as dendrites, receptors, and axon terminals, and so on for neurons; or antennae, sensory receptors, odorant glands, tactile sensitive surfaces, and so on for ants, down as many levels of description as one likes. Synapses are locations for interactions between neurons and can be viewed as processes in their own right. Each synapse is denoted by s_{ij} where i,j refer to the neurons that take part in the synapse. We can expand the neural process to $A_i(s_{ij}, s_{ik}, s_{il}, \dots)$. Given two neurons that participate in a ligand-gated synapse, we can write $A_i(s_{ij}, s_{ik}, s_{il}, \dots) \square A_j(s_{ji}, s_{jm}, s_{jn}, \dots) = A_i(s_{ij} \square s_{ji}, s_{ik}, s_{il}, \dots) \square A_j(s_{ji} \square s_{ij}, s_{jm}, s_{jn}, \dots)$. We can also write this as $A_i(s_{ij} \square s_{ji}, s_{ik}, s_{il}, \dots), A_j(s_{ji} \square s_{ij}, s_{jm}, s_{jn}, \dots)$ and allow the synapses to carry the interaction. A similar notation can be used for gap junctions. The neural-level interaction denotation must persist in the case of a G-protein-coupled receptor on account of the fact that the action alters the neuronal dynamics, and hence can induce effects outside of the receptor per se. Volume transmission requires the introduction of a global process, P, which expresses the diffusion dynamics of the volume transmitter. Each neuron potentially will possess receptors, p, for the volume transmitter, and some neurons will also possess releaser sites, r, for the volume transmitter. Interaction with the volume transmitter process will, in the general case, look like $A_i(s_{ij}, s_{ik}, s_{il}, \dots, p, r) \square P = A_i(s_{ij} s_{ik}, s_{il}, \dots p \square P, r \square P) \square P$. The presence of

a volume transmitter offers a possibility for a second contextuality type effect, because the coupling through the volume transmitter can enable an increase in correlations between neurons that are physically distant from each another. A similar pattern holds for collective intelligence systems. Therefore, there are "non-local" influences between the agents of these systems even though these influences remain causal.

Each interaction potentially induces a change in the state of the involved processes, and in some cases, changes the process itself. In process algebra, this is expressed in the concatenation of processes which represents the changes in processes over time. These transitions may be externally driven through interactions with the environment, or they may be internally driven due to interactions among subprocesses. In the case of both neurobehavioural regulatory systems and collective intelligence systems, these transitions may be at least transiently irreversible (and in some cases, such as the triggering of trap-jaw ants, permanently irreversible). The state of a process remains constant unless an interaction takes place which induces a transition. We can denote the duration of persistence of a process P by $^{(n)}P$, where n is the number of generations over which the process P persists (or remains active). If a transition takes place at step n, we can denote this as $^{(n)}P^{(0)}R$, and over the persistence of R, this becomes $^{(n)}P^{(m)}R$. The persistence of the concatenation is thus $n + m$. The irreversibility of these transitions ensures that concatenation is a non-commutative operation, which lays the foundation for contextual effects of a deeper kind.

Irreversible transitions are of paramount importance in these systems because they enable decisions to be made by individual agents and to ensure that the consequences of these decisions persist for a sufficient time so as to enable a transition to take place at the collective level. This irreversibility also has the consequence that the behaviour of these systems cannot be represented by a single probability distribution. A decision cannot be represented merely as a fluctuation, but rather induces a transition to a new probability distribution. This irreversibility implies that these systems are history-dependent, and different histories will give rise to different probabilities for subsequent behaviour. The generativity inherent in these systems enables these probabilities to be generated on the fly in response to changing contexts. There is no need to find some optimal one-size-fits-all distribution. Instead, the system can optimize locally with respect to individual contexts.

There is no reason to believe that there should be a single joint probability from which all of these context distinct probabilities may be derived. To see this, consider a toy model of a *Pogonomyrmex barbatus* colony. Workers can be broadly divided into two groups: midden workers that remain within the nest and carry out caring for nest mates and maintaining the wellbeing of the nest, and patrollers, that forage outside the nest, returning when a suitable food source has been located in order to recruit more patrollers. They do so by inducing midden workers to shift their task, a transition which is irreversible, at least over the course of a single day. This role transition may reset overnight, enabling the colony to revert to a low-energy-expenditure state until the next rich food source is encountered. Patrolling is energy-intensive; thus, a colony can conserve energy in low-resource environments by allocating lower numbers of patrollers and then reallocating workers when conditions warrant.

Consider the following situation: the colony is allowed to behave freely in the absence of a food source. The colony splits into two task groups: midden workers that remain within the nest, and patrollers that remain outside the nest searching for food. During this phase of observation, midden workers are tagged with a small "*m*", whereas patrollers are tagged with a small "*p*". The colony is allowed to remain in this state for some time, *t*. Next, food is introduced into the environment. Context 1 consists of the period from the end of the tagging until the first patrollers return to the nest, which marks the start of context 2. This persists until the patrollers leave the next. In reality, patrollers will come and go, but for simplicity, it is assumed that a fixed percentage, *r*, of patrollers returns to the nest and remains until a fixed percentage, *k*, of midden workers are induced to switch task to become patrollers. It is further assumed that all of the patrollers leave the nest more or less simultaneously. That moment marks the end of context 2 and the beginning of context 3.

Context 3 lasts until the next group of patrollers return to the nest. The period from this point until the beginning of context 4 provides an additional context, but the probability structure is more complicated, and for simplicity of presentation will be ignored. Context 4 begins when all of the patrollers return to the nest at the end of the day. Each context is initiated by a transition within the dynamics of the colony. Therefore, during each context, the colony process can be considered to be fixed. If we denote the process associated with context *i* by P^i, then we can describe the history of the colony as $^{(a)}P^1{}^{(b)}P^2{}^{(c)}P^3{}^{(d)}P^*{}^{(e)}P^4$, where *a,b,c,d,e* are random times reflecting the durations of each context. Note that the timing of these contexts is intrinsic to the colony dynamics. The observer chooses which context to examine, but they do not cause the context to appear. The observer does not create reality by their act of observation, nor do they create measurements; they simply sample the colony and perform a measurement. There are no non-objective or subjective features here; this reality is classical. Nevertheless, contexts matter.

During each context, one or more random variables can be measured. An ant is selected at random, and one of three questions is asked: (M/P) Is this ant a midden worker (1) or patroller (−1)?; (S/D) Did this ant change its allocated task (1) or does it continue in the same task (−1)?; (I/O) Was this ant inside the nest (1) or outside (−1)? If *m* is the number of midden workers and *p* is the number of patrollers at the start of the observations, then we obtain the following distributions of workers (*S*, same; *D*, different; *I*, inside; *O*, outside):

Context 1

	I	O
S	m	p
D	0	0

Context 2

	I	O
S	m − km + rp	p − rp
D	Km	0

Context 3

	I	O
S	m − km	p
D	0	km

Context 4

	I	O
S	m − km + p	0
D	km	0

For context 1, the marginal (non-normalized) distributions associated with each of the questions are: $M/P = (m, p)$, $S/D = (m + p, 0)$, $I/O = (m, p)$. For context 2, we have $M/P = (m − km, p + km)$, $S/D = (m − km + p, km)$, $I/O = (m + rp, p − rp)$. For context 3, we have $M/P = (m − km, p + km)$, $S/D = (m − km + p, km)$, $I/O = (m − km, p + km)$. For context 4, we have $M/P = (m − km, p + km)$, $S/D = (m − km + p, km)$, $I/O = (m + p, 0)$. It is obvious that these joint distributions are all distinct. Moreover, it is also easy to show that the joint distributions for contexts 1 and 4 are given by the product of the marginal distributions for S/D and I/O, but this is not true of contexts 2 and 3. The two random variables, S/D and I/O, exhibit statistical independence in contexts 1 and 4, but not in contexts 2 and 3, as a consequence of the transiently irreversible interactions that take place between patrol and midden workers.

The evolution of the colony throughout the period of observation is governed by a non-stationary probability structure. Nevertheless, this probability structure is characterized by five periods of transience, during each of which the probability is stationary. The generative and fungible nature of the colony dynamics allows for these periods of transiently stable behaviour to emerge, the contextual dependence arising due to the effect of the (relatively) irreversible transitions triggered by interactions between the colony and the environment (detection of food), and within the colony (between patrollers and midden workers during recruitment). Overnight, the colony reverts to its original state, enabling these processes to repeat. These dynamical features give rise to contextual effects, preventing the existence of a single joint distribution and to varying degrees of statistical independence/dependence, which may lead to deeper contextuality effects. Indeed, transience plays a fundamental role in the dynamics of these complex systems [86].

Further analysis is needed to determine whether or not the probabilities across these four or related contexts can violate the CHSH inequality and exhibit inconsistent coupling. Preliminary work suggests not, at least in terms of observations of individual attributes. This is different from the quantum mechanical situation and consistent with the ideas of Maruyama [87]. However, there are many other aspects to the dynamics of a collective intelligence which are not reflected within this model, and which may lead to the kinds of entanglements of agent dynamics which result in inconsistent coupling. Experiments with natural systems will eventually be necessary to confirm any formal analysis. The fact that Dzhafarov and colleagues have seen evidence at the psychological level for this form of contextuality makes it likely that it will be observed at the level of the neurobehavioural regulatory or cortical systems, and from the discussion above, it is expected that an analogue will also be observed in at least some collective intelligence systems. It is hoped that the process algebra, which reflects more of the dynamics of these systems, particularly their generativity and transience, will prove useful in addressing these questions.

The obvious place to look is at the level of collective decision making, which is not merely a statistical averaging over individual attributes and actions, but is an emergent expression of complex interactions between the individual workers and their environment. One possibility is to search for violations of transitivity in paired choice preferences. Transitivity over choices is expected for a perfectly rational agent. This means that if the agent prefers A over B, B over C, and C over D, then it should also prefer A over D. However, neither individual workers, social insect colonies, nor human beings, are perfectly rational. A personal example illustrates the possibility. My preference between two items is made based on two criteria: taste and propensity to cause gastroesophageal reflux. If the difference in propensity to cause reflux differs by less than 50%, I will select by taste. If the difference in propensity is greater than 50%, then I will choose the item with the lower propensity. Consider the following foods described by a pair (taste (0–100), propensity (0–100)): Double-fudge chocolate ice cream (DFC) (100,100), Fudge chocolate (FC) (75,75), Chocolate (C) (50,50), and Pistachio (P) (25,25). By my criteria, I will make the following choices if presented with pairs of items: DFC > FC, FC > C, C > P, but I will also choose P > DFC, and thus violate transitivity. The reason for the violation is that decisions are not based upon single attributes but rather on multiple attributes which compete rather than cooperate. This is a frequent scenario whenever social insects must make decisions, such as in nest emigration.

If I am perfectly consistent in these choices, then it is easy to construct an experiment in which paired choices are presented, assigning a value of 1 or −1 to either choice, and then forming the CHSH formula for the expectation values for the four pairs above, which will yield its maximal value of 4. Although idealized, this argument does suggest that a fruitful place to look for contextuality is collective decision making among social insects.

One point that seems repeatedly to be missed in discussions of entanglement, quantum probability, and Bell's inequalities is the fact that entangled particles are not statistically independent. Statistically independent processes have joint probabilities given by simple products. They are independent. Period. Entangled particles are not statistically indepen-

dent. Their joint probability is given by a quantum state such as $1/2 |0> |1> + 1/2 |1>$ $|0>$, which is not a product such as $1/2(|0> + |1>) + 1/2(|0> + |1>)$. It does not matter that in a Bell experiment the two observers Alice and Bob *believe* that the particles they are observing are statistically independent—they are not. In the derivation of Bell's theorem and all such inequalities, it is also assumed when developing the classical hidden variable case that the particles involved are statistically independent (as are the hidden variables). However, entangled particles are not independent. Moreover, within the process algebra framework, the actual occasions that form the foundation for a fundamental particle are not statistically independent—instead, the local process strengths are informationally entangled and therefore statistically dependent.

Both collective intelligence systems and neurobehavioural regulatory systems involve large numbers of agents whose dynamics are complexly intertwined. Thus, they appear to be ideal candidates for the search for stricter forms of contextuality such as those determined by violations of the Bell-type inequalities. Collective intelligence systems seem ideal due to their accessibility and ease of experimental manipulation. Time will tell if quantum-mechanical-type contextuality will be observed in these systems.

4. Conclusions

The prevailing view regarding contextuality and the violation of the Bell inequalities, particularly the CHSH inequality, is that it is a uniquely defining feature of the quantum realm. Classical phenomena are distinguished by being constrained by the limitations of Kolmogorov probability, and so should never give rise to correlations capable of violating the CHSH inequality. Quantum phenomena, on the other hand, follow Born probability, and thus can give rise to correlations which can violate the CHSH inequality. However, quantum phenomena are themselves said to be constrained; therefore, their degree of violation of the CHSH inequality is bound by the so-called Cirel'son bound. The region between the Cirel'son bound and the maximum value, which can be achieved by the CHSH inequality, appears to be a no-man's zone, which no naturally occurring phenomena can ever achieve, somewhat akin to the situation in the theory of computation and the Turing limit. However, as noted previously, recent studies in human cognition have demonstrated violations of the CHSH inequality in a manner consistent with quantum mechanical violations [18,19], and even with violations of the Cirel'son bound [20]. Thus, violation of CHSH per se is no longer a defining characteristic of quantum systems, but there is still debate as to its significance ontologically. Some authors [11,81–85] argue that the fundamental problem lies in a misapplication of Kolmogorov probability theory at the classical level, thus failing to take contextuality into account. This, in turn, leads to a misunderstanding of the implications of contextuality, such as non-locality and non-realism [4]. Others argue that although the CHSH inequality may be violated in both classical and quantum systems, the reasons for these violations differ, and the classical and quantum realms remain fundamentally distinct [87].

Human cognition may be relatively easy to study at the psychological level, but an attempt to understand it at a deep dynamical level, which is necessary in order to understand the origins of contextuality and CHSH violations, is extraordinarily difficult. It would be useful if simpler systems could be found whose dynamics are more amenable to experimental manipulation and analysis. A comparison of neurobehavioral regulatory systems and collective intelligence systems reveals a broad range of dynamical similarities, which suggests that collective intelligence systems might yield insights into the relationship between dynamics at the individual (lower, neuronal) level and that at the collective (higher, psychological) level which would aid in understanding the reasons behind contextuality and CHSH violations in decision making. Collective decision making has been studied in detail within the collective intelligence community, particularly involving social insects. Social insect colonies, the archetypal example of collective intelligence, are highly amenable to experimental study. Thus, a demonstration of contextuality in the decision making of social insect colonies would provide a setting in which to study mechanisms

of contextuality at the classical level, which may offer insights into the sources of contextuality in neurobehavioural regulatory systems and, ultimately, human decision making. An understanding of classical contextuality may, in turn, lead to a better understanding of quantum contextuality. At the very least, it demonstrates that acausal (superluminal) non-locality is not necessary for contextuality or the violation of the CHSH inequality.

Moving the comparison between the neurobehavioural regulatory systems and collective intelligence from metaphorical to a rigorous dynamical/structural one requires the development of new theoretical, mathematical, philosophical, and experimental methodologies. Sulis [55–57] reviewed the experimental literature on collective intelligence as manifested in the behaviour of social insect colonies. On the philosophical side, Minati [88] provides an excellent overview of systems theoretical ideas related to collective dynamics and suggests an approach based in constructivism. Trofimova [44,89] developed a related approach called functional constructivism and has proposed a formal language for its description and analysis. Sulis [90,91] developed a parallel approach based upon the idea of process and has proposed a process algebra for its description and analysis (see Appendix A for details). This has been applied to the case of non-relativistic quantum mechanics and is now being extended to collective intelligence systems. The hope is that by using a common language which can describe both classical and quantum systems, the mathematical duality in their descriptions, which leads to a conceptual duality in their proposed ontologies, may be eliminated, and the two domains can be coherently unified within a single conceptual and formal framework.

Appendix A

Appendix A.1. Process Theory

The core ideas of Whitehead's Process Theory [5] have been presented elsewhere [14, 90–94], but are summarized here concisely for reference. Described as a philosophy of organism, Whitehead proposed a metaphysics in which reality consisted of an ever-changing flux of phenomena organized into coherence by some form of underlying subjectivity. The subjectivity in Whitehead's theory is termed "prehension", which loosely refers to the incorporation of prior information into the newly emerging elements of reality.

Whitehead called these most primitive elements of reality "actual occasions". According to Whitehead, a process consists of a sequence of events having a coherent temporal structure in which relationships between events are considered more fundamental than the events themselves. Becoming is a fundamental aspect process, whereas being and substance arise from the actions of process. In process theory, entities are considered to be generated as opposed to simply existing. Whitehead's actual occasions are transient entities: they come into being, exist long enough to pass on whatever information they represent, and then fade away. There is a subjective, meaning-laden thread linking these events. The entities that make up our observable reality are emergent from these actual occasions.

Processes generate the actual occasions that constitute space–time, and as such may be considered to exist outside of space and time. The idea that quantum phenomena might possess features which exist outside of space–time has been suggested by Bancal and Gisin

and colleagues [16], and by Aerts and Sozzo and their colleagues [95]. They can exist either in a state of activity, in which they generate actual occasions, or they can exist in a state of inactivity, in which they are merely potentialities for a future state of activity. Transitions between these states depend upon the flux of actual occasions in the moment and the interactions among the currently active processes.

Alterations in the characteristics of processes occur through interactions among processes, dependent upon their compatibility [96] and triggered by the appearance of specific actual occasions.

The formal representation of the process in process algebra has been described in detail elsewhere [90–94], and so will merely be summarized here. Process algebra rests upon the realization that mathematical structures can be *generated* with the use of combinatorial games (in particular, Ehrenfeucht–Fraïssé games) [97–99], together with the fact that the Hilbert space of NRQM is a reproducing kernel Hilbert space [100]. Given a reproducing kernel Hilbert space $H(X)$ with base space X, one can find a discrete subspace Y of X (sampling subspace), and a Hilbert space $H(Y)$ on Y, such that each function in $H(Y)$ can be lifted to a function in $H(X)$ via interpolation. Interpolation means that if $\Psi(z)$ is a function in $H(X)$, then for each $y \in Y$ there exists an interpolation function $\Psi_y(z)$ on $H(X)$, such that $\Psi(z) = \sum_{y \in Y} \Psi(y)\Psi_y(z)$.

One can use either Whittaker–Shannon–Kotel'nikov sinc interpolation theory (for Y being a discrete lattice), or Fechtinger–Gröchenik interpolation theory may be used instead [100] (for non-uniform spaces satisfying the Beurling density [101]). Processes can be modeled heuristically as (epistemologically equivalent [90,92]) combinatorial games, which generates a discrete space of primitive events from which the larger events emerge via interpolation. The discrete subsets Y are considered to be fundamental, X is an *interpretation* selected by an observer, the elements of $H(Y)$ are the ontological state (wave) functions, and the elements of $H(X)$ are derived (emergent) through an (arbitrary) interpolation procedure. The important point is that the elements of the space Y are created in distinct generations, and the value of the function at each point is determined by propagating information from prior elements by means of a causal propagator, K. Thus, $\Psi(y) = \sum_i K(y,i)\,\Psi(i)$, where the sum is over immediately prior elements i. Probabilities are given by the Born rule, $\Psi^*(y)\Psi(y) = \sum_{ij} K(y,i)K(y,j)\,\Psi^*(i)\Psi(j)$, which clearly has a non-Kolmogorov structure.

The discrete subsets are called *causal tapestries* and their individual points are called *informons*. A detailed description of informons is given in [90–94], and the interested reader is referred there because the details are not essential to the discussion in this paper.

A fundamental tenet of this model is that a process does not change state unless in interaction with other compatible processes. The concept of compatibility between interacting complex systems was first proposed by Trofimova [96]. Compatibility $\Xi(\mathbf{P},\mathbf{M})$ is conjectured to be a function of fixed factors (e.g., mass, charge, coupling constants) and of the local compatibilities. The probability of an interaction taking place $\Pi(\mathbf{P},\mathbf{M})$ is conjectured to be a function of the compatibility, $\Pi(\mathbf{P},\mathbf{M}) = \chi(\Xi(\mathbf{P},\mathbf{M}))$. The precise form of these functions depends upon the particular case, but it can be expected to depend, in part, on the local process strength $\Psi^*(y)\Psi(y)$.

Appendix A.2. Process Algebra

Process algebra is the formal language for describing interactions between processes.

There are additional technical aspects such as the process covering map and the configuration space covering map, whose details can be found elsewhere [90–94]

An important concept is that of *epistemological equivalence*. Epistemological equivalence of two processes, **P** and **Q**, means that their global Hilbert space interpretations, $\Psi^P(z)$, $\Psi^Q(z)$, respectively, are equal, i.e., $\Psi^P(z) = \Psi^Q(z)$.

If two processes are epistemologically equivalent, then the specifics of generation do not matter. They generate the same emergent state functions and therefore will yield the same predictions. This is useful because processes can be modeled heuristically based upon mathematical convenience, just so long as they are epistemologically equivalent to

any real processes. In particular, one can use processes based upon combinatorial games which have particularly valuable characteristics [97–99].

Processes may influence one another in two different ways. The first (*coupling*) involves the generation of individual informons, their relative timing, as well as the sources of information which enter their generation. Coupling results in epistemologically equivalent processes; thus, properties are unaltered. The second (*interaction*) involves the activation or inactivation of individual processes and the creation of new processes. Epistemological equivalence is broken, and properties are altered.

Two processes, P_1, P_2, may be independent, meaning that neither constrains the actions of the other in any way. This relationship is denoted simply by the comma ",". Compound processes ($R > 1$) can be formed from primitive processes ($R = 1$) by various coupling operations. A coupling affects timing and information flow. Two processes may generate informons concurrently (*products*) during each round, or sequentially (*sums*), with only one process generating informons during a given round. Information from either or both processes may enter into the generation of a given informon (free), or information incorporated into an informon by a process may only come from informons previously generated by that process (*exclusive*). This leads to four possible operators:

1. Free sequential (free sum): $P_1 \hat{\oplus} P_2$;
2. Exclusive sequential (exclusive sum): $P_1 \oplus P_2$;
3. Free concurrent (free product): $P_1 \hat{\otimes} P_2$;
4. Exclusive concurrent (exclusive product): $P_1 \otimes P_2$.

The operation of concatenation is used to denote processes that act in successive generation cycles. Thus, $P_1 \cdot P_2$ (or simply $P_1 P_2$) indicates that P_1 acts during the first generation cycle, whereas P_2 acts during the second generation cycle.

Interactions break epistemological equivalence, and can do so in myriad ways. Interactions between processes may activate an inactive process or inactivate an active process. In addition, an interaction among processes P_1, P_2, ... , P_n may generate a new process, P, which can be described in functional form as $F(P_1, P_2, \ldots, P_n) = P$. If $\Theta(P_1, P_2, \ldots, P_n)$ describes a coupling among P_1, P_2, ... , P_n, then the functional relationship may be described using the operation of concatenation, as $\Theta(P_1, P_2, \ldots, P_n) P$.

There are potentially so many different types of interactions; therefore, a set of generic operators comparable to those above are used to indicate the presence of an interaction with the specifics to be spelled out if known. Thus, there are:

1. Free sequential (free interactive sum);
2. Exclusive sequential (exclusive interactive sum);
3. Free concurrent (free interactive product);
4. Exclusive concurrent (exclusive product).

Independence, sums, and products are commutative, associative, and distributive operations. Concatenation is non-commutative and non-associative in general. The zero process, O, is the process that does nothing.

The basic rules for applying these operations in combining processes are the following:

(1) The free sum is only used for single systems and combining states which possess identical property sets (pure states);

(2) The exclusive sum is used for single systems and combining states which possess distinct property sets (mixed states);

(3) The free product is used for multiple systems which possess distinct characters (scalar, spinorial, vectorial, and so on) such as coupling a boson and a fermion. It is unclear whether two bosons might couple via a free product;

(4) The exclusive product is used for multiple systems which possess the same character, such as coupling two bosons or two fermions.

Appendix B

Taken from [4]. Dzhafarov, Zhang and Kujala provide a concise summary of the *Contextuality by Default* model in [82]. Following the notation in that paper, each random variable is associated with the quantity, *q*, being measured and the context, *a*, within which the measurement is made, and denoted, R_q^a. Consider two measurements, *q*,*q*′, and two contexts, *a*,*b*. For a fixed context, *a*, the pair $R_q^a, R_{q'}^a$ is termed *bunch*, representing the collection of measurements associated with a specific context. It is reasonable to believe that such a pair is jointly distributed. For a fixed measurement, *q*, the pair R_q^a, R_q^b is termed a *connection* for *q*.

The most basic form of contextuality occurs when no joint distribution can be found for a connection. In such a case, they are said to be *inconsistently connected*. This is the situation of *Contextuality by Default*. Dzhafarov considers this to be the most trivial form of contextuality because it is so ubiquitous. Dzhafarov has developed a more restricted notion of contextuality, in line with contextuality in physics. He considers couplings between bunches. For example, given two bunches, $R_q^a, R_{q'}^a$ and $R_q^b, R_{q'}^b$, a coupling is a set of jointly distributed random variables (*A*,*B*,*X*,*Y*), subject to certain constraints, such that (*A*,*B*) is distributed as $R_q^a, R_{q'}^a$ and (*X*,*Y*) is distributed as $R_q^b, R_{q'}^b$. The constraints involve *A*,*X* and *B*,*Y*, which correspond to measurements of *q* and *q*′, respectively. A measurement, *q*, is considered to be *context-independent* if, among all couplings (*A*,*B*,*X*,*Y*), we have $Pr(A \neq X) = 0$. It can be shown that such a coupling may not exist even if the system is consistently connected.

Now, considering all couplings (*A*,*X*) for just the connection $R_q^a, R_{q'}^b$, the minimal value *m*′ can be found for $Pr(A \neq X)$. Then, considering the global coupling (*A*,*B*,*X*,*Y*), the minimal value *m* for $Pr(A \neq X)$ can again be found. If *m* = *m*′, the system is non-contextual, and if *m* > *m*′, then the system is contextual. This form of contextuality is analogous to that found in physics and gives rise to similar types of inequalities.

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