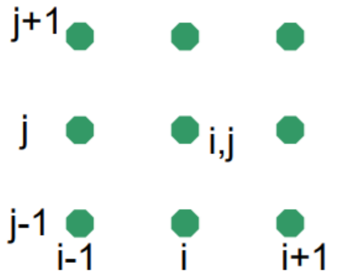
Slope, Aspect, and Curvatures

For this study, a multi-scale approach by fitting quadratic parameters to any size window (via least squares) was used to derive slope, aspect and curvatures. This method was proposed and implemented by Wood [1].

Slope algorithm



Wood and Evans both fit a quadratic surface to the points of the form:

$$z = ax^2 + by^2 + cxy + dx + ey + f \tag{1}$$

Thus

$$\frac{\delta z}{\delta x} = 2ax + cy + d \tag{2}$$

$$\frac{\delta z}{\delta y} = 2by + cx + e \tag{3}$$

and if we use a local coordinate system then we can set

$$x, y = 0 \text{ and so } \delta z/\delta x = d;\ \delta z/\delta y = e \tag{4}$$

The quadratic does not pass through all the points, and the parameters are found by matrix algebra.

In this method, all 9 cells are used to derive slope.

For more details, please see [1].

The key contents are described below.

Evans [2] considers five terrain parameters that may be defined for any two dimensional continuous surface.

elevation

slope, aspect

profile convexity, plan convexity

These correspond to groups of 0, 1st and 2nd order differentials, where the 1st and 2nd order functions have components in the XY and orthogonal planes. Whilst higher derivatives may also be extracted, there is no evidence that these have any geomorphological meaning. Additionally, higher order derivatives must be based on surface patches modelled by higher order polynomials if any component is to be extracted. The higher the order of the polynomial, the greater the number of points required to uniquely identify the necessary coefficients. Since it is only the property of the surface at the central point of each local patch that is required, the effect of generalization increases with the order of polynomial.

Evans goes on to approximate the surface using a bivariate quadratic function in the form:

$$z = ax^2 + by^2 + cxy + dx + ey + f \tag{5}$$

This can be written in the form of the general conic:

$$ax^2 + 2hxy + by^2 + 2jx + 2ky + m = 0 \tag{6}$$

Where $h=c/2$, $j=d/2$, $k=e/2$, and $m=f\text{-}z$.

Instances of the general conic fall into one of three types, depending on the values of the coefficients *a, b* and *h* [3]:

	$ab - h^2 > 0$		*elliptic*
if,	$ab - h^2 = 0$	conic is	*parabolic*
	$ab - h^2 < 0$		*hyperbolic*

Slope and Aspect

The rate of change of elevation in both the *x* and *y* directions can be used to identify the direction and magnitude of steepest gradient. These two parameters ca n be found by taking the partial first order derivatives of *(1)* above, with respect to *x* and *y*. Slope (magnitude) can be found by combining the two component partial derivatives:

$$\frac{dz}{dxy} = \sqrt{\left(\frac{\delta z}{\delta x}\right)^2 + \left(\frac{\delta z}{\delta y}\right)^2} \tag{7}$$

The partial derivatives for *x* and *y* are given as

$$\begin{aligned} \frac{\delta z}{\delta x} &= 2ax + cy + d \\ \frac{\delta z}{\delta y} &= 2by + cx + e \end{aligned} \tag{8}$$

Since we are only interested in the slope at the central point of the quadratic surface, by adopting a local coordinate system with the origin located at the point of interest, we can combine *(3)* and *(4)* where $x = y = 0$ *giving,*

$$\frac{dz}{dxy} = \sqrt{d^2 + e^2} \tag{9}$$

This slope value is more usually represented in degrees, giving:

$$slope = \arctan\left(\sqrt{d^2 + e^2}\right) \tag{10}$$

This definition is consistent with that reported in the literature (e.g. [2,4-6]). Likewise, aspect is simply the polar angle described by the two orthogonal partial derivatives:

$$aspect = \arctan\left(\frac{e}{d}\right) \tag{11}$$

Profile and Plan Curvature (Quadratic surface model)

Profile and plan curvature measures (of dimension [L^{-1}]) are defined as follows,

$$profc = \frac{-200(ad^2 + be^2 + cde)}{(e^2 + d^2)(1 + d^2 + e^2)^{1.5}} \tag{12}$$

$$planc = \frac{200(bd^2 + ae^2 - cde)}{(e^2 + d^2)^{1.5}} \tag{13}$$

Mass Balance Index

The mass-balance index is derived from transformed f(k, ht, n) values (Equation 14). High positive MBI values occur at convex terrain forms, like upper slopes and crests, while lower MBI values are associated with valley areas and concave zones at lower slopes. Balanced MBI values close to zero can be found in midslope zones and mean a location of no net loss or net accumulation of material [7].

$$\mathrm{MBI} = \begin{cases} f(k) \times [1 - f(n)] \times [1 - f(ht)]\ for\ f(k) < 0 \\ f(k) \times [1 + f(n)] \times [1 + f(ht)]\ for\ f(k) > 0 \end{cases} \quad with\ \mathrm{MBI} \in [-1, 3] \tag{14}$$

Where MBI = mass-balance index, k = mean curvature, ht = vertical distance to channel network, and n = slope.

The attributes were transformed according to Equation 15 [7,8]:

$$\mathrm{f(x)} = \frac{x}{(|x| + T_x)} \quad with\ x = k, n, ht, h, f(k) \in [-1, 1], \mathrm{f(n, ht, h)} \in [0, 1] \tag{15}$$

Where Tx = transfer constant, and h = elevation.

References

1. Wood, J. The geomorphological characterisation of digital elevation models. Ph.D. Thesis, Department of Geography, University of Leicester, U.K., 1996.
2. Evans, I.S. *An integrated system of terrain analysis and slope mapping*; Final Report on Grant DA-ERO-591-73-G0040; Department of Geography, University of Durham: England, 1979; p 192.
3. Stephenson, G. *Mathematical methods for science students*, 2nd ed ed.; Longman: Harlow, England, 1973; pp. 528.
4. Skidmore, A.K. A comparison of techniques for calculating gradient and aspect from a gridded digital elevation model. *International Journal of Geographical Information Systems* **1989**, *3*, 323-334, doi:10.1080/02693798908941519.
5. Zevenbergen, L.W.; Thorne, C.R. Quantitative analysis of land surface topography. *Earth Surface Processes and Landforms* **1987**, *12*, 47-56, doi:10.1002/esp.3290120107.
6. Evans, I.S. An integrated system of terrain analysis and slope mapping. *Zeitschrift fur Geomorphologie* **1980**, *36*, 274-295.
7. Möller, M.; Volk, M.; Friedrich, K.; Lymburner, L. Placing soil-genesis and transport processes into a landscape context: A multiscale terrain-analysis approach. *Journal of Plant Nutrition and Soil Science* **2008**, *171*, 419-430, doi:10.1002/jpln.200625039.
8. Friedrich, K. Multivariate distance methods for geomorphographic relief classification. In *Land Information Systems—Developments for Planning the Sustainable Use of Land Resources*, Heineke, H.J., Eckelmann, W., Thomasson, A.J., Jones, R.J.A., Montanarella, L., Buckley, B., Ed. The European Soil Bureau, Joint Research Centre: Ispra, Italy, 1998; Vol. EUR17729EN, pp. 259-266. Available online: https://esdac.jrc.ec.europa.eu/ESDB_Archive/eusoils_docs/esb_rr/n04_land_information_systems/contents.html