

Article

Was the 2025 DAX Crash Endogenous? Evidence from the Log-Periodic Power Law Model

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Abstract

In this article, we investigate whether the crash of the German DAX index following the U.S. tariff announcement in April 2025 is consistent with pre-existing endogenous market fragility rather than a purely exogenous shock. The analysis is conducted within the Log-Periodic Power Law (LPPL) framework using the Filimonov–Sornette (FS) specification, complemented by shrinking-window estimation, Ornstein–Uhlenbeck residual diagnostics, surrogate time-series analysis, and a GARCH-based Monte Carlo false-positive assessment. The results reveal a statistically stable critical period preceding the observed market collapse, within which the tariff announcement occurred and was followed by a pronounced market decline. Overall, the findings suggest that the market operated in a regime of elevated systemic fragility, where the tariff announcement may have acted as a triggering event within an already critical state. This study contributes to the literature on financial critical phenomena by providing evidence that LPPL-based critical windows may be interpreted as periods of heightened systemic vulnerability rather than precise crash forecasts. From a risk management perspective, such periods may be informative for identifying conditions under which markets are particularly sensitive to external disturbances.

Keywords: econophysics; log-periodic power law model; financial bubbles; endogenous and exogenous crashes; complex systems; U.S. tariffs announcement

1. Introduction

Financial markets are widely regarded as complex systems characterized by nonlinear interactions and the frequent emergence of extreme events with substantial economic consequences (Mantegna and Stanley 1999; Sornette 2003; Potirakis et al. 2013). Understanding and anticipating such events remains a central challenge in financial economics and complex systems research. While traditional approaches often interpret market crashes as stochastic anomalies or exogenous shocks (Fama 1970; Malkiel 1973), an alternative literature frames them as emergent phenomena arising from endogenous dynamics, interpretable within the framework of phase transitions and critical phenomena (Ilinski 1999; Plerou et al. 2003; Yalamova and McKelvey 2011; Zitis et al. 2022).

Within this perspective, financial crashes may result either from endogenous instabilities generated by nonlinear interactions among market participants or from exogenous shocks such as geopolitical or policy events. Endogenous dynamics are typically associated with self-reinforcing feedback mechanisms, herding behavior, and increasing deviations of prices from fundamental values, leading to regimes of heightened fragility and critical sensitivity to small perturbations (Petersen et al. 2010; Battiston et al. 2016). Exogenous



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crashes, by contrast, are precipitated by external shocks such as geopolitical conflicts, natural disasters, or abrupt political decisions. These events are typically unforeseeable and lack reliable early warning indicators (Nayar and Islam 2024).

A recent example of a policy decision that triggered significant market turbulence over a short period was the announcement of new tariffs by the United States (U.S.). On 2 April 2025, the U.S. President announced the so-called “Liberation Day” tariffs, imposing sudden duties on a broad range of imported goods, citing national economic security and the need to protect domestic industries. Financial markets reacted immediately and sharply. The S&P 500 plunged 4.8% on 3 April, followed by a further 5.9% decline on 4 April and an additional 0.23% drop on 7 April. By 8 April, the index had suffered a cumulative loss of approximately 12% from its level on 2 April. The ripple effects extended beyond the U.S., with European markets also tumbling. Germany’s DAX index fell 3.01% on 3 April, 4.95% on 4 April, and another 4.13% on 7 April, culminating in a total loss of 11.62% by 8 April. This sell-off represented one of the sharpest short-term declines observed in recent years, fueled by fears of an escalating trade war and its potential impact on Germany’s export-driven economy. Global markets mirrored this volatility, reflecting widespread concern over intensifying trade tensions. The steepest sectoral declines were observed in technology, manufacturing, and consumer goods—industries highly sensitive to trade policy shifts. This episode underscored how abrupt political decisions, particularly those affecting international economic relations, can act as potent exogenous shocks, triggering destabilization and short-term systemic stress across globally interconnected financial systems.

While the decision to impose tariffs represents a typical exogenous event capable of triggering market shocks, it raises a broader question regarding the underlying state of the financial system at the time of the announcement. Specifically, the key issue is whether the market had already entered a phase of endogenous instability prior to the tariff declaration, such that an external shock may have acted primarily as a catalyst accelerating an ongoing fragile dynamic. To investigate this question, we employ the LPPL framework to analyze intraday data from the DAX index Contract for Difference (CFD). The DAX index is selected as a major European benchmark with pronounced sensitivity to international trade and global macroeconomic developments. The LPPL framework has been widely used to identify speculative bubble dynamics and estimate critical times associated with financial instability and market crashes (Sornette et al. 2015; Zhang et al. 2016; Ji and Zhang 2024). Originating from the work of Sornette and collaborators in the 1990s, the model conceptualizes financial crashes as critical phenomena analogous to phase transitions in physical systems (Sornette et al. 1996; Sornette and Johansen 1997). This approach was later formalized in the Johansen–Ledoit–Sornette (JLS) framework, which incorporates heterogeneous interacting agents and herding-driven feedback mechanisms as drivers of market instability (Johansen et al. 2000).

Despite its widespread application, the LPPL framework has been subject to several methodological criticisms, including sensitivity to calibration windows, parameter instability, false positives, retrospective fitting, and limited reproducibility of estimated critical times (Feigenbaum 2001; Brée and Joseph 2013; Geraskin and Fantazzini 2013). In particular, concerns have been raised that LPPL-like signatures may arise spuriously from noisy financial data and that estimation results can depend strongly on the choice of fitting window and optimization procedure. To address these limitations, the present study adopts five complementary methodological strategies. First, we employ the FS reformulation (2013), which reduces the dimensionality of the nonlinear optimization problem and improves estimation stability by mitigating sensitivity to local minima. Second, we implement a shrinking-window approach to assess the stability and convergence of estimated critical

times across multiple calibration intervals. Third, we apply Ornstein–Uhlenbeck residual diagnostics to evaluate whether the residual dynamics remain consistent with a stationary mean-reverting process. Fourth, we conduct a surrogate time-series analysis based on randomized residuals in order to examine the robustness of the inferred LPPL structure under stochastic perturbations. Finally, we perform a GARCH-based Monte Carlo false-positive analysis to evaluate whether critical windows comparable to those identified by the FS model could emerge by chance under a conventional volatility-clustering process. Collectively, these procedures enhance the statistical robustness, stability, and reproducibility of the inferred critical behavior.

In summary, this study examines whether the crash of the German DAX index following the U.S. tariff announcement in April 2025 can be interpreted through the lens of pre-existing endogenous market dynamics rather than as a purely exogenous shock. More specifically, we investigate whether the market had already been transitioning toward a regime of elevated fragility prior to the announcement, exhibiting patterns consistent with speculative bubble dynamics and the emergence of a critical market state. To the best of our knowledge, this is among the first studies to analyze this recent episode using an LPPL-based critical phenomena framework focused on the identification of a critical time interval rather than a single crash date. Accordingly, the analysis shifts the interpretation of LPPL-based inference away from point forecasting and toward the characterization of time periods associated with heightened systemic vulnerability. From a financial perspective, the results contribute to the literature on systemic risk and market instability by suggesting that LPPL-type dynamics may provide informative signals regarding the buildup of endogenous market fragility prior to major corrections. Such signals may therefore be of practical relevance for investors and risk managers operating under conditions of elevated uncertainty, nonlinear dynamics, and increased tail risk.

2. Methods

This section outlines the methodological framework adopted in the study. Section 2.1 introduces the theoretical formulation of the LPPL model and its modified version by Filimonov and Sornette (2013), which captures the endogenous dynamics of speculative bubbles through power-law growth and log-periodic oscillations. Section 2.2 presents the shrinking-window estimation procedure used to examine the stability and convergence of the inferred critical times across multiple calibration intervals. Section 2.3 describes the analysis of residual dynamics via the Ornstein–Uhlenbeck framework and unit-root tests to assess mean-reversion, while Section 2.4 details the surrogate time-series approach, in which residuals are randomly reshuffled and reintegrated into the LPPL trajectory to test the stability of the estimated critical times. Section 2.5 introduces a GARCH-based Monte Carlo false-positive analysis, in which synthetic price series generated under a volatility-clustering null model are used to evaluate the likelihood of obtaining critical regions similar to those identified in the empirical FS analysis.

2.1. Model Description

Financial systems are inherently heterogeneous, consisting of agents who interpret and respond to information differently (Bury 2013; Kristoufek 2013). Nevertheless, investors often exhibit herd-like behavior, collectively buying or selling in ways that can amplify volatility and contribute to the formation of bubbles and crashes (Bikhchandani and Sharma 2001; Chiang and Zheng 2010; Zitis et al. 2023a, 2023b). The JLS model and its extensions (Johansen et al. 2000) focus on the positive feedback loops generated by mutual imitation among investors, identifying herd behavior as a key driver in bubble formation. This feedback mechanism leads to super-exponential price increases accompanied by cyclical

oscillations around the upward trend. As imitation intensifies and surpasses a critical threshold, growing demand pushes prices upward in a self-reinforcing manner, causing market dynamics to become dominated more by investor sentiment than by fundamental values. This inherently unsustainable process may culminate in a critical juncture, where the mispricing either smoothly transitions to a new regime or abruptly collapses in a market crash. The JLS model suggests that during a price bubble, the observed price trajectory of an asset becomes disconnected from its intrinsic fundamental value.

Building on this framework, the present study adopts the LPPL approach originally developed within the JLS framework (Johansen et al. 2000) and later reformulated by Filimonov and Sornette (2013) to improve estimation stability and reduce parameter degeneracy. In this formulation, bubble dynamics are represented as the superposition of a power-law acceleration and log-periodic oscillations that intensify as the system approaches a critical time t_c —interpreted as the theoretical endpoint of the bubble regime—associated with the most probable time of regime change, e.g., crash or transition. For empirical implementation, this study uses the FS specification, expressed as (Filimonov and Sornette 2013):

$$\ln p(t) = A + B(t_c - t)^m + C_1(t_c - t)^m \cos(\omega \ln(t_c - t)) + C_2(t_c - t)^m \sin(\omega \ln(t_c - t)), \quad (1)$$

where $p(t)$ is the asset price, and $A, B, C_1, C_2, m, \omega$ are parameters of the model explained in Appendix A.

This formulation improves numerical stability by eliminating the nonlinear dependence between the phase parameter and the angular log-frequency present in the standard LPPL representation. The parameters C_1 and C_2 correspond to an amplitude–phase decomposition of the oscillatory component and are related to the standard LPPL parameters through $C_1 = C \cos \phi$ and $C_2 = C \sin \phi$. To guarantee the physical and economic viability of the model’s calibrations, standard filtering constraints are imposed on the key parameters, specifically requiring $0.1 \leq m \leq 0.9$, $6 \leq \omega \leq 13$, and $B < 0$ (Filimonov and Sornette 2013). All empirical estimations in this study are based on the FS specification. Key aspects of the JLS/LPPL model, the FS reparameterization of the LPPL function, and the interpretation of the model parameters are provided in Appendix A.

2.2. Shrinking-Window Estimation Procedure

The FS specification is estimated using a shrinking-window procedure designed to assess the stability of parameter estimates across a sequence of calibration windows. Specifically, the model is fitted over a series of overlapping subsamples of the data, where the endpoint of the time series is kept fixed, and the starting point is progressively shifted forward in increments of 50 observations (Figure 1). This generates a sequence of calibration windows of decreasing length, allowing the model to be evaluated across different temporal segments of the sample.

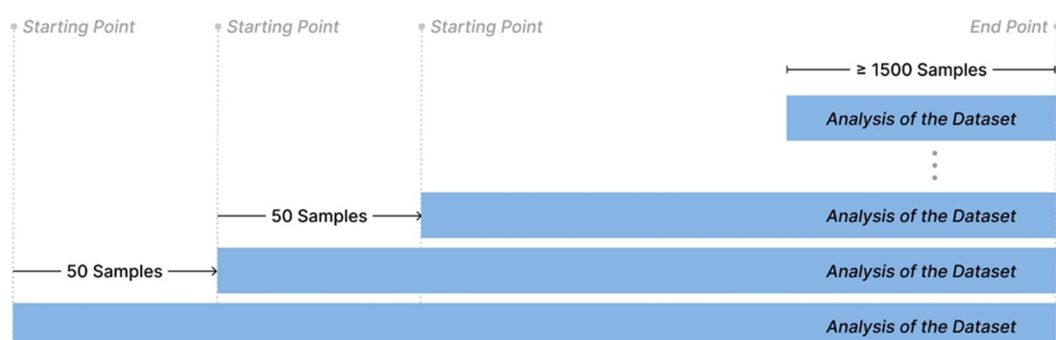


Figure 1. The shrinking-window estimation procedure.

The procedure is repeated until the length of the estimation window remains greater than or equal to 1500 observations. This constraint ensures that all calibrations are performed on sufficiently large samples, thereby maintaining comparability across estimations and avoiding very short-window effects. For each calibration window, the FS model is estimated, and the coefficient of determination R^2 is computed as a descriptive measure of in-sample goodness-of-fit. R^2 is used exclusively for comparative purposes across windows and is not interpreted as evidence of predictive performance or model validity. The primary focus of the analysis is the distribution and temporal stability of the estimated critical times t_c across the sequence of shrinking windows. The convergence properties of these estimates are further examined in the empirical results section.

2.3. Ornstein–Uhlenbeck and Unit Root Tests

Following the volatility-confined LPPL model proposed by Lin et al. (2014), the logarithmic price of a financial asset during a bubble regime can be decomposed into two components: a deterministic LPPL trajectory capturing the endogenous speculative growth, and a stochastic residual component that follows a mean-reverting Ornstein–Uhlenbeck (O–U) process. In this formulation, the O–U dynamics constrain the volatility of the system, preventing unbounded deviations from the LPPL trajectory and thereby ensuring a consistent stochastic framework for explosive yet bounded growth.

Testing whether the residuals of the LPPL model follow an O–U process is equivalent to testing whether they exhibit $AR(1)$ -type mean reversion, where the autoregressive coefficient $|\phi| < 1$ implies stability around a long-term mean. Therefore, the O–U property of the residuals can be empirically verified by applying unit-root tests to the residual series.

In this study, we employ both the Augmented Dickey–Fuller (ADF) and Phillips–Perron (PP) tests to examine the null hypothesis H_0 that the residual series is non-stationary (i.e., has a unit root). Rejection of H_0 indicates that the residuals are stationary and thus consistent with the O–U mean-reversion hypothesis.

2.4. Surrogate Time-Series Approach via Residual Randomization

To further assess the robustness of the model, we employ a surrogate time-series procedure based on the random reshuffling of the residual component. Following the standard surrogate-data framework (Theiler et al. 1992), we generate shuffled-residual surrogate series that preserve the empirical distribution of the noise while eliminating all temporal structure. Specifically, only the residual component is randomly shuffled, while the deterministic FS structure remains unchanged. The surrogate series are therefore constructed by permuting the residuals of the original FS fit and reinjecting them into the deterministic FS trajectory to produce artificial price paths. Re-estimating the FS model on these surrogate series allows us to examine whether the inferred critical times remain stable once autocorrelation and long-range dependence in the residuals are removed. Although detailed diagnostics are not reported here, standard techniques—the detrended fluctuation analysis (DFA) and Hurst’s exponent estimation (Hurst 1951; Kwapień and Drożdż 2012; Costa et al. 2019; Zitis et al. 2024)—indicate that reshuffling effectively removes temporal dependence, yielding surrogate residuals free of autocorrelation and long-range memory. Complete diagnostic results are available in the Supplementary Materials.

2.5. GARCH-Based False Positive Analysis

Following a similar approach proposed by Lin et al. (2014), we perform a false-positive analysis based on a synthetic time series generated from a GARCH process. The objective is to examine whether FS critical windows similar to those identified in the empirical analysis may arise under a standard volatility-clustering mechanism without the presence of endogenous bubble dynamics.

The logarithmic returns are modeled using a $GARCH(1, 1)$ process:

$$\begin{aligned} r_t &= \mu + \varepsilon_t, \\ \varepsilon_t &= \sigma_t z_t, \\ \sigma_t^2 &= \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \end{aligned} \quad (2)$$

where $z_t \sim t_\nu(0, 1)$ denotes a standardized Student-t innovation process. The model is calibrated on the observed DAX returns and subsequently used to generate Monte Carlo synthetic return series. These returns are transformed into synthetic price trajectories by recursively reconstructing the price process from the simulated returns.

The same shrinking-window estimation procedure described in Section 2.2 is then applied to each simulated price series. In addition, the residual stationarity diagnostics described in Section 2.3 are performed in order to ensure methodological consistency between the empirical and simulated analyses. For each simulation, the resulting distribution of estimated critical times t_c defines a critical interval $[t_{c,min}, t_{c,max}]$ with width $W = t_{c,max} - t_{c,min}$.

The width of the empirical FS critical interval is denoted by W_{Real} , while t_E denotes the event date, i.e., the crash of the DAX index following the tariff-announcement shock. A simulated trajectory is classified as a false positive when the event date belongs to the simulated critical interval and the corresponding interval width is not larger than the empirical one, i.e., $t_E \in [t_{c,min}, t_{c,max}] \cap W \leq W_{Real}$. The false-positive probability is estimated as $FPR = P(t_E \in [t_{c,min}, t_{c,max}] \cap W \leq W_{Real})$, and is computed as the proportion of Monte Carlo simulations satisfying the above condition.

3. Data and Results

The DAX is Germany's primary stock market index and one of the most important benchmarks in European financial markets. Comprising 40 of the largest and most liquid German companies listed on the Frankfurt Stock Exchange, the DAX serves as a crucial barometer of the health and performance of Germany's economy, which is Europe's largest and most export-driven economy. Due to Germany's significant economic influence within the Eurozone, fluctuations in the DAX often have a ripple effect across other European markets, impacting investor sentiment and regional financial stability. As a result, the DAX is closely monitored by global investors and policymakers alike, serving as a key indicator of broader economic trends within Europe. For our analysis, we utilized intraday data from the CFD on the DAX index, sampled at 4-h intervals, spanning the period from 31 March 2020 to 24 April 2025. The financial time series were retrieved from Dukascopy Bank, a Swiss Forex Bank and an ECN broker with its headquarters in Geneva. The data are publicly available at <https://www.dukascopy.com/swiss/english/marketwatch/historical/> (last accessed on 7 January 2026).

In our analysis, we applied a shrinking-window estimation procedure (Figure 1) to evaluate the stability and convergence properties of the FS model across different calibration intervals (Jiang et al. 2010; Geraskin and Fantazzini 2013). Specifically, the FS model was estimated over a sequence of windows with varying starting dates and a fixed endpoint, allowing the inferred critical dynamics to be examined across multiple temporal calibrations. This procedure ultimately generated more than 120 model calibrations. From this ensemble, we retained the top 20% of models in terms of R^2 values. Previous studies have shown that shrinking-window approaches may improve the robustness and consistency of LPPL-based inference relative to single-window calibrations, which can produce unstable or highly window-dependent results (Sornette et al. 2013). In this context, the shrinking-window framework allows us to examine whether the estimated critical dynamics exhibit convergence toward a common critical region across multiple calibration intervals. It is

important to highlight that the selection of the endpoint was deliberate rather than arbitrary. We designated 21 February 2025—the last trading day (Friday) before the German federal elections on Sunday, 23 February 2025—as the cutoff for our analysis. This choice was made to ensure that our findings remained untainted by market reactions to the election results, thereby preserving the integrity of the dynamics under investigation.

Following the methodology described above, we obtained 25 fitted FS models. The empirical fits, presented in Figure 2 and Table 1, indicate that the FS specification provides a consistent representation of the observed price dynamics across all calibration intervals. Each curve in Figure 2 represents a fitted FS trajectory obtained from a distinct shrinking-window calibration applied to the observed price series. All fitted models closely replicate the index dynamics, each achieving an R^2 greater than 0.95. The estimated parameter ranges remain relatively stable across windows, with the power-law exponent m varying between 0.56 and 0.90 and the log-frequency parameter ω ranging between 8.33 and 10.76, values that are broadly consistent with those reported in previous LPPL applications (e.g., Filimonov and Sornette 2013; Sornette et al. 2013). The estimated critical times, as obtained from the full set of calibrations, define a critical window between 19 March and 23 April 2025—an interval of approximately 25 trading days—highlighted by the light gray region in Figure 2a,b. Within this window, the U.S. tariff announcement and the subsequent market decline occurred, as indicated by the red dashed line in the same figures. Notably, despite variations in window length and starting dates, the estimated critical times exhibit convergence toward a common temporal region, a feature that has been identified in the literature as a characteristic empirical signature of bubble-like dynamics (Johansen and Sornette 2001). This stability is further supported by the robustness of the results to the model-selection threshold, defined in terms of the ranking based on R^2 , as varying the cutoff from 20% to 15% and 25% yields only minor shifts in the critical window (approximately ± 5 trading days relative to the baseline specification). It is also important to emphasize that all model estimations were truncated on 21 February 2025 (vertical light-blue line in Figure 2), ensuring that the analysis is based exclusively on information available prior to the onset of the identified critical window.

Table 1. Presents the time interval analyzed by each model, the resulting parameter estimates, and the corresponding R^2 value.

Model	Starting Point	End Point	A	B	C1	C2	ω	m	t_c	R^2
1	07-Mar-22	21-Feb-25	10.08	−0.0050	−0.0002	−0.0006	10.77	0.56	23-Apr-25	0.95
2	30-Mar-22	21-Feb-25	10.08	−0.0049	−0.0004	−0.0005	10.73	0.56	22-Apr-25	0.95
3	11-Apr-22	21-Feb-25	10.08	−0.0048	−0.0006	−0.0001	10.64	0.57	22-Apr-25	0.96
4	22-Apr-22	21-Feb-25	10.08	−0.0046	−0.0004	0.0003	10.54	0.57	21-Apr-25	0.96
5	05-May-22	21-Feb-25	10.08	−0.0045	−0.0001	0.0005	10.45	0.57	17-Apr-25	0.96
6	17-May-22	21-Feb-25	10.07	−0.0043	0.0003	0.0004	10.36	0.58	16-Apr-25	0.96
7	27-May-22	21-Feb-25	10.07	−0.0039	0.0004	−0.0003	10.15	0.59	11-Apr-25	0.96
8	09-Jun-22	21-Feb-25	10.05	−0.0030	−0.0002	0.0003	9.73	0.62	07-Apr-25	0.96
9	21-Jun-22	21-Feb-25	10.04	−0.0024	0.0003	0.0001	9.49	0.64	03-Apr-25	0.96
10	01-Jul-22	21-Feb-25	10.04	−0.0022	0.0002	−0.0001	9.37	0.66	02-Apr-25	0.96
11	14-Jul-22	21-Feb-25	10.03	−0.0021	0.0002	−0.0002	9.36	0.66	02-Apr-25	0.96
12	26-Jul-22	21-Feb-25	10.03	−0.0019	0.0000	−0.0002	9.27	0.67	01-Apr-25	0.96
13	05-Aug-22	21-Feb-25	10.02	−0.0013	−0.0002	0.0000	9.06	0.71	28-Mar-25	0.96
14	18-Aug-22	21-Feb-25	10.00	−0.0007	0.0001	0.0001	8.77	0.79	25-Mar-25	0.96
15	30-Aug-22	21-Feb-25	9.98	−0.0004	0.0001	0.0000	8.62	0.85	24-Mar-25	0.96
16	09-Sep-22	21-Feb-25	9.98	−0.0003	0.0001	0.0000	8.58	0.88	24-Mar-25	0.96
17	22-Sep-22	21-Feb-25	9.97	−0.0003	0.0000	0.0000	8.55	0.90	21-Mar-25	0.96
18	04-Oct-22	21-Feb-25	9.98	−0.0004	0.0001	0.0000	8.58	0.85	21-Mar-25	0.96
19	14-Oct-22	21-Feb-25	10.00	−0.0007	0.0001	0.0000	8.61	0.79	24-Mar-25	0.96
20	27-Oct-22	21-Feb-25	10.01	−0.0011	0.0001	−0.0001	8.59	0.73	24-Mar-25	0.96
21	08-Nov-22	21-Feb-25	10.02	−0.0015	0.0001	−0.0001	8.55	0.70	21-Mar-25	0.96
22	18-Nov-22	21-Feb-25	10.01	−0.0015	0.0001	−0.0001	8.56	0.70	21-Mar-25	0.96

Table 1. Cont.

Model	Starting Point	End Point	A	B	C1	C2	ω	m	t_c	R^2
23	30-Nov-22	21-Feb-25	10.01	−0.0013	0.0002	0.0000	8.62	0.72	24-Mar-25	0.96
24	13-Dec-22	21-Feb-25	10.01	−0.0011	0.0002	0.0000	8.65	0.73	24-Mar-25	0.95
25	05-Jan-23	21-Feb-25	10.02	−0.0018	−0.0002	−0.0002	8.34	0.68	19-Mar-25	0.95

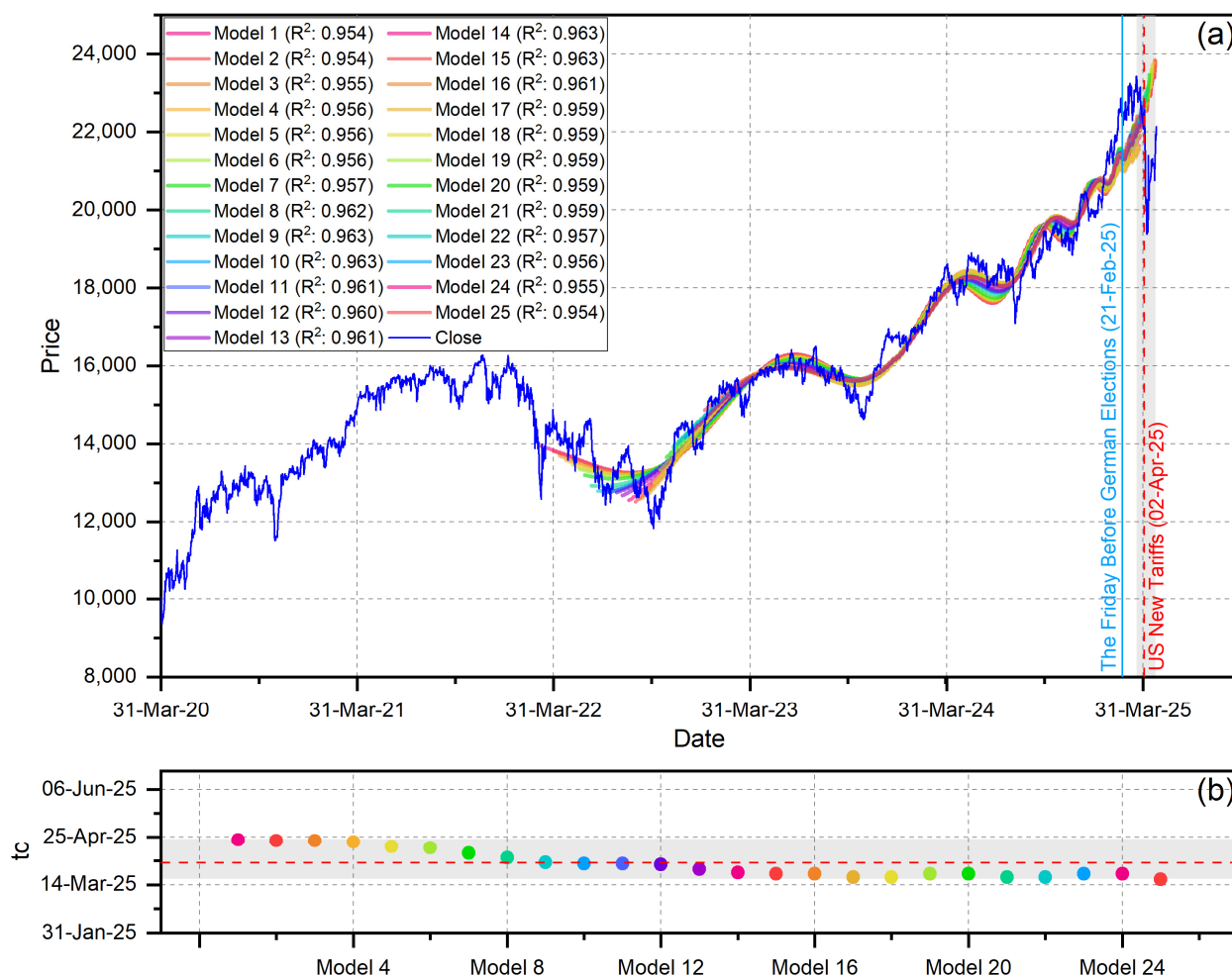


Figure 2. (a) The blue curve shows the 4-h intraday closing prices of the DAX CFD from 31 March 2020 to 24 April 2025. The 25 colored curves represent FS model fits obtained from different shrinking windows. The vertical light-blue line marks the common endpoint of all estimation windows (21 February 2025), while the vertical dashed red line indicates the date of the U.S. tariff announcement. The gray shaded region denotes the critical period estimated by the models (19 March–23 April 2025). (b) The colored points represent the estimated critical times (t_c) for each model. The dashed red line marks the tariff announcement date, and the gray shaded region indicates the estimated critical window.

To evaluate the robustness of the inferred FS dynamics, we did not restrict our evaluation to the confirmation of the parameters (m and ω), the convergence of the model, or the high R^2 values. Instead, we broadened our analysis by incorporating additional statistical tests to further substantiate the validity and robustness of our results. Specifically, we analyzed the residuals within an Ornstein–Uhlenbeck (O–U) framework, as suggested in the LPPL model with bounded volatility (Lin et al. 2014). This approach evaluates whether deviations from the deterministic LPPL trajectory behave as bounded, mean-reverting fluctuations rather than unstructured noise. Unit-root tests, including the Augmented Dickey–Fuller (ADF) and Phillips–Perron (PP) tests, were applied to the

residuals—computed as the difference between the logarithmic observed prices and the corresponding LPPL-fitted logarithmic values. In all cases, both tests rejected the unit-root null hypothesis, which is consistent with residual stationarity (Table 2).

Table 2. Reports $AR(1)$ estimates (ϕ) and unit root test statistics (ADF, PP) for residuals from rolling FS model calibrations. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Model	Starting Point	End Point	$AR(1)$ Coefficient (ϕ)	Standard Error of ϕ	ADF Test Statistic	PP Test Statistic
1	07-Mar-22	21-Feb-25	0.992	0.002	−2.529 **	−14.143 **
2	30-Mar-22	21-Feb-25	0.993	0.002	−3.020 **	−18.007 **
3	11-Apr-22	21-Feb-25	0.992	0.002	−2.862 **	−17.000 **
4	22-Apr-22	21-Feb-25	0.992	0.002	−2.689 **	−17.745 **
5	05-May-22	21-Feb-25	0.992	0.002	−2.730 **	−17.078 **
6	17-May-22	21-Feb-25	0.993	0.002	−2.832 **	−16.541 **
7	27-May-22	21-Feb-25	0.993	0.002	−3.272 **	−17.834 **
8	09-Jun-22	21-Feb-25	0.993	0.002	−2.677 **	−19.686 **
9	21-Jun-22	21-Feb-25	0.991	0.002	−2.578 **	−15.386 **
10	01-Jul-22	21-Feb-25	0.991	0.002	−2.630 **	−14.612 **
11	14-Jul-22	21-Feb-25	0.992	0.002	−2.574 **	−14.177 **
12	26-Jul-22	21-Feb-25	0.992	0.002	−2.478 **	−13.420 **
13	05-Aug-22	21-Feb-25	0.992	0.002	−2.482 **	−13.345 **
14	18-Aug-22	21-Feb-25	0.992	0.002	−2.426 **	−13.113 **
15	30-Aug-22	21-Feb-25	0.990	0.002	−2.539 **	−12.464 **
16	09-Sep-22	21-Feb-25	0.991	0.002	−2.401 **	−11.971 **
17	22-Sep-22	21-Feb-25	0.991	0.002	−2.926 **	−12.409 **
18	04-Oct-22	21-Feb-25	0.991	0.002	−2.831 **	−14.882 **
19	14-Oct-22	21-Feb-25	0.991	0.002	−2.608 **	−15.928 **
20	27-Oct-22	21-Feb-25	0.990	0.002	−2.479 **	−13.257 **
21	08-Nov-22	21-Feb-25	0.990	0.002	−2.341 **	−12.264 **
22	18-Nov-22	21-Feb-25	0.990	0.002	−2.113 **	−10.635 **
23	30-Nov-22	21-Feb-25	0.990	0.002	−2.109 **	−10.444 **
24	13-Dec-22	21-Feb-25	0.990	0.002	−2.077 **	−10.172 **
25	05-Jan-23	21-Feb-25	0.990	0.002	−1.980 **	−9.162 **

Residuals also exhibited strong first-order autocorrelation, with $AR(1)$ coefficients ϕ ranging from 0.990 to 0.993. Although the residual series remain mathematically stationary, since the estimated coefficients are below unity, the dynamics exhibit a high degree of persistence, indicating slow mean reversion and gradual adjustment toward equilibrium. This behavior is consistent with a slowly decaying feedback mechanism around the deterministic LPPL trajectory. To further quantify this persistence, we compute the implied shock half-life, defined as $h = \ln(0.5)/\ln(\phi)$. Given the 4-h sampling frequency, the estimated half-life ranges between 69 and 99 periods, corresponding to approximately 11.5 to 16.5 calendar days. This indicates that deviations from the LPPL trend dissipate gradually but remain transitory, confirming bounded rather than explosive dynamics. Within this framework, the deterministic LPPL component captures the underlying price trajectory, while the residuals represent persistent yet mean-reverting stochastic deviations around this path.

To further assess the robustness of the model—focusing specifically on the stability of the estimated critical times—we employed a surrogate time-series methodology. This approach is based on the hypothesis that, if the FS model adequately captures the dominant deterministic component of price dynamics, stochastic residual fluctuations should not materially alter the estimation of the critical time t_c . Under this premise, variations in the noise structure of the data should lead only to minor deviations in the inferred critical times. Accordingly, we constructed surrogate time series that preserve the deterministic

LPPL backbone while systematically perturbing the stochastic component through the model residuals. This procedure allows us to directly evaluate the sensitivity of the estimated critical times to noise and, hence, to assess their statistical stability. Specifically, we employed the random shuffle technique on the residual time series in order to destroy any existing autocorrelation, long-range dependence, or persistence/antipersistence effects they might exhibit. For each residual series, we repeated this procedure one hundred times, generating 100 randomly shuffled versions. This process was carried out for all residual series derived from the 25 fitted models, each corresponding to the shrinking-window estimation framework. Subsequently, we constructed a surrogate DAX-price time series by reintegrating the randomly shuffled residuals into the corresponding deterministic LPPL trajectories. This allowed us to generate 100 surrogate price series for each of the 25 original estimation windows. Finally, we applied the FS model to each surrogate price series and compared the estimated critical times t_c with the corresponding t_c values obtained from fitting the FS model to the actual DAX prices (Figure 3).

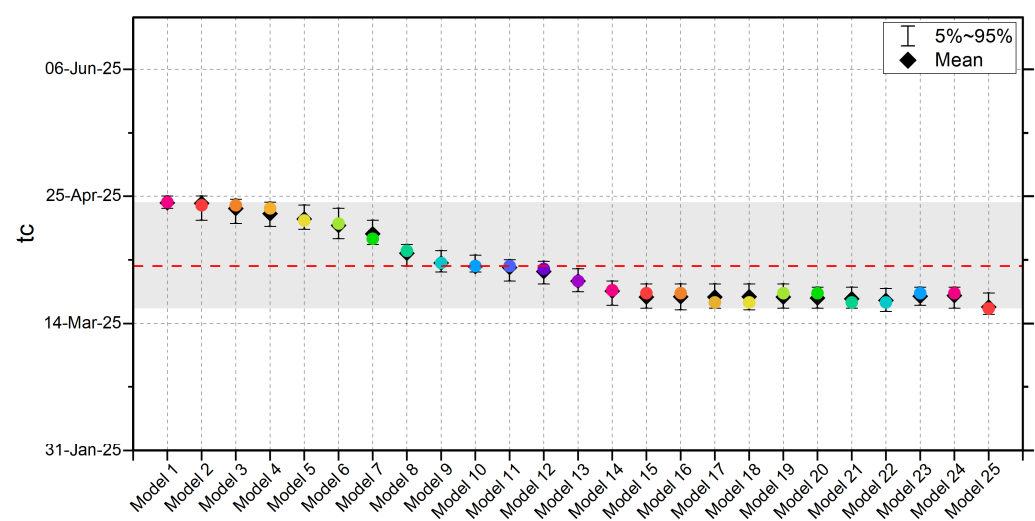


Figure 3. Colored circles indicate t_c estimates across the 25 model specifications. Black diamonds and error bars denote the mean and 5th–95th percentile interval of the surrogate time-series distribution. The gray shaded region denotes the critical period estimated by the models (19 March–23 April 2025).

As shown in the results of Figure 3, the critical points estimated from the models applied to the real data largely coincide with the mean of the corresponding distribution of critical points obtained from the models applied to the artificial time series. Moreover, in all cases, the critical points estimated from the empirical data lie within the 5–95% confidence interval of the distribution derived from the artificial time series. Consequently, even for the results obtained from the models applied to the artificial time series, the critical region remains stable—specifically, from 19 March to 23 April 2025.

These findings indicate that the estimation of t_c is not materially influenced by the stochastic structure of the residuals. In addition, the distribution ranges remain exceptionally narrow, further supporting the view that the residuals do not significantly affect the estimation of the critical point. Overall, the results are broadly consistent with the initial hypothesis and support that the FS models reliably capture the dominant deterministic component of the price dynamics without overfitting short-term stochastic noise.

Subsequently, we evaluated the FPR through a Monte Carlo analysis designed to address the following question: could a standard volatility-clustering process, represented by a $GARCH(1,1)$ model, generate by chance a critical region exhibiting both the temporal alignment and the precision observed in the empirical FS results? More specifically, we examined whether synthetic GARCH-generated price series could produce a critical inter-

val that contains the event date associated with the DAX crash t_E while simultaneously exhibiting a width smaller than or equal to that of the empirical critical region $W \leq W_{Real}$.

To address this question, 1000 Monte Carlo simulations were generated from the estimated $GARCH(1,1)$ model. In order to ensure full consistency with the empirical analysis, exactly the same estimation procedure was applied to both the real and synthetic datasets. Specifically, the FS model was estimated using the shrinking-window framework described in Section 2.2, while only calibrations belonging to the top 20% of the R^2 distribution and satisfying the residual stationarity criteria described in Section 2.3 were retained. The resulting distributions of critical times were then used to construct simulation-specific critical regions and evaluate the false-positive probability.

The analysis yielded a false-positive rate of 5.79%, indicating that only a small fraction of the GARCH-generated trajectories produced critical regions that simultaneously matched the empirical event date and exhibited a comparable degree of temporal precision. This result suggests that the critical region identified in the empirical FS analysis is unlikely to be a spurious consequence of volatility clustering alone and provides additional evidence supporting the statistical significance of the detected critical period.

4. Discussion

In this study, we examined whether the crash of the German DAX index following the U.S. tariff announcement in April 2025 can be interpreted in terms of pre-existing endogenous market dynamics rather than as a purely exogenous shock. The empirical results indicate that the FS model consistently identified a common critical period across multiple shrinking-window calibrations. Notably, the U.S. tariff announcement and the subsequent crash of the index occurred within the estimated critical window, despite the fact that all model estimations had been truncated several weeks earlier.

This naturally raises the question of whether a model based exclusively on endogenous market dynamics can predict an exogenous shock. Our findings suggest a different interpretation. More specifically, the price trajectory appears to have already entered an advanced phase of endogenous bubble formation prior to the announcement, characterized by super-exponential growth accompanied by accelerating log-periodic oscillations and increasing systemic fragility. Within the FS framework, such behavior is generally associated with endogenous market dynamics driven by positive feedback mechanisms, imitation effects, and herding behavior among investors (Johansen et al. 2000; Sornette 2003). These mechanisms may progressively decouple prices from underlying fundamental values while simultaneously increasing the system's sensitivity to external disturbances.

It is noteworthy that this interpretation is also broadly consistent with the prolonged upward trajectory of the DAX during the study period, despite macroeconomic indicators suggesting that the German economy was simultaneously experiencing recessionary pressure and persistent stagnation. Specifically, real GDP growth crawled at just 0.2% in 2025 following a prolonged downturn, while the manufacturing sector faced severe structural challenges, resulting in declining investment and widespread job losses in the sector (European Commission 2025). In such a regime, even relatively limited external disturbances may be sufficient to trigger the crash of an already mature and fragile speculative structure. From this perspective, the tariff announcement can be interpreted not as the sole cause of the collapse, but rather as an external trigger acting upon a system that had already entered a state of elevated endogenous instability.

This interpretation is consistent with the perspective proposed by Gustavsson et al. (2016), who argue that LPPL critical times should be understood as periods of elevated systemic vulnerability rather than precise deterministic crash dates. Similar interpretations have also been reported in previous empirical studies. For example, Song et al. (2022)

identified LPPL signatures in several major global stock indices prior to the COVID-19 market collapse and suggested that pre-existing endogenous speculative dynamics may have amplified the effects of the external shock.

To enhance the robustness of the analysis and address the methodological concerns frequently raised in the LPPL literature (Feigenbaum 2001; Brée and Joseph 2013; Geraskin and Fantazzini 2013), we did not rely solely on goodness-of-fit measures or parameter admissibility conditions. Instead, we implemented a combination of complementary validation procedures. First, we employed the FS reformulation in order to improve estimation stability and reduce sensitivity to local minima within the nonlinear optimization process. Second, we implemented a shrinking-window methodology to assess the stability and convergence of the estimated critical times across multiple calibration intervals, thereby directly addressing concerns related to window sensitivity and parameter instability. Third, we examined the residuals of the FS model within an Ornstein–Uhlenbeck framework. Although the estimated $AR(1)$ coefficients were close to unity, the residual series remained stationary according to the ADF and PP tests, suggesting bounded and slowly mean-reverting dynamics. This slow mean-reverting behavior may reflect heterogeneous adjustment speeds and delayed information diffusion in financial markets, resulting in persistent but bounded deviations around the deterministic LPPL trajectory. Fourth, we employed a surrogate time-series approach based on shuffled residuals to evaluate the robustness of the inferred critical times under perturbations of the stochastic component. The relatively narrow distribution of surrogate critical times indicates that the estimated critical region was not materially driven by short-term stochastic fluctuations. Finally, we performed a GARCH-based Monte Carlo false-positive analysis to assess whether critical regions comparable to the empirical one could arise by chance under a standard volatility-clustering null model. The estimated false-positive rate of 5.79% indicates that such outcomes occur relatively infrequently, implying that the observed alignment between the FS critical region and the subsequent market event is unlikely to be attributable solely to random fluctuations. While this does not constitute proof of predictive power, it provides a statistical benchmark demonstrating that the detected critical window is substantially more informative than would be expected under a conventional GARCH-driven data-generating process. Taken together, these complementary procedures suggest that the estimated LPPL dynamics are not solely an artifact of a specific calibration choice or noise structure but exhibit a degree of statistical robustness across alternative empirical specifications. Nevertheless, these results should be interpreted with appropriate caution, given the complexity of financial market dynamics.

It is also important to note that the LPPL framework is not the only approach available for studying financial instability and market crashes. Alternative methods, including GARCH models, regime-switching frameworks, Hawkes processes, and structural break models, capture important features such as volatility clustering, regime transitions, endogenous event excitation, and abrupt statistical changes. However, the LPPL framework offers a distinct advantage when the primary objective is to investigate speculative bubble dynamics and the emergence of endogenous market fragility prior to major market disruptions. In particular, unlike conventional volatility-based approaches, the LPPL model explicitly attempts to capture the super-exponential growth and accelerating log-periodic structures theoretically associated with positive feedback mechanisms and herding behavior in financial markets. For this reason, the LPPL framework may provide complementary insights into the buildup of systemic instability that are not directly observable through traditional econometric models alone.

From a practical risk-management perspective, the LPPL-based identification of critical periods may be used as a complementary early-warning tool for portfolio monitoring. In

particular, the estimated critical window can be interpreted as a regime of elevated downside risk, during which traditional risk measures such as Value-at-Risk (VaR) may underestimate tail risk due to increasing market instability and nonlinear dynamics. In this context, LPPL signals may inform dynamic risk adjustments, such as tightening stop-loss levels, reducing leverage exposure, or increasing hedging intensity as the system approaches the inferred critical region. Rather than providing precise trading signals, the model offers a probabilistic indication of heightened vulnerability, which can be integrated with conventional risk models to enhance decision-making under extreme market conditions.

Finally, several limitations should be acknowledged. The LPPL framework, while widely applied in the study of financial bubbles, has been associated in the literature with sensitivity to calibration windows, potential model non-uniqueness, possible overfitting, retrospective interpretability, dependence on market regimes, and limited generalizability across assets and time periods. Although the robustness checks implemented in this study were designed to address some of these concerns, they cannot fully remove the intrinsic uncertainties associated with nonlinear bubble modeling and finite-sample calibration. Accordingly, the results should be interpreted as indicators of a critical market regime rather than as precise forecasts of market crashes. Given the complexity and adaptive nature of financial systems, LPPL-based signals are best understood as probabilistic measures of elevated systemic vulnerability. In practical applications, such signals should therefore be used in conjunction with established econometric and risk management frameworks, rather than as standalone predictive tools, in order to support more robust and informed decision-making.

5. Conclusions

This study investigated whether the 2025 crash of the German DAX index following the U.S. tariff announcement can be understood in terms of pre-existing endogenous market dynamics using the FS formulation of the LPPL model. The analysis focused on identifying whether the market exhibited structural features consistent with the buildup of financial instability and elevated endogenous fragility prior to the event, rather than on predicting specific exogenous events.

The main finding of the study is that the observed market dynamics appear consistent with the presence of an underlying regime of increasing fragility before the tariff announcement, where external shocks may act as triggers rather than fundamental causes of market corrections. In this sense, the results support an interpretation of financial instability as a gradual and endogenous process rather than a purely exogenous phenomenon.

More broadly, the study contributes to the literature on financial criticality by proposing an integrated framework for LPPL-based analysis that emphasizes robustness and stability of inference. The combination of FS specification with complementary validation techniques aims to improve the reliability of bubble detection in empirical applications. From a practical perspective, the identification of stable critical regimes may provide useful complementary information for risk monitoring and stress testing in financial markets. Future research could extend this analysis to other financial markets, alternative temporal resolutions, and comparative modeling approaches such as GARCH, Hawkes, or regime-switching frameworks.

Supplementary Materials: The following supporting information can be downloaded at <https://www.mdpi.com/article/10.3390/risks14070145/s1>. Figure S1: Detrended Fluctuation Analysis (DFA) scaling exponent α for polynomial detrending orders $m = 1, 2, 3$, and 4 for original and surrogate residual series. Figure S2: Hurst exponent H estimation for the original and shuffled/surrogate residual series.

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Appendix A. Relation Between the JLS/LPPL and FS Models

The Johansen–Ledoit–Sornette (JLS) model (Johansen et al. 2000) assumes that the asset price, $p(t)$, follows a jump-diffusion process with a time-varying drift, $\mu(t)$, in the presence of discrete discontinuous jumps:

$$\frac{dp(t)}{p(t)} = \mu(t)dt + \sigma(t)dW - \kappa dj. \quad (A1)$$

In this expression, $\sigma(t)$ denotes the volatility, dW is the infinitesimal increment of a standard Wiener process, and dj represents a discontinuous jump process describing the occurrence of a crash. The parameter κ quantifies the relative amplitude of the crash. The expected value of the jump process is determined by the crash hazard rate $h(t)$, defined as the conditional probability per unit time that a crash occurs, given that it has not yet occurred: $E[dj] = h(t)dt$.

The JLS model assumes that two classes of agents are present in the market: rational investors and noise traders. The latter exhibits herding behavior that can generate positive feedback mechanisms and destabilize asset prices. Following Johansen et al. (2000), the crash hazard rate is modeled as a power-law divergence modulated by log-periodic oscillations:

$$h(t) = \alpha(t_c - t)^{m-1} [1 + \beta \cos(\omega \ln(t_c - t) - \phi')]. \quad (A2)$$

The power-law component, $\alpha(t_c - t)^{m-1}$, captures the increasing instability associated with positive feedback effects during the bubble regime. The log-periodic correction accounts for the hierarchical organization of market participants and the resulting alternation between phases of accelerating and decelerating growth.

Under the no-arbitrage (martingale) condition, the expected price dynamics must compensate investors for the crash risk associated with the hazard rate. Using $E[dj] = h(t)dt$ and $E[dW] = 0$, Johansen et al. (2000) show that the drift of the process is proportional to the crash hazard rate, leading to:

$$\mu(t) = \kappa h(t). \quad (A3)$$

Following Johansen et al. (2000), combining Equations (A2) and (A3) and integrating the resulting expected growth rate over time leads to the Log-Periodic Power Law (LPPL) expression for the expected logarithm of the asset price:

$$E[\ln p(t)] = A + B(t_c - t)^m + C(t_c - t)^m \cos(\omega \ln(t_c - t) - \phi). \quad (A4)$$

The coefficients B and C are given by: $B = -\frac{\kappa\alpha}{m}$, $C = -\frac{\kappa\alpha\beta}{\sqrt{m^2 + \omega^2}}$.

Note that the phase parameter ϕ differs from ϕ' because of the integration of Equation (A2) and is related to it through a constant phase shift. Equation (A4) describes

the expected trajectory of the logarithm of the asset price during the bubble regime and is valid only up to the critical time t_c . The critical time corresponds to the most probable termination of the bubble and the transition toward another market regime, which may involve either a crash or a substantial change in growth dynamics.

The LPPL model (A4) contains three linear parameters (A, B, C) and four nonlinear parameters (m, ω, t_c, ϕ). The exponent, m , plays a central role in determining the nature of the bubble dynamics. The condition $m > 0$ ensures that the cumulative crash probability remains finite and that the expected log-price remains bounded as t approaches t_c . The condition $m < 1$ implies that the crash hazard rate accelerates and diverges as the critical time is approached. Together, these requirements yield the characteristic LPPL condition: $0 < m < 1$. Furthermore, the condition $B < 0$ guarantees a faster-than-exponential growth regime, which is a defining feature of speculative bubbles.

Based on extensive empirical investigations of historical bubbles, Johansen et al. (2000) proposed restricting the calibration to: $0.1 \leq m \leq 0.9, 6 \leq \omega \leq 13, B < 0$.

These constraints correspond to stylized empirical characteristics of bubble regimes. The restriction on ω prevents the log-periodic oscillations from becoming either too rapid, in which case they mainly fit noise, or too slow, in which case they become indistinguishable from the underlying trend (Huang et al. 2000). Following van Bothmer and Meister (2003), the additional condition $|C| < 1$ is often imposed to ensure that the oscillatory component remains bounded and that the implied hazard rate remains positive.

To improve estimation stability and reduce parameter degeneracy, Filimonov and Sornette (2013) proposed a reformulation of the LPPL model. Expanding the cosine term in Equation (A4) gives (Filimonov and Sornette 2013; Filimonov et al. 2017):

$$E[\ln p(t)] = A + B(t_c - t)^m + C(t_c - t)^m \cos(\omega \ln(t_c - t)) \cos \phi + C(t_c - t)^m \sin(\omega \ln(t_c - t)) \sin \phi. \quad (\text{A5})$$

Introducing the new parameters:

$$C_1 = C \cos \phi, C_2 = C \sin \phi, \quad (\text{A6})$$

Equation (A4) can be rewritten as:

$$E[\ln p(t)] = A + B(t_c - t)^m + C_1(t_c - t)^m \cos(\omega \ln(t_c - t)) + C_2(t_c - t)^m \sin(\omega \ln(t_c - t)). \quad (\text{A7})$$

As shown by Filimonov and Sornette (2013), this transformation absorbs the phase parameter into the linear parameters C_1 and C_2 , reducing the number of nonlinear parameters from four (m, ω, t_c, ϕ) to three (m, ω, t_c). The resulting optimization landscape is considerably smoother and contains substantially fewer local minima, leading to more stable and reliable calibrations. In the transformed formulation, the condition $|C| < 1$ becomes $C_1^2 + C_2^2 < 1$.

This reparameterization preserves the mathematical structure of the LPPL model while significantly improving its numerical implementation.

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