

Article

Empirical Optimization of the Stumpf Method Parameter for Satellite-Derived Bathymetry

Ausiàs Roch-Talens ¹, Josep E. Pardo-Pascual ¹, Jaime Almonacid-Caballer ¹, Ángel Balaguer-Beser ¹ and Carlos Cabezas-Rabadán ^{2,*}

¹ Geo-Environmental Cartography and Remote Sensing Group (CGAT-UPV), Department of Cartographic Engineering, Geodesy and Photogrammetry, Universitat Politècnica de València, Camí de Vera s/n, 46022 Valencia, Spain; aurocta@topo.upv.es (A.R.-T.); jepardo@cgf.upv.es (J.E.P.-P.); jaialca@upv.edu.es (J.A.-C.); abalague@mat.upv.es (Á.B.-B.)

² Department of Geography, Universitat de València, Avinguda de Blasco Ibáñez 28, 46010 Valencia, Spain

* Correspondence: carlos.cabezas@uv.es

Highlights

- Replacing $n = 1000$ with the site-specific optimum raised mean R^2 from 0.70 to 0.84 and cut RMSE by ~22% (~18 cm).
- Across 31 nearshore scenarios at 18 coastal sites, the conventional Stumpf $n = 1000$ was suboptimal in 26 of them. Adjusting the parameter n to the range $n \approx 1$ –3 recovers most of that gain with no calibration.
- Hue angle and chlorophyll-*a* predict how much tuning n will improve SDB accuracy in a given scene.
- Small- n tuning extends usable Sentinel-2 SDB to greener, less clear waters, broadening imagery for beach monitoring.

Abstract

Satellite-Derived Bathymetry (SDB) based on the Stumpf log-ratio method routinely uses a fixed parameter $n = 1000$, a convention that has rarely been evaluated systematically. This study assesses empirically how SDB error varies with n across 31 bathymetric scenarios at 18 coastal sites with contrasting morphological and water quality conditions using Sentinel-2 imagery, ACOLITE atmospheric correction and in situ reference depths in the -0.5 to -6 m range. For each scenario, an optimal n was retrieved by minimizing RMSE against the reference bathymetry, and the response of the error to n was characterized. Two stable regions (a left and a right plateau) are identified in the error-versus- n curve, with most scenarios (26 of 31) reaching their optimum on the left plateau at small n values. Replacing $n = 1000$ with the site-specific optimum, the mean R^2 increases from 0.70 to 0.84 and reduces RMSE by 21.9% on average (~18 cm). A single generalized value $n_{\text{median(LP)}} = 2.2$, transferable across sites, recovers most of this gain; the unscaled, parameter-free case $n = 1$ alone already reduces RMSE by 15.9% on average relative to $n = 1000$ and outperforms it in 25 of the 31 cases. The margin of improvement obtainable by tuning n is strongly correlated (Spearman) with hue angle, chlorophyll-*a* and Trophic State Index, while turbidity and Secchi depth show no correlation, consistent with the loss of reliability of these last two indicators in optically shallow waters. The benefit of using a small n is greatest in greener, higher-chlorophyll waters, whereas in very clear, blue waters (hue $> 160^\circ$, Chl-*a* $< 2 \mu\text{g/L}$) $n = 1000$ remains a defensible choice. These results support replacing $n = 1000$ by $n \approx 2$ –3 as the practical default for Stumpf-based SDB in the studied depth range and provide a water-quality-based criterion to anticipate the expected benefit of

Academic Editor: Antonino Maltese

Received: 31 May 2026

Revised: 17 July 2026

Accepted: 25 July 2026

Published: 5 August 2026

Copyright: © 2026 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

parameter tuning. These improvements make it possible to use imagery from less clear waters for SDB extraction, broadening the range of images suitable for beach monitoring.

Keywords: SDB; optically shallow waters; water optical properties; Sentinel-2; nearshore morphological characterization; parameter optimization

1. Introduction

Coastal bathymetry, the measurement of water depth in shallow marine environments, is a fundamental input for the study of coastal morphodynamic processes and habitat mapping, as well as the assessment of coastal vulnerability to sea level rise and storm events [1]. The nearshore zone is of particular relevance due to its highly dynamic character: it constitutes the primary pathway of sediment exchange between the beach front and the inner shelf, and its morphology directly governs short- and long-term shoreline change [2]. Despite its importance, remote sensing techniques for accurate and temporally frequent bathymetric mapping are still less mature than those already developed for the emerged beach, such as shoreline position and foredune morphology (e.g., [3–8]). Traditional hydrographic survey methods (including single-beam and multibeam echosounders) are costly, logistically demanding, and often constrained by navigational hazards in very shallow water [9,10]. Airborne LiDAR bathymetry constitutes a substantial advance in spatial coverage and vertical accuracy over traditional acoustic surveys; nevertheless, its prohibitive operational cost and demanding logistical requirements render it largely incompatible with the systematic, basin-scale monitoring that contemporary coastal management increasingly demands [11].

The use of freely available medium-resolution satellite imagery to characterize the dynamics of coastal areas has reached a high level of maturity, although almost all of these efforts have focused on studies of shoreline evolution, whether in applications for regional-scale studies [7,8,12], for local studies of the impacts of major storms [5], or for human interventions in the coastal system [6]. Satellite-Derived Bathymetry (SDB) has emerged as a cost-effective and temporally flexible alternative for mapping water depth in optically shallow coastal waters [13,14]. By exploiting the differential attenuation of solar radiation at various wavelengths as it propagates through the water column and reflects from the seabed, multispectral satellite imagery can be used to retrieve bottom depth over large areas and at high revisit frequency [15]. Since the pioneering works of Lyzenga [16], SDB techniques have been progressively refined and applied to a growing range of coastal environments worldwide, benefiting from the increasing availability of medium- and high-resolution satellite missions such as Landsat, WorldView, PlanetScope, VEN μ S, and Sentinel-2 (e.g., [17–21]).

Certainly, obtaining bathymetries derived from satellite imagery does not allow the level of accuracy that can currently be achieved with more traditional techniques or with LiDAR. Nevertheless, thanks to the frequency with which optical imagery is available [22], it is to be expected that they will yield results with sufficient precision to detect the position and displacement of submerged bars, as well as some of the most significant changes that may occur in the submerged profile of a beach. In this context, recent frameworks have successfully demonstrated that long-term satellite observations can track nearshore bar dynamics and morphodynamic beach states by analyzing wave-breaking patterns directly from optical imagery [21,23]. At present, however, it is not easy to take advantage of all the images, or even most of them [24]. Many of them present features (changes in water color, foam, suspended sediment, high turbidity, etc.) that hinder their use for deriving depths, representing a significant limitation on one of the main attractions

associated with the use of these images. For that reason, some authors have proposed not using a sole image from a specific date as a source of reflectance information, but rather a composite created from several images recorded over a relatively short period [13,25,26]. Advancing methodologies that expand the number of usable images would therefore be a significant step forward in making optical imagery more useful for monitoring submerged near-shore morphologies.

SDB methods can broadly be classified into physics-based and empirical approaches. Focusing on empirical methods, they rely on establishing statistical relationships between spectral reflectance and known water depth values through regression analysis, offering operational simplicity and benefiting from site-specific calibration. Among empirical approaches, the log-linear model suggested by Lyzenga [16] and especially the band ratio method proposed by Stumpf et al. [15] have become widely adopted reference techniques due to their robustness (Equation (1)).

$$Z = a \cdot \frac{\ln(n \cdot R_{w \text{ blue}})}{\ln(n \cdot R_{w \text{ green or red}})} + b \quad (1)$$

Equation (1): Stumpf Method.

Where R_w is the water reflectance (dimensionless) usually from the RGB bands; a and b stand for slope and intercept, respectively; n is an arbitrary number, which, for values between 500 and 1500, was assessed in [15] as presenting a Root Mean Square Error (RMSE) lower than 40 cm up to depths over 20 m for the study case by Lubin et al. [27] in a coral sandy area.

The literature usually fixes n at 1000 [13,15], arguing that this value keeps both logarithms positive under all conditions and yields a linear response. Since reflectance theoretically takes values in the interval (0, 1), the function $\ln(n \cdot R_w)$ would be negative for a small n ; setting $n = 1000$ ensures that $n \cdot R_w > 1$, so both numerator and denominator are positive and the ratio is well-defined. However, even when the argument of the logarithm is smaller than 1 (and both terms therefore become negative), the quotient itself remains positive, since a negative numerator is divided by a negative denominator.

As an essential parameter, n could be optimized to other positive values, or at least its behavior should be explored. However, very few related studies have addressed this issue. Some site-specific studies have analyzed this possibility of improvement (e.g., [28]) or applied different n values such as $n = 10,000$ (e.g., [29]) or $n = 100$ (e.g., [30]). Nevertheless, no analysis of n for different oceanographical or morphological conditions is available in the literature. Taking this into account, there is a scientific gap in the application of empirical approaches for deriving SDBs, and it is necessary to shed some light on a possible optimization of the n parameter depending on local conditions. This optimization would improve the explicability of the SDB models, as it enhances the ability to explain and characterize the underlying physical phenomena.

To simplify the determination of the optimal n value, this study focuses exclusively on Sentinel-2 imagery and the blue–green ratio combination. This choice aligns with the previous literature [31] as these data offer a 10 m spatial resolution suited to the scale of the analyzed features, alongside a high temporal frequency that maximizes the probability of acquiring useful imagery.

This study aims to optimize the Stumpf model by evaluating how SDB errors vary with n across 31 bathymetric scenarios under diverse morphological and oceanographic conditions. From these tests, a generalized ready-to-use n value is proposed, and the local conditioning factors and their correlation with water conditions are analyzed to better understand the model's behavior. Furthermore, the correlation between the improvements achieved over the conventional $n = 1000$ value and the local water conditions is analyzed

to assess SDB extraction across different coastal environments. The wide range of conditions analyzed avoids conclusions drawn solely from local areas.

2. Methodology

The strategy proposed here is to build SDB models at sites representing a large diversity of hydrological and physical characteristics. For each site, the optimal n value is determined, and the causes of variation from the conventional $n = 1000$ value are identified. This makes it possible to determine whether there is a universal n value that can be applied across all sites, or if site-specific conditions require calibrations.

To sum up, the methodology of this work follows three main steps (Figure 1): (1) image selection and preprocessing, (2) SDB retrieval and optimization of the n parameter, and (3) the analysis of the relationship between water quality and error results.

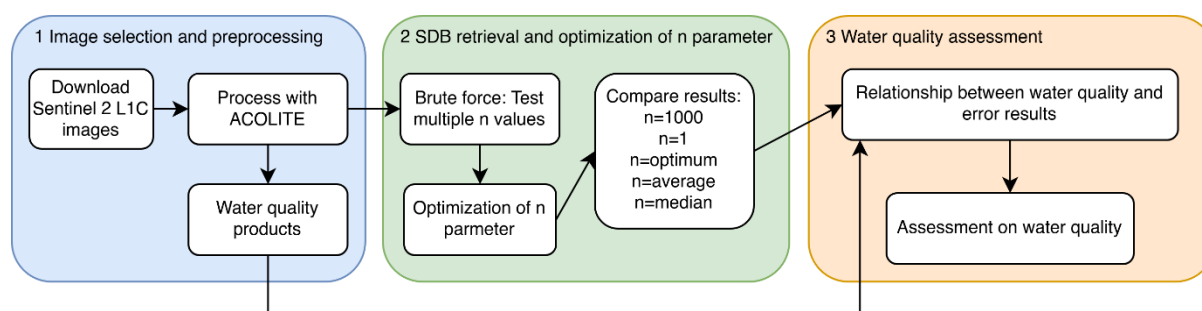


Figure 1. Workflow divided into three phases: (1) image selection and preprocessing, (2) SDB retrieval and n parameter optimization, and (3) water quality assessment.

2.1. Test Sites

The selection of the test sites (Figure 2) was determined by: (a) the availability of recent, accurate bathymetries obtained through field campaigns by official bodies (in most of the cases for coastal monitoring purposes) and (b) the geographical distribution of the sites covering a wide range of coastal types, allowing us to evaluate the model's performance under contrasting and highly diverse environmental conditions. In order to do so, 31 scenarios distributed on 18 sites have been analyzed. These sites (Figure 2, Table 1) encompass both US coasts, the Great Lakes and diverse European coasts (Ireland, northern and eastern Spain). Regarding tidal level, no correction has been made; as the models trained here are instantaneous to a specific water level, it is not required to apply any offset correction. For specific details of each study site, the reader is referred to Tables 1 and 2, which include information about the provider of bathymetry as well as further morphological and water quality information.

Regarding the morphological and hydrodynamic conditions of the test sites, most of the studied beaches are located on coastal barriers adjacent to lagoons or near river outlets. For the USA sites, raster bathymetric datasets were downloaded from the Elevation Datasets of the National Oceanic and Atmospheric Administration Digital Coast [32]. Twelvemile Beach, located on the coast of Lake Superior, features a small creek that discharges into this lake, occasionally altering water coloration [33]. The North Carolina sites are situated around Pamlico Sound but with different characteristics. While Duck and Nags Head are located on the seaward side of the barrier island, Hatteras Inlet sits directly exposed to the inlet's mouth. The tidal ebb from the lagoon induces higher turbidity and chlorophyll-*a* concentrations at this site; indeed, it displays the highest chlorophyll levels in this study (Table 2). Lastly, Diamond Shoals is a shallow sandbank extending over 10 km off Cape Hatteras. This is the only site where a single bathymetric dataset was evaluated under two distinct scenarios using different satellite images but the same reference

bathymetry. These images, acquired 20 days apart, capture nearly identical morphological states but contrasting water qualities: the earlier imagery exhibits a prominent blue hue, whereas the latter is clearly influenced by greener waters outflowing from the lagoon (Table 2). Figure 3 illustrates these differences using an RGB composition and also shows other water quality parameters, such as chlorophyll and hue angle, which better represent the water quality differences.

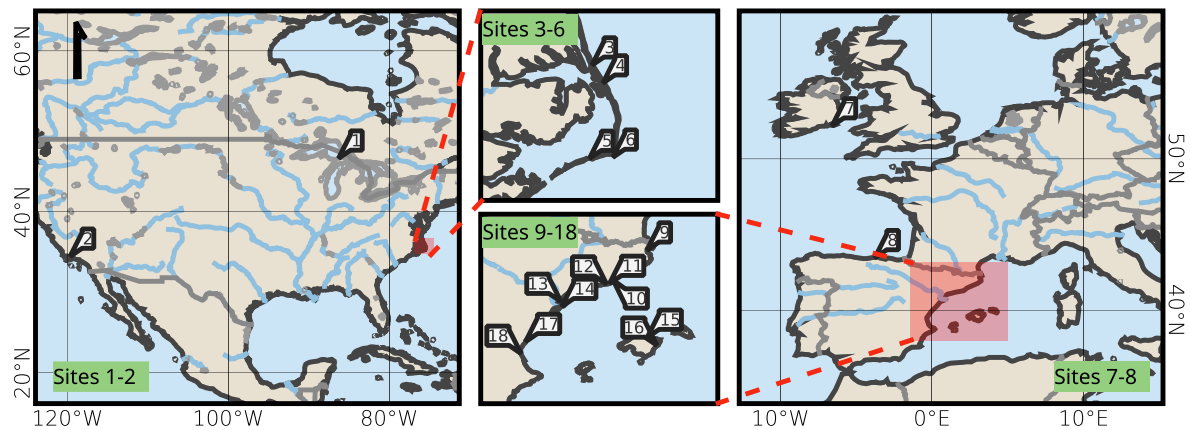


Figure 2. Study sites. The left panel shows the study sites within the USA, with an inset in the upper-middle panel zooming in on North Carolina. The right panel displays the European sites, with an inset in the lower-middle panel focusing on the north-western Mediterranean coast.

Table 1. Overview of study site characteristics.

ID	Site	Region/Country	Coastal Setting	Tidal Regime
1	Twelvemile Beach	Michigan (Lake Superior), USA	Great Lakes shoreline with a small creek discharging into the lake, occasionally altering water coloration.	Microtidal (<0.05 m)
2	More Mesa Beach	Santa Bárbara, California, USA	USA Pacific coast. Sandy beach backed by a coastal cliff.	Mesotidal (~2.5 m)
3	Duck Southern Shores	North Carolina (Pamlico Sound), USA	USA Atlantic coast. Seaward side of the barrier island.	Microtidal (~1.3 m)
4	Nags Head	North Carolina (Pamlico Sound), USA	USA Atlantic coast. Seaward side of the barrier island.	Microtidal (~1.3 m)
5	Diamond Shoals	North Carolina (Cape Hatteras), USA	USA Atlantic coast. Shallow sandbank extending over 10 km off Cape Hatteras.	Microtidal (~1.3 m)
6	Hatteras Inlet	North Carolina (Pamlico Sound), USA	USA Atlantic coast. Directly exposed to the inlet's mouth; tidal ebb from the lagoon increases turbidity and chlorophyll-a.	Microtidal (~1.3 m)
7	Kilmore Quay and Ballytaige Bay	Wexford, Ireland	Irish Atlantic coast. Ballytaige Bay: dune system backed by an inland marshy area. Kilmore Quay: partially submerged moraine forming a linear feature extending into the sea.	Macrotidal (~3.5 m)
8	El Puntal interior and exterior	Cantabria, Spain	Spanish Atlantic coast. 4-km sandy spit at the entrance of Santander Bay, divided into inner and outer sections.	Macrotidal (~3.8 m)
9	Sant Pere Pescador	Girona (Gulf of Roses), Spain	Spanish Mediterranean coast. Adjacent to a wetland system near the mouth of the Fluvià river.	Microtidal (~0.2 m)
10	La Barceloneta and Badalona	Barcelona, Spain	Spanish Mediterranean coast. Fronts the Barcelona metropolitan area.	Microtidal (~0.2 m)

11	Llobregat	Barcelona, Spain	Spanish Mediterranean coast. Comprehends the Llobregat river delta, fronting a marshy environment with strong anthropogenic influence.	Microtidal (~0.2 m)
12	Gavà	Barcelona, Spain	Beach placed in the southern part of the Llobregat delta.	Microtidal (~0.2 m)
13	Península del Fangar	Tarragona, Spain	Spanish Mediterranean coast. Ebro delta, near the river mouth; high water clarity.	Microtidal (~0.2 m)
14	L'Eucaliptus	Tarragona, Spain	Spanish Mediterranean coast. Ebro Delta, near the river mouth; high water clarity.	Microtidal (~0.2 m)
15	Muro	Mallorca (Balearic Islands), Spain	Spanish Mediterranean coast. Clear waters, white sand and seagrass meadows.	Microtidal (~0.2 m)
16	Cala Millor	Mallorca (Balearic Islands), Spain	Spanish Mediterranean coast. Clear waters, white sand and seagrass meadows.	Microtidal (~0.2 m)
17	L'Almardà	València, Spain	Spanish Mediterranean coast. Coarse gravel beach.	Microtidal (~0.2 m)
18	El Saler	València, Spain	Spanish Mediterranean coast. Barrier beach protecting the L'Albufera de València lagoon; influenced by the Túria river mouth and the lagoon inlets.	Microtidal (~0.2 m)

Table 2. Summary of studied sites and scenarios, including the pairs of bathymetry data and S2 imagery employed in the analysis, alongside their associated water quality parameters (Chl, hue, zSD...). Note: Currently, ACOLITE algorithms do not support Secchi disk depth retrieval for Sentinel-2C, which may lead to data gaps for scenario 18, image date 24 April 2025.

Scenario	Beach	Area (km ²)	Date of Bathymetry	Provider's Bathymetry	S2 Grid	Date of S2 Image	Turbidity (NTU)	Chl-a (µg/L)	Hue Angle (°)	z_SD (m)	TSI	TS
1	Twelvemile Beach	2.07	From 10 June 2019 to 21 August 2019	NOAA	16TES	15 June 2019	0.7 ± 0.4	7.0 ± 1.5	113.0 ± 7.8	4.7 ± 0.6	44	M
2	More Mesa Beach	0.35	From 19 June 2015 to 29 September 2015	NOAA	11SKU	14 November 2015	1.7 ± 0.3	1.1 ± 0.1	193.4 ± 3.9	7.3 ± 0.4	31	O-M
3	Duck Southern Shores	1.58	From 24 August 2018 to 28 August 2018	NOAA	18SVE	6 August 2018	4.3 ± 0.5	3.6 ± 0.6	134.7 ± 7.1	3.8 ± 0.3	42	M
4	Nags Head	1.86	From 24 August 2018 to 28 August 2018	NOAA	18SVE	6 August 2018	4.0 ± 0.5	4.0 ± 0.6	128.1 ± 10.8	3.8 ± 0.4	42	M
5	Hatteras Inlet	5.86	From 2 October 2018 to 15 October 2018	NOAA	18SVE	3 October 2018	5.1 ± 0.9	10.6 ± 3.0	75.3 ± 5.7	2.1 ± 0.4	51	E
6.1	Diamond Shoals	4.93	From 24 August 2018 to 28 August 2018	NOAA	18SVE	6 August 2018	3.0 ± 0.6	2.4 ± 0.5	160.2 ± 7.4	4.6 ± 0.5	39	O-M
6.2	Diamond Shoals	4.93	From 24 August 2018 to 28 August 2018	NOAA	18SVE	26 August 2018	2.3 ± 0.4	3.9 ± 1.1	133.6 ± 13.0	4.4 ± 0.5	42	M
7.1	Ballytaige Bay	1.91	2017–2018	GSI and MII	29UPT	28 May 2018	2.8 ± 0.5	2.9 ± 0.4	150.1 ± 8.0	4.8 ± 0.4	39	O-M
7.2	Kilmore Quay	2.61	2017–2018	GSI and MII	29UPT	28 May 2018	2.4 ± 0.3	1.2 ± 0.3	191.6 ± 8.4	6.6 ± 0.5	33	O-M
8.1	Puntal exterior	0.74	April 2024	IHC	30TVP	20 April 2024	1.3 ± 0.6	3.2 ± 0.9	150.2 ± 16.3	6.0 ± 1.0	39	O-M
8.2	Puntal interior	0.35	April 2024	IHC	30TVP	20 April 2024	1.0 ± 0.6	2.9 ± 0.6	155.0 ± 7.4	6.3 ± 1.3	37	O-M
9	Sant Pere Pescador	2.72	2021–2024	ICGC	31TEG	30 August 2022	0.3 ± 0.3	3.3 ± 0.5	155.7 ± 6.5	6.0 ± 0.4	38	O-M
10.1	Badalona	1.35	May 2023	AMB	31TDF	7 April 2023	2.6 ± 1.2	3.8 ± 0.7	141.6 ± 11.0	4.0 ± 0.5	42	M
10.2	Barceloneta	0.29	May 2023	AMB	31TDF	7 April 2023	2.0 ± 0.9	3.2 ± 0.8	155.7 ± 17.7	4.9 ± 0.7	40	O-M
11.1	Llobregat	2.18	2021–2024	ICGC	31TDF	7 January 2023	4.6 ± 0.6	1.5 ± 0.3	181.9 ± 7.1	4.9 ± 0.4	36	O-M
11.2	Llobregat	2.14	May 2023	AMB	31TDF	12 April 2023	2.3 ± 0.5	2.1 ± 0.4	167.2 ± 5.7	5.3 ± 0.4	37	O-M

12.1	Gavà	1.77	May 2023	AMB	31TDF	12 April 2023	1.8 ± 0.4	2.0 ± 0.3	168.5 ± 4.7	5.7 ± 0.4	36	O-M
12.2	Gavà	1.79	2021–2024	ICGC	31TDF	9 September 2023	1.7 ± 1.4	1.8 ± 0.3	175.2 ± 6.8	5.8 ± 0.5	35	O-M
13	Península del Fangar	3.83	2021–2024	ICGC	31TCF	16 July 2022	1.9 ± 0.7	4.8 ± 1.5	129.3 ± 11.4	4.7 ± 0.5	42	M
14	L'Eucaliptus	3.08	2021–2024	ICGC	31TCF	16 July 2022	1.9 ± 0.6	3.6 ± 0.9	146.8 ± 12.5	5.0 ± 0.6	40	O-M
15	Muro	0.20	21 March 2023	SOCIB	31TDF	25 March 2023	2.7 ± 0.7	3.0 ± 0.2	141.5 ± 9.9	3.8 ± 0.6	41	M
16.1	Cala Millor	0.48	15 March 2017	SOCIB	31TDF	10 April 2017	2.8 ± 1.3	1.6 ± 0.3	169.2 ± 16.4	4.0 ± 1.3	38	O-M
16.2	Cala Millor	0.36	1 November 2017	SOCIB	31TDF	19 November 2017	4.5 ± 0.9	1.6 ± 0.3	169.5 ± 10.4	3.9 ± 0.7	38	O-M
16.3	Cala Millor	0.44	23 October 2018	SOCIB	31TDF	25 October 2018	3.4 ± 0.9	2.1 ± 0.4	157.6 ± 15.0	4.3 ± 1.0	39	O-M
16.4	Cala Millor	0.38	20 May 2019	SOCIB	31TDF	20 May 2019	3.4 ± 1.2	1.9 ± 0.5	164.9 ± 17.2	4.1 ± 1.0	39	O-M
17.1	L'Almardà	2.76	3 November 2020	DGCM	30SYJ	28 August 2020	3.3 ± 0.7	2.2 ± 0.4	166.3 ± 7.3	4.8 ± 0.4	38	O-M
17.2	L'Almardà	0.09	28 November 2024	CGAT (UPV)	30SYJ	26 September 2024	2.0 ± 0.3	1.8 ± 0.2	176.5 ± 3.4	5.5 ± 0.3	36	O-M
18.1	El Saler	2.72	23 September 2018	DGCM	30SYJ	23 September 2018	2.3 ± 1.4	3.6 ± 0.8	149.1 ± 14.2	5.0 ± 0.6	40	M
18.2	El Saler	2.66	23 August 2023	DGCM	30SYJ	23 August 2023	2.4 ± 0.7	4.7 ± 1.1	130.1 ± 10.5	4.4 ± 0.5	42	M
18.3	El Saler	0.06	14 May 2024	CGAT (UPV)	30SYJ	30 March 2024	3.7 ± 0.3	2.2 ± 0.2	161.8 ± 3.9	4.3 ± 0.2	39	O-M
18.4	El Saler	0.02	30 June 2025	CGAT (UPV)	30SYJ	24 April 2025	1.7 ± 0.5	3.1 ± 0.4	146.9 ± 6.1	—	—	—

In Europe, the Irish study sites accessed via the INFOMAR viewer [34] are located along the South Wexford Coast, which experiences an expected macro-tidal range of 3.5 m. Ballytaige Bay features a dune system backed by an inland marshy area. Nearby, Kilmore Quay presents a distinct coastal morphology due to a partially submerged moraine that forms a linear feature extending into the sea.

For the Spanish northern coast, El Puntal is a 4 km sandy spit at the entrance of Santander Bay formed by the interaction of estuarine and marine dynamics with a tidal range around 3.8 m. Because of its different water conditions, this site has been divided into the inner and outer parts of the spit. These data have been obtained by the Environmental Hydraulics Institute of Cantabria (IHC).

For the Mediterranean Sea, the tide range is around 20 cm. This region includes 10 sites, most of which are sandy beaches, except for l'Almardà beach. Almost all of them present very clear waters; however, turbid examples also occur, some strongly affected by chlorophyll-rich inputs from the adjacent coastal lagoons, as occurs at Saler beach. The reference bathymetry for these sites was obtained from publicly available datasets of the Institut Cartogràfic i Geològic de Catalunya [35] and the Barcelona Metropolitan Area [36], except for the case of sites 17 (l'Almardà) and 18 (El Saler), for which bathymetries were acquired by the Spanish General Directorate of Coast and Sea or directly by the authors.

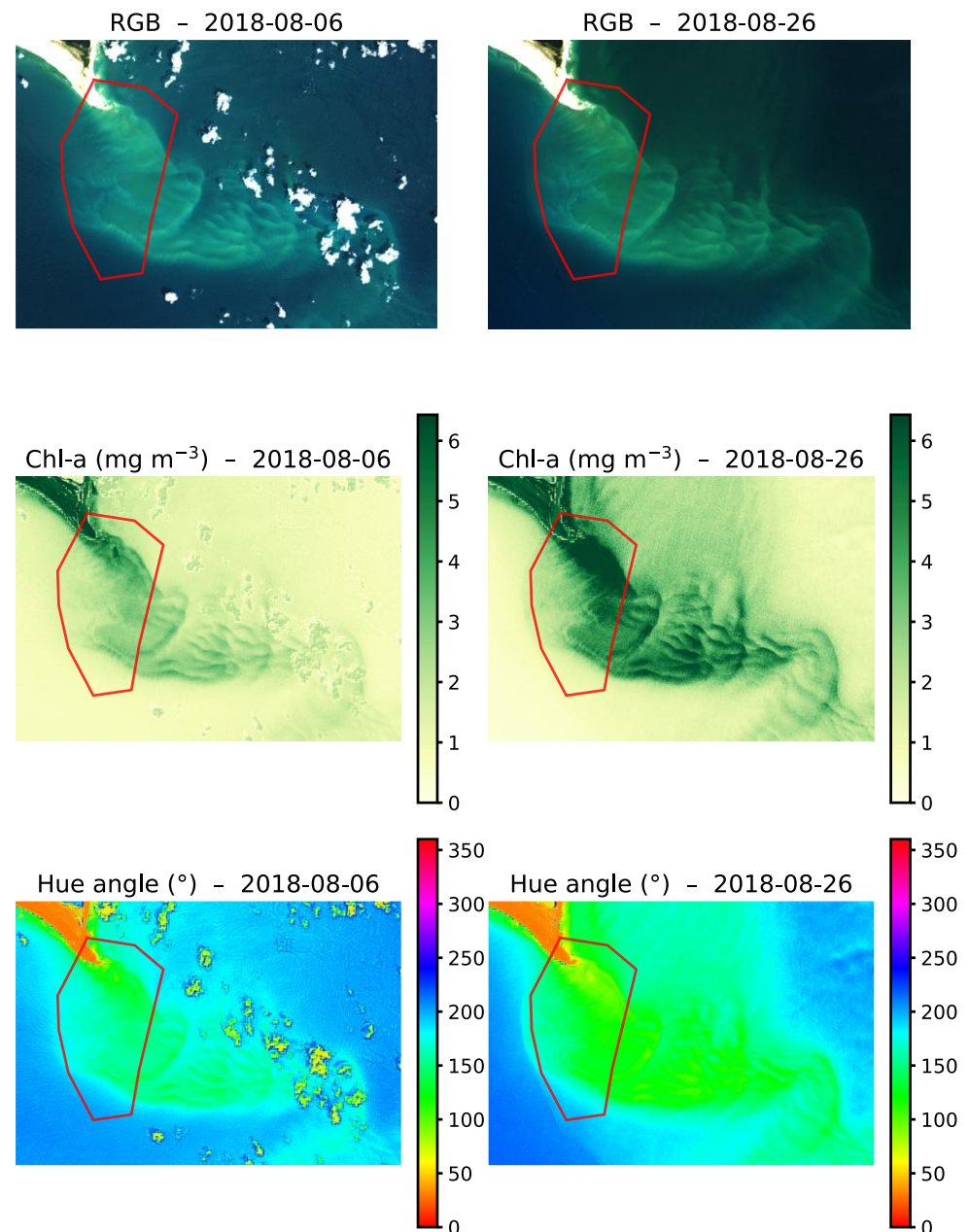


Figure 3. Comparison between two near dates with different water conditions in the study site of Diamond Shoals (red polygon).

2.2. Image Selection and Preprocessing

To pursue the objective of empirically improving the n parameter for multiple sites, suitable and close-in-time Sentinel-2 images were manually selected for each analyzed bathymetry (see Table 2). To be considered suitable, satellite scenes must meet strict quality criteria: they must be completely cloud-free in the study zone (including cirrus) and free of breaking waves and foam presence in the surf zone. Additionally, images are excluded if they contain floating obstacles (e.g., macroalgae, boats, and wakes), severe sun glint or high turbidity driven by sediment transport.

The image product L1C of the Sentinel-2 satellite has been downloaded from the Copernicus Data Space Ecosystem, as shown in Table 2. As SDBs require very accurate reflectance signals, the ACOLITE tool v20260421 [37], which has been developed for marine remote sensing applications, was employed for the correction of the atmospheric effects.

To ensure the reproducibility of the results, the default configuration was modified as follows. All intermediate NetCDF products (L1R, L2R and L2W) were set to be deleted after processing to reduce disk and memory usage, and the output resolution was fixed at 10 m so that every spectral band was resampled to a common pixel size. For atmospheric correction, dark spectrum fitting (DSF) was configured in tiled mode, and a default residual glint correction was applied over the 1500–2400 nm wavelength range. The built-in land mask was disabled, as this process was applied later within the SDB retrieval workflow. The L2W parameters extracted from each scene included the full set of water-leaving reflectance, turbidity, chlorophyll-a, Secchi disk depth and hue angle, which together support both the SDB computation and the subsequent water quality assessment. Finally, an RGB composite was exported as a GeoTIFF for visualization purposes.

2.3. SDB Retrieval and Optimization of n Parameter

In this study, several places with different water conditions have been chosen to validate our results. Table 2 shows the list of studied places, their bathymetry, the approximate studied area, and their corresponding Sentinel-2 image.

As water quality determines the transparency of the water column and, therefore, the applicability of SDB methods, we retrieved several water quality parameters: water turbidity (also referred to as TUR) and chlorophyll-a concentration (also referred to as chl_oc3). These parameters have been chosen over other ones due to their key importance for water quality mapping [38]. To calculate them, we used the built-in option in ACO-LITE software to obtain the chlorophyll index and the turbidity implemented by Vanhellemont and Ruddick [37], following Nechad et al. [39]. Secondly, the hue angle [40,41] and the Secchi disk depth implemented by [37] were also retrieved.

The analysis focuses on the nearshore, i.e., the zone of the coast affected by wave action. The depth of the study areas was restricted to an upper limit of -0.5 m in order to minimize land–water mixed pixels in the satellite images, as well as to reduce radiometric disturbances associated with foam in the wave-breaking zone. Regarding the lower limit, it is defined by the closure depth of each site, which depends on the local wave climate. Since our primary interest lies in the most dynamic part of the profile, this lower limit was fixed at -6 m [2,42]. Some sites might have a deeper closure depth, but this threshold is probably enough to capture the gross of the sediment transport. Also, the errors on SDBs might be higher than the bathymetric changes. Furthermore, this procedure standardizes the input datasets, as several reference bathymetries do not extend into deeper waters.

As previously mentioned, the value is usually defined as $n = 1000$, with the argument that both logarithms will be positive in all conditions and get a linear response from the data. However, this scaling is a numerical convenience rather than a physical requirement. For $R_w \in (0, 1)$, $\ln(n \cdot R_w)$ only changes sign; changing the value of “ n ” does not alter the sensitivity of the function $\ln(n \cdot R_w)$ with respect to R_w , since it only shifts it vertically and only affects the numerical scale of the values. The derivative $d[\ln(n \cdot R_w)]/d R_w = 1/R_w$ is independent of n , so any positive n preserves the same sensitivity of the logarithm to depth. The limiting case $n = 1$ is therefore included in the analysis as the unscaled, parameter-free form of the model: although $\ln(R_w)$ becomes negative throughout $(0, 1)$, the quotient $\ln(R_{w \text{ blue}})/\ln(R_{w \text{ green}})$ remains positive (negative divided by negative) and well-defined as long as neither logarithm equals zero (i.e., $R_w \neq 1$). Since $n = 1$ already works correctly and does not introduce arbitrary parameters, any choice of $n > 1$ should demonstrate a real improvement. Otherwise, the most parsimonious option is to keep $n = 1$. The choice of values for $n > 1$ in Equation (1) may lead to two mathematical instabilities in the quotients of logarithms. First, a number/0 undefined value, and second, a 0/0 indeterminate form. These instabilities are inherent to the method, and the model will not be able to

predict in these situations. Nonetheless, it is possible to analyze them and avoid these voids of prediction capacity.

Both instabilities restrict the explanatory capacity of the model. To understand why these instabilities appear, it is important to see it geometrically: With the logarithm, the exponential attenuation of radiation is linearized, and as the value of n changes, the logarithm of the water reflectance shifts towards line $Y = 0$. When the R_w values with $\ln(n \cdot R_w) = 0$ are within the range related to our working depth values (from -0.5 to -6 m), instability occurs, and numerical problems arise. As green and blue bands exhibit different attenuation properties, their linearized representations intersect, introducing mathematical instabilities near this intersection point. The undefined division by zero occurs when the denominator is zero: the anti-logarithm equals 1, and the quotient diverges towards $\pm\infty$. The same condition arises for the logarithm in the numerator (blue band) and might produce a $0/0$ indetermination, but the most restrictive instability is the division by zero, caused by the green band (Equation (2)).

In practice, our models do not produce exact divergences: because the radiometric and bathymetric data carry errors and dispersion, the instability manifests as a marked increase in scatter and non-linearity. The transition is gradual: first, a few points exert leverage on the linear regression as the green logarithm bends the point cloud; then, a fully unpredictable region appears (red zone in the Results, Figure 4); and finally, linearity is recovered, but with the opposite sign. As Equation (2) shows, the unstable value of n is the inverse of the green reflectance, and nearby values also yield poor fits because the quotient is no longer linear. These effects are analyzed in more detail in the Sections 3 and 4.

$$\ln(R_{W \text{ green}} \cdot n) = 0 \rightarrow R_{W \text{ green}} \cdot n = 1 \rightarrow n = \frac{1}{R_{W \text{ green}}} \quad (2)$$

To characterize these instabilities, two complementary approaches were used: an analytical (theoretical) derivation and a numerical (empirical) optimization. From the analytical standpoint, as has been mentioned, the instability begins and ends with the green logarithm (since green attenuates at shallower depths, it has a steeper slope when it is linearized): so the lower bound of the instability interval is the inverse of the maximum green reflectance, and the upper bound is the inverse of the minimum green reflectance (Equation (3)).

$$\text{Unstable interval: } \left[\frac{1}{\max R_{W \text{ green}}}, \frac{1}{\min R_{W \text{ green}}} \right] \quad (3)$$

On the other hand, the numerical optimization tests every value of n within a given range and selects the optimal parameter as the one yielding the lowest RMSE or the highest R^2 (as happens in our study sites). The instability interval has been defined operationally in this study as follows: it begins at the first n value for which R^2 falls to 0.1 or below and ends at the last n value for which R^2 remains below 0.1. The optimum n for each scenario was then identified automatically. Improvements in RMSE and R^2 are defined by Equations (4) and (5):

$$\text{Improvement of RMSE (\%)} = \frac{RMSE_{\text{reference}} - RMSE_{\text{to compare}}}{RMSE_{\text{reference}}} \cdot 100 \quad (4)$$

$$\text{Improvement } R^2 (\%) = \frac{R^2_{\text{to compare}} - R^2_{\text{reference}}}{R^2_{\text{reference}}} \cdot 100 \quad (5)$$

In the results shown here, the RMSE and R^2 values obtained with $n = 1000$ were considered the reference values and compared against alternative configurations, including the site-specific optimum n and the parameter-free case $n = 1$, which serves as the natural

lower bound and the simplest form of the log-ratio model (see argument above). More descriptors like median or average were analyzed further in the following paragraphs.

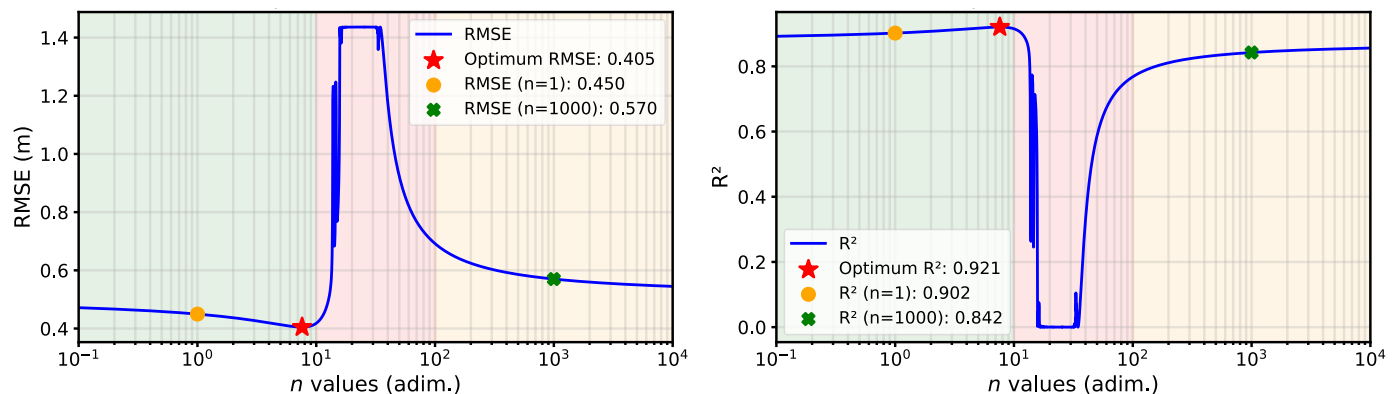


Figure 4. Regression performance (RMSE and R^2) across a continuous range of n values at l’Eucliptus Beach (E Spain, scenario 14).

The numerical optimization was performed by brute force, testing every value of n from 0.1 to 10,000 in steps of 0.1; the value yielding the lowest RMSE (or, equivalently, the highest R^2) was taken as the optimum for each scenario. The Stumpf method was applied using Sentinel-2 Band 3 (green, 560 nm) as the denominator and Band 2 (blue, 490 nm) as the numerator of the logarithmic ratio. For every tested n , a linear regression was fitted between the resulting band ratio and the co-registered in situ depths within the defined depth range (−0.5 m to −6 m), yielding site-specific coefficients a (slope) and b (intercept). Model performance was assessed through RMSE (in meters) and R^2 , with the optimal n defined as that minimizing the RMSE.

To provide practically applicable reference values that do not require site-specific tuning, several summary statistics of the distribution of optimum n values were computed: the global mean and median, and the mean and median restricted to a subset of cases (the restriction is described in Section 3, where better performance is reported for this subset). These reference values were then applied uniformly across all study cases to assess their generalization against both the site-specific optimum and $n = 1000$.

2.4. Water Quality Assessment

To categorize the water types of the study sites, Table 2 summarizes the mean (μ) and standard deviation (σ) of the chlorophyll, turbidity, hue angle and Secchi disk depth parameters. These indicators were computed between −4 m and −6 m to minimize near-shore artifacts such as wave breaking, foam and seabed reflectance.

Because these products/indices were originally developed for offshore (optically deep) waters, their values in shallow coastal environments may be partially affected and biased by bottom (seabed reflectance) contributions [43,44]. Consequently, they are used here as qualitative descriptors rather than absolute measures. It is also important to acknowledge that it is not possible to measure the water quality offshore to avoid seabed reflectivity disruption because water quality varies especially from inshore to offshore.

Nonetheless, as an approximate way to classify the studied areas, some of these indicators were combined (chl_oc3 and Secchi disk depth) to get the Trophic State Index, TSI [45], defined in this case as appears on the North American Lake Management Society [46]:

$$TSI(SD) = 60 - 14.41 \ln(SD) \quad (6)$$

$$TSI(CHL) = 9.81 \ln(CHL) + 30.6 \quad (7)$$

$$TSI = \frac{TSI(CHL) + TSI(SD)}{2} \quad (8)$$

with CHL being the chlorophyll average and SD being the average Secchi disk depth. Trophic state is classified according to TSI values as oligotrophic ($TSI < 30$), oligotrophic–mesotrophic ($30 \leq TSI < 40$), mesotrophic ($40 \leq TSI < 50$), or eutrophic ($50 \leq TSI < 60$). Under this classification, the studied scenarios fall predominantly in the oligotrophic–mesotrophic range (20 scenarios), followed by mesotrophic (9 scenarios) and a single eutrophic case (1 scenario).

3. Results

The results presented here follow a three-step approach. Firstly, the original Stumpf method (Equation (1)) is optimized and analyzed by determining the optimal multiplier n through a brute-force search. Secondly, the best-fitting n values are generalized by spotting statistical patterns in the results. Finally, the improvements are assessed in relation to local oceanographic conditions.

3.1. Assessment of n Parameter: Optimum Value and Generalization

A systematic sensitivity analysis of the Stumpf method (Equation (1)) was performed by testing values of n between 0.1 and 10,000 (0.1 step) for the blue/green band ratio. As justified by the underlying physical and numerical behavior of the model (see Section 2.3), $n = 1$ was integrated into this assessment to serve as the baseline configuration against which any alternative choices of $n > 1$ must be evaluated.

A representative example of this analysis is shown in Figure 4, where the behavior of the model is presented in a particular test site using RMSE and R^2 as goodness-of-fit metrics. In this example, the conventional configuration $n = 1000$ is marked with a green x-cross, the baseline value $n = 1$ with an orange dot, and the optimal value with a red star. The optimal value is automatically extracted by identifying the minimum (for RMSE) or maximum (for R^2) of the optimization function.

An analysis of the sensitivity plots across the diverse scenarios reveals the consistent mathematical patterns of the functions. Firstly, the function typically displays two distinct stable regions: two plateaus (left and right) are detected (green and yellow zones, respectively), separated by a central unstable area (red shaded zone). While the particular values defining the inflection points of the function may change, its general pattern is repeated across all sites, making the position of the unstable area scenario-dependent. These two plateaus contain the optimum values for all the study cases. However, the right plateau does not always host the absolute optimum. In fact, for 26 of 31 analyzed scenarios, the left plateau (LP) yields better results than the right one within the tested range of n . On the other hand, the unstable noisy behavior observed in the central (red) area corresponds to the numerical instabilities caused by the log-ratio quotient. These instabilities occur when the antilogarithm composed of the green reflectance multiplied by n approaches unity, causing the denominator to diverge towards zero, as has been mentioned (Equations (2) and (3)). In the example at l'Eucaliptus (Figure 4), optimizing this parameter shifts the model away from this unstable zone, revealing that an optimal $n = 7.60$ outperforms the conventional default $n = 1000$.

We have analyzed the differences between the RMSE considering $n = 1$ and the RMSE considering a fixed value of $n = 1000$. The Shapiro–Wilk normality test indicates that these differences do not follow a normal distribution (p -value = 0.0017), and thus we have applied the Wilcoxon signed-rank test (p -value < 0.001), indicating that the median of said

differences is different from zero. On the other hand, the Shapiro–Wilk normality test for the differences between R^2 ($n = 1$) and R^2 ($n = 1000$) obtains a p -value < 0.001 , and the Wilcoxon signed-rank test a p -value < 0.001 , so the R^2 and RMSE values considering $n = 1$ were statistically different from those obtained using $n = 1000$. This last test also indicates that R^2 ($n = 1$) is greater than R^2 ($n = 1000$) (p -value < 0.001) and the RMSE ($n = 1$) is less than the conventional configuration RMSE ($n = 1000$) (p -value < 0.001).

To validate these findings globally, the predicted Satellite-Derived Bathymetries (SDB) configured with alternative n values were compared against in situ reference data across the full dataset (using Equations (4) and (5)). The resulting model performance metrics and their relative gains over the $n = 1000$ baseline are summarized in Table 3. The results of the improvement reached with each n value with respect to $n = 1000$ show how calibrating the model to its site-specific optimum increases the average R^2 from 0.70 ± 0.23 to 0.84 ± 0.10 , an absolute gain of $+0.14 \pm 0.18$. Simultaneously, RMSE decreases by $21.9 \pm 16.7\%$, equivalent to a vertical accuracy of 18 ± 20 cm, dropping from 0.698 to 0.515 m. When compared with the baseline, it can be observed that $n = 1$ already outperforms $n = 1000$, so transitioning towards the optimum n only provides a marginal additional improvement: R^2 increases from 0.82 ± 0.12 to 0.84 ± 0.10 ($+0.02$ in absolute terms), and RMSE decreases by just $6.0 \pm 6.6\%$, equivalent to 4 ± 4 cm. Comparing with the median of n values that fell on the left plateau ($n = 2.2$), which represents the best single transferable parameter across all study sites, R^2 reaches 0.83 ± 0.11 , only 0.02 below the per-site optimum, while RMSE decreases by $16.8 \pm 19.3\%$ relative to $n = 1000$, equivalent to 15 ± 20 cm.

Table 3. Average SDB model performance (R^2 and RMSE) across diverse n values, with percentage of improvement relative to $n = 1000$.

n Setting	n Value	R^2 (Mean $\pm$ SD)	ΔR^2	ΔR^2 (%)	RMSE (Mean $\pm$ SD) (m)	Δ RMSE (m)	Δ RMSE (%)
$n = 1000$	1000	0.703 ± 0.232	—	—	0.698 ± 0.254	—	—
$n = \text{opt}$	optimum	0.845 ± 0.103	+0.142	+20.2%	0.515 ± 0.135	−0.184	−26.3%
$n = 1$	1	0.822 ± 0.120	+0.118	+16.8%	0.551 ± 0.147	−0.147	−21.1%
$n = \text{mean}$	1045.2	0.704 ± 0.231	+0.001	+0.1%	0.697 ± 0.253	−0.001	−0.2%
$n = \text{median}$	3.0	0.816 ± 0.108	+0.113	+16.0%	0.568 ± 0.158	−0.130	−18.7%
$n = \text{mean (LP)}$	2.9	0.818 ± 0.107	+0.115	+16.3%	0.564 ± 0.154	−0.134	−19.2%
$n = \text{median (LP)}$	2.2	0.827 ± 0.109	+0.123	+17.5%	0.548 ± 0.143	−0.151	−21.6%

As outlined in Section 2.3, the instability interval has been characterized through two complementary approaches (Figure 5): the theoretical interval, analytically derived from Equation (3) (red bars); and the empirical interval, which is obtained numerically as defined in Section 2.4 (blue bars).

In the scenarios of Twelvemile Beach (scenario 1) and Hatteras Inlet (4), the empirical interval shows no upper bound within the tested range of n values: R^2 remains below 0.1 beyond the instability and fails to recover as n increases. These specific environments are incompatible with the conventional $n = 1000$ setting, although they can still yield reasonably accurate SDB results when configured with other lower values of n .

By definition, the per-site optimum n values (marked with a red star in Figure 5) lie outside the instability interval. The red star represents these optimum n values; for some scenarios, it lies outside the plot range (truncated at $n = 500$ for visualization purposes).

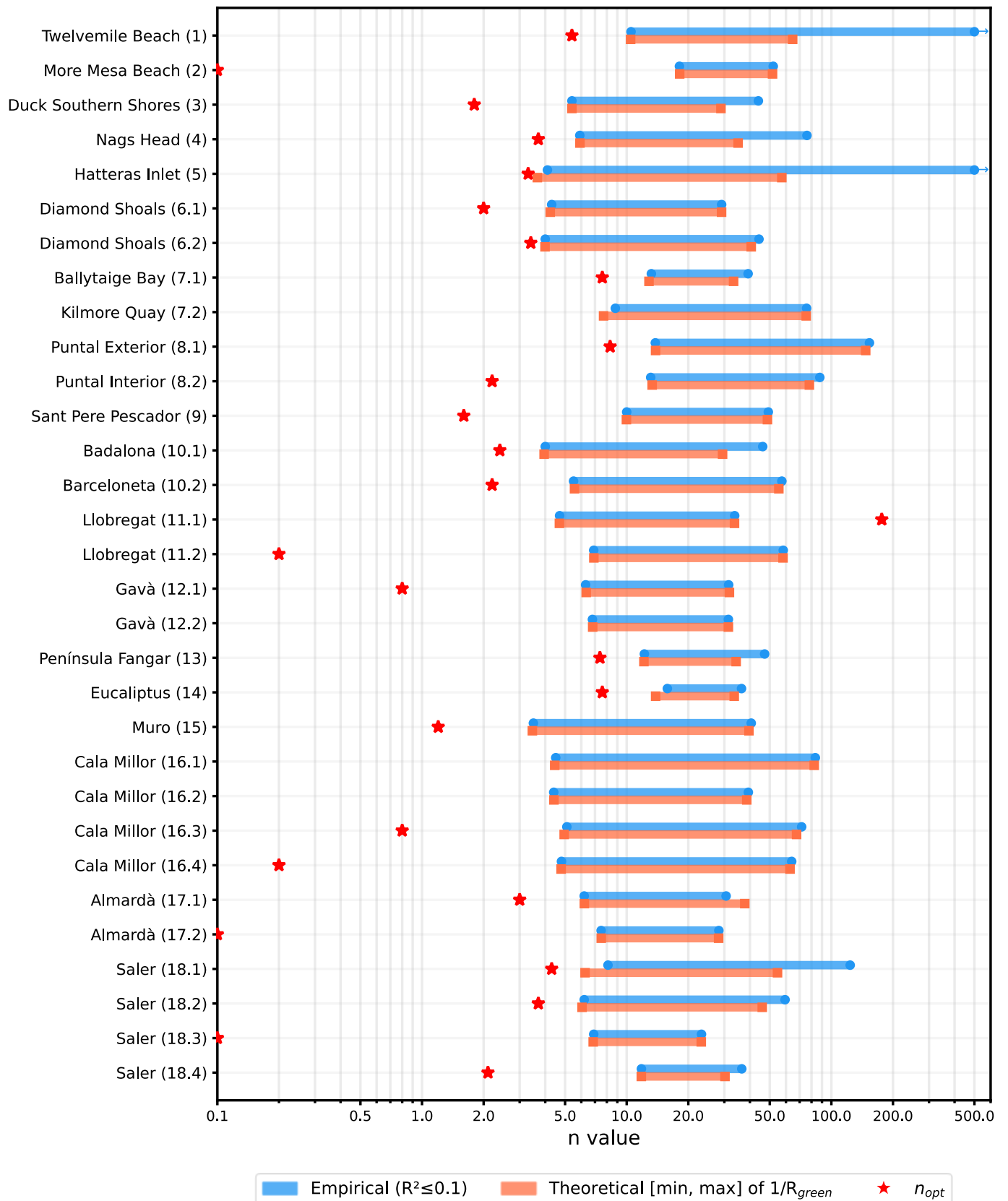


Figure 5. Theoretical (red) vs. empirical (blue) instabilities across all studied scenarios, identified by site and S2 image date. The red star marks the optimum n value (truncated at $n = 500$ for visualization). The small blue arrow indicates that the instability goes beyond $n = 500$.

As the usage of the optimum n value for each scenario is not feasible, as it might be unknown for new oceanographic conditions or sites, the generalization potential of these parameters was evaluated. To address this limitation, optimum n values were generalized using the results obtained across all analyzed scenarios. To do this, median and average optimal n values have been calculated for all study scenarios as representative descriptor parameters. Furthermore, as the optimum n value is usually on the left plateau of the function, the median and average values have also been calculated for all the scenarios in which the optimum n value is on the LP. All descriptors have been tested on all study scenarios (Tables 3 and 4). The best result is achieved with the median of n values that fell on the left plateau, with an average R^2 value of 0.827 ± 0.109 and RMSE of 0.548 ± 0.143 m. The LP median offers similar results to the LP average.

Considering the full set of scenarios (Figure 6), an ideal error distribution was defined ($n = n_{\text{opt}}$), representing the target behavior to which the results obtained from the proposed generalizations (i.e., employing $n = 1$, $n = 1000$, and $n = \text{median}$ for the LP) should converge. The distribution of RMSE results using the conventional $n = 1000$ (green) exhibits a wider spread of RMSE values, indicating clearly sub-optimal performance. By contrast, $n = 1$ and the median of the left plateau yield the nearest distributions to the ideal results.

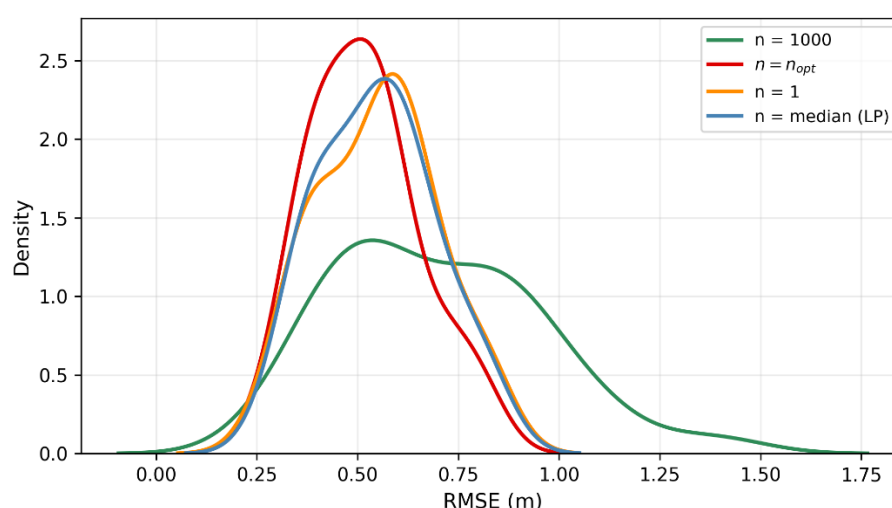


Figure 6. Histogram errors (expressed as RMSE, in m) when employing diverse generalization approaches for defining n value across all scenarios.

To analyze these improvements in each scenario, Figure 7 compares the RMSE obtained using the conventional $n = 1000$ (reference case) against the RMSE obtained using $n = n_{\text{opt}}$, $n = 1$, and $n = \text{median LP}$. The green region represents an improvement relative to the reference (ideal) result, the 1:1 line corresponds to an equal result, and the red area refers to a worsening of the results. The first subplot of Figure 7 compares the results obtained when using the $n = \text{optimum}$ value, so no scenario showed a deterioration of the results. The second and third subplots show the comparisons for $n = 1$ and $n = \text{median LP}$, respectively. Both approaches yield very similar results, with about 25 to 26 showing improvements or unchanged performance compared to only 5 to 6 that got worse results.

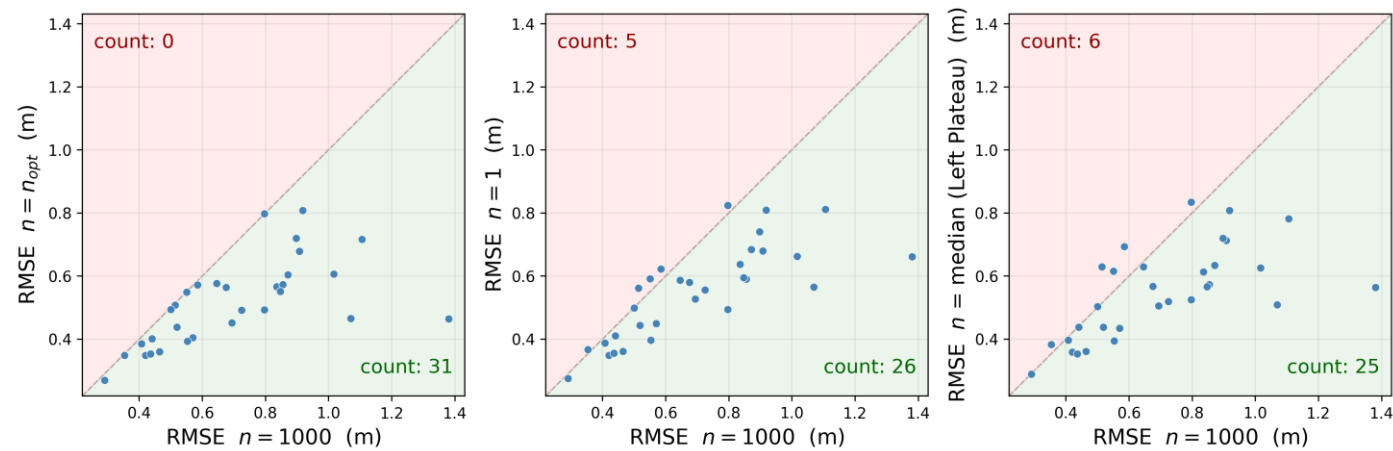


Figure 7. Comparison between RMSE results obtained from the conventional $n = 1000$ vs. n optimal, $n = 1$, and $n = \text{median (LP)}$. Green region below line 1:1 represents an improvement, while red region above the line reflects a deterioration in performance.

Table 4. Results for all studied scenarios. The median and mean LP values used were $n = 2.2$ and $n = 2.9$, respectively. Green cells represent improvements (positive values) compared to $n = 1000$, while red cells represent deterioration in performance (negative values).

Scenario	Beach	S2 Date	N opt	n med (LP)	n avg (LP)	RMSE n = 1000	RMSE n = opt	RMSE n = 1	RMSE n = med	RMSE n = avg	RMSE n = med LP	RMSE n = avg LP	% imp. opt vs. 1000	% imp. n = 1 vs. 1000	% imp. med LP vs. 1000
1	Twelvemile Beach	15 June 2019	5.4	2.2	3.1	1.38	0.46	0.66	0.51	1.38	0.56	0.52	+66.4%	+52.1%	+59.1%
2	More Mesa Beach	14 November 2015	0.1	2.2	3.1	0.50	0.49	0.50	0.51	0.50	0.50	0.51	+1.2%	+0.3%	−0.7%
3	Duck Southern Shores	6 August 2018	1.8	2.2	3.1	0.55	0.39	0.40	0.41	0.55	0.39	0.40	+28.8%	+28.2%	+28.6%
4	Nags Head	6 August 2018	3.7	2.2	3.1	1.02	0.61	0.66	0.61	1.01	0.63	0.61	+40.3%	+34.8%	+38.4%
5	Hatteras Inlet	3 October 2018	3.3	2.2	3.1	0.87	0.60	0.68	0.61	0.87	0.63	0.61	+30.6%	+21.5%	+27.2%
6.1	Diamond Shoals	6 August 2018	2.0	2.2	3.1	0.52	0.44	0.44	0.46	0.52	0.44	0.46	+15.7%	+14.6%	+15.6%
6.2	Diamond Shoals	26 August 2018	3.4	2.2	3.1	0.72	0.49	0.56	0.49	0.72	0.52	0.50	+32.1%	+23.2%	+28.4%
7	Ballytaige Bay	28 May 2018	7.6	2.2	3.1	0.69	0.45	0.53	0.49	0.69	0.50	0.49	+34.9%	+23.9%	+27.2%
7	Kilmore Quay	28 May 2018	2148.7	2.2	3.1	0.80	0.80	0.82	0.84	0.80	0.83	0.84	+0.0%	−3.4%	−4.7%
8	Puntal exterior	20 April 2024	8.3	2.2	3.1	1.11	0.72	0.81	0.76	1.10	0.78	0.77	+35.2%	+26.6%	+29.3%
8	Puntal interior	20 April 2024	2.2	2.2	3.1	0.92	0.81	0.81	0.81	0.92	0.81	0.81	+12.0%	+11.8%	+12.0%
9	Sant Pere Pescador	30 August 2022	1.6	2.2	3.1	0.46	0.36	0.36	0.36	0.46	0.36	0.36	+22.4%	+22.2%	+22.2%
10.1	Badalona	7 April 2023	2.4	2.2	3.1	0.90	0.72	0.74	0.74	0.90	0.72	0.74	+19.8%	+17.6%	+19.8%
10.2	Barceloneta	7 April 2023	2.2	2.2	3.1	0.85	0.57	0.59	0.59	0.85	0.57	0.58	+33.0%	+30.9%	+33.0%

11.1	Llobregat	7 January 2023	176.3	2.2	3.1	0.55	0.55	0.59	0.65	0.55	0.62	0.64	+0.3%	−7.2%	−11.7%
11.2	Llobregat	12 April 2023	0.2	2.2	3.1	0.41	0.39	0.39	0.41	0.41	0.40	0.41	+5.5%	+4.8%	+2.7%
12.1	Gavà	12 April 2023	0.8	2.2	3.1	0.42	0.35	0.35	0.38	0.42	0.36	0.38	+17.2%	+17.1%	+14.6%
12.2	Gavà	9 September 2023	10,000.0	2.2	3.1	0.35	0.35	0.37	0.40	0.35	0.38	0.40	+1.3%	−3.8%	−8.4%
13	Península del Fangar	16 July 2022	7.4	2.2	3.1	0.84	0.57	0.64	0.60	0.83	0.61	0.60	+32.4%	+23.8%	+26.6%
14	Eucaliptus	16 July 2022	7.6	2.2	3.1	0.57	0.40	0.45	0.43	0.57	0.44	0.43	+29.0%	+21.1%	+23.7%
15	Muro	25 March 2023	1.2	2.2	3.1	0.80	0.49	0.49	0.81	0.79	0.52	0.75	+38.1%	+38.0%	+34.1%
16.1	Cala Millor	10 April 2017	10,000.0	2.2	3.1	0.59	0.57	0.62	0.80	0.58	0.69	0.79	+2.3%	−6.3%	−18.5%
16.2	Cala Millor	19 November 2017	10,000.0	2.2	3.1	0.51	0.51	0.56	0.72	0.51	0.63	0.71	+1.2%	−9.3%	−22.2%
16.3	Cala Millor	25 October 2018	0.8	2.2	3.1	0.91	0.68	0.68	0.78	0.91	0.71	0.77	+25.3%	+25.2%	+21.6%
16.4	Cala Millor	20 May 2019	0.2	2.2	3.1	0.65	0.58	0.59	0.70	0.64	0.63	0.69	+10.7%	+9.1%	+2.6%
17.1	l'Almardà	28 August 2020	3.0	2.2	3.1	0.68	0.56	0.58	0.56	0.68	0.57	0.56	+16.6%	+14.2%	+16.1%
17.2	l'Almardà	26 September 2024	0.1	2.2	3.1	0.29	0.27	0.28	0.31	0.29	0.29	0.31	+7.0%	+5.1%	+0.3%
18.1	Saler	23 September 2018	4.3	2.2	3.1	1.07	0.47	0.56	0.48	1.07	0.51	0.48	+56.5%	+47.2%	+52.4%
18.2	Saler	23 August 2023	3.7	2.2	3.1	0.85	0.55	0.60	0.55	0.85	0.57	0.55	+34.9%	+29.8%	+33.2%
18.3	Saler	30 March 2024	0.1	2.2	3.1	0.44	0.40	0.41	0.48	0.44	0.44	0.47	+8.8%	+6.7%	+0.6%
18.4	Saler	24 April 2025	2.1	2.2	3.1	0.44	0.35	0.36	0.36	0.44	0.35	0.36	+18.9%	+18.4%	+18.9%

3.2. Assessment of Water Quality

To identify relationships between the results and the water quality parameters, an extensive analysis was carried out. Spearman's correlation coefficient was used because the n = optimal results showed clearly non-normal distribution, including outliers, and were not expected to exhibit strictly linear relationships. Figure 8 shows the Spearman's correlation matrix between all the water quality parameters studied (rows) and the RMSE results obtained with different n values, as well as the corresponding improvements compared to n = 1000 results. As first impressions, turbidity and Secchi depth showed no correlation with any of the performance metrics. In contrast, hue angle, chlorophyll-a and the Trophic State Index (TSI) exhibited the highest correlations, not in the results themselves but in the improvement relative to n = 1000 instead. This has interesting applications because, as Figure 8 shows, with previous knowledge of the water quality parameters offered by ACOLITE, an approximate improvement with respect to n = 1000 could be estimated.

The hue angle and Chl-a exhibited a strong but inverse relationship. This behavior can be explained by the fact that increasing Chl-a concentrations are generally associated with lower hue angle values, and vice versa. Finally, the TS index, as a combination of the Chl-a (highly related) and the Secchi disk depth (poorly related), has slightly inferior correlation.

Regarding the collinearity between variables, Table 5 shows some statistically significant multiple linear regression models for predicting RMSE and R^2 using a stepwise procedure to select statistically significant water quality parameters as predictors and a variance inflation factor (VIF) below 10.

The RMSE model, considering a fixed value of n = 1000, has moderate predictive capacity (adjusted R^2 of 39.60%) using TSI and hue angle ($^\circ$) as predictors. However, the percentage improvement in RMSE using the optimal n compared to n = 1000 can be predicted with these same independent variables with a good fit (adjusted R^2 jumps to 64.86%), indicating that the model is quite robust in predicting where and by how much the error will be reduced. The negative coefficients of the model indicate that the higher the TSI (more eutrophic/turbid waters) and the larger the hue angle, the lower the percentage improvement achieved with optimization.

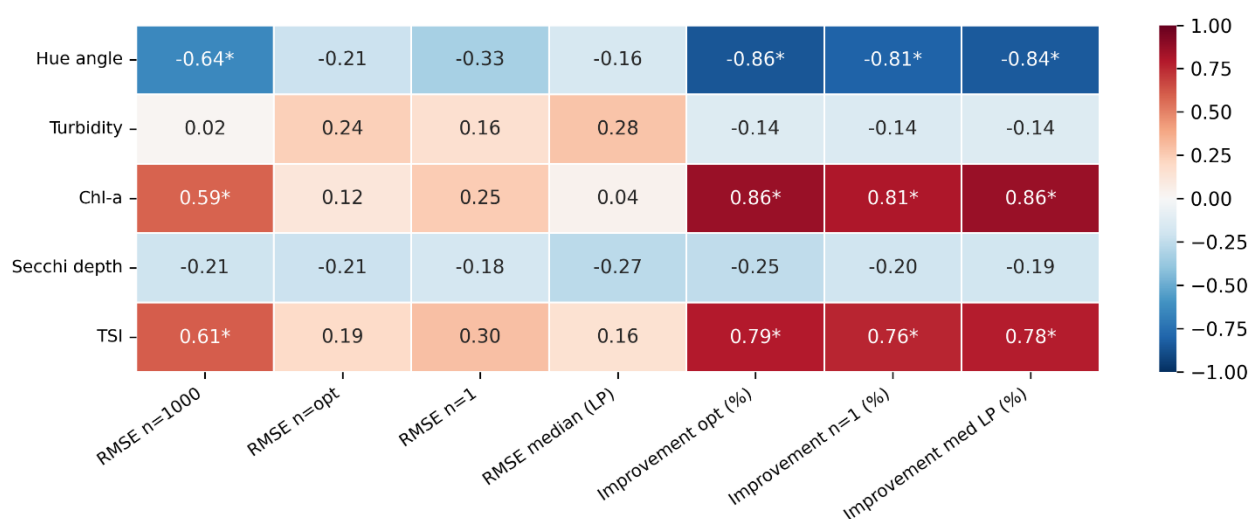


Figure 8. Spearman correlation matrix of results and water quality parameters. The asterisk (*) symbol indicates correlations significantly different from zero with at least a 95% confidence level.

Whether using $n = 1000$ or the mean of the optimal n LP values, the variation of R^2 in different scenarios can be predicted using Chlorophyll-a (Chl-a) as a predictor with high explanatory power. The chlorophyll-a concentration alone is able to explain almost 70% of the variance in the fit of the analyzed system, indicating that the presence of more chlorophyll worsens the fit of the bathymetry estimates with both values of n .

Table 5. Statistics of multivariate linear models for some RMSE and R^2 results using water quality parameters as predictors and a stepwise procedure.

Dependent Variable (Units)	Formula	Coefficient	<i>p</i> Value	Adj- R^2 %	RMSE	MAE	VIF
RMSE $n = 1000$ (m)	Intercept	0.82891	0.0004				-
	TSI	0.014994	0.0408	39.60%	0.1972 m	0.1656 m	8.49
	Hue angle (°)	0.002125	0.0002				8.49
RMSE improv. opt vs. 1000 (%)	Intercept	239.956	0.0000				-
	TSI	-2.64093	0.0019	64.86%	10.094%	6.823%	1.98
	Hue angle (°)	-0.774202	0.0000				1.98
R^2 $n = 1000$	Intercept	1.02631	0.0000	69.44%	0.1288	0.1035	-
	Chl-a (µg/L)	-0.106764	0.0000				1.0
R^2 $n = \text{mean LP}$	Intercept	1.02639	0.0000	69.47%	0.1283	0.1031	-
	Chl-a (µg/L)	-0.106446	0.0000				1.0

As mentioned earlier, the correlation between Chl-a/hue angle and the improvements of the results by changing the n value is significant. Figure 9 shows a scatter plot between these variables. Although there is a strong correlation between these variables, several apparent outliers are present (e.g., Hatteras (5), Saler (18.1) or Cala Millor (16.1 and 16.2)), apparently worsening the results. However, these should not be interpreted as erroneous observations but as a possible real case. For example, and more particularly, Hatteras (5) has the highest chlorophyll levels by far among all studied scenarios. Nonetheless, as the results show, it has clearly improved with the proposed modifications.

Although we will consider all points as inliers (including those that appear to be outliers from a statistical point of view), a RANSAC fit has been applied to obtain an approximate linear model, which is used to make possible recommendations for use in Section 4. Also, bias and RMSE are shown in the figure. These regressions suggest that clearer (bluer) waters offer less potential for improvement through n optimization, whereas greener waters exhibit a greater margin for performance enhancement.

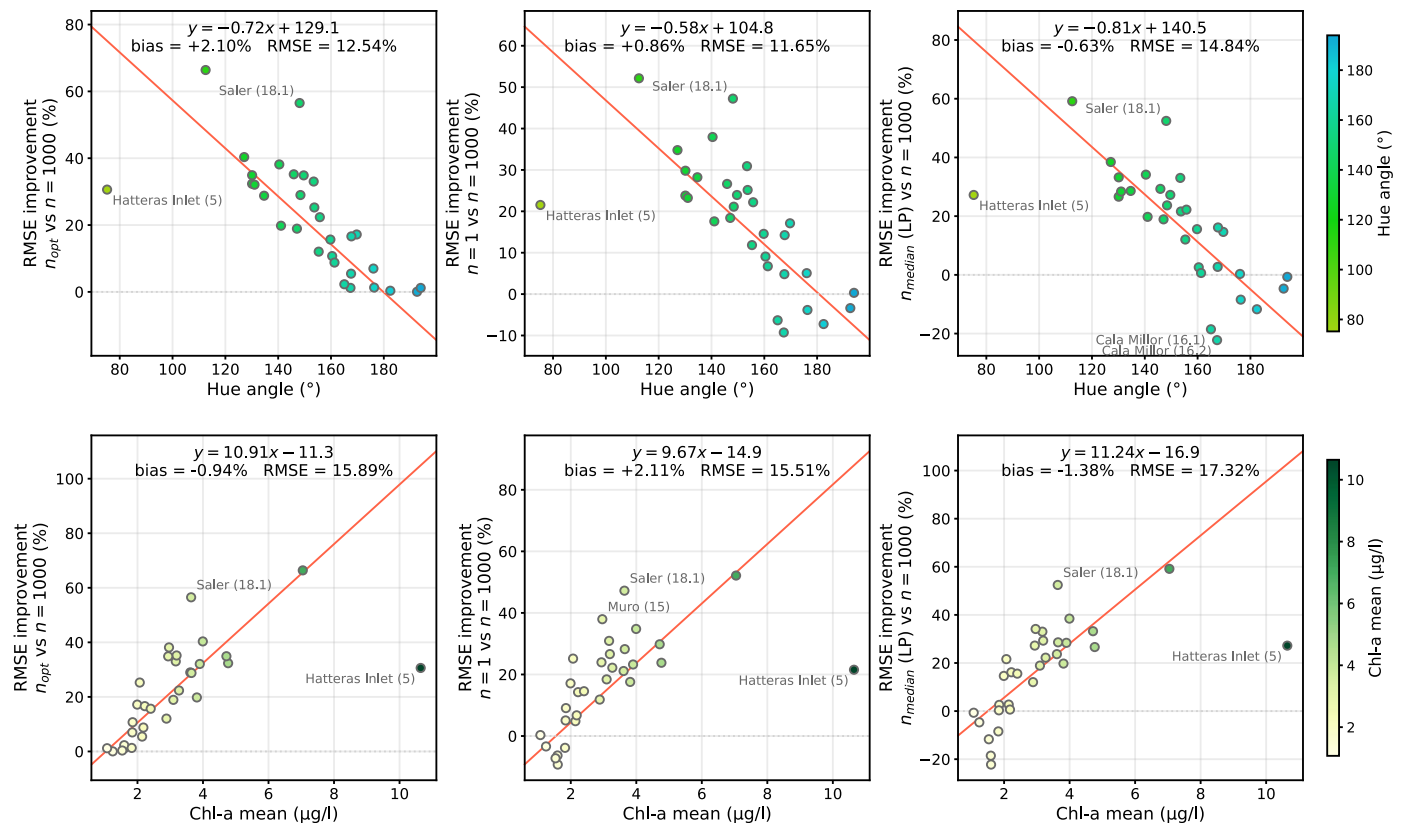


Figure 9. Scatterplots showing the relationship between the hue angle or chlorophyll and RMSE improvements for each scenario.

4. Discussion

The first contribution of this work is to challenge the routine use of $n = 1000$ in the Stumpf log-ratio method. Across 31 study cases spanning 18 locations with diverse morphological and hydrodynamic characteristics, $n = 1000$ was suboptimal in 26 of them, while switching to a site-specific optimum improved R^2 from 0.70 to 0.84 and reduced RMSE by 21.9% on average, equivalent to a reduction of depth error of roughly 18 cm. Although $n = 1000$ has become a de facto convention since Stumpf et al. [15] introduced it as a means of scaling the observed reflectances (thereby keeping both logarithms positive and the quotient stable in most plausible reflectance ranges), its dominance in the literature appears to rest more on convention than on systematic evaluation.

Only a few studies have explicitly questioned this value. Zhu et al. [28] tested alternative n values for Stumpf-based SDB in shallow waters and reported improvements over the default; their work, however, was confined to a single optically clear environment in the Paracel Islands (atoll-type, very transparent waters) and used a different platform and atmospheric correction chain, so the reflectance magnitudes may not directly be comparable with the ones proposed in the present study. Although previous studies differ substantially from the present work, they reported the same instability behavior explained in the Section 2. Additionally, Casal et al. [29] adopted $n = 10,000$ and Eugenio et al. [30] and Frugier et al. [21] used $n = 100$ in their respective study areas (Maspalomas (Canary Islands) and Cabrera Island (Balearic Islands), with WorldView-2 and -3, and Duck (North Carolina) with Venüs), illustrating that, in practice, n may be adjusted on a case-by-case basis without an agreed criterion. The multi-site assessment presented here provides that criterion: for the -0.5 to -6 m depth range, n values around the left plateau of the optimization ($n \approx 1-3$) outperform $n = 1000$ in the large majority of cases, regardless of the specific local conditions. Within this range, $n = 1$ deserves particular attention, as it represents the

unscaled, parameter-free form of the model, as shown in Section 2.3. Setting $n = 1$ removes a free parameter without losing information: both logarithms remain negative throughout the $(0, 1)$ interval and their quotient stays positive and well-defined, while the operating point is placed firmly on the left plateau in the majority of the scenarios. The exception appears in those whose green reflectance is low enough to push the instability above $n = 1$ (the five right-plateau cases discussed below). In practice, $n = 1$ reduces RMSE relative to $n = 1000$ in 25 of the 31 scenarios and performs similarly to the per-site optimum in most of them, which makes it the natural default whenever no water quality information is available to justify a different choice. The magnitude of this gain is substantial: switching from $n = 1000$ to $n = 1$ raises R^2 from 0.70 ± 0.23 to 0.82 ± 0.12 , and reduces RMSE by approximately 15.9% on average (around 14 cm of depth error), which is the bulk of the 21.9% gain achievable with per-site optimization (theoretical maximum). In other words, $n = 1$ alone provides roughly three-quarters of the improvement margin achievable through full per-scenario optimization with respect to the conventional $n = 1000$ at no calibration cost. The generalized $n_{\text{median(LP)}} = 2.2$ is preferable when a small safety margin away from the $n = 1$ boundary is desired, but the gain over $n = 1$ is modest, and the theoretical argument for employing $n = 1$ is the cleanest. Additionally, studying the instability interval as proposed here could be used for evaluating future scenario applications.

Not all scenarios benefit from a small n . Five of the 31 scenarios yielded an optimum on the right plateau ($n_{\text{opt}} > 1000$). This is the case for scenarios 7.2 (Kilmore Quay), 11.1 (Llobregat), 12.2 (Gavà), and 16.1 and 16.2 (Cala Millor). In these scenarios, $n = 1$ and $n_{\text{median(LP)}} = 2.2$ do not improve over $n = 1000$, and in some cases, they degrade it noticeably. As an example, scenario 16.2 (Cala Millor) worsens by 22.2% when the LP-median is applied, and 11.1 (Llobregat) by 11.7%. Using $n = 1$ would yield a lower degradation. The instability interval defines the operational boundary of the small n recommendation and constitutes precisely the regime in which $n = 1000$ remains a defensible default. The cases where $n_{\text{median(LP)}}$ underperforms $n = 1000$ are therefore not failures of the generalization but expected behavior for scenarios whose optimum lies on the right plateau.

A second contribution of this work is showing that the interval of n values that produce instabilities in the log-ratio can be characterized both analytically and empirically, and that the two characterizations agree. The analytical interval is derived directly from Equation (2), with bounds $1/\max(R_{\text{green}})$ and $1/\min(R_{\text{blue}})$, as introduced in Section 2.3 (Equation (3)). The empirical interval is obtained numerically as the region of n where the regression collapses ($R^2 < 0.1$). Figure 5 shows that, across the 31 scenarios, the two intervals overlap closely and, by construction, the per-site optimum n_{opt} always lies outside both. The agreement is not trivial: it confirms that the loss of accuracy at an intermediate n is governed by the instability caused by the denominator (a property of the model, not of the data) and that the position of the interval can be predicted from the scene's reflectance distribution alone, without running the full optimization. The analytical–empirical concordance therefore closes the loop between the theoretical derivation of Section 2.3 and the numerical results of Section 3.1 and provides a sensor- and water-aware criterion for delimiting the interval that the operational $R^2 < 0.1$ rule approximates.

A third contribution of this work is showing that water quality predicts the margin of improvement obtainable by tuning n . Hue angle, Chl-*a*, and TSI all correlate strongly (Spearman) with the percentage improvement over $n = 1000$, whereas turbidity and Secchi depth do not. As mentioned in the Methodology section, the developers of these indices [39,40] and of ACOLITE [37] warned about the use of these water quality indicators in regions where the sea bottom is visible, and the broader literature confirms that products developed for optically deep waters are biased by bottom reflectance in shallow environments [43,44]. An example of this interference is shown in Figure 10 for the case of Twelvemile Beach (1). The true-color RGB image illustrates how the turbidity band is

clearly biased by lakebed reflectance, while Chl-a and the hue angle remain less affected. Twelvemile Beach represents the most pronounced case in our dataset. Replacing $n = 1000$ with the per-site optimum reduces RMSE by 66.4% (from 1.38 m to 0.46 m), and the generalized n_{median} (LP) = 2.2 alone recovers most of that gain (RMSE 0.56 m, a 59.1% improvement). This confirms the pattern seen above: sites where the conventional n performs worst are exactly those whose optimum lies well below 1000, meaning a single small transferable n captures most of the achievable benefit.

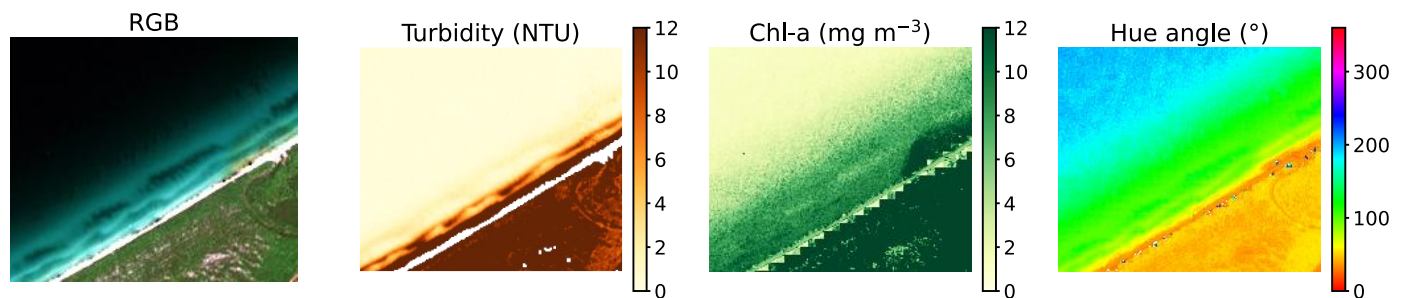


Figure 10. Lake Superior (Michigan, date 15 June 2019) example: turbidity is strongly biased by lakebed reflectance, whereas Chl-a and hue angle are less affected. Note: predictions are not valid for inland pixels.

Bluer, clearer water leaves less room for improvement because $n = 1000$ already performs reasonably well under such conditions, whereas greener waters benefit most from lower n values. The physical reasoning behind this is consistent with the band attenuation framework introduced in Section 2.3: in greener waters, the green reflectance is higher, suggesting that the green logarithm reaches its instability at lower n values, and the operating point therefore shifts decisively to the left plateau. In clearer, bluer waters, the logarithm of the green band remains far from its zero-crossing across the whole tested range, the left plateau is correspondingly narrower or vanishes, and the optimum migrates to the right plateau where $n = 1000$ sits. Hue angle and Chl-a, which jointly describe the green-versus-blue radiometric balance of the water, therefore predict not the absolute SDB error but rather the regime in which a given scene operates, which is precisely the information a practitioner needs to decide whether tuning n is likely to provide a meaningful benefit.

Taken together, these findings provide preliminary support for a practical recommendation: adopting $n \approx 1\text{--}3$ as the default configuration in Stumpf-based SDB, yielding the best results with $n = 2.2$ in the studied scenarios. Conversely, it is recommended, according to these scenarios, to reserve $n = 1000$ for the specific case of very clear, blue waters (hue $> 160^\circ$, chl-a $< 2 \mu\text{g/L}$) where it works well (Figure 9). This reframes n from an arbitrary convention into a parameter with a defensible numerical and empirical basis.

To test this threshold, the results were recalculated by applying this maximum threshold of $2 \mu\text{g/L}$ of chlorophyll to the battery of scenarios used in this work. This new calculation shows that if $n = 1000$ is used, the average RMSE provides lower performance (0.77 m compared to 0.69 m) than if it is calculated with all scenarios. A predictable result if we consider that, normally, green waters provide worse SDB results. Regarding the improvements, if $n = \text{optimum}$ is used, the decrease in average RMSE goes from -26.3% to -32.4% (-0.18 m to -0.25 m) or from -21.6% to -29.1% (from -0.15 m to -0.23 m) for the case of $n = \text{median}$ (LP) (2.2). These changes reinforce the motivation proposed in this article to make images with greener waters more usable to increase the quality and availability of SDBs for coastal monitoring.

The observed lack of correlation with turbidity and Secchi depth is also informative. As presented in the Section 2, the ACOLITE turbidity and Secchi retrievals [47,48] lose

reliability in shallow waters where the sea-bottom signal contaminates the water column estimate, which is exactly the regime targeted by SDB. Figure 10 illustrates this for Lake Superior: turbidity reads as anomalously high above the visible sandy bottom, while Chl-a and hue angle, computed from spectral ratios less sensitive to bottom albedo, remains coherent. The Spearman correlations should therefore be read not as evidence that turbidity is unrelated to SDB performance, but rather as evidence that turbidity cannot be reliably measured in the same scenes where SDB is meaningful. This constitutes primarily a methodological constraint, not a physical one.

This decouples the choice of n from the in situ water quality measurements that are often unavailable in operational SDB workflows: a practitioner cannot predict the resulting RMSE from a TSI value but can predict whether tuning n is worth the effort. This could potentially increase the usage of SDB algorithms in less clear waters and open new opportunities for the near-shore sediment transport modelling over a wider range of areas and satellite images.

Several limitations should be kept in mind when interpreting these results. First, the analysis is restricted to the -0.5 to -6 m depth range, chosen for its relevance to nearshore sediment transport and beach modelling; the behavior of n outside this range, in particular at greater depths where bottom reflectance weakens, may differ, and the optimum could plausibly migrate towards higher values as the green band saturates. Second, the analysis uses Sentinel-2 imagery and the blue/green ratio exclusively; other sensors (WorldView, PlanetScope) and other band combinations (e.g., blue/red) may yield different optima and different plateau structures, and the transferability of $n \approx 1-3$ to those settings remains an open question. Third, the empirical instability interval was defined operationally as the region where $R^2 < 0.1$; a more principled, sensor- and water-aware definition would strengthen the link between the theoretical and empirical intervals shown in Figure 5. Finally, the in situ bathymetries used for validation come from heterogeneous sources (LiDAR, multibeam, RTK surveys) with their own error budgets, which propagate into the apparent RMSE improvements reported here.

Several lines of future research follow naturally. A deeper mathematical understanding of the bathymetric function is still missing. In particular, an explanation of why the left and right plateaus are asymmetric and whether an analytical derivation of the optimum n is feasible from the joint distribution of blue and green reflectance. The role of alternative band combinations (notably blue/red) also deserves further investigation. Extending the analysis to deeper waters and to other sensors (WorldView, PlanetScope, Landsat-9) would test the robustness of the small n recommendation across spatial resolutions and spectral configurations. Finally, the predictive use of hue angle and Chl-a as proxies for the expected magnitude of improvement could be operationalized into a simple decision rule that, given an ACOLITE-derived water quality estimate, recommends either tuning n or retaining $n = 1000$, closing the gap between the radiometric diagnosis and the choice of model parameter.

5. Conclusions

This study has assessed empirically how the n parameter of the Stumpf log-ratio method controls SDB accuracy across 31 bathymetric scenarios at 18 coastal sites with contrasting morphological and water quality conditions, in the -0.5 to -6 m depth range. The following conclusions can be drawn:

The conventional value $n = 1000$ is suboptimal in 26 of the 31 scenarios analyzed. Replacing it with the site-specific optimum raises R^2 from 0.70 to 0.84 and reduces RMSE by 21.9% on average, equivalent to roughly 18 cm of depth error.

(i) The response of RMSE to n shows two stable regions, a left plateau at small n values and a right plateau at large n . The optimum lies on the left plateau in the large

majority of scenarios. A single generalized value, $n_{\text{median(LP)}} = 2.2$, is transferable across sites and recovers most of the gain achievable by per-scenario optimization.

(ii) The five scenarios whose optimum sits on the right plateau ($n_{\text{opt}} > 1000$) all correspond to very transparent waters, where the green logarithm does not reach its instability within the tested range. These cases define the operational boundary of the small n recommendation and the regime in which $n = 1000$ remains a defensible default.

(iii) The interval of n values that yield instabilities is calculated both analytically (with bounds $1/\max(R_{\text{green}})$ and $1/\min(R_{\text{green}})$ derived from the model itself) and empirically, as the region where $R^2 < 0.1$. The two characterizations show a strong agreement across the 31 scenarios, and the per-site optimum n_{opt} always lies outside both. This concordance turns the position of the instability into a quantity that can be predicted from the scene's reflectance distribution alone, without running the full RMSE-versus- n optimization.

Water quality indicators are strongly associated with the margin of improvement obtainable by tuning n . Hue angle, Chl-*a* and TSI correlate strongly (Spearman) with the percentage improvement over $n = 1000$, whereas turbidity and Secchi depth do not, a behavior consistent with the loss of reliability of these last two indicators in optically shallow waters where the sea-bottom signal contaminates the water column estimate.

Taken together, these findings support a practical recommendation: use $n \approx 1\text{--}3$ ($n = 2.2$ yielded the best results in tested scenarios) as the default in Stumpf-based SDB within the studied depth range, and reserve $n = 1000$ for very clear, blue waters (which in our dataset were characterized by hue $> 160^\circ$, Chl-*a* < 2). The choice of n is thus shifted from an arbitrary convention to a quantity with a defensible numerical and empirical basis, and the proposed water quality criterion allows practitioners to anticipate, from readily available remote-sensing indicators, whether tuning n is worth the effort in a given scene. Future work should extend the analysis to greater depths, to other sensors (WorldView, PlanetScope, Landsat-9) and to alternative band combinations, and pursue an analytical derivation of the optimum n from the joint distribution of blue and green reflectance.

Author Contributions: Conceptualization, A.R.-T., J.E.P.-P., J.A.-C. and C.C.-R.; methodology, A.R.-T., J.E.P.-P., J.A.-C., C.C.-R. and Á.B.-B.; software, A.R.-T.; validation, A.R.-T., C.C.-R. and Á.B.-B.; formal analysis, A.R.-T., J.E.P.-P., C.C.-R. and Á.B.-B.; investigation, A.R.-T.; data curation, A.R.-T.; writing—original draft preparation, A.R.-T.; writing—review and editing, J.E.P.-P., C.C.-R., J.A.-C. and Á.B.-B.; visualization, A.R.-T.; supervision, J.E.P.-P., Á.B.-B. and C.C.-R.; funding acquisition, J.E.P.-P. All authors have read and agreed to the published version of the manuscript.

Funding: This research has been funded by the project VETCOCLIM (PID2023-151587OB-I00) granted by MICIU/AEI/10.13039/50110001103 and by the SIMONPLA subproject integrated within the ThinkinAzul programme by the Ministry of Science and Innovation with funds from the European Union NextGeneration EU (PRTR-c17.I1) and Generalitat Valenciana.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The reference bathymetric datasets analyzed in this study are openly available from their respective providers: NOAA Digital Coast (<https://coast.noaa.gov>) for the US sites; the INFOMAR programme of Geological Survey Ireland and the Marine Institute (<https://www.infomar.ie>) for the Irish sites; the Institut Cartogràfic i Geològic de Catalunya (<https://www.icgc.cat>) and the Àrea Metropolitana de Barcelona for the Catalan sites; and SOCIB (Balearic Islands Coastal Observing and Forecasting System, <https://www.socib.es>) for the Balearic sites. Bathymetric data for El Puntal (Santander) were provided by the Environmental Hydraulics Institute of Cantabria (IHCantabria) and, for the Valencian sites, by the Direcció General de la

Costa y el Mar and CGAT-UPV; these datasets are available from the respective providers upon reasonable request. Sentinel-2 Level-1C imagery is freely available from the Copernicus Data Space Ecosystem (<https://dataspace.copernicus.eu>), and atmospheric correction was carried out with the open-source ACOLITE software (<https://github.com/acolite/acolite>, accessed on 25 May 2026). The derived SDB results supporting the findings of this study are available from the corresponding author upon reasonable request.

Acknowledgments: The authors recognize the utility of the data supplied by the Instituto de Hidráulica Ambiental de Cantabria and the Dirección General de la Costa y el Mar, as well as the publicly available data provided by the National Oceanic and Atmospheric Administration, the Geological Survey of Ireland, l'Àrea Metropolitana de Barcelona, Sistema d'Observació Costaner de les Illes Balears and the Institut Cartogràfic i Geològic de Catalunya.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Cesbron, G.; Melet, A.; Almar, R.; Lifermann, A.; Tullot, D.; Crosnier, L. Pan-European satellite-derived coastal bathymetry—Review, user needs and future services. *Front. Mar. Sci.* **2021**, *8*, 740830. <https://doi.org/10.3389/fmars.2021.740830>.
2. Nicholls, R.; Birkemeier, W.; Lee, G. Evaluation of depth of closure using data from Duck, NC, USA. *Mar. Geol.* **1998**, *148*, 179–201. [https://doi.org/10.1016/S0025-3227\(98\)00011-5](https://doi.org/10.1016/S0025-3227(98)00011-5).
3. Almonacid-Caballer, J.; Cabezas-Rabadán, C.; Gorkovchuk, D.; Palomar-Vázquez, J.; Pardo-Pascual, J.E. Re-using historical aerial imagery for obtaining 3D data of beach-dune systems: A novel refinement method for producing precise and comparable DSMs. *Remote Sens.* **2025**, *17*, 594. <https://doi.org/10.3390/rs17040594>.
4. Burvingt, O.; Castelle, B.; Marieu, V.; Lubac, B.; Nicolae Lerma, A.; Robin, N. Using Pleiades satellite imagery to monitor multi-annual coastal dune morphological changes. *Remote Sens.* **2025**, *17*, 1522. <https://doi.org/10.3390/rs17091522>.
5. Cabezas-Rabadán, C.; Pardo-Pascual, J.E.; Palomar-Vázquez, J.; Roch-Talens, A.; Guillén, J. Satellite observations of storm erosion and recovery of the Ebro Delta coastline, NE Spain. *Coast. Eng.* **2024**, *188*, 104451. <https://doi.org/10.1016/j.coastaleng.2023.104451>.
6. Cabezas-Rabadán, C.; Pardo-Pascual, J.E.; Palomar-Vázquez, J.; Cooper, A. A remote monitoring approach for coastal engineering projects. *Sci. Rep.* **2025**, *15*, 2955. <https://doi.org/10.1038/s41598-025-86485-y>.
7. Mao, Y.; Vos, K.; Cagigal, L.; Bodin, V.; Harley, M.D.; Splinter, K.D. A forty-year regional-scale dataset of shoreline change and nearshore wave conditions in Southeast Australia. *Sci. Data* **2026**, *13*, 484. <https://doi.org/10.1038/s41597-026-06859-3>.
8. Vos, K.; Harley, M.D.; Splinter, K.D.; Simmons, J.A.; Turner, I.L. Sub-annual to multi-decadal shoreline variability from publicly available satellite imagery. *Coast. Eng.* **2019**, *150*, 160–174. <https://doi.org/10.1016/j.coastaleng.2019.04.004>.
9. International Hydrographic Organization. C-55: *Status of Hydrographic Surveying and Nautical Charting Worldwide*; IHO Publication C-55; International Hydrographic Organization: Monaco, 2022. Available online: <https://iho.int/en/iho-c-55> (accessed on 26 May 2026).
10. Costa, B.M.; Battista, T.A.; Pittman, S.J. Comparative evaluation of airborne LiDAR and ship-based multibeam SoNAR bathymetry and intensity for mapping coral reef ecosystems. *Remote Sens. Environ.* **2009**, *113*, 1082–1100. <https://doi.org/10.1016/j.rse.2009.01.015>.
11. Klemas, V. Beach profiling and LIDAR bathymetry: An overview with case studies. *J. Coast. Res.* **2011**, *27*, 1019–1028. <https://doi.org/10.2112/JCOASTRES-D-11-00017.1>.
12. Briceño de Urbaneja, I.C.; Pardo-Pascual, J.E.; Cabezas-Rabadán, C.; Aguirre, C.; Martínez, C.; Pérez-Martínez, W.; Palomar-Vázquez, J. Characterization of multi-decadal beach changes in Cartagena Bay (Valparaíso, Chile) from satellite imagery. *Remote Sens.* **2024**, *16*, 2360. <https://doi.org/10.3390/rs16132360>.
13. Caballero, I.; Stumpf, R.P. Towards routine mapping of shallow bathymetry in environments with variable turbidity: Contribution of Sentinel-2A/B satellites mission. *Remote Sens.* **2020**, *12*, 451. <https://doi.org/10.3390/rs12030451>.
14. González-Villanueva, R.; Heredia, S.; Alejo, I.; Villaceros, N.; Nombela, M.Á.; Caballero, I. Assessing the temporal stability of satellite-derived bathymetry in intertidal flats using Sentinel-2A/B/C. *Coast. Eng.* **2026**, *209*, 105043. <https://doi.org/10.1016/j.coastaleng.2026.105043>.
15. Stumpf, R.P.; Holderied, K.; Sinclair, M. Determination of water depth with high-resolution satellite imagery over variable bottom types. *Limnol. Oceanogr.* **2003**, *48*, 547–556. https://doi.org/10.4319/lo.2003.48.1_part_2.0547.

16. Lyzenga, D.R. Shallow-water bathymetry using combined lidar and passive multispectral scanner data. *Int. J. Remote Sens.* **1985**, *6*, 115–125. <https://doi.org/10.1080/01431168508948428>.
17. Pacheco, A.; Horta, J.; Loureiro, C.; Ferreira, Ó. Retrieval of nearshore bathymetry from Landsat 8 images: A tool for coastal monitoring in shallow waters. *Remote Sens. Environ.* **2015**, *159*, 102–116. <https://doi.org/10.1016/j.rse.2014.12.004>.
18. Eugenio, F.; Marcello, J.; Martin, J. High-resolution maps of bathymetry and benthic habitats in shallow-water environments using multispectral remote sensing imagery. *IEEE Trans. Geosci. Remote Sens.* **2015**, *53*, 3539–3549. <https://doi.org/10.1109/TGRS.2014.2377300>.
19. Poursanidis, D.; Traganos, D.; Reinartz, P.; Chrysoulakis, N. On the use of Sentinel-2 for coastal habitat mapping and satellite-derived bathymetry estimation using downscaled coastal aerosol band. *Int. J. Appl. Earth Obs. Geoinf.* **2019**, *80*, 58–70. <https://doi.org/10.1016/j.jag.2019.03.012>.
20. Casal, G.; Monteys, X.; Hedley, J.; Harris, P.; Cahalane, C.; McCarthy, T. Assessment of empirical algorithms for bathymetry extraction using Sentinel-2 data. *Int. J. Remote Sens.* **2019**, *40*, 2855–2879. <https://doi.org/10.1080/01431161.2018.1533660>.
21. Frugier, S.; Almar, R.; Bergsma, E.W.J.; Bak, S.A. Standalone color-based bathymetry over 10 years at Duck (NC, USA) from optical satellite imagery and wave breaking analysis. *Coast. Eng.* **2026**, *203*, 104855. <https://doi.org/10.1016/j.coastaleng.2025.104855>.
22. Li, J.; Roy, D.P. A global analysis of Sentinel-2A, Sentinel-2B and Landsat-8 data revisit intervals and implications for terrestrial monitoring. *Remote Sens.* **2017**, *9*, 902. <https://doi.org/10.3390/rs9090902>.
23. Frugier, S.; Almar, R.; Bergsma, E.W.J.; Gran, M.; Ruessink, G. Multi-site assessment of microtidal wave-dominated active beach state and morpho-sedimentary parameters using optical satellite imagery. *Sci. Rep.* **2026**, *16*, 10949. <https://doi.org/10.1038/s41598-026-45638-3>.
24. Ashphaq, M.; Srivastava, P.K.; Mitra, D. Evaluation of satellite-derived bathymetry from high and medium-resolution sensors using empirical methods. *Remote Sens.* **2022**, *14*, 772. <https://doi.org/10.3390/rs14030772>.
25. Traganos, D.; Poursanidis, D.; Aggarwal, B.; Chrysoulakis, N.; Reinartz, P. Estimating satellite-derived bathymetry (SDB) with the Google Earth Engine and Sentinel-2. *Remote Sens.* **2018**, *10*, 859. <https://doi.org/10.3390/rs10060859>.
26. Duan, Z.; Cheng, L.; Mao, Q.; Song, Y.; Zhou, X.; Li, M.; Gong, J. MIWC: A multi-temporal image weighted composition method for satellite-derived bathymetry in shallow waters. *ISPRS J. Photogramm. Remote Sens.* **2024**, *218*, 430–445. <https://doi.org/10.1016/j.isprsjprs.2024.10.009>.
27. Lubin, D.; Li, W.; Dustan, P.; Mazel, C.H.; Stamnes, K. Spectral signatures of coral reefs: Features from space. *Remote Sens. Environ.* **2001**, *75*, 127–137. [https://doi.org/10.1016/S0034-4257\(00\)00161-9](https://doi.org/10.1016/S0034-4257(00)00161-9).
28. Zhu, J.; Hu, P.; Zhao, L.; Gao, L.; Qi, J.; Zhang, Y.; Wang, R. Determine the Stumpf 2003 model parameters for multispectral remote sensing shallow water bathymetry. *J. Coast. Res.* **2020**, *102*, 54–62. <https://doi.org/10.2112/SI102-007.1>.
29. Casal, G.; Hedley, J.D.; Monteys, X.; Harris, P.; Cahalane, C.; McCarthy, T. Satellite-derived bathymetry in optically complex waters using a model inversion approach and Sentinel-2 data. *Estuar. Coast. Shelf Sci.* **2020**, *241*, 106814. <https://doi.org/10.1016/j.ecss.2020.106814>.
30. Eugenio, F.; Marcello, J.; Mederos-Barrera, A.; Marques, F. High-resolution satellite bathymetry mapping: Regression and machine learning-based approaches. *IEEE Trans. Geosci. Remote Sens.* **2022**, *60*, 1–14. <https://doi.org/10.1109/TGRS.2021.3135462>.
31. Caballero, I.; Stumpf, R.P. Retrieval of nearshore bathymetry from Sentinel-2A and 2B satellites in South Florida coastal waters. *Estuar. Coast. Shelf Sci.* **2019**, *226*, 106277. <https://doi.org/10.1016/j.ecss.2019.106277>.
32. National Oceanic and Atmospheric Administration. Coastal LiDAR Point Cloud and DEM Datasets—NOAA Digital Coast. Available online: <https://noaa-nos-coastal-lidar-pds.s3.amazonaws.com/laz/index.html> (accessed on 26 May 2026).
33. National Ocean Service. *Do the Great Lakes Have Tides?* National Oceanic and Atmospheric Administration: Washington, DC, USA, 2024. Available online: <https://oceanservice.noaa.gov/facts/gltides.html> (accessed on 26 May 2026).
34. Geological Survey Ireland; Marine Institute of Ireland. INFOMAR Seabed Data Viewer. Available online: <https://experience.arcgis.com/experience/3f2815ec89e745d2b65630429d06385c/> (accessed on 26 May 2026).
35. Institut Cartogràfic i Geològic de Catalunya. Batimetria—ICGC. Available online: <https://www.icgc.cat/ca/Geoinformacio-imapes/Dades-i-productes/Elevacions/Batimetria> (accessed on 26 May 2026).
36. Àrea Metropolitana de Barcelona. Geoportal de Cartografia. Available online: <https://geoportalcartografia.amb.cat/AppGeoportalCartografia2/?locale=ca> (accessed on 26 May 2026).
37. Vanhellemont, Q.; Ruddick, K. ACOLITE for Sentinel-2: Aquatic applications of MSI imagery. In *Proceedings of the 2016 ESA Living Planet Symposium*; ESA SP-740; European Space Agency: Paris, France, 2016.

38. Caballero, I.; Román, A.; Tovar-Sánchez, A.; Navarro, G. Water quality monitoring with Sentinel-2 and Landsat-8 satellites during the 2021 volcanic eruption in La Palma (Canary Islands). *Sci. Total Environ.* **2022**, *822*, 153433. <https://doi.org/10.1016/j.scitotenv.2022.153433>.
39. Nechad, B.; Dogliotti, A.; Ruddick, K.; Doxaran, D. Particulate backscattering and suspended matter concentration retrieval from remote-sensed turbidity in various coastal and riverine turbid waters. In *Proceedings of the Living Planet Symposium 2016*; ESA SP-740; European Space Agency: Paris, France, 2016; p. 419.
40. van der Woerd, H.J.; Wernand, M.R. Hue-angle product for low to medium spatial resolution optical satellite sensors. *Remote Sens.* **2018**, *10*, 180. <https://doi.org/10.3390/rs10020180>.
41. Pitarch, J.; Vanhellemont, Q. The QAA-RGB: A universal three-band absorption and backscattering retrieval algorithm for high resolution satellite sensors. Development and implementation in ACOLITE. *Remote Sens. Environ.* **2021**, *265*, 112667. <https://doi.org/10.1016/j.rse.2021.112667>.
42. Aragonés, L.; Pagán, J.I.; López, I.; Serra, J.C. Depth of closure: New calculation method based on sediment data. *Int. J. Sediment Res.* **2017**, *33*, 198–207. <https://doi.org/10.1016/j.ijsrc.2017.12.001>.
43. Cannizzaro, J.P.; Carder, K.L. Estimating chlorophyll a concentrations from remote-sensing reflectance in optically shallow waters. *Remote Sens. Environ.* **2006**, *101*, 13–24. <https://doi.org/10.1016/j.rse.2005.12.002>.
44. Katlane, R.; Doxaran, D.; ElKilani, B.; Trabelsi, C. Remote sensing of turbidity in optically shallow waters using Sentinel-2 MSI and PRISMA satellite data. *PFG J. Photogramm. Remote Sens. Geoinf. Sci.* **2024**, *92*, 431–447. <https://doi.org/10.1007/s41064-023-00257-9>.
45. Carlson, R.E. A trophic state index for lakes. *Limnol. Oceanogr.* **1977**, *22*, 361–369. <https://doi.org/10.4319/lo.1977.22.2.0361>.
46. North American Lake Management Society. Trophic State Equations—The Robert Carlson Secchi Dip-In. Available online: <https://www.nalms.org/secchidipin/monitoring-methods/trophic-state-equations> (accessed on 26 May 2026).
47. Nechad, B.; Ruddick, K.; Neukermans, G. Calibration and validation of a generic multisensor algorithm for mapping of turbidity in coastal waters. In *Proceedings of SPIE Europe Remote Sensing*; International Society for Optics and Photonics: Bellingham, WA, USA, 2009; Volume 7473, 74730H. <https://doi.org/10.1117/12.830700>.
48. Lee, Z.P.; Carder, K.L.; Arnone, R.A. Deriving inherent optical properties from water color: A multiband quasi-analytical algorithm for optically deep waters. *Appl. Opt.* **2002**, *41*, 5755–5772. <https://doi.org/10.1364/AO.41.005755>.

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.