Abstract: This article presents a new, in-house developed method of selecting a variant of the transport system in the underground of a mine, using multi-variant decision support, taking into account the specificity of an underground mining plant. The implementation of the method should facilitate the selection of the most optimal transport system, ensuring continuity and the lowest operating costs. Seven functional criteria are proposed herein, which may be of a stimulant or destimulant nature. Each criterion was assigned a specific scoring weight reflecting the level of significance, with the sum of the weights being 100. The highest scores for the variants in the individual criteria go to those characterized by the following traits: the shortest transport time, the highest compatibility with the transport system already existing in the mine, transport routes with the greatest coverage communication, allow workers to be transported to the front of the excavation as quickly as possible, are most compatible with the existing transport systems in terms of the reinforcement and removal of longwalls, have a drive with the lowest operational hazard, have the least negative impact on the atmosphere of workings (exhaust gas emissions), and those that will ensure the best functioning of transport in emergency situations involving risk or uncertainty. For each criterion, a scoring formula based on specific parameters is provided. The method was used to select the optimal variant of the transport system in one of the mines, where four long walls were cut and four long galleries were drilled. Out of ten variants, the variant that should ensure the highest degree of reliable transport operation and continuity of operation has been determined using seven usability criteria.

Keywords: hard coal mine; underground transport; multi-variant decision support; optimization of transport solutions in hard coal mining

1. Introduction

A number of works are carried out in underground excavations, where their implementation requires the provision of various materials, parts of machines, or devices. Due to the geological and mining conditions in Polish mines, the total length of excavation tunnels is many times greater than in mines located in other countries. Hence, the degree of complexity of transport systems in Polish mines is much more complex than in foreign mines. For this reason, it is necessary to use sophisticated optimization methods to design transport systems. The need to deliver a specific material, in a specific quantity, and in a specific place is a necessity from the point of view of the rational course of the mining production process [1]. Therefore, as a result of designing the transport sub-system, there are several variants that meet the technical requirements and each of them is therefore possible to implement. The problem then arises when attempting to choose the best variant and generates the following question: on what basis can we conclude that a specific solution will be the best?
During a review of the scientific literature relating to the rules for designing transport systems in underground hard coal mines, a research gap was noticed regarding the mutual relations between exploitation processes and the functioning of transport systems, especially in terms of determining the quality factors of contemporary, modern transport systems. In terms of this situation, there are no clear guidelines for the design of material transport systems in underground mines, and standard tools for the selection of means of transport have not been implemented in mines. The article describes an attempt to solve this problem.

Although the literature contains examples of the use of multi-criteria analysis to optimize the operation of mines, they do not refer to modern, complex transport systems. An example is the 2009 research by Naghadehi et al. [2] relating to bauxite mines in Iran. The approach developed by the authors allowed for the selection of the best mining methods and the development of their ranking, taking into account technological, economic, and safety criteria for the mining crews.

Similar studies were also conducted by Saki et al. (2020) [3], using the example of a zinc mine in Iran. The authors included as many as 50 criteria, namely, geomechanical, geometrical, technical, economic, environmental, and social. Then, based on expert opinions and the integration of results, they determined that the best mining method for the examined mine would be cut-and-fill mining.

Multi-criteria methods in mining are also used to assess the risk of underground construction. Haghshenas et al. (2022) [4], in investing threats in drilling underground mining tunnels, take into account twelve factors at that time, i.e., machinery failure, lack of machinery, design mistakes, lack of experience of contractor, squeezing, instability of wall, water inflow, face tunnel instability, swelling of rock, gas emission, construction delay, and changes of price. In risk assessment, in addition to multi-criteria analysis, the authors also use expert opinions.

Issues in this field also include publications on the transport of mining by-products and waste by Banghua et al. (2016) [5]; Caneda-Martínez et al. (2019) [6]; An et al. (2021) [7]; and Skubacz et al. (2017) [8]. The literature also discusses the issue of threats to human health and life related to transport in mines in the following: Burgess-Limerick [9] and Gautam et al. (2017) [10].

Accordingly, the literature and, above all, practice lack more detailed solutions relating to the individual aspects of the design and development of underground mines. Meanwhile, these are very complex, difficult, and risky tasks, resulting in serious consequences for health and life. Design errors can also be a source of serious financial losses. For these reasons, the authors of this study took up the topic of selecting and optimizing transport systems in underground mining.

Due to the specificity of the underground mining of hard coal deposits and the uncertainty and unpredictability involved therein, the selection of the appropriate design solution is influenced by numerous different dependencies. Therefore, the authors proposed a method for selecting systems and means of underground material transport based on multi-criteria decision theory methods. This article presents the assumptions of multi-criteria decision support and the procedure to enable the selection of the “best” variant, i.e., optimal for the considered criteria. Thus, the authors proposed the use of seven criteria called “usability criteria”, assessed using an established scoring method. Their selection was made based on a thorough analysis of the functioning of transport systems in 13 hard coal mines operating in the Polish Upper Silesian Coal Basin. They all use means of self-propelled transport: track railways with electric and diesel locomotives, suspended railways, and floor railways. In all mines, transport systems are used (to varying extents) for the regular transport of “typical” materials used in the working faces, heavy loads (in particular during reinforcement and decommissioning works), and for the transport of crew. The main subjects of the research were the systems of regular material transport, while issues related to the transport of people and heavy loads were considered in the areas of
technological and organizational interconnection, with the possibility of the simultaneous use of the same transport systems.

The originality of the proposed research results from two key circumstances. The first is the lack of research on complex underground transport systems in mines with extensive and complex corridor excavations. The second one concerns the inclusion in the designed and optimized transport system of the compilation of all means of transport of materials in connection with the technological and organizational transport of people. Systematizing and universalizing the issues of the underground transport of materials proposed in the article is therefore a valuable alternative to the intuitive solutions used so far.

The further structure of the article is subordinated to the implementation of defined research goals. Following their introduction, the adopted research methodology is presented, taking into account the multi-criteria analysis used to integrate the optimization criteria and the original criteria for assessing the material transport system in hard coal mines. This methodology constitutes an essential part of this scientific communication and is, in accordance with the essence of the scientific publication defined in this way, an introduction to in-depth empirical research, which the authors intend to continue in the near future. The entire methodological considerations end with a summary containing the most important conclusions, a description of the advantages of the proposed method over the solutions used so far, as well as research limitations and directions for further research.

2. Materials and Methods

2.1. Multi-Criteria Analysis as a Basis for the Selection and Optimization of the Transport System

Multi-criteria decision support is a scientific discipline derived from operation research called multi-criteria analysis or multi-criteria decision making [11,12]. In 1951, T. Koopmans proposed the notion of the so-called “causative agent”, i.e., a non-dominated solution being the grounds of the multi-criteria decision support theory [13]. P. Vincke interpreted this issue as solving complex decision problems where multiple, often antithetic perspectives must be considered [14]. According to M. Zeleny, the multi-criteria decision support is defined as decision-making with respect to many criteria being independent objectives [15]. B. Roy defines decision support as an activity which, based on analytical models, helps find elements of answers to the questions posed by the decision-maker in the decision-making process [16]. When solving complex problems entailing numerous criteria, the standard approach to the “optimum” property becomes outdated as it is impossible to obtain optimum outcomes (solutions), i.e., the best ones from all perspectives simultaneously. Given these circumstances, it seems much more realistic to adopt a trade-off solution which considers both the decision-maker’s preferences and the profit and loss analysis [17,18].

By analyzing the criteria represented by the decision-maker’s aspirations, it can be concluded that he wants a solution that will meet his requirements to the highest extent. We then talk about usability—a term that was first introduced by H. H. Gossen in the 19th century [19]. One of the methods of solving multi-criteria problems taking into account the usefulness of a given variant is the use of a multi-attribute utility function, which involves aggregating various criteria into one maximized utility function, wherefrom a global utility criterion is then obtained [14].

The multi-attribute utility theory is based on the assumption that all the variants of a given problem are comparable, meaning that the decision-maker will always prefer one of every pair of variants or will consider them equivalent [20]. Assessment measures corresponding to the specified goal as precisely as possible must be preferred. To ensure comparisons between solutions, they should also be measurable (if they cannot be expressed by quantities, substitute indicators should be used). The decision-maker often finds some criteria more important than others; in such a case, a popular method of multi-criteria optimization is the weight method. It consists of reducing the problem to a one-criterion task by introducing the size of preferences (weights) for the objective function of each criterion [21]. At present, in particular for repeatable problems, the decision-maker is often
supported by a ready-made analytical tool, usually a computer program, which enables them to solve complex decision problems.

Multi-criteria decision support methods may be used to assess transport system solution variants [22,23]. The basic criteria for transport project assessment apply to three groups: functional, economic, and environmental. A different division of criteria is usually used in the case of urban and national transport, where the criteria are grouped as technical, environmental, marketing, and those related to spatial development [24]. Cost–benefit analysis is also often used in the evaluation of a transport system [25].

The mine transport system also has numerous assessment criteria which are sometimes contradictory. To solve the decision-making problem of selecting an optimum variant, it is advisable to use multi-criteria methods considering the specific nature of the mine. These methods can be used for many complex decision-making problems related to a mining plant (mine) [26,27]. For example, multi-criteria models for planning and controlling material needs in a mining enterprise are described in [28]—based on data from the IT material supply system, various models reflecting the actual material needs of mines were indicated.

In the developed method, a weight-scoring method was used to evaluate individual variants, in which a linear rating scale was adopted. The score for individual criteria is calculated in two ways. For criteria where the variants are described by specific, comparable values, e.g., time or length, the score is awarded by scaling (proportionally) the range of the parameter value from the highest to the lowest value, with the highest parameter value being ascribed the maximum score (weight) and the lowest one zero. The second method is based on the additional score connected with specific conditions. The additional points obtained by meeting specific conditions are summed up one by one, and then, just as in the first case, they are subject to proportional assessment (linear dependence).

Depending on the parameter, there are stimulants, in which the increased parameter value results in an increased score, and destimulants, in which the increased parameter value results in a decreased score. All usability criteria are assessed with increasing scores, i.e., the better the value within a given parameter according to a given criterion, the higher the score [29].

Then, for the stimulant:

\[ p_{kui} = (u_i - u_{\text{min}}) \times s_{uj} = (u_i - u_{\text{min}}) \times \frac{w_{kuj}}{u_{\text{max}} - u_{\text{min}}} \]  

(1)

and for the destimulant:

\[ p_{kui} = (u_{\text{max}} - u_i) \times s_{uj} = (u_{\text{max}} - u_i) \times \frac{w_{kuj}}{u_{\text{max}} - u_{\text{min}}} \]  

(2)

where

\( p_{kui} \) — score for “j” criterion in “i” variant [point U];
\( u_i \) — parameter value in “i” variant [value];
\( u_{\text{min}} \) — minimum parameter value for “j” criterion — \( u_{\text{min}} = \min (u_1, \ldots u_i, \ldots u_n) \);
\( u_{\text{max}} \) — maximum parameter value for “j” criterion — \( u_{\text{max}} = \max (u_1, \ldots u_i, \ldots u_n) \);
\( s_{ui} \) — basic parameter score in the \( u_{\text{min}} \)–\( u_{\text{max}} \) range for “j” criterion;
\( w_{kuj} \) — weight, maximum score for “j” criterion [point U].

Each criterion should be ascribed a specific weight of \( w_{kui} \geq 0 \) score, which the designer deems to reflect the significance level, in line with the following requirements:

\[ w_u = 100 \text{ points}; \ w_u > 50\% \ w_u \]  

(3)

\[ w_u = w_{\text{un}} + \sum_{j=1}^{n_{\text{ij}}} \ w_{uj} \]  

(4)
The rest of the article presents individual evaluation criteria along with the principles of their use.

2.2. KU1 Criterion—Lead Time of the Transport Task

The criterion is used to assess the lead time of the full transport cycle. It is assumed that there is some initial transport set ready to start a new task and that, to avoid "empty miles", the load will be carried also on the way back. In real circumstances, this is usually the case, although the load is not always collected from the unloading site. The complete cycle is composed of the initial handling, maneuver works, loading, load transport, unloading and loading in the destination, return, and all the activities required to be ready for subsequent transport:

\[ T_{zt} = t_{ot} + t_m + t_{z1} + t_{j1} + t_{r1} + t_{z2} + t_{j2} + t_{r2} \] (5)

\[ T_{zt} = t_{ot} + t_m + \frac{2 \times s_t}{60 \times v_u} + (1 + l) \times (t_z + t_r) \] (6)

where

- \( T_{zt} \) — transport task duration [min];
- \( t_{ot} \) — technical handling duration [min];
- \( t_m \) — maneuver work duration [min];
- \( t_{z1} \) — duration of loading at the dispatch station [min];
- \( t_{j1} \) — duration of transport to the collection station [min];
- \( t_{r1} \) — duration of unloading at the collection station [min];
- \( t_{z2} \) — duration of loading at the collection station [min];
- \( t_{j2} \) — duration of transport to the dispatch station (return) [min];
- \( t_{r2} \) — duration of unloading at the dispatch station (upon return) [min];
- \( s_t \) — length of the transport distance to the collection (dispatch) station [m];
- \( v_u \) — transport speed — \( v_u = \frac{s_t \times 60}{t} \) [m/s];
- \( l_p \) — payload use coefficient: \( 1 \geq l_p \geq 0 \) [1].

In the “i” variant, the length of the transport distance to the most distant collection point is \( s_{idi} = \max (s_{i1}, \ldots, s_{in}) \), and the transport distance length to the average collection point should be determined as a mode calculated based on the range of distance lengths to the collection point (if there is more than one mode, the one with the highest value should be selected):

\[ s_{tpi} = D_O (s_{i1}, \ldots, s_{in}) \] (7)

The transport duration to the mid-way and furthermost stations:

\[ t_{ztp} = t_{ot} + \frac{2s_{tpi}}{v_u} + (1 + l) \times (t_z + t_r) \] (8)

\[ t_{ztd} = t_{ot} + \frac{2s_{idi}}{v_u} + (1 + l) \times (t_z + t_r) \] (9)

where

- \( s_i \) — “i” distance to the collection point in the “i” variant [m];
- \( n \) — number of possible transport routes to the collection points [pc.].

Due to the possible large discrepancies between the transport distance to different collection points, it is advisable to calculate the transport duration to the point situated at an average distance from the basic dispatch station and to the most distant collection point. Moreover, it should be considered that the return usually takes place with the cargo not using the total payload, which results in a shorter loading and unloading duration. Such a case was considered by applying the payload use coefficient, the value of which is determined by the developer.
The summary of the score for the KU1 criterion, broken down into the transport duration to the point situated at an average distance (KU1A) and to the one at an extreme distance (KU1B), is presented in Table 1.

**Table 1. KU1 criterion—lead time of the transport task.**

<table>
<thead>
<tr>
<th>Variants</th>
<th>Transport Duration to the Collection Point Situated at an Average Distance [min]</th>
<th>Weight: $w_{ku1A}$ [Point U]</th>
<th>Transport Duration to the Collection Point Situated at an Extreme Distance [min]</th>
<th>Weight: $w_{ku1B}$ [Point U]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_1$</td>
<td>$t_{tp1}$</td>
<td>$P_{ku1A1}$</td>
<td>$t_{td1}$</td>
<td>$P_{ku1B1}$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$W_i$</td>
<td>$t_{tpi}$</td>
<td>$P_{ku1Ai}$</td>
<td>$t_{tdi}$</td>
<td>$P_{ku1Bi}$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$W_n$</td>
<td>$t_{tpn}$</td>
<td>$P_{ku1An}$</td>
<td>$t_{tdn}$</td>
<td>$P_{ku1Bn}$</td>
</tr>
</tbody>
</table>

Source: own work.

The value with the highest score in the criterion being a de-stimulant is the shortest transport duration, as this variant obtains the maximum score. The other variants obtain proportional scores:

$$P_{ku1Ai} = \left(\frac{t_{tp max} - t_{tp i}}{t_{tp max} - t_{tp min}}\right) \times \frac{w_{ku1A}}{t_{tp max} - t_{tp min}}$$  \hspace{1cm} (10)

$$P_{ku1Bi} = \left(\frac{t_{td max} - t_{td i}}{t_{td max} - t_{td min}}\right) \times \frac{w_{ku1B}}{t_{td max} - t_{td min}}$$  \hspace{1cm} (11)

where
- \(P_{ku1Ai}\)—score for KU1A criterion in “i” variant [point U];
- \(P_{ku1Bi}\)—score for KU1B criterion in “i” variant [point U];
- \(w_{ku1A}\)—weight, maximum score for KU1A criterion [point U];
- \(w_{ku1B}\)—weight, maximum score for KU1B criterion [point U];
- \(t_{tp max}, t_{td max}\)—the longest transport duration to the point situated at an average (extreme) distance [min];
- \(t_{tp min}, t_{td min}\)—the shortest transport duration to the point situated at an average (extreme) distance [min];
- \(t_{tp i}, t_{td i}\)—the transport duration to the point situated at an average (extreme) distance in “i” variant [min].

2.3. KU2 Criterion—Compatibility of Transport Systems

This criterion is used to assess the conformity of the designed subsystem with the existing transport system in a mine, referring both to the trackbed and the rolling stock.

With respect to the trackbed, it is assessed whether trains or multiple units can run on it. If applicable, the second condition should be considered, i.e., the permissible trackbed carrying capacity. A higher score will be awarded to variants enabling the trouble-free crossing of multiple units in the existing and designed transport system. The travel possibility is conditional on the trackbed carrying capacity, expressed

- For suspended rail in [kJ];
- For underground rail and rack railway in axle load [kJ/axle].

With respect to the rolling stock, the criterion refers to the conformity of the tractor units or engines (drive type). If it is fulfilled, the subsequent step is to assess the type conformity (of the manufacturer).

Regarding the staff’s experience relating to the operation, maintenance, and repairs, and the reduction in the number of the required spare parts, the compatibility criterion translates into a higher score for the variant where the selected means of transport are
identical to the ones already possessed. It should be stressed that awarding a high weight to this criterion may hamper the implementation of new engineering solutions.

The summary of the KU2 criterion score and additional score referring to the trackbed and the rolling stock is presented in Tables 2 and 3.

Table 2. KU2 criterion—compatibility of transport systems.

<table>
<thead>
<tr>
<th>Variants</th>
<th>Trackbed and Rolling Stock</th>
<th>Weight: w_{ku2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>W_1</td>
<td>R_{ck1}</td>
<td>P_{ku21}</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>W_i</td>
<td>R_{cki}</td>
<td>P_{ku2i}</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>W_n</td>
<td>R_{ckn}</td>
<td>P_{ku2n}</td>
</tr>
</tbody>
</table>

Source: own work.

Table 3. Additional score for KU2 criterion.

<table>
<thead>
<tr>
<th>Scope</th>
<th>Additional Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_{k1}—track passability</td>
<td>2</td>
</tr>
<tr>
<td>R_{k2}—track carrying capacity</td>
<td>1</td>
</tr>
<tr>
<td>R_{k3}—rolling stock kind conformity</td>
<td>2</td>
</tr>
<tr>
<td>R_{k4}—rolling stock type conformity</td>
<td>1</td>
</tr>
</tbody>
</table>

Source: own work.

Individual variants are assessed based on the total additional score obtained as a result of assessing individual “R_{ki}” ranges. The highest score will be awarded to the variant obtaining the highest score in the additional assessment (this is a stimulant). The other variants are assessed proportionately:

\[
p_{ku2i} = \frac{(R_{cki} - R_{k \text{ min}}) \times w_{ku2}}{R_{k \text{ max}} - R_{k \text{ min}}}\tag{12}
\]

where

- \(p_{ku2i}\): score for KU2 criterion in “i” variant [point U];
- \(w_{ku2}\): weight, maximum score for KU2 criterion [point U];
- \(R_{cki}\): additional score for “i” variant [D];
- \(R_{k \text{ max}}\): maximum additional score, \(\max (R_{ck1}, \ldots, R_{cki}, \ldots, R_{ckn})\) [point D];
- \(R_{k \text{ min}}\): minimum additional score, \(\min (R_{ck1}, \ldots, R_{cki}, \ldots, R_{ckn})\) [point D].

2.4. KU3 Criterion—Continuous Communication

To be able to monitor and control individual processes, the cutting-edge logistic systems are monitored using information systems allowing for the recording of the vehicle circulation and material flow in real time [30]. The underground rail supplied by a slide busbar has long used analogue radio communication, ensuring continuous communication of the train staff with the dispatcher. Today’s transport systems with combustion engines or battery-driven means of transport use wireless solutions based on leaky feeders or wireless network technology, e.g., according to IEEE 802.11 (WLAN) [31]. Due to the inability to use electromagnetic wave propagation through the rock mass, the range of radio connection is limited to below one hundred meters depending on the local propagation conditions [32], meaning that the “wireless communication”, as a matter of fact, refers to several meters to the leaky feeder or up to one hundred meters for serial access points or directional antenna. Digital communication systems enable the sending of expanded data packages which may be used to develop traffic control, security, and material shipment management processes, ensuring not only voice communication but also the ability to determine the
location of the means of transport, identify individual transport units and the delivery
destinations allocated to them, the description of the transported material, etc. [31]. This
corresponds to the definition by M. Christopher concerning LIS, i.e., logistics information
system, considering four functions, including planning, coordination, monitoring, and
logistic process control [33]. Such a system is particularly desirable for underground
transport systems, where the transport needs are often highly changeable.

The criterion assesses whether the communication, construed as the ability to send
messages between the dispatcher and the multiple unit or train operators during travel, is
guaranteed. This enables the monitoring of the transport task performance in real time, as
well as the rapid change or implementation of a new task in sudden emergency situations.

The summary of KU3 criterion score is presented in Table 4.

Table 4. KU3 criterion—continuous communication.

<table>
<thead>
<tr>
<th>Variants</th>
<th>Continuous Communication Coverage [%]</th>
<th>Weight: $w_{ku3}$ [Point U]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_1$</td>
<td>$l_1$</td>
<td>$p_{ku3 \cdot 1}$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$W_i$</td>
<td>$l_i$</td>
<td>$p_{ku3 \cdot i}$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$W_n$</td>
<td>$l_n$</td>
<td>$p_{ku3 \cdot n}$</td>
</tr>
</tbody>
</table>

Source: own work.

The parameter value allocated to individual variants is expressed as a percentage of
the total length of the route sections covered by the communication to the total route length:

$$L_{ti} = \frac{\sum_{j=1}^{n_l} l_{tj}}{L_{ti}} \times 100\%$$  \hspace{1cm} (13)

where

- $L_{ti}$—continuous communication coverage of transport routes in “$i$” variant [%];
- $l_{tj}$—”$j$” section of transport routes with continuous communication [m];
- $n_l$—number of sections with continuous communication [pc.];
- $L_{ti}$—total length of the transport routes in “$i$” variant [m].

The highest score will be awarded to the variant with the highest communication
coverage of the transport routes (this is a stimulant). The other variants are assessed
proportionately:

$$p_{ku3 \cdot i} = \frac{(l_{t_i} - l_{t_{\min}})}{l_{t_{\max}} - l_{t_{\min}}} \times w_{ku3}$$  \hspace{1cm} (14)

where

- $p_{ku3 \cdot i}$—score for KU3 criterion in “$i$” variant [point U];
- $w_{ku3}$—weight, maximum score for KU3 criterion [point U];
- $l_{t_{\max}}$—maximum continuous communication coverage of transport routes [%];
- $l_{t_{\min}}$—minimum continuous communication coverage of transport routes [%];
- $l_{t_i}$—continuous communication coverage of transport routes in “$i$” variant [%].

2.5. KU4 Criterion—Co-Use with Other Transport Tasks

This criterion considers using the transport subsystem to transport passengers or
heavy-duty transport during the reinforcement and liquidation of the walls. This is consid-
ered solely when it is assumed that such a (co-use) situation will take place.

For passenger transport, the variant assessment is based on the transport duration
(just the travel duration is considered):

$$t_{jl} = \frac{s_{jl}}{v_{jl}}$$  \hspace{1cm} (15)
where

\( s_j \) — distance of the staff transport [m];

\( v_{jl} \) — useful speed of passenger transport [m/s];

\( s_{jl \ p} \) — passenger station at the average distance [m];

\( s_{jl \ d} \) — furthermost (extreme) passenger station [m];

\( t_{jl \ p} \) — transport duration to the passenger station at the average distance —

\[ t_{jl \ p} = \frac{s_{jl \ p}}{v_{jl}} \times 60 \text{ [min]} \]

\( t_{jl \ d} \) — transport duration to the passenger station at the extreme distance (furthermost) —

\[ t_{jl \ d} = \frac{s_{jl \ d}}{v_{jl}} \text{ [min]} \].

In the “i” variant, the distance of the passenger transport to the furthermost passenger station is

\[ s_{jl \ di} = \max (s_{1i}, \ldots, s_{ni}) \].

In the “i” variant, the distance to the passenger station at the average distance should be determined as a mode determined from the range of passenger transport distances to the passenger station (if there is more than one mode, select the one with the highest value):

\[ s_{jl \ p} = D_O (s_{1i}, \ldots, s_{ni}) \]  \hspace{1cm} (16)

The summary of KU4A criterion score is presented in Table 5.

### Table 5. KU4A criterion — using for passenger transport.

<table>
<thead>
<tr>
<th>Variants</th>
<th>Transport Duration to the Passenger Station at an Average Distance [min]</th>
<th>Weight: ( w_{ku4A1} ) [Point U]</th>
<th>Transport Duration to the Passenger Station at an Extreme Distance [min]</th>
<th>Weight: ( w_{ku4A2} ) [Point U]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_1 )</td>
<td>( t_{jl \ p 1} )</td>
<td>( P_{ku4A1 \ 1} )</td>
<td>( t_{jl \ d 1} )</td>
<td>( P_{ku4A2 \ 1} )</td>
</tr>
<tr>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( W_i )</td>
<td>( t_{jl \ p i} )</td>
<td>( P_{ku4A1 \ i} )</td>
<td>( t_{jl \ d i} )</td>
<td>( P_{ku4A2 \ i} )</td>
</tr>
<tr>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( W_0 )</td>
<td>( t_{jl \ p n} )</td>
<td>( P_{ku4A1 \ n} )</td>
<td>( t_{jl \ d n} )</td>
<td>( P_{ku4A2 \ n} )</td>
</tr>
</tbody>
</table>

Source: own work.

The highest score will be awarded to the variant with the shortest travel duration (this is a stimulant). The other variants are assessed proportionately.

If there are multiple passenger stations, for this criterion it is advisable to calculate values for the point at an average and at an extreme distance (just as for the KU1 criterion):

\[ P_{ku4A1 \ i} = \left( \frac{t_{jl \ p \ max} - t_{jl \ p i}}{t_{jl \ p \ max} - t_{jl \ p \ min}} \right) \times \frac{w_{ku4A1}}{t_{jl \ p \ max} - t_{jl \ p \ min}} \]  \hspace{1cm} (17)

\[ P_{ku4A2 \ i} = \left( \frac{t_{jl \ d \ max} - t_{jl \ d i}}{t_{jl \ d \ max} - t_{jl \ d \ min}} \right) \times \frac{w_{ku4A2}}{t_{jl \ d \ max} - t_{jl \ d \ min}} \]  \hspace{1cm} (18)

where

\( P_{ku4A1 \ i} \) — score for KU4A1 criterion in “i” variant [point U];

\( P_{ku4A2 \ i} \) — score for KU4A2 criterion in “i” variant [point U];

\( w_{ku4A1} \) — weight, maximum score for KU4A1 criterion [point U];

\( w_{ku4A2} \) — weight, maximum score for KU4A2 criterion [point U];

\( t_{jl \ p \ max} \) — longest passenger transport duration to the station at an average distance [min];

\( t_{jl \ d \ max} \) — longest passenger transport duration to the station at an extreme distance [min];

\( t_{jl \ p \ min} \) — shortest passenger transport duration to the station at an average distance [min];

\( t_{jl \ d \ min} \) — shortest passenger transport duration to the station at an extreme distance [min];

\( t_{jl \ p \ i} \) — passenger transport duration to the station at an average distance in “i” variant [min];
- passenger transport duration to the station at an extreme distance [min].

In this case, the possibility of using the transport subsystem to transport heavy machine and equipment components with no need to dismount them or no need to dismount them in the scope higher than the one assumed in the mine, including but not limited to relating to wall reinforcement and liquidation, is assessed. The assessment is performed by awarding an additional score (similar to KU2 criterion). The score is obtained relating to

- The minimum carrying capacity of the trackbed enabling to transport concentrated load;
- No need to expand the loading gauge. It is met if the selected means of transport enable the transport of devices in the assumed dismounting state.

The variant awarded the highest additional score obtains the highest assessment and the one meeting the fewest conditions obtains the lowest. Intermediate variants are assessed proportionately.

The summary of the KU4B criterion score and additional score referring to the trackbed and the rolling stock is presented in Tables 6 and 7.

**Table 6. KU4B criterion—using for reinforcement and liquidation.**

<table>
<thead>
<tr>
<th>Variants</th>
<th>Trackbed and Rolling Stock [Point D]</th>
<th>Weight: w_{ku4B} [Point U]</th>
</tr>
</thead>
<tbody>
<tr>
<td>W_1</td>
<td>R_{czl 1}</td>
<td>P_{ku4B 1}</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>W_n</td>
<td>R_{czl n}</td>
<td>P_{ku4B n}</td>
</tr>
</tbody>
</table>

Source: own work.

**Table 7. Additional score for KU4B criterion.**

<table>
<thead>
<tr>
<th>Condition</th>
<th>Additional Score [Point D]</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_{zl1}—track carrying capacity</td>
<td>1</td>
</tr>
<tr>
<td>R_{zl2}—loading gauge</td>
<td>2</td>
</tr>
<tr>
<td>R_{zl3}—rolling stock use</td>
<td>1</td>
</tr>
</tbody>
</table>

Source: own work.

The individual variants are assessed based on the total additional score obtained in individual “R_{zl}” ranges. The highest score will be awarded to the variant obtaining the highest score in the additional assessment (this is a stimulant). The other variants are assessed proportionately:

\[ P_{ku4B i} = (R_{zl i} - R_{zl min}) \star \frac{w_{ku4B}}{R_{zl max} - R_{zl min}} \]  \hspace{1cm} (19)

where

- \( P_{ku4B i} \) —score for KU4 criterion in “i” variant [point U];
- \( w_{ku4B} \) —weight, maximum score for KU4B criterion [point U];
- \( R_{zl i} \) —additional score for “i” variant [point D];
- \( R_{zl max} \) —maximum additional score, max (R_{zl1}, ..., R_{zln}) [point D];
- \( R_{zl min} \) —minimum additional score, min (R_{zl1}, ..., R_{zln}) [point D].

2.6. **KU5 Criterion—Safety**

This criterion assesses the potential threat level resulting from the adopted solution. The classification refers to the drive type, with the highest score awarded to self-propelled
vehicles. A lower score refers to vehicles powered by means of a trailing cable or conductor or by a slide busbar, and the lowest to the rope traction (Table 8).

Table 8. Additional score for KU5 criterion.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Additional Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rb1—self-propelled</td>
<td>4</td>
</tr>
<tr>
<td>Rb2—wired propulsion—slide busbar</td>
<td>3</td>
</tr>
<tr>
<td>Rb3—wired drive—trailing conductor/hose *</td>
<td>2</td>
</tr>
<tr>
<td>Rb4—rope traction</td>
<td>1</td>
</tr>
</tbody>
</table>

* Including a pneumatic and hydraulic drive. Source: own work.

The additional score is determined based on the formula:

$$R_{cbi} = \frac{\sum_{i=1}^{n_{ut}} (R_{b1}, R_{b2}, R_{b4})}{n_{ut}}$$  \hspace{1cm} (20)

where

- $R_{cbi}$—KU5 criterion additional score for “i” variant [point D];
- $n_{ut}$—number of transport system types (differing in terms of drives) [pc.].

The formula considers the fact that a transport subsystem may be configured from several transport systems with different drives. As in the previous criteria with the additional score, the highest assessment will be given to the variant with the highest score (this is a stimulant):

$$p_{ku5 i} = (R_{cb i} - R_{b \ min}) \ast \frac{w_{ku5}}{R_{b \ max} - R_{b \ min}}$$  \hspace{1cm} (21)

where

- $p_{ku5 i}$—score for KU5 criterion in “i” variant [point U];
- $w_{ku5}$—weight, maximum score for KU5 criterion [point U];
- $R_{b \ max}$—maximum additional score [point D];
- $R_{b \ min}$—minimum additional score [point D];
- $R_{cbi}$—KU5 criterion additional score for “i” variant [point D].

2.7. KU6 Criterion—Nuisance

This is used to assess the adverse impact of the means of transport on the air in the mine workings, including but not limited to combustion engines, which emit gas directly to the mine air and generate higher waste heat than the electrical drives. Each variant is ascribed a total power of engines installed in all selected combustion tractors and engines:

$$p_{ku6 i} = (P_{st \ max} - P_{st \ i}) \ast \frac{w_{ku6}}{P_{st \ max} - P_{st \ min}}$$  \hspace{1cm} (22)

where

- $p_{ku6 i}$—score for KU6 criterion in “i” variant [point U];
- $w_{ku6}$—weight, maximum score for KU6 criterion [point U];
- $P_{st \ max}$, $P_{st \ min}$—maximum (minimum) total power of combustion engines [kW];
- $P_{st \ i}$—total power of combustion engines in the means of transport selected in “i” variant [point U].

The variant with the lowest total combustion engine power obtains the highest score and the one with the highest total power obtains the lowest score (this is a destimulant). The other variants are assessed proportionately.
2.8. KU7 Criterion—Operation in Circumstances Exceeding the Initial Assumptions

The basic task of the underground transport system is the reliable delivery of the required materials to the faces and indicated places in the mine workings. Due to the geological and mining conditions and existing natural threats, the underground operation may entail transport needs different to the assumed design assumption [34]. Such circumstances are termed “exceeding the initial assumptions”.

Two cases will be assumed, including one relating to the changed number of collection points and the other relating to the changed number of available means of transport (tractors). In both cases, the adverse impact will be measured by the number of non-delivered transport units which threatens continuous production.

In the first case, we may consider the following:

- The reduction in the collection point number—with a certain increase in the transport capacity.
- The increase in the collection point number—accompanied by the increased number of non-delivered transport units.
- An additional assumption is the unchanged number of multiple units and the transport cycle duration.

The number of transport units non-delivered to every additional collection point must be determined by simulating the addition of a collection point with an average material consumption when compared to the base model. An assumption was made that the quantity of materials delivered to the additional collection point will be equal to the arithmetic mean of the quantity of materials delivered to base collection points:

\[ Z_{pd} = \frac{\sum_{i=1}^{n_{po}} z_{pi}}{n_{po}} \]  
(23)

where

- \( Z_{pd} \) — material consumption of an additional collection point [pc., t.u./change];
- \( n_{po} \) — base number of collection points [pc.];
- \( z_{pi} \) — material consumption of “i” base collection point [pc., t.u./change].

The second case refers to the multiple unit tractors. The circumstances exceeding the initial assumptions, analogous to the discussed in the first case, may entail the following:

- The “extra” number of multiple units,
- The shortage of multiple units, e.g., resulting from a tractor failure.

The payload of the additional multiple units is calculated as an arithmetic mean of the used units’ payload. In the shortage of units (e.g., due to a failure), the payload should be determined as a mode calculated based on the payload range of the selected tractors (if there is more than one mode, it is necessary to select the one with the highest value):

\[ L_d = \frac{\sum_{i=1}^{n_{zt}} l_{zi}}{n_{zt}} \]  
(24)

\[ L_d = D_o \left( l_{z1}, \ldots l_{zi}, \ldots l_{zn} \right) \]  
(25)

where

- \( L_d \) — payload of the additional multiple unit [pc. t.u.];
- \( n_{zt} \) — base number of multiple units [pc.];
- \( l_{zi} \) — payload of the “i” base transport unit [pc. t.u.].

KU7A Criterion—Operation in Uncertain Circumstances

If the designer is not able to determine the probability of the circumstances exceeding the initial assumptions, the further procedure will refer to uncertain circumstances. This means that the knowledge of the effects is limited, values beyond the decision-maker’s
control are random variables with an unknown probability distribution, and it is assumed that solely the value set of those variables is known (Table 9).

**Table 9.** Set of possible data in uncertainty conditions.

<table>
<thead>
<tr>
<th>Variants</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>...</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>b₁ (+)</td>
<td>b₂ (0)</td>
<td>b₃ (−)</td>
<td>...</td>
<td>bₘ (−)</td>
</tr>
<tr>
<td>Number of Additional Collection Points [pcs.]</td>
<td>w₁₁</td>
<td>w₁₂</td>
<td>w₁₃</td>
<td>w₁ ...</td>
<td>w₁ₘ</td>
</tr>
<tr>
<td>Number of Non-Delivered Transport Units [t.u.]</td>
<td>w₂₁</td>
<td>w₂₂</td>
<td>w₂₃</td>
<td>w₂ ...</td>
<td>w₂ₘ</td>
</tr>
</tbody>
</table>

Source: own work. Assumption: b₁ < b₂ < b₃ < ... < bₘ, where bₘ—number of additional collection points in “m” circumstances [pc.], w₁ₘ—number of non-delivered transport units in “i” variant in “m” circumstances [t.u.].

The selection of the optimum action variant is particularly important at that point, and requires making additional assumptions which may be expressed by different selection rules. In more complex circumstances, the optimum solution is selected based on multiple assessment criteria [35]. The four criteria are adopted, including the one by Wald, Hurwicz, Savage, and Laplace.

**Wald’s (maximin) criterion**

It is termed max (min) criterion as well. It stipulates as follows: “first the minimum value of e.g., a disbursement is determined and then a solution is indicated where its minimum is the highest”. Another (optimistic) approach is the max (max) criterion: “first the maximum value is determined, and then the solution is indicated where its maximum is the highest” [36].

In conditions exceeding the initial assumptions:

- **K₁A max. (min) criterion**—an optimum variant where the number of non-delivered transport units is the lowest in the circumstances with the highest shortage of those units, i.e., the variant being “the best in the worst circumstances”;
- **K₁B max. (max) criterion**—an optimum variant where the transport capacity increase will be the highest in the circumstances with the lowest number of collection points, i.e., the variant being “the best in the best circumstances” which may, e.g., allow for a reduction in the number of non-delivered transport units.

The above criteria consider extreme circumstances (Table 10).

**Table 10.** Minimum and maximum numbers of non-delivered transport units in individual variants.

<table>
<thead>
<tr>
<th>Variant</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>W₁</td>
<td>w₁₁</td>
<td>w₁ₘ</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Wᵢ</td>
<td>wᵢ₁</td>
<td>wᵢₘ</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Wᵢₙ</td>
<td>wᵢₙ₁</td>
<td>wᵢₙₘ</td>
</tr>
</tbody>
</table>

Source: own work.

Assumptions:

wᵢₘ = min (wᵢ₁₁, wᵢ₁₂, ... , wᵢₘₘ);
wᵢₘ = max (wᵢ₁₁, wᵢ₁₂, ... , wᵢₘₘ);
w₁ₘ < |w₁ₘ |;
w₁ₘ = max (w₁₁₁, w₁₂₂, ... , w₁ₘₘ);
w₁ₘ = max (w₁₂₁, w₁₂₂, ... , w₁ₘₘ).
The “i” variant is an optimum solution according to the max (min) criterion if the condition \( W_{OK1A} = w_{i \text{min}} \) is met. The “i” variant is an optimum solution according to the max (max) criterion if the condition \( W_{OK1B} = w_{i \text{max}} \) is met, where:

- \( W_{OK1A} \): optimum value in the max (min) criterion [t.u.];
- \( W_{OK1B} \): optimum value in the max (max) criterion [t.u.].

Hurwicz’s criterion

Pessimism and optimism are two extreme attitudes which is why it is worth considering the introduction of a value reflecting the designer’s pessimism (optimism). This additional value is called a caution coefficient. The higher its value is, the higher the risk aversion. It is a subjective belief of the designer that the most adverse circumstances may occur [35].

In the \(<0, 1>\) range, the designer specifies the value of the caution coefficient “h” corresponding to their subjective assessment of the probability with which the most adverse circumstances will occur, i.e., the highest number of non-delivered transport units. The probability of the occurrence of the most favorable circumstances, i.e., a reduction in the collection point number, is “1 – h”. An optimum solution is the variant where the number of non-delivered transport units is the lowest for the assumed caution coefficient. This criterion also considers extreme circumstances, but is examined in comparison to the subjective probability of their occurrence.

The anticipated number of non-delivered transport units \( H_i \) in individual variants is calculated based on the following formula [37]:

\[
H_i = w_{i \text{min}} \times h + w_{i \text{max}} \times (1 - h)
\]

where

- \( w_{i \text{min(max)}} \): minimum (maximum) number of non-delivered transport units in “i” variant [pc.];
- \( h \): caution coefficient.

According to Hurwicz’s criterion, the “i” variant is an optimum solution if the following condition is met: \( W_{OK2} = \max (H_1, \ldots, H_i, \ldots, H_n) \), with the assumption that \( w_{i \text{max}} < |w_{i \text{min}}| \), where \( W_{OK2} \): number of non-delivered units in the optimum solution according to Hurwicz’s criterion.

The influence of the “h” caution coefficient on the optimum variant choice is presented in the diagram of the dependence of the anticipated value of “H” on that coefficient, depicting the function behavior corresponding to each and every variant (Figure 1). The optimum variant is the upper envelope determined from the sections between the intercepts of subsequent variant functions. In those points, the anticipated value obtains the maximum for the subsequent values of the caution coefficient: the highest surplus of transport capacity, and for negative values it is the lowest number of non-delivered transport units.

To draw a function graph, it is necessary to adopt an assumption for the boundary conditions: \( h = 0 \) and \( h = 1 \). The functions \( H_1, \ldots, H_i, \ldots, H_n \) are linear. The caution coefficient value in their intercepts in the “i” and “m” variants is calculated based on the following formula:

\[
H_i (h) = H_m (h)
\]

Savage’s criterion

The basis for this criterion is the “sense of loss”. Its author proposes to minimize loss (minimize lost profits) when compared to the optimum decision which would be made if the future circumstances were known. A higher score is awarded to the variant with the lower relative loss, i.e., the difference between the highest number of non-delivered transport units and the surplus transport capacity obtained in the absence of a collection point. This criterion considers circumstances exceeding the initial assumptions broadly and, to a certain extent, enables the reduction of adverse circumstances.
The anticipated number of non-delivered transport units “Hi” in individual variants is calculated based on the following formula [37]:

\[ H_i = w_{i,1} \times h + w_{i,2} \times (1-h) \]  

(26)

where

\[ w_{i,\text{min}} (\text{max}) \]—minimum (maximum) number of non-delivered transport units in “i” variant [pc.];

\[ h \]—caution coefficient.

According to Hurwicz’s criterion, the “i” variant is an optimum solution if the following condition is met: \( W_{\text{OK2}} = \max (H_1, \ldots, H_i, \ldots, H_n) \), with the assumption that \( w_{i,\text{max}} < |w_{i,\text{min}}| \), where \( W_{\text{OK2}} \) —number of non-delivered units in the optimum solution according to Hurwicz’s criterion.

The influence of the “h” caution coefficient on the optimum variant choice is presented in the diagram of the dependence of the anticipated value of “H” on that coefficient, depicting the function behavior corresponding to each and every variant (Figure 1). The optimum variant is the upper envelope determined from the sections between the intercepts of subsequent variant functions. In those points, the anticipated value obtains the maximum for the subsequent values of the caution coefficient: the highest surplus of transport capacity, and for negative values it is the lowest number of non-delivered transport units.

To draw a function graph, it is necessary to adopt an assumption for the boundary conditions: \( h = 0 \) and \( h = 1 \). The functions \( H_1, \ldots H_i, \ldots H_n \) are linear. The caution coefficient value in their intercepts in the “i” and “m” variants is calculated based on the following formula:

\[ H_i (h) = H_m (h) \]  

(27)

Figure 1. Impact of the “h” caution coefficient value on the “N” number of transported transport units for three variant examples, i.e., \( w_A, w_B, w_C \). Intercepts—\( h_1, h_2, h_3, h_3 \), optimum solution—\( w_0 \). (A) is an optimum solution—\( w_A, w_B, w_C \) depending on “h” caution coefficient, (B) is an optimum solution \( w_A \) independent from “h” caution coefficient. Source: own study based on [38].

In this criterion, a relative loss is calculated in every circumstance, creating a matrix of items being the difference between the maximum loss and the loss in a given variant (Table 11).

Table 11. Relative loss matrix for the Savage criterion.

<table>
<thead>
<tr>
<th>Variant</th>
<th>Circumstances Exceeding the Initial Assumptions</th>
<th>Max [t.u.]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>( W_1 )</td>
<td>( s_{11} )</td>
<td>( s_{12} )</td>
</tr>
<tr>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>( w_i )</td>
<td>( s_{i1} )</td>
<td>( s_{i2} )</td>
</tr>
<tr>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>( W_n )</td>
<td>( s_{n1} )</td>
<td>( s_{n2} )</td>
</tr>
</tbody>
</table>

Source: own study based on [25].

According to Savage’s criterion, the “i” variant is an optimum solution if the following condition is met: \( W_{\text{OK3}} = \min (S_{\text{max} 1}, \ldots, S_{\text{max} i}, \ldots S_{\text{max} n}) \) when

\[
S_{\text{max} i} = \max (s_{i1}, s_{i2}, \ldots, s_{im}) \\
S_{i1} = \max (w_{i1}, \ldots, w_{i1}, \ldots, w_{i1}) = w_{i1}
\]  

(28)

where

\( s_{im} \)—number of non-delivered transport units in “i” variant in “m” circumstances [t.u.].

For the adopted assumptions, the maximum value \( S_{\text{max} 1} \) corresponds to the absence of the collection point.
Laplace’s criterion
It assumes that if there is no data or premises indicating the most probable circumstances, the principle of the equal probability of the occurrence of each of them is used [37]. The variant with the lowest number of non-delivered units will be the optimum one.

The assumption of the equal probability of occurrence for every circumstance set, where just one of them corresponds to the base one, is an apparent contradiction, as the most probable circumstances are the design assumptions. This should be interpreted to mean that the circumstances exceeding the initial assumptions are short term and the goal is to maintain the production continuity as per the work schedule.

In practice, a table listing the variants of possible transport system solutions with the numbers of non-delivered transport units for each of them is created (Table 12).

Table 12. Laplace’s criterion—numbers of non-delivered transport units in individual variants.

<table>
<thead>
<tr>
<th>Variant</th>
<th>Circumstances Exceeding the Initial Assumptions</th>
<th>( l_{max} ) [t.u.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_1 )</td>
<td>( l_{11} )</td>
<td>( l_{12} )</td>
</tr>
<tr>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>( W_i )</td>
<td>( l_{i1} )</td>
<td>( l_{i2} )</td>
</tr>
<tr>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>( W_n )</td>
<td>( l_{n1} )</td>
<td>( l_{n2} )</td>
</tr>
</tbody>
</table>

Source: own work.

According to Laplace’s criterion:

\[
\text{\( l_{max} i = \frac{1}{m} \sum_{i=1}^{m} (l_{i1}, l_{i2}, \ldots, l_{im}) \)}
\] (29)

The “\( i \)” variant is an optimum solution if the following condition is met: \( W_{OK4} = \max (L_1, \ldots, L_i, \ldots, L_n) \).

Based on the discussed Wald’s, Hurwicz’s, and Laplace’s criteria, the subsequent utility criteria can be specified as KU7A—operation in uncertainty circumstances.

All the discussed uncertainty criteria should be summarized. Each of them is ascribed “\( z \)” point weight by the designer, reflecting his perception of their significance, in line with the condition: \( \sum (z_m, z_{mm}, z_H, z_S, z_L) = W_{KU7A \ pkt \ U} \), where

\( z_m \)—max (min) criterion weight [point N];
\( z_{mm} \)—max (max) criterion weight [point N];
\( z_H \)—Hurwicz’s criterion weight [point N];
\( z_S \)—Savage’s criterion weight [point N];
\( z_L \)—Laplace’s criterion weight [point N].

This will be the weight of points in the KU7A criterion (Table 13). If the designer awards the total score \( W_{KU7A} \) to just one criterion, the variants will be assessed based on this criterion.

Table 13. Determination of additional score in KU7A criterion.

<table>
<thead>
<tr>
<th>Variant</th>
<th>Min (max) ( z_m )</th>
<th>Max (max) ( z_{mm} )</th>
<th>Hurwicz’s ( z_H )</th>
<th>Savage’s ( z_S )</th>
<th>Laplace’s ( z_L )</th>
<th>Weight: ( w_{KU7A} ) [Point U]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_1 )</td>
<td>( k_{1m} )</td>
<td>( n_{1m} )</td>
<td>( k_{1mm} )</td>
<td>( n_{1mm} )</td>
<td>( k_{1H} )</td>
<td>( n_{1H} )</td>
</tr>
<tr>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>( W_i )</td>
<td>( k_{im} )</td>
<td>( n_{im} )</td>
<td>( k_{imm} )</td>
<td>( n_{imm} )</td>
<td>( k_{iH} )</td>
<td>( n_{iH} )</td>
</tr>
<tr>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>( W_n )</td>
<td>( k_{nm} )</td>
<td>( n_{nm} )</td>
<td>( k_{nmm} )</td>
<td>( n_{nmm} )</td>
<td>( k_{nH} )</td>
<td>( n_{nH} )</td>
</tr>
</tbody>
</table>

Source: own work.
For min (max), max (max), Hurwicz’s, and Laplace’s criteria (stimulants), the score is calculated based on the following formula:

$$N_{ji} = (K_{ji} - K_{j \min}) \cdot \frac{z_j}{K_{j \max} - K_{j \min}}$$  \hspace{1cm} (30)

while for Savage’s criterion (destimulant), it is based on

$$N_{ji} = (K_{j \max} - K_{ji}) \cdot \frac{z_j}{K_{j \max} - K_{j \min}}$$  \hspace{1cm} (31)

where

- $N_{ji}$—score for “j” uncertainty criterion in “i” variant [point U];
- $z_j$—weight, maximum score for “j” criterion [point U];
- $K_{ji}$—number of transport units in “j” uncertainty criterion in “i” variant [pc. t.u.];
- $K_{j \min}$—minimum number of transport units in “j” uncertainty criterion [pc. t.u.];
- $K_{j \max}$—maximum number of transport units in “j” uncertainty criterion [pc. t.u.].

The score for KU7A is calculated based on

$$p_{\text{KU7A}|i} = \sum (N_{im}, N_{imm}, N_{iH}, N_{iS}, N_{iL})$$  \hspace{1cm} (32)

If the designer is able to specify the probability of the occurrence of specific circumstances exceeding the initial assumptions, they shall ascribe it to each of them, which is a procedure carried out in risk circumstances. This situation creates more convenient conditions to make decisions. In such a case, Table 9 is expanded by means of columns concerning the “$\alpha$” probability of each circumstance’s set occurrence (Table 14).

### Table 14. Set of possible data in risk conditions.

<table>
<thead>
<tr>
<th>Variants</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>…</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$b_1$ (+)</td>
<td>$b_2$ (0)</td>
<td>$b_3$ (−)</td>
<td>$b_…$ (−)</td>
<td>$b_m$ (−)</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>$\alpha_2$</td>
<td>$\alpha_3$</td>
<td>…</td>
<td>$\alpha_m$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of Additional Collection Points [pcs.]</th>
<th>$w_{11}$</th>
<th>$w_{12}$</th>
<th>$w_{13}$</th>
<th>$w_{1…}$</th>
<th>$w_{1m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
</tr>
<tr>
<td>$W_i$</td>
<td>$w_{i1}$</td>
<td>$w_{i2}$</td>
<td>$w_{i3}$</td>
<td>$w_{i…}$</td>
<td>$w_{im}$</td>
</tr>
<tr>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
</tr>
<tr>
<td>$W_n$</td>
<td>$w_{n1}$</td>
<td>$w_{n2}$</td>
<td>$w_{n3}$</td>
<td>$w_{n…}$</td>
<td>$w_{nm}$</td>
</tr>
</tbody>
</table>

Source: own work. Assumption: $\alpha_1 + \alpha_2 + \alpha_3 + … + \alpha_m = 1$, where $\alpha_i$—probability of “i” circumstance set occurrence.

**Graphic method**

The ranges of the anticipated value variability may be determined using a graphic method by plotting a function depicting the dependence of the anticipated value on the probability of the circumstances occurrence in individual variants (Figure 2) [35]. The function behavior may be simulated, assuming that the base circumstances’ $\alpha_0$ probability is known. The analysis will refer to the circumstances deviating from the base ones, i.e., events which may take place with $(1 - \alpha_0)$ probability. To carry out a graphic interpretation, it is necessary to assume

$$\alpha_0 + \alpha_+ + \alpha_- = 1$$

$$\alpha_- = (1 - \alpha_0) - \alpha_+ \quad \alpha_+ = (1 - \alpha_0) - \alpha_-$$  \hspace{1cm} (33)
where
\( \alpha_0 \)—probability of base circumstances occurrence [1];
\( \alpha_+ \)—probability of circumstances with surplus transport capacity [1];
\( \alpha_– \)—probability of circumstances with non-delivered transport units [1].

For data in Table 14:
\[
\alpha_0 = \alpha_2, \\
\alpha_+ = \alpha_1, \\
\alpha_– = \sum_{i=0}^{m} \alpha_i
\]

The boundary limit of the graph is the range of the analyzed “\( \alpha_z \)” probability of the occurrence of circumstances exceeding the initial assumptions: \( 0 < \alpha_z < 1 - \alpha_0 \).

**Figure 2.** Impact of “a” probability of the occurrence of exceeding the initial assumptions on the selection of the optimum variant for three variant examples, i.e., \( w_A, w_B, w_C \). Intercepts—probability values \( a_1, a_1', a_B \) probability of base condition occurrence. Behavior of the \( w_o \) optimum solution: (A) is a probability of a base circumstance set occurrence \( a_B = 0.5 \), (B) is a probability of a base circumstance set occurrence \( a_B < 0.5 \) (\( a_B = 0.2 \)). Source: own study based on [35].

Graph interpretation: Depending on the probability value \( \alpha_z \), the optimum value is determined as the upper envelope based on the sections between the dependence intercept for every variant, analogous to the graph depicting the caution coefficient impact in Hurwicz’s criterion (the intercepts are determined in the same way). By changing the \( \alpha_0 \) value, it is possible to generate subsequent mappings for
- \( \alpha_0 > 0.5 \)—circumstances exceeding the initial assumptions will be less probable than the base circumstance set;
- \( \alpha_0 < 0.5 \)—circumstances exceeding the initial assumptions will be more probable than the base circumstance set;
- \( \alpha_0 = 0 \)—the base circumstance set is not considered, solely circumstances exceeding the initial assumptions will occur.

Bayes’ criterion
The anticipated value is the weighted average of variables where the weights are the probabilities of individual circumstance set occurrence. The optimum variant is the
solution where the “B” number of non-delivered transport units determined in this way is the lowest:

\[ B_i = \alpha_1 * w_{i1} + \alpha_2 * w_{i2} + \alpha_3 * w_{i3} + \ldots + \alpha_m * w_{im} \]

\[ w_{R1} = \max (B_1, \ldots, B_i, \ldots, B_n) \] (34)

where

- \( B_i \)—anticipated value in “i” variant of B criterion [t.u.];
- \( w_{R1} \)—anticipated value for the optimum variant [t.u.].

The “i” variant is the optimum one if \( w_{R1} = B_i \).

Highest probability rule

If the probability of any circumstance set occurrence is equal to or higher than the total probability for the other circumstance sets, this optimum solution is determined for this set. Referring to the previous assumptions (Table 14), i.e., ascribing probability to the base circumstance set, i.e., the one being the basis for the subsystem design, it is obvious that this will be the circumstance set with the highest occurrence probability:

\[ \alpha_{\text{max}} = \max (\alpha_3, \ldots, \alpha_m) \]

\[ \alpha_{\text{max}} \geq \alpha_3 + \ldots + \alpha_m \] (35)

The rule is used solely on the circumstance sets with additional collection points or shortages of tractors, i.e., in adverse conditions. The anticipated value for all options for the circumstances with \( \alpha_{\text{max}} \) probability of occurrence is calculated:

\[ W_{R2} = \max (R_{R1}, \ldots, R_{Ri}, \ldots, R_{Rn}) \]

\[ R_{Rij} = \alpha_{\text{max}} * W_{ij} \]

where

- \( W_{R2} \)—anticipated value of the optimum variant in the highest probability rule;
- “i” variant is the optimum one if \( w_{R2} = R_{Ri} \) condition is met.

This criterion is not sensitive to the number of non-delivered transport units in the circumstances with the lower occurrence probability.

Lost profit analysis

The lost profit notion is equivalent to the relative loss notion introduced in Savage’s criterion. A matrix is created, comparing the relative losses to every circumstance set where the difference between the best and the analyzed variant is analyzed (Table 15).

**Table 15. Lost profit matrix.**

<table>
<thead>
<tr>
<th>Variant</th>
<th>Circumstances Exceeding the Initial Assumptions</th>
<th>Max [t.u.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>W_1</td>
<td>( e_{11} ) ( e_{12} ) ( e_{13} ) \ldots ( e_{1m} )</td>
<td>E_1</td>
</tr>
<tr>
<td>\ldots</td>
<td>\ldots \ldots \ldots \ldots \ldots \ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>W_i</td>
<td>( e_{i1} ) ( e_{i2} ) ( e_{i3} ) \ldots ( e_{im} )</td>
<td>E_i</td>
</tr>
<tr>
<td>\ldots</td>
<td>\ldots \ldots \ldots \ldots \ldots \ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>W_n</td>
<td>( e_{n1} ) ( e_{n2} ) ( e_{n3} ) \ldots ( e_{nm} )</td>
<td>E_n</td>
</tr>
</tbody>
</table>

Source: own study based on [35].

In the lost profit analysis, the “i” variant is an optimum solution if the following condition is met: \( W_{OR3} = \min (E_1, \ldots, E_i, \ldots, E_n) \).

\[ E_i = \alpha_1 * e_{i1} + \alpha_2 * e_{i2} + \alpha_3 * e_{i3} + \ldots + \alpha_m * e_{im} \]

\[ e_{i1} = \max (w_{i1}, \ldots, w_{i1}, \ldots, w_{n1}) \]

\[ w_{i1} \] (37)

where

- \( E_i \)—lost profit amount [pc. t.u.];
- \( \alpha_i \)—probability of “i” circumstances occurrence [1];
$e_{im}$—relative loss of transport units in “$i$” variant in “$m$” circumstances [pc. t.u.];

$w_{im}$—number of non-delivered transport units in “$i$” variant in “$m$” circumstances [pc. t.u.].

Based on the discussed Bayes’ criterion, the highest probability rule and the lost profit analysis, another utility criterion can be specified: $KU7B$—operation in risk circumstances.

All the criteria to assess the circumstances exceeding the initial assumptions in the risk conditions should be summarized. Each of them can be ascribed a “$z$” point weight by the designer, reflecting their perception of their significance, in line with the condition:

$$\sum(z_{B}, z_{p}, z_{uk}) = W_{KU7B} \text{ pkt U} \text{, where}$$

- $z_{B}$—Bayes’ criterion weight [point U];
- $z_{p}$—highest probability criterion weight [point U];
- $z_{uk}$—lost profit criterion weight [point U].

This will be the weight of the points in the $KU7B$ criterion (Table 16). If the designer awards the total score $W_{KU7B}$ to just one criterion, the variants will be assessed based on it.

### Table 16. Determination of additional score in $KU7B$ criterion.

<table>
<thead>
<tr>
<th>Variant</th>
<th>Risk Criteria</th>
<th>Weight: $w_{KU7B}$ [Point U]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_1$</td>
<td>$K_{R1B}$</td>
<td>$R_{iB}$</td>
</tr>
<tr>
<td></td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$W_i$</td>
<td>$K_{RiB}$</td>
<td>$R_{iB}$</td>
</tr>
<tr>
<td></td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$W_n$</td>
<td>$K_{RnB}$</td>
<td>$R_{nB}$</td>
</tr>
<tr>
<td></td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Source: own work.

For the Bayes’ criterion and the highest probability criterion (stimulants), the score is calculated based on the following formula:

$$R_{ji} = (K_{Rji} - K_{Rj min}) * \frac{z_j}{K_{Rj max} - K_{Rj min}} \quad (38)$$

while for the lost profit criterion (destimulant), it is based on

$$R_{ji} = (K_{Rj max} - K_{Rji}) * \frac{z_j}{K_{Rj max} - K_{Rj min}} \quad (39)$$

where

- $R_{ji}$—score for “$j$” risk criterion in “$i$” variant [point U];
- $K_{Rji}$—anticipated number of transport units in “$j$” risk criterion in “$i$” variant [pc. t.u.];
- $z_j$—weight, maximum score for “$j$” criterion [point U];
- $K_{Rj max}$—maximum anticipated value of transport units in “$j$” risk criterion, $K_{Rj max} = max (K_{Rj1}, \ldots, K_{Rji}, \ldots, K_{Rjn})$ [pc. t.u.];
- $K_{Rj min}$—minimum anticipated value of transport units in “$j$” risk criterion, $K_{Rj min} = min (K_{Rj1}, \ldots, K_{Rji}, \ldots, K_{Rjn})$ [pc. t.u.].

The score for $KU7B$ is calculated based on

$$P_{ku7B i} = \sum(R_{iB}, R_{ip}, R_{iuk}) \quad (40)$$

In the design process, just one assessment criterion relating to the operation in conditions exceeding the initial assumptions can be used:

- If the designer is unable to determine the probability of individual circumstance set occurrence, they use the $KU7A$ criterion;
- If they are able to determine that probability, they use the $KU7B$ criterion.
As indicated in point 3, every criterion is ascribed a particular weight reflecting the significance level, in line with the requirements stipulated in Formulas (3) and (4). An example of the score distribution between individual criteria is presented in Table 17.

Table 17. Examples of proposed weights for the utility criteria.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>KU1A</th>
<th>KU1B</th>
<th>KU2</th>
<th>KU3</th>
<th>KU4A1</th>
<th>KU4A2</th>
<th>KU4B</th>
<th>KU5</th>
<th>KU6</th>
<th>KU7A</th>
<th>KU7B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Score [point U]</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>3</td>
<td>10</td>
<td>51</td>
<td>51</td>
</tr>
</tbody>
</table>

Source: own work.

The total score of 100 points \( w_u \) was assumed to make it easier to apply the score. For the score of the criterion of operations in conditions exceeding the initial assumptions \( w_{un} \), the values above 50% \( w_{u} \) were assumed as this is the most important utility constituent responsible for ensuring the continuous production process.

3. Conclusions

The result of designing a new material transport system in underground mine excavations is usually the development of several variants, either technical or organizational. To obtain the opportunity to select the optimum solution, it is necessary to specify the transport system functionality in terms of covering the transport needs in the operation process and reliable operation. Analyzing the nature and variability of those needs, and considering the specific properties of the mine, it is possible to use specific, predetermined utility criteria, including those functioning in super-planned states, analyzed in relation to the uncertainty or risk. This will enable the simulation of the transport system operation in the case of sudden changes or failures, which are not infrequent due to the geological and mining conditions of mine operations. The method takes into account the systemic interdependencies of transport processes with the process of underground mining operations. By determining the value of the score weights for each criterion, the designer of a new material transport system is able to select the optimal solution based on repeatable numerical calculations. The values specified for individual criteria depend on the subjective assessment of the designer and their familiarity with the operation of the transport system layouts at a given mine.

The proposed original in-house method uses methods and models known both in theory and in practice. There is no description in the specialist literature of solutions showing how to select systems and means of underground material transport, while maintaining criteria that guarantee the smooth underground exploitation of the deposit, maintaining safety and cost rationality. So far, the design of these solutions has been intuitive, which did not allow for the rationalization of the projects in question. The results of the conducted research complement the level of knowledge of underground transport in hard coal mines. An important benefit is the possibility of developing a repeatable procedure for the selection of underground transport systems and means.

In practice, the method was used to select the optimal variant of the transport system in a new part of the seam of one of the mines, where it was planned to cut four walls with runs up to 1.5 km and drill four tunnels varying in length from 1,315 to 1,520 m. Of the ten variants identified, using seven usability criteria, the variant was indicated that should ensure the highest degree of reliable transport operation and continuity of the operation.

The main research limitation of the proposed method is its high complexity, requiring the collection of a significant amount of data and a certain subjectivity that always accompanies tools that use weights to assess the individual selection criteria. However, due to the lack of methods supporting the process of optimizing decisions about the design and use of complex transport systems in hard coal mines, these limitations are not of great importance.

Further research should concern the repeated empirical verification of the designed method and its further improvement based on the obtained results. Additionally, an interesting research direction may be the assessment of the economic profitability of individual...
variants of transport systems, which is particularly important in the context of the need to maintain the profitability of mining enterprises in the European Union.

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