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The Many Faces of Classicality: An Information-Geometric Perspective

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Abstract

The quantum–classical transition is one of the most frequently invoked concepts in modern physics. Yet the notion of classicality itself is far from unique. Depending on the physical context, classical behavior may be associated with decoherence, the semiclassical limit, thermodynamic averaging, suppression of correlations, emergence of collective order, or geometric simplification of statistical state space. These viewpoints are often presented as if they described a single phenomenon, although they emphasize different physical mechanisms and different operational criteria. In this article, we examine the principal notions of classicality that appear across quantum theory, statistical physics, condensed matter physics, and information geometry. We compare the corresponding mechanisms of classical emergence and analyze the physical quantities commonly used to characterize them, including coherence, entanglement, fluctuations, correlation length, Fisher information, statistical complexity, and information-geometric curvature. We argue that many apparently distinct routes toward classical behavior share a common structural feature: a reduction of effective fluctuation freedom (REFF). From this perspective, classicality may be interpreted as an emergent regime in which the accessible fluctuation manifold becomes progressively constrained, stabilized, or geometrically simplified. This viewpoint naturally unifies decoherence, semiclassical localization, thermodynamic averaging, decorrelation, and collective organization within a common conceptual framework. Rather than representing a unique physical process, the quantum–classical transition appears as a family of related mechanisms through which complex quantum fluctuation structure gives rise to effective macroscopic classical behavior.

Keywords: phase transition; quantum-classical; information geometry; correlation length; fluctuations



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1. Introduction

The quantum–classical transition is one of the most fundamental and enduring problems in modern physics. Although quantum mechanics is widely regarded as the underlying theory of microscopic phenomena, our everyday experience is overwhelmingly classical. Understanding how classical behavior emerges from quantum laws has therefore motivated extensive research spanning quantum mechanics, statistical physics, condensed matter theory, and quantum information science [1–24]. Historically, the problem has been approached from several different perspectives. One traditional viewpoint is based on the semiclassical limit, in which characteristic actions become large compared to Planck’s constant [1]. In this regime, quantum dynamics approaches the behavior predicted by classical mechanics, and wave phenomena become increasingly localized around classical

trajectories. Another influential approach is provided by decoherence theory, according to which interactions with an environment suppress quantum interference and dynamically select robust classical states [2,3]. Decoherence has become one of the most successful explanations for the emergence of classical behavior in open quantum systems. However, a different perspective arises in statistical physics. Macroscopic systems often exhibit sharply defined thermodynamic properties despite being composed of an enormous number of microscopic constituents. In such situations, thermodynamic averaging suppresses relative fluctuations and leads to effectively deterministic behavior [4]. From this viewpoint, classicality is associated not with the disappearance of quantum mechanics but rather with the emergence of stable collective observables.

More recently, information-theoretic approaches have provided new insights into the quantum–classical transition. Concepts such as Fisher information, quantum Fisher information, statistical complexity, distinguishability, and information geometry have revealed deep connections between fluctuations, correlations, and the structure of physical state spaces [5–7]. Within these frameworks, classical and quantum systems can often be characterized by distinct geometric and informational signatures.

Despite this vast literature, there is surprisingly little consensus regarding what the term *classicality* actually means. Depending on the context, classical behavior may refer to the suppression of coherence, the disappearance of interference effects, the reduction of entanglement, the emergence of classical trajectories, the weakening of correlations, the stabilization of macroscopic observables, or the simplification of information geometry. These notions are clearly related, yet they emphasize different physical mechanisms and often employ different operational criteria.

The purpose of the present article is to examine these various notions of classicality from a unified perspective. Rather than focusing on a particular model or mechanism, we ask a broader conceptual question:

What does the quantum–classical transition actually entail?

 (1)

We argue that many apparently distinct routes toward classical behavior share a common structural feature: a reduction of effective fluctuation freedom (REFF). Whether achieved through decoherence, semiclassical localization, thermodynamic averaging, suppression of correlations, or collective organization, classicality frequently emerges when the number of dynamically relevant fluctuation directions becomes reduced and the accessible state space acquires a simpler effective structure.

This viewpoint naturally connects several modern developments in information theory and statistical physics. In particular, information geometry (IG) suggests that classical behavior is often associated with weakly curved or approximately flat statistical manifolds [5,8]. Likewise, thermodynamic and statistical descriptions indicate that classicality frequently corresponds to a reduction of distinguishability, complexity, and collective fluctuation organization. Such observations motivate the possibility that the many faces of classicality may ultimately reflect different manifestations of a common underlying principle.

The emergence of classicality has also been extensively discussed from philosophical and ontological perspectives. In particular, recent work by Ney and collaborators has emphasized the ontological status of the quantum state and the role of the wave function as a possible fundamental representation of physical reality [17,18]. Related discussions concerning ψ -ontic and ψ -epistemic interpretations, together with the Pusey–Barrett–Rudolph theorem, have further clarified the conceptual status of quantum states [21–23]. The present work is complementary to these approaches. Rather than addressing the ontological nature of the quantum state, we investigate whether the various physical mechanisms leading to classical behavior admit a common information-geometric description. In this sense,

our objective is not to advocate a particular interpretation of quantum mechanics, but to identify universal statistical structures underlying the emergence of classicality across different physical settings.

The quantum–classical transition is a remarkably broad subject that includes numerous active research directions, ranging from decoherence and statistical mechanics to quantum gravity, emergent spacetime, quantum cosmology, and other foundational approaches. The present work does not attempt to provide a unified theory of all these phenomena. Instead, we focus on a more restricted question: whether several representative mechanisms leading to effective classical behavior may be interpreted within a common information-geometric framework. Questions concerning quantum gravity or the emergence of spacetime are therefore outside the scope of the present analysis.

The paper is organized as follows. Section 2 reviews the principal notions of classicality that appear throughout the literature. Section 3 discusses the major mechanisms that generate classical behavior. Section 4 examines information-theoretic and geometric approaches to the quantum–classical transition. Section 5 proposes a unified interpretation based on the REFF. Finally, Section 6 summarizes the main conclusions and discusses possible directions for future work.

2. The Many Meanings of Classicality

The notion of classicality occupies a central position in physics, yet no universally accepted definition exists. Depending on the physical context, the term may refer to distinct phenomena, different mathematical limits, or alternative operational criteria. Before discussing the quantum–classical transition, it is therefore useful to review the principal meanings that classicality has acquired across different branches of physics.

2.1. Classicality as the Semiclassical Limit

Historically, the most traditional interpretation associates classicality with the semiclassical limit of quantum mechanics. In this viewpoint, classical behavior emerges when the characteristic action scale S of a system greatly exceeds Planck's constant:

$$S \gg \hbar. \quad (2)$$

Under such conditions, wave-like effects become comparatively small, and the dynamics can often be approximated by classical trajectories. The correspondence principle provides the conceptual basis for this interpretation, asserting that quantum predictions should reproduce classical mechanics in the appropriate limit [1].

From this perspective, classicality is identified with the recovery of Newtonian dynamics from the underlying quantum description.

2.2. Classicality as Decoherence

A second and highly influential interpretation arises from decoherence theory. Here, the emphasis is placed not on the limit $\hbar \rightarrow 0$, but on the interaction between a system and its environment.

Environmental coupling suppresses quantum interference by damping off-diagonal elements of the density matrix,

$$\rho_{ij} \rightarrow 0, \quad i \neq j, \quad (3)$$

thereby selecting preferred states that behave approximately classically [2,3].

In this framework, classicality is associated with the loss of coherence and the effective disappearance of observable quantum interference effects.

2.3. Classicality as Thermodynamic Stability

In statistical physics, classical behavior often emerges through the law of large numbers. Macroscopic systems contain enormous numbers of microscopic constituents, and collective averaging suppresses relative fluctuations.

For an extensive observable A , one typically finds

$$\frac{\Delta A}{A} \propto \frac{1}{\sqrt{N}}, \quad (4)$$

where N denotes the number of particles. As N becomes large, fluctuations become negligible relative to the mean value [4].

Within this viewpoint, classicality is identified with the stability and predictability of macroscopic observables rather than with the disappearance of the underlying quantum description.

2.4. Classicality as Decorrelation

Another notion frequently encountered in statistical physics and information theory associates classical behavior with weak correlations.

The characteristic correlation length ξ measures the spatial extent over which fluctuations remain statistically linked. When

$$\xi \rightarrow 0, \quad (5)$$

degrees of freedom become approximately independent.

Many classical statistical models, including the ideal gas, exhibit precisely this property. The resulting fluctuation structure is simple, additive, and weakly organized [9].

From this perspective, classicality corresponds to the suppression of collective correlations.

2.5. Classicality as Geometric Flattening

IG provides a further viewpoint. Statistical states may be represented as points on a manifold equipped with a metric that quantifies distinguishability between neighboring states [5].

In many cases, strongly correlated systems exhibit nontrivial geometric structure characterized by finite curvature. By contrast, idealized classical systems are often associated with weakly curved or flat statistical manifolds [8].

Consequently, one may regard classicality as a regime in which information-geometric curvature becomes negligible:

$$R \rightarrow 0. \quad (6)$$

Within this interpretation, classical behavior emerges through geometric flattening of the underlying fluctuation manifold.

2.6. Classicality as Collective Rigidity

Perhaps the least appreciated notion of classicality arises from collective organization. Strong interactions can lock many microscopic degrees of freedom into coherent macroscopic structures.

Examples include ferromagnets, superconductors, superfluids, and other ordered phases. Although their microscopic origin is fundamentally quantum, these systems often behave macroscopically as rigid collective objects governed by a small number of effective variables [10,11].

In such cases, classicality does not emerge from weak correlations but rather from extremely strong correlations. Independent fluctuation directions disappear because many microscopic constituents move collectively.

2.7. A Possible Conceptual Tension

If one assumes that (1) quantum phenomena are inherently microscopic, whereas (2) macroscopicity is intrinsically associated with classical behavior [note tht (2) is not may not be generally valid], the preceding discussion reveals an important conceptual tension. Some notions of classicality emphasize the suppression of correlations, while others emerge precisely because correlations become extremely strong.

For example,

$$\text{decorrelation} \implies \text{classicality} \quad (7)$$

and

$$\text{collective locking} \implies \text{classicality} \quad (8)$$

appear to represent opposite physical mechanisms.

This observation suggests that classicality may not correspond to a unique physical process. Instead, it may encompass a family of emergent regimes sharing certain macroscopic features despite arising from different microscopic origins.

In the following section we examine these mechanisms in greater detail and investigate whether a common structural principle underlies their apparent diversity.

The natural next step is to move from the *definitions* of classicality (the present Section 2) to the physical mechanisms that produce it, emphasizing that different routes lead to similar macroscopic outcomes.

3. Mechanisms of Classical Emergence

The discussion in the previous section reveals that classicality admits multiple interpretations. A natural question therefore arises: Are these notions connected by a common underlying mechanism, or do they represent fundamentally distinct routes from quantum behavior to classical behavior? Although no universal answer is presently known, several of the most important mechanisms leading to classicality share a common tendency: the progressive REFF. In what follows, we briefly review the principal routes through which such reduction may occur.

3.1. Decoherence and the Suppression of Interference

Perhaps the most widely studied mechanism is decoherence. A quantum system interacting with an environment becomes entangled with external degrees of freedom, causing the suppression of phase coherence between different components of the wave function [2,3].

The reduced density matrix evolves toward a form in which off-diagonal elements become negligible,

$$\rho_{ij} \rightarrow 0, \quad i \neq j. \quad (9)$$

As interference effects disappear, only a restricted set of robust states remains experimentally accessible. The system therefore loses part of its original quantum freedom and behaves increasingly classically.

From a fluctuation perspective, decoherence may be viewed as a reduction of accessible quantum alternatives.

3.2. Semiclassical Localization

A second route toward classicality arises when the characteristic action of a system becomes large compared with Planck's constant. In this regime,

$$S \gg \hbar, \quad (10)$$

and quantum wave packets become increasingly localized in phase space and follow trajectories that approximate classical motion [1].

The effective uncertainty associated with quantum fluctuations becomes small relative to macroscopic scales. Consequently, the accessible region of phase space narrows around classical paths.

Classical behavior therefore emerges through localization rather than through environmental interactions.

3.3. Thermodynamic Averaging

Macroscopic systems provide another important route toward classical behavior. Even when microscopic constituents obey quantum laws, collective averaging suppresses relative fluctuations.

For extensive observables,

$$\frac{\Delta A}{A} \sim \frac{1}{\sqrt{N}}, \quad (11)$$

so that fluctuations become increasingly negligible as the system size grows [4].

In this case, classicality emerges through the statistical stabilization of collective observables. The underlying quantum description remains intact, but its effects become difficult to detect at macroscopic scales.

3.4. Decorrelation and Independent Fluctuations

Classical statistical behavior is frequently associated with weak correlations. When the correlation length satisfies

$$\xi \rightarrow 0, \quad (12)$$

fluctuations become increasingly local and statistically independent.

The resulting system approaches the behavior of idealized classical models in which different degrees of freedom contribute additively to thermodynamic properties [9].

Here, classicality emerges through the disappearance of collective fluctuation structure.

3.5. Collective Locking and Emergent Rigidity

Interestingly, the opposite limit may also produce classical behavior. Strong interactions can generate collective organization that effectively suppresses independent fluctuations.

Examples include ferromagnetic order, superconductivity, superfluidity, and other forms of collective matter [10,11]. In such systems, many microscopic degrees of freedom become locked into coherent macroscopic structures governed by a relatively small number of collective variables.

The resulting behavior often appears highly classical despite originating from strong quantum correlations.

In this case, classicality emerges not through the disappearance of correlations but through their overwhelming dominance.

3.6. A Common Feature

At first sight, the mechanisms discussed above appear unrelated. Decoherence suppresses coherence, semiclassical limits suppress quantum uncertainties, thermodynamic

averaging suppresses relative fluctuations, and decorrelation suppresses collective organization, while collective order arises precisely because correlations become strong.

Despite these differences, all mechanisms share a common structural property: they reduce the number of dynamically relevant fluctuation directions available to the system.

Symbolically,

$$\text{classical emergence} \implies \text{REFF}. \quad (13)$$

The reduction may occur because fluctuations become statistically independent, because they are averaged away, because interference is suppressed, or because they become locked into collective modes. Nevertheless, the resulting macroscopic behavior is characterized by a simpler and more constrained fluctuation structure.

This observation motivates the search for a more general framework capable of describing these apparently diverse mechanisms within a common language. IG is a natural candidate for such a framework, since it quantifies distinguishability, correlations, and fluctuation organization through geometric quantities defined directly on the statistical manifold.

In the next section, we examine how information-theoretic and geometric concepts may provide a unified description of classical emergence.

4. Information Geometry and the Structure of Classicality

The diversity of mechanisms discussed in the previous section naturally raises the question of whether a common mathematical framework exists for describing the emergence of classical behavior. IG is a particularly attractive candidate because it characterizes statistical systems directly in terms of distinguishability, fluctuations, and correlations [5].

The central idea of IG is to regard a family of physical states as a differentiable manifold whose coordinates are given by a set of control parameters,

$$\theta = (\theta^1, \theta^2, \dots, \theta^n). \quad (14)$$

The local distinguishability between neighboring states is quantified by a metric tensor. For classical probability distributions, this metric is given by the Fisher–Rao metric,

$$g_{ij} = \left\langle \frac{\partial \ln p}{\partial \theta^i} \frac{\partial \ln p}{\partial \theta^j} \right\rangle, \quad (15)$$

or

$$g_{ij} = \langle \partial_i \ln p, \partial_j \ln p \rangle = \int p(x; \theta) \partial_i \ln p \partial_j \ln p dx.$$

while quantum systems admit analogous constructions based on the quantum Fisher information [6,7].

The metric provides a measure of the statistical distance between neighboring states,

$$ds^2 = g_{ij} d\theta^i d\theta^j, \quad (16)$$

thereby transforming distinguishability into a geometric concept.

4.1. Thermodynamic Length and Structural Change

One of the most important quantities derived from the information metric is the thermodynamic length [12,13],

$$L = \int \sqrt{g_{ij} d\theta^i d\theta^j}. \quad (17)$$

Thermodynamic length measures the cumulative distinguishability between neighboring equilibrium states along a given path in parameter space.

Unlike ordinary geometric distance, thermodynamic length possesses a direct physical interpretation: it quantifies the amount of structural change experienced by a system as external parameters vary. Large values of L indicate substantial reorganization of fluctuations, while short lengths correspond to relatively minor changes.

The concept therefore provides a natural bridge between statistical distinguishability and physical evolution.

4.2. Curvature and Correlations

Beyond distances, IG also characterizes the intrinsic organization of fluctuations through geometric curvature.

The scalar curvature R measures the extent to which the statistical manifold deviates from local Euclidean geometry. In many thermodynamic systems, curvature is closely related to correlation structure [8].

A widely used scaling relation takes the form

$$R \sim \zeta^d, \quad (18)$$

where ζ denotes the correlation length, and d an effective dimensionality.

Although the precise relation depends on the system under consideration, the essential point is that curvature frequently acts as a geometric measure of collective fluctuation organization. Large values of $|R|$ typically indicate strong correlations, whereas weakly correlated systems often exhibit small curvature. IG therefore establishes a direct connection between geometric structure and statistical organization.

4.3. Complexity and Distinguishability

Information-theoretic approaches have also introduced measures designed to quantify the balance between order and disorder. Examples include the López–Ruiz–Mancini–Calbet statistical complexity [14], Fisher–Shannon complexity [15], and related quantities.

Although these measures differ in detail, they generally attain small values both for perfectly ordered states and for completely random states, reaching larger values in intermediate regimes characterized by nontrivial organization.

This observation reinforces the idea that geometry, distinguishability, and complexity represent complementary aspects of the same underlying fluctuation structure.

4.4. Geometric Flattening

The preceding considerations suggest a possible geometric characterization of classicality.

Many idealized classical statistical models are described by weakly curved or exactly flat manifolds. The classical ideal gas provides the best-known example, possessing

$$R = 0. \quad (19)$$

By contrast, strongly correlated systems typically exhibit nontrivial curvature generated by interactions, coherence, or collective organization.

This motivates the hypothesis that classical emergence may often be accompanied by geometric flattening,

$$R \rightarrow 0. \quad (20)$$

In such a regime, the statistical manifold progressively approaches a locally Euclidean structure, correlations become less important, and neighboring fluctuation directions become increasingly independent.

From this perspective, classicality is not necessarily defined by the disappearance of quantum mechanics itself. Instead, it corresponds to a simplification of the underlying fluctuation geometry.

4.5. Toward a Unified Interpretation

The information-geometric viewpoint reveals a remarkable convergence among several notions introduced earlier.

Decoherence reduces distinguishability between quantum alternatives. Thermodynamic averaging suppresses relative fluctuations. Decorrelation weakens collective organization. Semiclassical localization restricts the accessible region of phase space. Collective order confines fluctuations to a small number of macroscopic modes.

Although these mechanisms differ physically, they all alter the geometric structure of the corresponding state manifold.

This observation suggests that classicality may be understood as a progressive reduction of effective fluctuation complexity. IG is the natural language for describing this reduction, because it simultaneously incorporates distinguishability, correlations, complexity, and geometric organization.

In the following section, we develop this idea further and propose a unified interpretation of classicality based on the REFF.

5. A Unified Interpretation: Classicality as REFF

The preceding discussion suggests that the various notions of classicality encountered in the literature may not be as unrelated as they initially appear. Although decoherence, semiclassical localization, thermodynamic averaging, decorrelation, and collective ordering arise from distinct physical mechanisms, they all tend to reduce the effective freedom of fluctuations.

This observation motivates the central hypothesis of the present work:

$$\boxed{\text{Classicality} \iff \text{REFF.}} \quad (21)$$

The statement is intentionally broad. It does not identify classicality with a particular microscopic mechanism. Instead, it proposes that classical behavior emerges whenever the number of dynamically relevant fluctuation directions becomes significantly reduced.

Refine now the effective fluctuation dimension idea as the participation ratio of the eigenvalues of the Fisher information metric,

$$D_{\text{eff}} = \frac{(\sum_{i=1}^n \lambda_i)^2}{\sum_{i=1}^n \lambda_i^2}, \quad (22)$$

where λ_i denote the eigenvalues of the Fisher (or thermodynamic) metric.

This quantity possesses several desirable mathematical properties.

- It is a standard participation ratio widely used in statistical physics, condensed matter theory, and information geometry.
- It satisfies the bounds

$$1 \leq D_{\text{eff}} \leq n,$$

where n is the dimension of the statistical manifold.

- If all fluctuation directions are equally important, $\lambda_1 = \dots = \lambda_n$, then

$$D_{\text{eff}} = n,$$

indicating that all directions contribute equally to statistical distinguishability.

- If only a single fluctuation direction remains statistically relevant,

$$\lambda_1 \gg \lambda_{i>1},$$

then

$$D_{\text{eff}} \approx 1,$$

indicating an effectively one-dimensional fluctuation structure.

The above further shows how this quantity behaves in each representative physical example: environmental decoherence suppresses independent quantum fluctuations, thermodynamic averaging reduces relative fluctuations through the law of large numbers, collective ordering confines fluctuations to a few macroscopic order parameters, and semiclassical localization restricts the dynamics to a reduced set of collective coordinates. In every case, the Fisher spectrum becomes progressively more concentrated, leading to a reduction of D_{eff} .

Consequently, the central hypothesis of the paper is now formulated as the quantitative statement that the emergence of classical behavior is accompanied by a progressive reduction in the effective number of statistically distinguishable fluctuation directions.

5.1. Accessible Fluctuation Directions

Consider a system described by a family of equilibrium states parameterized by

$$\theta = (\theta^1, \theta^2, \dots, \theta^n). \quad (23)$$

The information metric determines the distinguishability of neighboring states,

$$ds^2 = g_{ij} d\theta^i d\theta^j. \quad (24)$$

The eigenvalues of the metric characterize the sensitivity of the system to perturbations in different directions of parameter space.

Large eigenvalues correspond to highly distinguishable fluctuation directions, whereas small eigenvalues indicate directions that are difficult to resolve experimentally.

When one or more eigenvalues become very small,

$$\lambda_{\min} \rightarrow 0, \quad (25)$$

the corresponding fluctuation directions effectively disappear from the observable description of the system.

The accessible fluctuation manifold then acquires a reduced effective dimensionality.

5.2. Quantifying Effective Fluctuation Freedom

The preceding discussion suggests that classical emergence is associated with a reduction in the number of dynamically relevant fluctuation directions. This observation motivates the introduction of a quantitative measure of *effective fluctuation freedom*.

Within the information-geometric framework, the local fluctuation structure is encoded in the Fisher–Rao metric g_{ij} (or its quantum counterpart). Let λ_i denote the eigenvalues of the metric tensor. These eigenvalues quantify the distinguishability of fluctuations along different directions in parameter space. Large eigenvalues correspond to highly resolvable fluctuation modes, whereas small eigenvalues represent directions that contribute little to observable state discrimination.

A natural measure of the effective number of fluctuation directions is the participation ratio

$$D_{\text{eff}} = \frac{(\sum_i \lambda_i)^2}{\sum_i \lambda_i^2}. \quad (26)$$

This quantity satisfies

$$1 \leq D_{\text{eff}} \leq n, \quad (27)$$

where n is the dimensionality of the parameter manifold.

If all fluctuation directions contribute equally ($\lambda_i = \lambda$), one obtains

$$D_{\text{eff}} = n, \quad (28)$$

indicating maximal fluctuation freedom. By contrast, if a single eigenmode dominates,

$$\lambda_1 \gg \lambda_{i>1}, \quad (29)$$

then

$$D_{\text{eff}} \approx 1, \quad (30)$$

revealing a strong reduction of the accessible fluctuation manifold.

This definition provides a quantitative realization of the central hypothesis proposed in the present work. Classical emergence may be viewed as a process in which

$$D_{\text{eff}} \downarrow, \quad (31)$$

regardless of the microscopic mechanism responsible for the reduction. Decoherence suppresses distinguishable quantum alternatives, thermodynamic averaging diminishes observable fluctuations, decorrelation eliminates collective fluctuation structure, and collective ordering locks many microscopic degrees of freedom into a small number of macroscopic modes. In all cases, the effective dimensionality of fluctuation space decreases.

Although the quantity D_{eff} should presently be regarded as a heuristic indicator rather than a universal measure, it illustrates how the notion of effective fluctuation freedom can be formulated quantitatively within information geometry and may provide a useful starting point for future investigations. This definition is attractive because: 1. It is basis-independent (depends only on metric eigenvalues). 2. It is already familiar in physics as a participation-ratio type measure. This converts our central statement

$$\text{Classicality} \iff \text{REFF}$$

from a qualitative slogan into a mathematically testable hypothesis.

5.3. Classicality Through Decorrelation

The most familiar route to classical behavior arises through the suppression of correlations. If the characteristic correlation length satisfies

$$\xi \rightarrow 0, \quad (32)$$

then fluctuations become increasingly local and statistically independent.

Information-geometric arguments often suggest a corresponding reduction of curvature,

$$R \rightarrow 0, \quad (33)$$

and the manifold approaches an approximately Euclidean structure.

In this situation, classicality emerges because collective fluctuation organization disappears.

5.4. Classicality Through Collective Locking

Interestingly, the opposite mechanism may lead to a similar macroscopic outcome.

Strong interactions can force many microscopic degrees of freedom to fluctuate collectively rather than independently. Examples include ferromagnetic order, superconductivity, superfluidity, and other forms of emergent organization.

In such systems, independent fluctuation directions are not removed because correlations vanish. Rather, they disappear because correlations become so strong that many microscopic variables behave as a single collective entity.

Symbolically,

$$\text{strong correlations} \implies \text{collective locking} \implies \text{reduced fluctuation freedom.} \quad (34)$$

Thus, weak correlations and strong correlations may both produce classical behavior, albeit through entirely different microscopic mechanisms.

5.5. Geometric Interpretation

The information-geometric viewpoint provides a natural way to unify these apparently opposite scenarios.

In both cases, the accessible fluctuation manifold becomes simpler.

For decorrelation,

$$\tilde{\zeta} \rightarrow 0 \implies R \rightarrow 0. \quad (35)$$

For collective locking,

$$\lambda_{\min} \rightarrow 0, \quad (36)$$

and the effective dimension of fluctuation space decreases.

The common feature is therefore not the microscopic origin of the process, but the resulting simplification of fluctuation geometry.

The system progressively loses dynamically relevant directions and becomes describable by a smaller set of effective variables.

5.6. Classicality Revisited

Within this framework, classicality should not be viewed as the opposite of quantumness.

Indeed, some forms of classical behavior emerge precisely because quantum correlations become extremely strong. Ordered phases of matter provide familiar examples in which macroscopic rigidity originates from collective quantum organization. Ordered phases of matter provide familiar examples. Ferromagnets, superconductors, superfluids, Bose–Einstein condensates, and other collective quantum states originate from highly correlated microscopic dynamics. In such systems, individual particles can no longer be regarded as independent entities. Instead, large numbers of microscopic degrees of freedom become locked into coherent collective configurations described by a relatively small set of macroscopic variables [10,11].

The resulting macroscopic behavior often appears remarkably classical. A ferromagnet, for example, behaves as if characterized by a single collective magnetization vector. Likewise, a superfluid may be described by a macroscopic order parameter possessing a well-defined phase throughout the sample. Although these phenomena are fundamentally quantum in origin, their observable behavior exhibits a degree of rigidity and predictability usually associated with classical systems.

From the viewpoint of fluctuations, strong quantum correlations suppress many independent modes of variation. Rather than fluctuating separately, microscopic constituents

fluctuate collectively. Consequently, the effective number of accessible fluctuation directions becomes dramatically reduced. Symbolically,

$$\text{Strong quantum correlations} \implies \dim(\mathcal{F}_{\text{eff}}) \text{ Minim.} \implies D_{\text{eff}} \downarrow, \quad (37)$$

or, even better,

$$\text{Strong quantum correlations} \implies \lambda_2, \lambda_3, \dots, \lambda_n \rightarrow 0 \implies D_{\text{eff}} = \frac{(\sum_i \lambda_i)^2}{\sum_i \lambda_i^2} \downarrow. \quad (38)$$

where λ_i are the eigenvalues of the Fisher information metric.

We then assert that:

$\lambda_2, \lambda_3, \dots, \lambda_n \rightarrow 0 \implies D_{\text{eff}} \downarrow$, which is more consistent with the information-geometric framework to be developed later.

The emergence of classical behavior in such systems therefore differs fundamentally from the conventional decoherence scenario. In decoherence, classicality arises because quantum coherence is destroyed. In contrast, ordered phases retain a highly quantum microscopic structure, yet display classical macroscopic properties because the underlying correlations constrain the system to move collectively.

The essential transition is therefore not

$$\text{quantum} \rightarrow \text{classical}, \quad (39)$$

but rather

$$\text{high-dimensional fluctuation organization} \rightarrow \text{reduced effective fluctuation structure}. \quad (40)$$

From this perspective, the many faces of classicality discussed throughout this article may be regarded as different manifestations of a common structural tendency toward fluctuation reduction, geometric simplification, and effective macroscopic organization.

6. Physical Illustrations

The general ideas developed in the preceding sections become more transparent when applied to representative physical systems. Although the microscopic mechanisms responsible for the emergence of classical behavior differ considerably from one system to another, they may all be analyzed within the same information-geometric framework.

For each example discussed below we explicitly identify:

1. The physical mechanism responsible for classicality;
2. The relevant statistical manifold;
3. The natural information-geometric metric;
4. The parameters characterizing the system;
5. The REFF.

Each representative system discussed below is analyzed according to the same information-geometric scheme. Specifically, we identify (i) the statistical manifold $\mathcal{M}$, (ii) the coordinates θ^μ parametrizing that manifold, (iii) the Fisher (or thermodynamic) metric $g_{\mu\nu}$, (iv) the physical mechanism responsible for classicality, and (v) the evolution of the effective fluctuation dimension D_{eff} . This common structure allows apparently distinct manifestations of classical behavior to be compared quantitatively within a unified geometric framework.

This unified description allows apparently unrelated physical phenomena to be compared quantitatively.

6.1. Ideal Classical Gas

The ideal gas provides the simplest realization of classical statistical behavior.

Since particles do not interact, the microscopic degrees of freedom are statistically independent. Consequently, the equilibrium probability distribution factorizes into one-particle contributions.

The statistical manifold is the Gibbs family

$$p(x; \beta, V, N),$$

parametrized by the inverse temperature β , volume V , and particle number N .

The corresponding Fisher–Rao metric is

$$g_{ij} = \langle \partial_i \ln p, \partial_j \ln p \rangle,$$

which coincides with the thermodynamic fluctuation metric.

Because microscopic correlations are absent, the Ruppeiner scalar curvature satisfies

$$R = 0,$$

a well-known signature of the ideal gas.

Within the present framework, classicality arises through complete decorrelation. Every fluctuation direction is statistically independent, and the information manifold is globally flat.

6.2. Decohering Qubit

The simplest quantum example is provided by a two-level system coupled to an external environment.

Its density matrix may be written as

$$\rho(\gamma) = \frac{1}{2} \begin{pmatrix} 1 & e^{-\gamma} \\ e^{-\gamma} & 1 \end{pmatrix},$$

where γ denotes the accumulated decoherence parameter.

The statistical manifold consists of the one-parameter family

$$\mathcal{M} = \{\rho(\gamma)\}.$$

Neighboring quantum states are distinguished through the quantum Fisher metric,

$$ds^2 = F_Q(\gamma) d\gamma^2,$$

with

$$F_Q(\gamma) = \frac{e^{-2\gamma}}{1 - e^{-2\gamma}}.$$

Since

$$\frac{dF_Q}{d\gamma} < 0,$$

the distinguishability between neighboring states decreases monotonically during decoherence.

The physical mechanism responsible for classicality is the suppression of quantum interference through environmental entanglement.

Information-geometrically, the statistical manifold contracts as coherence disappears. In multiparameter systems, several Fisher eigenvalues become progressively negligible. Therefore, the effective fluctuation dimension

$$D_{\text{eff}} = \frac{(\sum_i \lambda_i)^2}{\sum_i \lambda_i^2}$$

decreases continuously.

6.3. Thermodynamic Averaging

Macroscopic thermodynamic systems provide another familiar route toward classicality. Here, the microscopic dynamics remains quantum mechanical, but the observable behavior becomes increasingly deterministic because of the law of large numbers.

The equilibrium manifold is

$$p(x; \beta, \mu, h, \dots),$$

whose coordinates are intensive thermodynamic variables.

The thermodynamic metric is

$$g_{ij} = \beta^2 \langle \delta X_i \delta X_j \rangle,$$

where X_i denote extensive observables.

For a macroscopic system,

$$\frac{\Delta X}{X} \sim N^{-1/2},$$

so that relative fluctuations vanish in the thermodynamic limit.

The covariance matrix becomes increasingly concentrated around its dominant directions, causing many fluctuation modes to become statistically irrelevant.

Consequently,

$$D_{\text{eff}}$$

decreases as the system approaches macroscopic deterministic behavior.

6.4. Collective Ordering in Condensed Matter

Ordered phases provide an apparently opposite route toward classicality.

Examples include the following:

- Ferromagnets;
- Superconductors;
- Superfluids;
- Bose–Einstein condensates.

The equilibrium states are Gibbs states,

$$\rho = \frac{e^{-\beta H}}{Z},$$

whose natural coordinates are temperature, magnetic field, chemical potential, or interaction strength.

The information metric is

$$g_{ij} = \partial_i \partial_j \ln Z.$$

Unlike the ideal gas, correlations are now extremely strong.

Large numbers of microscopic variables fluctuate collectively through a small number of macroscopic order parameters.

Therefore, the relevant statistical degrees of freedom become

$$\text{spin fluctuations} \longrightarrow \text{magnetization},$$

or

$$\text{particle fluctuations} \longrightarrow \text{condensate order parameter}.$$

The Fisher spectrum becomes increasingly anisotropic, many eigenvalues approach zero, and

$$D_{\text{eff}}$$

is substantially reduced.

Thus, classical behavior emerges not through the disappearance of correlations but through collective organization.

6.5. Semiclassical Localization

The correspondence principle provides a fourth representative example.

A localized wave packet

$$\psi(x; q, p, \sigma)$$

is described by its centroid position q , momentum p , and spatial width σ .

Therefore, the statistical manifold possesses coordinates

$$(q, p, \sigma).$$

The Fisher information metric measures the distinguishability between neighboring wave packets.

As environmental decoherence or coarse graining suppresses rapidly oscillating interference patterns, the wave packet remains localized.

The evolution becomes effectively describable by only a few collective coordinates.

Consequently, many microscopic fluctuation directions disappear from the information metric, reducing

$$D_{\text{eff}}.$$

6.6. Unified Interpretation

The examples discussed above originate from fundamentally different physical mechanisms.

Nevertheless, they all possess the same information-geometric structure.

Each system is characterized by

- A statistical manifold;
- A set of physically meaningful coordinates;
- An information metric;
- A spectrum of Fisher eigenvalues;
- An effective fluctuation dimension.

The microscopic mechanisms responsible for classicality differ substantially.

The resulting geometric evolution, however, exhibits the same general feature: the progressive reduction of statistically distinguishable fluctuation directions.

From this viewpoint,

$$\boxed{\text{Classicality} \iff \text{REFF.}}$$

The present work therefore does not propose a new microscopic mechanism for the quantum–classical transition. Rather, it suggests that many apparently distinct manifestations of classical behavior may be viewed as different realizations of a common information-geometric simplification process.

7. Decohering Qubit

Consider a two-level quantum system interacting with an environment. The density matrix may be written as

$$\rho = \begin{pmatrix} p & d \\ d^* & 1-p \end{pmatrix}, \quad (41)$$

where d measures quantum coherence.

As decoherence proceeds,

$$d \rightarrow 0, \quad (42)$$

the off-diagonal elements disappear and interference effects become unobservable [2,3].

Information-geometrically, neighboring quantum states become increasingly difficult to distinguish through phase-sensitive measurements. The effective fluctuation structure simplifies and approaches that of a classical probability distribution.

This example illustrates classicality through suppression of coherence.

7.1. Decohering Qubit: A Quantitative Information-Geometric Illustration

The simplest realization of the quantum–classical transition is provided by a single qubit interacting with its environment.

For simplicity, we consider a pure dephasing model in which the density operator takes the form

$$\rho(\gamma) = \frac{1}{2} \begin{pmatrix} 1 & e^{-\gamma} \\ e^{-\gamma} & 1 \end{pmatrix}, \quad (43)$$

where the parameter $\gamma \geq 0$ measures the accumulated decoherence. The off-diagonal elements encode the quantum coherence of the state. As the interaction with the environment increases,

$$\gamma \rightarrow \infty,$$

the coherence term satisfies

$$e^{-\gamma} \rightarrow 0,$$

and the density matrix approaches the classical statistical mixture

$$\rho_{\infty} = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The statistical manifold associated with this family of states is

$$\mathcal{M} = \{\rho(\gamma)\},$$

which is parametrized by the single coordinate

$$\theta = \gamma.$$

The distinguishability between neighboring quantum states is measured by the quantum Fisher information,

$$ds^2 = F_Q(\gamma) d\gamma^2, \quad (44)$$

where

$$F_Q(\gamma) = \frac{e^{-2\gamma}}{1 - e^{-2\gamma}}. \quad (45)$$

Equation (45) shows that

$$\frac{dF_Q}{d\gamma} < 0,$$

so that the statistical distinguishability of neighboring quantum states decreases monotonically during decoherence.

From an information-geometric viewpoint, the statistical manifold contracts continuously as coherence is lost. The environment therefore does not merely suppress interference; it also reduces the amount of statistically accessible information carried by the quantum state.

Although the present example involves only a single parameter, it illustrates the general mechanism operating in higher-dimensional systems. There, the Fisher information metric possesses several eigenvalues

$$\{\lambda_i\},$$

whose relative magnitudes quantify the statistically distinguishable fluctuation directions.

As decoherence progresses, one expects several eigenvalues to become progressively smaller, reflecting the suppression of independent quantum fluctuations. This motivates the definition of the effective fluctuation dimension

$$D_{\text{eff}} = \frac{(\sum_i \lambda_i)^2}{\sum_i \lambda_i^2}, \quad (46)$$

which is the well-known participation ratio of the Fisher spectrum.

This quantity satisfies

$$1 \leq D_{\text{eff}} \leq n,$$

where n is the dimension of the statistical manifold. Equal Fisher eigenvalues yield

$$D_{\text{eff}} = n,$$

whereas a single dominant eigenvalue gives

$$D_{\text{eff}} = 1.$$

The quantum–classical transition may therefore be interpreted as a progressive reduction of the effective number of statistically distinguishable fluctuation directions. In this sense, environmental decoherence contracts not only the quantum coherence of the state but also the effective dimensionality of its information-geometric description.

7.2. Concrete Application: Quantifying the Reduction of Effective Fluctuation Freedom During Qubit Decoherence

The previous subsection introduced the information-geometric description of a decohering qubit. We now illustrate the proposed framework through a simple quantitative calculation.

Consider the one-parameter family of density matrices

$$\rho(\gamma) = \frac{1}{2} \begin{pmatrix} 1 & e^{-\gamma} \\ e^{-\gamma} & 1 \end{pmatrix}, \quad (47)$$

where γ denotes the accumulated decoherence.

The corresponding quantum Fisher information is

$$F_Q(\gamma) = \frac{e^{-2\gamma}}{1 - e^{-2\gamma}}. \quad (48)$$

As the off-diagonal coherence decays, neighboring quantum states become progressively less distinguishable.

The effective fluctuation freedom is characterized by the participation ratio

$$D_{\text{eff}} = \frac{(\sum_i \lambda_i)^2}{\sum_i \lambda_i^2}, \quad (49)$$

where λ_i denote the eigenvalues of the Fisher metric.

For the present one-parameter example,

$$D_{\text{eff}} = 1,$$

while the magnitude of the accessible fluctuation direction is measured by the Fisher eigenvalue itself,

$$\lambda(\gamma) = F_Q(\gamma).$$

Thus, the decrease of F_Q directly quantifies the contraction of the information manifold. Table 1 shows representative numerical values.

Table 1. Quantum Fisher information for a decohering qubit. Increasing decoherence suppresses statistical distinguishability, illustrating the progressive REFF.

γ	$e^{-\gamma}$	$F_Q(\gamma)$	Interpretation
0.20	0.819	2.03	Nearly coherent
0.50	0.607	0.58	Moderate decoherence
1.00	0.368	0.156	Strong decoherence
2.00	0.135	0.0187	Almost classical
3.00	0.050	0.0025	Effectively classical

The numerical results clearly exhibit the monotonic reduction of the quantum Fisher information as decoherence progresses. Because the Fisher information measures statistical distinguishability, its decrease corresponds to a contraction of the information manifold.

Within the present framework, this contraction provides a quantitative realization of the central hypothesis proposed throughout this work: classicality emerges through a reduction of effective fluctuation freedom.

Although the present example involves only a single qubit, the same construction immediately generalizes to higher-dimensional quantum systems. There, the complete spectrum of Fisher eigenvalues evolves during decoherence, allowing the participation ratio D_{eff} to decrease continuously from values comparable to the dimension of the statistical manifold toward much smaller values. This suggests that the effective fluctuation dimension constitutes a natural order parameter for the quantum–classical transition.

7.3. Quantitative Case Study: Information-Geometric Collapse of a Decohering Qubit

To illustrate the proposed information-geometric interpretation of the quantum–classical transition, we consider one of the simplest examples: a single qubit undergoing pure dephasing.

The density matrix may be written as

$$\rho(\gamma) = \frac{1}{2} \begin{pmatrix} 1 & e^{-\gamma} \\ e^{-\gamma} & 1 \end{pmatrix}, \quad (50)$$

where the dimensionless parameter $\gamma \geq 0$ measures the accumulated decoherence. For $\gamma = 0$, the state is pure, whereas $\gamma \rightarrow \infty$ corresponds to complete loss of coherence.

7.3.1. Bloch-Vector Representation

Equation (50) corresponds to the Bloch vector

$$\mathbf{r}(\gamma) = (e^{-\gamma}, 0, 0),$$

whose length decreases continuously from

$$|\mathbf{r}| = 1$$

to

$$|\mathbf{r}| = 0.$$

Thus, the quantum state moves from the surface of the Bloch sphere toward its center.

7.3.2. Quantum Fisher Metric

For a one-parameter family of mixed qubit states, the quantum Fisher information is

$$F_Q = \left| \frac{d\mathbf{r}}{d\gamma} \right|^2 + \frac{(\mathbf{r} \cdot \frac{d\mathbf{r}}{d\gamma})^2}{1 - |\mathbf{r}|^2}.$$

Using

$$\mathbf{r} = (e^{-\gamma}, 0, 0),$$

we obtain

$$\frac{d\mathbf{r}}{d\gamma} = (-e^{-\gamma}, 0, 0),$$

which yields

$$F_Q(\gamma) = \frac{e^{-2\gamma}}{1 - e^{-2\gamma}}.$$

The associated information metric is therefore

$$ds^2 = F_Q(\gamma) d\gamma^2.$$

7.3.3. Eigenvalue of the Metric

Since only one independent parameter is present, the Fisher metric possesses a single nonzero eigenvalue,

$$\lambda(\gamma) = F_Q(\gamma).$$

As decoherence progresses,

$$\lambda(\gamma) = \frac{e^{-2\gamma}}{1 - e^{-2\gamma}} \longrightarrow 0, \quad (\gamma \rightarrow \infty).$$

Hence, the statistical distinguishability between neighboring quantum states decreases monotonically.

7.3.4. Effective Fluctuation Dimension

The present work interprets classicality as a progressive reduction of the number of statistically relevant fluctuation directions.

A convenient quantitative measure is the effective fluctuation dimension

$$D_{\text{eff}} = \frac{(\sum_i \lambda_i)^2}{\sum_i \lambda_i^2},$$

where λ_i are the eigenvalues of the Fisher metric.

For the present one-parameter model,

$$D_{\text{eff}} = 1,$$

reflecting the existence of a single fluctuation direction. More generally, however, decoherence of an n -parameter quantum state suppresses some eigenvalues much faster than others,

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n,$$

so that the effective dimension decreases continuously,

$$D_{\text{eff}} < n.$$

In the classical limit, only a small number of statistically significant directions remain accessible.

For greater detail, we proceed as follows. The quantity

$$D_{\text{eff}} = \frac{(\sum_i \lambda_i)^2}{\sum_i \lambda_i^2}, \quad (51)$$

where $\{\lambda_i\}$ denote the eigenvalues of the Fisher information metric, is known in statistical physics as the *participation ratio* (or, equivalently, the inverse participation ratio of the normalized spectrum). It provides a quantitative measure of the effective number of statistically relevant fluctuation directions contributing to the dynamics.

Equation (51) possesses several desirable mathematical properties. First, it is bounded according to

$$1 \leq D_{\text{eff}} \leq n,$$

where n is the dimension of the statistical manifold. Consequently, D_{eff} may be interpreted as an effective statistical dimension.

Second, if all fluctuation directions contribute equally, namely

$$\lambda_1 = \lambda_2 = \cdots = \lambda_n,$$

then

$$D_{\text{eff}} = n,$$

indicating that the full statistical manifold is effectively explored.

Conversely, if only one eigenvalue is appreciable while all others are negligible,

$$\lambda_1 \gg \lambda_2, \dots, \lambda_n,$$

then

$$D_{\text{eff}} \approx 1,$$

showing that the dynamics are effectively confined to a single fluctuation direction.

For the one-parameter decohering qubit considered here, only one independent eigenvalue is present, and therefore

$$D_{\text{eff}} = 1.$$

More generally, however, multi-parameter quantum systems undergoing decoherence typically exhibit a progressive suppression of a subset of the Fisher eigenvalues. As a consequence, the effective dimension decreases continuously during the quantum–classical transition,

$$D_{\text{eff}} < n,$$

providing a quantitative measure of the reduction of effective fluctuation freedom proposed throughout this work.

We therefore propose D_{eff} as a natural information-geometric order parameter for the quantum–classical transition. Unlike purely qualitative descriptions of fluctuation suppression, the participation ratio furnishes a rigorous, basis-independent measure of the number of statistically distinguishable fluctuation modes that remain dynamically accessible. In this sense, the emergence of classicality may be viewed as the progressive contraction of the effective statistical dimension of the underlying information manifold.

7.3.5. Interpretation

This simple calculation illustrates the central hypothesis proposed in this work. The emergence of classicality is accompanied by a contraction of the IG associated with the quantum state. As coherence decays, neighboring quantum states become progressively less distinguishable, the Fisher metric decreases, and the effective number of accessible fluctuation directions is reduced.

Rather than viewing decoherence solely as suppression of off-diagonal density-matrix elements, the information-geometric perspective reveals it as a continuous collapse of the statistical manifold itself. This interpretation naturally connects quantum decoherence with the broader hypothesis advanced throughout this paper, namely that diverse manifestations of classicality correspond to a universal reduction of effective fluctuation freedom.

7.4. Collective Order in Condensed Matter

Ordered phases provide a fundamentally different route toward classical behavior.

In ferromagnets, superconductors, superfluids, and Bose–Einstein condensates, strong microscopic interactions generate collective quantum states characterized by macroscopic order parameters [10,11,16].

A ferromagnet, for example, may contain an enormous number of interacting spins. Nevertheless, its macroscopic behavior is largely described by a single collective magnetization vector.

In this situation, classicality emerges not because correlations disappear, but because they become overwhelmingly strong. Independent microscopic fluctuation directions become locked together, reducing the effective dimensionality of the fluctuation manifold.

Symbolically,

$$\text{strong quantum correlations} \implies \text{collective locking} \implies \text{reduced fluctuation freedom.} \quad (52)$$

This example demonstrates that classical behavior may arise from strong quantum organization rather than from its absence.

8. Thermodynamic Averaging

Macroscopic systems provide another familiar illustration.

For an extensive observable A , one generally finds

$$\frac{\Delta A}{A} \sim \frac{1}{\sqrt{N}}, \quad (53)$$

where N is the number of microscopic constituents [4].

As N increases, relative fluctuations become negligible and observables acquire effectively deterministic values.

The resulting behavior is classical despite the underlying microscopic dynamics remaining quantum mechanical. Here, classicality emerges through statistical averaging and the suppression of observable fluctuations.

Information-Geometric Flattening

The previous examples suggest a common pattern. Whether classicality arises through decorrelation, decoherence, collective locking, or thermodynamic averaging, the effective fluctuation manifold becomes progressively simpler.

This tendency may be expressed geometrically through a reduction of information-geometric complexity. In many situations, the scalar curvature decreases,

$$R \rightarrow 0, \quad (54)$$

or the effective dimensionality of fluctuation space becomes reduced through the disappearance of relevant directions.

From this perspective, geometric flattening is not itself a separate mechanism of classicality. Rather, it may be viewed as a common geometric manifestation of several distinct physical routes toward classical behavior.

These examples support the central thesis of the present work: classicality can often be interpreted as an REFF, regardless of the microscopic mechanism responsible for its emergence.

9. Information-Geometric Characterization of Representative Mechanisms of Classicality

The central hypothesis of this work is that apparently distinct mechanisms leading to classical behavior share a common information-geometric structure. Although the underlying microscopic physics differs substantially among quantum decoherence, thermodynamic averaging, collective ordering, and semiclassical localization, all of them may be interpreted as exhibiting a progressive reduction of the effective number of statistically distinguishable fluctuation directions.

Within the present framework, each physical realization is described by

1. A statistical manifold $\mathcal{M}$ of admissible states;
2. A set of coordinates θ_μ parametrizing that manifold;
3. An information metric $g_{\mu\nu}$;
4. An effective fluctuation dimension D_{eff} constructed from the spectrum of the metric.

This common mathematical language allows the different manifestations of classicality to be compared quantitatively.

9.1. Decohering Qubit

The simplest realization of the quantum–classical transition is provided by a single qubit interacting with its environment.

The density operator may be written as

$$\rho(\gamma) = \frac{1}{2} \begin{pmatrix} 1 & e^{-\gamma} & e^{-\gamma} & 1 \end{pmatrix},$$

where the decoherence parameter γ measures the accumulated loss of phase coherence.

The physical mechanism responsible for classicality is environmental entanglement. Interaction with uncontrolled environmental degrees of freedom suppresses the off-diagonal elements of the density matrix, thereby destroying quantum interference.

The statistical manifold consists of the family

$$\mathcal{M} = \rho(\gamma),$$

which is one-dimensional and parametrized by

$$\theta = \gamma.$$

The distinguishability of neighboring quantum states is quantified by the quantum Fisher information metric,

$$ds^2 = F_Q(\gamma) d\gamma^2.$$

As shown in the preceding subsection,

$$F_Q(\gamma) = \frac{e^{-2\gamma}}{1 - e^{-2\gamma}},$$

which decreases monotonically during decoherence.

Consequently, neighboring quantum states become progressively less distinguishable. In information-geometric terms, the statistical manifold contracts continuously as coherence is lost.

The corresponding participation ratio

$$D_{\text{eff}} = \frac{(\sum_i \lambda_i)^2}{\sum_i \lambda_i^2}$$

measures the effective number of statistically relevant fluctuation directions. For multi-parameter systems, decoherence suppresses several Fisher eigenvalues, producing a continuous reduction of D_{eff} and therefore of the effective fluctuation freedom.

9.2. Thermodynamic Averaging

Macroscopic thermodynamics represents another route toward classicality. Unlike decoherence, the suppression of fluctuations originates from the collective averaging over an enormous number of microscopic degrees of freedom.

The statistical manifold is the equilibrium manifold

$$\mathcal{M} = p(x; \beta, \mu, \dots),$$

whose coordinates are intensive thermodynamic variables, such as inverse temperature β , chemical potential μ , magnetic field h , or pressure.

The natural metric is the Fisher–Rao metric,

$$g_{ij} = \langle \partial_i \ln p, \partial_j \ln p \rangle,$$

which is equivalent to the thermodynamic metric derived from equilibrium fluctuations.

Its components satisfy

$$g_{ij} = \beta^2 \langle \delta X_i \delta X_j \rangle,$$

where X_i denote the extensive observables.

As the number of particles increases,

$$N \rightarrow \infty,$$

relative fluctuations decrease according to

$$\frac{\Delta X}{X} \sim N^{-1/2}.$$

The statistical manifold therefore becomes increasingly concentrated around its equilibrium point.

Information-geometrically, the spectrum of the covariance matrix collapses, reducing the participation ratio and confining the dynamics to a lower-dimensional region of state space.

9.3. Collective Ordering in Condensed Matter

Collective ordering phenomena such as ferromagnetism or superconductivity provide another important manifestation of emergent classical behavior.

The relevant statistical manifold consists of equilibrium Gibbs states,

$$\rho(\beta, h) = \frac{e^{-\beta H(h)}}{Z},$$

parametrized by temperature and external field.

The associated thermodynamic metric is

$$g_{ij} = \partial_i \partial_j \ln Z,$$

whose components coincide with equilibrium susceptibilities.

The emergence of long-range order below the critical temperature suppresses many microscopic fluctuations.

Instead of fluctuating independently, large numbers of microscopic degrees of freedom become correlated through a common order parameter, such as the spontaneous magnetization.

Consequently, only a limited number of collective modes remain statistically relevant.

The reduction of independent fluctuation channels is reflected in the eigenvalue spectrum of the Fisher metric.

Away from the critical point, many eigenvalues become negligible, producing a substantial decrease of the effective dimension

$$D_{\text{eff}}.$$

Thus, ordered phases may be interpreted as lower-dimensional statistical manifolds generated through spontaneous organization.

9.4. Semiclassical Localization

The correspondence between quantum and classical mechanics provides a fourth example.

In semiclassical systems, localized wave packets evolve approximately along classical trajectories.

The statistical manifold is formed by families of wave packets,

$$\psi(x; q, p, \sigma),$$

whose coordinates correspond to their centroid position, momentum, and width.

The Fisher information metric associated with these probability densities measures the distinguishability between neighboring wave packets.

Quantum evolution tends to increase the complexity of interference patterns.

However, environmental interactions or coarse graining suppress highly oscillatory components, causing the wave packet to remain localized.

As localization proceeds, only a few collective coordinates remain necessary to characterize the state.

The remaining microscopic interference degrees of freedom become statistically irrelevant. Accordingly, the Fisher eigenvalue spectrum becomes increasingly sparse, leading once again to a reduction of the effective fluctuation dimension.

9.5. Unified Information-Geometric Interpretation

Despite their distinct microscopic origins, the four examples considered above possess the same mathematical structure.

Each system is characterized by

- A statistical manifold $\mathcal{M}$;
- Coordinates θ_μ describing admissible states;
- An information metric $g_{\mu\nu}$ quantifying statistical distinguishability;
- A spectrum of Fisher eigenvalues λ_i ;
- An effective fluctuation dimension D_{eff} measuring the number of statistically relevant degrees of freedom.

The quantum–classical transition may therefore be interpreted as a progressive contraction of the information manifold driven by the suppression of statistically distinguishable fluctuation directions.

This information-geometric viewpoint unifies decoherence, thermodynamic averaging, collective ordering, and semiclassical localization within a common quantitative framework, despite the fundamentally different microscopic mechanisms responsible for each phenomenon.

10. Conclusions and Open Questions

The quantum–classical transition is often presented as a single phenomenon. The analysis developed in this work suggests a different picture. Rather than possessing a unique definition, classicality appears in several distinct forms across quantum theory, statistical physics, condensed matter physics, and information geometry.

We have reviewed a number of widely used notions of classicality, including the semiclassical limit, decoherence, thermodynamic averaging, decorrelation, geometric flattening, and collective rigidity. Although these mechanisms differ substantially in their microscopic origin, they exhibit a common tendency toward the REFF.

This observation motivated the central proposal of the present article:

$$\boxed{\text{Classicality} \iff \text{REFF.}} \quad (55)$$

Within this framework, classical behavior emerges whenever the number of dynamically relevant fluctuation directions becomes reduced. Such reduction may occur through suppression of coherence, weakening of correlations, thermodynamic averaging, localization in phase space, or the formation of strongly correlated collective structures.

Distinguishability, fluctuations, complexity, thermodynamic length, and curvature become different manifestations of a common geometric organization. In this context,

geometric flattening may be interpreted as one possible route to classicality, while collective locking and dimensional reduction provide another.

An intriguing consequence of this viewpoint is that classicality need not always arise from the disappearance of correlations. In some cases, such as ideal gases or weakly interacting systems, classical behavior emerges through decorrelation. In other situations, including ordered phases and strongly interacting many-body systems, classical macroscopic organization may result from overwhelmingly strong correlations. Weak correlations and strong correlations therefore represent opposite microscopic mechanisms that can produce remarkably similar macroscopic outcomes.

The analysis presented here raises several open questions.

First, is there a quantitative measure capable of characterizing effective fluctuation freedom in a model-independent manner? Information-geometric quantities such as scalar curvature, thermodynamic length, Fisher information, and metric eigenvalues may provide partial answers, but a universally accepted measure remains elusive.

Second, what is the precise relationship between geometric flattening and classicality? While many classical statistical systems possess weakly curved or flat information manifolds, it remains unclear whether flattening is merely a frequent signature of classical behavior or a more fundamental criterion.

Third, can strong collective correlations and weak correlations be described within a single geometric framework? The existence of multiple routes to classicality suggests that both phenomena may correspond to different forms of effective reduction in fluctuation space.

An important limitation of the present work should also be emphasized. The framework developed here concerns information-geometric and statistical descriptions of representative mechanisms leading to effective classical behavior. It is not intended as a theory of quantum gravity, nor does it address approaches in which spacetime or gravity themselves emerge from more fundamental quantum structures. Whether the information-geometric ideas developed here can be extended to such settings remains an interesting topic for future research.

Finally, one may ask whether the notion of classicality itself should be replaced by a broader concept based on fluctuation organization. Such a reformulation would shift attention away from the traditional opposition between quantum and classical physics and toward the geometric and informational structures governing collective behavior.

The answer to these questions remains open. Nevertheless, the present analysis suggests that the many faces of classicality may ultimately be understood as different manifestations of a common tendency toward simplification, stabilization, and reduction of accessible fluctuation structure.

In this sense, the quantum–classical transition may be viewed less as a single physical process than as a family of mechanisms through which complex fluctuation organization gives rise to effective macroscopic order.

It is important to distinguish the type of unification proposed here from the notion of a fundamental microscopic unification of quantum and classical physics. The present work does not claim that the various mechanisms leading to effective classical behavior originate from a single underlying physical process. Rather, it shows that these mechanisms admit a common information-geometric description in terms of the reduction of statistically relevant fluctuation directions. The proposed unification is therefore conceptual and geometric rather than microscopic. Whether a deeper dynamical unification exists remains an open problem that lies beyond the scope of the present investigation.

As a final note, we emphasize that the present contribution is not a new model, but a new way of organizing and comparing the many meanings of classicality.

The proposed framework is experimentally testable because the effective fluctuation dimension can, in principle, be inferred from reconstructed Fisher information metrics (FIM) or thermodynamic covariance matrices. Consequently, the hypothesis that classicality corresponds to a REFF yields quantitative predictions that may be examined across quantum-optical, condensed-matter, and statistical-mechanical systems.

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