

Article

Measuring the Performance of Bank Loans under Basel II/III and IFRS 9/CECL

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Abstract: In the last two decades, both internal and external risk management of banks have undergone significant developments. Banking supervision encourages banks to use a risk-based approach for computing minimum regulatory capital. Accounting rules have been tightened requiring more timely loss reserves for impaired loans. In this article, we propose a comprehensive scheme for calculating the profitability of a loan that could be used both for setting risk-based interest rates when originating a loan and for accurately determining the profitability of existing clients. The scheme utilizes the credit models developed for regulatory purposes and takes the impact of regulation on loan performance into account. We show that accounting loan loss provisions cannot be applied in a performance measurement scheme because they do not reflect the true economic loss. In addition, we demonstrate that it is crucial to measure loan performance over the full life cycle of a loan. Restricting profitability measurement to a time horizon of one year as often observed in practice could be misleading. Although our focus is on profitability measurement, the framework could be applied in a wider context, i.e., for macroeconomic stress tests, bank balance sheet projections, capital management, or evaluating the impact of securitizing parts of a bank's loan portfolio.

Keywords: Basel II; IFRS 9; CECL; RAROC; performance measurement; stress testing

JEL Classification: G21

1. Introduction

The profitability of banks is currently on a decline in many countries worldwide. One reason is the historically low-interest rates, which are even negative in many jurisdictions (Altavilla et al. 2018). An adverse accompaniment of low interest rates is so-called zombie firms that can survive only by receiving new credit at low rates but are not profitable enough to ever pay back these loans. Should interest rates increase mildly, they very likely would have to declare bankruptcy (McGowan et al. 2018). These firms are a substantial burden to the banking system and intensify the pressure on bank profitability.

For this reason, an accurate assessment of a loan's profitability is crucial for a bank as in the present environment, the margin for errors is small. The profitability of a loan is influenced by a number of factors. First, the credit quality of a borrower, which is expressed by its default probability, is a key component. Second, the quality of the collateral, which is measured by a loss rate conditional on borrower default is important. Third, the regulatory environment, which defines capital buffers and the timing for building reserves has an impact on loan performance. Other factors that might apply to specific loan segments only, like prepayment and the drawing of credit lines, are non-negligible.

Since the 2000s, a number of initiatives have been undertaken by regulators and accounting boards worldwide to increase the stability of the financial system. First, the Basel Committee on Banking Supervision (BCBS) allowed banks under the Basel II accord to use internal risk parameters to compute minimum required capital to bring capital requirements more in alignment with economic risk (BCBS 2006). Second, Basel III improved the quality of bank capital and gave supervisors more flexibility in setting capital levels for individual institutions (BCBS 2011). In addition, the accounting boards IASB (International Accounting Standards Board) and FASB (Financial Accounting Standards Board) have released new standards on building loan loss provisions (LLP) for impaired loans, IFRS 9 (IASB 2014) and CECL (FASB 2016), respectively. These reforms have a direct impact on the volume of loans a bank can originate and their profitability.

To comply with these new regulations, banks have undertaken substantial investments in data collection and credit risk modeling. New risk teams have been established that utilized the data and estimated quantitative models for measuring various aspects of credit quality, mainly the likelihood of a borrower's default and the loss severity in case of a default. In addition, models for the probability of prepayment and estimated credit conversion factors for undrawn credit lines might be available.

In this article, we will show how to utilize these models to build a multi-period projection of future capital requirements, loan loss provisions, and credit risk parameters and discuss its application for loan profitability measurement in detail. The main ingredient for this projection is a macroeconomic scenario, which makes the assumptions underlying the profitability projections transparent. To measure the performance of a loan, we propose a scheme based on RAROC (risk-adjusted return on capital). Basically, RAROC is computed as interest income minus all costs (funding, operational, expected loss) divided by the capital that is used as a buffer against unexpected losses. RAROC is an established measure in banking practice, which is usually applied in a one-period framework. We will extend it to multi-period projections ensuring that the full lifetime of a loan is considered when measuring its profitability.

The main contribution of this article is threefold. First, we will propose a comprehensive scheme for loan performance measurement that is based on the risk parameters banks have estimated for Basel and IFRS 9/CECL purposes. The scheme enables a credit risk manager to make decisions about loan applications and the suitability of a bank's interest rate-setting process taking into account all internal and external factors affecting a bank's business. Second, we will show that LLP under IFRS 9/CECL is not suitable for direct inclusion into a loan profitability scheme. In general, they are different from the true economic expected loss that a bank is suffering in its lending business when all cost components are considered appropriately. Third, we will demonstrate that an accurate picture of a loan's performance can only be obtained if the full lifetime of the loan is analyzed, including past periods. Just looking at the current year might give a misleading picture, especially for loans with collateral that changes its value over time.

Although the focus of this article is on profitability measurement, the underlying projections of capital, loss provisions, and credit risk parameters could be utilized in a much wider context. Since the projections are based on a macroeconomic scenario, it can be easily used for stress tests by using stress scenarios as inputs. Furthermore, it could be used for evaluating the impact of securitization or sales transactions of a loan portfolio on both profitability and capital requirements. This could help banks in optimizing their balance sheets. Finally, since the scheme is comprehensive and applies both the Basel and the IFRS 9 credit models, it ensures that a consistent modeling framework is used for decision making.

Our proposed framework is directly implementable in banks which should make our article particularly valuable for people working in financial risk management or regulation and for academics interested in practical applications. Our approach is designed for banks operating in developed countries in North America, Europe and parts of Asia (e.g., China, South Korea, Singapore). For less developed countries the proposed framework might be too complex and has to be reduced and simplified to be applicable.

The structure of this article is as follows. The next section provides a literature review. Section 3 briefly reviews the latest accords developed by regulators and accounting boards as far as they are relevant for loan performance measurement. In Section 4, the framework for loan performance measurement is developed and its parameterization is discussed. In Section 5 a detailed example of residential mortgages is presented using stylized but realistic credit risk models¹. The final section concludes.

2. Literature Review

Risk-adjusted performance measures (RAPM) are widely used in modern finance. Most literature on RAPM deals with measuring the performance of stock portfolios and mutual funds. An overview on RAPMs developed for this purpose is provided by [Le Sourd \(2007\)](#). The focus of this article is risk-adjusted performance measurement of loans where the most popular performance measures are RORAC (Return On Risk Adjusted Capital) and RAROC (Risk-Adjusted Return On Capital). RORAC relates the loan's interest rate to the contribution of a loan to the risk capital of a bank's loan portfolio. This capital is computed using a model like [Gupton et al. \(1997\)](#) and the contribution of a single loan to portfolio capital is done along the lines of [Tasche \(2007\)](#).

RAROC was introduced in [Zaik et al. \(1996\)](#) and is defined as the loan's interest rate reduced by various cost components, in particular a component for loss risk, divided by capital. The definition of RAROC is not unambiguous. The ambiguity is in the definition of capital. It could be either the capital prescribed by regulatory rules, the regulatory capital or the contribution of a loan to portfolio capital, the economic capital². RAROC in the latter context is studied in [Crouhy et al. \(1999\)](#) where a one-period model is used. It can be shown that capital budgeting rules based on RAROC lead to economically desirable results ([Stoughton and Zechner 2007](#)). Compared to ROROC, RAROC is more suitable for decision making in loan origination as pointed out by [Punjabi and Dunsche \(1998\)](#). Besides that, the measure itself shows a reasonable behavior under extreme scenarios, i.e., a loan's RAROC as a function of its interest rate has a maximum and cannot become arbitrarily large when a bank increases rates ([Engelmann and Pham 2020](#)).

From a practical perspective, the preference was in using economic capital when the RAROC measure was introduced because it was considered as better reflecting true economic risk compared to regulatory capital rules ([Crouhy et al. 1999](#); [Zaik et al. 1996](#)). As long as economic capital is always higher than regulatory capital, this does not pose a problem in practical applications. In recent years, however, regulatory rules have been tightened, resulting in higher capital requirements. This leads to situations where regulatory capital exceeds economic capital making a consistent application of economic capital in performance measurement infeasible. The consequence is a shift in banking practice from using regulatory instead of economic capital for performance measurement ([Ita 2016](#); [Klaassen and Van Eeghen 2018](#)). A similar trend can be observed in the insurance sector where Solvency II capital is applied in performance measures ([Baione et al. 2020](#); [Braun et al. 2018](#)).

RAROC can be considered as a bottom-up performance measure, which is computed from various basic information on the borrower and the loan facility. Alternative approaches try to improve the performance of a bank portfolio by using the past performance of a loan product as the input of a statistical forecasting model, which should help to predict future performance ([Andreeva et al. 2007](#)). Another study that is related to profitability measurement is by [Stein \(2005\)](#), who suggests an improvement of the loan origination process by integrating credit scoring models with loan performance measures. The reason why we do not follow these approaches is the limit in their

¹ One weakness of this article is that we did not utilize real data. Although one of the authors has done some implementation work of this framework in practice, both the results and the data could not be utilized for research purposes due to confidentiality restrictions.

² Some authors call the resulting performance measure when economic capital is used RARORAC (Risk-Adjusted Return On Risk-Adjusted Capital) to avoid misunderstandings.

applicability. Methods that require the performance of a loan product as input are applicable only to short-term credit products like store cards which were analyzed in [Andreeva et al. \(2007\)](#). For long-term products like 30-year mortgages, these methods are not feasible for performance measurement and bottom-up approaches like RAROC are more suitable.

Although RAROC is studied and applied since more than three decades, most published studies use a one-period framework and a rather simplistic parameterization. We are not aware of any research that combines both the credit risk modeling for Basel regulation and the new accounting standards into a multi-period loan performance measurement scheme. In this article, we apply RAROC in a multi-period framework that explicitly takes into account changing risk parameters, capital requirements and loan loss provisions over the lifetime of a loan.

The most important ingredients of our proposed loan performance scheme are the credit risk parameters default probability, loss given default, and exposure at default for credit products containing credit lines. Numerous studies address the statistical aspects of credit scoring and estimating these parameters for Basel regulation and provisioning³. Overviews with a focus on the Basel guidelines include [Engelmann and Rauhmeier \(2011\)](#) and [Ong \(2007\)](#). The most complex risk parameter for IFRS 9 is the term-structure of default probabilities. It could be either estimated by techniques from survival analysis ([Banasik et al. 1999](#); [Malik and Thomas 2010](#)) or by translating a macroeconomic scenario into default probabilities, which is more common in IFRS 9 models ([Skoglund 2017](#); [Xu 2016](#)). The estimation of LGD is covered, for instance, in [Loterman et al. \(2012\)](#) and [Tobback et al. \(2014\)](#), while LGD and EAD specifically for IFRS 9 purposes is treated in [Chawla et al. \(2016a\)](#).

In addition to the statistical estimation of credit risk parameters, their impact is studied by various authors. An evaluation of the impact of Basel regulation on loan prices is done in [Repullo and Suarez \(2004\)](#). The impact of the Basel regulation on the variability of bank capital is analyzed in [Gordy and Howells \(2006\)](#); [Repullo and Suarez \(2013\)](#); [Saurina and Trucharte \(2007\)](#). A similar analysis for evaluating capital fluctuation under IFRS 9 and the likelihood for bank recapitalization is done in [Abad and Suarez \(2018\)](#), while the new Basel rules for provisions are empirically studied in [Krüger et al. \(2018\)](#). All these studies focus on capital levels, not on performance measurement.

Although the focus of this article is profitability measurement, the suggested multi-period projections of risk parameters, bank capital and LLP underlying the loan performance scheme could be applied in a wider context. It could also be used for stress testing providing a detailed loan-level picture of the most vulnerable parts of a portfolio under stress. This is in contrast to typical stress testing approaches that focus on the global picture only ([Kapinos and Mitnik 2016](#); [Montesi and Papiro 2018](#); [Skoglund and Chen 2016](#)). In addition, our framework could be used for evaluating the full bank balance sheet, i.e., profit, capital, LLP instead of focusing on isolated aspects like capital only. This is complementing the existing literature on balance sheet projection and stress testing ([Birge and Júdice 2013](#); [Busch et al. 2017](#); [Hasan et al. 2011](#); [Skoglund and Chen 2020](#)).

3. Recent Developments in Accounting and Banking Supervision

The two major aspects of IFRS 9/CECL and the Basel regulation having an impact on loan performance are loan loss provisioning and minimum capital requirements, respectively. Each will be discussed in a separate subsection.

3.1. Loan Loss Provisioning under IFRS 9 and CECL

IFRS 9 prescribes a three-stage algorithm for provisioning:

- **Stage 1:** Normally performing loans, banks have to reserve one-year expected loss

³ A good source of the latest developments is the paper archive of the bi-annual credit scoring conference in Edinburgh: <https://crc.business-school.ed.ac.uk/category/conference-papers/>

- **Stage 2:** Loan with substantially deteriorated credit quality, banks have to reserve lifetime expected loss
- **Stage 3:** Defaulted loans, banks have to build a specific loan loss provision

IFRS 9 does not prescribe the criteria for determining when a loan's credit quality is deteriorated substantially. This has to be defined internally by a bank. Some suggestions are made by [Chawla et al. \(2016b\)](#). CECL can be roughly considered as a special case of IFRS 9 where all loans are treated as Stage 2 loans ([European Systemic Risk Board 2019](#)). For this reason, the focus of this article will be on IFRS 9.

Consider a fixed-rate n -year loan with interest rate z . Denote with p_i the probability that a borrower will default between today and year i , l_i the loss given default associated with a default in year i , and N_i the outstanding balance in year i . In this context, LLP for Stage 1 loans is computed as

$$LLP^1 = p_1 \cdot l_1 \cdot N_1. \quad (1)$$

For Stage 2 loans, LLP is computed as lifetime expected loss. It is defined as the difference between the present value of all future cash flows of a loan and the expected present value of future cash flows. The calculation of the latter includes default probabilities and loss rates and leads, therefore, to a lower result than the former. As shown in [Engelmann \(2020\)](#) the following expression for LLP can be derived:

$$LLP^2 = \sum_{i=1}^n \frac{1}{(1+z)^{i-1}} \cdot (p_i - p_{i-1}) \cdot l_i \cdot N_i, \quad (2)$$

where $p_0 = 0$. A key assumption in the derivation of (2) is that l_i is measured relative to outstanding balance plus one interest amount. This means, in case of a default, a bank can expect to receive $(1 - l_i) \cdot N_i \cdot (1 + z)$.

Banks are required to reserve LLP at the beginning of a year before any interest income is generated. This means that LLP has to be funded by bank capital. The exact relation of provisions and capital will be discussed in the next subsection. Over the year, expected losses should be covered by a component of the interest rate income, the expected loss margin. When the expected loss margin is well-designed, it should cover observed losses on average. Only in those years where something unexpected happens, LLP and bank capital have to be utilized to preserve depositors' funds.

3.2. Minimum Capital Requirements under Basel II/III

Minimum capital requirements have a crucial impact on loan performance. The higher the capital required to back a loan against unexpected losses, ceteris paribus, the lower is the return on capital a bank can generate. Basel II defines three different frameworks of increasing complexity for calculating regulatory capital, the standardized approach, the foundation internal ratings-based (IRB) approach, and the advanced IRB approach. Throughout this article, we assume a bank is applying the advanced IRB approach. Its cornerstone is a formula for computing required minimum regulatory capital K_{\min} :

$$K_{\min} = EAD \cdot LGD \cdot \left(\Phi \left(\frac{\Phi^{-1}(PD) + \sqrt{\rho} \cdot \Phi^{-1}(0.999)}{\sqrt{1-\rho}} \right) - PD \right), \quad (3)$$

where EAD is the exposure at default, LGD the loss given default, PD the one-year default probability, Φ the cumulative standard normal distribution, and ρ the asset correlation which is prescribed by supervisors ([BCBS 2006](#)).

How is the formula parameterized? Since its parameters are similar to p_1 and l_1 in (1), one might suppose $PD = p_1$ and $LGD = l_1$. However, this is not true. IFRS 9 requires risk parameters to be forward-looking ([IASB 2014](#)). Such parameters are called point-in-time (PIT) estimates, i.e., best possible predictions of their real-world realizations over the forecast horizon. Basel II, however, requires for default probabilities "long-term averages" and for loss given default an estimate corresponding to

an economic downturn. This means different sets of risk parameters have to be estimated for IFRS 9 and Basel II.

Long-term average default probabilities are also referenced through-the-cycle (TTC) default probabilities, i.e., default probabilities representing the average default rate over an economic cycle. From a statistical point of view, TTC probabilities are more complex to estimate because of the long time horizon they refer to. In addition, a direct comparison with realized default rates is not meaningful because by construction TTC probabilities should underestimate default rates during a recession and overestimate default rates during a boom. An established procedure in practice is adapting (3) to transform PIT probabilities into TTC probabilities (Aguais et al. 2007; Carlehed and Petrov 2012):

$$p_1 = \Phi \left(\frac{\Phi^{-1}(PD) + \sqrt{\rho} \cdot Z}{\sqrt{1 - \rho}} \right), \quad (4)$$

where Z is a standard normally distributed systemic factor. While for capital calculation under Basel II Z was set to the 99.9% quantile, for the transformation between p_1 and PD the systemic factor Z has to be estimated and represents the state of the economy.

To determine Z , one could use a time series of default rates d_i observed in a particular sector like all Vietnamese corporations or all Dutch mortgages. An observed default rate can be interpreted as the realization of an average PIT PD because, by definition, these PDs are forecasts of default rates. Computing $\Phi^{-1}(d_i)$ and estimating the parameters PD and ρ should lead to reasonable values for Z_i if PD is stable over time and the data history is sufficiently long. If predictions of Z_i are needed, a time series model could be built that links $\Phi^{-1}(d_i)$ with macroeconomic factors X and predicts Z_i under an assumed macroeconomic scenario. This procedure will be illustrated in Section 5.

We conclude this section describing a recent amendment to Basel II linking capital requirements with loss provisions. In (3) expected loss $EL_B = PD \cdot LGD \cdot EAD$ (Basel EL) is subtracted from the term involving the normal distribution. Economically, this means that supervisors have assumed that expected losses of a loan portfolio are covered by loss provisions, which should be offset by a component of interest income, the expected loss margin. Only losses beyond expectations are backed by capital. This rule was changed in BCBS (2019) where the Basel Committee requires banks to provision at least EL_B . If LLP is less than Basel EL on portfolio levels, additional capital is required while if it is higher than Basel EL capital can be released up to a cap of 0.6% of risk-weighted assets (RWA). RWA is computed as $12.5 \cdot K_{\min}$ with K_{\min} from (3).⁴ This means that (3) has to be adjusted to reflect this additional requirement:

$$\tilde{K}_{\min} = K_{\min} - \min(LLP - EL_B, 0.06 \cdot RWA). \quad (5)$$

A further complication in the practical implementation of this rule is that if portfolio LLP is lower than total Basel EL a bank has to build an additional capital buffer in Core Tier 1 capital. If LLP is higher than Basel EL the difference will be added to Tier 2 capital up to a cap of 0.6% of RWA which makes this rule asymmetric. For a discussion of the impact of this regulation on these bank capital components see Krüger et al. (2018).

4. A Framework for Loan Performance Measurement

In this Section, the framework for loan performance measurement will be developed. We will consider profitability on a loan-by-loan instead of a portfolio basis allowing banks to identify their

⁴ Using (3) for capital calculations implicitly assumes that a bank is required to have at least 8% of RWA, which might not be true after the introduction of new capital buffers in BCBS (2011). The actual percentage, however, might differ between banks depending on the policy of its national supervisor. Therefore, we stick to 8% for the purpose of this article.

most valuable customers. The performance of a loan will be measured by RAROC (risk-adjusted return on capital) which is calculated by the following scheme:

$$\begin{array}{l}
 \text{Interest Rate Income} \\
 - \text{Funding Costs} \\
 - \text{Expected Loss Coverage} \\
 - \text{Operational Costs} \\
 \hline
 \div \text{Allocated Capital Buffer} \\
 \hline
 = \text{RAROC}
 \end{array}$$

To get a correct picture of a loan's performance, it is essential to take the full lifetime of a loan into account. Risk parameters might change over time and looking into one single period only might be misleading. The lifetime of a loan can be split into past periods, the current period and future periods. The key difference is that in future periods the IFRS 9 stage of a loan is unknown, which makes the calculation of expected profitability more demanding.

We start with the past periods. We denote the interest rate in period i with z_i . For a fixed-rate loan z_i is always the fixed-rate z . For a floating-rate loan, $z_i = \lambda_i + s$, where λ_i is the realized Libor rate in this period and s a period-independent spread. The outstanding balance is N_i , the funding costs f_i , the operational costs c_i , the coverage for expected losses ELC , and the required capital $\bar{K}_{\min,i}$. The expected loss coverage is designed to cover losses in loan balance, operational costs and funding costs. It is computed as

$$ELC_i = \frac{N_i \cdot d_{r,i} \cdot l_i \cdot (1 + z_i) + N_i \cdot d_{r,i} \cdot (f_i + c_i - z_i)}{1 - d_{r,i}}, \quad (6)$$

where $d_{r,i}$ is the realized default rate of all borrowers in a borrower's rating grade r . The expected loss coverage (which is a realized quantity for past periods) is computed as the sum of realized loss in deposits $N_i \cdot d_{r,i} \cdot (1 + f_i)$ and the realized loss in operational costs $N_i \cdot d_{r,i} \cdot c_i$ minus the expected recovery from collateral liquidation of defaulted loans $(1 - l_i) \cdot N_i \cdot d_{r,i} \cdot (1 + z_i)$. The result is divided by $1 - d_{r,i}$ because only surviving borrowers can make up for the loss. Remember from Section 3.1 that we use the convention that l_i measures the loss rate of the outstanding balance plus one interest rate. Note that the estimated loss rate l_i could be replaced with some realization if it was known. However, since this usually takes more time to observe than the default rate, we suggest sticking with the risk parameters that were used in the past if no better alternative is available.

The intuition behind using the default rate of a rating grade $d_{r,i}$ is that within a rating grade, all loans have similar default probabilities. The surviving loans have to cover the losses of the defaulting loans on average. Roughly speaking, the rating grade is interpreted as an "insurance pool" and (6) measures the realized loss per surviving borrower the pool has to cover.

If a loan was in Stage j in period i , we find for its RAROC

$$RAROC_i = \frac{(z_i - f_i - c_i) \cdot N_i - ELC_i}{\bar{K}_{\min,i} + LLP_i^j}, \quad (7)$$

We relate the risk-adjusted return to regulatory capital (RegCap). Other research uses economic capital (ECap) instead (Crouhy et al. 1999) which is computed from a credit portfolio model that, at least in theory, more accurately reflects the true economic risk of a loan. There are, however, conflicts of using a pure ECap model with regulation that are difficult to resolve. If ECap is lower than RegCap, a bank has to allocate by law RegCap which reduces a loan's RAROC. Only if ECap is higher than RegCap it might be safely used to penalize loans with large contributions to portfolio risk. Over time ECap might be fluctuating considerably if a bank's portfolio changes, which is difficult to anticipate

at loan origination. For these reasons, we prefer regulatory capital in our performance measurement scheme. This is supported by the latest developments in banking practice, where a shift from economic to regulatory capital in bank management can be observed (Ita 2016; Klaassen and Van Eeghen 2018). Note that we implement (5) on loan level, although it is implemented on portfolio levels in the Basel accord. The motivation is penalizing loans that contribute to high LLP, which could eventually require a bank to build provisions so high that it will breach the $0.6\% \cdot RWA$ cap.

For the current period, we can use exactly the same formulas because the stage of a borrower is known. However, the realized default rates of the portfolio are still unknown. Therefore, the PIT PD p_1^j has to be used instead of $d_{r,i}$ in (6) leading to

$$ELC_i^j = \frac{N_i \cdot p_1^j \cdot l_i \cdot (1 + z_i) + N_i \cdot p_1^j \cdot (f_i + c_i - z_i)}{1 - p_1^j}, \tag{8}$$

which has to be used in (7).

To compute expected RAROC for future periods, we have to estimate a number of quantities. First, we do not know the stage of a loan in the future. We denote by t_i the probability that a loan is in Stage 2 in period i . Depending on Stage $j = 1, 2$, we need parameters p_i^j , the cumulative PIT default probabilities until period i , l_i , the PIT loss rate in period i , expected balance \hat{N}_i , exposure at default EAD_i , TTC probabilities PD_i^j , downturn LGDs LGD_i , expected interest rate \hat{z}_i , expected funding costs \hat{f}_i and operational costs c_i . Note, that EAD_i might be different from \hat{N}_i . By the expected balance \hat{N}_i the outstanding loan amount that earns interest should be modeled while EAD_i refers to the outstanding exposure in case of a default. This could include additional expected drawings from credit lines. In any case $EAD_i \geq \hat{N}_i$ ⁵.

For future periods, ECL depending on stage j is computed as

$$ELC_i^j = \frac{N_i \cdot p^j(i|i-1) \cdot l_i \cdot (1 + z_i) + N_i \cdot p^j(i|i-1) \cdot (f_i + c_i - z_i)}{1 - p^j(i|i-1)}, \tag{9}$$

where $p^j(i|i-1)$ is the default probability of a borrower in period i conditional that he survived until period $i-1$. Since we design our performance measurement scheme period-by-period, a calculation of RAROC in a future period is only meaningful conditional on the survival of a borrower. From the term-structure of default probabilities p_i the conditional probability $p(i|i-1)$ is computed as $p(i|i-1) = \frac{p_i - p_{i-1}}{1 - p_{i-1}}$. Expected RAROC in a future period i is then calculated as

$$RAROC_i = \frac{(1 - t_i) \cdot \left((z_i - \hat{f}_i - c_i) \cdot \hat{N}_i - ELC_i^1 \right) + t_i \cdot \left((z_i - \hat{f}_i - c_i) \cdot \hat{N}_i - ELC_i^2 \right)}{(1 - t_i) \cdot \left(\bar{K}_{\min,i}^1 + LLP_i^1 \right) + t_i \cdot \left(\bar{K}_{\min,i}^2 + LLP_i^2 \right)}. \tag{10}$$

Note that we have defined RAROC in future periods i as expected income minus costs divided by expected capital. A natural alternative would have been taking expectations over RAROC conditional on Stage $j = 1, 2$. We refrained from using this alternative because it could overstate RAROC. Suppose RAROC is positive in Stage 1 and negative in Stage 2 under very high capital requirements. This would result in a small negative RAROC conditional on Stage 2 and the RAROC conditional on Stage 1 will dominate the calculation. The outcome would be a positive RAROC despite the fact that expected interest income minus costs might be quite small or even negative. For this reason, we consider (10) the economically more meaningful alternative.

⁵ We have assumed that aside from PDs which clearly have to be stage-dependent, all other risk parameters are stage-independent. This might not be true in general. For instance, in loan segments where prepayments are possible without penalty, prepayment probabilities might depend on credit quality making \hat{N}_i stage-dependent.

To summarize all the period RAROCs $RAROC_i$ into a single performance measure, we use capital-weighted RAROC:

$$\overline{RAROC} = \frac{\sum_{i=1}^n RAROC_i \cdot \left((1-t_i) \cdot \left(\bar{K}_{\min,i}^1 + LLP_i^1 \right) + t_i \cdot \left(\bar{K}_{\min,i}^2 + LLP_i^2 \right) \right)}{\sum_{i=1}^n \left((1-t_i) \cdot \left(\bar{K}_{\min,i}^1 + LLP_i^1 \right) + t_i \cdot \left(\bar{K}_{\min,i}^2 + LLP_i^2 \right) \right)} \quad (11)$$

The motivation for using a weighted average is giving those periods where more capital is required more weight. We abstain from introducing discount factors because they dampen potentially negative effects in the future. To ensure the stability of a bank's business over time, there should be no incentives created to favor higher short-term gains over lower profit or even losses in the future. A second argument is that the use of discount factors would imply that for past periods we have to compound interest. This would be inconsistent with reality since banks mainly distribute profit to its shareholders instead of reinvesting it.

The profitability measure (11) has several applications. It can be used by a bank to evaluate the profitability of its clients or to monitor existing portfolios. In addition, it can play an important role in loan origination. For this purpose, banks define an internal profitability target $RAROC_{target}$ they would like to achieve. Usually, this is a management decision possibly guided by an application of the CAPM to determine the required return of a bank's shareholders. This approach has some shortcomings (Crouhy et al. 1999) and alternatives based on the Merton model (Merton 1974) exist, leading to borrower-specific hurdle rates (Miu et al. 2016). However, the proposed alternatives can be applied sensibly only in markets where borrowers have listed equity, which is true only for a small minority. Therefore, we stick to the assumption of a borrower-independent target profitability $RAROC_{target}$.

In markets where a bank is a price taker, i.e., where it has very limited power to set the interest rate z due to strong competition, it could use (11) to define an acceptance rule for loan applications. Only those applications should be approved where $\overline{RAROC} \geq RAROC_{target}$. When a bank is a price setter which could be the case in more specialized markets, it could define the hurdle rate z_h by solving the optimization problem

$$\begin{aligned} z_h = \min z \\ \text{s.t. } \overline{RAROC} \geq RAROC_{target} \end{aligned} \quad (12)$$

A bank should offer loans only at an interest rate z_h or higher to ensure it fulfills its profitability requirements.

The remainder of this Section is devoted to discussing how the parameters needed for applying (11) are estimated. This will be done in two separate subsections, where the first one is devoted to credit risk parameters. The second subsection deals with the internal costs for funding and operations.

4.1. Estimation of Credit Risk Parameters

The exact design of a credit risk model depends on factors like portfolio segment, data availability, data quality, or predictive power of risk factors. For the purpose of this article, we assume a generic modeling framework that could be realized if data is available for more than one economic cycle and the cyclical behavior of macroeconomic factors is well reflected in the data. This assumption reflects an ideal world from a modeling point of view and may not be feasible in all financial institutions and for all portfolios. How to make adjustments if these assumptions are not fulfilled cannot be stated in general but has to be decided on a case-by-case basis.

To be more precise, we assume that a macroeconomic scenario $X_{k,i}$ for future periods $i = 1, \dots, n$ exists where X_k , $k = 1, \dots, l$ are l macroeconomic factors that have a strong link to the credit risk in a particular loan segment. For instance, X could be GDP growth for a corporate portfolio, it could be the

unemployment rate for a retail portfolio or it could be a house price index for a residential mortgage portfolio. In addition to the future scenarios, the past realizations of $X_k, X_{k-m}, \dots, X_{k,0}$ are known. The macroeconomic scenario could be generated by an econometric model or by expert judgment. Usually, there is some opinion over the next 1–5 years, and after that, the scenario converges to some neutral long-term average since predicting over time horizons of 5 years or more is too unreliable.

For the calculation of TTC default probabilities, it is essential to understand in which point of the cycle the economy is. This could be done by computing abstract systemic factors Z and applying (4). A necessary input for this procedure is realized default rates $d_i, i = -m, \dots, 0$ that are representative for the market the bank is operating in. For instance, for a corporate portfolio d_i could be country-wide default rates obtained by some government agency. From the time-series d_i the parameters B and ρ of the slightly transformed Equation (4)

$$\Phi^{-1}(d_i) = \frac{B + \sqrt{\rho} \cdot Z_i}{\sqrt{1 - \rho}} \tag{13}$$

can be estimated using the techniques described in Carlehed and Petrov (2012) and the references therein. Once, these parameters are known, Z_i can be computed as

$$Z_i = \frac{\Phi^{-1}(d_i) \cdot \sqrt{1 - \rho} - B}{\sqrt{\rho}}. \tag{14}$$

To obtain future predictions of Z , a link between the macroeconomic scenario $X_{k,i}$ and Z is needed. This could be established by estimating a time series model like

$$\Phi^{-1}(d_i) = \alpha_0 + \sum_{k=1}^l \alpha_k \cdot X_{k,i-1} + \epsilon_i. \tag{15}$$

Once the parameters $\alpha_0, \dots, \alpha_l$ are estimated, (14) and (15) can be applied to translate the macroeconomic scenario into a scenario value for the systemic factor Z .

To compute PIT credit risk parameters, we assume that at least one of the macroeconomic variables X_k is included in the model equation. In a generic setup, a bank could have a model for the PIT default probability in the next period, p_1 , the PIT loss rate l_1 , the PIT prepayment rate cpr_1 , and the PIT credit conversion factor ccf_1 ⁶. All these parameters are estimated conditional on current (period 0) macroeconomic data and risk factors. The estimation of these parameters is done by the equations

$$p_1^j = g_p(\mathbf{X}_0, \mathbf{Y}_0^p; j), \tag{16}$$

$$l_1 = g_l(\mathbf{X}_0, \mathbf{Y}_0^l), \tag{17}$$

$$cpr_1 = g_{cpr}(\mathbf{X}_0, \mathbf{Y}_0^{cpr}), \tag{18}$$

$$ccf_1 = g_{ccf}(\mathbf{X}_0, \mathbf{Y}_0^{ccf}). \tag{19}$$

where $\mathbf{X} = (X_1, \dots, X_l)$ is the vector of macroeconomic factors and $\mathbf{Y}^p, \mathbf{Y}^l, \mathbf{Y}^{cpr}$, and \mathbf{Y}^{ccf} are risk factors related to the borrower, the loan or the collateral that is backing the loan. These risk factors can differ from model to model. The estimation of p has to include the IFRS 9 stage $j = 1, 2$. A possible solution is presented in the next Section. The functions g_p, g_l, g_{cpr} , and g_{ccf} are the link functions between the risk parameters and the risk factors. For p, cpr and ccf logistic regression is a common choice, while for l a linear regression or some other form of regression function could be used.

⁶ Whether all these parameters are necessary and meaningful depends on the particular portfolio and the legal environment the bank is operating in. If the loans do not include credit lines, ccf is not needed and if prepayment without penalty is prohibited, there might be no use for cpr .

These risk models can be used to compute risk parameters in all future periods i of a loan. This results in

$$p^j(i|i-1) = g_p(\mathbf{X}_{i-1}, \mathbf{Y}_{i-1}^p; j), \tag{20}$$

$$l_i = g_l(\mathbf{X}_{i-1}, \mathbf{Y}_{i-1}^l), \tag{21}$$

$$cpr(i|i-1) = g_{cpr}(\mathbf{X}_{i-1}, \mathbf{Y}_{i-1}^{cpr}), \tag{22}$$

$$ccf_i = g_{ccf}(\mathbf{X}_{i-1}, \mathbf{Y}_{i-1}^{ccf}). \tag{23}$$

Note that by p_i^j we denoted the cumulative default probability until period i . The risk models deliver PIT parameters in period i , i.e., in the case of default probabilities, the probability of default in period i conditional on survival in period $i-1$, $p^j(i|i-1)$. Cumulative default and prepayment probabilities are then computed as

$$p_i^j = 1 - \prod_{k=1}^i (1 - p^j(k|k-1)),$$

$$cpr_i = 1 - \prod_{k=1}^i (1 - cpr(k|k-1)).$$

In (20)–(23) we have used \mathbf{Y}_i^* as inputs. Does that mean we need in addition to the macroeconomic scenarios also scenarios for the risk factors \mathbf{Y} ? Remember that \mathbf{Y} is borrower- or loan-specific, making this a much more demanding task. There is no clear answer to this question. In most practical cases, \mathbf{Y} is period-independent or changes deterministically over periods. This will become clear from the detailed example provided in Section 5. However, it cannot be ruled out that in some cases, it might be beneficial to define some scenarios also for a particular risk factor Y .

The main purpose of all these risk models is to provide the inputs for (10). In particular, we need the four quantities \hat{N}_i , ELC_i^j , LLP_i^j and $\bar{K}_{\min,i}^j$. According to the agreed amortization schedule, the outstanding balance of a loan in period i is N_i . To compute $\bar{K}_{\min,i}^j$ we need PD_i^j , LGD_i , EAD_i and LLP_i^j as inputs. The required parameters for (10) are computed as

$$\hat{N}_i = (1 - cpr_{i-1}) \cdot N_i, \tag{24}$$

$$PD_i^j = \Phi \left(\Phi^{-1} \left(p^j(i|i-1) \right) \cdot \sqrt{1 - \rho} - \sqrt{\rho} \cdot Z_i \right), \tag{25}$$

$$LGD_i = h(l_i), \tag{26}$$

$$EAD_i = \hat{N}_i + ccf_i \cdot (L - \hat{N}_i), \tag{27}$$

$$LLP_i^1 = p^1(i|i-1) \cdot l_i \cdot \hat{N}_i, \tag{28}$$

$$LLP_i^2 = \sum_{k=i}^n \frac{1}{(1+z)^{k-i}} \cdot \left(p^2(k|i-1) - p^2(k-1|i-1) \right) \cdot l_k \cdot \hat{N}_k. \tag{29}$$

$$ELC_i^j = \frac{\hat{N}_i \cdot p^j(i|i-1) \cdot l_i \cdot (1+z_i) + \hat{N}_i \cdot p^j(i|i-1) \cdot (\hat{f}_i + c_i - z_i)}{1 - p^j(i|i-1)} \tag{30}$$

where L is the credit limit of a loan facility in case, there is some credit lines included and $p^2(k|i-1)$ the term structure of default probabilities of a borrower conditional on survival until period $i-1$ and being in Stage 2. The function h translates PIT LGDs into regulatory downturn LGDs. This might be a function prescribed by regulators as in the US or it might be something defined internally by a bank as in other jurisdictions. An explanation of why (28) and (29) with \hat{N}_i instead of N_i are still correct can be found in Engelmann (2020).

To complete the explanation of (10), we have to know where t_i , \hat{f}_i and c_i come from. The parameter t_i depends on the staging rule implemented by a bank. It could either be derived from the existing models or it could be estimated by a separate model, e.g., a logistic regression where instead of the event “moving to default” the event “moving to Stage 2” will be modeled. The example in Section 5 will clarify how this could work. The remaining cost parameters \hat{f}_i and c_i will be discussed in the next subsection.

4.2. Determination of Cost Components

A bank’s operational costs have to be covered by fees or interest income. How to allocate costs to a certain product is an internal procedure that might differ from bank to bank. For the purpose of this article, we assume that operational costs are modeled as a percentage of the outstanding loan balance. It might also be modeled as an absolute amount or a combination of both. There might also be cases where a borrower pays some fees in addition to interest. To include all these cases in (10) is quite straightforward.

More interesting are funding costs. This is the interest a bank has to pay to its depositors and bond investors, which is, in general, a term-structure that is provided by the treasury department. A stylized example is given in Table 1.

Table 1. The example term structure of funding rates.

Expiry	Interest Rate
1 Year	12M Libor + s_1
2 Years	12M Libor + s_2
⋮	⋮
n Years	12M Libor + s_n

The table means that if a bank would originate an i -year loan, it would have to pay 12M Libor + s_i on the funds to its depositors every year. There might be alternative ways a treasury quotes these rates depending on the institutional setup. We use Table 1 as an example to illustrate how to derive funding costs for every year from the data of this table. A generalization to different market conventions is not difficult.

The reason why a treasury quotes funding rates in this way is that swap rates in the interbank market are much more volatile than the spread a bank has to pay over interbank market rates for its funding. To compute future expected funding costs \hat{f}_i , swap rates S_i for each expiry $i = 1, \dots, n$ are collected; and discount factors and forward rates are bootstrapped from them. For the purpose of this article, we assume swaps exchange once per year a fixed rate for a 12M Libor rate; and the payment frequencies of swaps and loans are identical.

The calculation of \hat{f}_i is a two-step procedure. First, discount factors for the interbank curve have to be computed from the swap rates S_i observed in the market. For current market quotes, the value of the floating leg equals the value of the fixed leg resulting in

$$1 - \delta_i^M = \sum_{j=1}^i S_j \cdot \delta_j^M,$$

where δ_j^M is the discount factor corresponding to year j . These discount factors can be computed iteratively using the bootstrap algorithm

$$\delta_1^M = \frac{1}{1 + S_1},$$

$$\begin{aligned} \delta_2^M &= \frac{1 - S_2 \cdot \delta_1^M}{1 + S_2}, \\ &\vdots \\ \delta_n^M &= \frac{1 - S_n \cdot \sum_{j=1}^{n-1} \delta_j^M}{1 + S_n}. \end{aligned}$$

From δ_i^M , we can compute forward rates $\hat{\lambda}_i$ that a bank expects to pay on its funding, $\hat{\lambda}_i = \delta_{i-1}^M / \delta_i^M - 1$. Using the forward rates $\hat{\lambda}_i$ and the spread s_i in Table 1 allows us to compute \hat{f}_i using a similar bootstrap procedure.

The condition for bootstrapping the funding curve δ_i is that the present value of future cash flows for depositors equals the deposited amount:

$$1 = \sum_{j=1}^i (\hat{\lambda}_j + s_j) \cdot \delta_j + \delta_i,$$

which leads to the bootstrapping algorithm

$$\begin{aligned} \delta_1 &= \frac{1}{1 + \hat{\lambda}_1 + s_1}, \\ \delta_2 &= \frac{1 - (\hat{\lambda}_1 + s_2) \cdot \delta_1}{1 + \hat{\lambda}_2 + s_2}, \\ &\vdots \\ \delta_n &= \frac{1 - \sum_{j=1}^{n-1} (\hat{\lambda}_j + s_n) \cdot \delta_j}{1 + \hat{\lambda}_n + s_n}. \end{aligned}$$

Future expected funding costs are then computed as $\hat{f}_i = \delta_{i-1} / \delta_i - 1$. These expected funding costs \hat{f}_i are suitable for measuring the profitability of floating-rate loans. Therefore, we denote them by $\hat{f}_{float,i}$ for clarity. For fixed-rate loans, a bank commonly uses a swap to manage interest rate risk. This results in fixed rates \hat{g}_i a bank has to pay depending on the maturity of its funds. They are computed from the relation

$$\sum_{j=1}^i \hat{f}_{float,j} \cdot \delta_j = \hat{g}_i \cdot \sum_{j=1}^i \delta_j \Rightarrow \hat{g}_i = \frac{\sum_{j=1}^i \hat{f}_{float,j} \cdot \delta_j}{\sum_{j=1}^i \delta_j}.$$

The funding costs $f_{fix,i}$ a bank has to stem in each period for a fixed-rate loan can be computed from the loan's amortization schedule

$$\hat{f}_{fix,i} \cdot N_i = \sum_{j=i}^n \hat{g}_j \cdot (N_j - N_{j+1}), \tag{31}$$

where $N_{n+1} = 0$. The intuition is to split the loan into parts with different maturities according to its amortization schedule and charge \hat{g}_j on the part maturing in period j .

5. An Example for Residential Mortgages

We provide a fully worked-out example for residential mortgages for illustration. The data and models we are using are not derived from real-world data. They are a bit simpler and stylized compared to real-world models but still contain the most important features. We consider a 10-year fixed-rate annuity loan as an example, that pays interest and amortization annually. We assume the

loan has just been originated; and we measure its expected future performance using the scheme described in Section 4.

We start with the funding curve and the calculation of future funding costs $\hat{f}_{fix,i}$ for the years $i = 1, \dots, 10$. The results are summarized in Table 2. Both swap rates S and funding spreads s are increasing with expiry leading to a rather steep curve for \hat{f}_{float} . Since we use a fixed-rate loan in our example, the rates \hat{f}_{fix} will be used for computing funding costs later using (31). Operational costs are assumed at $c = 0.50\%$ of the outstanding balance.

Table 2. Funding curve and calculated values for \hat{f}_{float} and \hat{f}_{fix} .

Year	S (%)	s (%)	δ^M	$\hat{\lambda}$ (%)	δ	\hat{f}_{float} (%)	\hat{f}_{fix} (%)
1	1.00	0.100	0.9901	1.000	0.9891	1.100	1.100
2	1.20	0.100	0.9764	1.403	0.9745	1.503	1.300
3	1.30	0.110	0.9619	1.504	0.9588	1.635	1.410
4	1.40	0.120	0.9458	1.710	0.9413	1.861	1.520
5	1.50	0.135	0.9280	1.917	0.9218	2.115	1.634
6	1.70	0.150	0.9030	2.764	0.8950	2.994	1.849
7	1.90	0.165	0.8750	3.204	0.8650	3.468	2.063
8	2.10	0.180	0.8441	3.659	0.8321	3.957	2.276
9	2.30	0.200	0.8106	4.132	0.7961	4.517	2.494
10	2.50	0.220	0.7748	4.626	0.7578	5.062	2.712

Next, we describe the stylized macroeconomic data. We assume that three macroeconomic factors are used in modeling credit risk for residential mortgages, the unemployment rate (UR), the growth in a house price index ($HPIgr$) and the mortgage rate (MR). The unemployment rate should be an indicator for default rates since growth in unemployment should result in a higher number of people unable to repay their loans. Changes in house prices are usually modeled by an index since modeling prices of individual properties is, in most cases, not feasible. This variable should affect both default rates and loss rates. The mortgage rate in a macroeconomic sense is not the interest rate of an individual mortgage but some average rate for currently originated mortgages. Central banks often publish such rates. If a loan market permits prepayments, this interest rate is usually a good indicator of the prepayment likelihood. For these three macroeconomic factors, we assume the scenario in Table 3. The value for Year 0 would be a realized value, while for Years 1 to 9 the values reflect a scenario; in this example, the gradual return from an economic boom to a neutral state.

Table 3. Scenario for the macroeconomic factors UR , $HPIgr$, and MR .

Year	UR (%)	$HPIgr$ (%)	MR (%)
0	3.00	2.00	5.00
1	3.00	2.00	5.00
2	3.50	1.50	4.80
3	4.00	1.00	4.60
4	4.50	0.50	4.40
5	5.00	0.50	4.20
6	5.00	0.00	4.00
7	5.00	0.00	4.00
8	5.00	0.00	4.00
9	5.00	0.00	4.00

For the determination of Z_i for each future year by means of (14), our stylized macroeconomic model is

$$\Phi^{-1}(d_i) = -2.5 + 5.0 \cdot UR_{i-1} - 2.0 \cdot HPIgr_{i-1}. \quad (32)$$

Furthermore, we assume that the parameters B and ρ in (14) have been estimated as $B = -2.25$ and $\rho = 3\%$. Using the macro scenario in Table 3 allows the calculation of a scenario for Z which is displayed in Table 4. Note, that negative Z reflect booms while positive Z signals recession.

Table 4. Probit-transformed global default rate and corresponding value of Z .

Year	$\Phi^{-1}(d)$	Z
1	-2.39	-0.60
2	-2.39	-0.60
3	-2.355	-0.40
4	-2.32	-0.20
5	-2.285	0.00
6	-2.26	0.14
7	-2.25	0.20
8	-2.25	0.20
9	-2.25	0.20
10	-2.25	0.20

To measure the profitability of a mortgage, we need additional information. We assume that the original balance $N_1 = 500,000$ is equal to the house price, i.e., the loan-to-value (LTV) at origination was 100%. The loan amortizes at 2% per year and its fixed interest rate z is 3.5%. This leads to an annual annuity payment of $A = 500,000 \cdot (2\% + 3.5\%) = 27,500$. For Basel II and IFRS 9, we assume the bank has developed the following risk models for borrower and collateral:

$$\text{logit}(p(i|i-1)) = -6.0 + 3.0 \cdot AD_{i-1} + 4.0 \cdot UR_{i-1} + LTV_{i-1} + 2.0 \cdot DSC_{i-1}, \quad (33)$$

$$l_i = 0.01 + 0.5 \cdot \max(LTV_{i-1} - 0.80, 0.0), \quad (34)$$

$$\text{logit}(cpr(i|i-1)) = 0.02 + 0.5 \cdot (z - MR_{i-1}) - 0.01 \cdot LTV_{i-1}, \quad (35)$$

$$\text{logit}(ar(i|i-1)) = -5.0 + 2.0 \cdot UR_{i-1} + 2.0 \cdot DSC_{i-1}, \quad (36)$$

$$\text{logit}(cr(i|i-1)) = 1.0 - 4.0 \cdot UR_{i-1} - 2.0 \cdot DSC_{i-1}, \quad (37)$$

where $\text{logit}(x) = \log\left(\frac{1-x}{x}\right)$ is the logistic transformation and AD an arrears dummy, i.e., $AD = 1$ if the borrower's payments are in arrears for 10 days or more and $AD = 0$ otherwise. The risk factor DSC is the debt service coverage ratio and is computed as the ratio of all annual payments a borrower has to make on all his loan products divided by his net income. To be meaningful all loans of a borrower including those from other banks have to be included. Whether this information is easily available or not depends on the institutional setup of a country. In some countries (e.g., Malaysia) the central bank supports the collection of this data, while in other countries credit bureaus might be useful sources of information.

Model Equations (33)–(35) have been expected from the discussions in Section 4. There is no model for ccf because mortgages usually come without credit lines making EAD equal to the outstanding balance. The other two model equations are related to the staging rule required for IFRS 9. One option to model “deteriorated credit quality” is using “payment missing for 10 days or more” as a criterion. The probability for being in Stage 2 is in this case equal to the probability of being in arrears but not yet in default at the end of a year. This probability is modeled by the arrears rate $ar(i|i-1)$ which is the probability that a loan is in arrears in Year i conditional on being performing in Year $i-1$. Once a loan is in arrears, it could either move into default or back to Stage 1. The cure rate $cr(i|i-1)$ models the probability that a loan is in Stage 1 in Year i conditional on being in Stage 2 in Year $i-1$. This allows the calculation of cumulative probabilities $ps_i(j)$ of a currently performing loan for being in Stage $j = 1, 2, 3$ in Year i by the following algorithm:

$$\begin{aligned}
ps_1(2) &= ar(1|0), \\
ps_1(3) &= p(1|0), \\
ps_1(1) &= 1 - ps_1(2) - ps_1(3), \\
&\vdots \\
ps_i(2) &= ps_{i-1}(1) \cdot ar(i|i-1) + ps_{i-1}(2) \cdot (1 - cr(i|i-1) - p(i|i-1, AD = 1)), \\
ps_i(3) &= ps_{i-1}(3) + ps_{i-1}(1) \cdot p(i|i-1, AD = 0) + ps_{i-1}(2) \cdot p(i|i-1, AD = 1), \\
ps_i(1) &= 1 - ps_i(2) - ps_i(3)
\end{aligned}$$

In Year i the Stage 2 loans consist of loans that moved from Stage 1 to Stage 2 or were in Stage 2 already at the end of Year $i - 1$ but did neither default nor cure. This is possible since a loan can be in arrears, pay back the outstanding amount during the year and move into arrears again. The higher default probability for a loan in Stage 2 is reflected by the arrears indicator AD in (33). The probability t_i is the likelihood that a loan is in Stage 2 conditional on survival in Year i . From the above, it can be easily computed as $t_i = \frac{ps_i(2)}{1 - ps_i(3)}$.

To make the framework of Section 5 applicable, we have to explain how to project the risk factors LTV and DSC over 10 years and how to compute downturn LGD for regulatory capital calculations. For LTV we project the house price into the future using the house price index growth scenario that was already used in the macroeconomic model (32). The outstanding loan balance is computed according to the amortization schedule of the annuity. For DSC we assume that the net annual income of the borrower is 100,000 and this value stays constant over the 10 years. We further assume that the mortgage is the only loan of the borrower. Then, DSC is computed as the ratio of the annual interest and amortization payments divided by net income. Finally, we have to define a rule for downturn LGD. In some countries, like the US, this is prescribed by regulators, in other countries, this has to be defined by the bank. We assume downturn LGD is defined by the bank and it is computed using (34) under a worst-case scenario of a 25% decline of the house price. The resulting risk parameters in this setup are presented in Table 5.

Finally, we have all parameters at hand that are needed to compute loan performance. We can compute ELC^j , LLP^1 and LLP^2 as well as required minimum capital conditional on Stage $j = 1, 2$. The results are displayed in Table 6. Computing $RAROC$ of this loan using (11) leads to 8.586%. We see that period-by-period $RAROC$ takes values between 7.26% and 10.24%. This means that looking into the performance of the loan in a single period and ignoring past and future periods could be quite misleading. Usually, credit risk parameters change with the loan balance. Loans could become less risky over time as in the mortgage example over the first seven years but could also become riskier if the collateral value is declining or the macroeconomic environment is expected to deteriorate.

Table 5. Projection of house price (*HP*), loan balance (*N*), *LTV* (in %), *DSC* (in %), *PIT PD* conditional on *AD* = 0 (in %) by (33), *PIT PD* conditional on *AD* = 1 (in %) by (33), *LTV_{dt}* in the 25% house price decline scenario, *TTC PD* conditional on *AD* = 0 (in %), *TTC PD* conditional on *AD* = 1 (in %), *PIT loss rate l* (in %), downturn *LGD* (in %) by (34), prepayment probability (in %) by (35), arrears rate (in %) by (36), cure rate (in %) by (37), Stage 2 probability (in %).

Year	<i>HP</i>	<i>N</i>	<i>LTV</i>	<i>DSR</i>	<i>p_{AD=0}</i>	<i>p_{AD=1}</i>	<i>PD_{AD=0}</i>	<i>PD_{AD=1}</i>	<i>LTV_{dt}</i>	<i>l</i>	<i>LGD</i>	<i>cpr</i>	<i>ar</i>	<i>cr</i>	<i>t</i>
1	500,000	500,000	100.0	27.5	1.30	20.3	1.84	23.7	133.3	11.00	27.7	0.25	1.23	58.2	0.00
2	510,000	490,000	96.1	27.5	1.25	19.6	1.77	23.0	128.1	9.04	25.1	0.29	1.22	58.2	1.24
3	517,650	479,650	92.7	27.5	1.23	18.8	1.60	21.1	123.5	7.33	22.8	0.42	1.24	57.7	1.53
4	522,827	468,938	89.7	27.5	1.22	18.1	1.46	19.4	119.6	5.85	20.8	0.55	1.25	57.2	1.63
5	525,441	457,851	87.1	27.5	1.21	17.4	1.33	17.7	116.2	4.57	19.1	0.68	1.26	56.7	1.68
6	528,068	446,375	84.5	27.5	1.21	17.0	1.24	16.8	112.7	3.26	17.4	0.80	1.27	56.2	1.72
7	528,068	434,498	82.3	27.5	1.18	16.4	1.18	16.0	109.7	2.14	15.9	0.93	1.27	56.2	1.74
8	528,068	422,206	80.0	27.5	1.15	16.1	1.15	15.6	106.6	1.00	14.3	0.95	1.27	56.2	1.75
9	528,068	409,483	77.5	27.5	1.13	15.8	1.13	15.3	103.4	1.00	12.7	0.97	1.27	56.2	1.76
10	528,068	396,315	75.1	27.5	1.10	15.5	1.10	15.0	100.1	1.00	11.0	1.00	1.27	56.2	1.77

Table 6. Projection of expected loan balance, expected interest income, expected funding costs, expected operational costs, *ECL* conditional on Stag 1, *LLP* conditional on Stage 1, adjusted minimum capital conditional on Stage 1, *RAROC* conditional on Stage 1 (in %), *ECL* conditional on Stage 2, *LLP* conditional on Stage 2, adjusted minimum capital conditional on Stage 2, *RAROC* conditional on Stage 2 (in %), total expected *RAROC* (in %).

Year	\hat{N}	$z \cdot \hat{N}$	$\hat{f} \cdot \hat{N}$	$c \cdot \hat{N}$	<i>ELC</i> ¹	<i>LLP</i> ¹	\bar{K}_{min}^1	<i>RAROC</i> ¹	<i>ELC</i> ²	<i>LLP</i> ²	\bar{K}_{min}^2	<i>RAROC</i> ²	<i>RAROC</i>
1	500,000	17,500	12,592	2500	718	715	22,340	7.3	13,853	26,757	69,948	-11.84	7.33
2	488,775	17,107	12,482	2444	551	552	19,368	8.18	10,640	20,265	64,309	-10.00	7.26
3	477,067	16,697	12,347	2385	427	431	16,114	9.27	7922	14,926	57,374	-8.24	8.19
4	464,438	16,255	12,196	2322	326	332	13,457	10.19	5807	10,648	51,226	-6.58	9.09
5	450,949	15,783	12,028	2255	244	250	11,287	10.84	4171	7258	45,789	-5.03	9.75
6	436,663	15,283	11,840	2183	165	172	9513	11.23	2767	4607	40,855	-3.32	10.20
7	421,624	14,757	11,621	2108	99	107	8164	11.14	1633	2720	36,451	-1.55	10.24
8	405,897	14,206	11,367	2029	40	47	7024	10.79	651	1,533	32,173	0.47	10.06
9	389,924	13,647	11,078	1950	39	44	5890	9.71	640	1085	27,383	-0.07	9.01
10	373,707	13,080	10,749	1869	38	41	4819	8.66	622	577	22,897	-0.68	7.97

6. Discussion

In this article, we discussed the measurement of loan performance, taking into account the latest Basel regulation and the recent revision of accounting standards, IFRS 9 and CECL, respectively. We have discussed the different risk parameters that are needed for each framework, through-the-cycle, point-in-time and downturn parameters, and shown how they can be obtained in principle. We have proposed a framework where the performance of a loan is measured period-by-period using RAROC as the performance measure. Taking a weighted average over all periods results in a particular loan's total performance. An example illustrated the application of this framework for residential mortgage lending.

The contribution of this article has been threefold. First, we have provided a comprehensive scheme for loan performance measurement that takes into account all relevant regulatory rules and utilizes the credit risk parameters that have been estimated for regulatory and accounting purposes. It allows banks to make more informed decisions about loan applications and supports them in monitoring their existing loan portfolios. Second, we have shown that LLP computed under IFRS 9 or CECL are not suitable as a measure for expected loss and cannot be used in the numerator of a RAROC scheme. Instead, we have derived an expression for the expected loss coverage (ELC) which reflects the true economic loss. Third, we have seen that for accurate measurement of the loan performance, it is insufficient to consider the current or the next year only, but the full lifetime of a loan has to be taken into account.

A limitation of this approach is that it strongly relies on econometric models, especially models that relate macroeconomic factors to risk measures related to borrower and loan facility, like PD, LGD, CPR, or CCF. If such models are unavailable or their predictive power is weak, this approach might not be applicable or does not lead to reliable results. Furthermore, we have set the focus on residential mortgages in this article. Extensions to more general loan structure, including credit lines, are left for future research.

Finally, we remark that the framework we have proposed could be used for numerous risk management applications beyond performance measurement. It could be used for balance sheet stress testing where a macroeconomic stress scenario is provided and the evolution of provisions, capital, and profit can be monitored under this scenario over time. In addition, the impact of the proposed new regulation can be evaluated to see whether a bank is severely impacted or not. An example would be [BCBS \(2017\)](#) where a new floor on regulatory capital is introduced and our proposed framework could be very helpful in assessing its impact. Finally, if a bank seeks capital relief by securitizing or selling parts of its portfolio, the multi-period projections could be valuable in deciding about portfolio size and the optimal selection of loans.

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Abbreviations

The following abbreviations are used in this manuscript:

RAROC	risk-adjusted return on capital
RORAC	return on risk-adjusted capital
PD	default probability
LGD	loss given default

EAD exposure at default
 CCF credit conversion factor
 CPR conditional prepayment rate

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