Risks **2013**, *1*, 57-80; doi:10.3390/risks1020057

OPEN ACCESS

risks

ISSN 2227-9091

www.mdpi.com/journal/risks

Article

A Welfare Analysis of Capital Insurance

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Received: 26 July 2013; in revised form: 29 August 2013 / Accepted: 6 September 2013 / Published: 17 September 2013

Abstract: This paper presents a welfare analysis of several capital insurance programs in a rational expectation equilibrium setting. We first explicitly characterize the equilibrium of each capital insurance program. Then, we demonstrate that a capital insurance program based on aggregate loss is better than classical insurance, when big financial institutions have similar expected loss exposures. By contrast, classical insurance is more desirable when the bank's individual risk is consistent with the expected loss in a precise way. Our analysis shows that a capital insurance program is a useful tool to hedge systemic risk from the regulatory perspective.

Keywords: capital insurance; welfare; equilibrium

Classification: **JEL** G11, G12, G13

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1. Introduction

This paper presents a welfare analysis of recently proposed capital insurance programs in a rational expected equilibrium setting. The idea of capital insurance is motivated by the desire to resolve "too big to fail" issues. As those "too big to fail" banks or companies that are "financial in nature" (hereafter, banks)[1] expect capital injection from the central bank in times of financial distress, the banks might act

[1]Under the standards set forth in section 113 of the Dodd-Frank Act, a bank holding company or "non-bank financial company" poses a potential systemic risk if "material financial distress at the company, or the nature, scope, size, scale, concentration, interconnectedness, or mix of the activities of the company, could pose a threat to the financial stability of the United States." Therefore, we focus only on these companies with systemic risks (too big to fail).

in a risk-taking manner and put the central bank, the regulator and all taxpayers in a fragile financial position. In a capital insurance program (see [11]), the bank pays some amount as a premium or reserve to the central bank, which, in turn, would inject funds into the banks during future financial failures. A capital insurance program is motivated by the desire to protect taxpayers and the economy as a whole in the presence of a big financial predicament. Our purpose is to study whether this capital insurance idea works or not from the perspective of its welfare.

Capital insurance is very different from the current capital regulation implemented in BASEL II and BASEL III. It is also different from the Dodd-Frank Act, which posits several prudential standards and new stringent capital requirements to banks with systemic risks. According to the capital regulation requirement, the amount of capital reserve or the economic capital amount depends on the risk of the loss portfolio and the riskiness of the bank itself. The riskier the bank, the higher the economic capital; the economic capital is higher for a bank with a weak credit situation than for a strong counterpart, while assuming the portfolio to be identically the same. Therefore, the economic capital idea depends on both the individual bank's riskiness and the individual loss portfolio.

By contrast, capital insurance, in essence, is an insurance contract, and the capital insurance idea casts all banks together at the market level. On the one side, the central bank is an insurer of the contract and receives an insurance premium with the obligation to inject funds to save the bank in financial distress. On the other side, the bank is an insured in this contract agreement. As the central bank represents the taxpayer in this structure, the insurer of the contract is a taxpayer, and the premium represents a special purpose tax in the sense described by Acharya *et al.* [1]. In contrast to the traditional insurance contract, the contract redemption is contingent on the aggregate loss, and the insured event is contingent on a systematic event in the economy.

The rational expected equilibrium of the capital insurance program is explained as follows. The central bank issues insurance contracts to the banks, and the banks purchase these contracts, which are placed on the market. The central bank predicts the correct optimal demand from the banks with a given premium structure, so the central bank maximizes the welfare with the premium structure as characterized. Consequently, both the demand (from the banks) and the supply (from the central bank) are determined uniquely in a rational expectation equilibrium.

In this paper, we assume that the insurance contract payout has been placed as proposed by the capital insurance program. Therefore, we do not address the optimal capital insurance design problem. Instead, we consider two capital insurance programs. In the first one, the insurance contract insures the *aggregate loss* of all banks. In the second one, each bank buys insurance that depends on the aggregate loss of all banks, except for the insured bank's own loss portfolio. For comparison purposes, we further consider the situation in which each bank purchases insurance that relies on its own loss portfolio. This is "classical insurance" by the terminology in this paper, and it has the same indemnity as the traditional coinsurance contract. As the premium structure depends on all the loss portfolios of the banks, those loss portfolios together affect each bank's coinsurance demand. Therefore, classical insurance in our setting is different from the traditional coinsurance contract in equilibrium.

We demonstrate that many factors affect the welfare analysis and the chosen capital insurance program. *First*, the proposed two capital insurance programs are distinguished from each other by the correlation structure. A low correlation environment ensures the low welfare of the contract based on the aggregate loss, except for the individual bank's loss. Therefore, the aggregate insurance is better than the other one. In fact, when each loss portfolio can be observed completely by all banks and the central bank and the bank does not manipulate the book loss, aggregate insurance ensures a higher welfare than another one, in general.

Second, both the specific risk and the systematic risk components of the individual loss are important ingredients for comparing the classical insurance and the aggregate insurance contracts. These two components play a crucial role in the classical demand analysis of the coinsurance contract (for the mean-variance insured); see [9]. We demonstrate that the way in which each bank's specific risk and systematic risk components behave together in the market has a significant effect on the comparison analysis. When a higher individual risk corresponds to a higher expected loss per each volatility unit, we say that the market displays an *ordering loss market*. Otherwise, the market is a *disordering loss market*.[2] We show that classical insurance works better in the ordering loss market, while aggregate insurance is more beneficial to the central bank in the disordering loss market. Hence, our result is significantly disparate from the optimal sharing rules in a pure exchange market.[3] The optimal insuring rule, in our equilibrium, relies on the aggregate loss portfolio in a more complicated way. Literally, the way in which the loss portfolios are connected to each other implies a different welfare outcome of the insurance program.

Third, the way in which the systematic risk is distributed among each bank is also captious for a comparative welfare analysis of the insurance contracts. If each bank contributes equally or very closely to each other in the total systematic risk, we show that aggregate insurance ensures a higher welfare. Therefore, it is a more desirable insurance program than the classical insurance one. Wagner [14] shows that diversification might enhance systemic risk, while it reduces each institution's individual probability of failure; so, a full diversification is not always beneficial from the systemic perspective. According to our result, aggregate insurance offers a solution in a full diversification situation to reduce the systemic risk.

The remainder of the paper is organized as follows. Section 2 introduces the setting and characterizes the equilibrium. Section 3 presents the comparison of three types of capital insurance programs by the welfare analysis developed in the equilibrium. Section 4 offers a discussion and the implications of our theoretical results. Moreover, we explain how to implement the capital insurance program in practice and how to identify the "too big to fail" banks from the regulatory perspective. Section 5 briefly describes the conclusions of the analysis conducted. All proofs are stated in Appendix A, and Appendix B presents the equilibrium in a general situation and identifies these "too big to fail" banks by using this capital insurance program.

[2] Precisely, when a risk-adjusted covariance of the loss portfolio is co-monotonic to the Sharpe ratio of the loss portfolio, we say it is an ordering loss market. If both of these sequences are counter-monotonic to each other, we say that the market is a disordering loss market. See Propositions 4, 6 and 7 below.

[3] By [4], the optimal sharing rules must increase with respect to the aggregate endowment. Our setting is different from Borch's equilibrium setting in the presence of the central bank.

2. The Model

There are N big banks indexed by $i = 1, \cdots, N$ in a one-period economic world. Each bank is endowed with a loss portfolio, $X_1, \cdots, X_N$, respectively. These loss portfolios are defined on the same state space Ω, and all banks have the same beliefs on the nature of state. This common belief is represented by one probability measure, P, on the state space. However, these bank's loss portfolios can be significantly different. We assume that each bank is risk-averse, and the preference of risk is interpreted by a utility function, $U_i(\cdot)$. The bank's initial wealth is given by W_0^i for each bank, $i = 1, \cdots, N$, respectively.

There is a government entity, such as the Financial Stability Oversight Council (FSOC) in the Dodd-Frank Act or a central bank, which sells the insurance contract to each bank. Each bank is either voluntarily or enforced to purchase the insurance contract by paying a particular amount as a premium, and a fund commitment is guaranteed by a central bank in a bad business situation in the future. The premium amount can be treated as a special tax purpose rate for each bank as suggested by Acharya *et al.* [1]. The fund commitment offered by the government entity is the indemnity of the insurance. Alternatively, these insurance contracts can be issued by a reinsurance company which is able to diversify the reinsurance risk. For simplicity, we name the insurer as a *regulator*.

The prototype insurance structure has the indemnity, $I_i(X, X_i)$, which depends on the individual book loss, X_i, and the aggregate loss, X. The aggregate loss $X = \sum_{i=1}^N X_i$. This insurance contract is called "capital insurance", as it depends on the aggregate loss being realized in the future. The capital insurance contract is different from the classical contracts in which $I_i(X, X_i)$ is irrelevant to the aggregate loss, X, and, instead, depends on the individual loss, X_i. Following the classical insurance literature ([2,13]), we assume that the insurance premium is determined by $(1+\rho)\mathbb{E}[I(X, X_i)]$, where ρ is a load factor. For simplicity, we assume that the loss factor is the same across the bank industry, but it is possible to consider a bank-specific premium structure in the extended analysis. The loss factor is characterized by the regulator in equilibrium, which will be explained shortly.

Given a load factor, ρ, each bank chooses the best available insurance contract to maximize the expected utility (see [2]):

$$\mathbb{E}[U_i(W^i)] = \mathbb{E}\left[U_i\left(W_0^i - X_i + I_i(X, X_i) - (1+\rho)\mathbb{E}[I_i(X, X_i)]\right)\right] \tag{1}$$

The regulator is risk-neutral and receives the premium for each contract. The welfare of the regulator is:

$$W^r = \sum_i (1+\rho)\mathbb{E}\left[I_i(X, X_i)\right] - \sum_i I_i(X, X_i) - \sum_i c\left(I_i(X, X_i)\right) \tag{2}$$

where $c\left(I_i(X, X_i)\right)$ represents the cost for the regulator to issue the contract, $I_i(X, X_i)$. The cost can be fixed, a constant percentage of the indemnity or can depend on a drastic market event. To focus on the analysis of the insurance program, we assume that the cost structure is a constant for each bank. The regulator's objective is to determine the best premium structure given the optimal demand for each bank (with any, a given load structure, ρ), as well as to maximize the expected welfare. Clearly, the insurance $I_i^*(X, X_i)$ in equilibrium depends on both the demand (from all banks) and the supply (from the regulator) and relies on the load factor, ρ^*, proposed by the regulator. We do not distinguish between the welfare and the expected welfare when there is no confusion in the rest of this paper.

In this paper, we focus on the following three capital insurance programs:

- *Aggregate Insurance*: $I_i(X, X_i) = \beta_i X$, where $\beta_i \geq 0$.
- *Classical Insurance*: $I_i(X, X_i) = \beta_i X_i$, where $\beta_i \geq 0$.
- *Aggregate-Cross Insurance*: $I_i(X, X_i) = \beta_i \hat{X}_i$, where $\hat{X}_i = \sum_{j \neq i} X_j$ is the total loss, except for the insured bank's loss, and $\beta_i \geq 0$.

In each case, bank i chooses the best coinsurance parameter, β_i. The optimal β is written as $\beta(\rho)$ to highlight its dependence on the load factor. The first insurance contract depends solely on the aggregate loss, X, so it is called "aggregate insurance". The coinsurance parameter, β_i, represents the percentage of the aggregate loss that is insured for the bank, i. Clearly, this coinsurance parameter depends on how much the individual bank's loss risk contributes to the aggregate loss, as will be seen later. The second insurance contract is a standard one, initiated by [2], and is termed "classical insurance". However, the premium structure in the traditional insurance contract is either given exogenously or depends on the specific loss portfolio in equilibrium. Therefore, our classical insurance is different from those traditional insurance contracts in a rational expectation equilibrium. The last insurance contract is motivated differently. Because of the possibility of the bank's manipulation of the loss report on X_i, as discussed in [6] in a similar context, there is a moral hazard issue in case $I_i(X, X_i)$ is related to X_i. To resolve it, Kashyap *et al.* [11] introduces the aggregate-cross insurance idea in which the bank insures the total risks of all banks, except for the bank's own risk. The aggregate-cross insurance contract is inspired by the idea outlined in [11].

In what follows, we impose two assumptions to simplify the discussions.

Assumption I: Each bank is a mean-variance agent with the reciprocal of the risk aversion parameter, $\gamma_i > 0$. *We also assume zero (or constant) cost structure for each contract.*[4]

Assumption II: There exists no asymmetric information between each bank and the regulator. The loss portfolio, X_i, *is equivalently identified by the bank and the regulator, and both the bank and the regulator make a decision based on the same interpretation of the loss portfolio.*

We now move to present our equilibrium analysis on each capital insurance program. We also examine how these loss portfolios affect each insurance contract, as well as the welfare. Moreover, we examine which insurance contract is desirable from the perspectives of the regulator and the bank.

2.1. Aggregate Insurance

We characterize the equilibrium precisely for the aggregate insurance. We start with the bank i's rational decision by assuming that the insurance contract has been placed on the market.

[4] We follow the same mean-variance setting as in [12], in which the aggregate uncertainty insurance is considered, as we focus on the aggregate or systematic risk.

2.1.1. Optimal Load for Bank i

Bank i's objective is to find suitable coinsurance parameter β_i to maximize:

$$\max_{\beta_i \geq 0} \mathbb{E}[W^i] - \frac{1}{2\gamma_i} Var(W^i) \tag{3}$$

where $W^i = W_0^i - X_i + \beta_i X - (1+\rho)\mathbb{E}[\beta_i X]$ is the terminal wealth for the bank, i. Given the load factor, ρ, the optimal β_i for the bank, i, is[5]:

$$\beta^{i,a}(\rho) = \frac{Cov(X_i, X) - \rho\mathbb{E}(X)\gamma_i}{Var(X)} \tag{4}$$

if $Cov(X_i, X) - \rho\mathbb{E}(X)\gamma_i \geq 0$; otherwise, $\beta^{i,a}(\rho) = 0$. The symbol, "a", represents the "aggregate insurance". We use $\beta^{i,a}(\rho)$ to highlight the effect of the load factor, ρ, for the bank, i.

2.1.2. Optimal Load Factor for the Regulator

The regulator predicates the demand from the bank, i, as $\beta^{i,a}(\rho)X$ correctly for each bank, $i = 1, \cdots, N$. Therefore, by plugging Equation (4) into Equation (2) and assuming that $Cov(X_i, X) \geq \rho\mathbb{E}(X)\gamma_i$, the welfare is:

$$\mathbb{E}(W^r) = \rho\mathbb{E}(X) - \rho^2 \sum_i \frac{\gamma_i \mathbb{E}(X)^2}{Var(X)}. \tag{5}$$

By using Formula (5) and its first-order condition, the best load factor is determined by the regulator as:

$$\rho^{*,a} = \frac{1}{2\sum_i \gamma_i} \frac{Var(X)}{\mathbb{E}(X)}. \tag{6}$$

Consequently, under this premium structure, we obtain the following characterization of the equilibrium.

Proposition 1 *Assume for each $i = 1, \cdots, N$,*

$$\frac{Cov(X_i, X)}{Var(X)} \geq \frac{1}{2} \frac{\gamma_i}{\sum_i \gamma_i}, \tag{7}$$

then, the optimal load factor, $\rho^{*,a}$, is given by Equation (6); the welfare for the aggregate insurance is:

$$\mathbb{E}(W^{*,a}) = \frac{1}{4\sum_i \gamma_i} Var(X) \tag{8}$$

and the best coinsurance parameter for the bank, i, in this aggregate insurance contract is:

$$\beta^{i,a} = \frac{Cov(X_i, X)}{Var(X)} - \frac{1}{2} \frac{\gamma_i}{\sum_i \gamma_i}. \tag{9}$$

[5] It is easy to see that $Var(W^i) = Var(X_i) + \beta_i^2 Var(X) - 2\beta_i Cov(X_i, X)$. Then, $\beta^{i,a}(\rho)$ follows from the first-order condition in Equation (3).

Proof: Under Condition (7) and the choice of $\rho^{*,a}$ by Equation (6), we observe that $Cov(X_i, X) \geq \rho\gamma_i E[X]$. Therefore, $\beta^{i,a}(\rho)$ is given by Equation (4), and the equilibrium welfare is obtained in Equation (5). Then, the equilibrium follows from the standard first-order condition. In general, if condition (7) does not hold for each $i = 1, \cdots, N$, it means that $\beta^{i,a}(\rho)$ is a "corner solution", and the equilibrium welfare is changed accordingly. A general solution is presented in Appendix B. □

There are several remarkable points about aggregate insurance by using Proposition 1. *First*, the welfare estimated by the regulator depends on the *variability* of the aggregate loss, the systematic risk. The higher the variability, the higher the expected welfare. The smaller the variability, or alternatively, the more stable the aggregate loss is, the smaller the welfare. More interestingly, the welfare does not depend on the expected aggregate loss, $\mathbb{E}[X]$. Therefore, only the aggregative risk variability contributes to the welfare. Hence, Proposition 1 supports the aggregate insurance idea to reduce the systemic risk.

Second, the optimal coinsurance parameter, $\beta^{i,a}$, for bank i is the difference between the "beta", $\frac{Cov(X_i,X)}{Var(X)}$,[6] and the individual risk aversion parameter, γ_i, comparing with the total risk aversion among the banks, $\sum_i \gamma_i$. The higher the beta, the larger $\beta^{i,a}$; so the bank, i purchases insurance proportional to the systematic risk. It is intuitively appealing, because a higher beta implies a larger contribution of the bank, i, to the systematic risk or the bank, i, has a higher systemic risk. To hedge the systemic risk, the bank needs to insure a larger amount of the systematic risk. Moreover, the relationship between the bank i's risk aversion and the other bank's risk preferences is also important for the aggregate insurance. Higher $\frac{\gamma_i}{\sum_i \gamma_i}$ implies less risk aversion of the bank, i, and, thus, a smaller β_i.

Third, note that[7]:

$$\sum_i \beta^{i,a} = \frac{1}{2}, \tag{10}$$

the total aggregate insurance indemnity for the regulator, is $\sum_i I_i(X, X_i) = \frac{1}{2}X$. This states that exactly half of the systematic risk is insured in this program. The number $1/2$ comes from the mean-variance setting and does not have any specific meaning. However, a crucial insight at this point is that the aggregate loss is not fully insured in this equilibrium insurance market, which is similar to the classical result for the standard coinsurance contract.

2.2. Classical Insurance

For comparative purposes, we next consider the classical insurance, $I_i(X, X_i) = \beta_i X_i$. By the same idea, we characterize $\beta^{i,c}(\rho)$, ρ^* and the welfare sequentially. The equilibrium is summarized as follows.

[6] It is the beta in the capital asset pricing model when the loss variable is replaced by the return variable.
[7] Since $X = \sum_i X_i$, $\sum_i Cov(X_i, X) = Var(X)$.

2.2.1. Optimal Load for Bank i

$$\beta^{i,c}(\rho) = \max\left\{1 - \frac{\rho\mathbb{E}(X_i)\gamma_i}{Var(X_i)}, 0\right\} \tag{11}$$

where the symbol, "c", represents "classical insurance".

2.2.2. Optimal Load Factor for the Regulator

Given the above optimal load factor, $\beta^{i,c}(\rho)$, and assuming $\frac{\rho\mathbb{E}(X_i)\gamma_i}{Var(X_i)} \leq 1, i = 1, \cdots, N$, the welfare is obtained as follows.

$$\mathbb{E}(W^r) = \rho\mathbb{E}(X) - \rho^2 \sum_i \frac{\gamma_i \mathbb{E}(X_i)^2}{Var(X_i)} \tag{12}$$

Therefore, the optimal load factor from the regulator's perspective is:

$$\rho^{*,c} = \frac{1}{2} \frac{\mathbb{E}(X)}{\sum_i \frac{\gamma_i(\mathbb{E}(X_i))^2}{Var(X_i)}} \tag{13}$$

We have the following result.

Proposition 2 *Assume that for each $i = 1, \cdots, N$:*

$$\frac{\mathbb{E}(X)}{\sum_i \frac{\gamma_i(\mathbb{E}(X_i))^2}{Var(X_i)}} \frac{\gamma_i \mathbb{E}(X_i)}{Var(X_i)} \leq 2, \tag{14}$$

then, the optimal load factor is determined in Equation (13). The welfare of the classical insurance is:

$$\mathbb{E}(W^{*,c}) = \frac{1}{4} \frac{\mathbb{E}(X)^2}{\sum_i \frac{\gamma_i \mathbb{E}(X_i)^2}{Var(X_i)}} \tag{15}$$

and the best coinsurance parameter for the bank, i, in this classical insurance contract is:

$$\beta^{i,c} = 1 - \frac{1}{2} \frac{\mathbb{E}(X)}{\sum_i \frac{\gamma_i(\mathbb{E}(X_i))^2}{Var(X_i)}} \frac{\gamma_i \mathbb{E}(X_i)}{Var(X_i)} \tag{16}$$

Proof: The same as the proof of Proposition 2. □

According to Proposition 2, the welfare estimated by the regulator in classical insurance depends on both the expectation and the variance of individual loss, as well as the expectation of the aggregate loss, whereas the variability of the aggregate loss does not contribute to the estimated welfare directly. In fact, the correlation structure of the loss portfolios, $(X_1, \cdots, X_n)$, is not involved in the insurance contract at all. Therefore, the welfare depends only on the marginal distribution, but not on the joint distribution of the loss portfolios. Obviously, this should be seen as a limitation of classical insurance to address systemic risk. We will compare classical insurance with aggregate insurance in detail in the next section.

It is interesting to look at the optimal coinsurance parameter, β_i, for the bank, i, in the classical insurance contract. While keeping the risks on other banks fixed, the higher $Var(X_i)$, the higher β_i. More insurance is required for a higher individual risk. It is straightforward to verify that for large values of $\mathbb{E}[X_i]$, the optimal coinsurance parameter is increasing with respect to the increase of $\mathbb{E}[X_i]$. As the premium structure depends on all loss portfolios, $\{X_1, \cdots, X_n\}$, the risks of other banks affect the classical insurance demand in this setting[8].

2.3. Aggregate-Cross Insurance

At last, we consider the aggregate-cross insurance $I_i(X, X_i) = \beta_i \hat{X}_i$. By definition, it focuses on the insurance of all banks, except the insured bank in the market.

2.3.1. Optimal Load for Bank i

It is easy to derive $\beta^{i,ac}(\rho)$ in this situation as:

$$\beta^{i,ac}(\rho) = \max\left\{\frac{Cov(X_i, \hat{X}_i) - \rho\mathbb{E}(\hat{X}_i)\gamma_i}{Var(\hat{X}_i)}, 0\right\} \tag{17}$$

where the symbol, "ac", represents "aggregate-cross insurance".

2.3.2. Optimal Load Factor for the Regulator

By plugging Formula (17) into Formula (2) and assuming that $Cov(X_i, \hat{X}_i) \geq \rho\mathbb{E}(\hat{X}_i)\gamma_i$, we have:

$$\mathbb{E}(W^r) = \rho\sum_i \mathbb{E}(\hat{X}_i)\frac{Cov(X_i, \hat{X}_i) - \rho\mathbb{E}(\hat{X}_i)\gamma_i}{Var(\hat{X}_i)} \tag{18}$$

and:

$$\rho^{*,ac} = \frac{1}{2}\frac{\sum_i \mathbb{E}(\hat{X}_i)\dfrac{Cov(X_i, \hat{X}_i)}{Var(\hat{X}_i)}}{\sum_i \dfrac{\gamma_i\mathbb{E}(\hat{X}_i)^2}{Var(\hat{X}_i)}}. \tag{19}$$

Therefore, we obtain the following proposition, the proof of which is similar to Propositions 1 and 2.

Proposition 3 *Assume for each $i = 1, \cdots, N$:*

$$\frac{\sum_i \mathbb{E}(\hat{X}_i)\dfrac{Cov(X_i, \hat{X}_i)}{Var(\hat{X}_i)}}{\sum_i \dfrac{\gamma_i\mathbb{E}(\hat{X}_i)^2}{Var(\hat{X}_i)}}\mathbb{E}(\hat{X}_i)\gamma_i \leq 2Cov(X_i, \hat{X}_i). \tag{20}$$

[8] It is different from a traditional insurance contract on individual loss exposure. The load factor for a traditional insurance contract is either given exogenously or depends on the specific loss vector in equilibrium. Classical insurance in our setting, however, is characterized in a rational expectation equilibrium with banks and a regulator

Then, the welfare of the aggregate-cross insurance is:

$$\mathbb{E}(W^{*,ac}) = \frac{1}{4}\frac{\left(\sum_i \mathbb{E}(\hat{X}_i)\dfrac{Cov(X_i,\hat{X}_i)}{Var(\hat{X}_i)}\right)^2}{\sum_i \gamma_i \dfrac{\mathbb{E}(\hat{X}_i)^2}{Var(\hat{X}_i)}} \tag{21}$$

and the best coinsurance parameter for the bank, i, in this aggregate-cross insurance contract is:

$$\beta^{i,ac} = \frac{Cov(X_i,\hat{X}_i)}{Var(\hat{X}_i)} - \frac{1}{2}\frac{\sum_i \mathbb{E}(\hat{X}_i)\dfrac{Cov(X_i,\hat{X}_i)}{Var(\hat{X}_i)}}{\sum_i \dfrac{\gamma_i \mathbb{E}(\hat{X}_i)^2}{Var(\hat{X}_i)}} \frac{\mathbb{E}(\hat{X}_i)\gamma_i}{Var(\hat{X}_i)}. \tag{22}$$

By Proposition 3, the expected welfare in an aggregate-cross insurance contract depends positively on the covariance between the individual bank's loss, X_i, and the aggregate loss, except for the insured bank's loss, $\hat{X}_i$, for each bank, i. The intuition is simple: higher correlation coefficient $corr(X_i, \hat{X}_i)$ results in higher expected welfare from the regulator's perspective.

In contrast to classical insurance, aggregate-cross insurance depends on the correlation structure of the loss portfolios. We see easily that when X_i and $\hat{X}_i$ are uncorrelated for each i, both the estimated welfare and the optimal coinsurance β for bank i in this aggregate-cross insurance contract are equal to zero. In particular, when all banks' loss portfolios are independent, there is no necessity to buy the aggregate-cross insurance.

The next result illustrates the main insights of these three insurance contracts when the loss risk factors are uncorrelated. We say one contract is *preferred* to another one, as long as the former has higher welfare than the latter.

Proposition 4 *Assume that the loss portfolios are uncorrelated, i.e., $Cov(X_i, X_j) = 0\ \forall i \neq j$. Then, both the aggregate insurance and the classical insurance are preferred to the aggregate-cross insurance. Moreover:*

1. *If the* risk-adjusted variance *vector,* $\left(\frac{Var(X_i)}{\gamma_i}\right)$*, and the Sharpe ratio vector,* $\left(\frac{\mathbb{E}[X_i]}{\sqrt{Var(X_i)}}\right)$*, are co-monotonic*[9]*, then the classical insurance is preferred to the aggregate insurance.*

2. *If the* risk-adjusted variance *vector,* $\left(\frac{Var(X_i)}{\gamma_i}\right)$*, and the Sharpe ratio vector,* $\left(\frac{\mathbb{E}[X_i]}{\sqrt{Var(X_i)}}\right)$*, are counter-monotonic, and there exists one* "too big to fail" *bank in the sense that* $\mathbb{E}[X]^2$ *is close to* $\sum_i \mathbb{E}[X_i]^2$*, then the aggregate insurance is preferred to the classical insurance.*

Proof: See Appendix A. □

[9] Given two vectors $a = (a_1, \cdots, a_n), b = (b_1, \cdots, b_n)$, a and b are counter-monotonic if $(a_i - a_j)(b_i - b_j) \leq 0, \forall i, j$, and at least one inequality is strict; a and b are co-monotonic if $(a_i - a_j)(b_i - b_j) \geq 0, \forall i, j$, and at least one inequality is strict.

There are several points in Proposition 4. First of all, the relationship between the risk-adjusted variance and the Sharpe ratio across the banks plays a crucial role in comparing classical insurance and aggregate insurance. As each X_i represents the loss portfolio, we assume a positive expected loss in our analysis. Its variance, $Var(X_i)$, represents the individual risk of the bank, i. Similarly, we use the terminology "Sharpe ratio" to represent the expected loss per each volatility unit. Both the risk-adjusted variance, $\frac{Var(X_i)}{\gamma_i}$, and the Sharpe ratio represent two important factors to characterize the loss risk for bank i.

Secondly, when these individual banks' risk-adjusted variance has the same order as the Sharpe ratio, i.e., a higher risk-adjusted variance is consistent with a higher Sharpe ratio, we say that the risk-adjusted variance is co-monotonic to the Sharpe ratio. In this case, the bank sector is in an ordering loss market, because a higher expected loss ensures a higher variance. Proposition 4 states that classical insurance is a better contract from the regulator's perspective in the ordering loss market.

Thirdly, in the disordering loss market in which a higher risk-adjusted variance is always linked to a smaller Sharpe ratio, in the presence of a few banks with very large expected loss, Proposition 4 ensures that aggregate insurance is the more beneficial insurance contract. To explain this, say bank 1 is big enough, such that $\mathbb{E}[X_1] \gg \mathbb{E}[X_2], \cdots, \mathbb{E}[X_n]$[10]. In this case, bank 1's expected loss is so big that the total expected systematic loss, $\mathbb{E}[X]$, is close to $\mathbb{E}[X_1]$; then, $\mathbb{E}[X]^2$ is close enough to $\sum \mathbb{E}[X_i]^2$. Therefore, the aggregate insurance issued to other banks with small losses together would benefit the regulator.

We next move to the more interesting situation in which each loss contributes to the systematic risk, so that these loss portfolios are correlated.

3. Systemic Risk and Comparative Analysis

In this section, we examine closely which insurance contract should be preferred to another one from the perspective of the regulator, as well as the bank. For this purpose, we assume that the contribution of each bank to the market risk is given exogenously. It is natural to examine the question in a one-factor model. A multi-factor model shares the same insights as a one-factor model.

Suppose $X_i = \alpha_i Y + \epsilon_i$, where ϵ_i is white noise with zero mean and variance σ_i^2. Y represents a market (or systematic) risk factor, and each ϵ_i represents the specific risk of bank i. The aggregate loss $X = \sum_i \alpha_i Y + \sum_i \epsilon_i = \alpha Y + \epsilon$, where $\alpha = \sum_{i=1}^n \alpha_i$. Write $\hat{X}_i = \hat{\alpha}_i Y + \hat{\epsilon}_i$, where $\epsilon = \sum_{i=1}^n \epsilon_i$, $\hat{\alpha}_i = \sum_{j=1, j\neq i}^n \alpha_j, \hat{\epsilon}_i = \sum_{j=1, j\neq i}^n \epsilon_j$.

We first consider one special case for which specific risks equal zero. By using Equations (8), (15) and (21), we have the following result.

Proposition 5 *If there is no specific risk in the market, then the welfare is equivalent for all three types of insurance contracts. Precisely, if each $\sigma_i = 0$, then:*

$$\mathbb{E}(W^{*,a}) = \mathbb{E}(W^{*,c}) = \mathbb{E}(W^{*,ac}) = \frac{Var(Y)}{4\sum_i \gamma_i}\alpha^2 > 0 \tag{23}$$

[10] We write $x \gg y$ to denote $y/x \to 0$.

In general, when the systematic risk factor is highly volatile, that is, $Var(Y)$ is high, then these three contracts offer the same welfare asymptotically. Precisely, when $Var(Y) \to \infty$[11]*:*

$$\mathbb{E}(W^{*,a}) \sim \mathbb{E}(W^{*,c}) \sim \mathbb{E}(W^{*,ac}) \sim \frac{Var(Y)}{4\sum_i \gamma_i}\alpha^2. \tag{24}$$

Proof: See Appendix A. □

Proposition 5 states that if $Var(Y)$ is extremely large relative to a company's specific risk, then from the regulator's perspective, the welfare of all three types of insurance contracts is almost identical and positively depends on both $Var(Y)$ and the aggregate contribution of all banks to the market risk, $\sum_i \alpha_i$. Alternatively, when the individual risks are immaterial compared to the systematic risk, these three contracts, in essence, provide the same welfare. Therefore, the capital insurance idea does not work particularly well under some circumstances with an extremely high systemic risk factor or extremely small specific risks.

Proposition 6 *If the risk-adjusted individual risk vector, $\left(\frac{Var(X_i)}{\gamma_i}\right)$, is co-monotonic to the Sharpe ratio vector, $\left(\frac{\mathbb{E}[X_i]}{\sqrt{Var(X_i)}}\right)$, then the classical insurance is preferred to the aggregate insurance in the sense that $\mathbb{E}[W^{*,a}] < \mathbb{E}[W^{*,c}]$.*

If the risk-adjusted individual risk vector, $\left(\frac{Var(X_i)}{\gamma_i}\right)$, is counter-monotonic to the Sharpe ratio vector, $\left(\frac{\mathbb{E}[X_i]}{\sqrt{Var(X_i)}}\right)$, and the expected aggregate loss, $\mathbb{E}[X]$, is large enough, then the aggregate insurance is preferred to the classical insurance in the sense that $\mathbb{E}[W^{*,c}] < \mathbb{E}[W^{*,a}]$.

Proof: See Appendix A. □

Proposition 6 has the same insight as Proposition 4, but Proposition 6 holds in a general correlated market environment. In the ordering loss market, such that a higher risk-adjusted variance corresponds to a Sharpe ratio, classical insurance works better. In the disordering loss market, however, the aggregate insurance contract should be preferred to the classical one when the expected total risk, $\mathbb{E}[X]$, is a big concern. Indeed, both Proposition 4 and Proposition 6 demonstrate in different market situations that aggregate insurance is a good design when the individual risk and the Sharpe ratio display a negative relationship for each bank.

To finish this section, we compare aggregate-cross insurance with classical insurance.

Proposition 7 *If the expected losses across the banks are fairly close, the risk-adjusted variance is co-monotonic to the Sharpe ratio, and the risk-adjusted correlated variance, $\rho_i^2\frac{Var(X_i)}{\gamma}$, is co-monotonic to the Sharpe ratio of its dual risk, $\frac{\mathbb{E}[\hat{X}_i]}{\sqrt{Var(\hat{X}_i)}}$, where ρ_i is the correlation coefficient between X_i and $\hat{X}_i$ for each $i = 1, \cdots, N$; then $\mathbb{E}[W^{*,ac}] < \mathbb{E}[W^{*,c}]$.*

Proof: See Appendix A. □

As shown in Proposition 6, the classical insurance is preferred to the aggregative insurance when risk-adjusted variance is co-monotonic to the Sharpe ratio. Therefore, Proposition 7 shows us that

[11] By two functions, $f \sim g$, we mean that $\lim_{Var(Y)\to\infty} \frac{f}{g} = 1$.

both the aggregative-type of insurances (i.e., aggregate and aggregate-cross insurance contracts) are not supportive under the situations described in Proposition 7.

4. Discussion

Under what circumstance should capital insurance programs be implemented and how? In this section, we show several important insights based on our theoretical results.

4.1. Disordering Loss Market and Ordering Loss Market

According to Proposition 4 and Proposition 6, based on our welfare analysis, the aggregate insurance contract should be insured by the regulator in the disordering loss market. When the individual risk of loss, $Var(X_i)$, is mismatched with the expected loss per unit, the loss in each bank displays the disordering loss market.

There are two important situations in which the disordering loss market occurs. The *first* situation is when the contribution to the aggregate loss of each back is fairly close, and each bank has a fairly close preference for risk. In other words, when the aggregate loss is almost equally distributed among the banks, it is a disordering loss market. To see this, we assume $\gamma_i = \gamma$ for all i. Clearly, the risk-adjusted variance, $\frac{Var(X_i)}{\gamma_i}$, is counter-monotonic to $\frac{\mathbb{E}[X_i]}{\sqrt{Var(X_i)}}$. Therefore, both Proposition 4 and Proposition 6 ensure that aggregate insurance is better than the classical insurance contract.

We describe the *second* situation in a one-factor model. We argue that when the individual risk mainly comes from the specific risk in each bank, this is another example of the disordering loss market. Write $X_i = \alpha_i Y + \epsilon_i, i = 1, \cdots, N$. When a higher individual risk, $Var(X_i)$, corresponds to a higher $\frac{Var(\epsilon_i)}{Var(X_i)}$, the market can be described as the "*disordering loss market*". To demonstrate, we assume, again, $\gamma_i = \gamma$ for all i. Note that $\frac{Var(X_i)}{\mathbb{E}[X_i]^2} = Var(Y) + \left(\frac{\sigma_i}{\alpha_i}\right)^2$, and $\frac{Var(\epsilon_i)}{Var(X_i)}$ is increasing with respect to $\frac{\sigma_i}{\alpha_i}$. Then, under this assumption, $\frac{Var(X_i)}{\gamma_i}$ is co-monotonic to $\frac{Var(X_i)}{\mathbb{E}[X_i]^2}$ and, thus, counter-monotonic to $\frac{\mathbb{E}[X_i]}{\sqrt{Var(X_i)}}$; this is a disordering loss market. Hence, aggregate insurance is a better insurance program when specific risk plays a dominate role inside individual risk.

Table 1 demonstrates the first situation as described. There are 10 big banks in the market, and each bank has the same expected loss as $\alpha_i = 0.1$ for all $i = 1, \cdots, 10$. For simplicity, we assume that the variance of the systematic risk factor, Y, equals one and each $\gamma_i = 1$. However, the specific risk in each bank varies from 10% to 40%. Table 1 displays the negative relationship between the risk-adjusted variance and the Sharpe ratio of the loss portfolios among these 10 banks. Therefore, Table 1 shows one example of the disordering loss market, and we know that aggregate insurance is a preferred program by Proposition 6. Moreover, by numerical computations, $\frac{Cov(X_i,X)}{Var(X)} > 0.06 > \frac{1}{2N}$ for each $i = 1, \cdots, N$. Hence, the equilibrium of the aggregate insurance is given explicitly in Proposition 1.

The second situation is shown in Table 2, in which $\frac{\alpha}{\sigma}$ is increasing with respect to α. In this case, these banks have different expected loss, ranging from $0.1\mathbb{E}[Y]$ to $0.55\mathbb{E}[Y]$. As shown, there is a negative relationship between the risk-adjusted variance and the Sharpe ratio of the loss portfolios among these 10 banks; hence, Table 2 shows another example of the disordering loss market. By numerical computations, $\frac{Cov(X_i,X)}{Var(X)} > 0.08 > \frac{1}{2N}$ for each $i = 1, \cdots, N$. Hence, the equilibrium of the aggregate insurance is given explicitly in Proposition 1.

Table 1. Example 1 of a disordering loss market. This table displays a disordering loss market when each bank has the same expected loss in a one-factor model. Therefore, aggregate insurance is a better capital insurance program by Proposition 6. It can be checked that the condition in Proposition 1 is satisfied, so the equilibrium of the aggregate insurance is given in Proposition 1. We assume $\gamma_i = 1$ for each $i = 1, \cdots, N$. There are $N = 10$ banks.

Bank	α	σ	Risk-Adjusted Variance	Sharpe Ratio
1	0.1	0.40	0.170	0.243
2	0.1	0.35	0.133	0.275
3	0.1	0.30	0.100	0.316
4	0.1	0.26	0.078	0.359
5	0.1	0.23	0.063	0.399
6	0.1	0.20	0.050	0.447
7	0.1	0.18	0.042	0.486
8	0.1	0.15	0.033	0.555
9	0.1	0.12	0.024	0.640
10	0.1	0.10	0.020	0.707

Table 2. Example 2 of a disordering loss market. This table displays a disordering loss market when the percentage of specific risk in individual risk increases with respect to individual risk. Therefore, aggregate insurance is a better insurance program than the classical insurance program by Proposition 6. It can be checked that the condition in Proposition 1 is satisfied, so the equilibrium of the aggregate insurance is given in Proposition 1. We assume $\gamma_i = 1$ for each $i = 1, \cdots, N$. There are $N = 10$ banks.

Bank	α	σ	Risk-Adjusted Variance	Sharpe Ratio
1	0.10	0.200	0.050	0.447
2	0.15	0.315	0.122	0.430
3	0.20	0.440	0.234	0.414
4	0.25	0.575	0.393	0.399
5	0.30	0.720	0.608	0.385
6	0.35	0.875	0.888	0.371
7	0.40	1.040	1.242	0.359
8	0.45	1.215	1.679	0.347
9	0.50	1.400	2.210	0.336
10	0.55	1.595	2.847	0.326

On the other hand, when the individual risk, $Var(X_i)$, is opposite of the percentage of the specific risk, $\frac{\sigma_i^2}{Var(X_i)}$, classical insurance is better. In general, when a higher systemic risk corresponds to a smaller specific risk, classical insurance is better than aggregate insurance. Table 3 displays an example of the ordering loss market in which the classical insurance program should be preferred to aggregate insurance.

Table 3. An Example of an ordering loss market. This table displays an ordering loss market when the percentage of specific risk in individual risk decreases with respect to individual risk. Therefore, classical insurance is a better insurance program than the aggregate insurance program by Proposition 6. We assume $\gamma_i = 1$ for each $i = 1, \cdots, N$. There are $N = 10$ banks.

Bank	α	σ	Risk-adjusted Variance	Sharpe ratio
1	0.10	0.400	0.170	0.243
2	0.15	0.350	0.145	0.394
3	0.20	0.300	0.130	0.555
4	0.25	0.260	0.130	0.693
5	0.30	0.230	0.143	0.794
6	0.35	0.200	0.163	0.868
7	0.40	0.180	0.192	0.912
8	0.45	0.150	0.225	0.949
9	0.50	0.120	0.264	0.972
10	0.55	0.100	0.313	0.984

Through these examples, we have shown that specific risk is critical in comparing those capital insurance programs. If the specific risks can be ignored, these three insurance contracts offer similar welfare. Equivalently, when the systematic risk is *extremely large*, it does not matter which capital insurance program should be issued, as is demonstrated by Proposition 5.

4.2. Low Correlation Market and High Correlation Market

The correlation structure affects the capital insurance program. On the one hand, we have seen by Proposition 4 that aggregate-cross insurance is not a good choice in a low-correlated market. A low correlation parameter comes from large specific risks. In other words, if specific risks are sufficiently large enough compared with the systemic risk component, aggregate-cross insurance does not add welfare. On the other hand, when the specific risks are very small, Proposition 5 ensures that aggregate-cross insurance does not add welfare over aggregate insurance either. Low specific risks correspond to a high (or even perfectly correlated) correlation coefficient among the loss portfolios. Therefore, aggregate-cross insurance does not work better in either a low or a high correlation environment under Assumption I and Assumption II.

Actually, in the absence of asymmetric information, we argue that aggregate-cross insurance does not work better than aggregate insurance in general. To see this, we assume that α_i is the same for all i and σ_i is the same for all i. Then, each pair of banks has the same correlation coefficient written as τ. By straightforward calculation, we have:

$$\mathbb{E}(W^{*,ac}) = \tau^2 \mathbb{E}(W^{*,a}) = \tau^2 \mathbb{E}(W^{*,c}). \tag{25}$$

Therefore, the lower the correlation coefficient τ, the smaller expected welfare of the aggregate-cross insurance. Overall, $\mathbb{E}(W^{*,ac}) < \mathbb{E}(W^{*,a}) = \mathbb{E}(W^{*,c})$. When all banks contribute to systematic risk equally, then specific risks are also similar; the aggregate-cross insurance is not as good as the two other insurance programs.

4.3. Systemic Risk

There are many different interpretations about systemic risk. Some authors suggest using the default probability of the whole financial system (see, for instance, [10]). Other authors suggest using Shapley values to estimate systemic risk (see [3]).[12] It is beyond the scope of this paper to develop a systemic risk theory, as we focus on the effect of capital insurance. Rather, we indicate that aggregate insurance is a useful tool to deal with systemic risk by using two interpretations of systemic risk.

First, we view systemic risk as the likelihood of aggregate loss meeting a threshold. Precisely, the higher probability $P(X \geq L)$, the higher the systemic risk. In aggregate insurance, post-aggregate insurance becomes (by using Equation (10)):

$$\sum X_i - \sum \beta_i X = \frac{1}{2} X. \tag{26}$$

Clearly, the *ex post* aggregate loss is smaller than the *ex ante* aggregate loss, X. Therefore, aggregate insurance, indeed, reduces systemic risk.

Second, we consider the systemic risk for each individual bank in a one-factor model. Before purchasing aggregate insurance, the systematic risk contribution of the bank, i, is α_i. We assume that γ_i is the same across the banks. Then, the coinsurance percentage for the bank, i, is:

$$\beta_i \geq \frac{\alpha_i \alpha Var(Y)}{\alpha^2 Var(Y) + \sigma^2} - \frac{1}{2N}. \tag{27}$$

Hence, the contribution to the systematic risk of the bank, i, after purchasing aggregate insurance, is:

$$\alpha_i - \beta_i \alpha \leq \frac{\alpha_i \sigma^2}{\alpha^2 Var(Y) + \sigma^2} + \frac{1}{2} \frac{\alpha}{N}. \tag{28}$$

When the number of banks, N, is large enough or when the variability of the systemic risk, $Var(Y)$, is sufficiently large, we see that $\alpha_i - \beta_i \alpha < \alpha_i$. Therefore, the systemic risk of each bank, i, is reduced after purchasing aggregate insurance.

4.4. Identification and Implementation of "Too Big to Fail"

Suppose the disordering loss market occurs; according to our theory, the aggregate insurance program is a desired regulatory tool to solve the "too big to fail" issue. Nevertheless, there are two fundamental questions to be solved as follows.

1. How to implement the aggregate insurance program, i.e., how to characterize the equilibrium in a general situation.

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[12] See [5,7,8].

2. How to distinguish the "too big to fail" banks that are forced to purchase aggregate insurance from the other banks. Alternatively, how to identify those "too big to fail" banks.

We illustrate our solutions to these questions by an example, while a general solution is given in Appendix B.

To explain the answers to the questions above, we consider 15 banks, and the loss portfolio of each bank follows a one-factor model. The systematic risk factor is represented by Y with $\mathbb{E}[Y] = Var(Y) = 1$. Each bank has the same expected loss, $0.05\mathbb{E}[Y]$, but the specific risk varies differently. In fact, σ_i moves from 40% to 12%. Proposition 6 implies that aggregate insurance is more desirable than classical insurance. It is also easy to see that $\frac{Cov(X_i,X)}{\gamma_i}$ is decreasing from $i = 1$ to $i = 15$. However, as shown in Table 4, condition (7) in Proposition 1 is not always satisfied. To be precise, for the last five banks, $\frac{Cov(X_i,X)}{Var(X)} < \frac{1}{2N}, i = 11, 12, 13, 14, 15$.

Table 4. Example 3 of a disordering loss market. This table displays a disordering loss market when each bank has the same expected loss in a one-factor model. Therefore, aggregate insurance is a better program by Proposition 6. However, the condition in Proposition 1 is not satisfied, as shown for $i = 11, 12, \cdots, 15$. There are $N = 15$ banks, and each $\gamma_i = 1$.

Bank	α	σ	**Risk-Adjusted Variance**	**Sharpe Ratio**	$\frac{Cov(X_i,X)}{Var(X)}$
1	0.05	0.40	0.1625	0.124	0.1170
2	0.05	0.38	0.1469	0.130	0.1070
3	0.05	0.36	0.1321	0.138	0.0990
4	0.05	0.34	0.1181	0.145	0.0907
5	0.05	0.32	0.1049	0.154	0.0829
6	0.05	0.30	0.0925	0.164	0.0755
7	0.05	0.28	0.0809	0.176	0.0686
8	0.05	0.26	0.0701	0.189	0.0622
9	0.05	0.24	0.0601	0.204	0.0563
10	0.05	0.22	0.0509	0.222	0.0509
11	0.05	0.20	0.0425	0.243	0.0459
12	0.05	0.18	0.0349	0.268	0.0414
13	0.05	0.16	0.0281	0.298	0.0374
14	0.05	0.14	0.0221	0.336	0.0338
15	0.05	0.12	0.0169	0.385	0.0307

Appendix B presents a general solution of the equilibrium without Condition (7). The equilibrium problem and how to identify the "too big to fail" problem are solved simultaneously. As the risk-adjusted covariance sequence, $\frac{Cov(X_i,X)}{\gamma_i}$, is decreasing for $i = 1, \cdots, N$, we know that the sequence, $\frac{\sum_{j=1}^{i} Cov(X_j,X)}{2\sum_{j=1}^{i}\gamma_j}$, is decreasing for $i = 1, \cdots, N$, as well. The first step is to find a unique number, n, such that:

$$\frac{Cov(X_i, X)}{\sum_{k=1}^{n} Cov(X_k, X)} \geq \frac{\gamma_i}{2\sum_{k=1}^{n}\gamma_k}, i = 1, \cdots, n \tag{29}$$

and

$$\frac{Cov(X_i, X)}{\sum_{k=1}^{n} Cov(X_k, X)} < \frac{\gamma_i}{2\sum_{k=1}^{n}\gamma_k}, i = n+1, \cdots, N. \tag{30}$$

In this example, we find out that $n = 13$ (see Table 5). Therefore, the first 13 banks, but not the first 10 banks, are "too big to fail" banks that should be required to purchase the aggregate insurance. The last two banks can be ignored in this aggregate insurance program. The second step is to determine the optimal load factor, ρ^*, in the aggregate insurance program, which is:

$$\rho^* = \frac{1}{\mathbb{E}[X]} \frac{\sum_{i=1}^n Cov(X_i, X)}{2\sum_{i=1}^n \gamma_i} = 0.081. \tag{31}$$

At last, the optimal co-insurance parameters for the first 13 banks are:

$$\beta^{i,a}(\rho^*) = \frac{Cov(X_i, X) - \rho^*\gamma_i \mathbb{E}[X]}{Var(X)}, i = 1, \cdots, 13. \tag{32}$$

The last two banks do not buy the aggregate insurance as $\beta^{i,a}(\rho^*) = 0, i = 14, 15$. The equilibrium and relevant computation are displayed by Table 5. We observe that the optimal co-insurance parameter decreases with respect to $\frac{Cov(X_i,X)}{\gamma_i}$, a measure of the systemic risk of these "too big to fail".

Table 5. Implementation of Example 3. This table displays the equilibrium of Example 3. We note that when i starts from 14, $\frac{Cov(X_i,X)}{\gamma_i}$ is strictly greater than $\frac{\sum_{j=1}^i Cov(X_j,X)}{2\sum_{j=1}^i \gamma_j}$. Then, the last two banks are not "too big to fail". The optimal load factor is $\rho^* = 8.1\%$.

Bank	$\frac{Cov(X_i,X)}{\gamma_i}$	$\frac{Cov(X_i,X)}{Var(X)}$	$\frac{\sum_{j=1}^i Cov(X_j,X)}{2\sum_{j=1}^i \gamma_j}$	$\beta^{i,a}$
1	0.1975	0.1170	0.09875	8.10 %
2	0.1819	0.1070	0.09485	7.18 %
3	0.1671	0.0990	0.09108	6.30 %
4	0.1531	0.0907	0.08745	5.47 %
5	0.1399	0.0829	0.08395	4.69 %
6	0.1275	0.0755	0.08058	3.95 %
7	0.1159	0.0686	0.07735	3.27 %
8	0.1051	0.0622	0.07425	2.63 %
9	0.0951	0.0563	0.07128	2.03 %
10	0.0859	0.0509	0.06845	1.49 %
11	0.0775	0.0459	0.06575	0.99 %
12	0.0699	0.0414	0.06318	0.54 %
13	0.0631	0.0374	0.06075	0.14 %
14	0.0571	0.0338	0.05845	0
15	0.0519	0.0307	0.05628	0

5. Conclusion

In this paper, we present a welfare analysis of several capital insurance programs in equilibrium. We show that aggregate insurance ensures a higher welfare if each big bank has similar systematic risk. The classical insurance program, however, has a higher welfare when the individual bank's risk is positively related to the expected loss per each volatility unit. In general, aggregate-cross insurance does not add more welfare if there exists no asymmetric information concern. Overall, we demonstrate that the capital insurance program is a useful regulatory tool to address the "too big to fail" issue.

Acknowledgements

We are grateful to two anonymous referees for comments which greatly improved the exposition.

Conflicts of Interest

The authors declare no conflict of interest.

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Appendix A. Proofs

The proofs rely on the following simple lemma.

Lemma 1 *Given positive numbers b_i, c_i, κ_i for each $i = 1, \cdots, n$,*

1. *If the vector $\kappa = (\kappa_i)$ is co-monotonic to the vector $\frac{b}{c} = (\frac{b_i}{c_i})$, then:*

$$\frac{\sum_{i=1}^n b_i\kappa_i}{\sum_{i=1}^n b_i} > \frac{\sum_{i=1}^n c_i\kappa_i}{\sum_{i=1}^n c_i}.$$

2. *If the vector $\kappa = (\kappa_i)$ is counter-monotonic to the vector $\frac{b}{c} = (\frac{b_i}{c_i})$, then:*

$$\frac{\sum_{i=1}^n b_i\kappa_i}{\sum_{i=1}^n b_i} < \frac{\sum_{i=1}^n c_i\kappa_i}{\sum_{i=1}^n c_i}.$$

Proof: $\sum b_i\kappa_i \sum c_i - \sum b_i \sum c_i\kappa_i = \sum_{i,j} b_i\kappa_i c_j - \sum_{i,j} b_j c_i \kappa_i = \sum_{i,j}(b_i c_j - b_j c_i)\kappa_i = \sum_{i,j,i<j}(b_i c_j - b_j c_i)(\kappa_i - \kappa_j) = \sum_{i,j,i<j} c_i c_j \left(\frac{b_i}{c_i} - \frac{b_j}{c_j}\right)(\kappa_i - \kappa_j)$. □

Given a vector $a = (a_1, \cdots, a_n)$, we use $VAR(a) = \sum a_i^2 - (\sum a_i)^2$ to represent the variability of the vector, a. A small $VAR(a)$ means that those components in a are close to each other. Similarly, we write $\mathbb{E}[a] = \sum a_i$. It is easy to see that $VAR(a) = \frac{1}{2}\sum(a_i - a_j)^2$.

Lemma 2 *Given two sequences of positive numbers, $a_i, b_i, i = 1, 2, \cdots, n$,*

- *If those numbers, $a_1, \cdots, a_n$, are close enough in the sense that $VAR(a) \le \mathbb{E}[a]^2 VAR(b)/\mathbb{E}[b]^2$, then $\frac{\sum a_i^2}{\sum b_i^2} \le \frac{(\sum a_i)^2}{(\sum b_i)^2}$.*
- *If those numbers, $b_1, \cdots, b_n$, are close enough in the sense that $VAR(b) \le \mathbb{E}[b]^2 VAR(a)/\mathbb{E}[a]^2$, then $\frac{\sum a_i^2}{\sum b_i^2} \ge \frac{(\sum a_i)^2}{(\sum b_i)^2}$.*

Proof: By straightforward calculation, we obtain:

$$\begin{aligned}\sum a_i^2(\sum b_i)^2 - (\sum a_i)^2 \sum b_i^2 &= \frac{1}{2}\left\{\sum_{i,j,k}(a_i - a_j)^2 b_k^2 - \sum_{i,j,k} a_i^2(b_j - b_k)^2\right\} \\ &= \sum b_i^2 VAR(a) - \sum a_i^2 VAR(b). \end{aligned} \tag{A-1}$$

When the numbers, a_i, are close enough, the first term in (A-1) is dominated by the second term. This is the first case. It is the classical Cauchy-Schwartz inequality when $a_1 = \cdots = a_n$. In the second case, the second term is close to zero. □

Proof of Proposition 4. Under the uncorrelated assumption, $\mathbb{E}[W^{*,a}] = \frac{\sum Var(X_i)}{4\sum \gamma_i}$. As $\mathbb{E}[X_i] \ge 0$ for each i, we have $\mathbb{E}[W^{*,c}] \ge \frac{\sum \mathbb{E}[X_i]^2}{4\sum_i \gamma_i \mathbb{E}[X_i]^2/Var(X_i)}$. For each $i \ne j$, if $\frac{Var(X_i)}{\gamma_i} < \frac{Var(X_j)}{\gamma_j}$, then by the co-monotonic assumption, $\frac{\mathbb{E}[X_i]}{\sqrt{Var(X_i)}} \le \frac{\mathbb{E}[X_j]}{\sqrt{Var(X_j)}}$. So, $\frac{\mathbb{E}[X_i]^2}{Var(X_i)} \le \frac{\mathbb{E}[X_j]^2}{Var(X_j)}$. Therefore:

$$\frac{Var(X_i)}{\mathbb{E}[X_i]^2} \ge \frac{Var(X_j)}{\mathbb{E}[X_j]^2}. \tag{A-2}$$

This means that vectors $\left(\frac{Var(X_i)}{\gamma_i}\right)$ and $\left(\frac{Var(X_i)}{\mathbb{E}[X_i]^2}\right)$ are counter-monotonic. Then, by Lemma 1, we obtain (using $b_i = \gamma_i, c_i = \gamma_i \mathbb{E}[X_i]^2/Var(X_i)$ and $\kappa_i = Var(X_i)/\gamma_i$):

$$\frac{\sum \mathbb{E}[X_i]^2}{\sum_i \gamma_i \mathbb{E}[X_i]^2/Var(X_i)} > \frac{\sum Var(X_i)}{\sum \gamma_i}. \tag{A-3}$$

We have proven the first part. As for the second part, assume that the risk-adjusted variance is counter-monotonic to the Sharpe ratio vector. Then, by the same idea, we have that:

$$\frac{\sum \mathbb{E}[X_i]^2}{\sum_i \gamma_i \mathbb{E}[X_i]^2/Var(X_i)} < \frac{\sum Var(X_i)}{\sum \gamma_i} = \mathbb{E}[W^{*,a}]. \tag{A-4}$$

Therefore, when $\mathbb{E}[X]^2$ is close to $\sum \mathbb{E}[X_i]^2$, we obtain that $\mathbb{E}[W^{*,c}] \leq \mathbb{E}[W^{*,a}]$. The proof is complete. □

Proof of Proposition 5. The welfare of each insurance contract in the one-factor model is computed as follows.

$$\mathbb{E}(W^{*,a}) = \frac{1}{4}\frac{\alpha^2 Var(Y) + \sigma^2}{\sum_i \gamma_i} \tag{A-5}$$

$$\mathbb{E}(W^{*,c}) = \frac{1}{4}\frac{\alpha^2}{\sum_i \gamma_i \dfrac{\alpha_i^2}{\alpha_i^2 Var(Y) + \sigma_i^2}} \tag{A-6}$$

and:

$$\mathbb{E}(W^{*,ac}) = \frac{1}{4}\frac{\left(\sum_i \hat{\alpha}_i E(Y) \dfrac{\alpha_i \hat{\alpha}_i Var(Y)}{\hat{\alpha}_i^2 Var(Y) + \hat{\sigma}_i^2}\right)^2}{\sum_i \gamma_i \dfrac{\hat{\alpha}_i^2 E(Y)^2}{\hat{\alpha}_i^2 Var(Y) + \hat{\sigma}_i^2}} = \frac{1}{4}\frac{\left(\sum_i \dfrac{\alpha_i \hat{\alpha}_i^2 Var(Y)}{\hat{\alpha}_i^2 Var(Y) + \hat{\sigma}_i^2}\right)^2}{\sum_i \gamma_i \dfrac{\hat{\alpha}_i^2}{\hat{\alpha}_i^2 Var(Y) + \hat{\sigma}_i^2}}. \tag{A-7}$$

Clearly, when the total $\sigma^2 = 0$, the welfare is identical for all three types of contracts. The second part follows from the same idea. □

Proof of Proposition 6.

First, note that $\alpha^2 \geq \sum \alpha_i^2$, and the function $f(x) \equiv \frac{x^2 Var(Y) + \sigma^2}{x^2}$ is decreasing with respect to x. Then:

$$\frac{\alpha^2 Var(Y) + \sigma^2}{\alpha^2} \leq \frac{\sum(\alpha_i^2 Var(Y) + \sigma_i^2)}{\sum \alpha_i^2}. \tag{A-8}$$

To prove $\mathbb{E}[W^{*,a}] < \mathbb{E}[W^{*,c}]$ under the co-monotonic condition, it suffices to show that:

$$\frac{\sum(\alpha_i^2 Var(Y) + \sigma_i^2)}{\sum \gamma_i} < \frac{\sum \alpha_i^2}{\sum_i \gamma_i \dfrac{\alpha_i^2}{\alpha_i^2 Var(Y) + \sigma_i^2}}. \tag{A-9}$$

In fact, by using the co-monotonic relationship between the risk-adjusted variance and the Sharpe ratio, the risk-adjusted variance is counter-monotonic to the vector, $\left(\frac{Var(X_i)}{\mathbb{E}[X_i]^2}\mathbb{E}[Y]^2\right)$. Note that $\mathbb{E}[X_i] = \alpha_i \mathbb{E}[Y]$ and $Var(X_i) = \alpha_i^2 Var(Y) +$

σ_i^2. Then, the last inequality (A-9) follows from Lemma 1 for $b_i = \gamma_i$, $c_i = \gamma_i \dfrac{\alpha_i^2}{\alpha_i^2 Var(Y) + \sigma_i^2}$, and $\kappa_i = Var(X_i)/\gamma_i$.

If the risk-adjusted variance is counter-monotonic to the Sharpe ratio across the banks, then by the same proof, we obtain:

$$\frac{\sum(\alpha_i^2 Var(Y) + \sigma_i^2)}{\sum \gamma_i} > \frac{\sum \alpha_i^2}{\sum_i \gamma_i \dfrac{\alpha_i^2}{\alpha_i^2 Var(Y) + \sigma_i^2}}. \tag{A-10}$$

For a large positive number, x, $f'(x) = -\frac{2\sigma^2}{x^3}$ is close to zero; so, the curve $y = f(x)$ is almost flat. Then, for a large $\mathbb{E}[X]$, the numbers, $\frac{\alpha^2 Var(Y)+\sigma^2}{\alpha^2}$ and $\frac{\sum(\alpha_i^2 Var(Y)+\sigma_i^2)}{\sum \alpha_i^2}$, are close enough that:

$$\frac{\alpha^2 Var(Y) + \sigma^2}{\alpha^2} \sim \frac{\sum(\alpha_i^2 Var(Y) + \sigma_i^2)}{\sum \alpha_i^2} > \frac{\sum \gamma_i}{\sum_i \gamma_i \dfrac{\alpha_i^2}{\alpha_i^2 Var(Y) + \sigma_i^2}}.$$

Equivalently, $\mathbb{E}[W^{*,a}] > \mathbb{E}[W^{*,c}]$. □

Proof of Proposition 7. As the risk-adjusted variance is co-monotonic to the Sharpe ratio across each bank, Lemma 1 yields that:

$$\frac{1}{4}\frac{\sum \mathbb{E}[X_i]^2}{\sum \gamma_i \mathbb{E}[X_i]^2/Var(X_i)} > \frac{1}{4}\frac{\sum Var(X_i)}{\sum_i \gamma_i}. \tag{A-11}$$

By using the Cauchy-Schwartz inequality, $Var(X_i)Var(\hat{X}_i) \geq Cov(X_i, \hat{X}_i)^2$ for each i. We obtain:

$$\frac{1}{4}\frac{\sum \mathbb{E}[X_i]^2}{\sum \gamma_i \mathbb{E}[X_i]^2/Var(X_i)} > \frac{1}{4}\frac{\sum Cov(X_i, \hat{X}_i)^2/Var(\hat{X}_i)}{\sum_i \gamma_i}. \tag{A-12}$$

Note that $\frac{Cov(X_i,\hat{X}_i)^2}{Var(\hat{X}_i)\gamma_i} = \rho_i^2 \frac{Var(X_i)}{\gamma}$, where ρ_i is the correlation coefficient between X_i and $\hat{X}_i$. If the Sharpe ratio of the "dual" risk $\frac{\mathbb{E}[\hat{X}_i]}{\sqrt{Var(\hat{X}_i)}}$ is counter-monotonic to the risk-adjusted correlated variance, $\rho_i^2 \frac{Var(X_i)}{\gamma}$, then $\rho_i^2 \frac{Var(X_i)}{\gamma}$ is co-monotonic to $\frac{Var(\hat{X}_i)}{\mathbb{E}[\hat{X}_i]^2}$. Again by Lemma 1 (for $b_i = \gamma_i, c_i = \gamma_i \frac{\mathbb{E}[\hat{X}_i]^2}{Var(\hat{X}_i)}$ and $\kappa_i = \rho_i^2 \frac{Var(X_i)}{\gamma}$), we have:

$$\frac{\sum Cov(X_i, \hat{X}_i)^2/Var(\hat{X}_i)}{\sum_i \gamma_i} > \frac{\sum \mathbb{E}[\hat{X}_i]^2 \frac{Cov(X_i,\hat{X}_i)^2}{Var(\hat{X}_i)^2}}{\sum \gamma_i \frac{\mathbb{E}[\hat{X}_i]^2}{Var(\hat{X}_i)}} \tag{A-13}$$

By combining (A-12) with (A-13) together, we obtain:

$$\frac{\sum \mathbb{E}[X_i]^2}{\sum \gamma_i \frac{\mathbb{E}[X_i]^2}{Var(X_i)}} > \frac{\sum \mathbb{E}[\hat{X}_i]^2 \frac{Cov(X_i,\hat{X}_i)^2}{Var(\hat{X}_i)^2}}{\sum \gamma_i \frac{\mathbb{E}[\hat{X}_i]^2}{Var(\hat{X}_i)}}. \tag{A-14}$$

Equivalently:

$$\frac{\sum \mathbb{E}[X_i]^2}{\sum \mathbb{E}[\hat{X}_i]^2 \frac{Cov(X_i,\hat{X}_i)^2}{Var(\hat{X}_i)^2}} > \frac{\sum \gamma_i \frac{\mathbb{E}[X_i]^2}{Var(X_i)}}{\sum \gamma_i \frac{\mathbb{E}[\hat{X}_i]^2}{Var(\hat{X}_i)}}. \tag{A-15}$$

When $\mathbb{E}[X_i]$ is distributed equally, or the expected losses are fairly close enough, Lemma 2 ensures that:

$$\frac{(\sum \mathbb{E}[X_i])^2}{\left(\sum \mathbb{E}[\hat{X}_i]^2 \frac{Cov(X_i,\hat{X}_i)}{Var(\hat{X}_i)}\right)^2} > \frac{\sum \mathbb{E}[X_i]^2}{\sum \mathbb{E}[\hat{X}_i]^2 \frac{Cov(X_i,\hat{X}_i)^2}{Var(\hat{X}_i)^2}}. \tag{A-16}$$

Finally, by using (A-15) and (A-16), we obtain:

$$\frac{(\sum \mathbb{E}[X_i])^2}{\left(\sum \mathbb{E}[\hat{X}_i]^2 \frac{Cov(X_i,\hat{X}_i)}{Var(\hat{X}_i)}\right)^2} > \frac{\sum \gamma_i \frac{\mathbb{E}[X_i]^2}{Var(X_i)}}{\sum \gamma_i \frac{\mathbb{E}[\hat{X}_i]^2}{Var(\hat{X}_i)}}. \tag{A-17}$$

By using Proposition 2 and Proposition 3, we obtain that $\mathbb{E}[W^{*,c}] > \mathbb{E}[W^{*,ac}]$. □

Appendix B. A General Solution of the Equilibrium of Aggregate Insurance

The regulator's problem is to solve the optimal load factor, such as:

$$\rho^* = argmax_{\{\rho \geq 0\}} \rho \sum_{i=1}^{N} \max\left(\frac{Cov(X_i,X) - \rho\gamma_i \mathbb{E}[X]}{Var(X)}, 0\right) \tag{B-1}$$

and the optimal coinsurance percentage for each bank $i = 1, \cdots, N$ is:

$$\beta^{i,a}(\rho^*) = \max\left(\frac{Cov(X_i,X) - \rho^*\gamma_i \mathbb{E}[X]}{Var(X)}, 0\right). \tag{B-2}$$

For this purpose, we reorder the bank index and still use $i = 1, \cdots, N$, such that:

$$\frac{Cov(X_1,X)}{\gamma_1} \geq \frac{Cov(X_2,X)}{\gamma_2} \geq \cdots \geq \frac{Cov(X_N,X)}{\gamma_N}. \tag{B-3}$$

In other words, we examine the risk-adjusted covariance of the loss portfolio with the aggregate loss for each bank and reorder these banks from the largest risk-adjusted covariance one to the lowest risk-adjusted covariance one. Intuitively, a large risk-adjusted covariance of the loss portfolio with the aggregate loss ensures a large systemic risk. Therefore, the regulator pays more attention to these banks and makes sure those banks purchase the aggregate insurance to resolve the issue of "too big to fail". The risk-adjusted covariance $\frac{Cov(X_i,X)}{\gamma_i}$ and the beta $\frac{Cov(X_i,X)}{Var(X)}$ can be viewed as two measures of the systemic risk.

The next lemma is trivial.

Lemma 3 *Given a decreasing sequence, $\frac{a_i}{b_i}$, for $i = 1, \cdots, N$ and $a_i, b_i > 0$, the sequence, $\frac{c_i}{d_i}$, is also decreasing, where $c_i = \sum_{j=1}^{i} a_j, d_i = \sum_{j=1}^{i} b_j$.*

By using Lemma 3 and (B-3), we have:

$$\frac{Cov(X_1, X)}{2\gamma_1} \geq \frac{\sum_{j=1}^{2} Cov(X_j, X)}{2\sum_{j=1}^{2}\gamma_j} \geq \cdots \geq \frac{\sum_{j=1}^{N} Cov(X_j, X)}{2\sum_{j=1}^{N}\gamma_j}. \tag{B-4}$$

By comparing these two decreasing sequences, $\left\{\frac{Cov(X_i,X)}{\gamma_i}; i = 1, \cdots, N\right\}$ and $\left\{\frac{\sum_{j=1}^{i} Cov(X_j,X)}{2\sum_{j=1}^{i}\gamma_j}; i = 1, \cdots, N\right\}$, we can easily find a unique number, n, such that:

$$\frac{Cov(X_i, X)}{\gamma_i} \geq \frac{\sum_{k=1}^{n} Cov(X_k, X)}{2\sum_{k=1}^{n}\gamma_k}, i = 1, \cdots, n \tag{B-5}$$

and:

$$\frac{Cov(X_i, X)}{\gamma_i} < \frac{\sum_{k=1}^{n} Cov(X_k, X)}{2\sum_{k=1}^{n}\gamma_k}, i = n+1, \cdots, N. \tag{B-6}$$

Equivalently:

$$\frac{Cov(X_i, X)}{\sum_{k=1}^{n} Cov(X_k, X)} \geq \frac{\gamma_i}{2\sum_{k=1}^{n}\gamma_k}, i = 1, \cdots, n \tag{B-7}$$

and:

$$\frac{Cov(X_i, X)}{\sum_{k=1}^{n} Cov(X_k, X)} < \frac{\gamma_i}{2\sum_{k=1}^{n}\gamma_k}, i = n+1, \cdots, N. \tag{B-8}$$

Define:

$$\rho^* = \frac{1}{\mathbb{E}[X]}\frac{\sum_{i=1}^{n} Cov(X_i, X)}{2\sum_{i=1}^{n}\gamma_i}. \tag{B-9}$$

It is easy to see that:

$$\beta^{i,a}(\rho^*) = \frac{Cov(X_i, X) - \rho^*\gamma_i\mathbb{E}[X]}{Var(X)}, i = 1, \cdots, n \tag{B-10}$$

and $\beta^{i,a}(\rho^*) = 0, i = n+1, \cdots, N$. Moreover, ρ^* is the optimal solution of the following problem:

$$\max_{\rho} \rho \sum_{i=1}^{n}\left(\frac{Cov(X_i, X) - \rho\gamma_i\mathbb{E}[X]}{Var(X)}\right).$$

Finally, it is straightforward to check that $\{\rho^*, \beta^{i,a}(\rho^*), i = 1, \cdots, N\}$ is the optimal solution in equilibrium. The bank, i, is considered to be "too big to fail" for $i = 1, \cdots, n$. $\beta^{i,a}(\rho^*) > 0$ for these "too big to fail" banks. The other bank, i, such that $i = n+1, \cdots, N$, does not buy the aggregate insurance.