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A Steering-Vector-Based Matrix Information Geometry Method for Space–Time Adaptive Detection in Heterogeneous Environments

Runming Zou , Yongqiang Cheng *, Hao Wu , Zheng Yang , Xiaoqiang Hua  and Hanjie Wu 

College of Electronic Science and Technology, National University of Defense Technology, Changsha 410073, China; zourunming18@nudt.edu.cn (R.Z.); wuhao13c@nudt.edu.cn (H.W.); yangzheng18@nudt.edu.cn (Z.Y.); huaxiaoqiang12@nudt.edu.cn (X.H.); wuhanjie21@nudt.edu.cn (H.W.)

* Correspondence: yqcheng@nudt.edu.cn

Abstract: Plagued by heterogeneous clutter, it is a serious challenge for airborne radars to detect low-altitude, weak targets. To overcome this problem, a novel matrix information geometry detector for airborne multi-channel radar is proposed in this paper. The proposed detector applies the given steering vector and array structure information to the matrix information geometry detection method so that it can be used for space–time adaptive detection. While improving the detection performance, the matrix information geometry detector’s original anti-clutter advantage is enhanced as well. The simulation experiment results indicate that the proposed detector has advantages in several of the properties related to space–time adaptive detection, while its computational complexity does not increase significantly. Moreover, experiment results based on the measured data verify the superior performance of the proposed method. Sea-detecting data-sharing-program data, mountaintop data, and phased-array radar data are employed to verify the performance advantage of the proposed method in heterogeneous clutter and the ability for weak target detection.

Keywords: information geometry; space–time adaptive detection; heterogeneous clutter; airborne radar



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1. Introduction

Due to the movement of a radar platform, airborne radar clutter appears to be significantly broadened in the frequency domain, which is characterized by two-dimensional coupling in space–time. In such a case, it is difficult for the traditional clutter suppression and target detection methods to achieve satisfactory detection performance. In order to solve this problem, space–time adaptive processing (STAP) has been developed and is widely used in airborne radar clutter suppression [1]. By transmitting and receiving multiple pulses from an array radar, STAP filters the clutter in both the spatial and temporal dimensions. Unfortunately, although STAP can effectively improve the airborne radar’s clutter suppression ability when the homogeneous training samples are sufficient [2,3], it suffers from heterogeneous clutter in a complex environment, and in such a case, its clutter suppression effectiveness greatly affects the performance of target detection. In recent years, the space–time adaptive detection (STAD) [4,5] method has been studied by many scholars. Different from the existing filter-then-CFAR detection methods [6], STAD is a target detection method that jointly processes the received signal in both the spatial and temporal domains and directly outputs the detection statistics. Compared to STAP-based airborne radar target detection methods, STAD has the following advantages [7]: (1) It usually has CFARness and does not require specialized constant false alarm rate (CFAR) techniques. (2) It has better detection performance, especially under low signal-to-clutter ratio (SCR) conditions.

It can be seen that STAD is a radar target detection technique for multi-channel radar. Multi-channel adaptive signal detection was first investigated by Kelly in 1986 [8].

In [8], Kelly proposed Kelly's generalized likelihood ratio test (KGLRT) to detect a rank-one signal in a homogeneous environment. A rank-one signal refers to a target with a known steering vector but an unknown amplitude [6]. Based on the generalized likelihood ratio test (GLRT) criterion, Chen and Robey proposed the adaptive matched filter (AMF) independently [9,10]. According to the Rao detector and Wald detector, De Maio proposed the corresponding detectors in [11,12] and proved that the AMF is equivalent to the Wald detector. The probability of detection (P_d) of the Rao detector obtained by De Maio (DMRao) [11] is lower than those of the KGLRT and AMF detectors, but it is superior to them in terms of rejecting mismatched signals.

It is worth highlighting that the aforementioned algorithms are considered in homogeneous environments [13–18]; however, since airborne radar usually works in a downward-looking state [19], it faces a severely heterogeneous clutter environment. Heterogeneous clutter refers to clutter for which its characteristics are inconsistent with that of the cell under test (CUT), such as clutter with a different power [20] or probability density function (PDF). When detecting the target, heterogeneous clutter interferes with the detector's estimation of the clutter, thus affecting the correctness of the detection results. The irregular terrain and sea surface make the environment heterogeneous, and the available training samples are drastically reduced, leading to performance degradation of existing algorithms in heterogeneous clutter environments [21]. To solve this problem, a class of clutter covariance matrix (CCM) estimation methods and detection algorithms [22,23] are investigated. Many studies have improved the estimation method of the CCM to enhance the robustness of detectors in heterogeneous environments [24–28], but these methods usually need iterative operations and are accompanied by heavy computational burden. In addition, based on the generalization of the Euclidean center, some geometric-center-based CCM estimation methods are presented in [29–31]. These methods have shown distinct superiority in radar signal processing, but their algorithm complexity is too complicated as well.

Recently, related studies on radar target detection methods based on information geometry received a lot of attention for their distinct advantages in hetero-clutter environments [32–44]. Specifically, the matrix information geometry (MIG) detector, which was pioneered by Barbaresco [45] and developed by Cheng et al. [46], shows a novel detection scheme for target detection. Thanks to the excellent discriminative ability of the geodesic distance on the matrix manifold, the MIG detector has outstanding performance in hetero-clutter environments [47]. Originally, the Riemann distance was adopted by the MIG detector [45,48], but its computational burden is too heavy. To overcome this problem, many other measurements have been introduced into the MIG detector [49]. Typically, the Kullback–Leibler divergence (KLD) is widely used [50]. While the MIG detector has made great progress and proved its advantages in hetero-clutter environments [51–53], which matters much in STAD, it is not applied in STAD because

- The given steering vector of the target is abandoned by existing MIG detectors, causing underutilized information in STAD. In addition, the MIG detector fails to suppress the clutter in the spatial and temporal domains; therefore, the performance of the MIG detector can be further improved for airborne radar.
- The various existing MIG detectors are designed for single-channel radar. For multi-channel radar, there is a strong correlation between channels, which has not been considered before. This problem is especially accentuated when the dimensionality of the data matrix increases. At present, MIG detectors can only work with dimensionality reduction algorithms, which put a price on performance loss.

To apply MIG methods on STAD, a novel MIG detector is proposed in this paper. Inspired by the existing STAD methods and the STAP method, we employ the space–time steering vectors to describe the target information during each temporal Doppler scan and the correlation between channels. Based on the GRLT, we deduce the detection formula of the proposed method and analyze its performance. The superiority of the proposed

method is demonstrated by experimental results. Particularly, the principal contributions of this paper are listed as follows.

- We provide a novel MIG detector for STAD. The proposed steering-vector-based matrix information (S-MIG) detector introduces the steering vector into target detection, overcoming the shortcomings of the MIG detector and making it feasible to apply information geometry methods to STAD.
- We perform a detailed analysis and validation of the performance of the S-MIG detector, especially the important property that matters in target detection. The mathematical derivation and experimental results illustrate that the proposed S-MIG detector has competitive performance, while its computational burden is comparable to existing methods.
- We describe a novel detection scheme for detectors based on matrix information geometry. In cases for which the target's steering vector is given, a detection scheme that is a hyperbola on the Hermitian positive definite (HPD) manifold is proposed, and many detection methods based on various metrics can be extended from it.

The remainder of this paper is organized as follows. Section 2 reviews the signal model of space–time coupling clutter for STAD and introduces the preliminaries of matrix information geometry. In Section 3, the proposed steering-vector-based MIG detector is deduced, and its geometry explanation is given. In addition, several important properties of the detector are analyzed in Section 3 as well. Section 4 presents experimental results based on both simulated and real data. Finally, Section 5 provides a brief conclusion of this paper.

The following notations are adopted in this paper: the math italic x , lowercase italic x , and uppercase bold $\mathbf{A}$ denote scalars, vectors, and matrices, respectively; $\bar{x}$ is the conjugate of x . Symbols $(\cdot)^{-1}$ and $(\cdot)^H$ denote the inverse and the conjugate transpose of matrices, respectively. $E(\cdot)$ denotes the statistical expectation, and $\otimes$ denotes the Kronecker product. $\mathbf{A} \succ 0$ represents that $\mathbf{A}$ is a positive definite matrix. Constant matrix $\mathbf{I}$ indicates the identity matrix.

2. Problem Formulation and Preliminaries of MIG

2.1. Signal Model and Detection Model

Consider a side-looking airborne radar that is a uniform linear array (ULA) with N elements. Assuming that each antenna receives P coherent pulses, the degree of freedom (DoF) M of this system is $M = NP$. As shown in Figure 1, we appoint the parameters of the radar as follows. The airborne platform travels with a velocity v , and the azimuth angle and the elevation angle between the radar and clutter are θ and φ , respectively.

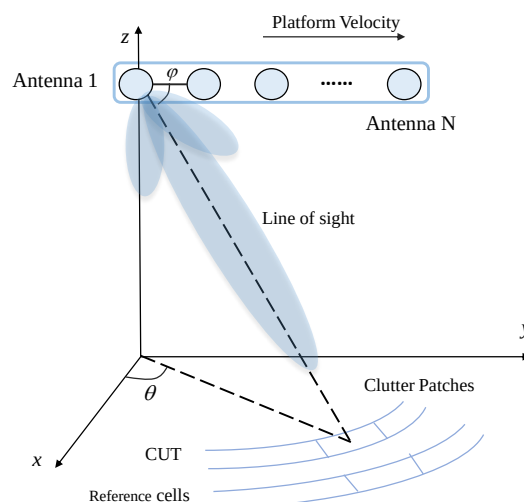


Figure 1. Geometric model of airborne radar system.

Denote $\mathbf{x}_k$ as the received signal of the ULA in the k th range cell, which is composed of the target echo $\mathbf{x}_t$ (if the target exists), clutter signal $\mathbf{x}_c$, and noise $\mathbf{n}$. According to the Wald clutter model of airborne radar [54], the clutter received by antennas is the summation of the echo of each clutter patch: namely,

$$\begin{aligned}\mathbf{x}_k &= \alpha_t \mathbf{s}_t + \mathbf{x}_{ck} + \mathbf{n} \\ &= \alpha_t \mathbf{s}_t + \sum_{i=1}^{N_c} \alpha_i \mathbf{s}_i + \mathbf{n}\end{aligned}\quad (1)$$

where N_c is the number of clutter patches in the k th range cell; α_t and $\mathbf{s}_t$ are the reflection coefficient and the steering vector, respectively, of the target; analogously, α_i and $\mathbf{s}_i$ are the reflection coefficient and the steering vector of the i th clutter patch in k th range cell, respectively.

The steering vector is the Kronecker product of the spatial steering vector $\mathbf{s}_s(f_s)$ and the temporal steering vector $\mathbf{s}_d(f_d)$, which are determined by the position of the tangential and radial Doppler. The space-time steering vector is written as

$$\mathbf{s} = \mathbf{s}_t(f_d) \otimes \mathbf{s}_s(f_s) \quad (2)$$

where

$$\begin{aligned}\mathbf{s}_t(f_d) &= [1, e^{j2\pi f_d}, \dots, e^{j(P-1)2\pi f_d}]^T \\ \mathbf{s}_s(f_s) &= [1, e^{j2\pi f_s}, \dots, e^{j(N-1)2\pi f_s}]^T\end{aligned}\quad (3)$$

and the temporal Doppler $f_d = 2vT/\lambda \cos \theta \cos \varphi$ and the spatial Doppler $f_s = d/\lambda \cos \theta \cos \varphi$; λ is the wavelength, and d is the distance between adjacent array elements.

In radar target detection, the received signals in different range cells can be divided into two sorts: the data in the CUT (also referred to as the primary data), which may include the target echo and the reference range cells located on both sides of the CUT, and the data in the reference range cells, which are called secondary data. Based on this classification, the problem of radar target detection can be expressed as a binary hypothesis problem:

$$\begin{aligned}\mathcal{H}_0 &: \begin{cases} \mathbf{x}_{\text{CUT}} = \mathbf{x}_c + \mathbf{n} \\ \mathbf{x}_k = \mathbf{x}_{ck} + \mathbf{n} \quad k = 1, \dots, K \end{cases} \\ \mathcal{H}_1 &: \begin{cases} \mathbf{x}_{\text{CUT}} = \alpha_t \mathbf{s}_t + \mathbf{x}_c + \mathbf{n} \\ \mathbf{x}_k = \mathbf{x}_{ck} + \mathbf{n} \quad k = 1, \dots, K \end{cases}\end{aligned}\quad (4)$$

where $\mathbf{x}_{\text{CUT}}$ is the data in the CUT, and $\mathbf{x}_k, k = 1, \dots, K$ represent the data in the reference range cells. K is the number of reference cells, and $\mathbf{x}_c$ is the clutter in the CUT and is usually unknown. The clutter in the CUT and the reference range cells are supposed to be independent and identically distributed (IID) so that the secondary data $\mathbf{x}_k, k = 1, \dots, K$ can be used to estimate $\mathbf{x}_c$.

To make use of the correlation between different array elements and pulses, the clutter covariance matrix of the CUT is often employed to improve the detection performance, which is shown as

$$\mathbf{R}_k = E[(\mathbf{x}_k \mathbf{x}_k^H)] = \begin{bmatrix} x_0 & \bar{x}_1 & \dots & \bar{x}_{M-1} \\ x_1 & x_0 & \dots & \bar{x}_{M-2} \\ \vdots & \ddots & \ddots & \vdots \\ x_{M-1} & x_{M-2} & \dots & x_0 \end{bmatrix} \quad (5)$$

where $x_k = \sum_{i=k+1}^M x_i x_{i+k}, 0 \leq k \leq M-1$.

The problem of radar target detection in the form of the covariance matrix is expressed as

$$\begin{aligned} \mathcal{H}_0 : & \begin{cases} \mathbf{R}_{\text{CUT}} = \mathbf{R}_c + \mathbf{R}_n \\ \mathbf{R}_k = \mathbf{R}_{ck} + \mathbf{R}_n \quad k = 1, \dots, K \end{cases} \\ \mathcal{H}_1 : & \begin{cases} \mathbf{R}_{\text{CUT}} = \mathbf{R}_t + \mathbf{R}_c + \mathbf{R}_n \\ \mathbf{R}_k = \mathbf{R}_{ck} + \mathbf{R}_n \quad k = 1, \dots, K \end{cases} \end{aligned} \quad (6)$$

where $\mathbf{R}_{\text{CUT}}$, $\mathbf{R}_c$, $\mathbf{R}_n$, $\mathbf{R}_k$, $\mathbf{R}_{ck}$, and $\mathbf{R}_t$ are the covariance matrices constructed by $\mathbf{x}_{\text{CUT}}$, $\mathbf{x}_c$, $\mathbf{n}$, $\mathbf{x}_k$, $\mathbf{x}_{ck}$, and $\alpha_t \mathbf{s}_t$, respectively.

2.2. Preliminaries of Matrix Information Geometry

According to Formula (5), the covariance matrices of the secondary data are Hermitian and Toeplitz. In addition, these matrices are positive definite matrices as well. Therefore, the matrices utilized in radar signal processing are located on the HPD manifold, which is defined as follows:

$$\mathcal{M}_{H++} = \left\{ \mathbf{R} \in \mathcal{T}(M, \mathbb{C}) \mid \mathbf{R}^H = \mathbf{R}, \mathbf{R} \succ \mathbf{0} \right\} \quad (7)$$

where $\mathcal{M}_{H++}$ denotes the HPD manifold, and $\mathcal{T}(M, \mathbb{C})$ is the set of $M \times M$ dimensional complex Toeplitz matrices.

The HPD manifold is a set constructed by all HPD matrices and has a certain spatial structure, which is distinct from the Euclidean space. On the HPD manifold, the different matrices of secondary data respond to different points, and their relationship in statistic characters can be expressed by the geometric measurement on the HPD manifold. Because the manifold is a curved space, the geodesic distance between two matrices $\mathbf{P}, \mathbf{Q} \in \mathcal{M}$ on the manifold $\mathcal{M}$ is called the Fisher information distance, which is defined as

$$\mathcal{D}(\mathbf{P}, \mathbf{Q}) = \inf \{ \mathcal{L}(\zeta) \mid \zeta : [a, b] \rightarrow \mathcal{M}, \zeta(a) = \mathbf{P}, \zeta(b) = \mathbf{Q} \} \quad (8)$$

where $\mathcal{L}(\zeta)$ is the length of ζ , which is a sufficiently smooth curve on $\mathcal{T}(M, \mathbb{C})$. Specifically, the Fisher information distance on the HPD manifold is called the Riemann distance (RD), which is

$$\mathcal{D}_{\text{Rie}}(\mathbf{P}, \mathbf{Q}) = \left\| \log(\mathbf{P}^{-1} \mathbf{Q}) \right\|_{\text{F}} \quad (9)$$

where $\log(\cdot)$ is the logarithm of the matrix, and $\|\cdot\|_{\text{F}}$ is the Frobenius norm of the matrix. It is worth indicating that, while the Riemann distance is the true geodesic distance on the HPD manifold, many other measurements, such as geometric divergence, are widely used in practice. Some of these measurements do not strictly satisfy the definition of distance on the manifold, but they are easily calculated and have preferable detection performance in some cases. The most commonly used KL divergence on the HPD manifold is defined as

$$\mathcal{D}_{\text{KL}}(\mathbf{P}, \mathbf{Q}) = \text{tr}(\mathbf{P}^{-1} \mathbf{Q}) - \log(|\mathbf{P}^{-1} \mathbf{Q}|) - M \quad (10)$$

Based on various geometric measurements on the matrix manifold, different MIG detectors are studied, but their detection formulations can be summarized as

$$\mathcal{D}(\mathbf{R}_{\text{CUT}}, \hat{\mathbf{R}}) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma \quad (11)$$

where $\hat{\mathbf{R}}$ is the estimated value of the clutter covariance matrix by the secondary data, and γ is the detection threshold. $\mathcal{D}(\cdot)$ represents the geometric distance employed by the MIG detector.

3. Steering-Vector-Based Matrix Information Geometry Detector and Its Performance Analysis

In this section, the S-MIG detector for STAD is proposed from the likelihood ratio test, and its geometric explanation on the HPD manifold is given. In addition, as a target detection method, the performance of the S-MIG detector is analyzed in theory.

3.1. Steering-Vector-Based Matrix Information Geometry Detector

In statistical signal processing, the likelihood ratio test is a basic form of hypothesis testing. Denote $x_1, x_2, \dots, x_M$ as the M independent and identically distributed observations of $\mathbf{Q}$, and $\mathbf{q}$ is the probability density function. Assume that there are two hypotheses for $\mathbf{q}$, denoted as $\mathbf{p}_0$ and $\mathbf{p}_1$, respectively. The likelihood ratio of this hypothesis test problem is

$$L = \prod_{i=1}^M \frac{\mathbf{p}_1(x_i)}{\mathbf{p}_0(x_i)}, \quad (12)$$

and the normalized log-likelihood ratio is

$$l = \frac{1}{M} \ln L = \frac{1}{M} \sum_{i=1}^M \ln \frac{\mathbf{p}_1(x_i)}{\mathbf{p}_0(x_i)}. \quad (13)$$

Actually, l is the arithmetic average of log-likelihood ratio l_i . According to the strong law of large numbers, l convergences in probability to the expectation of $l_i, i = 1, \dots, M$: namely,

$$\begin{aligned} l = E[l_i] &= \int \mathbf{q}(x) \ln \frac{\mathbf{p}_1(x)}{\mathbf{p}_0(x)} dx \\ &= \int \mathbf{q}(x) \ln \left[\frac{\mathbf{p}_1(x)}{\mathbf{p}_0(x)} \frac{\mathbf{q}(x)}{\mathbf{q}(x)} \right] dx \\ &= \int \mathbf{q}(x) \left[\ln \frac{\mathbf{q}(x)}{\mathbf{p}_0(x)} - \ln \frac{\mathbf{q}(x)}{\mathbf{p}_1(x)} \right] dx \\ &= \int \mathbf{q}(x) \ln \frac{\mathbf{q}(x)}{\mathbf{p}_0(x)} dx - \int \mathbf{q}(x) \ln \frac{\mathbf{q}(x)}{\mathbf{p}_1(x)} dx. \end{aligned} \quad (14)$$

Denote $\mathcal{D}_{\text{KL}}(\mathbf{q}||\mathbf{p}_0)$ and $\mathcal{D}_{\text{KL}}(\mathbf{q}||\mathbf{p}_1)$ as the KLD from $\mathbf{q}_0$ to $\mathbf{p}$ and the KLD from $\mathbf{q}_1$ to $\mathbf{p}$, respectively; the formula mentioned above can be written as

$$l = \mathcal{D}_{\text{KL}}(\mathbf{q}||\mathbf{p}_0) - \mathcal{D}_{\text{KL}}(\mathbf{q}||\mathbf{p}_1) \quad (15)$$

Formula (15) reveals the equivalence between the likelihood ratio and the Kullback–Leibler divergence; therefore, the likelihood ratio test also has its KL divergence form, expressed as

$$\mathcal{D}_{\text{KL}}(\mathbf{q}||\mathbf{q}_0) - \mathcal{D}_{\text{KL}}(\mathbf{q}||\mathbf{q}_1) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\geq}} \gamma \quad (16)$$

Compare Formulas (13) and (16): the KLD can be interpreted as a measurement of probability. The larger $\mathcal{D}_{\text{KL}}(\mathbf{q}||\mathbf{p}_0)$ is, the lower probability that hypothesis $\mathbf{p}_0$ has of being true.

For data in matrix form, the detection formula for radar target detection is

$$\mathcal{D}_{\text{KL}}(\mathbf{R}_{\text{CUT}}, \mathbf{R}_{\text{c}}) - \mathcal{D}_{\text{KL}}(\mathbf{R}_{\text{CUT}}, \mathbf{R}_{\text{c}} + \mathbf{R}_{\text{t}}) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\geq}} \gamma \quad (17)$$

Noted that there is no a priori knowledge of the parameter α_t : the matrix $\mathbf{R}_t$ in Formula (17) is unknown. In this case, the generalized likelihood ratio test (GLRT) is a suitable means for circumventing such a priori uncertainty. The GLRT is

$$\max_{\alpha_t} l \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\geq}} \gamma \quad (18)$$

which is equal to

$$\max_{\alpha_t} (\mathcal{D}_{\text{KL}}(\mathbf{R}_{\text{CUT}}, \mathbf{R}_c) - \mathcal{D}_{\text{KL}}(\mathbf{R}_{\text{CUT}}, \mathbf{R}_c + \mathbf{R}_t)) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\geq}} \gamma \quad (19)$$

The gradient of the left-hand side (LHS) of Formula (19) is

$$\nabla_{\mathbf{R}_t} LHS = -\mathbf{R}_{\text{CUT}}^{-1} + \frac{1}{\mathbf{R}_{\text{CUT}}(\mathbf{R}_c + \mathbf{R}_t)} \cdot \mathbf{R}_{\text{CUT}} \quad (20)$$

From the equation $\nabla_{\mathbf{R}_t} LHS = 0$, the LHS of Formula (19) can be easily shown to be minimum at:

$$\mathbf{R}_{\text{CUT}} = \mathbf{R}_c + \mathbf{R}_t \quad (21)$$

The least-square solution of this contradictory equation is

$$\alpha_t \alpha_t^H = \frac{\langle \mathbf{R}_{\text{CUT}} - \mathbf{R}_c, \mathbf{s}_t \mathbf{s}_t^H \rangle}{\langle \mathbf{s}_t \mathbf{s}_t^H, \mathbf{s}_t \mathbf{s}_t^H \rangle} \quad (22)$$

where $\langle \cdot \rangle$ is the inner product of the matrix. Taking Formulas (22) and (10) into (17), we get the output of the S-MIG detector

$$\text{tr}(\mathbf{R}_{\text{CUT}}^{-1} \mathbf{R}_c) - \text{tr}(\mathbf{R}_{\text{CUT}}^{-1} (\mathbf{R}_c + \mathbf{R}_t)) - \log(|\mathbf{R}_{\text{CUT}}^{-1} \mathbf{R}_c|) + \log(|\mathbf{R}_{\text{CUT}}^{-1} (\mathbf{R}_c + \mathbf{R}_t)|) \quad (23)$$

in which

$$\mathbf{R}_t = \frac{\langle \mathbf{R}_{\text{CUT}} - \mathbf{R}_c, \mathbf{s}_t \mathbf{s}_t^H \rangle}{\langle \mathbf{s}_t \mathbf{s}_t^H, \mathbf{s}_t \mathbf{s}_t^H \rangle} \mathbf{s}_t \mathbf{s}_t^H \quad (24)$$

where $\text{tr}(\cdot)$ and $|\cdot|$ denote the trace and determinant of the matrix, respectively.

3.2. Geometric Explanation of the S-MIG Detector

Consider the situation that the PDF of the hypothesis that the target exists is unknown; detection Formula (16) is simplified to

$$\mathcal{D}(\mathbf{q} \parallel \mathbf{p}_0) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\geq}} \gamma, \quad (25)$$

which is consistent with the detection formula of the MIG detector (11). Therefore, the MIG detector can be regarded as a centroid detector on the manifold. The decision boundary of the MIG detector on the manifold is a circle, as shown in Figure 2. If the geometric distance between $\mathbf{R}_{\text{CUT}}$ and $\hat{\mathbf{R}}$ is greater or equal to the threshold γ , the target is considered to exist in the CUT. Therefore, the decision domain of hypothesis $\mathcal{H}_0$ is a solid circle, and others are the decision domain of hypothesis $\mathcal{H}_1$.

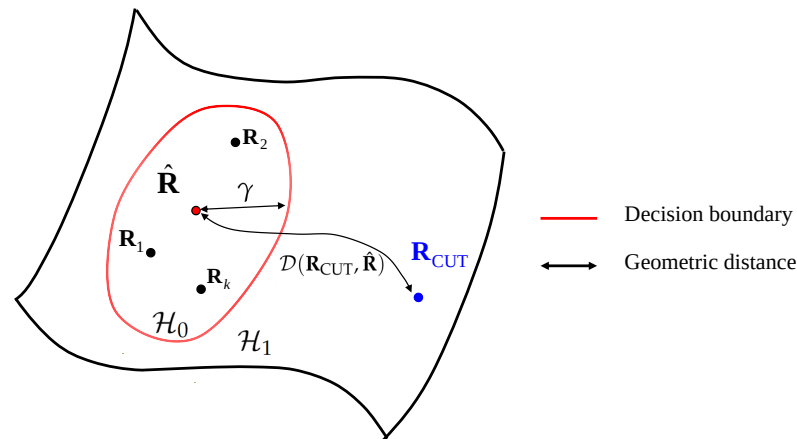


Figure 2. Geometric explanation of the MIG detector.

Different from the case that the target information is unknown, for the S-MIG detector, the target steering vector $\mathbf{s}_t$ is known and α_t can be estimated; thus, the PDF and covariance matrix of hypothesis $\mathcal{H}_1$ is utilizable. Benefiting from the target's steering vector, a rectification term is added to the MIG detector: from Formula (11) to Formula (17). On the manifold, the decision boundary of the S-MIG detector is a hyperbola defined by $\hat{\mathbf{R}}$, $\hat{\mathbf{R}} + \mathbf{R}_t$, and γ . The decision domain of the S-MIG detector is shown in Figure 3.

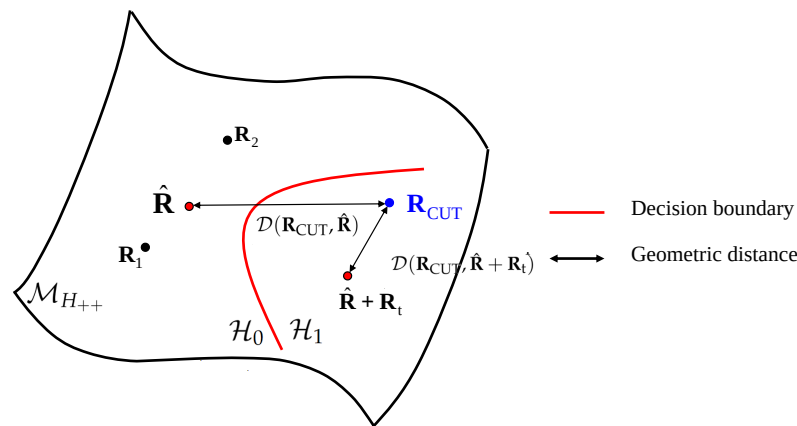


Figure 3. Geometric explanation of the S-MIG detector.

The geometric distance is an effective method to measure the similarities and differences between two matrices. The statistical meaning of the geometric distance can be interpreted as: the smaller the geometric distance is, the more similar the matrices are and the closer their statistical properties are. From this perspective, the MIG detector can be regarded as a detector that compares whether the CUT and the estimation of the clutter are “similar” enough—if they are not, the target exists, regardless of whether it is the target of interest or not. Likewise, the S-MIG detector is a detector that measures which hypothesis's likelihood is greater, focusing on the target of interest and the clutter. The S-MIG detector considers which hypothesis is more similar instead of caring about resemblance to hypothesis $\mathcal{H}_0$ as the MIG detector does. Therefore, the robustness of the S-MIG detector is better than that of the MIG detector. Much heterogeneous clutter will lead to false alarms in the MIG detector, but the wider decision domain of the S-MIG detector leads to more heterogeneous clutter remaining in the decision domain of hypothesis $\mathcal{H}_0$.

Based on the scheme analyzed in this subsection and shown in Figure 3, a variety of S-MIG detectors can be designed by using different geometric metrics. As for the selection of geometric metrics, the optimal metric is different in different clutter environments. In this paper, we focus on the KLD-based S-MIG detector.

3.3. Performance Analysis

This subsection is devoted to analyzing the detection performance of the S-MIG detector in theory, including the computation complexity and robustness. In this subsection, AMF [8], adaptive normalized matched filter (ANMF) [55], and MIG detectors are selected as comparison methods. The detection formulas of the AMF detector and the ANMF detector, respectively, are

$$\Lambda_{\text{AMF}} = \frac{|\mathbf{s}_t^H \mathbf{R}_c^{-1} \mathbf{x}_{\text{CUT}}|^2}{\mathbf{s}_t^H \mathbf{R}_c^{-1} \mathbf{s}_t} \quad (26)$$

$$\Lambda_{\text{ANMF}} = \frac{|\mathbf{s}_t^H \mathbf{R}_c^{-1} \mathbf{x}_{\text{CUT}}|^2}{(\mathbf{s}_t^H \mathbf{R}_c^{-1} \mathbf{s}_t)(\mathbf{x}_{\text{CUT}}^H \mathbf{R}_c^{-1} \mathbf{x}_{\text{CUT}})} \quad (27)$$

3.3.1. Computation Complexity Analysis

Adaptive radar target detectors usually include three steps: the construction of covariance matrices, the estimation of the clutter covariance matrix, and the calculation of the test statistic.

In step I of each detection operation, the primary data and K secondary data should be modeled as HPD matrices by Formula (5). In Formula (5), each element in the covariance matrix is calculated by $x_k = \sum_{i=k+1}^M x_i x_{i+k}$, $0 \leq k \leq M-1$; therefore, the computation complexity of each covariance matrix is $\mathcal{O}(M^2)$. Because the covariance matrix of $\mathbf{x}_{\text{CUT}}$ is not required in AMF or ANMF detectors, their computation complexity is $\mathcal{O}(KM^2)$, while the calculation complexity of MIG and S-MIG detectors is $\mathcal{O}((K+1)M^2)$.

In step II, the computation complexity of different CCM estimate methods is various. For the MIG detector, many information-geometry-based estimation methods have outstanding performance but heavy computation burden. To evaluate various detection methods, the algebraic mean of the secondary data is employed as the estimation result of the CCM in this paper, which is shown below.

$$\hat{\mathbf{R}} = \frac{1}{K} \sum_{i=1}^K \mathbf{R}_i \quad (28)$$

The computation complexity of the algebraic mean is $\mathcal{O}(KM)$.

In step III, note that the matrix inversion operation is contained in these four detectors, which occupies the main computing cost; thus, the computation complexity of (10), (26), and (27) is $\mathcal{O}(M^3)$. For the S-MIG detector, an extra operation (22) is added compared to the MIG detector, for which the computation complexity is $\mathcal{O}(M)$, and the computation complexity of Formula (23) is $\mathcal{O}(M^3)$.

In summary, the computation complexities of four detectors are listed in Table 1. In most cases, the DoF M is far greater than the number of reference cells K without dimension reduction; thus, the summary of the computation complexities of detectors is $\mathcal{O}(M^3)$.

Table 1. The computation complexities of different detectors.

Detector	Step I	Step II	Step III	Summary
AMF	$\mathcal{O}(KM^2)$	$\mathcal{O}(KM)$	$\mathcal{O}(M^3)$	$\mathcal{O}(M^3)$
ANMF	$\mathcal{O}(KM^2)$	$\mathcal{O}(KM)$	$\mathcal{O}(M^3)$	$\mathcal{O}(M^3)$
MIG	$\mathcal{O}((K+1)M^2)$	$\mathcal{O}(KM)$	$\mathcal{O}(M^3)$	$\mathcal{O}(M^3)$
S-MIG	$\mathcal{O}((K+1)M^2)$	$\mathcal{O}(KM)$	$\mathcal{O}(M^3)$	$\mathcal{O}(M^3)$

According to Table 1, the computation costs of the proposed S-MIG detector and other detectors are on the same scale, indicating that the complexity of the S-MIG detector is acceptable.

3.3.2. Robustness Analysis

For the reason that the order of magnitudes of different algorithms are conflicting and their metrics have different statistical implications, it is improper to assess the robustness of detectors in heterogeneous clutter by the normalized deviation of the detection statistic. In this paper, an influence function (IF) is proposed to evaluate the robustness of space–time adaptive detectors in heterogeneous environments, which is the unitized deviation of the detection statistic defined below.

The influence function of space–time adaptive detectors is

$$IF(m, n) = E\left[\frac{|\Lambda(\mathbf{R}_{\text{CUT}}, \hat{\mathbf{R}}) - \Lambda(\mathbf{R}_{\text{CUT}}, \mathbf{R}_c)|}{\text{mean}(\text{metrics}(\mathbf{R}_{\text{CUT}}, \mathbf{R}_c), \text{metrics}(\mathbf{R}_{\text{CUT}}, \mathbf{R}_j))}\right] \quad (29)$$

where $\mathbf{R}_j$ is the covariance matrix of heterogeneous clutter, and $\text{mean}(x, y)$ denotes the average of x and y ; m and $n = K - m$ are the number of the range cells of homogeneous clutter and heterogeneous clutter, respectively; $\text{metrics}(\cdot, \cdot)$ represents the metrics of detectors. For the AMF and ANMF detectors, $\text{metrics}(\mathbf{R}_{\text{CUT}}, \cdot) = \Lambda(\mathbf{x}_{\text{CUT}}, \cdot)$, and for the MIG detector and S-MIG detector, $\text{metrics}(\mathbf{R}_{\text{CUT}}, \cdot) = \mathcal{D}_{\text{KL}}(\mathbf{R}_{\text{CUT}}, \cdot)$. Based on the analysis of measured data, heterogeneous clutter may be localized or distributed [56], as shown in Figure 4. In the former case, the homogeneous clutter is disturbed by discrete mountains or man-made buildings, but the other terrain is consistent with homogeneous clutter. The covariance matrix of hetero-clutter can be expressed by

$$\mathbf{R}_j = \mathbf{R}_c + \sum \alpha_j \alpha_j^H \mathbf{s}_j \mathbf{s}_j^H \quad (30)$$

where α_j and $\mathbf{s}_j$ are the reflection coefficient and steering vector, respectively, of discrete interference. Usually, the steering vector $\mathbf{s}_j$ is considered separate from the target.

In the second case, where the hetero-clutter is distributed, the hetero-clutter has entirely different statistical properties and a covariance matrix from the homo-clutter. Distributed hetero-clutter occurs when two or more kinds of terrain are illuminated by airborne radar, such as a sea–land interface scene. In this case, we have

$$\mathbf{R}_j \neq \mathbf{R}_c. \quad (31)$$

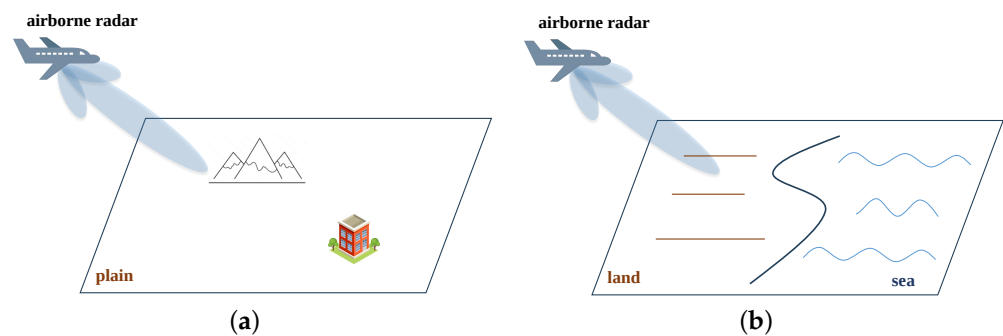


Figure 4. Schematic diagrams of hetero-clutter. (a) localized clutter. (b) distributed clutter.

Theorem 1. In heterogeneous clutter environments, the robustness of the S-MIG detector is superior to that of the MIG detector: namely,

$$IF_{\text{S-MIG}}(m, n) \leq IF_{\text{MIG}}(m, n) \quad (32)$$

Proof. See Appendix A. $\square$

The theorem introduced above indicates that in heterogeneous environments, the S-MIG detector will not be interfered with more seriously than the MIG detector. Considering that airborne radar usually works in heterogeneous environments, this property is of great significance.

4. Experimental Results and Analysis

In this section, the performance of the proposed S-MIG is compared with that of other STAD methods, such as the AMF and ANMF detectors; in addition, target detection methods based on STAP and CFAR are selected as comparison methods as well. It should be pointed out that the loaded sample covariance matrix (LSCM) algorithm is designated as the exclusive CCM estimate method; therefore, the detection performances of alternative methods are different because of their detection abilities rather than something else. The experiments in this section are divided into two parts: experiments based on simulated data to investigate the properties of the S-MIG detector and experiments based on measured data to validate the superiority of the proposed method.

4.1. Experiments Based on Simulated Data

Performance results are tested on simulated data in this subsection. In this subsection, we focus on three important properties of multi-channel detectors: direction sensitivity, robustness, and computational efficiency. Direction sensitivity represents the ability to suppress interference in other directions, and robustness reflects the reliability of detectors in heterogeneous environments. In addition, computational efficiency is vital because the application feasibility of the proposed method depends on it.

4.1.1. Directional Sensitivity

To assess the direction sensitivity of detectors, 10^3 independent trails are repeated by the standard Monte Carlo counting technique. In the simulation, the clutter shows the same covariance matrix Σ generated by

$$\Sigma = \Sigma_0 + \mathbf{I} \quad (33)$$

where

$$\Sigma_0(i, k) = \sigma_c^2 \rho^{|i-k|} e^{j2\pi f_{dc}(i-k)}, \quad i, k = 1, \dots, M \quad (34)$$

Here, ρ is the one-lag correlation coefficient, σ_c denotes the clutter-to-noise power ratio, and f_{dc} is the clutter Doppler frequency. In this part, we set $\sigma_c^2 = 20$ dB, $\rho = 0.6$, and $f_{dc} = 0.2$. The number of array elements N is 16, and the number of coherent pulses P is 10. These parameters are used in the following simulated experiments unless otherwise specified.

A simulated target with normalized spatial Doppler 0.55 and normalized temporal Doppler 0.3 is added to the clutter. The signal-to-clutter ratio (SCR) in this simulation is 5 dB. Figure 5 shows the normalized output of different directions (normalized spatial Doppler), which is obtained by detecting the target with different steering vectors. According to Figure 5, the detectors achieve the best performance when the steering vector is consistent with the target, and performance decreases if the directions are mismatched. The directional sensitivities of the AMF and ANMF detectors are similar but are worse than that of the S-MIG detector. The S-MIG detector still has better detection ability in the case of a small amount of direction mismatch, but the ability drops rapidly. The result indicates that for the S-MIG detector, a spot of target direction mismatch is tolerable, but interference from other directions will be efficiently suppressed, especially when the normalized spatial Doppler mismatch exceeds 0.3.

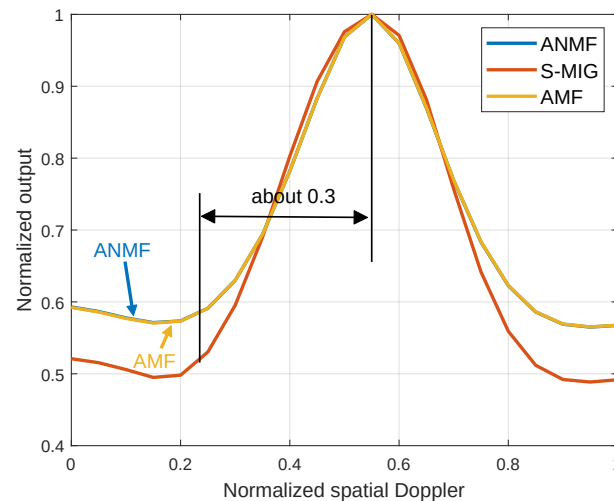


Figure 5. Normalized output in different directions.

4.1.2. Robustness

The robustness of detectors is studied by analyzing their influence functions in heterogeneous environments with different numbers of reference cells. In this experiment, the number of hetero-clutter range cells is set to 24. Considering the localized hetero-clutter and distributed hetero-clutter introduced in Section 3.3.2, two experiments are performed. The homogeneous clutter used in the experiments is given by Formula (33), and the clutter follows a K distribution. In case I, two interferences are added into homo-clutter to imitate discrete buildings. The covariance matrix of hetero-clutter is shown as Formula (30). In case II, the parameters in Formula (33) are entirely changed to establish a completely different covariance matrix. The parameters for the simulation experiments are listed in Table 2.

Table 2. Parameters for simulation.

Parameters	Clutter	Hetero-Clutter in Case I	Hetero-Clutter in Case II
N	4	4	4
P	6	6	6
n	-	24	24
ρ	0.6	0.6	0.1
σ^2	20 dB	20 dB	5 dB
f_{dc}	0.2	0.2	0.8
shape parameter v	0.4	0.4	0.4
scale parameter b	0.6	0.6	0.6
number of interference	0	2	0
(f_{d1}, f_{s1})	-	(0.2, 0.2)	-
(f_{d2}, f_{s2})	-	(0.2, 0.9)	-
(f_{dt}, f_{st})	(0.55, 0.55)	-	-

Figure 6 describes the IF performances of case I and case II. As seen in Figure 6a, in localized clutter, the IF curve of the S-MIG detector is superior to that of the MIG detector, which means that the detection result of the S-MIG detector will be less affected in a localized heterogeneous environment. The ANMF detector achieves the best performance and is followed by the AMF detector. However, in the distributed heterogeneous clutter environment, the AMF detector is inferior to other detectors, and the IF curves of the MIG and S-MIG detector are almost identical. Different from the ANMF detector, which has similar robustness in both cases, the MIG detector suffers from localized clutter much more than from distributed clutter. Compared to the MIG detector, the robustness of the proposed S-MIG detector is effectively improved.

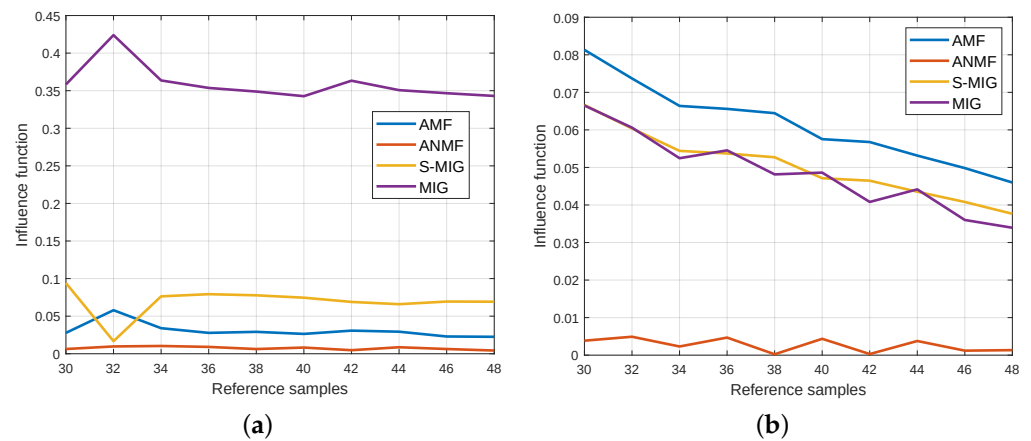


Figure 6. Influence function curves for different numbers of reference cells. (a) localized clutter. (b) distributed clutter.

4.1.3. Computational Efficiency

In this numerical simulation experiment, the average times for 500 runs of the proposed detector and the competitive methods are tested. To assess the run-times of various detectors, all these algorithms are performed based on the same estimate method. In addition, a CFAR detector is added after STAP. The experiment results are presented in Figure 7, which shows that there is no significant increase in computation time, and the practical run-times of the proposed method and the other methods are roughly equal. Under different DoFs, the run-time of the S-MIG detector is approximately 5 times that of the AMF detector and less than 2 times that of the MIG detector, which is close to that of the STAP-CFAR detector.

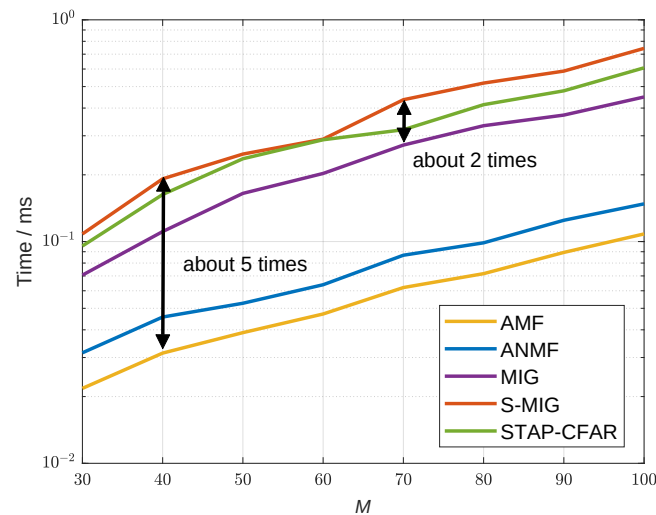


Figure 7. The average run-times for detectors.

4.2. Experiments Based on Measured Data

Experiments based on measured data are performed to investigate the weak target detection ability in heterogeneous environments. The experiment based on measured data includes three parts. The first part is based on the sea-detecting data-sharing program (SDDSP) [57,58], and the second is based on the mountaintop data set to verify the robustness of the S-MIG detector, as shown in Section 4.2.2. The third one is an experiment based on the PHA radar data set published in [59] to investigate the weak target detection ability, as shown in Section 4.2.3.

4.2.1. Real Data of the Sea-Detecting Data-Sharing Program

In order to investigate the suppression ability and robustness of different detectors, a real scene containing ground and sea clutter is considered. In this experiment, data cube 20191025165159_01_scanning is utilized, in which range cells from 200 to 500 are land–sea interface clutter, and range cells from 1 to 200 and from 500 to 1320 are land clutter and sea clutter, respectively, as presented in Figure 8. The measured data are expanded into multi-channel data by a phase shift. Three simulated targets are added in range cells 200, 400, and 800. Their SCR is -15 dB, and their normalized Doppler is 0.1. Both the clutter and target are located on the normal line of the antennae array. The number of reference samples is 24. Figure 9 presents the clutter suppression and target detection results of detectors with different types of clutter. As we can see, the proposed method achieves the best detection performance in three types of clutter environments. The STAP method can detect the target, but there are a lot of false alarms around it. The AMF detector and the MIG detector fail to detect the target in three clutter environments. The experimental results illustrate that the proposed method can suppress the different types of clutter and detect weak targets in the heterogeneous environment.

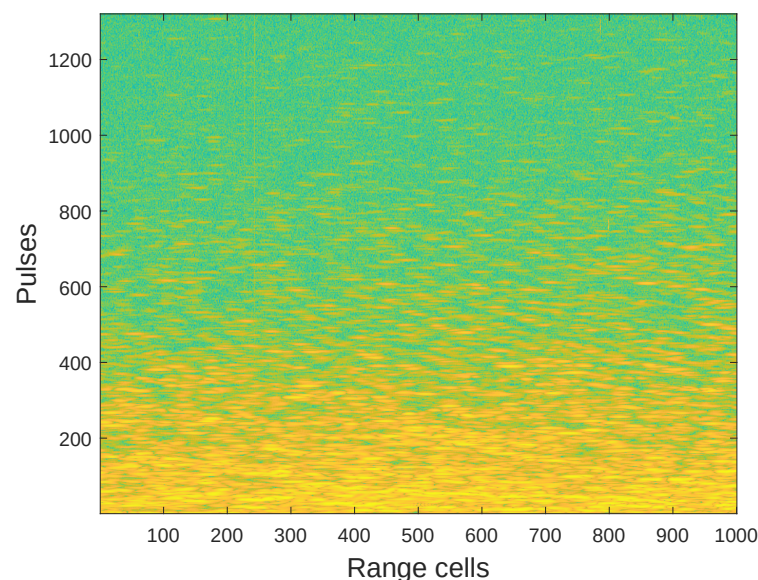


Figure 8. Measured data of SDDSP.

4.2.2. Real Data of the Mountaintop Program

The mountaintop data set was collected to support STAP studies at first and is now widely used in research on clutter suppression and target detection for airborne radars. The radar surveillance technology experimental radar (RSTER) is central to the mountaintop program. The antenna for the system is a 5 m wide by 10 m high horizontally polarized array made up of 14 column elements. Behind each element are an independent phase shifter, transmitter, and receiver. The RSTER is deployed on North Oscura Peak (NOP) on the White Sands Missile Range (WSMR). NOP is at the northeast corner of the WSMR and is 8000 feet above sea level and approximately 3500 feet above the desert floor [60–62], as shown in Figure 10. The site offers line-of-sight to a variety of terrain types including desert, bare and wooded hills, mountains, lava flows, and small suburban areas.

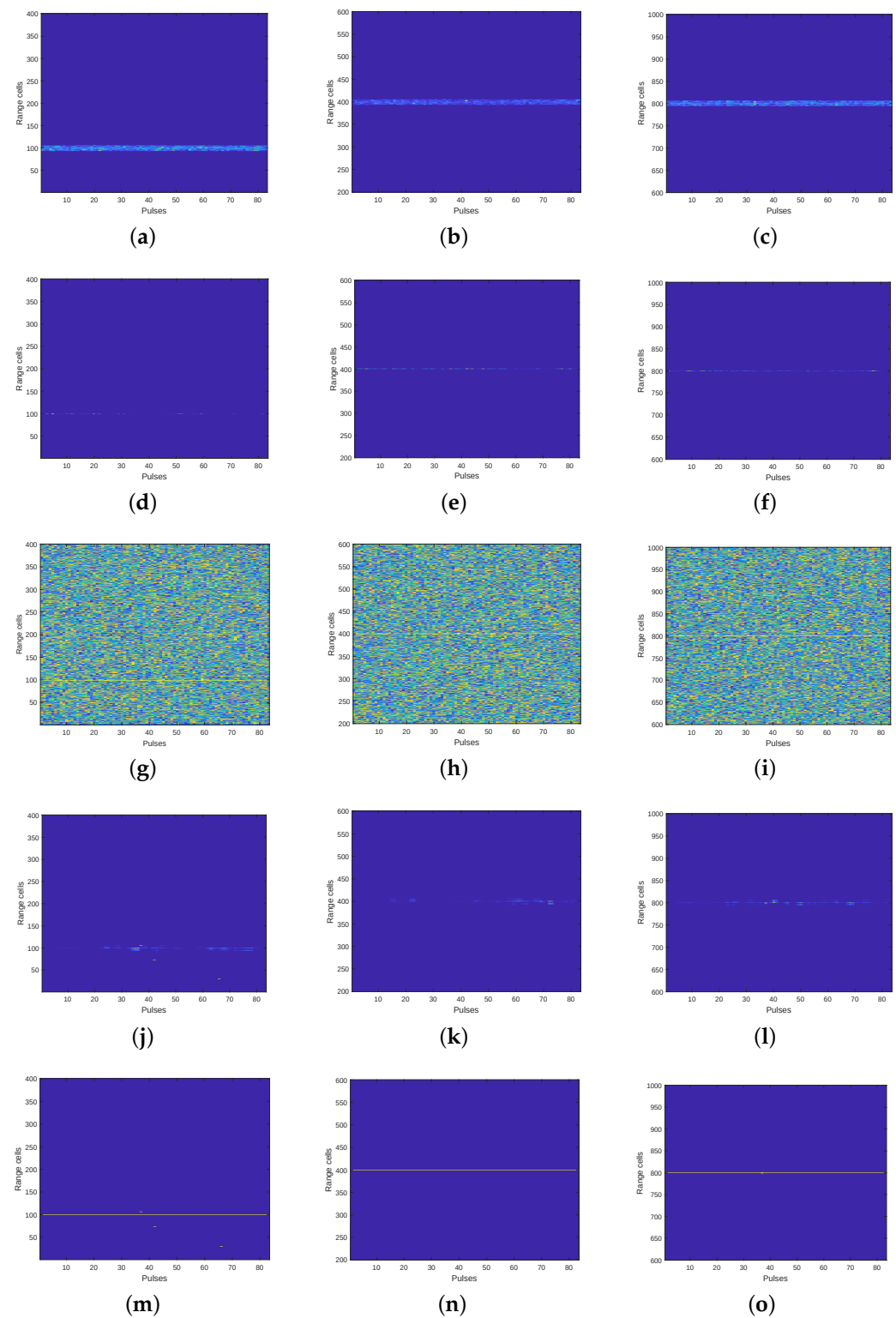


Figure 9. The clutter suppression and target detection performances of different methods in various clutter environments. (a) STAP, land clutter. (b) STAP, land-sea clutter. (c) STAP, sea clutter. (d) AMF, land clutter. (e) AMF, land-sea clutter. (f) AMF, sea clutter. (g) ANMF, land clutter. (h) ANMF, land-sea clutter. (i) ANMF, sea clutter. (j) MIG, land clutter. (k) MIG, land-sea clutter. (l) MIG, sea clutter. (m) S-MIG, land clutter. (n) S-MIG, land-sea clutter. (o) S-MIG, sea clutter.



Figure 10. The RESTER system (left) and its antenna (right) [63].

The data used in this paper are data file *stap3001v1.mat*, which contains 2 coherent processing intervals (cpi) and 404 range cells. A remotely sited broadband jammer provides the only signal in the data. In addition to the signal observed in the jammer's direction, some terrain-scattered jamming energy may be observed in the data. The jamming signal is pseudo-random noise with a bandwidth of 600 kHz, which is broadband relative to the radar's bandwidth, so it appears as broadband barrage jamming. The angle between the jammer and the radar direction is 42° (normalized spatial Doppler -0.31). Figure 11 presents the Fourier spectrum of the received signal. According to Figure 11, the power of the jammer signal is about 50 dB greater than that of the clutter.

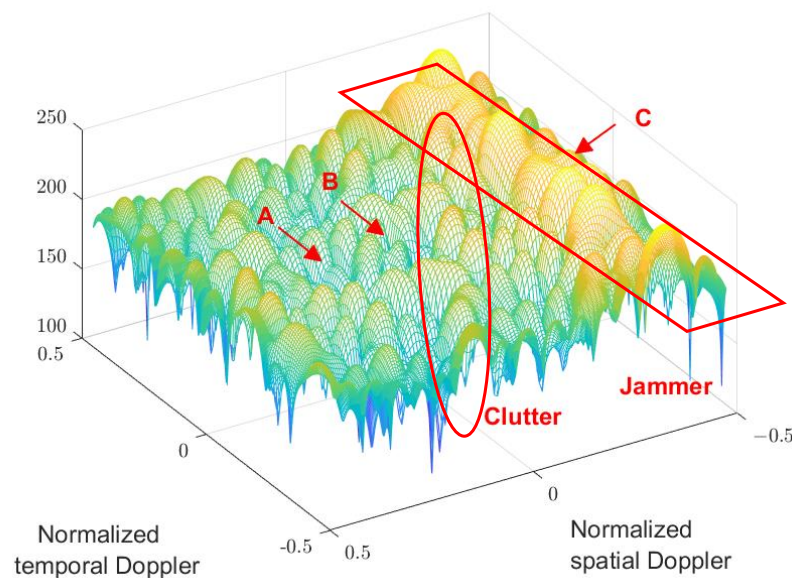
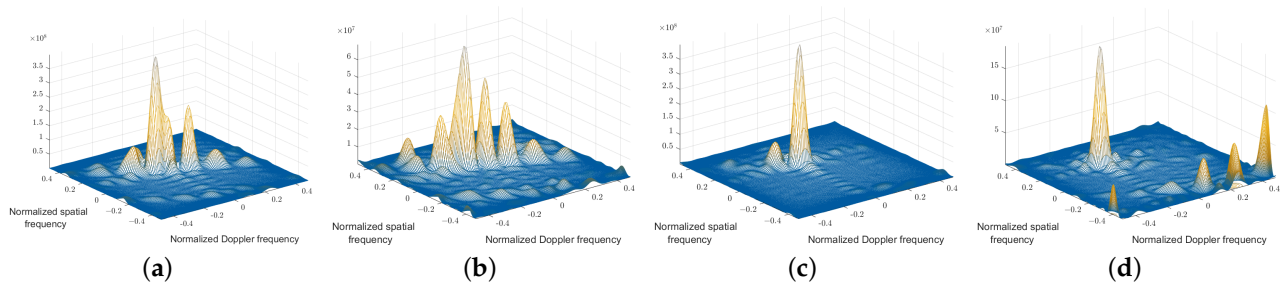


Figure 11. The power spectrum of the mountaintop data (dB).

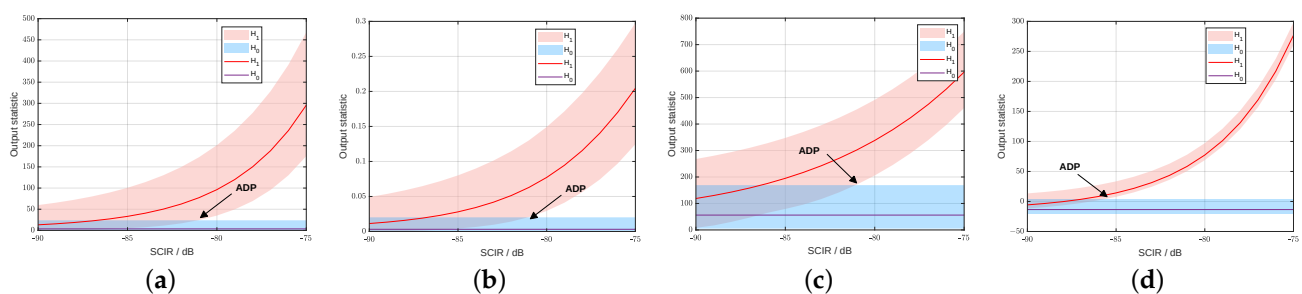
Three simulated targets with different Doppler are added into the clutter, and their detection performances under different signal-to-clutter-plus-interference ratios (SCIR) is analyzed. The Doppler of simulated targets and other parameters in this experiment are listed in Table 3. Their spatial Doppler were set up differently to test the effectiveness of various detectors on targets in different directions, where target A imitates an ordinary target, target B is a simulated slowly moving target whose Doppler is close to that of the clutter, and target C is a target near the jammer. As shown in Figure 12, the spectrum of clutter in different range cells is different, and the clutter is heterogeneous as well, which is challenging for detectors. First, the experimental results are shown in the form of output curves that focus on the average, minimum, and maximum outputs of detectors in hypotheses $\mathcal{H}_1$ and $\mathcal{H}_0$.

Table 3. Parameters for simulation.

Parameter	Symbol	Value
Doppler for target A	(f_{dA}, f_{sA})	$(0.25, -0.16)$
Doppler for target B	(f_{dB}, f_{sB})	$(0.01, -0.16)$
Doppler for target C	(f_{dC}, f_{sC})	$(-0.41, -0.16)$
DoF	M	224
number of reference cells	K	28
number of guard cells	-	1
CPI sequence number	cpi	1

**Figure 12.** The spectrum of the clutter in different range cells. (a) range cell 20. (b) range cell 27. (c) range cell 35. (d) range cell 46.

The output curves are presented in Figures 13–15. In these figures, the curves represent the average outputs for both hypotheses, and the shaded regions around the curves indicate the distribution intervals of the output statistics, whose upper bound and lower bound are the maximum and minimum, respectively. In this paper, we define the absolutely detected point (ADP) to assess the performance of detectors. The ADP refers to the SCIR that has its minimum output in hypothesis $\mathcal{H}_1$ greater than its maximum output in hypothesis $\mathcal{H}_0$, as marked in Figure 13. As we can see, the ADP of the proposed S-MIG detector is approximately 5 dB lower than those of the other detectors, meaning that a lower SCIR is needed for the proposed method to detect the target without a missed alarm. For target A, the S-MIG detector achieves the best performance and is followed by the AMF detector. As the target's Doppler approaches the clutter, the ADPs of detectors increase in Figure 14, but in such a case, the S-MIG detector still maintains advantageous performance. Furthermore, another unfavorable case is considered. The direction of simulated target C is close to the jammer, with a spatial Doppler difference of 0.1; here, the performance of the S-MIG detector is superior to those of the others as well. Particularly, the ADPs are listed in Table 4. The result proves that in heterogeneous environments, the proposed method can detect the target in a lower SCNR, which represents superior detection performance for weak targets.

**Figure 13.** Output statistic curves for different SCIRs for target A. (a) AMF. (b) ANMF. (c) STAP. (d) S-MIG.

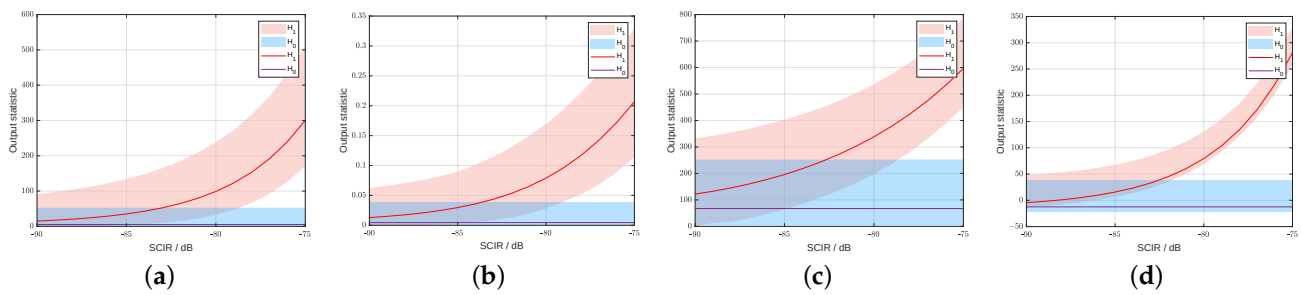


Figure 14. Output statistic curves for different SCIRs for target B. (a) AMF. (b) ANMF. (c) STAP. (d) S-MIG.

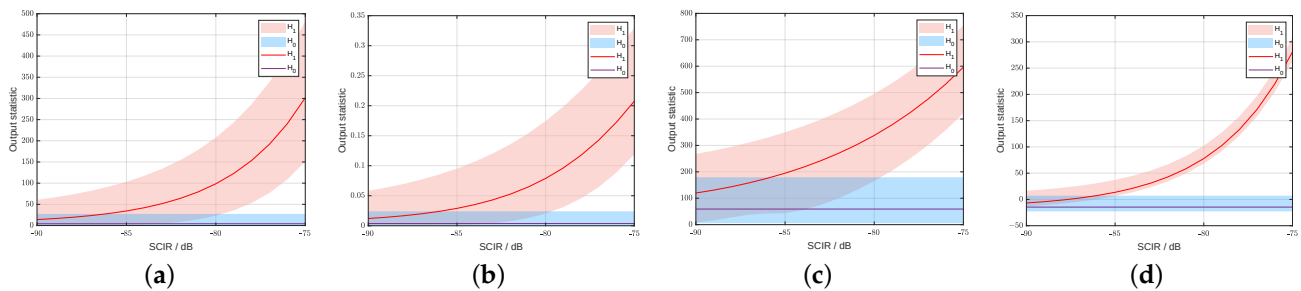


Figure 15. Output statistic curves for different SCIRs for target C. (a) AMF. (b) ANMF. (c) STAP. (d) S-MIG.

Table 4. ADPs for different detectors.

	AMF	ANMF	STAP	S-MIG
Target A	−81.06 dB	−81.05 dB	−81.08 dB	−85.63 dB
Target B	−78.69 dB	−79.03 dB	−78.62 dB	−81.76 dB
Target C	−79.65 dB	−79.66 dB	−79.63 dB	−84.88 dB

Furthermore, the probability-of-detection curves for the various detectors are presented in Figure 16. In this experiment, the simulated target is located in the direction of 90° , and its normalized Doppler is 0.15. To perform a Monte Carlo experiment, the received signals of different coherent processing intervals (CPI) are employed to calculate the Pd values of various detectors.

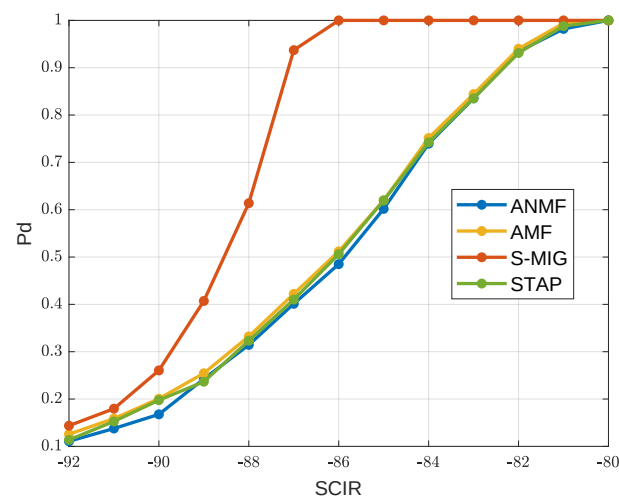


Figure 16. The Pd curves of the various detectors.

4.2.3. Real Data of the PHA Radar Database

In order to develop detection and tracking techniques, a series of low-altitude targets and radar clutter measurement trials were conducted by a high-resolution phased-array (PHA) radar in 2023 at Dongying, Shandong, China. The PHA radar is a Ku-band radar with a synthetic bandwidth of 1 GHz and detects targets from 300 to 2000 m. The PHA data set includes radar data for clutter measurement trails, migrating insect and bird observations, and drone observations [59].

In this experiment, the measured radar data of a single drone moving at a low speed is employed to verify the effectiveness of the proposed method, as shown in Figure 17. Here, the cooperative target is a DJI M300 drone that is moving horizontally, and the radar works in tracking mode. For the reason that there is no highlight clutter in the horizontal directions and the target is located at a low elevation angle (11.68°), spatial adaptive detection is performed in the pitch dimension to suppress the ground clutter.



Figure 17. The test scene, radar, and cooperative target [59].

Figures 18 and 19 present the detection results for different methods, including STAD and STAP. In this experiment, the DoF is set to 12, 24 reference cells are valuable, and 4 range cells adjacent to the CUT are designated as guard cells. As we can see, the performances of filter-then-detect methods (Figure 18a–d) are unsatisfactory. Meanwhile, the ANMF detector, which detects the target by phase information, fails to highlight the target. Without the utilization of the given steering vector, the target can be detected by the MIG detector, but it is accompanied by many false alarms caused by clutter (shown in Figure 18g). One can see that after being improved by the target steering vector, the clutter is effectively suppressed in Figure 18h. Both the S-MIG detector and AMF detector have the best performance. In detail, the output signal-to-clutter-plus-noise ratios (SCNRs) are listed in Table 5. The statistics in Table 5 are calculated by the average of the selected range cells in this experiment (excluding two range cells around the target). Moreover, the performance advantage of STAD is clearly presented in Figure 19. The figure demonstrates that the S-MIG detector and AMF detector achieve the best SCNR, followed by the MIG detector and STAP. The S-MIG detector has an SCNR gain of approximately 15 dB compared to the MIG detector and approximately 3 dB compared to the AMF detector.

Table 5. Output SCNRs of different methods.

Method	SCNR	Method	SCNR
STAP without CFAR	16.626 dB	AMF	28.831 dB
CA-CFAR after STAP	12.559 dB	ANMF	7.751 dB
OS-CFAR after STAP	12.255 dB	MIG	17.977 dB
GO-CFAR after STAP	12.888 dB	S-MIG	32.060 dB

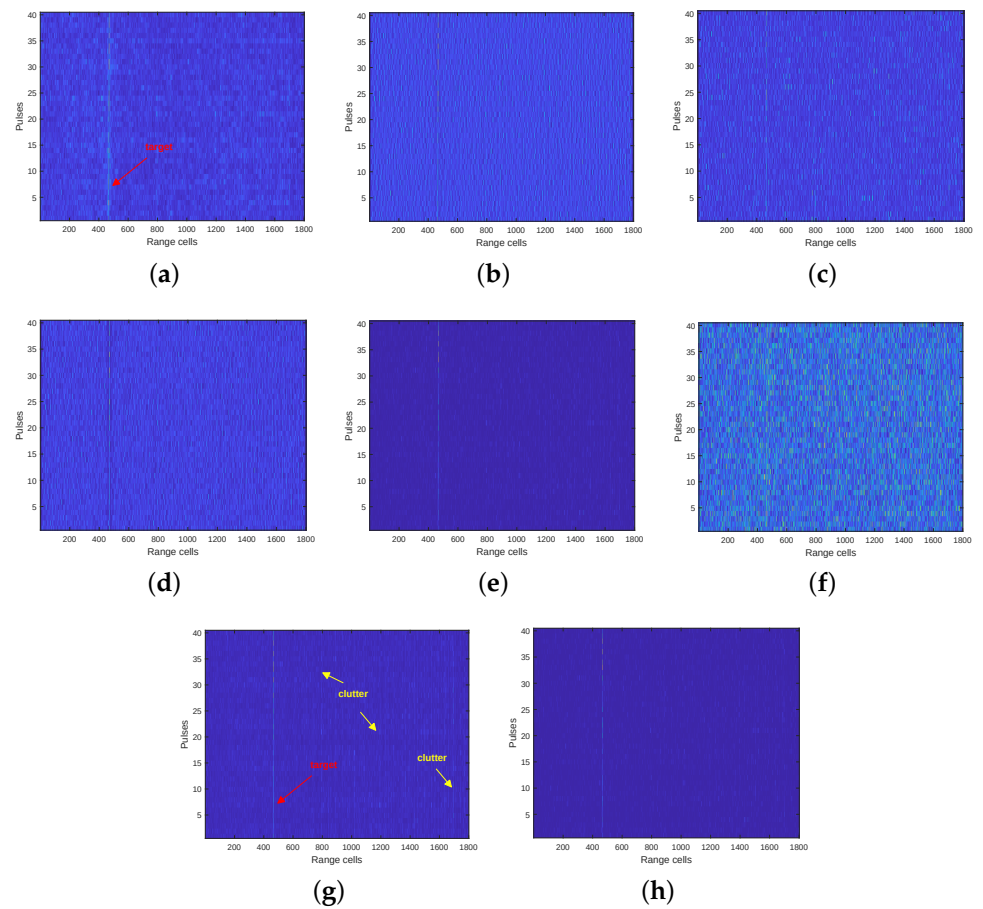


Figure 18. The range-pulse maps for different methods. (a) STAP without CFAR. (b) CA-CFAR after STAP. (c) OS-CFAR after STAP. (d) GO-CFAR after STAP. (e) AMF. (f) ANMF. (g) MIG. (h) S-MIG.

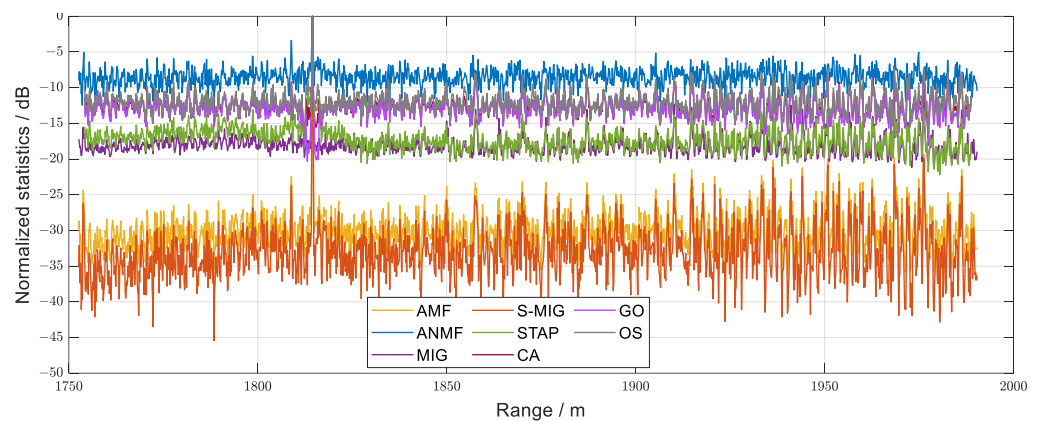


Figure 19. Normalized output statistics curves.

Then, to investigate the detection performance for weak moving targets in heterogeneous clutter, the clutter data from 1830 m to 1990 m are selected as the experimental environment, and a simulated target is added at 1885 m. To imitate a low-altitude drone hidden in ground clutter, the velocity of the target is set to 50 km/h, and the elevation angle is 30° . Figure 20a illustrates the amplitude of ground clutter in the range of 1800 to 2000 m. The ground clutter shows strong heterogeneity, as we can see, and the target is added in a position where the clutter power on both sides is significantly stronger than the CUT. In general, the number of homogeneous samples should be no less than two times the DoF to obtain a satisfying result [2]. However, considering that the clutter in this experiment

is heterogeneous, 16 neighboring reference cells, which is less than twice the DoF, are valuable in each detection to reduce the negative influence of hetero-clutter. The detection performances for detectors against different SCRs in this scene are presented in Figure 20b. In this experiment, the false alarm rate is 10^{-5} , and 10^6 trials are performed to determine the detection threshold γ . To calculate the Pd, 10^5 Monte Carlo experiments are performed. Compared to competitive methods, the Pd curve of the proposed method increases faster and has the best performance. According to the Pd curves, the AMF detector and ANMF detector perform better for a lower SCR, but STAP becomes better as the SCR increases. The detection performance of the S-MIG detector is an improvement of about 4 dB and 5 dB over the ANMF detector and the AMF detector, respectively. Moreover, the SCR is improved by about 6.5 dB over the MIG detector when Pd = 0.5. In addition, the Pd curves of the persymmetric adaptive normalized matched filter (P-ANMF), the recursive P-ANMF (RP-ANMF), the fixed point adaptive normalized matched filter (FP-ANMF), and the persymmetric FP-ANMF (PFP-ANMF) are presented in Figure 20b as well. These four methods improved the CCM estimation algorithm of the ANMF detector to detect the weak target better. As shown in Figure 20b, the performances of the ANMF detector based on different CCM estimation algorithms are different, but the ANMF detector is still inferior to the S-MIG detector, which utilizes the ordinary LSCM estimation method. Compared to the RP-ANMF detector, the S-MIG detector has a SCR gain of 2 dB.

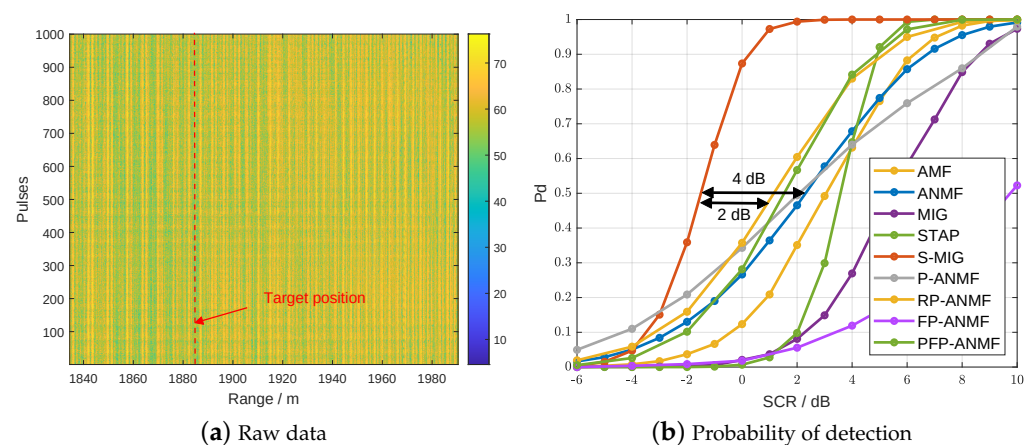


Figure 20. Ground clutter and Pd curves.

5. Conclusions

This paper focuses on the weak target detection problem in heterogeneous clutter for airborne radars. Based on the GRLT and information geometry, a novel detector that makes use of prior information is proposed to extend the MIG detector to airborne multi-channel radar. According to the theoretical deduction result, the decision boundary on the manifold for the GRLT ought to be a hyperbola rather than a circle when prior knowledge is known. Then, it is revealed that the proposed S-MIG detector has better directional sensitivity than traditional methods and better robustness than the MIG detector; furthermore, its computational efficiency maintains the same level as the comparison methods. The simulated results validate that the performance of the proposed method is better than others.

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Data Availability Statement: Data is contained within the article.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A. Proof of Theorem

Assume that there are K reference cells valuable for detectors; among them, m reference cells are homogeneous clutter and the other $n = K - m$ are heterogeneous.

The influence factors of the MIG detector and S-MIG detector, respectively, are

$$IF_{MIG} = E\left[\frac{|\mathcal{D}_{KL}(\mathbf{R}_{CUT}, \hat{\mathbf{R}}) - \mathcal{D}_{KL}(\mathbf{R}_{CUT}, \mathbf{R}_c)|}{\text{mean}(\mathcal{D}_{KL}(\mathbf{R}_{CUT}, \mathbf{R}_c), \mathcal{D}_{KL}(\mathbf{R}_{CUT}, \mathbf{R}_J))}\right] \quad (A1)$$

$$IF_{S-MIG} = E\left[\frac{|\mathcal{D}_{KL}(\mathbf{R}_{CUT}, \hat{\mathbf{R}}) - \mathcal{D}_{KL}(\mathbf{R}_{CUT}, \mathbf{R}_c) - \mathcal{D}_{KL}(\mathbf{R}_{CUT}, \mathbf{R}_{t1} + \hat{\mathbf{R}}) + \mathcal{D}_{KL}(\mathbf{R}_{CUT}, \mathbf{R}_{t0} + \mathbf{R}_c)|}{\text{mean}(\mathcal{D}_{KL}(\mathbf{R}_{CUT}, \mathbf{R}_c), \mathcal{D}_{KL}(\mathbf{R}_{CUT}, \mathbf{R}_J))}\right] \quad (A2)$$

$$\text{where } \mathbf{R}_{t1} = \frac{\langle \mathbf{R}_{CUT} - \hat{\mathbf{R}}, \mathbf{s}_t \mathbf{s}_t^H \rangle}{\langle \mathbf{s}_t \mathbf{s}_t^H, \mathbf{s}_t \mathbf{s}_t^H \rangle} \mathbf{s}_t \mathbf{s}_t^H \text{ and } \mathbf{R}_{t0} = \frac{\langle \mathbf{R}_{CUT} - \mathbf{R}_c, \mathbf{s}_t \mathbf{s}_t^H \rangle}{\langle \mathbf{s}_t \mathbf{s}_t^H, \mathbf{s}_t \mathbf{s}_t^H \rangle} \mathbf{s}_t \mathbf{s}_t^H.$$

Their difference is

$$\begin{aligned} \Delta(IF_{S-MIG}, IF_{MIG}) &= IF_{S-MIG} - IF_{MIG} \\ &= \frac{-\mathcal{D}_{KL}(\mathbf{R}_{CUT}, \mathbf{R}_{t1} + \hat{\mathbf{R}}) + \mathcal{D}_{KL}(\mathbf{R}_{CUT}, \mathbf{R}_{t0} + \mathbf{R}_c)}{\text{mean}(\mathcal{D}_{KL}(\mathbf{R}_{CUT}, \mathbf{R}_c), \mathcal{D}_{KL}(\mathbf{R}_{CUT}, \mathbf{R}_J))} \end{aligned} \quad (A3)$$

In the following proof, we focus on the numerator of (A3) foremost and deduce in two cases that the clutter is localized and distributed.

- In case I, according to Formula (30), we have

$$\begin{aligned} E[\hat{\mathbf{R}}] &= \frac{1}{K} \sum_{i=1}^K \mathbf{R}_i \\ &= \frac{m}{m+n} \mathbf{R}_c + \frac{n}{m+n} \mathbf{R}_J \\ &= \mathbf{R}_c + \frac{n}{m+n} \sum_{i=1}^{n_J} \alpha_{Ji} \alpha_{Ji}^H \mathbf{s}_{Ji} \mathbf{s}_{Ji}^H \end{aligned} \quad (A4)$$

where n_J is the number of interference clutter patches. Therefore, we have

$$\mathbf{R}_{t1} = \frac{\langle \mathbf{R}_{CUT} - \mathbf{R}_c - \frac{n}{m+n} \sum_{i=1}^{n_J} \alpha_{Ji} \alpha_{Ji}^H \mathbf{s}_{Ji} \mathbf{s}_{Ji}^H, \mathbf{s}_t \mathbf{s}_t^H \rangle}{\langle \mathbf{s}_t \mathbf{s}_t^H, \mathbf{s}_t \mathbf{s}_t^H \rangle} \mathbf{s}_t \mathbf{s}_t^H \stackrel{a}{=} \mathbf{R}_{t0} \quad (A5)$$

where Equation (a) is established on both the property of the matrix inner product that $\langle \mathbf{X} + \mathbf{Y}, \mathbf{Z} \rangle = \langle \mathbf{X}, \mathbf{Z} \rangle + \langle \mathbf{Y}, \mathbf{Z} \rangle$ and the equation that $\langle \mathbf{s}_{Ji} \mathbf{s}_{Ji}^H, \mathbf{s}_t \mathbf{s}_t^H \rangle = 0$ because $\mathbf{s}_{Ji}$ is orthogonal to $\mathbf{s}_t$.

Based on Formula (A5), the numerator of (A3) is

$$\begin{aligned} \Delta_{\text{num}}(IF_{S-MIG}, IF_{MIG}) &= -\mathcal{D}_{KL}(\mathbf{R}_{CUT}, \mathbf{R}_{t1} + \hat{\mathbf{R}}) + \mathcal{D}_{KL}(\mathbf{R}_{CUT}, \mathbf{R}_{t0} + \mathbf{R}_c) \\ &= -\mathcal{D}_{KL}(\mathbf{R}_{CUT}, \mathbf{R}_{t1} + \mathbf{R}_c + \frac{n}{m+n} \sum_{i=1}^{n_J} \alpha_{Ji} \alpha_{Ji}^H \mathbf{s}_{Ji} \mathbf{s}_{Ji}^H) \\ &\quad + \mathcal{D}_{KL}(\mathbf{R}_{CUT}, \mathbf{R}_{t1} + \mathbf{R}_c) \end{aligned} \quad (A6)$$

- In case II, as analyzed in Section 3.3.2, the hetero-clutter shows entirely different characteristics than the homo-clutter. We denote that

$$\mathbf{R}_j = \mathbf{R}_D \neq \mathbf{R}_c; \quad (\text{A7})$$

then

$$\begin{aligned} E[\hat{\mathbf{R}}] &= \frac{1}{K} \sum_{i=1}^K \mathbf{R}_i = \frac{m}{m+n} \mathbf{R}_c + \frac{n}{m+n} \mathbf{R}_D \\ &= \mathbf{R}_c + \frac{n}{m+n} (\mathbf{R}_D - \mathbf{R}_c). \end{aligned} \quad (\text{A8})$$

Similar to (A5), in case II, we have

$$\begin{aligned} \mathbf{R}_{t1} &= \frac{\langle \mathbf{R}_{\text{CUT}} - \hat{\mathbf{R}}, \mathbf{s}_t \mathbf{s}_t^H \rangle}{\langle \mathbf{s}_t \mathbf{s}_t^H, \mathbf{s}_t \mathbf{s}_t^H \rangle} \mathbf{s}_t \mathbf{s}_t^H \\ &= \frac{\langle \mathbf{R}_{\text{CUT}} - \mathbf{R}_c - \frac{n}{m+n} (\mathbf{R}_D - \mathbf{R}_c), \mathbf{s}_t \mathbf{s}_t^H \rangle}{\langle \mathbf{s}_t \mathbf{s}_t^H, \mathbf{s}_t \mathbf{s}_t^H \rangle} \mathbf{s}_t \mathbf{s}_t^H = \mathbf{R}_{t0} \end{aligned} \quad (\text{A9})$$

Furthermore, applying (A8) and (A9) to (A3),

$$\begin{aligned} \Delta_{\text{num}}(IF_{S-\text{MIG}}, IF_{\text{MIG}}) &= -\mathcal{D}_{\text{KL}}(\mathbf{R}_{\text{CUT}}, \mathbf{R}_{t1} + \hat{\mathbf{R}}) + \mathcal{D}_{\text{KL}}(\mathbf{R}_{\text{CUT}}, \mathbf{R}_{t1} + \mathbf{R}_c) \\ &= -\mathcal{D}_{\text{KL}}(\mathbf{R}_{\text{CUT}}, \mathbf{R}_{t1} + \mathbf{R}_c + \frac{n}{m+n} (\mathbf{R}_D - \mathbf{R}_c)) \\ &\quad + \mathcal{D}_{\text{KL}}(\mathbf{R}_{\text{CUT}}, \mathbf{R}_{t1} + \mathbf{R}_c) \end{aligned} \quad (\text{A10})$$

It can be seen that both Formulas (A6) and (A10) obey the following mathematical structure:

$$\Delta_{\text{num}}(IF_{S-\text{MIG}}, IF_{\text{MIG}}) = -\mathcal{D}_{\text{KL}}(\mathbf{A}, \mathbf{B} + \mathbf{C}) + \mathcal{D}_{\text{KL}}(\mathbf{A}, \mathbf{B}) \quad (\text{A11})$$

where $\mathbf{A} = \mathbf{R}_{\text{CUT}}$, and $\mathbf{B} = \mathbf{R}_{t1} + \mathbf{R}_c$. In case I, $\mathbf{C} = \frac{n}{m+n} \sum_{i=1}^{n_j} \alpha_{ji} \alpha_{ji}^H \mathbf{s}_{ji} \mathbf{s}_{ji}^H$. In case II, $\mathbf{C} = \frac{n}{m+n} (\mathbf{R}_D - \mathbf{R}_c)$.

According to (10), formula (A11) can be further written as

$$\begin{aligned} \Delta_{\text{num}}(IF_{S-\text{MIG}}, IF_{\text{MIG}}) &= -\text{tr}(\mathbf{A}^{-1}(\mathbf{B} + \mathbf{C})) + \log(|\mathbf{A}^{-1}(\mathbf{B} + \mathbf{C})|) \\ &\quad + \text{tr}(\mathbf{A}^{-1}\mathbf{B}) - \log(|\mathbf{A}^{-1}\mathbf{B}|) \\ &\stackrel{\text{b}}{=} \log\left(\frac{|\mathbf{A}^{-1}(\mathbf{B} + \mathbf{C})|}{|\mathbf{A}^{-1}\mathbf{B}|}\right) - \text{tr}(\mathbf{A}^{-1}\mathbf{C}) \end{aligned} \quad (\text{A12})$$

where (b) relies on the property of matrix trace $\text{tr}(\mathbf{M} + \mathbf{N}) = \text{tr}(\mathbf{M}) + \text{tr}(\mathbf{N})$.

Noted that in the binary hypothesis problem (6), $E[\mathbf{R}_{\text{CUT}}] = \mathbf{R}_c + \mathbf{R}_{t1}$, $\mathbf{A} = \mathbf{B}$ holds for both hypotheses (it should be emphasized that the noise of an adaptive detector is usually contained in the clutter). Therefore,

$$\begin{aligned} \Delta_{\text{num}}(IF_{S-\text{MIG}}, IF_{\text{MIG}}) &= \log(|\mathbf{I} + \mathbf{A}^{-1}\mathbf{C}|) - \text{tr}(\mathbf{A}^{-1}\mathbf{C}) \\ &\stackrel{\text{c}}{=} \log\left(\prod_{i=1}^M \lambda_i(\mathbf{I} + \mathbf{A}^{-1}\mathbf{C})\right) - \sum_{i=1}^M \lambda_i(\mathbf{A}^{-1}\mathbf{C}) \\ &= \sum_{i=1}^M \log(\lambda_i(\mathbf{I} + \mathbf{A}^{-1}\mathbf{C})) - \sum_{i=1}^M \lambda_i(\mathbf{A}^{-1}\mathbf{C}) \\ &\stackrel{\text{d}}{=} \sum_{i=1}^M (\log(1 + \lambda_i(\mathbf{A}^{-1}\mathbf{C})) - \lambda_i(\mathbf{A}^{-1}\mathbf{C})) \end{aligned} \quad (\text{A13})$$

where Equation (c) relies on the properties of the matrix determinant and matrix trace. Equation (d) is established on the formula $\lambda_i(\mathbf{I} + \mathbf{M}) = 1 + \lambda_i(\mathbf{M})$.

Because $\log(1 + x) \leq x$, we have $\Delta(IF_{S-\text{MIG}}, IF_{\text{MIG}}) \leq 0$: namely, $IF_{S-\text{MIG}} \leq IF_{\text{MIG}}$.

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