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Measuring Systemic Governmental Reinsurance Risks of Extreme Risk Events

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Abstract: This study presents an easy-to-handle approach to measuring the severity of reinsurance that faces a system of dependent claims, where the reinsurance contracts are of excess loss or proportional loss. The proposed approach is a natural generalization of common reinsurance methodologies providing a conservative framework that deals with the fundamental question of how much money should a government hold to prepare for natural or human-made extreme risk events that the government will cover? Although the ruin theory is commonly used for extreme risk events, we suggest a new risk measure to deal with such events in a new framework based on multivariate risk measures. We analyze the results for the log-elliptical model of dependent claims, which are commonly used in risk analysis, and illustrate our novel risk measure using a Monte Carlo simulation.

Keywords: loss retention; reinsurance policy; reinsurance claim; catastrophic events; risk management; financial simulation

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1. Introduction

Insurance policies promise to pay claims to the policyholders when accidents occur; however, if the insurer defaults, policyholders may lose their claims. To avoid potential insolvency, insurance companies typically manage residual risks through reinsurance arrangements that transfer all or part of their liabilities arising from sold insurance policies to another insurer. Hence, in cases of high aggregate claims, the reinsurer will pay the primary insurer part of their excess loss (Cai and Chi 2020).

However, while reinsurance arrangements help insurers reduce risk exposure and protect them against catastrophic losses and possible insolvency (Berger et al. 1992; Powell and Sommer 2007), they do not completely eliminate the risk. More specifically, an insurer may significantly reduce underwriting risk when reinsurance programs manage small and uncorrelated risks (Cummins and Trainar 2009); however, in cases of correlated risks, a single event can cause losses to many policyholders simultaneously, which might pose a significant risk of default for both the primary insurer and reinsurer (Cummins et al. 2021). In fact, the positive-dependence structure among the risks plays a critical role in the risk transfer to the reinsurer, as noted by Cai and Chi (2020), and thus the reinsurer should consider the correlation among the risks when managing his terminal risk, to avoid potential insolvency.

Large catastrophic events, in particular, significantly increase the insurer's risk of default, as noted by Cummins and Trainar (2009), and thus have a large potential to cause extreme losses to the reinsurer. More particularly, reinsurance is the traditional mechanism for hedging catastrophe events (Drexler and Rosen 2022; Zhao et al. 2021); thus, extreme catastrophe events, which evidently cause high losses to many policyholders (Cummins et al. 2021), may result in a large aggregate loss to the reinsurer and potential insolvency. Moreover, Cai and Wei (2012) and Zhao et al. (2021) argue that multiple reinsurance treaties are commonly positive-dependent, since the number of property losses and dead people in such catastrophes, for instance, are usually positively dependent; thus, if the reinsurer has many lines of business, one catastrophe may cause him severe and unexpected aggregate loss.

The impacts of catastrophe events on reinsurance is evident in recent natural disasters. For example, Cummins et al. (2004) document that Hurricane Andrew in 1992 and the Northridge Earthquake in 1994 resulted in a USD $30 billion loss in the reinsurance industry; Chang et al. (2018) show that Hurricanes Katrina and Wilma in 2005 resulted in an estimated USD $60 billion loss to the insurance and reinsurance industries; Cummins and Trainar (2009) also argue that World Trade Center terrorist attacks in 2001 and Hurricanes Katrina, Rita, and Wilma in 2005 seriously weakened the reinsurers' capital. Hence, if the reinsurers' programs are subject to highly correlated risks, the potential aggregate loss might exceed the reinsurers' capital, and the reinsurers will fail to pay the loss to the primary insurers. Since reinsurers typically manage several reinsurance programs with correlated risks, we argue that reinsurers may default, even if they have high capital reserves.

In practice, if the reinsurer fails to pay the promised claims, the primary insurer will face unexpected aggregate claims, resulting in a potential risk of default and a significant loss to many policyholders (Cai and Chi 2020; Boonen and Jiang 2022). To avoid the potential insolvency of the primary insurer, the reinsurer has to hold sufficient economic capital (Boonen 2017). Typically, insurers and reinsurers set their capital requirements based on the new Solvency II supervisory framework, which sets solvency capital standards based on the risks that the insurer and reinsurer face (Boonen 2017; Cai and Chi 2020). In this regard, an immense body of literature uses the common value-at-risk (VaR) measure, which quantifies the potential excess loss using a high quantile of the loss distribution, or the expected shortfall (ES), which quantifies the average of expected loss when it exceeds the VaR for reinsurance applications (Zhou and Wu 2009; Cai and Tan 2007; Cai et al. 2008; Chi and Tan 2011; Cai et al. 2014; Mao and Cai 2018; Cai and Chi 2020). Other studies use the well-known Tail Conditional Expectation (TCE) measure, which quantifies the average exceedance of the insurance claims given that the claim exceeds a certain quantile level, to derive optimal reinsurance arrangements that minimize the insurer's risk after reinsurance (Dhaene et al. 2006; Chi 2012; Chi and Tan 2011; Chi and Weng 2013). However, almost all of these studies assume only two agents in the market, that is, one insurer and one policyholder, thus assuming that the insurer (or reinsurer) is facing one risk.

On the contrary, in practice, most insurers have many lines of business with several reinsurance arrangements (Cai and Chi 2020; Zhao et al. 2021). Thus, the reinsurer should employ a risk management strategy that is associated with multiple and correlated risks. In this regard, the world of risks is in fact multivariate, and thus dealing with a univariate risk measure is inadequate, as noted by Landsman et al. (2016).

More recent studies have attempted to deal with multivariate risks to determine optimal reinsurance strategies (Cai and Wei 2012; Cai et al. 2014; Cheung et al. 2014; Zhu et al. 2014). We argue that these studies had two major limitations. First, most studies assume that the reinsurer will always be able to pay promised claims, regardless of the reinsurer's solvency; in fact, the reinsurer may also bear extreme losses, in cases of positive-dependent risks, as evident from Cummins et al. (2004) and Zhao et al. (2021). Second, the vector of risks in the common TCE measure does not consider the dependence structure among the various risks, which is fully determined by the variance–covariance structure of the underlying distributions of the risks, as noted by Landsman et al. (2016) and Cai et al. (2017). Hence, most of these studies do not consider the correlation among these risks, which evidently affects the reinsurer's capital reserve and risk of default (Froot 2007; Cummins and Trainar 2009; Cai and Chi 2020; Zhao et al. 2021).

In this study, we propose a novel systemic risk reinsurance measure (SRRM) for government reinsurance contracts for national disasters that quantifies the reinsurer's expected aggregate loss as a function of dependent multivariate risks. The proposed model has several advantages. By focusing on tail losses, we can directly capture the dependence structure of the losses. Furthermore, the model is easy to handle and is distributed because it is based on (nontrivial) conditional expectations. In the present work, we both model reinsurance contracts of systemic risks and consider extreme risk events by conditioning the model to be at a certain level of risk; this also allows measuring the tail distributions of the losses faced by governments when signing reinsurance contracts. Our approach does not assume a particular distribution for risks; therefore, it provides a flexible and general framework for modelling reinsurance between the government and private sectors.

2. Literature Review

The literature proposes several models for an optimal reinsurance design under different risk measures. These models suggest that the insurer has an optimal reinsurance program based on maximizing his terminal wealth's expected utility function and minimizing his retained risk, measured by various risk measures. For example, Verlaak and Beirlant (2003) and Kaluszka and Okolewski (2008) derive optimal reinsurance under premium principles based on the mean and variance of the reinsurer's share of the total claim amount. Other studies, such as Wang et al. (2005), Huang (2006), Zhou and Wu (2009), Cai et al. (2008), and Chi and Tan (2011), employ VaR and ES measures to assist insured parties in determining the optimal insurance policy. Furthermore, Cai and Tan (2007), Cai et al. (2008), and Chi and Tan (2011) derive optimal reinsurance strategies that minimize the insurer's VaR and ES. Other studies, including Cheung (2010), Chi (2012), Chi and Tan (2011), and Chi and Weng (2013), consider the TCE measure to derive an optimal one-period reinsurance model with a minimal insurer risk. However, almost all these studies assume a one-period reinsurance model with one loss variable, while in practice, most reinsurers have many lines of business.

Recent studies have extended univariate risk models to include several business lines and multivariate risk factors. Cheung et al. (2014) and Bernard et al. (2020) considered multivariate risks with given dependencies among these risks to model optimal multivariate reinsurance programs. Denuit and Vermandele (1998) and Cai and Wei (2012) show that the excess-of-loss treaty is the optimal reinsurance form for an insurer with dependent risks among a class of individualized reinsurance contracts. Cheung et al. (2014) proposed a minimax model that establishes an optimal stop-loss reinsurance that minimizes the total retained risk obtained by several dependent risks. Asimit et al. (2013b) and Zhu et al. (2014) study the optimal risk transfer among multiple reinsurance counterparties, where insurer risk is a function of quantile-based risk measure criteria, including VaR and ES. However, most studies on optimal reinsurance programs do not consider the likelihood of reinsurer insolvency; hence, they assume that the reinsurer will always be able to pay the loss in case of a catastrophic event. In practice, the reinsurer is also subject to the risk of default in extreme catastrophic events, as evident from Cummins et al. (2004).

Recent studies discuss the impact of the risk of default by the reinsurer and its impact on the insurer. Bernard and Ludkovski (2012) studied the impact of reinsurers' defaults by applying a multiplicative default risk model in which the probability of the reinsurer's default depends on the loss incurred by the insurer, concluding that reinsurance becomes unreliable in the presence of counterparty risk. Asimit et al. (2013a) studied the optimal reinsurance contract in the presence of the reinsurer's default and found that the reinsurer's risk of default affects the policyholders' welfare and the default risk may change the insurer's ideal arrangement. Cai et al. (2014) studied the impact of the reinsurer's initial capital and default risk on the insurer and derived an optimal reinsurance model that maximizes the expected utility of an insurer's terminal wealth or minimizes the insurer's VaR. However, these studies assume a constant initial capital for the reinsurer or derive the regulatory requirements of the reinsurer's initial capital reserve as a function of the reinsurer's promised amount.

In individual reinsurance programs, the reinsurer is mainly concerned with dependent risks, which may have a large impact on the risk of default by the reinsurer in the case of a catastrophic event. For example, the number of property losses and the number of deaths in earthquakes or hurricanes are usually dependent (Cai and Wei 2012). Hence, we argue that it is vital to describe the effect of dependent risks on the aggregate loss for the reinsurer.

In this study, we propose a novel methodology that measures the tail distributions of reinsurers' aggregate losses in extreme events when these risks are mutually dependent. We extend the concept of the MTCE risk measure (Landsman et al. 2016; Cai et al. 2017) to measure reinsurance systemic risk. We develop a novel SRRM that quantifies the reinsurer's excess aggregate loss under a system of dependent risks and different quantile levels of risks. We also provide a simulated example of quantifying this risk using a Monte Carlo simulation, which may be used as an actuarial tool for quantifying the reinsurer's risk and capital requirements.

3. Methodology

3.1. Systemic Excess Loss Reinsurance Model

Let $\mathbf{X} = (X_1, X_2, \ldots, X_n)^T$ be a random number vector of n independent claims $X_1, X_2, \ldots, X_n$ with value-at-risk (VaR) measures $VaR_{q1}(X_1), VaR_{q2}(X_2), \ldots, VaR_{qn}(X_n)$, respectively, where the VaR of each claim X_j has q_j-th quantile, $q_j \in (0,1)$, $j = 1, 2, \ldots, n$. The multivariate tail value at risk ($MTVaR$) is defined (Landsman et al. 2016):

$$MTVaR_q(\mathbf{X}) = E\big(\mathbf{X} \big| \mathbf{X} > VaR_{\mathbf{q}}(\mathbf{X})\big) = E\big(\mathbf{X} \big| X_1 > VaR_{q_1}(X_1), \ldots, X_n > VaR_{q_n}(X_n)\big). \quad (1)$$

here, $VaR_{\mathbf{q}}(\mathbf{X}) = \big(VaR_{q_1}(X_1), VaR_{q_2}(X_2), \ldots VaR_{q_n}(X_n)\big)^T$ is the $n \times 1$ vector of VaRs, and the inequality $\mathbf{U} \leq \mathbf{V}$ of two random vectors $\mathbf{U}, \mathbf{V} \in \mathbb{R}^n_+$ means that $U_i \leq V_i$ almost surely for every $i = 1, 2, \ldots, n$. We also define the aggregate loos of X, Y, and Z, respectively, by $S_X = X_1 + X_2 + \ldots + X_n$, $S_Y = Y_1 + Y_2 + \ldots + Y_n$, $S_Z = Z_1 + Z_2 + \ldots + Z_n$, where clearly, $S_X = S_Y + S_Z$.

We start by considering a reinsurance contract between the primary insurance company and the reinsurer, in which the reinsurer is obligated to pay the primary insurer all the excess losses in case of a catastrophic event. We let X_i be a univariate random insurance claim from catastrophic events, which represents the loss or the claim in line i for the policyholder $(i = 1, 2, \ldots, n)$. Under the reinsurance model, for the i-th business line, we let Y_i be the claims paid by the insurer party, namely:

$$Y_i = \begin{cases} X_i \text{ if } X_i \leq M_i \\ M_i \text{ if } X_i > M_i \end{cases}, \quad (2)$$

and we let Z_i be the excess loss paid by the reinsurer, namely:

$$Z_i = \begin{cases} 0 \text{ if } X_i \leq M_i \\ X_i - M_i \text{ if } X_i > M_i \end{cases} \quad (3)$$

Since we assume that, in general, $\mathbf{X} = (X_1, X_2, \ldots, X_n)^T$ is a vector of mutually dependent claims from n business lines, we define $\mathbf{Y} = (Y_1, Y_2, \ldots, Y_n)^T$ and $\mathbf{Z} = (Z_1, Z_2, \ldots, Z_n)^T$ as the respective vectors of payments for the primary insurer and reinsurer, derived from positive-dependent catastrophic events.

Assuming the dependence structure of $\mathbf{X}$, we argue that the correlation between reinsurance claims may potentially cause a high aggregate loss for the reinsurer, as in Cummins et al. (2004) and Cummins and Trainar (2009). Hence, we argue that reinsurer's capital may not be determined by simply taking $TCE_{\mathbf{q}}(\mathbf{X})$, as desired by regulatory requirements, which do not consider the aggregate loss of the reinsurer and do not consider mutually dependent risks.

The simplest measure of the reinsurer vector of risks **Z** is given by taking the expectation $E(\mathbf{Z})$, which gives the expected value of reinsurance-dependent claims $Z_1, Z_2, \ldots, Z_n$. Because $E(\mathbf{Z}) = E(\mathbf{X}) - E(\mathbf{Y})$, we can consider the expectation of **Y**, which is given by:

$$E(\mathbf{Y}) = E\big(\mathbf{X}I_{X_i \le M_i} + \mathbf{M}I_{X_i > M_i}\big), \tag{4}$$

where $I_{X_i \le M_i} = diag\big(1_{X_1 \le M_1}, 1_{X_2 \le M_2}, \ldots, 1_{X_n \le M_n}\big)$ and $I_{X_1 > M_1} = diag(1_{X_1 > M_1}, 1_{X_2 > M_2}, \ldots, 1_{X_n > M_n})$ are diagonal matrices with indicator components.

However, the problem with the measure in (4) is that it does not take into account the dependence structure between the risks, since the i-th element of $E(\mathbf{Y})$ is $E(Y_i)$, which is the i-th claim paid by the primary insurer independent of the other claims.

Instead, we suggest a different approach to measure the reinsurer's systemic risk, using conditioning on the expectation of **Y** that focuses on the values of the riskier claims of **X**. Following Landsman et al. (2016), we propose a quantile-based conditional expectation measure that quantifies the reinsurer's systemic risk by taking the tail systemic projection of **X** over **Y**. However, we extend Landsman et al. (2016) by considering the common tail systemic projection $\mathbf{X} \to \mathbf{X}\big|\cup_i^n X_i > VaR_q(X_i)$ and suggest quantifying the expectation of **Y** given the unification of the common tail region of **X**, namely:

$$E_{\mathbf{q},\mathbf{X}}(\mathbf{Y}) := E\big(\mathbf{Y}\big|\cup_i^n X_i > VaR_q(X_i)\big) = E\big(\mathbf{X}I_{X_i \le M_i} + \mathbf{M}I_{X_i > M_i}\big|\cup_i^n X_i > VaR_q(X_i)\big). \tag{5}$$

3.2. Systemic Proportional Reinsurance

For the same vector of claims **X**, we can have a proportional reinsurance contract. The most straightforward reinsurance contract is just paying for some percentage of the claim using a *quota share arrangement*:

$$E(\mathbf{X}) := E(\mathbf{Y}) + E(\mathbf{Z}) = E(A\mathbf{X}) + E\big(\overline{A}\mathbf{X}\big), \tag{6}$$

where $A = diag(\alpha_1, \alpha_2, \ldots, \alpha_n)$ is the matrix of the retention levels $0 \le \alpha_i \le 1$ for $i = 1, 2, \ldots, n$, and $\overline{A} = I_n - A = diag(1 - \alpha_1, 1 - \alpha_2, \ldots, 1 - \alpha_n)$, and I_n is the $n \times n$ identity matrix.

For a surplus proportional-based reinsurance model, a possible reinsurance contract is:

$$\mathbf{Y} = \begin{cases} A\mathbf{X} \text{ if } \mathrm{A}\mathbf{X} \le \mathbf{M} \\ \mathbf{M} \text{ if } \mathrm{A}\mathbf{X} > \mathbf{M} \end{cases}, \tag{7}$$

and the reinsurers' claims are then given by:

$$\mathbf{Z} = \begin{cases} \overline{A}\mathbf{X} \text{ if } \mathrm{A}\mathbf{X} \le \mathbf{M} \\ \mathbf{X} - \mathbf{M} \text{ if } \mathrm{A}\mathbf{X} > \mathbf{M} \end{cases}. \tag{8}$$

For the systemic proportional reinsurance model, we observe that the expected value of Y is given by:

$$E(\mathbf{Y}) = E\big(A\mathbf{X}I_{\alpha_i X_i \le M_i} + \mathbf{M}I_{\alpha_i X_i > M_i}\big). \tag{9}$$

We consider quantiles $\mathbf{q} \in [0,1]^n$ that satisfy the inequalities $VaR_{\mathbf{q}}(\mathbf{X}) \le \mathbf{M}$, since VaR is assumed to be a lower bound for the claims. We then define SRMM, which is given by:

$$\begin{aligned} &E_{\mathbf{q},\alpha,\mathbf{X}}(\mathbf{Y}) = E\big(\mathbf{Y}\big|\cup_i^n X_i > VaR_q(X_i)\big) \\ &= E\big(A\mathbf{X}I_{\alpha_i X_i \le M_i} + \mathbf{M}I_{\alpha_i X_i > M_i}\big|\cup_i^n X_i > VaR_q(X_i)\big) \\ &= AE\big(\mathbf{X}I_{\alpha_i X_i \le M_i}\big|\cup_i^n X_i > VaR_q(X_i)\big) + \mathbf{M}E\big(I_{\alpha_i X_i > M_i}\big|\cup_i^n X_i > VaR_q(X_i)\big). \end{aligned} \tag{10}$$

Lastly, following Chen and Yuen (2012) we propose evaluating the risk on the primary insurer and the reinsurer by measuring the SRRM over S_Y and over S_Z. Namely, we propose evaluating the reinsurer's risk by measuring the multivariate conditional tail expectation of the aggregate amount of claims, from a large insurance portfolio, given the unification of the common tail region of **X**.

3.3. Monte Carlo Simulation

To illustrate the proposed SRRM, we provide a simulated example of the reinsurer risk measure by calculating the quantile-based risk of the aggregated claims of $\mathbf{Z}$ under different quantile levels according to Equation (9). We exemplify this methodology by calculating the SRRM value of the aggregated claims for $i = 1, 2, \ldots, 10$ business lines, assuming positive-dependent risks.

To simulate insurance severity losses, in which losses are rare but each has a high financial value, we follow the strand of literature by assuming that these claims are modelled by a lognormal distribution (see Nowak and Romaniuk 2013). Hence, we start by assuming that the insurance claims $\mathbf{X}$ are lognormally distributed, with the respective vector means $\boldsymbol{\mu}$ and vector of standard deviations $\boldsymbol{\sigma}$. We construct $\boldsymbol{\mu}$ by generating random numbers between 0 and 1, which represent the average risk claims (in billions $), and we construct $\boldsymbol{\sigma}$ by generating random numbers between 0 and 0.1 which represent the standard deviations of the claims (in billions $).

Further, we generate $\mathbf{X}$ claims by assuming that the insurance claims among the reinsurance treaties are mutually dependent with the respective correlation matrix $\mathbf{r}$. To illustrate the effect of the system of risks on the reinsurer, we construct two variance–covariance matrices $\boldsymbol{\Sigma}$ and respective simulated datasets: (1) the first dataset considers independent risks (i.e., $\boldsymbol{\Sigma}$ is a diagonal matrix with respective standard deviations), while the second dataset considers positively dependent risks, as noted by Cai and Wei (2012). For the second dataset, we constructed $\boldsymbol{\Sigma}$ using the $\boldsymbol{\sigma}$ vector and by simulating a positively highly correlated correlation matrix $\mathbf{r}$, which considers random numbers that vary between 0.85 and 0.95. This implies that $\boldsymbol{\Sigma}$ was highly correlated. Finally, we simulated both datasets using Matlab's MVLOGRAND function (Lienhard 2023), in order to generate multivariate lognormal random numbers with correlation. We use $\boldsymbol{\mu}, \boldsymbol{\sigma}$ and the respective variance–covariance matrices $\boldsymbol{\Sigma}$ as inputs to the function, and generate one million lognormal random numbers, which represent the insurance claims X_i for business line.

Following Asimit et al. (2013b), we assume that the primary insurer is VaR regulated; thus, the reinsurance strategy will protect from extreme losses only. We assume that the primary insurer retains 95% of the loss in line i and the reinsurer covers the rest of the loss. Accordingly, we set the maximal payment M_i made by the insurer to be the inverse of the theoretical cumulative distribution of the lognormal distribution. Furthermore, for the i-th claim, we calculate the payments Z_i made by the reinsurer according to Equation (3) and the respective value of M_i, and we calculate the aggregate payments made by the reinsurer by taking different quantile levels, ranging from 0.05 to 0.95. Finally, based on our novel SRRM, we plot the results of the tail conditional expectation of the aggregate claims of $\mathbf{Z}$ as a function of these quantiles for both the positive-dependent and independent simulated dataset; we plot the difference between these conditional expectations.

4. Results

Table 1 shows the random correlation matrix used to generate the simulated correlated risks. The correlation matrix shows highly positive dependent risks, indicating that a large reinsurance claim resulting from one risk results in large reinsurance claims for other risks. Hence, we use a correlation matrix to generate dependent risks, which captures a system of multivariate risks on a reinsurer, resulting in several individualized reinsurance treaties.

To obtain the risk measure of the sum of the individualized reinsurance claims that captures the aggregate risk for the reinsurer, we first generate two simulated insurance claims for independent risks (by assuming the correlation matrix as an identity matrix) and positive-dependent risks using the correlated risks in Table 1. We then calculate the expected reinsurance claims using Equation (9).

Table 1. Correlation matrix for generating correlated lognormal risks.

Risk 1	Risk 2	Risk 3	Risk 4	Risk 5	Risk 6	Risk 7	Risk 8	Risk 9	Risk 10
1.000	0.904	0.890	0.920	0.885	0.924	0.932	0.929	0.901	0.903
0.904	1.000	0.895	0.859	0.865	0.889	0.893	0.945	0.938	0.859
0.890	0.895	1.000	0.903	0.909	0.918	0.939	0.883	0.909	0.861
0.920	0.859	0.903	1.000	0.876	0.920	0.889	0.917	0.865	0.864
0.885	0.865	0.909	0.876	1.000	0.894	0.927	0.894	0.870	0.918
0.924	0.889	0.918	0.920	0.894	1.000	0.890	0.933	0.891	0.900
0.932	0.893	0.939	0.889	0.927	0.890	1.000	0.927	0.925	0.869
0.929	0.945	0.883	0.917	0.894	0.933	0.927	1.000	0.933	0.900
0.901	0.938	0.909	0.865	0.870	0.891	0.925	0.933	1.000	0.865
0.903	0.859	0.861	0.864	0.918	0.900	0.869	0.900	0.865	1.000

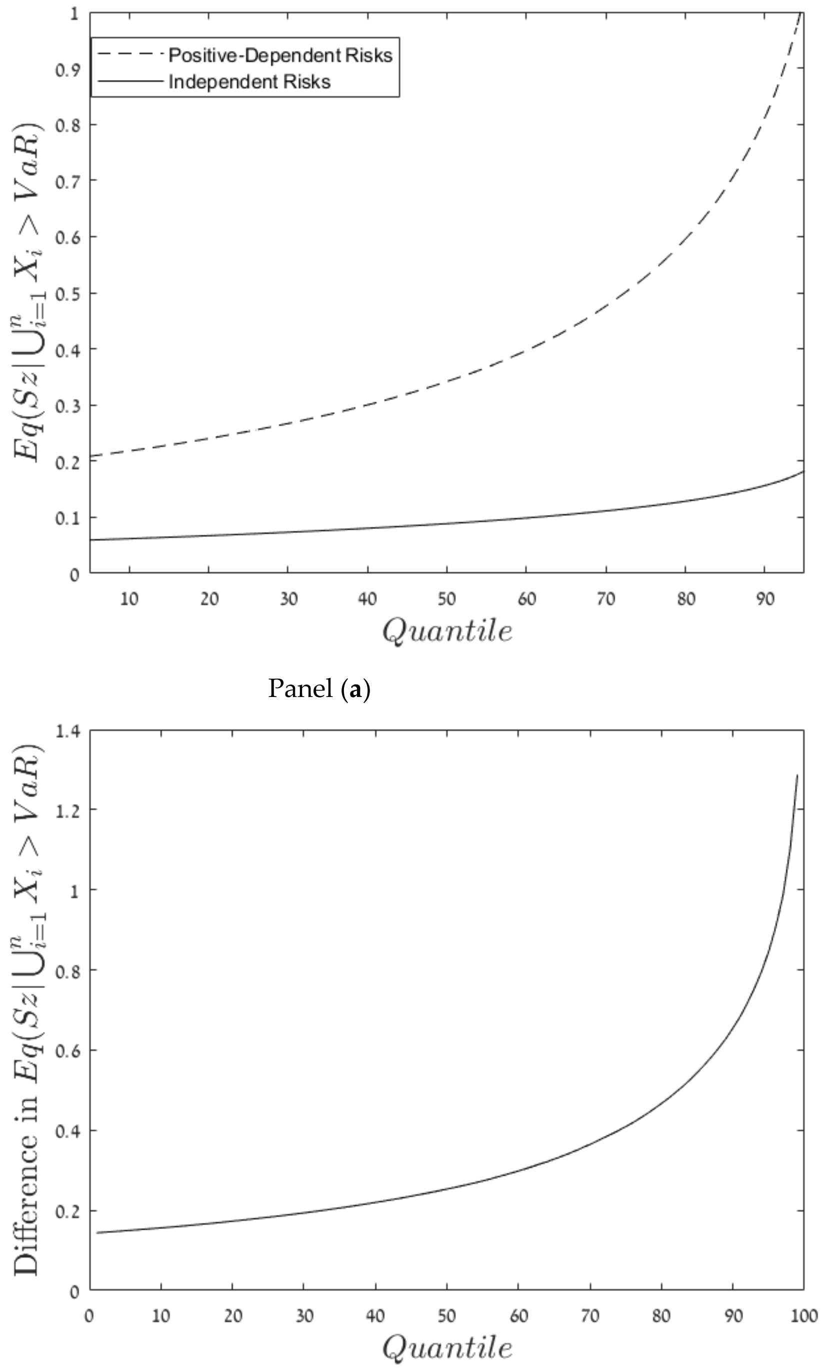
Table 2 provides the conditional expectation of reinsurance claims for independent and positive-dependent risks according to our novel SRRM in Equation (9) for different quantile levels of $\mathbf{q} = (0.1, 0.25, 0.5, 0.75, 0.9)$.

Table 2. SRRM for independent and positive-dependent reinsurance claims.

Percentile	Risk 1	Risk 2	Risk 3	Risk 4	Risk 5	Risk 6	Risk 7	Risk 8	Risk 9	Risk 10
Independent insurance claims										
10%	0.005	0.008	0.010	0.005	0.005	0.005	0.007	0.006	0.003	0.006
25%	0.006	0.010	0.011	0.006	0.006	0.006	0.008	0.007	0.003	0.007
50%	0.008	0.012	0.014	0.008	0.008	0.007	0.010	0.009	0.004	0.009
75%	0.010	0.016	0.019	0.011	0.010	0.010	0.014	0.012	0.005	0.012
90%	0.013	0.021	0.024	0.014	0.013	0.013	0.018	0.016	0.007	0.015
Positive-Dependent insurance claims										
10%	0.019	0.030	0.034	0.019	0.019	0.018	0.025	0.022	0.010	0.022
25%	0.022	0.035	0.039	0.022	0.022	0.021	0.029	0.026	0.012	0.025
50%	0.030	0.047	0.053	0.030	0.029	0.029	0.040	0.035	0.016	0.034
75%	0.047	0.072	0.082	0.046	0.045	0.045	0.062	0.055	0.024	0.051
90%	0.072	0.109	0.126	0.070	0.069	0.069	0.096	0.086	0.036	0.078

As expected, we find that the conditional expectation measure of $\mathbf{Z}$ is much higher in the case of positive-dependent risks than for independent risks for all reinsurance treaties. For example, for the 90% quantile level, the conditional expectation measure of the first risk is 0.072 for positive-dependent insurance risks compared to 0.013 for independent risks. Hence, these results suggest that if the reinsurer's capital reserve is determined by the TCE measure, according to regulatory requirements, then the reinsurer's capital reserve is highly affected by the correlation among the positive-dependent risks, as noted by Cummins and Trainar (2009) and Cai and Wei (2012). Thus, we find that a strong positive dependency between the system of risks (seen in disasters such as earthquakes, tornados, and hurricanes) significantly affects the individualized reinsurance risk; hence, it intuitively, significantly increases the systemic risk on the reinsurer, which is concerned with the aggregate risk of the entire portfolio.

Figure 1 depicts the conditional expectation of the reinsurer's aggregate risks $S_{\mathbf{Z}}$ for different quantile levels ranging from 1% to 99%. Panel (a) of the figure shows the tail conditional expectation of the aggregate sum of $\mathbf{Z}$ for both the independent and positive-dependent systems of risks. Panel (b) depicts the difference between the conditional expectations of the positive-dependent and independent datasets derived from our novel SRRM.



Panel (**a**)

Panel (**b**)

Figure 1. Tail conditional expectation of the reinsurer's aggregate risks, for different quantile levels. Panel (**a**) shows the conditional tail expectation of the aggregate loss, assuming positive-dependent and independent risks. Panel (**b**) shows the difference in the tail conditional expectation between positive-dependent and independent risks, for various quantile levels.

The results in panel (a) show that the simulated conditional expectation of the reinsurer's aggregate risks is much higher for a positive-dependent system of risks than for an independent system of risks for all quantile levels. The results in panel (b) also show that the difference in the tail-conditional expectation of aggregate loss, which captures the difference in the reinsurer's risk for multivariate positive-dependent risks against independent risks, grows as a function of the selected quantile level. Thus, the results suggest that, the reinsurer's risk is much higher than expected if the risks are not correlated in cases of positive-dependent risks. As noted, the difference in risk may not be reduced by using a higher quantile level.

Hence, the results suggest that if the reinsurer has several excess-of-loss reinsurance treaties from several business lines, then the reinsurer is subject to a much higher risk than the risk captured by the common TCE measure, which does not consider the correlation among the risks. Since Solvency II regulatory requirements commonly use the TCE measure to define the reinsurer's risk exposure and minimal capital requirements, the results suggest that the reinsurer needs to set much higher capital reserves to promise payments to the primary insurers. These results highlight the potential of the counterparty risk of default in a multivariate risk framework, since if the reinsurer uses the common TCE measure for setting capital requirements in common positive-dependent risks, the reinsurer may fail to pay the claims to the primary insurer.

5. Summary and Conclusions

Classical stop-loss reinsurance contracts are useful in protecting primary insurers from potential huge losses and the risk of default and in promising payments of insurance claims to policyholders. However, the classical reinsurance approach assumes that the reinsurer will always be able to pay claims to the primary insurers and ignores the possibility of counterparty default risk. More specifically, the potential reinsurer's default significantly rises if the reinsurer has several individualized reinsurance treaties with strong positive dependent risks, such as in cases of loss occurring from earthquakes or hurricanes (Cai and Wei 2012). Motivated by Solvency II regulatory requirements, we argue that the reinsurer should set a minimal capital reserve requirement to promise the payments of claims in positive-dependent risks, in accordance with his risk of exposure.

In this study, we developed a model that defines the minimum capital requirements from the reinsurer's point of view based on the reinsurer's risk exposure. To measure this risk, we extended the MTCE measure (Landsman et al. 2016) to evaluate the reinsurer's risk based on multivariate positive-dependent risks and provided a Monte Carlo simulation. The results show that the reinsurer's risk, captured by our novel SRRM, is significantly higher than the risk captured by the common TCE measure, which does not consider the correlation among the risks.

Thus, our framework provides an appealing actuarial tool for quantifying the potential loss to the reinsurer resulting from highly correlated risks. Since historical natural disasters have demonstrated the strong effect of multivariate and positive-dependent risks on reinsurers' capital reserves (Cummins et al. 2004; Cai and Chi 2020; Zhao et al. 2021), our framework might be a useful tool for assessing the potential loss to the reinsurer from catastrophic events that he reinsures. By quantifying the reinsurer's risk with a positive-dependent risk structure, our framework provides a more accurate guideline about minimum capital requirements for reinsurers with several reinsurance contracts.

Our framework takes into account the standard stop-loss reinsurance contracts, which do not capture more information about the aggregate risk associated with more complicated reinsurance contracts. Future research paths may expand the present study by tailoring the model to different reinsurance types and contracts, to quantify the effect of correlated risks on different reinsurers' risk exposure. Furthermore, future studies may extend the model to include different types of loss distributions.

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