Application of Quantum Cognition to Judgments for Medical Decisions

Sheng Yi 1,†, Meijuan Lu 2,† and Jerome Busemeyer 3,*

1 Franciscan Health Hospital, Franciscan Lafayette Faith, Hope and Love Cancer Center, 1425 Unity Place, Lafayette, IN 47905, USA; sheng.yi@franciscanalliance.org
2 Meng River Chinese Medicine Research Co., Ltd., Meng River 1950 East Greyhound Pass, Carmel, IN 46033, USA; ludai88@gmail.com
3 Department of Psychological and Brain Sciences, Indiana University, Bloomington, IN 47405, USA
* Correspondence: jbusemeyer@indiana.edu
† These authors contributed equally to this work.

Abstract: The psychology of judgment and decision making can provide useful guidance to the task of medical decision making. More specifically, we describe how a new approach to judgment and decisions, based on quantum probability theory, can shed new light on seemingly irrational judgments, as well as indicate ways to ameliorate these judgment errors. Five different types of probability judgment errors that occur in medical decisions are reviewed. For each one, we provide a simple account using theory from quantum cognition. We conclude by drawing the implications of quantum cognition for ameliorating these common medical probability judgment errors.

Keywords: quantum cognition; probability judgments; radiation oncology

1. Introduction

The main target of medical services is people, and the only way to confirm success is still to conduct empirical research on the human body—this is clinical research. Therefore, clinical research has become an important and necessary way for medical science to move from basic to clinical. A sound decision and accurate diagnosis and treatment can bring health opportunities to patients. From the current point of view, the treatment of many difficult diseases such as malignant tumors is still a problem that humans urgently need to overcome, but we are convinced that with the wisdom of humans and the hard work of everyone that the dawn has pierced the dark envelope like a sharp sword.

One area upon which we need to shed more light is decision making. Good decision making is essential for effective medical practice. Physicians must infer the appropriate diagnosis as well as decide the best treatment plan. The purpose of this article is to point out that research on the psychology of judgment and decision making may provide useful guidance to the task of medical decision making. More specifically, we describe how a new approach to judgment and decisions, based on quantum probability theory, can shed new light on seemingly irrational judgments, as well as indicate ways to ameliorate these judgment errors.

Over the past 50 years, decision scientists have identified several cognitive errors that affect human judgment and decision making, and many of these decision errors have also been demonstrated with medical practitioners. In general, these studies indicate that physicians are not immune to decision errors and that decision biases show up in medical domains as they do in other domains.

Some of these errors may be especially relevant to therapeutic radiation oncology decisions. Radiation oncologists differ widely in their aggressiveness and risk evaluations for tissue damage outside of the target areas depending on their age and training. Older generations tend to treat more aggressively than younger generations. Judgments...
concerning the probable outcomes of different radiation treatment plans may be prone to these biases. Using therapeutic radiation oncology as a setting, we describe five important probability judgments errors along with their theoretical accounts based on quantum cognition. We conclude with a discussion of the implications of quantum cognition for medical treatment decisions.

A recent review of the large literature supporting the application of quantum probability to human judgments is found in many previous publications. The present article is limited to describing possible implications of this theory for medical judgments. Although we argue that quantum probability provides a coherent account of these probability judgment errors, we acknowledge that quantum probability is not the only way to account for these findings.

2. Five Probability Judgment Errors Relevant to Medical Decisions

Below, we review five probability judgment errors that could reduce performance and effectiveness of medical decisions. These five biases include conjunction fallacies, disjunction fallacies, unpacking effects, order effects, and anchoring effects. First, we briefly define some concepts that are needed to describe these judgment biases.

When deciding a treatment, it is first necessary to consider the probabilities associated with different outcomes produced by the treatment plan. As an example, consider radiation treatments. In this case, the probability of successfully removing a tumor without extensive damaging of nontarget areas is an important consideration. In fact, this is an example of judging the probability of the conjunction of two events. One event is the successful removal of the tumor (call this event S) and the other event is the avoidance of damage to tissue outside the tumor (call this event A). Formally, the physician is required to judge the probability of S and A. These same two events can be used to form a disjunctive event by considering the probability that either the tumor is successfully removed or the tissue outside the tumor is not damaged, which is formally expressed as the probability of S or A. Breaking a general treatment outcome (successful removal) down into more specific options (successful removal with no damage, successful removal with little damage, and successful removal with more than little damage) is called unpacking an event into mutually exclusive and exhaustive parts. The total probability of successful removal is the sum of the probabilities of its component parts. Order dependence refers to the situation when the judgment of a probability of two events depends on the order that they are considered. For example, a judgment that first considers outside tissue damage and then considers the success of removal may not produce the same answer as the opposite order (first considering the successfulness of the removal and then considering outside tissue damage). An anchor refers to a piece of information that could be used to provide an initial guess at a probability of an event. Anchors may be relevant or irrelevant for a judgment, and in the latter case, they could interfere with good judgment.

2.1. Conjunction Errors

Conjunction errors refer to the empirical finding that people judge the probability of a conjunction A and B to be greater than the probability of a single event A. Clearly, according to standard probability theory, such judgments are errors because the probability of event A equals the total probability of event A and B plus the probability of event A and not B.

To illustrate the conjunction fallacy, consider the following study reported in [1]. They asked 115 Stanford and British Columbia undergraduates to answer the following question. A health survey was conducted in a representative sample of adult males in British Columbia of all ages and occupations. Mr. F was included in the sample. He was selected by chance from the list of participants. Which of the following statements is more probable? (check one): (H) Mr. F has had one or more heart attacks, (O and H) Mr. F has had one or more heart attacks and he is over 55 years old.
The result was that a majority (58%) chose the conjunction O and H over the single event H contained in the conjunction. These results are not limited to undergraduate students. Conjunction errors are also found with judgments by physicians. Tversky and Kahneman asked two groups of physicians (a total of 106) to answer the following question.

A 55-year-old woman had pulmonary embolism documented angiographically 10 days after a cholecystectomy. Please rank order the following in terms of the probability that they will be among the conditions experienced by the patient (use 1 for the most likely and 6 for the least likely). Naturally, the patient could experience more than one of these conditions. (D and H) dyspnea and hemiparesis, syncope and tachycardia, calf pain, (H) hemiparesis, pleuritic chest pain, hemoptysis.

The incidence of violations of the conjunction rule produced an average of 91%. These results were replicated and extended in several later studies with physicians \[2,3\].

Evaluating the conjunctions of events is a very common problem for radiation oncologist. For example, when treating a left breast cancer tumor, the physician needs to consider the probability that a dosage level successfully eliminates the tumor. At the same time, she/he needs to consider the probability that the dosage can damage the heart, given that dosage successfully eliminates the tumor.

Conjunction errors are not limited to medical scenarios, and they can also occur with intuitive physics situations that might be relevant to medical physics. In a recent article, \[4\] had participants view videos of physical scenarios and judged the probability that either a single event or a conjunction of two events would occur. Participants watched the trajectory of a simulated ball thrown in the air moving to the right. The trajectory suggested a small probability of the ball eventually falling into a modest hole in the ground. A second ball was seen to come from the orthogonal (down) direction with a high chance of colliding with the first ball, and if it did, this event would greatly increase the chance that the first ball would fall directly into the hole. Participants consistently rated conjunction events (second ball collides with first and first falls into hole) as more likely than the single event for the same scenes (first ball falls into hole whether or not the second collides with first).

Why do these probability judgment errors occur? Tversky and Kahneman argued that people (including physicians) intuitively do not rely on standard probability theory but instead use a judgmental heuristic called the “representativeness” heuristic. According to representativeness, people judge the probability of an event on the basis of the match between a model (provided by the background story) and an outcome (an event that is being judged). For example, the combined event dyspnea and hemiparesis matches the story about the woman with pulmonary embolism better than hemiparesis alone.

What might be of interest to medical physicists is that the representativeness heuristic can be formalized by a probability based on quantum theory \[5\]. Standard probability theory is based on a set of axioms proposed by Kolmogorov that defines events as subsets of a sample space. At about the same time, Von Neumann proposed an axiomatic foundation for quantum mechanics based on defining events as subspaces of a vector space. The collection of subsets must satisfy the strict axioms of Boolean algebra, whereas the collection of subspaces is only required to satisfy a more general set of partial Boolean algebra axioms \[6\]. Thus, there are two competing theories of probability based on different axioms. However, quantum probability provides a simple yet rigorous account of the conjunction fallacy as follows (using formalisms from quantum computing).

According to quantum theory, the background information or story produces a belief state, here symbolized as \(|S\rangle\), which is a unit length vector in the vector space (here we assume for simplicity a finite dimensional vector space). An event such as a “heart attack” is represented as a subspace in a vector space. Each subspace corresponds to a projector (here symbolized as \(H\), which is an idempotent and symmetric matrix). Another event, such as “over fifty”, is represented by another subspace that corresponds to another projector (symbolized here as \(O\), and \(O\) is the orthogonal complement so that \(O \cdot O = 0\), and \(O + \sim O = I\), where \(I\) is the identity matrix). The probability of the event “heart attack” is...
computed from the squared length of the projection of the state on the subspace for “heart attack”, \( \text{Prob}(H) = \|H \cdot |S\rangle\|^2 \). The probability of the sequence of events “over fifty and then heart attack” is computed from \( \text{Prob}(O, H) = \|H \cdot |S\rangle\|^2 \). The probability of \( H \) can then be decomposed into parts as follows [7]

\[
\text{Prob}(H) = \langle S|H|S\rangle = \|H \cdot |S\rangle\|^2 = \|H \cdot I \cdot |S\rangle\|^2 \\
= \|H \cdot (O+ \sim O) \cdot |S\rangle\|^2 \\
= \|H \cdot O \cdot |S\rangle + H \cdot \sim O|S\rangle\|^2 \\
= \|H \cdot O \cdot |S\rangle\|^2 + \|H \cdot \sim O|S\rangle\|^2 + \text{Int} \\
= \text{Prob}(O, H) + \text{Prob}(\sim O, H) + \text{Int}
\]

where \( \text{Int} \) refers to the cross-product term produced by squaring the sum

\[\text{Int} = \langle S| \cdot O \cdot H \cdot H \cdot \sim O \cdot |S\rangle + \langle S| \cdot O \cdot H \cdot H \cdot \sim O \cdot |S\rangle^*.\]

(* indicates the conjugate operation). Note that if the projectors commute, so that order does not matter, then the cross-product term is zero. If the projectors do not commute, then the \( \text{Int} \) can be positive or negative. If \( \text{Int} < -\text{Prob}(O, H) \), then we obtain the conjunction fallacy \( \text{Prob}(H) < \text{Prob}(O, H) \).

This account might seem post hoc at this point. However, once we adopt this account, we can derive several other important predictions. One is that we must predict order effects—the order that the events are evaluated must affect the probability judgment. In fact, research indicates that order effects do occur with these probability judgments [8]. Another important prediction is that the judged probability of the conjunction can only exceed that for one of the single events. In this example, \( \text{Prob}(O, H) \) can only exceed the lower probability \( \text{Prob}(H) \) so that \( \text{Prob}(O) > \text{Prob}(O, H) > \text{Prob}(H) \). This follows from the fact that according to quantum probability, if events are measured in the order \( O \) and then \( H \), then \( \text{Prob}(O, H) = \text{Prob}(O) \cdot \text{Prob}(H|O) \) and \( \text{Prob}(H|O) \leq 1 \), so that \( \text{Prob}(O, H) \leq \text{Prob}(O) \). Double conjunction errors are said to occur when the conjunction exceeds both single events. Although double conjunction errors are less frequently observed in empirical studies, they have been found in some cases [2], and so this prediction poses a challenge for the current formulation for quantum theory (however, see [9] for an alternative quantum model that is not required to make this prediction).

2.2. Disjunction Errors

Disjunction errors refer to the empirical finding that people judge the probability of a disjunction \( A \) or \( B \) to be less than the probability of a single event \( A \) contained in the disjunction [10,11]. Clearly, according to standard probability theory, such judgments are errors because the event \( A \) or \( B \) entails the event \( A \), and so the probability of event \( A \) or \( B \) must equal or exceed the probability of event \( A \) alone.

The same quantum model that was used to account for the conjunction error can also account for the disjunction error. The probability of either \( B \) occurring or then \( A \), in that order, is represented by

\[1 - \text{Prob}(\sim B, \sim A)\]

where \( \sim B \) is the negation of \( B \) and \( \sim A \) is the negation of \( A \). If interference results in \( \text{Prob}(\sim B, \sim A) > \text{Prob}(\sim A) \), then \( 1 - \text{Prob}(\sim B, \sim A) < 1 - \text{Prob}(\sim A) \), producing the disjunction error. A conjunction error produced by not \( B \) and then not \( A \) produces a disjunction error for \( B \) or \( A \).

2.3. Order Effects

As discussed above, the quantum cognition model must predict that the order for evaluating the events \( A,B \) is important for conjunction and disjunction errors. In fact, order effects have been found in medical decisions. Consider the following example from a
medical inference task [12]. Doctors were initially informed about a health complaint of a particular patient, and they were asked to estimate the likelihood that she/he had an infection on the basis of the patient’s history and physical exam (H) and (b) laboratory test results (L) presented in different orders. For the H and then L order, the initial estimate before any information was 0.67, the estimate after presenting H increased to 0.78, and then after presenting L it decreased to 0.51; in the other L and then H order, the initial estimate was again 0.67, and after presenting L it decreased to 0.44, and after presenting H it increased to 0.59. The difference at the end of presenting all the evidence changed across the two orders of presentation, revealing an order effect.

In radiation therapy decisions, order is also an important factor for evaluating events. For example, when treating a mediastirnum tumor, the radiation oncologist needs to evaluate risk of damage to the spinal cord, the heart, and the lung. Focusing on one risk, say the lung, changes the context for evaluating the another, say the heart.

The quantum model for order effects is quite simple:

\[
\text{Prob}(H,L) = \langle S| L \cdot H|S \rangle = \| L \cdot H \cdot |S \rangle \|^2 \\
\text{Prob}(L,H) = \langle S| H \cdot L|S \rangle = \| H \cdot L \cdot |S \rangle \|^2.
\]

Of course, there are previous traditional models that were proposed to account for order effects [13]. However, we tested and compared our quantum model with a popular anchor-adjustment model and found that our quantum model provided a superior account [14].

2.4. Unpacking Effects

Unpacking effects refer to the judgment error that occurs when a larger category event is broken down into smaller events whose union equals the larger event. The following is an example obtained from Stanford physicians reported in [15].

A well-known 22-year-old Hollywood actress presents to the emergency department with pain in the right lower quadrant of her abdomen of 12 h’ duration. Her last normal menstrual period was four weeks ago.

Half were asked to estimate probabilities for two diagnoses: gastroenteritis (G) and ectopic pregnancy (E), plus none of above (NS); the other half estimated the probabilities for five diagnoses that included the above two plus three others: gastroenteritis (G), ectopic pregnancy (E), appendicitis (A), pyelonephritis (P), pelvic inflammatory disease (I), plus none of above (NL). Clearly, the longer list is contained in the shorter list because none of the above for the shorter list includes the extra diagnoses for the longer list. The results show that the average probability given to the shorter list is significantly below that for the longer list. Breaking the short list into additional events increases the judged probability.

The unpacking of events is also an issue for radiation oncologists. For example, when considering left breast cancer tumor treatment, the event of damage to the heart may seem higher when this event is broken down into more specific events, such as damage to the left anterior descending artery of the heart or damage to the heart muscle or damage to other parts of the heart.

The quantum model for unpacking effects follows the same principles described above. Suppose A and B are mutually exclusive events, and event B can be decomposed into two events BC and BD. Then, the probability of A or B (two mutually exclusive events) can be decomposed as

\[
\| A|S \rangle \|^2 + \| B|S \rangle \|^2.
\]

and the event B can be decomposed further into events C and D with interference as before

\[
\text{Prob}(B,C) + \text{Prob}(B,D) + \text{Int}
\]
where $\text{Prob}(B, C)$ is interpreted as event $B$ is found and then special case $C$ is also found. If the interference $\text{Int} > 0$ is positive, then we obtain the unpacking effect.

2.5. Anchoring Effects

An anchoring judgment error occurs when an irrelevant cue is evaluated before a medical judgment is made that pulls the judgment toward the evaluation of the irrelevant cue. For example, in one study, [16], the researchers asked Iowa physicians to rate the chances that hypothetical patients had a pulmonary embolism and then formulated a treatment plan. After reading a vignette, the physicians were asked to decide if the chance of a pulmonary embolism was greater or less than an anchor. The anchor was low, 1%, for one group and it was high, 90%, for another group. Then, the physicians were asked to choose a treatment plan, which was rated by the experimenters for aggressiveness. The results show that the irrelevant anchor (low versus high) produces a significant effect on the aggressiveness of the later treatment decisions.

Anchoring effects may have an important influence on radiation therapy decisions as well. For example, the normal treatment decision for a left breast cancer tumor may not be heavily influenced by concern for damage to the esophagus. However, a radiation oncologist may temporarily become overly influenced by a recent medical article about damage to this location, and then make decisions that overcompensate for this risk.

A quantum cognition account of anchoring effects makes use of the projection postulate of quantum theory. Following a measurement, the new state of the system is formed by projecting the previous state onto the subspace representing the observed event and then normalizing this projection. Suppose the initial state, before the anchor is represented by $|S\rangle$, and we define $L$ as the projector following the answer to the low anchor and $H$ as the projector following the answer to the high anchor. Then, the new state following an answer to the low anchor question is

$$|S_L\rangle = \frac{L \cdot |S\rangle}{|L \cdot |S\rangle|}$$

likewise, the new state following the answer to the high anchor question is

$$|S_H\rangle = \frac{H \cdot |S\rangle}{|H \cdot |S\rangle|}$$

The new state, either $|S_L\rangle$ or $|S_H\rangle$ depending on the anchor, then is used to determine the later treatment decisions. Therefore, the difference in new states following the projections produces the anchoring effect. Recently, ref. [17] used this type of model to account for the anchoring type of effects.

3. Implications for Medical Practice and Conclusions

What implications can we draw from the quantum cognition account of probability judgment errors that occur in the medical decisions described above? Quantum probability is especially suited for dealing with vague and contextual situations that can arise in medical decision making. The essential ingredient producing most of the errors described above is the interference term, $\text{Int}$, which produces departures from the classical prediction derived from the law of total probability. Mathematically, this interference arises from the use of noncommuting projectors to represent events, $A \neq B$, $A \cdot B - B \cdot A \neq 0$, where $A, B$ are projectors. Psychologically, noncommuting projectors correspond to the judge relying on incompatible cognitive representations of events. An incompatible representation refers to the inability of the judge to simultaneously evaluate the two events, and instead, the judge needs to evaluate these events in a sequence. When that evaluation is order-dependent, then interference occurs. The dependence on order occurs because the evaluation of the first event changes the context that is then used to evaluate the second event. The judge needs to view the first event from a perspective (mathematically represented by one basis for
describing the event space), and then changing (by unitary rotation) to a new perspective
(a new and different basis) for evaluating the events.

When events are treated in this incompatible manner, it implies that the judge is failing
to use a single sample space to represent all the events. Essentially, one sample space is
used to represent one event, and a different sample space is then used to represent the
second event. This failure to represent events within a single sample space could occur for a
couple of reasons. One reason is that people have limited cognitive information-processing
capabilities. It is not feasible for humans to represent all events with a single space. For
example, if there are $n$ different questions and each question has $m$ answers, then the person
would need to form an $n^m$ joint probability space, which increases exponentially out of
a person's cognitive processing capability. A second reason is that the person may never
experience the events from a pair of questions simultaneously. In the latter case, the person
can only form a probability distribution for each question separately because no joint distri-
bution has ever been experienced. Prior research has indeed shown that experience with
joint events is needed to reduce the rate if there is occurrence of the conjunction error [18].
Additionally, the presentation of information in a format that encourages a compatible
representation of events can also reduce the occurrence of conjunction errors [19].

Probability judgment errors arise from an intuitive judgment process that may not
be directly accessible to conscious experience. Consequently, medical doctors may be
unaware of these biases, and real-world observational studies may not be capable to reveal
their occurrences. For this reason, controlled experiments are essential for uncovering and
understanding these judgment processes.

Quantum cognition is a new theory of judgment and decision making based on the
axioms of quantum rather than classical probability theory. In some ways, it relaxes
the axioms of commutativity and distributivity that are built into classical probability.
By relaxing these axioms, it provides a coherent account of several common probability
judgment errors that have been found in medical decision making. This article provided a
review of five different types of probability judgments errors along with a common account
of these errors based on quantum cognition. This article does not present a review of the
strong qualitative and quantitative tests that we conducted on quantum cognition. As we
mentioned at the beginning, this article is limited to exploring the applications to medical
judgments. It is not intended to be a review of the large literature supporting the use of
quantum probability theory to human judgments. A recent review of that work can be
found in [20].

Evidence-based science remains the best model of medical practice today. Increasing
the response of patients to treatment has become the dream of modern medicine. It should
be noted that evidence itself does not equal decision making, and decision making must
also take into account existing resources and people's value orientation. Medicine is
always on the quest for precision, and this article argues that quantum decision making
at every level of human cognition provides a way forward for precision medicine. Real-
world observational results based on big data cannot replace experimental evidence from
controlled experiments in confirming efficacy. This article wishes to inspire, in a skeptical
and critical manner, the true potential of precision medicine.

Author Contributions: Writing—original draft preparation, S.Y., M.L. and J.B. All authors have read
and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.
References


