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Influence of Stochastic Modeling for Inter-Frequency Clock Biases on Multi-Frequency Precise Point Positioning

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Abstract: The incorporation of multi-frequency signals into global navigation satellite systems (GNSS) has presented new possibilities for precise positioning and rapid ambiguity resolution. Inter-frequency clock bias (*IFCB*) pertains to the time-varying biases among distinct frequencies within carrier phase observations in GNSS signals. The appropriate handling of *IFCB* is critical in enhancing the accuracy and convergence time of precise point positioning (PPP) solutions. The focus of this study is on the proper modeling of phase *IFCB* in multi-GNSS multi-frequency PPP. In this paper, the optimal *IFCB* power spectral density value of 0.6 m/sqrt(s) is first determined. To obtain the optimal stochastic model for *IFCB*, a thorough comparison and analysis of the product correction and parameter estimation methods is conducted. Additionally, experiments are conducted on the effect of *IFCB* modeling on the performance of undifferenced and uncombined PPP using data from 130 multi-GNSS experiment stations across the globe over a period of one week in January 2022. The study reveals that the optimal power spectral density for *IFCB* is within [60, 0.006] m/sqrt(s), modeling *IFCB* as a random walk is feasible, and the PPP is comparable for the three *IFCB* processing schemes of product correction, random walk, and white noise. Meanwhile, it is not reasonable to treat *IFCB* as a random constant or neglect it in the multi-GNSS multi-frequency PPP. In the absence of product correction or for users who require immediate and continuous positioning solutions, modeling *IFCBs* as random walks can lead to more reliable positioning results and improved convergence performance.

Keywords: precise point positioning; inter-frequency clock bias; multi-GNSS; undifferenced and uncombined; stochastic modeling



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1. Introduction

The modernization of global navigation satellite systems (GNSS) presents both opportunities and challenges. A notable aspect is the enhanced availability of observable frequencies, which substantially increases the observations available to users [1]. This development also facilitates the provision of better resolutions in a range of fields, including wide-area precise positioning, low-orbit satellite orbiting, ionospheric monitoring, earthquake and tsunami monitoring and warning systems, and time transfer, among others. It is noteworthy that the satellite clock offset products provided by the International GNSS Service (IGS) are currently dependent on legacy dual-frequency ionosphere-free (*IF*) combinations (e.g., GPS L1/L2, BDS B1/B3 with GalileoE1/E5a). Careful evaluation of the impact of additional frequencies on positioning models is a fundamental requirement in constructing a multi-frequency observation model.

Employing multi-frequency observations in precise point positioning (PPP) can offer various advantages, but also poses several challenges. Montenbruck et al. initially observed deviations in carrier phase observations that were time-, signal- and satellite-dependent, such as the discrepancy between GPS L1/L2 and L1/L5 combined carrier phase *IF* observations [2]. Meanwhile, Pan et al. showed that the time-varying phase bias is assimilated by the satellite clock offset parameter in the precise clock estimation. Consequently, when

varying observations are employed, the satellite clock estimates will differ, and the distinction between them is referred to as the inter-frequency clock bias (*IFCB*) [3]. The error mentioned above could cause significant discrepancies of several tens of centimeters in the multi-GNSS multi-frequency PPP observation model [4]. The *IFCB* compensation method is a key issue in multi-GNSS multi-frequency PPP data processing. In general, there are three approaches to address the *IFCB* problem: The first is to determine the satellite clock offset through the addition of uncombined L5 observations to the conventional dual-frequency *IF* model [5], so that two sets of satellite clock offset products are available, e.g., the L5 uncombined satellite clock offset and conventional L1/L2 *IF* satellite clock offsets, where the L1/L5 *IF* satellite clock offset can be obtained through the conversion of the relationship. The second is based on the combined geometry-free and ionosphere-free (*GFIF*) carrier phase observations, where the *IFCB* of each station is obtained using the epoch difference strategy, and the satellite *IFCB* can be obtained through averaging the *IFCB* of all stations [6–9]. The last, but seldom used, option is to ignore the *IFCB*. Treating the *IFCB* as a parameter in observation is less. Currently, the second method is widely used for *IFCB* correction at the third frequency, but the receiver *IFCB* is also absorbed by the satellite *IFCB*. This requires substantial computational resources, while the reliability and accuracy of *IFCB* estimation are limited by the number of stations in the reference network, which also increases the probability of errors in the *IFCB* calculation results [10–13]. However, post and real-time phase *IFCB* products are available from few organizations, which has clearly hampered the development of the multi-GNSS multi-frequency PPP. This limits the real-time applicability of the approach, especially for users who require immediate and continuous positioning solutions.

With the arrival of the multi-GNSS multi-frequency era, *IFCB* has also become a hot spots of current research. Many scholars have studied and analyzed the impact of *IFCB* in GPS/BDS/Galileo triple frequency (TF) PPP. Montenbruck and Lin et al. studied the *IFCB* of GPS satellites using GPS TF observations, and showed that *IFCB* is a GPS bias with periodic and time-varying characteristics [14–17]. Chen and Montenbruck et al. showed that the *IFCB* amplitude varies up to 10–40 cm for GPS (L1/L2 and L1/L5), within 1–2 cm for Galileo (E1/E5a and E1/E5b), and within 1–3 cm for BDS-3 (B1/B3 and B1/B2a) [2,18]. Zhang found that the GPS Block IIF satellites' *IFCB* showed a variation period of more than two years, which is apparently consistent for two satellites placed in the same orbital plane [19]. Some scholars have also investigated the temporal characteristics of *IFCB* in depth, and have built different prediction models for *IFCB* correction [20]. For the current study, it is clear that *IFCB* is present at the L5 frequency of GPS Block IIF satellites, while Galileo and BDS-3 are barely affected [10–20]. An in-depth analysis of the additional signals is necessary to take full advantage of the benefits that additional frequencies bring. However, most of the studies in the literature tackle the characteristics of *IFCB* and its prediction model based on datasets of a few, or dozens, of stations [12–22], and there are no relevant unified criteria to evaluate the accuracy of the products. Additionally, the accuracy of the *IFCB* models and predictions may degrade over time; the prediction accuracy of *IFCB* models decreases when the time span is increased from a day to a week. This means that the accuracy of *IFCB* correction values may decrease over longer periods of time, potentially impacting the positioning accuracy. Most studies are limited to analyzing the temporal characteristics and periodicity of *IFCB* time series, but fewer studies consider *IFCB* as a parameter to be estimated and analyze the impact of stochastic models of *IFCB* parameters on the positioning performance of the multi-GNSS multi-frequency PPP. Treating *IFCB* as a parameter undoubtedly increases the computational load. Nonetheless, with the progression of scientific and technological advancements, this additional load does not pose a formidable challenge for future modern processors.

Most studies encountered in the literature focus primarily on the estimation and correction of *IFCB* for triple-frequency PPP. The stochastic modeling of *IFCB* plays a significant role in improving the understanding and impact of its parameters. The techniques proposed in this paper can be implemented directly in other positioning methods or scenarios

that involve triple-frequency observations. In this study, an analysis of the power spectral density relevant to *IFCB* is conducted and product correction methods are compared with parameter estimation methods. A comprehensive analysis is carried out to assess the impact (e.g., positioning accuracy, convergence time, and phase residuals) of stochastic *IFCB* modeling on PPP performance based on the International GNSS Service's (IGS) Multi-GNSS Experiment (MGEX) one-week precise-orbit and clock-offset products (Wuhan University) and data from 130 tracking stations. Additionally, the stochastic models which are employed to model the *IFCB* parameters in the multi-GNSS multi-frequency PPP are explicated using the Undifferenced and Uncombined (UDUC) PPP model [23–25].

The remainder of the article is structured as follows; the MGEX UDUC PPP model is first elaborated in Section 2, taking into account *IFCB*. In Section 3, experiments and data processing strategies are presented. The optimal power spectral density of the *IFCB* and the impact of different *IFCB* stochastic modeling on PPP performance is demonstrated and compared, followed by a discussion, in Section 4. The concluding Section 5 contains a summary of the findings.

2. Methods

2.1. Raw PPP Observation Equations

The raw GNSS PPP observation equation can be expressed as follows:

$$P_{r,i}^s = \rho_r^s + cdt_r - cdt^s + m_r^s \cdot Z_r + \mu_i^s \cdot I_{r,1}^s + b_{r,i}^s - b_i^s + e_{r,i}^s \quad (1)$$

$$L_{r,i}^s = \rho_r^s + cdt_r - cdt^s + m_r^s \cdot Z_r - \mu_i^s \cdot I_{r,1}^s + N_{r,i}^s + \varphi_{r,i} - \varphi_i^s + \varepsilon_{r,i}^s \quad (2)$$

where s represents the satellite PRN, r is the receiver, and i is the carrier frequency; $P_{r,i}^s$ and $L_{r,i}^s$ are the raw pseudorange and carrier phase in meters, respectively; ρ_r^s is the geometric distance; c is the speed of light; dt_r^s and dt^s represent the receiver and satellite clock offsets, respectively; $b_{r,i}^s$ and b_i^s represent the pseudorange hardware delay at the receiver and satellite ends, respectively; $\varphi_{r,i}^s$ and φ_i^s correspond to the phase hardware delay, respectively; μ_i^s is the ionospheric amplification factor, equal to $\mu_i^s = f_1^2 / f_i^2$; $I_{r,1}^s$ is the ionospheric delay at the first frequency; Z_r and m_r^s are the zenith tropospheric wet delay and the corresponding mapping function, respectively; $N_{r,i}^s$ is the phase ambiguity; $e_{r,i}^s$ and $\varepsilon_{r,i}^s$ represent the noise of pseudorange and carrier phase observations, respectively. In Equations (1) and (2), other errors, such as the antenna phase center offset (PCOs), tidal loads, phase wind up, and variations (PCVs), etc., were corrected in accordance with the existing model [26].

It is generally considered that the pseudorange hardware delay is relatively stable and varies little within a day [27]. Typically, since the initial receiver clock offset is usually calculated through the pseudorange, the delay b is considered to be absorbed by it. The phase hardware delay has obvious time-varying characteristics, which can be divided into constant and time-varying parts [28], as shown below:

$$\begin{cases} \varphi_{r,i} = \bar{\varphi}_{r,i} + \delta\varphi_{r,i} \\ \varphi_i^s = \bar{\varphi}_i^s + \delta\varphi_i^s \end{cases} \quad (3)$$

where $\bar{\varphi}$ represents the constant part of φ , which is absorbed by the ambiguity parameter, and $\delta\varphi$ corresponds to the time-varying part, which is absorbed by the receiver clock offset. IGS precise products are used for PPP data processing to minimize the impact of satellite orbit and clock errors.

For convenience, the following equation is introduced:

$$\left\{ \begin{array}{l} \alpha_{IFij} = \frac{f_i^2}{f_i^2 - f_j^2}, \beta_{IFij} = -\frac{f_j^2}{f_i^2 - f_j^2} \\ \mu_i = \frac{f_i^2}{f_i^2 - f_j^2}, \alpha_{IFij} + \beta_{IFij} = 1 \\ DCB_{ij}^s = b_i^s - b_j^s, DCB_{r,ij} = b_{r,i} - b_{r,j} \\ \delta DPB_{ij}^s = \delta \varphi_i^s - \delta \varphi_j^s, \delta DPB_{r,ij} = \delta \varphi_{r,i} - \delta \varphi_{r,j} \\ b_{IFij}^s = \alpha_{IFij} b_i^s + \beta_{IFij} b_j^s, b_{r,IFij} = \alpha_{IFij} b_{r,i} + \beta_{IFij} b_{r,j} \\ \varphi_{IFij}^s = \alpha_{IFij} \varphi_i^s + \beta_{IFij} \varphi_j^s, \varphi_{r,IFij} = \alpha_{IFij} \varphi_{r,i} + \beta_{IFij} \varphi_{r,j} \end{array} \right. \quad (4)$$

After introducing the precise satellite product expressions into Equations (1) and (2) and linearizing them, Equations (1) and (2) become:

$$\bar{P}_{r,i}^s = u_r^s \cdot \Delta x + c \bar{d}t_r + m_r^s \cdot Z_r + \mu_i^s \cdot \bar{I}_{r,1}^s + e_{r,i}^s \quad (5)$$

$$\bar{L}_{r,i}^s = u_r^s \cdot \Delta x + c \bar{d}t_r + m_r^s \cdot Z_r - \mu_i^s \cdot \bar{I}_{r,1}^s + \bar{N}_{r,i}^s + \varepsilon_{r,i}^s \quad (6)$$

where $\bar{P}_{r,i}^s$ and $\bar{L}_{r,i}^s$ represent the observed minus the calculated values; Δx is the incremental values with respect to the prior position; u_r^s denotes the line-of-sight (LOS) vector of unit length; $\bar{d}t_r$ is the receiver clock offset after re-parameterization, which is the same as for $\bar{N}_{r,i}^s$ and $\bar{I}_{r,1}^s$.

When the number of observable frequencies is greater than or equal to three, Equations (5) and (6) can be expressed as follows:

$$\left\{ \begin{array}{l} \bar{P}_{r,1}^s = u_r^s \cdot x + c \bar{d}t_r + m_r^s \cdot Z_r + \mu_1^s \cdot \bar{I}_{r,1}^s + e_{r,1}^s \\ \bar{P}_{r,2}^s = u_r^s \cdot x + c \bar{d}t_r + m_r^s \cdot Z_r + \mu_2^s \cdot \bar{I}_{r,1}^s + e_{r,2}^s \\ \bar{P}_{r,3}^s = u_r^s \cdot x + c \bar{d}t_r + m_r^s \cdot Z_r + \mu_3^s \cdot \bar{I}_{r,1}^s + IFB_r + e_{r,3}^s \\ \bar{L}_{r,1}^s = u_r^s \cdot x + c \bar{d}t_r + m_r^s \cdot Z_r + \bar{N}_{r,1}^s - \mu_1^s \cdot \bar{I}_{r,1}^s + \varepsilon_{r,1}^s \\ \bar{L}_{r,2}^s = u_r^s \cdot x + c \bar{d}t_r + m_r^s \cdot Z_r + \bar{N}_{r,2}^s - \mu_2^s \cdot \bar{I}_{r,1}^s + \varepsilon_{r,2}^s \\ \bar{L}_{r,3}^s = u_r^s \cdot x + c \bar{d}t_r + m_r^s \cdot Z_r + \bar{N}_{r,3}^s - \mu_3^s \cdot \bar{I}_{r,1}^s + IFCB_r^s + \varepsilon_{r,3}^s \end{array} \right. \quad (7)$$

where

$$\left\{ \begin{array}{l} \bar{d}t_r = dt_r + b_{r,IF12} + \delta \varphi_{r,IF12} \\ \bar{I}_{r,1}^s = I_{r,1}^s + \beta_{12} DCB_{r,12} - \beta_{12} (\delta DPB_{12}^s - \delta DPB_{r,12}) \\ IFB_r = b_{r,3} - b_{r,IF12} - \mu_3 \beta_{12} DCB_{r,12} \\ \delta \varphi_{r,j}^s = \mu_j \beta_{12} (\delta DPB_{12}^s + \delta DPB_{r,12}) - (\delta \varphi_{IF12}^s + \delta \varphi_{r,IF12}) \\ IFCB_r^s = (\delta \varphi_3^s - \delta \varphi_{IF12}^s - \mu_3 \beta_{12} \delta DPB_{12}^s) + (\delta \varphi_{r,3} - \delta \varphi_{r,IF12} - \mu_3 \beta_{12} \delta DPB_{r,12}) \\ \bar{N}_{r,i}^s = N_{r,i}^s + \bar{\varphi}_{r,i} - \bar{\varphi}_i^s - b_{i,IF12}^s - b_{r,IF12} + \mu_i \beta_{12} DCB_{r,12} \end{array} \right. \quad (8)$$

where IFB_r is the inter-frequency code bias, and $IFCB_r^s$ is the inter-frequency clock bias. The estimated parameters in our PPP model with $IFCB$ can be listed as:

$$X = \begin{bmatrix} x & \bar{d}t_r & Z_r & \bar{I}_{r,1}^s & IFB_r & IFCB_r^s & \bar{N}_{r,1}^s & \bar{N}_{r,2}^s & \bar{N}_{r,3}^s \end{bmatrix} \quad (9)$$

2.2. IFCB Estimation

In this part, the $IFCB$ estimation methods used in this study are described. They are divided into two types: one is the estimation of two combined IF observations (IF combination estimation), and the other is the modeling of the $IFCB$ parameters, which are estimated along with other parameters, such as position parameters (parametric estimation).

2.2.1. IF Combinations Estimation

Generally, the $IFCB$ is calculated through the difference of two IF combinations to obtain the $GFIF$ combination. Then the epoch-difference $GFIF$ is obtained, and finally the

satellite *IFCB* is calculated using the weighted average of the epoch-difference *GFIF* of the global network stations. The process of estimating *IFCB* is as follows:

First, the combined observation *GFIF* is calculated as follows:

$$GFIF = L_{r,IF12}^s - L_{r,IF13}^s = N_{GFIF} + B_{r,GFIF}^s + \delta B \quad (10)$$

where $L_{r,IFij}^s$ denotes the combined *IF* observations for the combination of frequency *i* and frequency *j* phase observations; N_{GFIF} denotes the phase ambiguity of the *GFIF*; $B_{r,GFIF}^s$ denotes the constant components of the pseudorange and phase hardware delay for the *GFIF* combination observations; δB denotes the *IF* combination *IFCB*, whose conversion relation with the uncombined *IFCB* is

$$\delta B = \left(f_3^2 / (f_1^2 - f_3^2) \right) \cdot IFCB \quad (11)$$

In the arc segment without cycle slip, the N_{GFIF} parameter can be eliminated through the epoch difference, while the hardware delay is generally considered to remain stable for short periods, so $B_{r,GFIF}^s$ is also eliminated. Then, the following equation is obtained:

$$\Delta \delta B_{(t,t-1)} = GFIF_t - GFIF_{t-1} \quad (12)$$

When there are *n* stations observing satellite *s*, the elevation angle of *s* can be used to calculate the current epoch satellite's *IFCB* using the weighted average equation as follows:

$$\Delta \bar{\delta B}_{(t,t-1)} = \frac{\left(\sum_{r=1}^n \Delta \delta B_{(t,t-1)} \cdot \omega_{r,(t,t-1)} \right)}{\sum_{r=1}^n \omega_{r,(t,t-1)}} \quad (13)$$

and

$$\omega_{r,(t,t-1)} = \begin{cases} 0 & E_{r,(t,t-1)}^s < 15^\circ \\ \sin E_{r,(t,t-1)}^s & 30^\circ > E_{r,(t,t-1)}^s \geq 15^\circ \\ 1 & E_{r,(t,t-1)}^s \geq 30^\circ \end{cases}$$

where $E_{r,(t,t-1)}^s$ is the average elevation angle of *s* between epoch *t* and epoch *t* − 1. The segment elevation angle weighting method is used. Then, the *IFCB* value for the *t*-th epoch can be obtained through the following equation:

$$\delta B_t = \delta \bar{B}_{t0} + \sum_{k=1}^{k=t} \Delta \delta B_{(k,k-1)} \quad (14)$$

where $\delta \bar{B}_{t0}$ is the reference *IFCB* of the satellite at epoch *t*₀. In this paper, 00:00:00 (GPS time) is set as the reference epoch every day, and the reference value of $\delta \bar{B}_{t0}$ is set to 0.01 m. The setting of $\delta \bar{B}_0$ causes a systematic bias in the float ambiguity, but does not affect the PPP positioning performance. However, it is added to the same benchmark of the third frequency bias product when performing PPP-AR [4].

2.2.2. Parameter Estimation

In multi-GNSS multi-frequency PPP, there are three randomness modeling methods for parameters: white noise, random walk, and random constant difference, expressed through Equations (15)–(17), respectively.

$$X_r(k) = X_r(k-1) + \omega, \omega \sim N(0, \sigma_\omega^2) \quad (15)$$

$$X_r(k) \sim N(0, \sigma^2) \quad (16)$$

$$X_r(k) = X_r(k-1) \quad (17)$$

Here, k represents the epoch. If the parameter is time-independent, white noise is a suitable modeling method. If the parameters fluctuate slightly over time, random walk is more suitable, while the random constant is suitable for modeling the behavior of parameters that do not change over time and is equivalent to a random walk with zero process noise. Random walk and white noise are two distinct stochastic processes that are differentiated by their respective characteristics. A random walk is a process that demonstrates trend and correlation, with each step's movement being dependent on the previous position, thereby introducing memory in the process. Conversely, white noise is a stochastic process that lacks any trend or correlation, with independent values at different time points. Although a random walk can occur as the result of white noise accumulation, their statistical properties exhibit notable differences. Thus, despite the connection between random walk and white noise, they are considered independent stochastic processes.

In this work, the above methods are adopted to estimate the *IFCB* and analyze its stochastic model in depth.

3. Processing Strategies and Data Sets

3.1. Data Set

To analyze the impact of stochastic *IFCB* modeling on multi-GNSS multi-frequency PPP performance, 30 s sample observation data from 130 stations of the MGEX network were selected, covering a one-week period of DOY (Day of Year) 18–24 January 2022. Figure 1 shows the geographical distribution of the selected stations. All stations were capable of tracking GEC constellations. The data from 105 MGEX stations (blue dots) were used to estimate *IFCB* products, and 25 stations (red dots) were used to analyze and verify the effect of the *IFCB* product correction method and *IFCB* estimation method on the positioning performance.

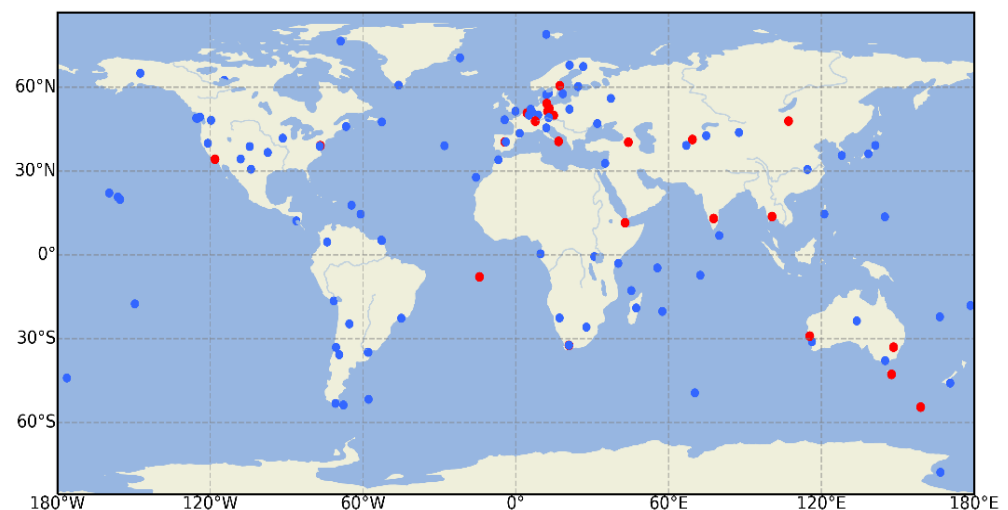


Figure 1. MGEX stations used in the experiment. Blue dots represent stations used to estimate *IFCB* by IF; red dots represent stations used for PPP validation.

3.2. Processing Strategy

The precise orbit and clock offset products provided by Wuhan University (WHU) were employed. The IGS absolute antenna phase center correction models (i.e., igs14.atx) were applied [29]. Phase center variation correction was not performed for BDS, as the corresponding model is not included in IGS14.atx. Since there is no receiver phase center offset and variation correction for BDS and Galileo signals, the corresponding GPS correction was used, which is consistent with the strategy adopted in BDS and Galileo for precise orbit determination and clock estimation [30]. The tropospheric ZWDs and the slant ionospheric delays were estimated as random walk process, and their spectral density values were empirically set to be 10^{-4} m/sqrt(s) and 4 cm/sqrt(s) divided by the cosine of the zenith

distance angles, respectively [31]. The elevation cutoff angle was set to 7° , and a weighting scheme related to the elevation angle ($a^2 / (b \cdot \sin(E))^2$) was applied to the observations. The ISB was modeled as a white noise process with a sigma of 60 m/sqrt(s), the value used by the open-source software GAMP [32], and the same process as the one for *IFB* was applied. The float phase ambiguities were estimated as constants for each continuous satellite arc. The standard deviation (STD) values of the carrier phase observations were all set to 0.003 m, and the ratio between the code and carrier phase observations was set to 100 [33]. More data processing strategies are shown in Table 1. The Turbo Edit method was used for cycle slip detection during preprocessing [32].

Table 1. Multi-GNSS PPP processing strategy.

Options	Processing Strategies
Observation	Uncombined observation
Signal	BDS-3:B1,B3,B2a;GPS:L1,L2,L5;GAL:E1,E5a,E5b
Parameter estimation	EKF
Observation interval	30 s
Weight distribution of observed values	Elevation-angle dependent model
Cut-off angle	7°
Satellite orbit	WHU precise ephemeris
Satellite clock	WHU precise clock offset
Phase center correction	IGS14.ATX
Phase center variation	GPS/Galileo
Phase windup	Model correction
Solid earth tide	
Ocean load	
Polar motion	
Relativistic effect	
Tropospheric delay	Dry delay (Corrected by Saastamoinen model) + Wet delay (Estimated as a Random walk)
Ionospheric delay	Random walk
ISB	White noise
<i>IFB</i>	
Receiver coordinates	Kinematic, Estimated as white noise
Receiver clock	White noise estimation
Ambiguity	Estimated as float constants for each arc

PPP tests were performed using the self-developed software. The position coordinates were modeled as white noise in kinematic PPP mode. The positioning performance was assessed with respect to the coordinates from weekly IGS solutions.

4. Experimental Validation

The previous theoretical analysis indicates that *IFCBs* stem from both the satellite- and receiver-dependent phase time-varying parts of the hardware delay, which is the difference between the satellite clock offsets for different *IF* combinations [25]. In this study, global data from 105 stations were collected for *IFCB* product estimation through methods considering different *IF* combinations. Subsequently, 25 stations were used to analyze the impact of *IFCB* stochastic modeling on the multi-GNSS multi-frequency PPP performance for simulated kinematic PPP. Based on the different *IFCB* processes, five schemes were

implemented: (a) ignoring the effect of GPS third frequency *IFCB*, (b) using *IFCB* product correction, (c) using random constants to estimate the *IFCB*, (d) using a white noise process to estimate the *IFCB*, and (e) using a random walk process to estimate the *IFCB*. Then, the third-frequency residual was analyzed using the five schemes. The results are summarized and discussed after the analysis of the findings.

In this study, one-week observation data from 25 stations were used to evaluate PPP performance. The kinematic PPP performance was evaluated using indices such as positioning accuracy, convergence time, the third frequency phase residuals' root mean square (RMS) and estimated *IFCB*. The position was considered to have converged only when the positioning error (east, north, up) of the epoch and the subsequent 20 epochs were within a 1 dm threshold [34,35]. For convenience, the five schemes mentioned above are marked as *IFCB_NO* (Neglect), *IFCB_PC* (Product Correction), *IFCB_CT* (random constant), *IFCB_WN* (white noise), and *IFCB_RW* (random walk), respectively. For the *IFCB_RW* scheme, the *IFCB* parameters were modeled as a random walk with a power density of 0.6 m/sqrt(s). The reason for using this value is explained in Section 4.1.2; for the first appearance of the satellite in the arc segment, the initial value of the *IFCB* was set to 0.01 m. It should be noted that the *IFCB* amplitudes of BDS-3 and Galileo are less than 2.0 cm, with negligible effects on multi-GNSS multi-frequency PPP positioning [4,14]; therefore, the effect of the third-frequency *IFCB* was ignored in this study. To facilitate the description, the *IFCBs* discussed in the following sections refer to the GPS L5 *IFCB*.

4.1. *IFCB* Estimation

4.1.1. IF Combinations Estimation (*IFCB_PC*)

Figure 2 shows the GPS L5 *IFCB* products obtained using the IF combinations estimation, where different colors represent the *IFCBs* of different satellites. From Figure 2, the single-day amplitude of GPS Block IIF satellite L5 *IFCB* is larger among all satellites and lies within the range of 10–20 cm, which is obviously a PPP error that cannot be ignored. Additionally, it has been demonstrated that the amplitude of GPS Block III L5 *IFCB* ranges from 1 to 3 cm within a single day, and the standard deviation of *IFCB* is about 1.5 mm in a single epoch, which is almost independent of *IFCB* [4]. Therefore, it is necessary to focus only on the effect of the GPS Block IIF satellites' L5 *IFCB* and to analyze the corresponding effect on the multi-GNSS multi-frequency PPP positioning (see Section 4.2). Since *IFCB* is considered a satellite- and receiver-related hardware delay between phase frequencies, which is caused by the difference between satellite clock offset calculated with different IF combinations, a thorough analysis of the satellite's internal and external configuration is required to fully understand the causes of its generation. The focus of this study is on the impact of *IFCB* on multi-GNSS multi-frequency PPP positioning performance, so the causes of *IFCB* are not discussed in this paper.

4.1.2. Parameter Estimation (*IFCB_CT*, *IFCB_WN*, *IFCB_RW*)

In order to analyze the effect of different stochastic models on *IFCB* results, in this section the *IFCB* is considered as a parameter and estimated lumped with other parameters, such as ambiguity, atmospheric error, and receiver clock offsets; corresponding time series are plotted. Taking BRUX station as an example, the *IFCB* daily series (DOY 18 January 2022) derived from the GPS/BDS-3 multi-GNSS multi-frequency PPP solutions are given in Figure 3, with different colors representing the *IFCB* of different satellites.

Figure 3 shows the *IFCB* results estimated via GPS/BDS-3 multi-frequency PPP using the *IFCB_CT*, *IFCB_WN*, and *IFCB_RW* schemes. It can be seen from Table 2 that the *IFCB* differences among the three *IFCB* process schemes (*IFCB_PC*, *IFCB_WN*, *IFCB_RW*) were less than 0.02 m for BRUX. Nonetheless, the *IFCB* differences between *IFCB_CT* and other schemes (*IFCB_PC*, *IFCB_WN*, and *IFCB_RW*) were significant. Comparing Figures 2 and 3, in the *IFCB_CT* scheme, the estimation of *IFCB* as a random constant does not fit the characteristics of *IFCB* and cannot truly reflect the change of its temporal characteristics, thus affecting the positioning accuracy and convergence performance of the

multi-GNSS multi-frequency PPP (see Section 4.2); therefore, the *IFCB_CT* scheme cannot be used to model *IFCB* variation.

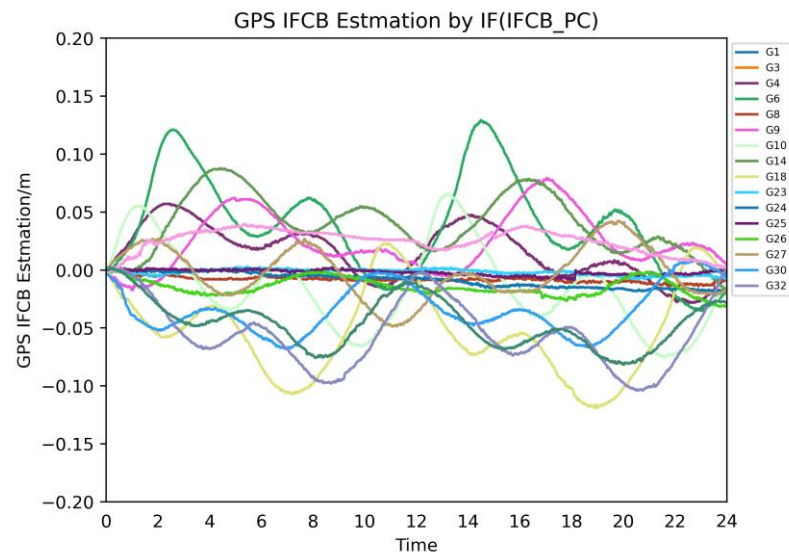


Figure 2. GPS L5 *IFCB* based on the combined IF observations of 105 stations (DOY 18 January 2022), where each color represents a satellite.

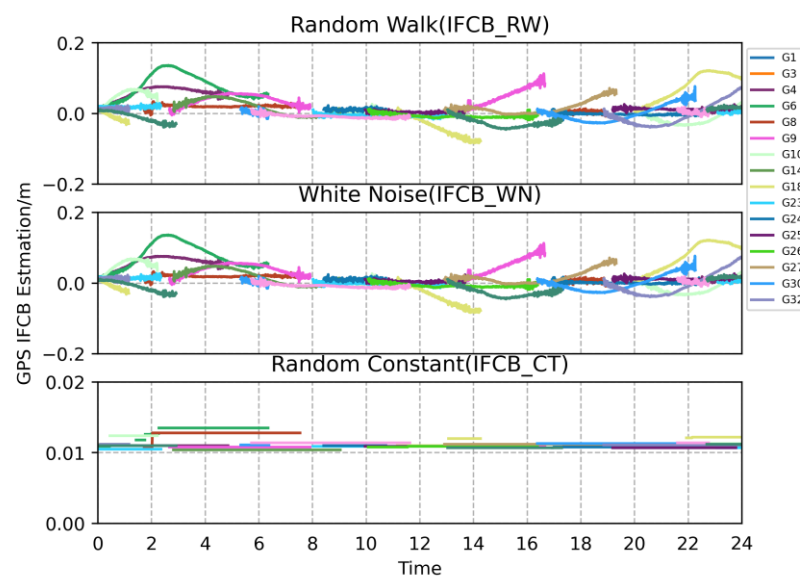


Figure 3. GPS *IFCB* for BRUX station based on the *IFCB* process Schemes *IFCB_RW*, *IFCB_WN* and *IFCB_CT* (DOY 18 January 2022), where each color represents a satellite.

The difference in satellite *IFCB* amplitudes between *IFCB_PC* and *IFCB_WN* was small, with an average difference of <1 cm, and the temporal characteristic change between them was similar, with an STD of <5 mm. Obviously, considering *IFCB* as a white noise process is consistent with the characteristic change of *IFCB*, which reflects the temporal characteristic change of *IFCB* more accurately and thus improves the multi-frequency PPP positioning accuracy and convergence. Therefore, the white noise process is suitable for describing the temporal characteristic change of *IFCB*. The same conclusion applies to the *IFCB_RW* scheme. It is worth noting that there are no studies on *IFCB*'s power spectral density (σ_{IFCB}) setting, so in order to obtain the optimal power spectral density value, the tests considered the range $\sigma_{IFCB} \in [60, 6 \times 10^{-5}]$ at the 25 stations (DOY 18 January 2022). The corresponding statistical results are shown in Table 3. As shown in Table 3, the convergence times in the σ_{IFCB} range within $[60, 6 \times 10^{-3}]$ are equal, and the *IFCB_PC*

results show that the inter-epoch variation and amplitude of *IFCB* are much smaller than 0.6 m, so $\sigma_{IFCB} = 0.6 \text{ m/sqrt(s)}$ is obviously a suitable choice for the *IFCB_RW* scheme. Consequently, this value was adopted chosen in this paper and the same value was used for the *IFCB_WN* scheme. The *IFCB_WN* scheme uses the same processing scheme as *IFCB_RW*. In conclusion, comparing Figures 2 and 3 and the results in Table 2, *IFCB_WN* and *IFCB_RW* reflect the variation of the temporal characteristics of *IFCB* accurately, which indicates that the processing methods of the schemes used in this study are reasonable.

Table 2. *IFCB* amplitudes of GPS satellites (DOY 18 January 2022).

PRN	<i>IFCB</i> Amplitude/m		
	<i>IFCB_PC</i>	<i>IFCB_WN</i>	<i>IFCB_RW</i>
G01	0.0860	0.0796	0.0795
G03	0.1660	0.1328	0.1328
G04	0.0150	0.0416	0.0413
G06	0.0980	0.0118	0.1175
G08	0.1410	0.1030	0.1029
G09	0.1050	0.0609	0.0608
G10	0.1420	0.2057	0.2056
G14	0.0130	0.0458	0.0415
G18	0.0200	0.0417	0.0427
G23	0.0110	0.0300	0.0314
G24	0.0340	0.0313	0.0313
G25	0.0920	0.0750	0.0749
G26	0.0770	0.1047	0.1046
G27	0.1070	0.1134	0.1133
G30	0.0430	0.0364	0.0364
G32	0.0840	0.0699	0.0699
AVG	0.0771	0.0740	0.0804
STD	0.0498	0.0492	0.0474

Table 3. Determination experiment of GPS L5 *IFCB* power spectral density at 25 stations (DOY 18 January 2022).

$\sigma_{IFCB} \text{ m/sqrt(s)}$		60	0.6	0.006	0.0006	0.00006
Convergence	AVERAGE		27.64		28.72	61.64
Time/min	MEDIAN		26.00		27.50	42.00

4.2. Three-Frequency PPP Performance

4.2.1. GPS-Only

The GPS-only results are given in Figure 4, which shows the convergence time box diagram for five *IFCB* process schemes. More than 24% of *IFCB_CT* schemes had a convergence time over 60 min; this performance was comparable with that of *IFCB_NO*. The average convergence times of the *IFCB_RW* and *IFCB_WN* schemes were comparable with that of *IFCB_PC*, with the difference being close to 1 min. The median convergence time improvement for the *IFCB_RW* and *IFCB_WN* schemes, compared to *IFCB_NO* was 43.0 min and 26.0 min, respectively. It is noteworthy that the convergence performance of GPS-Only PPP solutions was worst if the *IFCB_NO* or *IFCB_CT* schemes were adopted. This result was mostly caused by *IFCB* (see RMS values in Table 4).

Likewise, a clear difference in terms of weekly RMS was observed between the *IFCB_NO* scheme and the other four schemes (Table 4). The three *IFCB* process schemes (*IFCB_PC*, *IFCB_WN*, and *IFCB_RW*) were comparable in terms of positioning accuracy. Furthermore, the *IFCB_NO* and *IFCB_CT* schemes were generally worse than the other schemes.

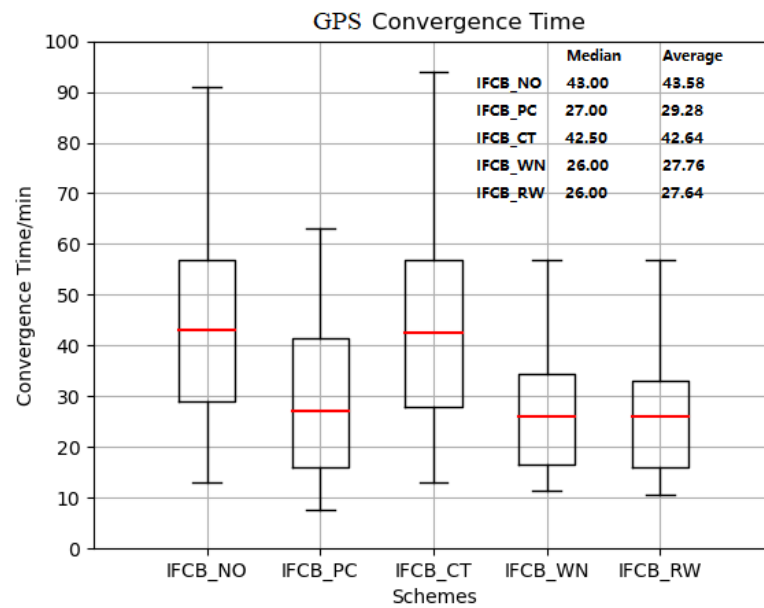


Figure 4. Comparisons of GPS-only convergence times for the five *IFCB* process schemes over 7 days.

Table 4. RMS (m) of GPS-only PPP solutions among different *IFCB* process schemes at 25 stations for one week.

RMS/m						Improvement/%			
<i>IFCB</i>	<i>IFCB_NO</i>	<i>IFCB_PC</i>	<i>IFCB_CT</i>	<i>IFCB_WN</i>	<i>IFCB_RW</i>	<i>IFCB_PC</i>	<i>IFCB_CT</i>	<i>IFCB_WN</i>	<i>IFCB_RW</i>
E	0.0512	0.0144	0.0229	0.0136	0.0136	71.9231	55.3678	73.3704	73.4434
N	0.0412	0.0123	0.0193	0.0117	0.0117	70.1304	53.2823	71.6625	71.6147
U	0.0601	0.0250	0.0345	0.0252	0.0252	58.4723	42.6752	57.9965	58.0559

4.2.2. GPS/BDS-3 and GPS/Galileo

Figure 5 shows the convergence time box diagram of GPS/BDS-3 and GPS/Galileo PPP for the five *IFCB* process schemes. Similar to GPS-only, the convergence performance of the *IFCB_PC*, *IFCB_RW*, and *IFCB_WN* schemes (about 15 min on average for GPS/BDS-3) was much shorter than that of the *IFCB_NO* and *IFCB_CT* schemes (about 20 min on average for GPS/BDS-3), which is a striking improvement. In general, the convergence performance of the *IFCB_PC*, *IFCB_RW*, and *IFCB_WN* schemes is comparable. From Figure 6, it is evident that the GPS/Galileo convergence performance is similar to that of the GPS/BDS-3 PPP solutions.

Figure 6 shows the comparisons of kinematic positioning errors for BRUX station for the five *IFCB* process schemes. Table 5 shows the weekly positioning accuracy of GPS/BDS-3 PPP at the 25 stations for the five *IFCB* schemes. Not surprisingly, the positioning accuracy of the *IFCB_NO* scheme is worse than that of the other schemes. Comparing *IFCB_NO* with the other three schemes (*IFCB_PC*, *IFCB_WN*, and *IFCB_RW*), the positioning accuracy is improved by 50.72% (from 2.05 cm to 1.01 cm) in the east, by 39.99% (1.55 cm to 0.93 cm) in the north, and by 52.72% (4.03 cm to 1.90 cm) in the up components. As shown in Table 5, the positioning accuracy of the multi-GNSS multi-frequency PPP solutions with *IFCB_PC*, *IFCB_WN*, and *IFCB_RW* is comparable among the five *IFCB* process schemes.

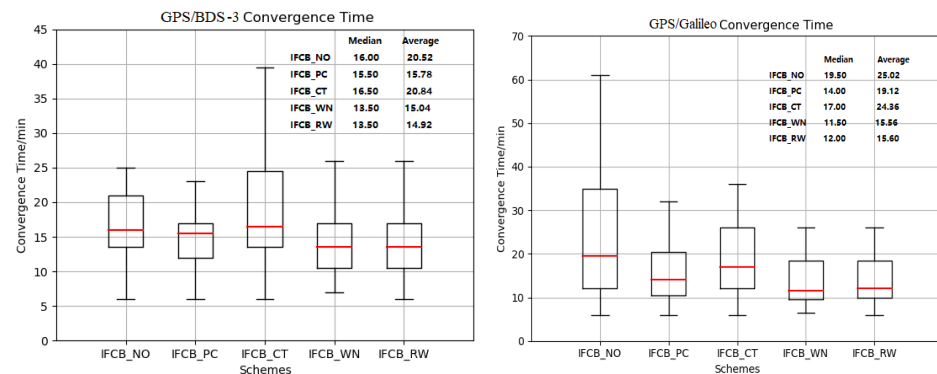


Figure 5. Comparisons of GPS/BDS-3 (Left) and GPS/Galileo (Right) convergence times for the five *IFCB* process schemes over 7 days.

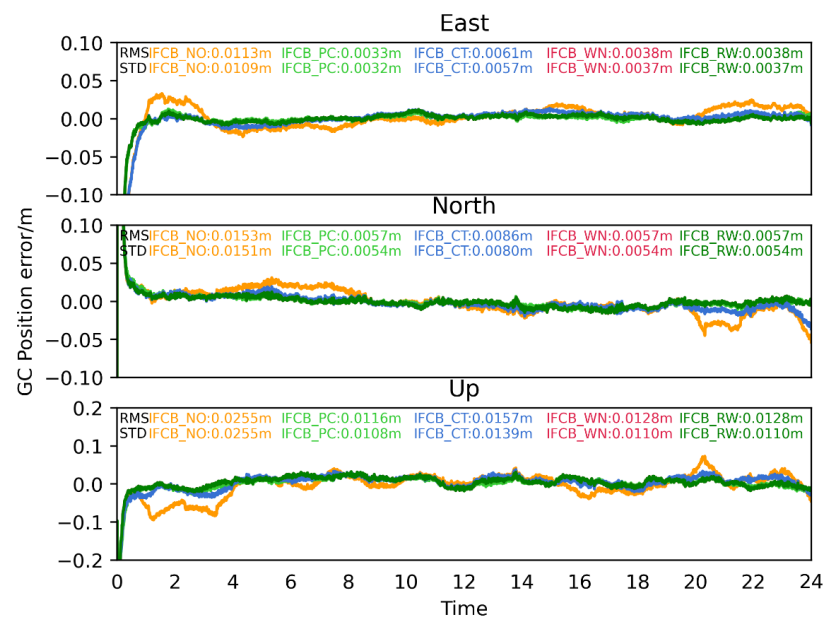


Figure 6. Comparisons of kinematic positioning errors for BRUX station for the five *IFCB* process schemes (DOY 18 January 2022).

Table 5. RMS (m) of GPS/BDS-3 and GPS/Galileo PPP solutions among different *IFCB* process schemes at 25 stations for one week.

Mode	RMS/m						Improvement/%			
	<i>IFCB</i>	<i>IFCB_NO</i>	<i>IFCB_PC</i>	<i>IFCB_CT</i>	<i>IFCB_WN</i>	<i>IFCB_RW</i>	<i>IFCB_PC</i>	<i>IFCB_CT</i>	<i>IFCB_WN</i>	<i>IFCB_RW</i>
GPS/BDS-3	E	0.0205	0.0100	0.0140	0.0101	0.0101	51.1308	31.6568	50.7240	50.7234
	N	0.0155	0.0095	0.0107	0.0093	0.0093	38.7091	30.8660	39.9950	39.9978
	U	0.0403	0.0197	0.0241	0.0190	0.0190	51.1071	40.1568	52.7273	52.7317
GPS/Galileo	E	0.0226	0.0102	0.0131	0.0094	0.0094	54.8057	41.8822	58.5311	58.5279
	N	0.0164	0.0094	0.0106	0.0089	0.0089	42.5375	35.0849	45.3270	45.3292
	U	0.0374	0.0211	0.0246	0.0205	0.0205	43.4080	34.2153	45.0044	45.0119

With regard to convergence as GPS/Galileo PPP solutions, one can see from Table 5 that the positioning accuracy of the *IFCB_PC*, *IFCB_RW*, and *IFCB_WN* schemes is improved by 58.53% (from 2.26 cm to 0.94 cm) in the east, 45.33% (1.64 cm to 0.89 cm) in the north, and by 45.00% (3.74 cm to 2.05 cm) in the up components compared with the *IFCB_NO* scheme. As shown in Table 5, the *IFCB_PC*, *IFCB_WN*, and *IFCB_RW* schemes are comparable among the five *IFCB* process schemes for positioning accuracy of PPP solutions.

4.2.3. GPS/BDS-3/Galileo

Figure 7 shows the convergence time box diagram of GPS/BDS-3/Galileo PPP for the five *IFCB* process schemes. It is evident that the differences in convergence performance are similar to those of the GPS/BDS-3 PPP solutions.

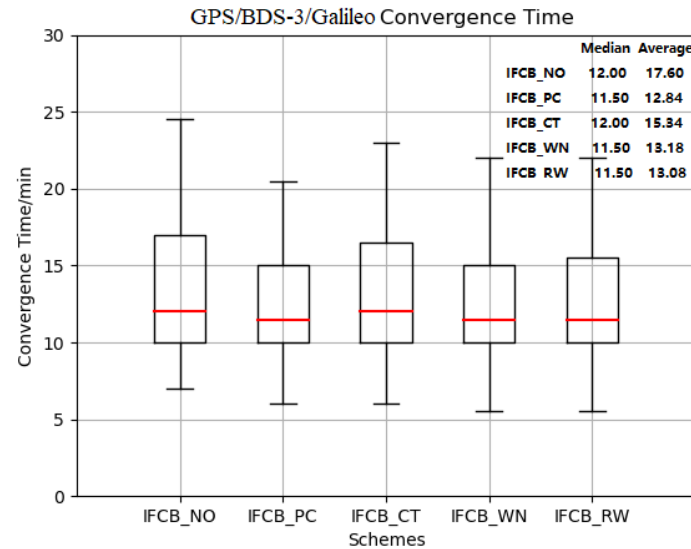


Figure 7. Comparisons of GPS/BDS-3/Galileo convergence times for the five *IFCB* process schemes over 7 days.

Comparing *IFCB_PC*, *IFCB_WN*, and *IFCB_RW* convergence times of GPS/BDS-3/Galileo PPP solutions, the average convergence time of *IFCB_NO* (or *IFCB_CT*) is 5 min less than that of *IFCB_WN* (and *IFCB_PC* or *IFCB_RW*). It is also evident, from Figure 7, that the median convergence times are comparable among the five *IFCB* process schemes. This is attributed to the increase in the number of BDS-3 and Galileo observations, in which the strength of the observation equation and the redundancy are increased. The positioning accuracy of the *IFCB_PC*, *IFCB_RW*, and *IFCB_WN* schemes is improved by 41.34% (from 1.37 cm to 0.81 cm) in the east, by 23.29% (1.10 cm to 0.84 cm) in the north, and by 37.92% (2.72 cm to 1.70 cm) in the up components compared to the *IFCB_NO* scheme. As shown in Table 6, the *IFCB_PC*, *IFCB_WN*, and *IFCB_RW* schemes are comparable among the five *IFCB* process schemes for positioning accuracy of PPP solutions.

Table 6. RMS (m) of GPS/BDS-3/Galileo PPP solutions among the five *IFCB* process schemes at the 25 stations over one week.

RMS/m						Improvement/%			
<i>IFCB</i>	<i>IFCB_NO</i>	<i>IFCB_PC</i>	<i>IFCB_CT</i>	<i>IFCB_WN</i>	<i>IFCB_RW</i>	<i>IFCB_PC</i>	<i>IFCB_CT</i>	<i>IFCB_WN</i>	<i>IFCB_RW</i>
E	0.0137	0.0082	0.0098	0.0081	0.0080	40.2152	28.6915	41.3393	41.7205
N	0.0110	0.0084	0.0087	0.0083	0.0083	23.2906	20.8129	24.0852	24.2910
U	0.0272	0.0175	0.0195	0.0169	0.0169	35.6965	28.2565	37.9158	37.9304

In conclusion, the above three sets of experiments show that it is feasible to use the parameter estimation method to deal with *IFCB*, and its effect on positioning accuracy and convergence performance is comparable to that of using product correction. Therefore, the white noise or random walk process can be used to describe *IFCB* parameters to accelerate the multi-GNSS multi-frequency PPP convergence in the absence of product correction or real-time applications.

4.3. Residual Analysis

Some unmodeled errors, such as *IFCB*, are reflected in the post residuals due to poor observation models or incomplete corrections. Figure 8 shows the post-phase residuals of the five *IFCB* process schemes of GPS L5 at BRUX station (DOY 18 January 2022), where different colors represent the residuals of different satellites. As previously concluded, the *IFCB_NO* and *IFCB_CT* schemes exhibit significant systematic bias effects in the GPS L5 phase residuals, while the *IFCB_PC*, *IFCB_WN*, and *IFCB_RW* schemes eliminate these effects. At the same time, to further evaluate the effect of *IFCB* on GPS L5 phase residuals, Figure 9 and Table 7 give the weekly RMS values of the GPS L5 phase residuals for the 25 stations. The histogram in Figure 9 shows the average GPS L5 phase residuals for one week at the 25 stations, while the line graph shows the percentage improvement of each station relative to the *IFCB_NO* scheme. Table 7 shows the average GPS L5 phase residual RMS values over one week of the five process schemes at the 25 stations. Compared to *IFCB_NO*, the RMS of the GPS L5 phase residuals of *IFCB_PC*, *IFCB_WN*, and *IFCB_RW* were improved from 0.3867 cm to 0.2311 cm, 0.0971 cm, and 0.0980 cm, respectively, which represent improvements of 40.25%, 74.89%, and 74.66%. The effect is especially pronounced in the case of the *IFCB_WN* and *IFCB_RW* schemes. Accordingly, it is concluded that the effect of the apparent systematic bias in the GPS L5 phase residuals can be eliminated using the *IFCB_PC*, *IFCB_WN*, or *IFCB_RW* schemes; the latter two schemes have significantly better residual values than the *IFCB_PC* scheme. The above conclusions indicate that the use of the estimation method results in an improved correction of the effects of *IFCB* compared to the use of product correction, and can help avoid satellite false rejects due to large residuals during data processing.

Table 7. RMS (cm) of GPS PPP L5 phase residuals among different *IFCB* process schemes at the 25 stations over one week.

RMS/cm					Promote/%			
<i>IFCB_NO</i>	<i>IFCB_PC</i>	<i>IFCB_CT</i>	<i>IFCB_WN</i>	<i>IFCB_RW</i>	<i>IFCB_PC</i>	<i>IFCB_CT</i>	<i>IFCB_WN</i>	<i>IFCB_RW</i>
0.3867	0.2311	0.3148	0.0971	0.0980	40.25	18.60	74.89	74.66

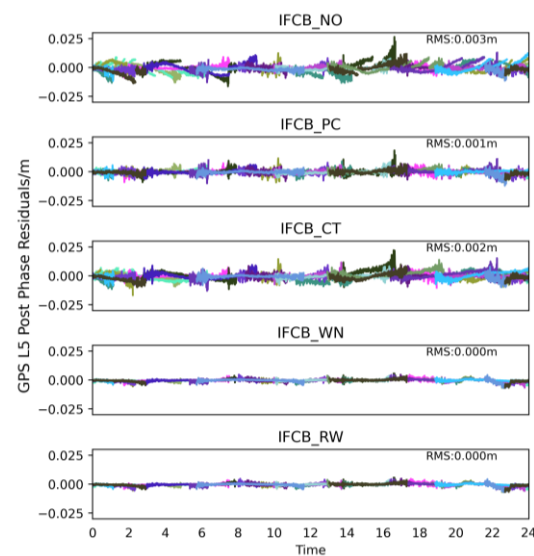


Figure 8. Comparisons of GPS L5 post phase residuals for BRUX station based on the five *IFCB* process schemes (DOY 18 January 2022), where each color represents a satellite.

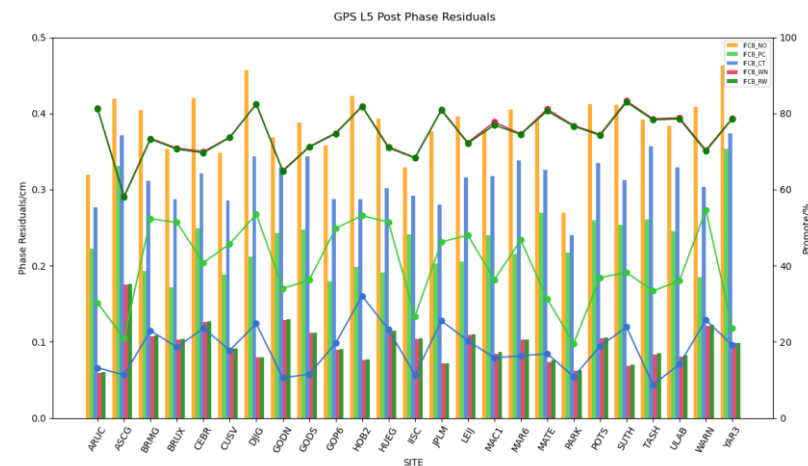


Figure 9. RMS error of GPS L5 phase residuals for the 25 stations over 7 days (RMS, unit: cm).

4.4. Discussion

The above results show that ignoring the effect of *IFCB* can seriously degrade the positioning performance of multi-GNSS multi-frequency PPP, and therefore the effect of *IFCB* must be considered. The IF combinations' *IFCB* estimation method averages the *IFCB* over all stations, and although this method uses epoch differences in the calculation process to eliminate most of the errors, there are still some unmodeled errors remaining in the epoch-difference *GFIF* observations, such as multipath, higher-order ionospheric delay, and others. This method does not consider the receiver *IFCB*, so the average receiver *IFCB* is absorbed by the satellite *IFCB*. If the number of stations in the network is limited, the receiver *IFCB* cannot be eliminated through averaging, so this method is not theoretically rigorous [25]. This is also the reason why the residuals of *IFCB_PC* are larger than those of *IFCB_RW* or *IFCB_WN*. This effect can be derived from the following equation:

Taking into account the unmodeled error τ_r^s in Equation (10), the following equation can be obtained:

$$GFIF = L_{r,IF12}^s - L_{r,IF13}^s = N_{GFIF} + B_{r,GFIF}^s + \delta\hat{B} + \tau_r^s \quad (18)$$

Then, the new epoch-difference *GFIF* is expressed as follows:

$$GFIF_t - GFIF_{t-1} = \Delta\delta\hat{B}_{(t,t-1)} + \tau_{r,(t,t-1)}^s \quad (19)$$

Therefore, the current epoch satellite *IFCB* becomes:

$$\Delta\delta\bar{B}_{(t,t-1)} = \frac{\sum_{r=1}^n (\Delta\delta\hat{B}_{(t,t-1)} + \tau_{r,(t,t-1)}^s) \cdot \omega_{r,(t,t-1)}}{\sum_{r=1}^n \omega_{r,(t,t-1)}} \quad (20)$$

Obviously $\tau_{r,(t,t-1)}^s$ causes an accumulation of errors, indicating that the *IFCB* estimated using the *IFCB_PC* method contains an unmodeled error τ_r^s for n stations, but this unmodeled error is small (RMS < 2 mm, see RMS values in Table 7) and can be neglected. The analysis in the previous sections shows that *IFCB_NO* and *IFCB_CT* have comparable convergence performance, but the positioning accuracy of the latter is improved. *IFCB_PC* has comparable positioning accuracy and convergence time with *IFCB_WN* and *IFCB_RW*, but the residuals of the latter two are smaller than that of the former. This indicates that the use of a proper *IFCB* stochastic model can achieve comparable performance with the use of the product, but the method of *IFCB* parameter estimation increases the computational effort on the user side and incurs higher requirements for user equipment. However, the impact on positioning performance is minimal. Therefore, the *IFCB_RW* or *IFCB_WN*

methods are recommended for processing *IFCB*, when an *IFCB* product is unavailable, in the case of users requiring real-time applications.

5. Conclusions

The incorporation of multi-frequency signals into GNSS has introduced new possibilities for precise positioning and rapid ambiguity resolution. The *IFCB* pertains to the variation in clock offsets among distinct frequencies of satellite signals, and its proper processing is of great importance, as it significantly impacts PPP accuracy when using multi-frequency signals. Neglecting the *IFCB* can result in reduced positioning accuracy and increased convergence time in both static and kinematic PPP solutions. In this study, the focus lies on the appropriate modeling for phase *IFCB* in multi-GNSS multi-frequency PPP based on the UDUC observation model. The optimal power spectral density applicable to *IFCB* is analyzed and studied, and the product correction methods are compared with parameter estimation methods. The impact of *IFCB* stochastic modeling on the performance of Undifferenced and Uncombined PPP is also evaluated from the aspects of convergence time and positioning accuracy through five *IFCB* process schemes, namely *IFCB_NO*, *IFCB_PC*, *IFCB_CT*, *IFCB_RW*, and *IFCB_WN*.

To validate these findings, an analysis of the effect of *IFCB* stochastic modeling on UDUC the multi-GNSS multi-frequency PPP performance was conducted using weekly data (DOY 18–24 January 2022) from 130 IGS MGEX tracking stations. Through this analysis, the following conclusions can be stated: (i) the optimal power spectral density for *IFCB* is 60, 0.006 m/sqrt(s); (ii) modeling *IFCB* as a random walk is feasible; (iii) the multi-GNSS multi-frequency PPP performance is comparable and best for the *IFCB_PC*, *IFCB_RW*, and *IFCB_WN* schemes, while the performance of the *IFCB_NO* and *IFCB_CT* schemes is poor; (iv) the third frequency phase residuals of the *IFCB_RW* and *IFCB_WN* scheme are smaller than those of the *IFCB_PC* scheme. However, any potential impacts on the troposphere, ionosphere, and float ambiguity are not discussed.

The methodology for estimating multi-GNSS satellite phase products is currently in a developmental stage, thus these products are not yet fully mature. The commonly utilized approach, which involves the use of *IFCBs* corrected via products, is applicable exclusively when the server provides such products. However, *IFCB* products are not readily accessible, especially for users who require immediate and continuous positioning solutions. Essentially, a random walk is an expansion and accumulation of white noise, and the *IFCB_RW* and *IFCB_WN* schemes are indistinguishable. However, compared to white noise, the random walk has the advantages of exhibiting trends, correlation, and memory, making it more suitable for modeling phenomena with directionality, time dependence, and historical dependence. Therefore, in the absence of *IFCB* products, modeling *IFCBs* as random walks can result in significantly more reliable performance.

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References

- Hein, G.W. Status, perspectives and trends of satellite navigation. *Satell. Navig.* **2020**, *1*, 22. [\[CrossRef\]](#) [\[PubMed\]](#)
- Montenbruck, O.; Hauschild, A.; Steigenberger, P.; Langley, R. Three's the Challenge: A Close Look at GPS SVN62 Triple-frequency Signal Combinations Finds Carrier-phase Variations on the New L5. *GPS World* **2010**, *21*, 8–19.
- Pan, L.; Zhang, X.; Guo, F.; Liu, J. GPS inter-frequency clock bias estimation for both uncombined and ionospheric-free combined triple-frequency precise point positioning. *J. Geod.* **2019**, *93*, 473–487. [\[CrossRef\]](#)
- Pan, L.; Li, X.; Zhang, X.; Li, X.; Lu, C.; Zhao, Q.; Liu, J. Considering Inter-Frequency Clock Bias for BDS Triple-Frequency Precise Point Positioning. *Remote Sens.* **2017**, *9*, 734. [\[CrossRef\]](#)
- Guo, J.; Geng, J. GPS satellite clock determination in case of inter-frequency clock biases for triple-frequency precise point positioning. *J. Geod.* **2018**, *92*, 1133–1142. [\[CrossRef\]](#)
- Montenbruck, O.; Hugentobler, U.; Dach, R.; Steigenberger, P.; Hauschild, A. Apparent clock variations of the Block IIF-1 (SVN62) GPS satellite. *GPS Solut.* **2012**, *16*, 303–313. [\[CrossRef\]](#)
- Li, H.; Li, B.; Xiao, G.; Wang, J.; Xu, T. Improved method for estimating the inter-frequency satellite clock bias of triple-frequency GPS. *GPS Solut.* **2016**, *20*, 751–760. [\[CrossRef\]](#)
- Li, H.; Zhou, X.; Wu, B.; Wang, J. Estimation of the inter-frequency clock bias for the satellites of PRN25 and PRN01. *Sci. China-Phys. Mech. Astron.* **2012**, *55*, 2186–2193. [\[CrossRef\]](#)
- Wang, Z.; Wang, R.; Wang, Y.; Hu, C.; Liu, B. Modelling and Assessment of a New Triple-Frequency IF1213 PPP with BDS/GPS. *Remote Sens.* **2022**, *14*, 4509. [\[CrossRef\]](#)
- Li, X.; Li, X.; Liu, G.; Feng, G.; Yuan, Y.; Zhang, K.; Ren, X. Triple-frequency PPP ambiguity resolution with multi-constellation GNSS: BDS and Galileo. *J. Geod.* **2019**, *93*, 1105–1122. [\[CrossRef\]](#)
- Xu, G.; Xu, Y. Applications of GPS theory and algorithms. In *GPS*; Springer: Berlin/Heidelberg, Germany, 2016.
- Xie, X.; Fang, R.; Geng, T.; Wang, G.; Zhao, Q.; Liu, J. Characterization of GNSS Signals Tracked by the iGMAS Network Considering Recent BDS-3 Satellites. *Remote Sens.* **2018**, *10*, 1736. [\[CrossRef\]](#)
- Zhang, F.; Chai, H.; Li, L.; Xiao, G.; Du, Z. Estimation and analysis of GPS inter-frequency clock biases from long-term triple-frequency observations. *GPS Solut.* **2021**, *25*, 126. [\[CrossRef\]](#)
- Pan, L.; Zhang, X.; Li, X.; Liu, J.; Li, X. Characteristics of inter-frequency clock bias for Block IIF satellites and its effect on triple-frequency GPS precise point positioning. *GPS Solut.* **2017**, *21*, 811–822. [\[CrossRef\]](#)
- Li, H.; Zhou, X.; Wu, B. Fast estimation and analysis of the inter-frequency clock bias for Block IIF satellites. *GPS Solut.* **2013**, *17*, 347–355. [\[CrossRef\]](#)
- Fan, L.; Shi, C.; Li, M.; Wang, C.; Zheng, F.; Jing, G.; Zhang, J. GPS satellite inter-frequency clock bias estimation using triple-frequency raw observations. *J. Geod.* **2019**, *93*, 2465–2479. [\[CrossRef\]](#)
- Thöelert, S.; Steigenberger, P.; Montenbruck, O. Signal analysis of the first GPS III satellite. *GPS Solut.* **2019**, *23*, 92. [\[CrossRef\]](#)
- Chen, Y.; Mi, J.; Gu, S.; Li, B.; Li, H.; Yang, L.; Pang, Y. GPS, BDS-3, and Galileo Inter-Frequency Clock Bias Deviation Time-Varying Characteristics and Positioning Performance Analysis. *Remote Sens.* **2022**, *14*, 3991. [\[CrossRef\]](#)
- Zhang, Z.; Pan, L. Modeling and predicting inter-frequency clock bias of BDS-2 GEO, IGSO and MEO satellites for triple-frequency precise point positioning. *GPS Solut.* **2022**, *26*, 89. [\[CrossRef\]](#)
- Gong, X.; Gu, S.; Lou, Y.; Zheng, F.; Yang, X.; Wang, Z.; Liu, J. Research on empirical correction models of GPS Block IIF and BDS satellite inter-frequency clock bias. *J. Geod.* **2020**, *94*. [\[CrossRef\]](#)
- Li, P.; Jiang, X.; Zhang, X.; Ge, M.; Schuh, H. GPS + Galileo + BeiDou precise point positioning with triple-frequency ambiguity resolution. *GPS Solut.* **2020**, *24*, 78. [\[CrossRef\]](#)
- Geng, J.; Guo, J.; Meng, X.; Gao, K. Speeding up PPP ambiguity resolution using triple-frequency GPS/BeiDou/Galileo/QZSS data. *J. Geod.* **2020**, *94*, 6. [\[CrossRef\]](#)
- Guo, F.; Zhang, X.; Wang, J.; Ren, X. Modeling and assessment of triple-frequency BDS precise point positioning. *J. Geod.* **2016**, *90*, 1223–1235. [\[CrossRef\]](#)
- Xiang, Y.; Gao, Y.; Shi, J.; Xu, C. Carrier phase-based ionospheric observables using PPP models. *Geod. Geodyn.* **2017**, *8*, 17–23. [\[CrossRef\]](#)
- Zhang, Y.; Nobuaki, K.; Chen, J.; Wang, H.; Wang, J. An improved method for BDS inter-frequency clock bias estimation. In Proceedings of the China Satellite Navigation Conference (CSNC), Beijing, China, 22–25 May 2019; Volume II.
- Kouba, J. *A Guide to Using International GNSS Service (IGS) Products*; Geodetic Survey Division; Natural Resources Canada: Ottawa, ON, Canada, 2015.
- Liu, T.; Zhang, B.; Yuan, Y.; Li, Z.; Wang, N. Multi-GNSS triple-frequency differential code bias (DCB) determination with precise point positioning (PPP). *J. Geod.* **2018**, *93*, 765–784. [\[CrossRef\]](#)
- Zhang, B.; Teunissen, P.J.; Yuan, Y. On the short-term temporal variations of GNSS receiver differential phase biases. *J. Geod.* **2017**, *91*, 563–572. [\[CrossRef\]](#)
- Reischung, P.; Schmid, R. IGS14/igs14.atx: A new framework for the IGS products. In Proceedings of the AGU Fall Meeting, San Francisco, CA, USA, 10–14 December 2017.
- Li, X.; Ge, M.; Dai, X.; Ren, X.; Fritsche, M.; Wickert, J.; Schuh, H. Accuracy and reliability of multi-GNSS real-time precise positioning: GPS, GLONASS, BeiDou, and Galileo. *J. Geod.* **2015**, *89*, 607–635. [\[CrossRef\]](#)

31. Li, P.; Zhang, X. Integrating GPS and GLONASS to accelerate convergence and initialization times of precise point positioning. *GPS Solut.* **2014**, *18*, 461–471. [[CrossRef](#)]
32. Zhou, F.; Dong, D.; Li, W.; Jiang, X.; Wickert, J.; Schuh, H. GAMP: An open-source software of multi-GNSS precise point positioning using undifferenced and uncombined observations. *GPS Solut.* **2018**, *22*, 33. [[CrossRef](#)]
33. Zhou, F.; Dong, D.; Li, P.; Li, X.; Schuh, H. Influence of stochastic modeling for inter-system biases on multi-GNSS undifferenced and uncombined precise point positioning. *GPS Solut.* **2019**, *23*, 59. [[CrossRef](#)]
34. Li, B.; Mi, J.; Zhu, H.; Gu, S.; Xu, Y.; Wang, H.; Yang, L.; Chen, Y.; Pang, Y. BDS-3/GPS/Galileo OSB Estimation and PPP-AR Positioning Analysis of Different Positioning Models. *Remote Sens.* **2022**, *14*, 4207. [[CrossRef](#)]
35. Lou, Y.; Zheng, F.; Gu, S.; Wang, C.; Guo, H.; Feng, Y. Multi-GNSS precise point positioning with raw single-frequency and dual-frequency measurement models. *GPS Solut.* **2016**, *20*, 849–862. [[CrossRef](#)]

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