A Measurement Method for Cislunar Spacecraft Based on Connected Element Interferometry and BeiDou-3 Interplanetary Link in Future Lunar Exploration

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Abstract: To meet the urgent need for high-precision tracking and reliable cataloging of non-cooperative targets in the Earth–Moon space, this paper proposes a GNSS Inter-Satellite Link and Connected Element Interferometry (CEI)-based measurement method for high-value cislunar space targets. Firstly, the general flow and basic scenario of the proposed method are given, followed by the mathematical model which, mainly includes four parts: (i) dynamical constraint equations for targets; (ii) GNSS-based interplanetary link for irradiation of targets; (iii) transmission loss equation of GNSS inter-satellite link signal in Earth–Moon space; (iv) CEI-based precision measurements of targets. On this basis, the full process link budget analysis is carried out, followed by the performance evaluation, which includes the reception performance of CEI receiving arrays and the measurement accuracy of targets. The feasibility of the proposed method is evaluated and verified in experiments, and it is illustrated that (i) for inter-satellite link visibility analysis, at least 20 satellites can simultaneously provide inter-satellite link signals to the Earth–Moon space targets, with a single GEO satellite up to 8.5 h continuously, while the chain access can be available at up to 73,000 km, with the angle ranging from $-80^\circ$ to $360^\circ$; (ii) the Max Duration of Chain Access for BD3-lunarprobe-CEI (from 24 March 2023 04:00:00.000 to 31 March 2023 10:00:00.000) is 50,998.804 s/day, with a Total Duration of 358,408.797 s in 7 days; (iii) for link budget and measurement accuracy analysis, even beyond the farthest Earth–Moon Lagrangian point, the $C/N_0$ will be above 56.1 dBHZ, while even approaching the distances of $4.5 \times 10^5$ km, the $\sigma_{DLL}$ and $\sigma_{FLL}$ will be below 5.345 m and $3.475 \times 10^{-4}$ m/s, respectively, and the final measurement error will remain at 62.5 m with the proposed method. The findings of this paper could play a key role in future increasingly serious space missions, such as Earth–Moon space situational awareness, and will have a broad application prospect, if put into actual testing and operations.

Keywords: future lunar exploration; Circular Space Situational Awareness; Inter-Satellite Link; Connected Element Interferometry; BeiDou-3; Circular Restricted Three-Body Problem

1. Introduction

Recent years have witnessed mankind accelerating into a new era of lunar exploration and development [1], and there will be more and more cislunar activities [2,3]. In the future, the number of unmanned lunar probes and manned spacecraft will increase and take more and more “one small step for individuals and one giant step for humankind” [4–6].
Cislunar space is a natural extension of the near-Earth orbit space. In a broad sense, it is a region dominated by the gravitational force of the Earth and the Moon, including near-Earth space, lunar gravitational space, and Earth–Moon transfer space; in a narrow sense, it refers to the Earth–Moon transfer space and lunar gravitational space outside the geosynchronous belt [7–10]. As the first stop towards deep space, the cislunar space contains various kinds of materials, energy, environments, locations, and other scarce strategic resources [11], which renders it as a strategic space for human survival and development and the main destination and outpost of space mission activities in the future, including deep space confrontation, in-orbit service and maintenance, deep space science and applications, and support for manned exploration far beyond near-Earth orbit [12,13]. It is foreseeable that space activities of all countries will further focus on the cislunar space.

After the announcement of the New Space Exploration Program in the early 21st century [14], in the future, NASA is planning to launch a nuclear-powered lunar rover to collect more samples from the moon and send them back to Earth by astronauts [15,16]. On 28 June 2022, the United States launched a new generation of the lunar probe “Capstone”, the first successful one launched in nearly a decade [17]. As the forerunner of the Artemis Project, this microwave oven-sized probe is advertised as “the world’s first lunar navigation satellite”, marking the substantial deployment phase of the U.S. version of GPS on the Moon [18].

Recently, China announced that its manned lunar exploration project has been launched, which plans to achieve the first landing of Chinese people on the moon before 2030. Furthermore, China will promote the fourth phase of the lunar exploration project [19–21]: around 2024, the Magpie Bridge-II Relay and CE-6 probe will be launched, to achieve the lunar back sampling return; around 2026 and 2028, CE-7 and CE-8 will be launched, to achieve the lunar south pole resource exploration and constitute the international lunar research station. Meanwhile, China is also demonstrating the construction of a constellation of lunar communication and navigation system—the Magpie Bridge remote integrated constellation system [22], which will be divided into three phases: pilot phase, basic phase, and expansion phase. The pilot phase will be completed before 2030 to support the fourth phase of the lunar exploration project and international lunar research stations; the basic phase will be accomplished before 2040 to realize regional navigation, service manned lunar exploration, international lunar exploration, etc., which can provide relay communication, navigation, and key information support for future lunar surface operations and more complex lunar exploration tasks [23].

On 27 December 2022, Korea’s first lunar orbiter, Moonwatch, successfully entered its scheduled orbit around the Moon, and will further help Project Artemis [24]. In early 2023, the Indian Space Organization announced that India’s third lunar exploration mission will be launched late this year [25]. The Indian Moonship III lander has successively passed electromagnetic compatibility and critical vibration environment tests and will undertake the relay communication mission and cooperate with the Moonship II propulsion module, thus jointly supporting India’s next lunar exploration mission (the lunar rover will work on the lunar surface for about half a month). Japan’s Shiraito-R, which successfully entered lunar orbit at the end of March 2023, carried the UAE’s first lunar rover and a small Japanese robot into the northern Atlas Crater on the lunar front at the end of April to study the movement of lunar soil, rock and dust, and plasma conditions on the lunar surface [26]. ESA has likewise launched plans to establish a network of navigation and communication satellites in orbit around the Moon [27,28].

In addition, several commercial space companies around the world have announced plans to apply jump robots to survey the lunar surface [29]. More lunar exploration missions are underway, with the general hope of obtaining rich lunar information and painting a delicate “portrait of the moon goddess” to lay a solid foundation for lunar resource exploration and development, lunar base construction, and long-term human presence on the Moon [30].
A schematic diagram of the relevant range in cislunar space is shown in Figure 1 (the figure contains the ground-based Connected Element Interferometry (CEI) observation array as well as non-cooperative spacecraft at significant cislunar orbital altitudes).

Given this, cislunar space has become an important base and destination for deep space explorations, which is also the main starting point of this study. The motivation and existing challenges of this research will be summarized as follows.

As future lunar exploration activities will place higher requirements on lunar probe orbiting accuracy, real-time and applicable scenarios such as Positioning, Navigation, and Timing (PNT) of cislunar space explorations \[31,32\] and the high-precision measurement, orbiting, and positioning of non-cooperative spacecraft in cislunar space will thus become a fundamental and cutting-edge research work. Nevertheless, the current means of determining its orbit still relies on ground-based measurements \[33\], including ground-based radar and optical telescopes, which are greatly restricted by geographical location and weather conditions around the stations, ground-based radio ranging, and velocimetry and interferometry, which requires rather long tracking time, especially for the transfer orbit between Earth and Moon \[34\].

Recently, the adoption of the short-time alternating differential measurement mode of radio source (Delta Differential One way Ranging, ΔDOR) has greatly improved the measurement accuracy of deep space probes and has been successfully applied in international deep space exploration missions \[35\]. The China VLBI Network (CVN) \[36\], led by the Shanghai Observatory of the Chinese Academy of Sciences (SOAS), has been successfully applied in China’s lunar exploration missions to support the high-precision orbit determination of lunar probes \[37\]. SOAS has also carried out the same beam interferometry observations of the two spinlets Rstar and Vstar of the Japanese lunar satellite SELENE and obtained differential phase time delays of ps magnitude to achieve ultra-high precision interferometry between lunar probes \[38\].

Furthermore, the farther the probe is from Earth, the worse geometry of ground-based measurement will become, thus limiting the orbiting accuracy and applicability of merely ground-based methods. With the development of cislunar space exploration, space-based measurement technology has emerged \[39\], including interplanetary ranging technology \[40\] and satellite-based GNSS leakage signal technology \[41\], which have been applied in the current Earth satellite navigation system and will undoubtedly be extended to cislunar space. In this field, a series of frontier explorations have been carried out, including the research of autonomous orbiting of constellations based on lunar leveling point orbit \[42\], the joint autonomous orbiting technology of leveling point detector and Earth navigation satellite \[43\], the autonomous orbiting of detector of cislunar triangle
leveling point [44], and the autonomous navigation and timing system of cislunar detector based on DRO-LEO formation [45]. However, the above studies are mainly limited to the realization of the autonomous real-time navigation of the cooperative cislunar spacecraft, and there is still a lack of reliable means for the effective tracking and measurement of the non-cooperative targets in cislunar space.

Conclusively, considering the limitations of existing space-based and ground-based cislunar space target measurement means, this research fully applies the advantages of GNSS interplanetary link signals in flexibility and reliability, with the integration of ground-based radio interferometry means, so as to achieve high-precision cislunar space target tracking and measurement. The specific exploration and verification will be introduced as follows.

As one of the most significant GNSS system, China’s BDS has successfully deployed ISL on BDS-3 series satellites [46]. One of the main roles of the ISL is to perform precision orbit setting and time synchronization of spacecraft [47], and the research on orbit and clock difference determination using BDS-3-measured ISL data, refinement of geometric observation models for the ISL, and modeling correction of system errors has just started [48]. The trajectory of the BeiDou satellites is shown in Figure 2. The space-division time-division access system and Ka-phase array beam shortening capability of the Beidou ISL system exert a strong ability to expand applications [49]. The ISL system of the BeiDou-3 harbors the following features [50,51]: (i) Ka-band inter-satellite link with high ranging accuracy, strong anti-jamming capability, and good confidentiality; (ii) Space-time division multiple access and double-way ranging (Beidou-3 adopts the space-time division multiple access system, which realizes point-to-point link building between satellites); (iii) High–medium link, medium–medium link, and star-ground link mixed link construction (The Beidou-3 constellation consists of 3 GEO satellites, 3 IGSO satellites, and 24 MEO satellites, each carrying an ISL payload to realize the link construction between high–medium orbit satellites and medium–medium orbit satellites, while the ground is equipped with anchoring station equipment to realize the star-ground link building with the space constellation).

![Ground Tracks of BDS Satellites](2023/04/16/01:00 BDT)

**Figure 2.** Trajectory of the BeiDou-3 satellites (16 April 2023, 01:00 BDT).

Taking full advantage of the unique characteristics of satellite navigation systems as a time and space reference and global coverage, given that the BeiDou interplanetary link system is fully configurable and rapidly configurable, the user spacecraft is regarded
as the expansion node of the BeiDou interplanetary link system to access the BeiDou interplanetary link system and use the on-board processor to process the interplanetary measurement data to get its own orbit, which can theoretically meet the demands of real-time orbiting of cooperative and non-cooperative spacecraft in the cislunar space range [52,53].

Based on the above analysis, this paper will utilize the inter-satellite link signal equipped on Beidou-3 satellites, combined with the ground-based CEI equipment, to achieve effective and reliable tracking measurements for non-cooperative spacecraft in cislunar space. The main contributions and structure of this paper are summarized as follows.

(i) A non-cooperative cislunar space targets measurement method based on CEI and BDS-3 inter-satellite link signals is proposed, with its general scenario and specific flow discussed in Section 2. To the best of our knowledge, this is the first time the collaboration of GNSS inter-satellite link signals and interferometry-based technology has been applied in non-cooperative cislunar space target tracking and measurement.

(ii) On this basis, in Section 3, the mathematical models of the proposed method are put forward, which mainly includes four parts: dynamical constraint equations for non-cooperative targets in cislunar space; BDS-based interplanetary link for irradiation of non-cooperative targets; transmission loss equation of BDS-3 inter-satellite link signal in cislunar space; CEI-based precision measurements of targets in cislunar space.

(iii) Based on the previous analysis, Section 4 will focus on the measurement errors of the inter-satellite link signals of Beidou-3 satellites in different orbits during the transmission process in the Earth–Moon space and the CEI observation process of the final received signals reflected by non-cooperative spacecraft. The error correction equation will be introduced to further correct the CEI observation equation of non-cooperative spacecraft in cislunar space.

(iv) Based on the above analysis, to give the feasibility demonstration of the proposed method, the link budget analysis and performance evaluation are presented in Sections 5 and 6, which focus on the carrier-to-noise ratio of the measurement of the target, and analysis of the pseudocode ranging and ranging rate of the GNSS-based interplanetary link signal, respectively.

(v) The research results of this paper (which are demonstrated by the experiments shown in Section 7) can achieve effective and reliable tracking and measurement for non-cooperative targets in cislunar space, indicating a promising potential in the increasingly critical cislunar space situational awareness missions in the future.

2. General Flow and Basic Scenario

In this section, the general flow and basic scenario of the proposed “CEI and BDS-3 inter-satellite link-based measurement method for non-cooperative targets in cislunar space” will be introduced, respectively.

2.1. General Flow of the Proposed Method

Figure 3 shows the general flow of the CEI and BDS-3 inter-satellite link-based measurement method for non-cooperative targets in cislunar space, which mainly consists of the following key steps:

(i) Firstly, the basic model is constructed, focusing on the whole process and key nodes of inter-satellite link-based irradiation of non-cooperative targets by BDS-3 satellites in each orbit, the transmission of signals reflected by targets in Earth–Moon space, and final reception by ground-based CEI arrays.

(ii) Then, we construct the kinetic constraint equations for the measurement of Earth–Moon space targets, the irradiation equations of targets based on the BDS-3 interplanetary link, the accurate measurement equations of targets based on CEI, and the error correction equations of the whole process.
(iii) Subsequently, the link budget of the whole process is performed, especially the establishment of an analytical equation of the load-to-noise ratio of the measurement link of the proposed method.

(iv) Finally, the performance evaluation of target measurement is carried out, especially the reception performance evaluation of CEI receiver arrays.

Figure 3. General flow of the CEI and BDS-3 inter-satellite link-based measurement method for cislunar non-cooperative targets.

2.2. Basic Scenario of the Proposed Method

Figure 4 illustrates an overview of the non-cooperative target tracking and measurement scenario in cislunar space based on the CEI and BDS-3 inter-satellite link.
Figure 4. Overview of the non-cooperative target tracking and measurement scenario in cislunar space based on the CEI and BDS-3 inter-satellite link.

In this model, firstly, based on the pre-observation data and operation law of the non-cooperative targets, the BDS-3 satellites located in each orbit will realize the irradiation of the non-cooperative targets in the cislunar space by applying the inter-satellite link signals between normal missions of inter-satellite link communication. Based on this, the BDS-3 interplanetary link signals reflected by the targets will be transmitted in cislunar space until they reach the ground. Then, the above signals will be reflected by targets and transmitted in cislunar space until they reach the ground. Based on the ground-based CEI receiver array, the above signals are received. Finally, accurate measurement information of the target will be obtained based on the correction of the signal observation error.

2.3. Assumptions and Notations

From the perspective of mechanism analysis, based on model construction, scenario analysis, and simulation iteration, the following assumptions need to be made in this paper:

(i) The research scenario in this paper can be reduced to a “three-body” problem that neglected the BDS-3 satellite as it has a negligible gravitational force on the targets;
(ii) The correction equation of the pseudocode ranging equation between Beidou-3 and targets in cislunar space will focus on large error factors, including antenna phase center offset and relativistic effects, while neglecting others;
(iii) The correction equation of the pseudocode ranging equation between the reflected signals and the ground-based CEI will focus on large error factors, including atmospheric delay errors, phase center offset of the station antenna, relativistic effects, and tidal effects, while neglecting others.

The main symbols used in this paper are illustrated in Table 1.
Table 1. Main symbols used in this study.

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{\text{earth}}$</td>
<td>Mass of the Earth</td>
</tr>
<tr>
<td>$m_{\text{moon}}$</td>
<td>Mass of the Moon</td>
</tr>
<tr>
<td>$x_i, y_i, z_i$</td>
<td>$x, y, z$-axis position coordinates of the $i$th cislunar targets</td>
</tr>
<tr>
<td>$d_i, d_i', d_i''$</td>
<td>$x, y, z$-axis velocity components of the $i$th cislunar targets</td>
</tr>
<tr>
<td>$dd_i, dd_i', dd_i''$</td>
<td>$x, y, z$-axis acceleration components of the $i$th cislunar targets</td>
</tr>
<tr>
<td>$r_{\text{BD}}$</td>
<td>position vector of BDS satellites</td>
</tr>
<tr>
<td>$r_{\text{space}}$</td>
<td>position vector of targets</td>
</tr>
<tr>
<td>$\delta t_{\text{BD}}$</td>
<td>clock difference of BDS satellites relative to BDT</td>
</tr>
<tr>
<td>$\delta t_{\text{space}}$</td>
<td>clock difference of targets relative to BDT</td>
</tr>
<tr>
<td>$\tau_{\text{BD}}$</td>
<td>hardware delay of BDS satellites</td>
</tr>
<tr>
<td>$e_{i, (\text{BD} \rightarrow \text{space})}$</td>
<td>other errors to be corrected</td>
</tr>
<tr>
<td>$n_{i, (\text{BD} \rightarrow \text{space})}$</td>
<td>observation noise</td>
</tr>
<tr>
<td>$L_{i, (\text{BD} \rightarrow \text{space})}$</td>
<td>interplanetary link transmission losses between the BDS satellites and targets</td>
</tr>
<tr>
<td>$L_{i, (\text{space} \rightarrow \text{CEI})}$</td>
<td>interplanetary link transmission losses between the targets and the ground-based CEI receiving array</td>
</tr>
<tr>
<td>$d_{i, (\text{BD} \rightarrow \text{space})}$</td>
<td>the distances between the GNSS satellite and the targets</td>
</tr>
<tr>
<td>$d_{i, (\text{space} \rightarrow \text{CEI})}$</td>
<td>the distances between the targets and the ground-based CEI receiving array</td>
</tr>
<tr>
<td>$\lambda_{\text{BD}}$</td>
<td>the carrier wavelength of the GNSS satellite</td>
</tr>
<tr>
<td>$f_{\text{BD}}$</td>
<td>the carrier frequency of the inter-satellite link of the Beidou-3 satellite</td>
</tr>
<tr>
<td>$w_{\text{antenna}}, (i, (\text{BD} \rightarrow \text{space})$</td>
<td>the antenna phase center offset of Beidou-3 satellites in the Solid Coordinate System (SCS)</td>
</tr>
<tr>
<td>$(P_x, P_y, P_z)$</td>
<td>the projection of $x, y, z$ in the Geocentric Inertial Coordinate System under the SCS</td>
</tr>
<tr>
<td>$c_{\text{dry}}, c_{\text{wet}}$</td>
<td>the zenith delay correction models for the dry and wet delay components</td>
</tr>
<tr>
<td>$m_{\text{dry}}(\varphi), m_{\text{wet}}(\varphi)$</td>
<td>mapping functions for the dry and wet delay components</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>observed elevation angle of the CEI array to targets</td>
</tr>
<tr>
<td>$C$</td>
<td>received signal power at the CEI receiving array</td>
</tr>
<tr>
<td>$N_0$</td>
<td>thermal noise power</td>
</tr>
<tr>
<td>$C/N_0$</td>
<td>the carrier-to-noise ratio</td>
</tr>
<tr>
<td>$T_{\text{transmit}}$</td>
<td>GNSS interstellar link signal transmit power</td>
</tr>
<tr>
<td>$G_{\text{transmit}}(\text{BD} \rightarrow \text{space}), G_{\text{receive}}(\text{space} \rightarrow \text{CEI})$</td>
<td>antenna gain of the GNSS interstellar link transmit and the CEI array receive toward the targets</td>
</tr>
<tr>
<td>$T_{\text{antenna}}, T_{\text{amplifier}}$</td>
<td>antenna and amplifier noise temperature</td>
</tr>
</tbody>
</table>

3. Construction of Mathematical Model

The mathematical model of the proposed BDS-3 Inter-Satellite Link and Connected Element Interferometry-based measurement method for cislunar space targets mainly includes four parts: (i) Dynamical constraint equations for non-cooperative targets in cislunar space; (ii) BDS-based interplanetary link for irradiation of non-cooperative spacecraft in cislunar space; (iii) Transmission loss equation of BDS-3 inter-satellite link signal in cislunar space; (iv) CEI-based precision measurements of targets in cislunar space. The derivation and analysis of the above mathematical model will be described as follows.

3.1. Dynamical Constraint Equations for Non-Cooperative Targets in Cislunar Space

The research scenario in this paper focuses on a “four-body” problem: the Earth, the Moon, BDS-3 satellites, and non-cooperative targets in cislunar space. For the convenience of study, the problem can be reduced to a “three-body” problem (which neglects the BDS-3 satellite as it has a negligible gravitational force on the targets due to its small
weight): the Earth, the Moon, and the non-cooperative spacecraft in cislunar space, and all three are considered as mass points. The corresponding dynamical constraint diagram of the non-cooperative spacecraft in cislunar space is shown in Figure 5. The line between the Earth and the Moon is used as the X-axis, and the origin of the coordinates is the center of gravity of both.

From the figure, it can be seen that the distances \( \lambda \) and \( 1 - \lambda \) of the Earth and the Moon from their respective centers of gravity satisfy:

\[
\lambda = \frac{m_{\text{moon}}}{m_{\text{earth}} + m_{\text{moon}}} \quad (1)
\]

\[
1 - \lambda = \frac{m_{\text{earth}}}{m_{\text{earth}} + m_{\text{moon}}} \quad (2)
\]

where \( m_{\text{earth}} \) and \( m_{\text{moon}} \) are the masses of the Earth and the Moon, respectively. In addition, from the figure, the distances between the Earth and the Moon and the \( k \)th cislunar space non-cooperative targets, respectively.

For \( k \) non-cooperative targets in cislunar space, the state vector containing information such as velocity and position can be expressed as:

\[
S = [x_l, y_l, z_l, x_i, y_i, z_i, x_j, y_j, z_j]^T, l = 1, 2, \ldots, k
\]

(5)

where \( x_l, y_l, z_l \) are the \( x, y, z \)-axis position coordinates of the \( l \)th cislunar space non-cooperative targets, respectively. \( x_i, y_i, z_i \) are the \( x, y, z \)-axis position coordinates, velocity components, and acceleration components of the \( \text{th} \) cislunar space non-cooperative targets, respectively. \( x_l, y_l, z_l \) satisfies:

\[
\begin{align*}
\frac{dd}{dt} x_l &= \frac{\partial p_l}{\partial x_l} + 2 \frac{dd}{dt} y_l \\
\frac{dd}{dt} y_l &= \frac{\partial p_l}{\partial y_l} - 2 x_l \\
\frac{dd}{dt} z_l &= \frac{\partial p_l}{\partial z_l}
\end{align*}
\]

(6)

where \( p_l \) is the potential function that satisfies:

\[
p_l = \frac{x_l^2 + y_l^2}{2} + \frac{1 - \lambda}{r_{es}} + \frac{\lambda}{r_{ms}} \quad (7)
\]

From (1) to (7), the dynamical constraint equations of non-cooperative spacecraft in cislunar space can be finally expressed as:

\[
\begin{align*}
S & = [x_l, y_l, z_l, x_i, y_i, z_i, x_j, y_j, z_j]^T, l = 1, 2, \ldots, k \\
\frac{dd}{dt} x_l &= x_l - \{(1 - \lambda)(x_l + \lambda)(y_l + \lambda)^2 + y_l^2 + z_l^2\}^2 \\
& \quad + \lambda(x_l + 1 - \lambda)[(x_l + 1 - \lambda)^2 + y_l^2 + z_l^2]^{-\frac{3}{2}} + 2 \frac{dd}{dt} y_l \\
\frac{dd}{dt} y_l &= y_l - y_l \{(1 - \lambda)(x_l + \lambda)^2 + y_l^2 + z_l^2\}^2 + \lambda[(x_l + 1 - \lambda)^2 + y_l^2 + z_l^2]^{-\frac{3}{2}} - 2 x_l \\
\frac{dd}{dt} z_l &= -z_l \{(1 - \lambda)(x_l + \lambda)^2 + y_l^2 + z_l^2\}^2 + \lambda[(x_l + 1 - \lambda)^2 + y_l^2 + z_l^2]^{-\frac{3}{2}}
\end{align*}
\]

(8)
3.2. BDS-Based Interplanetary Link for the Irradiation of Non-Cooperative Spacecraft in Earth–Moon Space

This subsection will focus on the establishment of the transmission equations of inter-satellite link signals in cislunar space for GNSS satellites located in different orbits, the transmission of inter-satellite link signals reflected by non-cooperating targets, the reception equations at the receiving end of the CEI array, and the inter-satellite link transmission loss equations, as described separately below.

3.2.1. The Transmission Equations of the Inter-Satellite Link Signals of Beidou-3 Satellites in Different Orbits in Cislunar Space

Based on the inter-satellite link signals of BDS satellites, the pseudocode ranging equations between BDS satellites located in different orbits and non-cooperative targets in cislunar space are shown below.

\[
D_{i(BD\rightarrow space)} = |\mathbf{r}_{space}(t_2) - \mathbf{r}_{BD}(t_1)| + c \cdot \delta t_{space}(t_2) - c \cdot \delta t_{BD}(t_1) + c \cdot \tau_{BD} + e_{i(BD\rightarrow space)} + n_{i(BD\rightarrow space)}
\]

(9)

where \(i = 1, i = 2, i = 3\) is the MEO, GEO, and IGSO orbit, respectively; \(t_1\) is the moment when the BDS satellite launches the interplanetary link, and \(t_2\) is the moment when the interplanetary link reaches the targets; \(\mathbf{r}_{BD}\) and \(\mathbf{r}_{space}\) is the position vector of BDS satellites and targets at different orbital altitudes in the geocentric inertial coordinate system; \(\delta t_{BD}\) and \(\delta t_{space}\) are the clock differences of BDS satellites and targets relative to BDS system time (BDT), respectively; \(\tau_{BD}\) is the hardware delay of BDS satellites at the time of launching the interplanetary link; \(e_{i(BD\rightarrow space)}\) is the other errors to be corrected, including antenna phase center shift, relativistic effects, etc.; and \(n_{i(BD\rightarrow space)}\) is the observation noise.
3.2.2. Transmission of Interplanetary Link Signals Reflected by Non-Cooperative Spacecraft in Cislunar Space and the Final Reception Equation at the Receiving End of the CEI Array

Imitating the above expression, we can obtain the pseudo-code ranging equation between the reflection of the BDS satellite’s inter-satellite link signal after shining on the targets and the ground CEI receiver array, as is shown below.

\[
D_{i(\text{space} \rightarrow \text{CEI})} = |r_{\text{CEI}}(t_{i3}) - r_{\text{space}}(t_{i2})| + c \cdot \delta_{\text{CEI}}(t_{i3}) - c \cdot \delta_{\text{space}}(t_{i2}) + c \cdot \tau_{\text{CEI}} + \epsilon_{i(\text{space} \rightarrow \text{CEI})} + n_{i(\text{space} \rightarrow \text{CEI})}
\]

where \(t_{i3}\) is the moment when the interstellar link reflected by the non-cooperative spacecraft reaches the CEI array on the ground; \(r_{\text{CEI}}\) is the position vector of the ground-based CEI observation array in the geocentric inertial coordinate system; \(\delta_{\text{CEI}}\) is the clock difference of the CEI observation array on the ground relative to the BDT; \(\epsilon_{i(\text{space} \rightarrow \text{CEI})}\) is the other errors to be corrected, including atmospheric delay errors (mainly ionospheric and tropospheric delay errors), relativistic effects, station antenna phase center shifts, tidal effects, etc.; and \(n_{i(\text{space} \rightarrow \text{CEI})}\) is the observation noise.

3.3. Transmission Loss Equation of BDS Inter-Satellite Link Signal in Cislunar Space

The inter-satellite link transmission losses between BDS satellites located in different orbits and non-cooperative targets in cislunar space is shown below:

\[
L_{i(\text{BD} \rightarrow \text{space})} = \left( \frac{4\pi d_{i(\text{BD} \rightarrow \text{space})}}{\lambda_{i\text{BD}}} \right)^2
\]

Similarly, the inter-satellite link transmission losses between the BDS satellites shining onto the targets then reflected by them and the ground-based CEI receiver array is shown as:

\[
L_{i(\text{space} \rightarrow \text{CEI})} = \left( \frac{4\pi d_{i(\text{space} \rightarrow \text{CEI})}}{\lambda_{i\text{BD}}} \right)^2
\]

where \(L_{i(\text{BD} \rightarrow \text{space})}\) and \(L_{i(\text{space} \rightarrow \text{CEI})}\) are the interplanetary link transmission losses between the GNSS satellite and targets in Earth–Moon space, and between the targets and the ground-based CEI receiving array after reflection, respectively; \(d_{i(\text{BD} \rightarrow \text{space})}\) and \(d_{i(\text{space} \rightarrow \text{CEI})}\) are the distances between the GNSS satellite and the targets, and between the targets and the ground-based CEI receiving array, respectively; and \(\lambda_{i\text{BD}}\) is the carrier wavelength of the GNSS satellite interplanetary link.

In the unit of dB, the above interstellar link transmission loss equations can be expressed as:

\[
L_{i(\text{BD} \rightarrow \text{space})} = 92.45 + 20 \lg f_{i\text{BD}} + 20 \lg d_{i(\text{BD} \rightarrow \text{space})}
\]

\[
L_{i(\text{space} \rightarrow \text{CEI})} = 92.45 + 20 \lg f_{i\text{BD}} + 20 \lg d_{i(\text{space} \rightarrow \text{CEI})}
\]

where \(f_{i\text{BD}}\) is the carrier frequency of the inter-satellite link of the Beidou-3 satellite, in GHZ, and the unit of \(d_{i(\text{BD} \rightarrow \text{space})}\) and \(d_{i(\text{space} \rightarrow \text{CEI})}\) is km.
3.4. CEI-Based Precision Measurements of Targets in Earth–Moon Space

Based on the previously established pseudocode ranging Equations (8) and (9) based on the BDS satellite-based interplanetary link signal, the orbiting equations for the CEI observation-based non-cooperative targets in cislunar space can be obtained as:

\[
D_{i\text{(BD→space)}} + D_{i\text{(space→CEI)}} = | \mathbf{r}_{CEI}(t_3) - \mathbf{r}_{space}(t_2)| \\
+ | \mathbf{r}_{space}(t_2) - \mathbf{r}_{BD}(t_1)| + c \cdot | \delta t_{CEI}(t_3) - \delta t_{BD}(t_1)| \\
+ c \cdot (\tau_{BD} + \tau_{CEI}) + (e_{i\text{(BD→space)}} + e_{i\text{(space→CEI)}}) \\
+ (n_{i\text{(BD→space)}} + n_{i\text{(space→CEI)}}) \\
\tag{15}
\]

\[
D_{i\text{(space→CEI)}} - D_{i\text{(BD→space)}} = | \mathbf{r}_{CEI}(t_3) - \mathbf{r}_{space}(t_2)| \\
- | \mathbf{r}_{space}(t_2) - \mathbf{r}_{BD}(t_1)| + c \cdot | \delta t_{CEI}(t_3) - \delta t_{BD}(t_1)| \\
- 2 \cdot (\delta t_{space}(t_2) + \delta t_{BD}(t_1)) \\
+ c \cdot (\tau_{CEI} - \tau_{BD}) + (e_{i\text{(BD→space)}} - e_{i\text{(space→CEI)}}) \\
+ (n_{i\text{(BD→space)}} - n_{i\text{(space→CEI)}}) \\
\tag{16}
\]

4. Observation Error Correction of the Whole Process

Based on the previous analysis, this section will focus on the measurement errors of the inter-satellite link signals of Beidou-3 satellites in different orbits during the transmission process in the Earth–Moon space and the CEI observation process of the final received signals reflected by non-cooperative spacecraft. The error correction equation will be introduced to further correct the CEI observation equation of non-cooperative spacecraft in cislunar space.

4.1. Correction Equation of the Observation Error during the Transmission of the Inter-Satellite Link Signals of Beidou-3 Satellites in Cislunar Space

As mentioned previously, the pseudocode ranging equation (Equation (9)) between Beidou-3 satellites in different orbits and non-cooperative spacecraft in cislunar space has errors that need to be corrected, mainly including antenna phase center offset, relativistic effects, etc. This step will focus on the analysis and correction of the above errors, and the specific equations are shown below.

4.1.1. Error Correction for Antenna Phase Center Shift of Beidou-3 Satellites Transmitting Interstellar Links

The phase center offset of the antenna of a Beidou-3 satellite transmitting an interplanetary link can be expressed in the geocentric inertial coordinate system as:

\[
\mathbf{s}_{i\text{(BD→space)}}^{\text{antenna}} = (p_x, p_y, p_z) \cdot \mathbf{w}_{i\text{(BD→space)}}^{\text{antenna}} \\
\tag{17}
\]

where \( \mathbf{w}_{i\text{(BD→space)}}^{\text{antenna}} \) is the antenna phase center offset of Beidou-3 satellites with different orbital altitudes in the solid coordinate system, while \( (p_x, p_y, p_z) \) is the projection of the three axes in the geocentric inertial coordinate system under the solid coordinate system. Then, the antenna phase center offset correction equation of BeiDou-3 satellites can be expressed as follows:

\[
e_{i\text{(BD→space)}}^{\text{antenna}} = \frac{\mathbf{r}_{BD} - \mathbf{r}_{space}}{|\mathbf{r}_{BD} - \mathbf{r}_{space}|} \cdot \mathbf{s}_{i\text{(BD→space)}}^{\text{antenna}} \\
\tag{18}
\]

where, as described previously, \( \mathbf{r}_{BD} \) and \( \mathbf{r}_{space} \) are the position vectors of Beidou-3 satellites and non-cooperative spacecraft in Earth–Moon space at different orbital altitudes in the geocentric inertial coordinate system, respectively.
Combining (17) and (18), the antenna phase center offset correction equation for the Beidou-3 satellite can be finally expressed as:

$$
\varepsilon_{\text{antenna}}^{(\text{i}(\text{BD} \rightarrow \text{space}))} = \frac{\mathbf{r}_{\text{BD}} - \mathbf{r}_{\text{space}}}{|\mathbf{r}_{\text{BD}} - \mathbf{r}_{\text{space}}|} \cdot (\mathbf{P}_s \times \mathbf{P}_y \times \mathbf{P}_z) \cdot \mathbf{w}_{\text{antenna}}^{(\text{i}(\text{BD} \rightarrow \text{space}))}
$$

(19)

4.1.2. Error Correction for Relativistic Effects

The effect of the Beidou-3 satellite clock time deviation on the pseudocode ranging values due to relativistic effects can be expressed as:

$$
\mathbf{s}_{\text{i}(\text{BD} \rightarrow \text{space})}^{\text{Relativity_1}} = \frac{2(\mathbf{r}_{\text{BD}} \cdot \mathbf{r}_{\text{space}} - \mathbf{r}_{\text{space}} \cdot \mathbf{r}_{\text{space}})}{c}
$$

(20)

where \( \mathbf{r}_{\text{BD}} \) and \( \mathbf{r}_{\text{space}} \) are the distances from the Beidou-3 satellite and the non-cooperative spacecraft in Earth–Moon space, respectively; \( \gamma \) is the post-Newtonian effect parameter; \( GM \) is the Earth’s gravitational constant; \( |\mathbf{r}_{\text{BD}}| \) and \( |\mathbf{r}_{\text{space}}| \) are the distances from the Beidou-3 satellite and the non-cooperative spacecraft in Earth–Moon space to the Earth’s center, respectively; \( \rho_{\text{i}(\text{BD} \rightarrow \text{space})} \) is the distance between the Beidou-3 satellite and the non-cooperative spacecraft in Earth–Moon space.

In summary, the total error correction equation for the relativistic effect can be obtained as follows:

$$
\mathbf{s}_{\text{i}(\text{BD} \rightarrow \text{space})}^{\text{Relativity_total}} = \frac{2(\mathbf{r}_{\text{BD}} \cdot \mathbf{r}_{\text{space}} - \mathbf{r}_{\text{space}} \cdot \mathbf{r}_{\text{space}})}{c} + (1 + \gamma) \frac{GM}{c^2} \ln \left( \frac{|\mathbf{r}_{\text{BD}}| + |\mathbf{r}_{\text{space}}| + \rho_{\text{i}(\text{BD} \rightarrow \text{space})}}{|\mathbf{r}_{\text{BD}}| + |\mathbf{r}_{\text{space}}| - \rho_{\text{i}(\text{BD} \rightarrow \text{space})}} \right)
$$

(22)

4.1.3. The Total Correction Equation for the Observation Error during the Transmission of the Inter-Satellite Link Signals of Beidou-3

By synthesizing the above analysis, the total correction equation for the observation error during the transmission of the inter-satellite link signals of Beidou-3 satellites can be obtained as:

$$
\varepsilon_{\text{i}(\text{BD} \rightarrow \text{space})} = \varepsilon_{\text{antenna}}^{(\text{i}(\text{BD} \rightarrow \text{space}))} + \mathbf{s}_{\text{i}(\text{BD} \rightarrow \text{space})}^{\text{Relativity_total}} + \mathbf{s}_{\text{i}(\text{BD} \rightarrow \text{space})}^{\text{Relativity_2}} + \mathbf{s}_{\text{i}(\text{BD} \rightarrow \text{space})}^{\text{Relativity_1}}
$$

(23)

4.2. Observation Error Correction Equation during CEI Observation of Inter-Satellite Link Signals Reflected by Non-Cooperative Spacecraft

As described previously, the pseudocode ranging equation (Equation (10)) between the Beidou-3 satellite’s interplanetary link signal reflected by the non-cooperative spacecraft and the ground-based CEI receiving array has errors that need to be corrected, mainly including atmospheric delay errors (mainly ionospheric and tropospheric delay errors), phase center offset of the station antenna, relativistic effects, tidal effects, etc. This subsection will focus on the analysis and correction of the above errors, and the specific equations are shown below.
4.2.1. Atmospheric Delay Error

The tropospheric delay correction equation is shown below:

$$s_{\text{Troposphere}} = c_{\text{dry}} \cdot m_{\text{dry}}(\varphi) + c_{\text{wet}} \cdot m_{\text{wet}}(\varphi)$$  \hspace{1cm} (24)

where $c_{\text{dry}}$ and $c_{\text{wet}}$ are the zenith delay correction models for the dry and wet delay components, respectively; $m_{\text{dry}}(\varphi)$ and $m_{\text{wet}}(\varphi)$ are the mapping functions for the dry and wet delay components, respectively; and $\varphi$ is the observed elevation angle of the CEI array to the non-cooperating spacecraft.

The ionospheric delay correction equation is shown below:

$$s_{\text{Ionosphere}} = \left[1 - \frac{R_e \cos \varphi}{R_e + H}\right]^{-\frac{1}{2}} \cdot \frac{40.3N_{\text{total}}}{c \cdot f_{\text{IBD}}}$$  \hspace{1cm} (25)

where, as mentioned above, $f_{\text{IBD}}$ is the carrier frequency of the Beidou-3 satellite interstellar link, $N_{\text{total}}$ is the total amount of electrons in the zenith direction, $R_e$ is the radius of the Earth, and $H$ is the height of the thin ionosphere.

4.2.2. Error Correction for the Phase Center Shift of the Station Antenna

Imitating the error correction process of the antenna phase center offset of the Beidou-3 satellite transmitting the interstellar link, the error correction equation for the phase center offset of the station antenna can be obtained as shown below:

$$c_{\text{antenna}} = \frac{r_{\text{CEI}} - r_{\text{Space}}}{|r_{\text{CEI}} - r_{\text{Space}}|} \cdot (P_x, P_y, P_z) \cdot w_{\text{antenna}}$$  \hspace{1cm} (26)

where $w_{\text{antenna}}$ is the station antenna phase center offset in the solid coordinate system, $(P_x, P_y, P_z)$ is the projection of the three axes in the solid coordinate system in the geocentric inertial coordinate system as described above, and $r_{\text{CEI}}$ and $r_{\text{Space}}$ are the position vectors in the geocentric inertial coordinate system of the CEI observation array and the non-cooperative spacecraft in Earth–Moon space located on the ground, respectively.

4.2.3. Relativistic Effect

Following the error correction process of the relativistic effect between the Beidou-3 satellite launching the interplanetary link and the non-cooperative spacecraft in Earth–Moon space that has been analyzed, the error correction equation of the relativistic effect between the non-cooperative spacecraft in the Earth–Moon space and the ground-based CEI observation array can be obtained as follows:

$$s_{\text{Relativity total}} = s_{\text{Relativity 1}} + s_{\text{Relativity 2}}$$  \hspace{1cm} (27)

$$s_{\text{Relativity total}} = 2 \frac{(r_{\text{Space}} - r_{\text{CEI}}) \cdot (r_{\text{Space}} - r_{\text{CEI}})}{c} + (1 + \gamma) \frac{GM}{c^2} \ln \frac{|r_{\text{CEI}}| + |r_{\text{Space}}| + \rho_{(\text{Space})}}{|r_{\text{CEI}}| + |r_{\text{Space}}| - \rho_{(\text{Space})}}$$

where $r_{\text{CEI}}$ and $r_{\text{Space}}$ are the velocities of the ground-based CEI array and the Earth–Moon space non-cooperative spacecraft in the geocentric inertial coordinate system, respectively; $\gamma$ is the post-Newton effect parameter; $GM$ is the Earth’s gravitational constant; $|r_{\text{CEI}}|$ and $|r_{\text{Space}}|$ are the distances from the ground-based CEI array and the Earth–Moon space non-cooperative spacecraft to the earth center, respectively; and $\rho_{(\text{Space})}$ is the distance between the ground-based CEI array and the Earth–Moon space non-cooperative spacecraft.

4.2.4. Tidal Effect

The tidal effect mainly causes the displacement of the CEI observation array, which affects the pseudocode ranging quantity between the non-cooperative spacecraft and the
Remote Sens. 2023, 15, 3744

4.2.5. The Total Correction Equation for the Observation Error during CEI Observations

Combining the above analysis, the total correction equation for the observation error during CEI observations of interstellar link signals reflected by non-cooperative spacecraft can be obtained as:

\[
e_i^{T_o}(\text{space} \rightarrow \text{CEI}) = s_{T_o}^{T_o} + s_{I_o}^{T_o} + s_{\text{antenna}}^{T_o} + s_{\text{Relativity_total}}^{T_o} + c_{\text{Tidal}}^{T_o}
\]

where \( GM_i \) is the gravitational constant of the tide-generating object (Moon at \( i = 1 \) and Sun at \( i = 2 \)); \( R \) and \( R_i \) are the geocentric positions of the CEI observing array and the tide-generating object, respectively; \( \hat{R} \) and \( \hat{R}_i \) are the unit vectors corresponding to \( R \) and \( R_i \); \( h^2 \) is the Love number; and \( l^2 \) is the Shida number.

5. Link Budget Analysis for the Measurements of Non-Cooperative Targets

The complete link schematic of this part is shown in Figure 6, which mainly includes the analysis of the load-to-noise ratio of the non-cooperative spacecraft tracking and measurement link in Earth–Moon space based on the CEI and BeiDou interplanetary link. The BeiDou-3 satellite transmits the inter-satellite link signal, which is reflected back by the Earth–Moon space target after irradiating it, and the signal is then received by the CEI array on the ground.

Figure 6. Complete link schematic for the measurements of non-cooperative targets.
The load-to-noise ratio of the non-cooperative spacecraft tracking and measurement link in Earth–Moon space based on the CEI and BeiDou interplanetary link can be expressed as follows:

\[ \frac{C}{N_0} = C - N_0 \]  
(30)

where \( C \) is the received signal power in \( dB \) at the CEI receiving array; \( N_0 \) is the thermal noise power in \( dB \); and the carrier-to-noise ratio \( C/N_0 \) is in \( dB - Hz \).

The received signal power \( C \) and the thermal noise power \( N_0 \) satisfy the following relations, respectively:

\[
C = T_{\text{transmit}} + G_{\text{transmit}}(BD\rightarrow space) + G_{\text{receive}}(space\rightarrow CEI) - L_{\text{reflect}} - L_{BD\rightarrow CEI} - L_{\text{receive}} 
\]  
(31)

\[
N_0 = 10\log_{10}[k_B(T_{\text{antenna}} + T_{\text{amplifier}})] 
\]  
(32)

where \( T_{\text{transmit}} \) is the GNSS interstellar link signal transmit power in \( dBW \); \( G_{\text{transmit}}(BD\rightarrow space) \) and \( G_{\text{receive}}(space\rightarrow CEI) \) are the antenna gain of the GNSS interstellar link transmit and the CEI array receive toward the targets in \( dBi \), respectively; \( L_{\text{reflect}} \) is the GNSS interstellar link signal reflection loss through the targets in \( dB \); \( L_{BD\rightarrow CEI} \) is the GNSS signal transmission loss in \( dB \) in Earth–Moon space; \( L_{\text{receive}} \) is the reception loss in \( dB \) at the CEI receiving array end; \( k_B = 1.3806452 \times 10^{-23} \) J/K; and \( T_{\text{antenna}} \) and \( T_{\text{amplifier}} \) are the antenna and amplifier noise temperature in \( K \), respectively.

\[
L_{BD\rightarrow CEI} = 20\log_{10} \left( \frac{4\pi f_{BD}(d_{(BD\rightarrow space)} + d_{(space\rightarrow CEI)})}{c} \right) 
\]  
(33)

where \( f_{BD} \) is the Beidou interplanetary link signal power, \( d_{(BD\rightarrow space)} \) is the distance between the Beidou-3 satellite transmitting the Beidou interplanetary link signal and the non-cooperative spacecraft, and \( d_{(space\rightarrow CEI)} \) is the distance between the non-cooperative spacecraft and the ground-based CEI receiver array.

The antenna noise temperature \( T_{\text{antenna}} \) and the amplifier noise temperature \( T_{\text{amplifier}} \) in (32) satisfy the following relations, respectively:

\[
T_{\text{antenna}} = T_{\text{antenna},1} + T_{\text{antenna},2} 
\]  
\[
= T_{\text{physical,antenna}} \cdot (e_{\text{antenna}}^{-1} - 1) + T_{\text{physical,antenna}} \cdot \left( \frac{25O_{\text{earth}} T_{\text{brightness,earth}} + O_{\text{moon}} T_{\text{brightness,moon}} + (4\pi - O_{\text{earth}} - O_{\text{moon}}) T_{\text{brightness,cosmic}}}{4\pi} \right) 
\]  
(34)

\[
T_{\text{amplifier}} = T_{\text{physical,amplifier}} \cdot (10^{N_{\text{figure}}} - 1) 
\]  
(35)

where \( T_{\text{antenna},1} \) is the ohmic loss caused by the antenna’s own defects; \( T_{\text{antenna},2} \) is the thermal noise captured by the antenna from the surrounding environment; \( T_{\text{physical,antenna}} \) is the physical temperature of the antenna; \( e_{\text{antenna}} \) is the antenna efficiency; \( O_{\text{earth}} \) and \( O_{\text{moon}} \) are the fixed angles subtracted by the Earth and the Moon, respectively, when the CEI receiving array is the viewing angle; \( T_{\text{brightness,earth}} \), \( T_{\text{brightness,moon}} \) and \( T_{\text{brightness,cosmic}} \) are the brightness temperatures of the Earth, the Moon, and the cosmic background, respectively; \( T_{\text{physical,amplifier}} \) is the physical temperature of the amplifier; and \( N_{\text{figure}} \) is the amplifier noise factor.
Combining (30) and (35), the load-to-noise ratio equation of the non-cooperative spacecraft tracking and measurement link in Earth–Moon space based on the CEI and BeiDou interplanetary link can be finally expressed as follows:

\[
\frac{C}{N_0} = T_{\text{transmit}} + G_{\text{transmit}}(\text{BD} \rightarrow \text{space}) + G_{\text{receive}}(\text{space} \rightarrow \text{CEI}) - L_{\text{reflect}} \\
-20\log_{10}\left(\frac{4\pi f_{\text{BD}}(d_{\text{BD} \rightarrow \text{space}}) + d_{\text{space} \rightarrow \text{CEI}}}{c}\right) - L_{\text{receive}} \\
-10\log_{10}\left(k_B \cdot \left[ T_{\text{physical antenna}} \cdot \left( \frac{1}{\text{antenna}} - 1 \right) \right] + T_{\text{physical antenna}} \right) \\
\left( \frac{25\Omega \text{brightness}_{\text{earth}} + \Omega \text{brightness}_{\text{moon}} + (4\pi - \Omega \text{earth} - \Omega \text{moon}) \text{brightness}_{\text{cosmic}}}{4} \right) \\
+ T_{\text{physical amplifier}} \cdot \left( 10^{N_{\text{figure}} - 1} \right) \right) \tag{36}
\]

6. Performance Analysis of the Proposed Method

This section mainly includes the reception performance evaluation of CEI receiving arrays, the geometric accuracy evaluation of non-cooperative spacecraft orbiting in Earth–Moon space based on the CEI and BeiDou interplanetary link, and the measurement accuracy evaluation of non-cooperative spacecraft in Earth–Moon space based on the CEI and BeiDou interplanetary link, which are analyzed as follows.

6.1. Reception Performance Evaluation of CEI Receiving Arrays

This subsection focuses on the evaluation of the reception performance of the CEI receiver array. The thermal noise in the phase-locked and frequency-locked loops of the receivers of the CEI receiver array affects the accurate measurement of the pseudocode ranging and ranging rate of the GNSS-based interplanetary link signal and the full process of the CEI measurement of targets, respectively. The corresponding 1σ uncertainty can be expressed as follows:

\[
\sigma_{\text{DLL}} = \lambda_{\text{code}} \sqrt{\frac{B_{\text{noise}}(1 + \frac{1}{T_{\text{integration}}C/N_0})}{2B_{\text{front-end}} T_{\text{chip}} C/N_0}} 
\]

\[
\sigma_{\text{FLL}} = \frac{\lambda_{\text{carrier}}}{2\pi T_{\text{integration}}} \sqrt{\frac{4B_{\text{noise}}}{C/N_0} \cdot \left( 1 + \frac{1}{T_{\text{integration}}C/N_0} \right)} \tag{38}
\]

where \(\lambda_{\text{code}}\) is the wavelength of the GNSS interstellar link signal ranging stream, \(\lambda_{\text{carrier}}\) is the carrier wavelength of the GNSS interstellar link signal, \(B_{\text{noise}}\) is the ranging code loop noise bandwidth, \(B_{\text{front-end}}\) is the loop bi-directional bandwidth, \(T_{\text{chip}}\) is the ranging stream period, and \(T_{\text{integration}}\) is the integration time.

For \(\lambda_{\text{code}}, \lambda_{\text{carrier}},\) and \(T_{\text{chip}}\) in the above Equations (37) and (38), there are:

\[
\lambda_{\text{code}} = \frac{c}{R_{\text{ranging chip}}} \tag{39}
\]

\[
\lambda_{\text{carrier}} = \frac{c}{f_{\text{BD}}} \tag{40}
\]

\[
T_{\text{chip}} = \frac{L_{\text{chip}} \lambda_{\text{code}}}{c} \tag{41}
\]

Therefore, the final reception performance evaluation equation of the CEI receiver array can be obtained as:

\[
\begin{cases}
\sigma_{\text{DLL}} = \frac{c}{R_{\text{ranging chip}}} \sqrt{\frac{B_{\text{noise}}}{2C/N_0} \cdot \frac{1}{T_{\text{front-end}} T_{\text{chip}} \lambda_{\text{code}} \cdot \left( 1 + \frac{1}{T_{\text{integration}}C/N_0} \right)}} \\
\sigma_{\text{FLL}} = \frac{c}{2\pi f_{\text{BD}} T_{\text{integration}}} \sqrt{\frac{4B_{\text{noise}}}{C/N_0} \cdot \left( 1 + \frac{1}{T_{\text{integration}}C/N_0} \right)}
\end{cases} \tag{42}
\]
6.2. The Geometric Accuracy Evaluation of Non-Cooperative Spacecraft Orbiting in Earth–Moon Space Based on CEI and BeiDou Interplanetary Link

The geometric accuracy of non-cooperative spacecraft orbiting in Earth–Moon space based on the CEI and BeiDou interplanetary link is an important indicator to judge the effectiveness of the method proposed in this paper, which can be expressed as follows:

\[ PDOP_{BD\rightarrow space\rightarrow CEI} = \sqrt{\sum_{i=1}^{j} \frac{1}{\lambda_i}} \]  

(43)

Let the observation matrix of the fixing process of this method be \( H \), the eigenvalues of \( H^T H \) be \( \lambda_i (i = 1, 2, \ldots, j) \), and \( PDOP_{BD\rightarrow space\rightarrow CEI} \) be the geometric accuracy of the full process of fixing.

6.3. The Range Accuracy Evaluation of Non-Cooperative Spacecraft in Earth–Moon Space Based on CEI and BeiDou Interplanetary Link

The non-cooperative spacecraft ranging accuracy in Earth–Moon space based on the CEI and BeiDou interplanetary link is another important metric to judge the effectiveness of this method, which can be expressed as follows:

\[ UERE_{BD\rightarrow space\rightarrow CEI} = \sqrt{\sigma_{clock\_error}^2 + \sigma_{ephemeris\_error}^2 + \sigma_{multipath\_error}^2 + \sigma_{receiver\_error}^2 + \sigma_{thermal\_error}^2} \]  

(44)

where \( \sigma_{clock\_error} \) is the BeiDou interstellar link clock difference; \( \sigma_{ephemeris\_error} \) is the BeiDou interstellar link ephemeris error; \( \sigma_{multipath\_error} \) is the multipath error of the BeiDou interstellar link throughout transmission; \( \sigma_{receiver\_error} \) is the receiver noise and resolution error of the CEI receiver array; and \( \sigma_{thermal\_error} \) is the thermal noise code tracking error, which is a jitter function of the altitude, acquisition, and tracking threshold, and the signal chip rate of the non-cooperative spacecraft.

7. Experimental Verification and Analysis

To validate the feasibility of the proposed method, in this paper, we will mainly focus on the BeiDou-3 Global Satellite Navigation System, which is now fully operational, as the main GNSS inter-satellite link simulation source, and the experiment mainly consists of five parts: (i) Simulation scenarios construction and analysis; (ii) Analysis of inter-satellite link visibility (from Beidou-3 constellation to lunarprobe-1); (iii) Inter-satellite link accessibility (from Beidou-3 constellation to cisilunar targets to CEI) analysis; (iv) Analysis of link budget of non-cooperative targets in Earth–Moon space; (v) Analysis of measurement accuracy of cisilunar targets with the proposed method.

7.1. Experimental Parameter Setting

The primary parameter setting of the following experiments is summarized as follows, and mainly includes the setting of the BeiDou-3 system, non-cooperative cisilunar targets (which will be denoted as lunarprobe-1), and CEI ground stations (which includes stations in Beijing and Kashi).

The orbit types and specific parameters of the BeiDou-3 are shown in Table 2.

Table 2. Parameter setting of the BeiDou-3 based scenario.

<table>
<thead>
<tr>
<th>Type</th>
<th>Number</th>
<th>Orbital Altitude</th>
<th>Orbital Inclination</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEO</td>
<td>24</td>
<td>21,528 km</td>
<td>55°</td>
</tr>
<tr>
<td>GEO</td>
<td>3</td>
<td>35,786 km</td>
<td>0°</td>
</tr>
<tr>
<td>IGSO</td>
<td>3</td>
<td>35,786 km</td>
<td>55°</td>
</tr>
</tbody>
</table>
### Table 3. The specific parameters of the BeiDou-3.

<table>
<thead>
<tr>
<th>Satellite Type</th>
<th>PRN</th>
<th>Clock Type</th>
<th>Launch Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEO-1</td>
<td>C19</td>
<td>Rubidium clock</td>
<td>5 November 2017</td>
</tr>
<tr>
<td>MEO-2</td>
<td>C20</td>
<td>Rubidium clock</td>
<td>5 November 2017</td>
</tr>
<tr>
<td>MEO-3</td>
<td>C21</td>
<td>Rubidium clock</td>
<td>12 February 2018</td>
</tr>
<tr>
<td>MEO-4</td>
<td>C22</td>
<td>Rubidium clock</td>
<td>12 February 2018</td>
</tr>
<tr>
<td>MEO-5</td>
<td>C23</td>
<td>Rubidium clock</td>
<td>29 July 2018</td>
</tr>
<tr>
<td>MEO-6</td>
<td>C24</td>
<td>Rubidium clock</td>
<td>29 July 2018</td>
</tr>
<tr>
<td>MEO-11</td>
<td>C25</td>
<td>Hydrogen clock</td>
<td>25 August 2018</td>
</tr>
<tr>
<td>MEO-12</td>
<td>C26</td>
<td>Hydrogen clock</td>
<td>25 August 2018</td>
</tr>
<tr>
<td>MEO-7</td>
<td>C27</td>
<td>Hydrogen clock</td>
<td>12 January 2018</td>
</tr>
<tr>
<td>MEO-8</td>
<td>C28</td>
<td>Hydrogen clock</td>
<td>12 January 2018</td>
</tr>
<tr>
<td>MEO-9</td>
<td>C29</td>
<td>Hydrogen clock</td>
<td>30 March 2018</td>
</tr>
<tr>
<td>MEO-10</td>
<td>C30</td>
<td>Hydrogen clock</td>
<td>30 March 2018</td>
</tr>
<tr>
<td>MEO-13</td>
<td>C32</td>
<td>Rubidium clock</td>
<td>19 September 2018</td>
</tr>
<tr>
<td>MEO-14</td>
<td>C33</td>
<td>Rubidium clock</td>
<td>19 September 2018</td>
</tr>
<tr>
<td>MEO-15</td>
<td>C34</td>
<td>Hydrogen clock</td>
<td>15 October 2018</td>
</tr>
<tr>
<td>MEO-16</td>
<td>C35</td>
<td>Hydrogen clock</td>
<td>15 October 2018</td>
</tr>
<tr>
<td>MEO-17</td>
<td>C36</td>
<td>Rubidium clock</td>
<td>19 November 2018</td>
</tr>
<tr>
<td>MEO-18</td>
<td>C37</td>
<td>Rubidium clock</td>
<td>19 November 2018</td>
</tr>
<tr>
<td>IGSO-1</td>
<td>C38</td>
<td>Hydrogen clock</td>
<td>20 April 2019</td>
</tr>
<tr>
<td>IGSO-2</td>
<td>C39</td>
<td>Hydrogen clock</td>
<td>25 June 2019</td>
</tr>
<tr>
<td>IGSO-3</td>
<td>C40</td>
<td>Hydrogen clock</td>
<td>5 November 2019</td>
</tr>
<tr>
<td>MEO-19</td>
<td>C41</td>
<td>Hydrogen clock</td>
<td>16 December 2019</td>
</tr>
<tr>
<td>MEO-20</td>
<td>C42</td>
<td>Hydrogen clock</td>
<td>16 December 2019</td>
</tr>
<tr>
<td>MEO-21</td>
<td>C43</td>
<td>Hydrogen clock</td>
<td>23 November 2019</td>
</tr>
<tr>
<td>MEO-22</td>
<td>C44</td>
<td>Hydrogen clock</td>
<td>23 November 2019</td>
</tr>
<tr>
<td>MEO-23</td>
<td>C45</td>
<td>Rubidium clock</td>
<td>23 September 2019</td>
</tr>
<tr>
<td>MEO-24</td>
<td>C46</td>
<td>Rubidium clock</td>
<td>23 September 2019</td>
</tr>
<tr>
<td>GEO-1</td>
<td>C59</td>
<td>Hydrogen clock</td>
<td>1 November 2018</td>
</tr>
<tr>
<td>GEO-2</td>
<td>C60</td>
<td>Hydrogen clock</td>
<td>9 March 2020</td>
</tr>
</tbody>
</table>

The parameters of the lunarprome-1 and CEI ground stations are shown in Tables 4 and 5, respectively.

### Table 4. Parameter setting of lunarprome-1.

<table>
<thead>
<tr>
<th>Maneuver Type</th>
<th>Duration (s)</th>
<th>Distance (km)</th>
<th>Orientation Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Launch</td>
<td>24</td>
<td>21,528 km</td>
<td>55°</td>
</tr>
<tr>
<td>Coast</td>
<td>3</td>
<td>35,786 km</td>
<td>0°</td>
</tr>
<tr>
<td>TransLunarInjection</td>
<td>3</td>
<td>35,786 km</td>
<td>55°</td>
</tr>
<tr>
<td>ToSwingBy</td>
<td>24</td>
<td>21,528 km</td>
<td>55°</td>
</tr>
<tr>
<td>ToPeriselene</td>
<td>3</td>
<td>35,786 km</td>
<td>0°</td>
</tr>
</tbody>
</table>

### Table 5. Parameter setting of CEI ground stations.

<table>
<thead>
<tr>
<th>Station</th>
<th>Latitude (Degree)</th>
<th>Longitude (Degree)</th>
<th>Altitude (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-1</td>
<td>40.072</td>
<td>116.257</td>
<td>0.041</td>
</tr>
<tr>
<td>B-2</td>
<td>39.908</td>
<td>116.42</td>
<td>0.049</td>
</tr>
<tr>
<td>B-3</td>
<td>40.1172</td>
<td>116.228</td>
<td>0.038</td>
</tr>
<tr>
<td>K-1</td>
<td>39.497</td>
<td>75.873</td>
<td>1.401</td>
</tr>
<tr>
<td>K-2</td>
<td>39.545</td>
<td>76.012</td>
<td>1.213</td>
</tr>
<tr>
<td>K-3</td>
<td>39.505</td>
<td>75.929</td>
<td>1.255</td>
</tr>
</tbody>
</table>

### 7.2. Simulation Scenarios Construction and Analysis

Based on the STK commercial software, the 2D simulation scenario of Beidou-3’s sub-star point trajectory from the perspective of Earth is shown in Figure 7, which reflects the coverage of the Beidou-3 constellation, while the specific location of the CEI measurement
station in the 2D simulation scenario is shown in Figure 8, which includes three CEI stations in Beijing (which have been denoted as B-1, B-2, and B-3 in Table 5) and another three in Kashi (which have been denoted as K-1, K-2, and K-3).

The joint scenario simulation of the ground-based CEI station, the BeiDou-3 constellation, and the non-cooperative spacecraft in Earth–Moon space is shown in Figures 9–13, with the farthest simulation distance of $4.9 \times 10^6$ km from the center of the Earth, and the simulation start time is 24 March 2023 04:00:00.000, which ends at 24 April 2023 04:00:00.000.

The spatial extent of the cislunar space studied in this paper (which highlights the Beidou-3 constellation) is presented in Figure 9, in earth inertial axes and moon inertial axes, respectively, followed by the simulation of a lunar exploration mission of the cislunar spacecraft (which will be called lunaprobe-1, from launch, maneuver to propagate (the whole process from launch in Earth, to Earth–Moon transfer maneuver, to the final lunar orbit)), which is illustrated in Figure 10. Figure 11 presents the significant performance indicators of lunaprobe-1 during the lunar exploration mission, which includes J2000 Classical Orbit Elements, inertial position and velocity, and LLA position.

**Figure 7.** The 2D simulation scenario of Beidou-3’s sub-star point trajectory.

**Figure 8.** The specific location of the CEI measurement station in the 2D simulation scenario.
Figure 9. The spatial extent of the Earth–Moon studied in this paper (which highlights the Beidou-3 constellation): (a) in earth inertial axes (b) in moon inertial axes.

Figure 10. Simulation of lunar exploration mission of the cislunar spacecraft (from launch, maneuver, to propagate): (a) in earth inertial axes (b) in moon inertial axes.
Figure 11. Key performance indicators of lunarprobe-1 during lunar exploration mission: (a) J2000 Classical Orbit Elements (b) inertial position and velocity (c) LLA position.

To be closer to the real lunar exploration missions, the constraints during the spacecraft flight has been added, and the adjusted trajectory of the lunarprobe-1 with more constraints in 100 iterations is shown in Figure 12. On this basis, to facilitate the analysis of link budget and measurement accuracy of the proposed method, the simulation scenario of link budget analysis in cislunar space based on the CEI and BeiDou-3 interstellar link has been constructed, as depicted in Figure 13.
Figure 12. The adjusted trajectory of lunar exploration mission of the cislunar spacecraft with more constraints (in 100 iterations): (a) in earth inertial axes (b) in moon inertial axes.

Figure 13. Simulation of link budget analysis in cislunar space based on CEI and BeiDou-3 interstellar link: (a) in earth inertial axes (b) in moon inertial axes.
7.3. Inter-Satellite Link Visibility (from Beidou-3 Constellation to Lunarpb-1) Analysis

To demonstrate the feasibility of the proposed method, this subsection will conduct a visibility analysis of the inter-satellite link (from Beidou-3 constellation to lunarprobe-1), during the simulation period (from 24 March 2023 04:00:00.000 to 24 April 2023 04:00:00.000).

The access time and chain access AER between the interstellar link of Beidou-3 constellation and lunarprobe-1 are illustrated in Figure 14 and Figure 15, respectively. The analysis time of the access time and chain access AER are 20 April 2023 23:06:15 and 21 April 2023 09:48:22.

![Figure 14. The access time between lunarprobe-1 and Beidou-3 constellation.](image)

![Figure 15. The chain access AER of the link from Beidou-3 constellation to lunarprobe-1.](image)

Figure 14 shows that for the entire BD-3 constellation (a total of 24 satellites), at least 20 satellites can simultaneously provide inter-satellite link signals to the Earth–Moon space targets, and a single GEO satellite can provide inter-satellite link signals for up to 8.5 h continuously. Meanwhile, Figure 15 confirms that the chain access can be available at up to 73,000 km, with the angle ranging from $-80^\circ$ to $360^\circ$. By comparison, similar work has been conducted in [54,55] to improve the assigning links of the inter-satellite links of GNSSs for the downlink of telemetry data in time and effective ranging under limited facilities and links, with the same simulation scene of BDS-3, and inter-satellite and satellite-ground visibility matrices were calculated. However, the final results in [54] illustrated that the number of visible satellites varied from 6 to 10, and the ratio of the proposed algorithms was <50% with more than 9 visible satellites, while [55] showed the number of visible satellites through ISL ranging from 10 to 24, with the maximum number of links being 17, and the average number of links was 9.6319.

Followed by the above simulation is the detailed information on link visibility, including the time nodes when the link started and ended, as well as the duration of each link,
which is presented in Figure 16 and Table 6. It can be seen that the Min Duration is 670.669 s (from 24 March 2023 04:00:00.000 to 24 March 2023 04:11:10.669), while the Max Duration is 620,793.171 s (from 24 March 2023 05:28:02.118 to 31 March 2023 09:54:35.289), and the Mean Duration is 36.14332 h, with a Total Duration of 5.782 days.

![Figure 16](image_url)

**Figure 16.** The accessibility and duration time of the link from Beidou-3 constellation to lunarpobe-1.

**Table 6.** Satellite-lunarpobe-1: Access Analysis.

<table>
<thead>
<tr>
<th>Access</th>
<th>Start Time (UTC)</th>
<th>Stop Time (UTC)</th>
<th>Duration (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>lunarpobe-1-To-GEO2</td>
<td>24 March 2023 06:04:58.670</td>
<td>31 March 2023 09:54:35.289</td>
<td>618,576.619</td>
</tr>
<tr>
<td>lunarpobe-1-To-IGSO1</td>
<td>24 March 2023 05:58:34.902</td>
<td>31 March 2023 09:54:35.289</td>
<td>618,960.387</td>
</tr>
<tr>
<td>lunarpobe-1-To-IGSO2</td>
<td>24 March 2023 05:58:34.902</td>
<td>31 March 2023 09:54:35.289</td>
<td>618,960.387</td>
</tr>
<tr>
<td>lunarpobe-1-To-IGSO3</td>
<td>24 March 2023 05:58:34.902</td>
<td>31 March 2023 09:54:35.289</td>
<td>618,960.387</td>
</tr>
<tr>
<td>lunarpobe-1-To-MEObasic11</td>
<td>24 March 2023 06:34:45.654</td>
<td>31 March 2023 09:54:35.289</td>
<td>616,789.635</td>
</tr>
<tr>
<td>lunarpobe-1-To-MEObasic12</td>
<td>24 March 2023 06:33:18.126</td>
<td>31 March 2023 09:54:35.289</td>
<td>616,877.163</td>
</tr>
<tr>
<td>lunarpobe-1-To-MEObasic13</td>
<td>24 March 2023 05:28:02.118</td>
<td>31 March 2023 09:54:35.289</td>
<td>620,793.171</td>
</tr>
<tr>
<td>lunarpobe-1-To-MEObasic14</td>
<td>24 March 2023 05:46:15.401</td>
<td>31 March 2023 09:54:35.289</td>
<td>619,699.888</td>
</tr>
<tr>
<td>lunarpobe-1-To-MEObasic16</td>
<td>24 March 2023 06:04:21.223</td>
<td>31 March 2023 09:54:35.289</td>
<td>618,614.066</td>
</tr>
<tr>
<td>lunarpobe-1-To-MEObasic17</td>
<td>24 March 2023 06:16:23.083</td>
<td>31 March 2023 09:54:35.289</td>
<td>617,892.206</td>
</tr>
<tr>
<td>lunarpobe-1-To-MEObasic18</td>
<td>24 March 2023 06:25:33.967</td>
<td>31 March 2023 09:54:35.289</td>
<td>617,341.323</td>
</tr>
<tr>
<td>lunarpobe-1-To-MEObasic22</td>
<td>24 March 2023 05:49:14.981</td>
<td>26 March 2023 08:52:50.257</td>
<td>183,815.277</td>
</tr>
<tr>
<td>lunarpobe-1-To-MEObasic23</td>
<td>24 March 2023 05:59:20.806</td>
<td>26 March 2023 07:17:58.043</td>
<td>177,517.237</td>
</tr>
<tr>
<td>lunarpobe-1-To-MEObasic24</td>
<td>24 March 2023 06:12:50.537</td>
<td>26 March 2023 18:29:12.379</td>
<td>216,981.842</td>
</tr>
<tr>
<td>lunarpobe-1-To-MEObasic26</td>
<td>24 March 2023 06:28:42.342</td>
<td>26 March 2023 15:16:35.871</td>
<td>204,473.529</td>
</tr>
<tr>
<td>lunarpobe-1-To-MEObasic27</td>
<td>24 March 2023 06:38:54.434</td>
<td>26 March 2023 13:40:24.555</td>
<td>198,090.121</td>
</tr>
<tr>
<td>lunarpobe-1-To-MEObasic31</td>
<td>24 March 2023 06:13:33.011</td>
<td>31 March 2023 09:54:35.289</td>
<td>618,062.278</td>
</tr>
<tr>
<td>lunarpobe-1-To-MEObasic32</td>
<td>24 March 2023 06:14:02.804</td>
<td>31 March 2023 09:54:35.289</td>
<td>618,032.485</td>
</tr>
<tr>
<td>lunarpobe-1-To-MEObasic33</td>
<td>24 March 2023 06:13:28.492</td>
<td>31 March 2023 09:54:35.289</td>
<td>618,066.797</td>
</tr>
<tr>
<td>lunarpobe-1-To-MEObasic34</td>
<td>24 March 2023 08:11:03.965</td>
<td>31 March 2023 09:54:35.289</td>
<td>611,011.324</td>
</tr>
<tr>
<td>lunarpobe-1-To-MEObasic35</td>
<td>24 March 2023 07:10:45.801</td>
<td>31 March 2023 09:54:35.289</td>
<td>614,629.488</td>
</tr>
<tr>
<td>lunarpobe-1-To-MEObasic36</td>
<td>24 March 2023 05:30:56.928</td>
<td>31 March 2023 09:54:35.289</td>
<td>620,618.361</td>
</tr>
<tr>
<td>lunarpobe-1-To-MEObasic37</td>
<td>24 March 2023 05:33:40.458</td>
<td>31 March 2023 09:54:35.289</td>
<td>620,454.831</td>
</tr>
<tr>
<td>lunarpobe-1-To-MEObasic38</td>
<td>24 March 2023 06:12:19.288</td>
<td>31 March 2023 09:54:35.289</td>
<td>618,136.001</td>
</tr>
</tbody>
</table>
7.4. Inter-Satellite Link Accessibility (from Beidou-3 Constellation to Lunarpobe-1 to CEI) Analysis

Based on the above inter-satellite link visibility (from Beidou-3 constellation to lunarpobe-1) analysis, in this subsection, the inter-satellite link accessibility (from Beidou-3 constellation to lunarpobe-1 to CEI) analysis will be given to further demonstrate the feasibility of the proposed method.

Figure 17 presents the accessibility of the link from the Beidou-3 constellation to lunarpobe-1 to CEI, while Figures 18 and 19 illustrates the chain access AER of the link (CEI stations located at Beijing and Kashi, respectively).

Figure 17. The accessibility of the link from Beidou-3 constellation to lunarpobe-1 to CEI.

Figure 18. The chain access AER of the link from Beidou-3 constellation to lunarpobe-1 to CEI (in Beijing Station).

Figure 19. The chain access AER of the link from Beidou-3 constellation to lunarpobe-1 to CEI (in Kashi Station).
It can be seen in Figure 17 that, with the collaboration of ISL of the Beidou-3 constellation, the link can be almost connected all the time, from 24 March 2023 04:00:00.000 to 24 April 2023 04:00:00.000. Meanwhile, in Figures 18 and 19, the GEO-1 satellite in the BeiDou-3 constellation was selected as the radiation source for transmitting the ISL, which illustrates that the distance between GEO-1 and lunarpobe-1 ranged from $0.5 \times 10^5 \text{ km}$ to $4.7 \times 10^5 \text{ km}$, while that between lunarpobe-1 and CEI stations ranged from $0.1 \times 10^5 \text{ km}$ to $4.5 \times 10^5 \text{ km}$, with azimuth and elevation fluctuating periodically, with only short jumps, and each duration remaining at almost 18 h.

The Complete Chain Access of BD3-lunarpobe-CEI (from 24 March 2023 04:00:00.000 to 31 March 2023 10:00:00.000) is shown in Table 7, which shows that the Min Duration is 18,282.217 s/day, while the Max Duration is 50,998.804 s/day, and the Mean Duration is 44,801.100 s/day, with a Total Duration of 358,408.797 s in 7 days.

Table 7. Chain–Chain-BD3-lunarpobe-CEI: Complete Chain Access.

<table>
<thead>
<tr>
<th>Access</th>
<th>Start Time (UTC)</th>
<th>Stop Time (UTC)</th>
<th>Duration (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>26 March 2023 04:45:27.093</td>
<td>26 March 2023 18:35:02.417</td>
<td>49,775.324</td>
</tr>
<tr>
<td>4</td>
<td>27 March 2023 04:49:01.082</td>
<td>27 March 2023 18:45:35.825</td>
<td>50,194.743</td>
</tr>
<tr>
<td>7</td>
<td>30 March 2023 04:52:37.628</td>
<td>30 March 2023 19:02:36.432</td>
<td>50,998.804</td>
</tr>
</tbody>
</table>

7.5. Link Budget Analysis

The link budget of non-cooperative targets is illustrated in Figures 20–22, with simulation parameters shown in Table 8, which mainly focus on the carrier-to-noise ratio ($C/N_0$ of the ISL during transmission) at the CEI receiver with the increasing distance of cis lunar space targets from Earth, in the case of variations of spacecraft RCS, CEI Receiving Antenna Aperture, and CEI Receiving Antenna Efficiency.

Table 8. Simulation parameters of link budget and measurement accuracy analysis.

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEI Receiving Antenna Aperture $D$ (Nominal value)</td>
<td>35 m</td>
</tr>
<tr>
<td>CEI Receiving Antenna Aperture $D$ (Simulation value)</td>
<td>35 m, 30 m, 25 m, 20 m, 15 m, 12 m, 10 m</td>
</tr>
<tr>
<td>CEI Receiving Antenna Efficiency $\eta$ (Nominal value)</td>
<td>0.56</td>
</tr>
<tr>
<td>CEI Receiving Antenna Efficiency $\eta$ (Simulation value)</td>
<td>0.76, 0.72, 0.68, 0.64, 0.6, 0.56, 0.52</td>
</tr>
<tr>
<td>spacecraft RCS (Nominal value)</td>
<td>$7 \text{ m}^2$</td>
</tr>
<tr>
<td>spacecraft RCS (Simulation value)</td>
<td>$9 \text{ m}^2$, $7 \text{ m}^2$, $5 \text{ m}^2$, $3 \text{ m}^2$, $1 \text{ m}^2$, $0.5 \text{ m}^2$, $0.1 \text{ m}^2$</td>
</tr>
</tbody>
</table>
Figure 20. Carrier-to-noise ratio at the CEI receiver in the case of spacecraft RCS variations (BD-3 in GEO/IGSO).

Figure 21. Carrier-to-noise ratio at the CEI receiver in the case of CEI Receiving Antenna Aperture variations (BD-3 in GEO/IGSO).

Figure 22. Carrier-to-noise ratio at the CEI receiver in the case of CEI Receiving Antenna Efficiency variations (BD-3 in GEO/IGSO).
It can be seen that when the distance of cislunar space targets to Earth range from $8 \times 10^4$ km (which has not reached Earth–Moon transfer orbit) to $45 \times 10^4$ km (which is beyond the farthest Earth–Moon Lagrangian point), the $C/N_0$ of ISL at the CEI receiver will reach from 93.2 dBHZ to 56.1 dBHZ, and the level will be slightly higher (approximately 0.95 dB higher) from the ISL radiation source on GEO and IGSO compared to that on MEO. The above analysis demonstrates that the carrier-to-noise ratio of this method meets the reception and observation requirements in the practical application of lunar exploration scenarios, which can achieve the effective tracking and measurement of cislunar spacecraft.

### 7.6. Measurement Accuracy Analysis

Based on the feasibility analysis above, this subsection will verify the measurement accuracy of the proposed method for cislunar spacecraft, which is illustrated in Figures 23–27, with similar simulation parameters shown in Table 8. The measurement accuracy of the pseudocode ranging (in Figures 23 and 24) and ranging rate (in Figures 25 and 26) from the CEI receiver will be presented, followed by the final measurement accuracy for cislunar spacecraft (in Figure 27).

**Figure 23.** Pseudocode ranging performance of CEI receiver arrays in the case of spacecraft RCS variations (BD-3 in GEO/IGSO).

**Figure 24.** Pseudocode ranging performance of CEI receiver arrays in the case of CEI Receiving Antenna Aperture variations (BD-3 in GEO/IGSO).
Figures 23 and 24 illustrate that when the distance of cis-lunar space targets to Earth range from $8 \times 10^4$ km to $45 \times 10^4$ km, the $\sigma_{DLL}$ of ISL at the CEI receiver will reach from 5.345 m to 6.917 m, and the level will be slightly lower (approximately 0.32 m) from the ISL radiation source on GEO and IGSO compared to that on MEO. Meanwhile, Figures 25 and 26 illustrate that the $\sigma_{FLL}$ of ISL at the CEI receiver will reach from $3.125 \times 10^{-4}$ m/s to $4.052 \times 10^{-4}$ m/s, and the level will be slightly lower (approximately $0.025 \times 10^{-4}$ m/s) from ISL radiation source on GEO and IGSO compared to that on MEO.

When RCS is 9 m$^2$ and CEI Receiving Antenna Aperture $D$ is 35 m, the $\sigma_{DLL}$ and $\sigma_{FLL}$ can remain at 5.345 m and $3.475 \times 10^{-4}$ m/s, even approaching the distances of $4.5 \times 10^5$ km.

In Figure 27, the final measurement accuracy for cis-lunar spacecraft through the proposed method has been given, which proves that the accuracy of the proposed method for cis-lunar spacecraft stays in the order of 75 m. When RCS is 9 m$^2$ and CEI Receiving Antenna Aperture $D$ is 35 m, the measurement error remain at 62.5 m, even approaching the distances of $4.5 \times 10^5$ km. By contrast, the method proposed in [56] achieved position accuracies of 100 m for the DRO, Earth-orbit, or lunar-orbit, through the Linked Autonomous Interplanetary Satellite Orbit Navigation (LiAISON) technique to analyze satellite-to-satellite tracking (SST) range measurements between a DRO satellite and an-
other satellite in cislunar space to determine their absolute orbital states. Furthermore, due to their method’s weak measuring geometry and extended orbital period, the Earth–Moon transfer orbit was less detectable.

Figure 27. Final measurement accuracy for cislunar spacecraft through proposed method (BD-3 in GEO/IGSO).

The above simulations and analysis illustrate that the proposed method could achieve effective and reliable tracking and measurement for non-cooperative targets in the Earth–Moon space, which indicates a promising potential in the increasingly critical Earth–Moon space situational awareness missions in the future.

8. Discussion

For the simplicity of discussion, only Beidou Navigation Satellite signals are applied as the GNSS sources in this research; however, the inter-satellite link signals of other navigation satellites (including global ones such as GPS, GLONASS, and GALILEO and regional ones such as QZSS and IRNSS) can also be used in the proposed method, but only through adjustment of the CEI receiving ends for different signal frequency and signal modulation regimes, with substantially equivalent tracking and measurement accuracy for the high-value cislunar space targets.

Furthermore, the impact of monitoring and measuring cislunar targets when there are interference signals on the GNSS Inter-Satellite Links should be noted. Previous studies have proven that the majority of the interference signals that the GNSS Inter-Satellite Link faces belong to narrow-band interference (NBI), followed by single frequency interference [57,58]. Currently, the technology of spread spectrum (SS) is mostly applied to cope with those interference signals, which can be categorized into four types [59,60]:

(i) adaptive finite-impulse response (FIR) filtering method, usually consisting of a linear predictive filter and a linear interpolation filter, whose suppression effect will substantially deteriorate on NBI signals, compared with that on single frequency interference ones.

(ii) frequency domain adaptive filtering method, which requires the fast Fourier transform (FFT) operation that greatly increases the computation complexity, and in some cases, spectral leakage may occur.

(iii) infinite impulse response (IIR) notch filtering method, which also increases the computational complexity through FFT and spectrum analysis, and its final interference suppression effect tends to be far from satisfactory as the notch filter bandwidth can hardly be precisely controlled by bandwidth parameters.

(iv) code-aided technique, which requires no prior information on interference, interference detection, and FFTs, as it calculates the anti-jamming weight based on the received signal and processes the signal directly in the time domain.
Based on the above analysis, the code-aided technique could be a more applicable method to deal with interference signals on the BeiDou-3 Inter-Satellite Link, and the relevant research could be a reference for future lunar missions, especially for the monitoring and measuring of high-value lunar space targets. To suppress narrowband interference (NBI) in direct sequence spread spectrum (DSSS) systems, in [61], Wang first applied the code-aided interference suppression technique to a direct sequence (DS) UWB system, and then improved the NBI suppression ability of the original code-aided technique by introducing a certain form of spreading sequence. Furthermore, it was suggested in [62] to use an adaptive code-aided technique based on the RLS algorithm, and an improved method was proposed to modify it for massively parallel signal processing. Meanwhile, [57] proposes an adaptive narrow-band interference (NBI) suppression approach for the inter-satellite links (ISLs) of GNSS based on a code-aided technique, and the concept of interference influence coefficient is proposed to evaluate the influence of residual interference on the carrier-to-noise ratio (CNR) at the receiving end.

It should also be confirmed that the proposed method in this research uses high-frequency radio frequency inter-satellite links, as they are the most commonly accessible ISL sources among all of the GNSS systems. However, with the emerging advancement of laser communication technology, laser inter-satellite links have appeared in some of the latest generations of GNSS ISL systems, which are expected to be the dominant signal forms for future GNSS ISL systems and could provide enhanced navigation and timing services. In [63], a laboratory demonstrator was developed to verify the optical inter-satellite linkages of the Kepler constellation, which links satellites in a GNSS constellation for optical range, time transfer, and data transmission. The recent initiatives to create orbits for a constellation of satellites linked by optical links were explored in [64], including progress on an orbit determination algorithm and a linking rule to schedule communication between constellation components in support of continuous and reliable synchronization capabilities. Considering this, future research could be focused on the application of laser inter-satellite links in the proposed method and the processing of signal sources mixing laser and high-frequency radio frequency ISL signals.

9. Conclusions

Aiming at the urgent requirement for high-precision tracking and reliable cataloging of non-cooperative targets in cislunar space, in this paper, a BeiDou-3 Inter-Satellite Link and Connected Element Interferometry (CEI)-based method is proposed for effective monitoring and measurement of high-value cislunar space targets. The main work of this paper is as follows:

(i) A non-cooperative cislunar space targets measurement method based on CEI and BDS-3 inter-satellite link signals is proposed, with its general scenario and specific flow.

(ii) The mathematical models of the proposed method are put forward, which mainly includes four parts: dynamical constraint equations for non-cooperative targets in cislunar space; BDS-based interplanetary link for irradiation of non-cooperative targets; transmission loss equation of BDS-3 inter-satellite link signal in cislunar space; CEI-based precision measurements of targets in cislunar space.

(iii) Based on the previous analysis, the measurement errors of the entire process has been analyzed, followed by the error correction equations.

(iv) To give the feasibility demonstration of the proposed method, the link budget analysis and performance evaluation are presented.

(v) Simulation results demonstrate that the proposed method could have potential in achieving long-term cataloging, effective tracking, and high-precision measurement of high-value unidentified targets in cislunar space, indicating a promising potential in the increasingly critical cislunar space situational awareness missions in the future.

Further works could be focused on the application of the proposed method to actual unidentified Earth–Moon space target tracking missions.
Supplementary Materials: The following supporting information can be downloaded at: https://www.mdpi.com/article/10.3390/rs15153744/s1, Text S1–S13: “Chain-BD3-lunarprobe-CEI Access AER” to “Satellite-lunarprobe-1-To-Satellite-GEO1 AER”; Excel X1 to X4: “SN0-link budget-1” to “SN0-link budget-4”.

Author Contributions: Z.G. and Y.J. conceived the idea; Z.G., X.L. and Y.W. conducted the experiments; Z.G. and C.L. completed the original manuscript; X.L. and Y.W. analyzed the results; Y.J., W.Y., H.M. and F.T. checked the original manuscript. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported in part by the Major Science and Technology Projects of Beijing under Grant: Z181100002918004, and in part by the National Key Laboratory of Science and Technology on Space Microwave under Grant: HTKJ2021KL504012.

Data Availability Statement: Some of the data are publicly available at http://www.beidou.gov.cn/xz/gfzx/ (accessed on 1 May 2023) and http://www.csno-tarc.cn/system/constellation (accessed on 4 May 2023). The data presented in this study are available on request from the corresponding author.

Acknowledgments: The authors would like to thank the anonymous reviewers and editors for their valuable suggestions to improve the quality of this work.

Conflicts of Interest: The authors declare no competing interests.

Abbreviations

The main abbreviations used in this paper are illustrated following:

<table>
<thead>
<tr>
<th>Abbreviations</th>
<th>Full Name</th>
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<tbody>
<tr>
<td>CSSA</td>
<td>Circular Space Situational Awareness</td>
</tr>
<tr>
<td>ISL</td>
<td>Inter-Satellite Link</td>
</tr>
<tr>
<td>CEI</td>
<td>Connected Element Interferometry</td>
</tr>
<tr>
<td>CRTBP</td>
<td>Circular Restricted Three-Body Problem</td>
</tr>
<tr>
<td>PNT</td>
<td>positioning, navigation, and timing</td>
</tr>
<tr>
<td>CVN</td>
<td>China VLBI Network</td>
</tr>
<tr>
<td>SOAS</td>
<td>Shanghai Observatory of the Chinese Academy of Sciences</td>
</tr>
<tr>
<td>GNSS</td>
<td>Global Navigation Satellite System</td>
</tr>
<tr>
<td>BDS</td>
<td>BeiDou Navigation Satellite System</td>
</tr>
<tr>
<td>LEO</td>
<td>Low Earth Orbit</td>
</tr>
<tr>
<td>GEO</td>
<td>Geostationary Earth Orbit</td>
</tr>
<tr>
<td>IGSO</td>
<td>Inclined Geosynchronous Orbit</td>
</tr>
<tr>
<td>MEO</td>
<td>Medium Earth Orbit</td>
</tr>
<tr>
<td>BDT</td>
<td>BDS system time</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>GLONASS</td>
<td>Global Navigation Satellite System</td>
</tr>
<tr>
<td>QZSS</td>
<td>Quasi-Zenith Satellite System</td>
</tr>
<tr>
<td>IRNSS</td>
<td>Indian Regional Navigational Satellite System</td>
</tr>
<tr>
<td>NBI</td>
<td>Narrow-Band Interference</td>
</tr>
<tr>
<td>SS</td>
<td>Spread Spectrum</td>
</tr>
<tr>
<td>FIR</td>
<td>Finite-Impulse Response</td>
</tr>
<tr>
<td>IIR</td>
<td>Infinite Impulse Response</td>
</tr>
<tr>
<td>DSSS</td>
<td>Direct Sequence Spread Spectrum</td>
</tr>
<tr>
<td>CNR</td>
<td>Carrier-to-Noise Ratio</td>
</tr>
<tr>
<td>RCS</td>
<td>Radar Cross Section</td>
</tr>
<tr>
<td>LiAISON</td>
<td>Linked Autonomous Interplanetary Satellite Orbit Navigation</td>
</tr>
<tr>
<td>SST</td>
<td>satellite-to-satellite tracking</td>
</tr>
<tr>
<td>DRO</td>
<td>Distant retrograde orbits</td>
</tr>
</tbody>
</table>

References


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