Article

Mechanical Design and Analysis of a Novel Three-Legged, Compact, Lightweight, Omnidirectional, Serial–Parallel Robot with Compliant Agile Legs

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Abstract: In this work, the concept and mechanical design of a novel compact, lightweight, omnidirectional three-legged robot, featuring a hybrid serial–parallel topology including leg compliance is proposed. The proposal focusses deeply on the design aspects of the mechanical realisation of the robot based on its 3D-CAD assembly, while also discussing the results of multi-body simulations, exploring the characteristic properties of the mechanical system, regarding the locomotion feasibility of the robot model. Finally, a real-world prototype depicting a single robot leg is presented, which was built by highly leaning into a composite design, combining complex 3D-printed parts with stiff aluminium and polycarbonate parts, allowing for a mechanically dense and slim construction. Eventually, experiments on the prototype leg are demonstrated, showing the mechanical model operating in the real world.

Keywords: legged robot; locomotion; spherical parallel manipulator; multi-body simulation; 3D printing; mechanical design

1. Introduction

Regarding legged locomotion of biological beings, exceptional ease can be observed in the traversal of not only flat, but also highly difficult uneven terrain, which is motivating researchers to study the concept of walking and running extensively. The replication of legged locomotion with walking machines is one central aspect in the field of robotics, which shows a wide variety of legged robots in its comprehensive literature, aiming to move a machine with a varying number of limbs over different types of terrain from one place to another, and sometimes additionally performing several tasks. Many robots share similarities with their biological counterparts, which is usually a preferable starting point, since a biologically based template might be able to benefit in regard to the achieved locomotion capabilities from the highly efficient examples found in nature.

Thus, without completeness, remarkable results have been shown regarding quadrupedal robots such as the Raibert quadruped [1], BigDog [2], Spot (Classic) and SpotMini [3], MIT Cheetah [4], ANYmal [5], HyQ [6], and the two-legged robots Atlas [7], ASIMO [8], ATRIAS [9,10], Cassie [11], and HRP [12]. Moreover, traversal can also be accomplished by a very different mechanical design; thus, examples are the Raibert Hopper [13], GOAT [14], Salto [15], STriDER [16], and the RHex hexapod [17]. In this regard, the robot presented and discussed in the following was not based entirely on biological examples, but shares some specific features that can be observed in some of the aforementioned robots, and thus might be considered important in the recreation of legged locomotion. Those features include series elasticities, compact, light, and optimised mechanical construction using 3D-printed parts, and a very specific mechanical design as a measure to tightly combine multiple mechanical structures, yielding a possibly agile legged robot design.
Hence, the basic template that is the foundation of the robot presented is the spring-loaded inverted pendulum model (SLIP) [18–20], which is studied intensely in the literature and resembles the gait of humans and animals quite closely. The SLIP model features a point mass, connected to a massless leg, comprising a linear spring. In the multi-legged context, the model is often expanded by additional legs connected to a central hip joint at the mass location. Since the robot in this article was targeting to be a close match of this model, a preliminary research [21] was performed by the authors of this work with a conceptual robot model that is recalled and explained briefly in the following, which then also explores the motivation and goal for the present research.

1.1. Preliminary Work and Research Motivation

Figure 1a shows the conceptual three-legged robot model derived in the previous work [21] and the corresponding joint degrees of freedom (DOFs). Since the model on the left side of Figure 1a was only partially realised as a real-world prototype in the previous work as shown in Figure 1b, a more complete and refined 3D-CAD model was developed for this work, also including an actual real-world prototype of one of its legs. Thus, building upon the results discovered in the previous work, the present work examined the refined construction in simulation and the build of real-world leg, whose results are discussed in the later sections of this work.

Figure 1. (a) Preliminary three-legged robot model. **Left:** 3D-model used in the simulation, featuring CAD-based hip mechanisms and simplified leg shapes. **Right:** Joint and body topology of the robot. **(b)** Real-world SPM hip unit actuated with Dynamixel Servos, including 3D-printed parts, stiff aluminium support structures, and the ceramic artificial load-support joint in the SPM centre. Multiple videos of the moving prototype model are included in the Supplementary Material.

As becomes obvious from Figure 1a, the three-legged robot model can be considered to be of the type 3SR, if all feet are in contact with the ground surface. Underline notation refers to actuated joints in contrast to non-actuated, passively connected joints. Thus, the system is statically stable in its default pose, which may result in a very robust system that is dynamically balanced during locomotion, but also features high precision for any future manipulation task that may be exercised in the statically stable tripodal standing pose. In this regard, based on the current topology and the assumed point contact model of the robot feet, the robot becomes underactuated, if less than three legs are in active contact with the ground plane. Regarding only one leg, the leg system is of the type SR. In any case, the leg joint selection results in one redundant DOF, yielding the possibility to freely tilt the leg around the virtual axis between the hip centre and foot point. The additional DOF
may reveal itself useful regarding walking performance or motion capabilities in difficult environments, and was therefore kept in the refined model.

A key aspect of the preliminary model is the construction of the hip joints as spherical parallel manipulator (SPM) units, which are manipulator structures that enable spherical motion through parallel linkages. Fundamentally, the concept—that the preliminary robot design was based on—aims to derive fast and agile legs as a measure to enable potentially highly dynamical locomotion. This originated the motivation to build the robot design around the implementation of an actuation scheme that removes all heavy components from the moving parts, reducing the inherent inertias and masses of the involved parts, yielding a parallel manipulator layout. In general, parallel mechanisms feature high stiffness, precision, and speed due to their relatively light moving parts, because the actuators themselves are in general not part of the articulated mechanism. Thus, the SPM is a mechanism featuring three parallel linkages, comprising a proximal and distal link, connecting the base-platform and tool-platform, which is depicted in Figure 1b. Each rotatory axis intersects in the virtual centre point of the manipulator; hence, the manipulator behaves as a spherical joint, allowing it to orient the tool-platform with respect to the base-platform by controlling the angular position of the proximal links that are connected to the respective actuators.

In this regard, the motivation for the conceptual design focussed on concentrating most of the robot mass in a narrow volume, centred in the robot torso. As a result, legs with low relative mass and inertia should reduce the required actuator torques for fast movements, yielding high agility. The parallel layout furthermore yields three DOFs alone for the hip joint; thus, with the knee joint, each leg has a total for four DOFs, not accounting for the additional passive DOF due to the included spring compliance that was added in the refined robot model of this work. Thus, combining those features in a mechanical model requires the dedicated development of specifically shaped parts, which was possible through using 3D printing and traditional manufacturing in conjunction with each other, which are discussed in the following as the main objective of the current work. Eventually, the resulting mechanical model possibly enables the application of control methods that—due to the specific layout of the robot and the resulting condensed mass distribution—can be based on a simplified or reduced-order dynamical model of the actual system, thus potentially allowing for efficient control methods in the future research and development of the legged robot presented.

1.1.1. The Spherical Parallel Manipulator as the Hip Joint

Regarding the SPM, the manipulator has been studied in many publications, e.g., [22–25], and often serves as fast and precise orientation device. Thus, it has been used as a high-speed-camera-orienting device—the Agile Eye [26]—and recently in the context of surgery applications [27,28]. Therefore, the SPM was used as the manipulator for the robot hip joints in the previous work, as the capability to quickly orient the attached upper leg showed promising results in the analysis examined. Considering the implementation as hip joints in walking robots, only a few other examples were found, as one can observe for the four-legged robot in [29] or the two-legged robot in [30]. In contrast to the present work, none of the cited robots explicitly targeted a compact and lightweight design, as they were also based on more traditional manufacturing with heavy metallic parts. Furthermore, the weight of the robot itself proved itself problematic regarding the stress upon the linkage structure of the SPM in [30], which then was the reason in [30] to include a pin in the SPM centre to constantly apply pressure on the upper leg as a measure to lower the stress on the parallel linkages.

Thus, in the previous work [21] and in contrast to other works applying the SPM in the context of legged locomotion, a more robust mechanical design was used, incorporating a ceramic high-precision low-friction ball–socket joint made for medical purposes [31] that is normally used in the surgery application called arthroplasty. The joint was placed conveniently in the available space in the centre of the SPM and served as a central spherical
support, capturing forces that otherwise would have to be supported by the outer linkage structure alone. Thus, as shown in the previous work, the central spherical support joint realised by the ceramic ball–socket joint captured substantial amounts of external forces induced into the hip joints, with the outer linkages only loaded by the torque transmitted by the actuators, which was a direct consequence of the non-redundant implementation of the passive support joint. In addition, this force decoupling allowed for the construction of a highly compact mechanism, as the required structural strength for the linkages themselves was lowered significantly.

The non-redundancy of the central support joint in the previous work was realised by a 3CCC structure of the SPM linkages. Many SPMs found in the literature feature a 3RRR structure that is actually mechanically overconstrained [27]; thus, it may yield problems in real-world applications. As a solution, a 3RCC design [27,32] yields a non-overconstrained mechanism, which is again overconstrained if combined with the central support joint, as examined in the previous work. The overconstrained nature of the system becomes obvious by realizing that the individual linkages in the 3RRR layout define a virtual spherical centre point all on their own, yielding a redundant definition of the centre point. Thus, in the previous work, the application of the 3CCC structure with the additional central S joint solves this problem and prevents the outer linkage structure from drifting apart, which is possible with the 3CCC structure alone, and can be shown by applying Grünbler’s extended formula ([33], p. 13, and [34], p. 243), which shows six DOFs for the 3CCC and three DOFs for the 3CCC+S configuration. The important result that was also furthermore used in the presented refined robot model is the 3CCC+S non-stressed SPM linkage that is only burdened by the torques to be transmitted. Yet, the actual mechanical model—in contrast to the simulation model with the 3CCC+S layout—still uses the overconstrained 3RRR structure, which is not problematic in this specific robot model as the inherent elasticity of the 3D-printed parts allows for the mitigation of minor manufacturing errors that are revealed in the mechanical assembly of overconstrained systems. One should note here that the specific implementation of the 3CCC+S structure in the simulation model representing the dynamical multi-body system of the robot is required at this point, as the system would otherwise become mathematically indeterminate.

As a final aspect regarding the central support joint, in the preliminary model, the actuators for the knee joints were placed in the upper leg, close to the virtual spherical hip joint, which is one of the key aspects that was improved for the new model in the present work. Thus, for the present model, the design of the former ball–socket load-support joint was redesigned as a measure to further improve the placement of the knee actuators in regard to the overall mass and inertia distribution. Still, this redesign did not alter the nature of the support joint behaving as a virtual spherical joint.

As a consequence, the leg structure—regarding mass and inertia—can be kept small in comparison to the robot torso that houses the actuators for the hip joints, as well as—additionally in the refined model—those for the knee joints. This new actuator location further concentrates the total robot mass in the robot torso, again approximating the SLIP model, which was additionally motivated by the fact that the shape of the SPM units allows for a tight arrangement, where the motors of one unit fit in the space between the motors of the neighbouring unit. This interlocking arrangement—one of the key design aspect that is possible due to the spatial location of the individual actuators—can be observed in the preliminary robot model depicted in Figure 1a and was also used in the current robot model. In this regard, the SPM was considered potentially promising by the authors of this paper as an option to construct a fast and agile robot.

1.1.2. Implementation of Series Elasticities

Since the preliminary model was investigated in regard to the resulting individual joint loads, while performing certain motions, the model did not comprise springs, which are essential in regard to energetically efficient locomotion, as they enable passive dynamics through the storage and exertion of energy, hence lowering the power requirements of the
actuators. Furthermore, compliance implements shock absorption at moments of impacts as the foot collides with the ground during walking or running, preventing the mechanical system and the actuators from taking damage. In general, compliant behaviour can be achieved in a passive or active fashion. The passive method creates compliance through practical elastic parts, e.g., mechanical springs, pneumatics, or hydraulics, yet active compliance can be realised through control methods applied to the actuators, e.g., virtual model control (VMC) [35].

For the current robot model, the implementation of mechanical springs was chosen, yielding a series elasticity within the robot structure, yet there is the possibility to further combine the passive springs with additional software-based elasticity of the actuators themselves in the future. Due to the parallel mechanism structure of the hip joint, a pure series elastic actuation (SEA) [36] was not employed. Directly coupling the spring element after the motor gears was considered by the authors of this paper as potentially compromising the overall structural robot compliance, if inserted before the nonlinear behaviour of the spherical parallel mechanism in the hip joint. In this regard, the selection of the classical compact DC motors with planetary gear heads—as opposed to, e.g., for direct drive motors—may limit the compliant behaviour for the most part to the mechanical springs alone, since the high gear reduction ratio necessary for achieving sufficient torques lowers the capability for back-driveability. Thus, agile and dynamic behaviour in uncertain environments requires a decoupling between the motor and the mechanical structure, as sudden externally introduced torques—due to contact situations generating impacts—may be too large to be coped with by the motors alone, possibly damaging the actuators due to their limited back-driveability. Hence, shock absorbency and energy storage have to be covered by the passive series elasticities, yet this takes away a certain amount of controllability, since the system now will be bound to certain natural frequencies, allowing the system to oscillate, which has to be considered during the motion and control of the legged robot. Interestingly, as shown later in the experiment sections, the design of the robot presented yielded a hybrid compliance behaviour; thus, the robot is theoretically able to perform fast and precise motions in its stiff non-compliant configuration and may also behave compliantly after passing a certain force-related threshold value, depending on the actual spring profiles used.

Regarding the legged robots found in the literature utilizing the SPM, the intentional addition of series elasticities was not present in the robot shown in [30], although the elastic coupling between the motors and the manipulator mechanism was investigated. For the SPM-based robot in [29], yet not further detailed, a series elasticity was included in the lower leg as one can observe in the photographs in the corresponding article.

In regard to the inclusion of leg elasticities, which are essential to resemble the SLIP model, the refined model design includes these by mechanical design changes, adding a further degree of freedom to each robot leg. Therefore, later sections of this article investigate the additional DOF that is realised due to the introduction of leg compliance and also discuss in detail the mechanical design that enables the additional DOF in conjunction with the altered design of the hip SPM and the reworked load-support joint, which previously was realised by the ceramic ball–socket joint.

2. Robot Design Foundations

With the results of the conceptual preliminary robot model in mind, the developed, more-refined robot model is presented in the following sections, while its features and design concepts are discussed in depth, considering the geometrical layout, mechanical realisation, and inclusion of springs to resemble the SLIP model. Based on the robot model presented, multi-body dynamics simulations are performed in the later sections of this study to further show the behaviour of the robot model considering the mechanics that enable the leg spring elasticities. Furthermore, a constructed real-world prototype model is shown, which proves the feasibility regarding the actual mechanical realisation of the robot, in contrast with the purely simulation-focussed analysis part of this paper.
A more focussed discussion of the robot model considering the more fundamental and analytical approach that was involved in the optimisation process, employed for the derivation of an overall suitable geometrical layout that targeted to maximise the workspace and other design criteria, will be part of a further proposal, as this is not the scope of this paper. Hence, this work aimed to express the results of the chosen mechanical design, while the actual derivation of the incorporated physical dimensions—based on the mathematical expression of certain design criteria—is another topic in itself. In this regard, Figure 2 displays a coloured cutaway view of the whole final leg model and its individual components, which is explained in detail in the following sections of this paper. In addition, a short overview of the goals and design concepts for the legged robot is given in Table 1, summarizing the essential specifications explored in the further sections.

![Figure 2. Cutaway view of the leg CAD model highlighting individual assemblies that are treated as rigid bodies, connected by joints. The figure also shows the naming convention for parts that are referred to in this article.](image)

<table>
<thead>
<tr>
<th>Project Goals</th>
<th>Required Design Foundations</th>
<th>Selected Design Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agile, highly dynamic, and compact legged robot.</td>
<td>Approximation of the SLIP model.</td>
<td>The SPM as the hip joint for agile motion with central universal joint as the load-support structure.</td>
</tr>
<tr>
<td>Ability to perform locomotion over flat and irregular terrain.</td>
<td>Dense mass concentration in the torso centre.</td>
<td>Interlocking actuator arrangement for a dense mass concentration and compact robot size, with all 12 actuators placed inside the torso.</td>
</tr>
<tr>
<td>Statically stable pose for additionally performed secondary tasks.</td>
<td>Leg structures with low mass and inertia.</td>
<td>Leg compliance via a 4 bar closed-loop linkage, located in series with the actuators.</td>
</tr>
<tr>
<td>Dynamically balanced, energy-efficient dynamics during locomotion.</td>
<td>Integrated leg compliance for storing and releasing energy.</td>
<td>Three legs as a requirement for statically stable standing.</td>
</tr>
<tr>
<td></td>
<td>Low total mass of the structural mechanism and actuators.</td>
<td>Intricate, complex shapes for densely interconnected structural parts via 3D printing.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Composite material design, light parts, and stiff structure.</td>
</tr>
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### 2.1. Actuator Placement and Selection

A good starting point for fast motions and resembling the SLIP model is the employment of parallel mechanisms for the hip joints, as this may be beneficial for the distribution of the overall robot mass that thereby can be effectively shifted towards the torso. Hence, the mass concentration is located at the robot torso centre and, by this measure, on the outside of the moving mechanism of the manipulator.

Considering the actuators required for the knee joints, the placement of these actuators inside the moving leg structure could still end up with inertias that are inappropriate for
achieving fast motions with the specific actuators selected for the hip joints. As a measure to reduce the total leg inertia, the placement of the knee actuators as depicted in Figure 3a is proposed, which is possible due to the fortunate structure of the parallel hip actuation. Here, the knee actuator was placed as near as possible to the robot centre on the torso side of the hip mechanism, which reduced the resulting inertia of the leg for rotatory motions of the hip joint.

As a consequence of moving the knee actuator from the upper leg—as it was located in the preliminary model—to the inside of the torso, the question considering the realisation of torque transmission from the actuator to the actual knee joint arises. Hence, an articulable axis through the centre of the hip mechanism and a screw-to-slider mechanism inside the upper leg were employed to enable the required torque transmission. As discussed in the later sections, the usage of the transmission axis did not allow for a simple spherical support joint in the centre of the spherical mechanism, as in the preliminary robot model, due to conflicting space requirements.

An important design aspect implemented as seen in Figure 2 was the placement of the knee actuator as part of the proximal Cardan assembly, which is separately depicted in Figure 3b. Consequently, as the hip mechanism performs spherical motion, the knee actuator itself will also partially follow with rotatory motion around axis $c_1$, since the actuator is seated and fixed inside the proximal Cardan part. Hence, torque exerted by the knee actuator does not work against the hip motion. Considering an alternative and naive placement of the knee actuator fixed to the base-platform, no mechanical decoupling to the tool-platform would exist; therefore, torques exerted by the knee actuator would directly and negatively impact the hip orientation. Similarly, if, e.g., an obstacle in the environment blocks the motion of the knee joint, which is required for the extension and retraction of the leg, the torque exerted by the knee actuator has to be captured by the mechanism. Thus, by placing the knee actuator structurally in series with the SPM actuators as depicted in Figure 2 as opposed to a rigid placement directly in the base-platform, the motor torques will be introduced intentionally into the manipulator structure itself without applying any disruptive and interfering torques into the spherical DOF between the tool- and base-platform of the hip joint mechanism.

For the base actuators the Maxon [37], DCX19S-GB-KL-18V motors were chosen with the planetary gear head GPX22-C-150:1, resulting in a maximum continuous power of 16 W, which is sufficient to deliver continuous approximate torques of max. 1.6 Nm at
a suitably low rotation speed of around 50 min\(^{-1}\). For the knee actuators, the Maxon DCX19S-GB-KL-18V motors with the appropriate gear head GPX19-C-35:1 were chosen. This motor gear head combination delivers continuous 0.39 Nm at 180 min\(^{-1}\). Performance data for both motors are valid under ideal thermal conditions and were derived from the data sheet provided by Maxon. The lower gear reduction ratio of the knee actuator compared to the base actuators was selected due to the additionally required lead screw pitch at the translation from rotatory motion into linear motion that is part of the knee actuation mechanism. The required torques and reduction ratios were derived by empirical testing in simulation. The weight of a single motor with gear head is \(m_b = 115\) g for a base actuator and \(m_k = 85\) g for a knee actuator.

2.2. Dense Interlocking Motor Placement

Due to the cylindrical actuators, a denser placement of the hip units is possible in contrast to the preliminary conceptual model. Table 2 shows the Z-X-Z Euler angles of the refined robot model, permitting a close and collision-free assembly. This allows for a small torso centre to hip centre radius of \(r_h = 89\) mm. Figure 4a,b depicts the geometric relationship between the involved reference frames, while Figure 5 shows the dense arrangement of the hip units via the CAD representation.

<table>
<thead>
<tr>
<th>Axis (\alpha)</th>
<th>(h_b^R_{R_1})</th>
<th>(h_b^R_{R_2})</th>
<th>(h_b^R_{R_3})</th>
<th>(h_f^R_{h_b})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Z_0)</td>
<td>112</td>
<td>-128</td>
<td>-8</td>
<td>101.006</td>
</tr>
<tr>
<td>(X_0)</td>
<td>115</td>
<td>115</td>
<td>115</td>
<td>7.479</td>
</tr>
<tr>
<td>(Z_1)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-104.288</td>
</tr>
</tbody>
</table>

Table 2. Z-X-Z Euler angles in degree (°) of the individual rotation matrices \(h_b^R_{R_i}\), expressing the relationship between the hip centre frames (hip base, \(hb\)) and the torso body reference frame (end-effector, \(ee\)) for the refined robot model. The orientation between the base-platform frame (\(hb\)) and the tool-platform frame (hip follower, \(hf\)) is expressed by \(hf^R_{hb}\) and is identical for each leg. Figure 4a,b shows the visual representation of the data.

Figure 4. (a) Hip reference frame, fixed to the base-platform. Red, green, and blue axes represent the \(x_,\ y_,\) and \(z_\) axes of the orthogonal reference system. Bold lines refer to the base-platform (\(hb\)) system and thin lines to the tool-platform (\(hf\)) system. Black lines depict the actuator (base) axes. Other axes are the link (cyan) and tool (magenta) axes. (b) Assembly depiction of the three connected SPM modules with the torso reference frame (\(ee\)) in the centre, also showing the collision-free arrangement of the motor axes.
The configuration of the SPM chosen for the robot followed the optimal design found in [22], which stated angles of $\pi/2$ for the proximal and distal link and $\pi/2$ between neighbouring axes ($u_1, u_2, u_3$) and ($w_1, w_2, w_3$). The naming convention used in this article refers to the axes in alphabetical order, with $u$ (base/actuator), $v$ (link) and $w$ (tool), which is different from most other publications and should be considered to avoid possible confusion. The vectors $u_j$ along the actuator axes are defined in the hip base frames ($hb$) as shown in Table 3, which therefore yields an explicit mathematical description of the spatial arrangement of the actuators in combination with the rotation matrices shown in Table 2.

One noticeable aspect following from the specific arrangement of the SPM units is the resulting angular offset between the tool-platform and base-platform. Thus, in the default standing pose, the reference frames for the base- and tool-platform are not aligned; hence, the SPMs are slightly offset from their ideal isotropic home configuration.

2.3. Motor Encoder Integration

Since it is required to be able to measure the motor axis angular positions, the integration of motor encoders is necessary. Due to space limitations and the need for absolute position information, the usual addition of motor encoders on the rear side of the actuators itself is omitted. Alternatively, a helical bevel gear connection to a neighbouring contactless magnet encoder RMB20 by RLS [38] was realised, incorporating gear $b_{prox}$, directly shaped into the proximal link, and gear $b_{enc}$, as shown in Figure 6a. Both rotatory axes intersect at the SPM centre and are oriented with $\gamma_{gears} = 18^\circ$. A reduction ratio was achieved, increasing the sensory basic absolute resolution of 12 bit (4096 counts per rotation) by the gear teeth ratio $r_{gears}$ of:

$$r_{gears} = \frac{z_{prox}}{z_{enc}} = \frac{26}{20} = 1.3,$$

TABLE 3. Vector components of the actuator axis vectors $u_j$, expressed in the hip base frame ($hb$), identical for each leg. Hat notation refers to vectors of unit-length.

<table>
<thead>
<tr>
<th>Actuator Axis $j$</th>
<th>$\hat{e}_x$</th>
<th>$\hat{e}_y$</th>
<th>$\hat{e}_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{u}_1$</td>
<td>$\sqrt{6}/3$</td>
<td>0</td>
<td>$\sqrt{3}/3$</td>
</tr>
<tr>
<td>$\hat{u}_2$</td>
<td>$-\sqrt{6}/6$</td>
<td>$\sqrt{2}/2$</td>
<td>$\sqrt{3}/3$</td>
</tr>
<tr>
<td>$\hat{u}_3$</td>
<td>$-\sqrt{6}/6$</td>
<td>$-\sqrt{2}/2$</td>
<td>$\sqrt{3}/3$</td>
</tr>
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not considering minor mechanical play between the gears.

![Figure 6](image_url)

**Figure 6.** (a) Cutaway view of the motor to encoder bevel gear construction. The bevel gears comprise helical teeth with a helix angle of 30°. Both gears themselves are cuttings of a spherical shell shape; thus, e.g., the front side of the small bevel gear is spherically shaped; hence, there is no obstruction with the equally spherical shape of the proximal link. (b) Backside of the real-world 3D-printed base-platform showing the encoder integration. The detail picture depicts the inner side, showing a close-up of the motor integration and the encoder bevel gear, printed with a layer resolution of 50 µm on the stereolithography based 3D printer Formlabs Form 2.

In the current real-world prototype, the SSI-protocol was used to gather data from the encoders. Since there are at least 12 angular encoders required without accounting for additional sensors in the future development of the robot, the interface was realised through a software implementation by fast toggling general purpose input–output ports (GPIOs) of an connected microcontroller. This enables the system to read a large number of sensors at the same time, which therefore delivers synchronous measurement data of the actuators and joints in the robot model. One should note that the knee actuator is not attached to an encoder in the current model, which will be realised in the future development of the robot by including a miniature linear encoder directly on the slider mechanism inside the leg tube.

### 2.4. Load-Support Joint Design

Considering the knee actuation concept, the straightforward solution of using a ball–socket connection, which was done for the prototype model, cannot be employed here, since the ball–socket design does not allow for the centre transmission axis to be placed. Regarding the preliminary model, for the centre support joint, an actual ceramic spherical hip joint by Ceramtec [39] was utilised that is usually employed in chirurgical implantation, called arthroplasty. Instead, despite the fact that a ball–socket connection features the advantages of extremely low friction, high accuracy, and the capability of transmitting very high external forces over its large contact surface, a Cardan (universal)-type joint will be established for the load-support joint, ensuring the additional space required for the central transmission axis. In addition, the Cardan joint enables a connection at all times, while the ball–socket joint in its current design is not locked into its socket and may lose the connection, which could of course be prevented by changing the socket form.

Figure 7a shows the mechanical implementation, also taking into account an additional bushing base joint between the Cardan construction and the hip base-platform, enabling three DOFs through the joint axes $c_1, c_2, c_3$, identical to the previous ball–socket solution. The base-platform and proximal Cardan assemblies are joined over axis $c_1$. Axis $c_2$ connects the proximal Cardan and Cardan cross-ring assembly. Axis $c_3$ is a shared axis between the Cardan cross-ring, distal Cardan, and tool-platform assemblies. All axes intersect in the SPM centre. In general, the non-homokinetic behaviour of a single Cardan joint at
increased tilt angles for axes $c_2$ and $c_3$ is objectively a drawback in comparison to the former ball–socket joint. Still, as the Cardan joint itself will not rotate at high speed around axis $c_1$—as this is not a profile expected from a desired walking behaviour—the introduced mechanical inefficiency of the Cardan joint was considered negligible, because instead, it is more likely to encounter minor oscillatory motions on axis $c_1$.

Figure 7. (a) Coloured CAD model of the Cardan section, depicting the joint axes $c_1$–$c_3$. Proximal and distal links of the SPM are not displayed. (b) Real-world model of the section, showing the mechanism in the assembled prototype with its pluggable motor connection shaft, containing 3D-printed and aluminium structural parts, and the hidden plastic bushings used for low-friction joints.

As the total volume necessary in the construction increases in contrast to the simple ball–socket model, which uses a very tiny version of the BIOLOX forte [31] joint of only 28 mm in diameter, the overall hip construction results in a slightly higher total diameter compared to the conceptual model. In addition, since the proximal Cardan part includes the knee motor and is furthermore mounted by a rotatory bushing on the base-platform, the resulting space requirement of the support stand has to be considered for the proximal links of the hip mechanism, in regard to avoiding internal collisions.

2.5. Spherical Part Design

Targeting a lightweight, low-inertia, and mechanically robust general mechanical design, each part of the hip mechanism is shaped as a spherical object, which is essential for achieving a compact and dense design. Therefore, a shell-type construction was employed, while the workspace was kept as big as possible by minimizing internal collisions as each part of the construction rotates as far as mechanically achievable on its own spherical layer. Figure 8a shows the concept.

The zeroth layer—as the most inner part of the spherical mechanism—is used for the transmission axis; the next two Layers 1 and 2 are occupied by the Cardan type support joint. The parts employ arc-shaped contours, which allow for robust parts and high dexterity, effectively reducing internal collisions as much as possible. Layer 3 is used for the tool-platform, followed by the distal links on Layer 4 and the proximal links, which occupy the two Layers 5 and 6. In [26], the proximal link was realised by two separated arcs of $90^\circ$, connected at a right angle to form a resulting $90^\circ$ link, which is necessary for preventing mechanical collisions. Otherwise, the proximal links would collide with their neighbouring links. Figure 9 shows the altered design of the proximal link designed for the proposed robot. Using 3D printing, the shape employed was optimised for collision avoidance and resembles the two-layered concept of [26], but focussed on a compact design. Furthermore,
the link was shaped to fit around the central support stand that occupies the middle of the base-platform; see Figure 8b for the composed arrangement. The last Layer 7 is used for the base-platform.

![Figure 8](image_url)

**Figure 8.** (a) Cutaway view and schematic depiction of the individual spherical Layers 0–7 of the rotating parts. Some parts share the same layer, which causes collisions in certain poses or marks workspace limitations. (b) Collision-free pose of the proximal links inside the base-platform at the rotated tool-platform. Only the base-platform, proximal, and distal links are displayed. Recesses are employed for weight reduction.

To the best of the author’s knowledge, there does not exist a spherical parallel manipulator with truly spherically parts as depicted in the figures. As can be observed for example in Figure 9, the surface of the parts is either vertically or tangential directed towards the centre of the SPM. By applying this design principle to each part of the leg assembly, a dense structural layout can be achieved. Hence, while in motion, each spherical shape fits between the spherical shapes of its previous and next shell, while the sides of the parts can only come in contact with surfaces that are oriented parallel with each other.

![Figure 9](image_url)

**Figure 9.** CAD model of a single proximal link. Each point of the surface is directed either vertical or tangential to the SPM centre, minimizing any possible interactions with other geometries and only allowing for surface tangential internal collisions, maximizing the contact plane.
2.6. Knee Torque Transmission Design

In the following, the transmission shaft concept is shown for an effective realisation of the knee actuation, which also was motivated by bringing the heavy parts of the robot model, namely the actuators, as near as possible to the torso centre. Starting from the knee actuator, recalling Figure 2, the proximal and the distal transmission axes are connected by an articulable coupling at the spherical centre of the hip mechanism. From there on, a lead screw transforms the rotatory motion into linear motion for actuating the knee joint by a slider mechanism; see Figure 10. Interestingly, as shown in the later real-world experiment, the motion of the leg does not end with the internal surface collision between the slider and lower leg assemblies, as depicted in the figure. Instead, further moving the slider results in the starting angular motion of the upper leg with respect to the tool-platform at the spring slot joint around axis $c_3$.

![Figure 10. Retracted (top) and expanded (bottom) pose. Base actuators are locked; therefore, only the knee actuator DOF is shown. The retracted pose is limited by a mechanical stop of the slider, and the expanded pose stops at the collision of the lower leg and slider assembly.](image)

Employing this construction over a linear actuator directly placed inside the upper leg reduces the inertia and may actually be preferred due to the possible self-locking behaviour of the lead screw at very low thread pitches, but also yields a more complicated mechanical construction. Eventually, a lead screw with a light plastic nut by IGUS [40] was employed here for a compact design, which shows very little friction; thus, it is easily possible to turn the screw by externally altering the position of the slider without experiencing noticeable resistance. The multi-helical lead screw comprises a thread pitch of 10 mm, which thus is equal to the linear motion of the slider for one full turn of the lead screw.

2.7. Homokinetic Joint Coupling

Since the knee actuator is located inside the robot torso, a transmission of the actuator torque to the actual knee joint is required, which implies that the transmission axis employed for this purpose has to cope with the varying angle between the base- and tool-platform. As a basic requirement for the design of the joint structure inside the transmission axis, a maximum tilt angle of 50° was considered, which is a value limited by the mechanical restrictions of the CAD model. Therefore, different available joint types were evaluated (see Figure 11a), intending to solve the arising problem of constructing a suitable transmission joint without becoming stuck or breaking under high tilt angles of the hip joint.

In summary, the mechanical properties of interest were high torque capability, low friction, suitable tilt angles without suffering from noticeably low efficiency at high angular deflections, and linearity between the input and output velocity independent of the tilt angle. Linear torque transmission is considered important regarding a reliable system response independent of the current hip orientation that is assumed in the controller side,
which otherwise might lead to possible vibrations induced in the mechanical system. In this regard, universal (Cardan) joints, either in single or double layout, were not suitable due to low efficiency, possible nonlinearity in certain conditions, and the very restricted spatial space inside the SPM.

Hence, since there was no suitable off-the-shelf solution at this miniature-scale available, a combination of two Rzeppa-type joints was selected, with both joints fixed in place to the upper leg side, respectively the base-platform side. Between those joints, a coupling as depicted in Figure 11b was constructed, offering linear play and therefore identical tilt angles of the sub-joints at varying hip joint angles. Thus, the resulting system offers relatively high-torque transmission capabilities, compact dimensions, low friction, even at higher tilt angles, and a homokinetic input–output relationship.

![Figure 11](a) Size and type comparison of available exemplary miniature couplings: Huco double universal joint, made of acetal/brass © [41], Huco single universal joint ©, and TVR Ball-X CV homokinetic Rzeppa-type joint © [42]. (b) The 3D-printed hexagonal middle transmission axis for normal operation © and the hollow version © for the combination with a heavy-duty metallic transmission shaft.

Figure 11a shows a cutaway view of the transmission joint from the CAD model and the geometric relationship. By observation, the cosine rule can be applied and yields the distance $c$ between the centre points of the Rzeppa joints,

$$c(\alpha) = \sqrt{2f^2(1 - \cos(\pi - \alpha))}.$$  

For the default non-tilted configuration, the centre distance equals $c(0) = 2f$, which gives the angular-dependent relative axial difference of the centre points,

$$d(\alpha) = c(\alpha) - 2f.$$  

Since the middle part of the connector rod has a higher diameter than the side parts, acting as a mechanical stop, the total required displacement of the rod, as depicted in Figure 12b, is distributed on both Rzeppa joints. This results in a very minor axial motion per side of the rod and allows for an overall deep fit of the rod ends inside the Rzeppa inner connector part. Furthermore, since axial displacement is required for the connector rod, a 3D-printed low-friction and self-lubricating plastic part was used here, employing the IGUS iglidur I3-material [40], which is specifically purposed for frictional contact situations, though long-term stress tests regarding possible wearing were not carried out on the current robot prototype. Yet, the surrounding universal-type joint construction is not negatively impacted by the inner transmission axis design, as the complete structure inside the SPM shows very little friction, which is promising regarding the possible agility of the leg itself. Still, further improvements may increase the individual joint ranges by reducing material at certain locations that currently determine the maximum workspace of the manipulator.
Figure 12. (a) Cutaway view and geometric dependencies of the double Rzeppa constant velocity (CV) transmission joint. Both CV joints are connected with R joints to the upper leg, respectively the proximal Cardan part, while the plastic centre part connects both CV joints with P joints. (b) Required relative axial displacement \( d(\alpha) \) of the Rzeppa centre points in dependence of the SPM manipulator tilt angle \( \alpha \) for the model-specific value \( f = 8.317 \) mm.

2.8. Introduction of Elasticities

Since compliant leg behaviour is necessary to approximate the SLIP model, enabling energy storage and shock absorbency, elasticities were introduced into the robot model by adding springs. Importantly, the relation between the angular position of the three base actuator axes per SPM at each hip joint and the resulting orientation of the tool-platform is nonlinear. Thus, directly inserting linear rotatory springs between the actuator axes and proximal links is difficult and may deviate too far from the intended behaviour of the SLIP model, as shown in Figure 13. Specifically, the torque applied externally into the actuator axes is dependent on the current orientation of the tool-platform with respect to the base-platform; thus, the resulting leg spring stiffness of the overall SPM would yield a configuration-dependent profile in this case. Hence, an alternative design is required, which aims to include the spring mechanism in series not with each individual actuator axis, but in series with the full parallel mechanism. In this regard, the next section first considers a theoretically possible spring implementation and its drawbacks, leading to the final design that was actually implemented in the robot model.

Figure 13. Targeted approximation of the SLIP model. The three-legged robot model (grey lines) should match as close as possible the depicted equivalent three-legged SLIP model (black lines) as shown above with spring stiffness \( k \) and main body mass \( m \).
2.8.1. Possible Three-DOF Spring Layout

In a first attempt and in view of the conceptual robot model from the previous section, the tool-platform was split into two parts, the inner and outer ring, and those parts were connected by pre-stressed contracting linear springs. A prototype was built and is depicted in Figure 14a. The inner ring is connected via a prismatic joint to the upper leg, decoupling the inner ring from vertical forces. By inserting the springs into the tool-platform, the springs are always rotated with the mechanism, overcoming the nonlinear kinematics of the SPM mechanics and acting between the tool-platform and upper leg. Therefore, the spring behaviour is independent of the actual orientation of the SPM mechanism and acts just relative to the spherical orientation difference of the upper leg and tool-platform. Figure 14b depicts the relative motion between both parts.

![Prototype of the divided tool-platform](image)

**Figure 14.** (a) Prototype of the divided tool-platform for full rotatory motions of the inner ring with respect to the outer ring, as long as the mechanical construction allows for no collisions between bodies or hindering the free contractions and elongations of the springs. (b) Planar depiction of the motion capability. Thus, the effect of rotation axes parallel with the paper plane through the spherical centre of the SPM is omitted.

Since the inner and outer rings move spherically around the SPM centre, a minor non-linearity between spring elongation and the orientation angle of the inner ring will appear. It was assumed that the spring connector joints at the inner and outer ring themselves are able to cope with the orientation deviation between both ring parts without collisions due to the space constraints, which also explains the usage of simple rubber bands in the depicted prototype.

Considering the proposed spring layout of Figure 14a with regard to the targeted SLIP model of Figure 13, some caveats are present. Regarding the SLIP model, the linear spring DOF should be employed between the foot and the hip, respectively torso, position. However, as can be observed in Figure 13, the actual distance between the leg tip and hip mainly depends on the knee joint angular position, while the hip orientation is only involved to a minor degree. In addition, due to the spherical coupling between the inner and outer ring, unwanted lateral displacement or twisting of the leg can occur, without any additional constraints employed. It remains to be seen in future investigations if the proposed spring layout of Figure 14a is actually beneficial for the robot locomotion, as the minor impact of the springs in regard to the hip–foot distance and the additional introduced leg deflection are regarded as major drawbacks. In this sense, the spring implementation was altered in the current robot model, as discussed in the next section. In general, the three-DOF spring design might be considered beneficial in other robot applications and is only possible due to the existence of the central spherical support joint in the first place. Thus, the design was included in this work as a noteworthy addition, but dropped for the further development.
2.8.2. Final Spring One-DOF Implementation

Regarding the limitations of the three-DOF hip spring implementation of Figure 14a, the design was changed, constraining the spring to only one DOF in the final robot model. Furthermore, a closed-loop structure over the leg mechanism was used, leading to a constraint kinematical relationship. The structure was realised in the actual design by the implementation of a shared revolute axis $c_3$ between the tool-platform, the distal Cardan part, and the Cardan cross-ring; see Figure 7a. Hence, the closed loop was composed of the tool-platform, the proximal Cardan part, the lower leg part, and the support axis; see Figure 15a. All axes $c_3$–$c_6$ of the closed-loop structure are in a parallel arrangement with each other, unrelated to the SPM orientation. By sharing one common revolute axis $c_3$, both the spring DOF and the distal part of the spherical DOF of the hip mechanism can be achieved without interference. Therefore, any motion, respectively change, in the orientation of the hip mechanism does not alter the distance between the SPM centre and the foot position, effectively decoupling spring-related leg compression from hip motion.

Figure 15. (a) Kinematical relationship of the closed-loop structure for the implementation of the elastic leg behaviour in planar representation. All revolute axes $c_3$–$c_6$ are parallel. Cyan marks the virtual polygon between joints. The red line shows the Dynema cord, connected at the yellow spots to the assemblies. (b) The distal Cardan part and tool-platform are able to independently rotate around the common axis $c_3$, and the motion is only limited by the mechanical limits of the slot inside the tool-platform. Without external forces, a Dynema cord presses the distal Cardan part against the lower side of the slot surface, which fixes the undisturbed pose of the leg mechanism.

Furthermore, the spring implementation does not mitigate the load-support joint functionality. Leg forces are still exerted in the Cardan load-support joint, while the tool-platform is able to transmit the torque for the spherical SPM motion over the shared revolute axis into the distal Cardan part. The relative rotatory motion between the distal Cardan part and the tool-platform is restricted by the spring forces. In the uncompressed state, the distal Cardan part is pressed against the slot surface of the tool-platform, constraining any loose motion of the spring mechanism; see Figure 15b. As external forces on the foot increase, the distal Cardan part rotates over the shared axis $c_3$ inside the tool-platform, until the other hard stop is reached.

Since the space inside the SPM is limited, a linear spring is included inside the polycarbonate leg tube and connected over low-friction pulley bars with a high-strength Dynema cord, as can be observed in Figures 2 and 15a. The cord connects the tool-platform on one spring side, while the other side is fixed to the distal Cardan part. In the default non-deflected pose, the spring is in a pre-stressed configuration.

Figure 16 shows the final leg mechanism reacting to external forces, exerted on the foot tip. The figure shows the extreme states of the leg structure and the lower and upper bound of the tool-platform spring slot depicted in Figure 15b, representing the minimal and maximal spring elongation at a typical pose of the leg manipulator. The resulting
virtual spring between the hip SPM centre and foot tip position is denoted by $k$, but does not inevitably obey a linear force-to-compression relationship.

Figure 16. Exemplary displacement of the foot tip position due to spring elongation. The angular positions of base and knee actuators are not changed between the minimal and maximal spring deflection poses; therefore, the pose differences are caused only by the spring DOF. The foot position is kept close to the virtual connection line between the uncompressed state and SPM centre in conjunction with the target SLIP model. The resulting virtual spring with stiffness $k$ is shown in green.

2.9. Support Axis Integration

The spring implementation relies on a closed-loop structure, involving the tool-platform and the support axis; refer to Figure 15a. Since the rotatory axis $c_4$ between those two parts has to be integrated, as well as regarding the space required for the moving SPM linkage structure, the asymmetrical layout of the SPM linkage is exploited here. Therefore, the joint placement was selected in view of minimizing possible collisions with the SPM linkage parts. In normal operation, the joint occupies the space that is not crossed by the linkage structure in general; see Figure 17a. Still, collisions may occur in certain configurations, which suggests that future investigations may consider the exact workspace resulting from this specific design. Figure 17b depicts the real-world model. The support axis was made of hardened steel to prevent possible bending, as the axis will be a highly stressed part, and was inserted tilted inside 3D-printed connectors due to the space requirements of the asymmetrical layout of the manipulator. Each rotatory axis of the closed loop was realised by a hidden plain bushing inserted into the 3D-printed parts, yielding low-friction motion, as depicted in the figures.

Figure 17. (a) Spatial position of the support axis joint for different extreme SPM orientations. Left: Sliding under the prox. link at upwards SPM rotation. Right: Using the space between distal links at SPM downwards rotation. (b) Real-world prototype.
2.10. Upper Leg Design

Figure 18 depicts the complete structure of the upper leg. For stability reasons, a rigid cylindrical polycarbonate tube was employed as the structural part for the upper leg. The outer diameter was only 32 mm; thus, the spring and screw–slider mechanism were integrated in a very constraint space of 27 mm in diameter, requiring a functionally nested design. As depicted in the figure, multiple cylindrical 3D-printed parts were inserted into the tube, while thin steel bars on the top side were used to align the parts and to fix them at certain distances from each other. The slider moves in two miniature linear recirculating ball bushings per bar that were press fit into the printed parts. The low friction lead screw aligns with the symmetry axis of the tube. Since much of the tight space is occupied by the slider mechanism, the linear tension spring was placed between the slider bars, with the Dynema cord close to the polycarbonate surface area. Multiple screw holes were prepared to additionally fix the parts to keep them from falling apart, yet surprisingly, the press-fit alignment bars in conjunction with the passive spring forces were sufficient to pull both ends of the upper leg model together on their own, not requiring any additional screws or bolts from the outside.

![Figure 18. Upper leg mechanism. Bars and bushings were press fit; thus, no additional gluing or screws were required. Due to the integrated design without any possibility to reach the inner parts inside the tube, the assembly of the leg requires multiple steps, including several partial back and forth pushes with partially disassembling and reassembling of the components, till all parts are fit together.](image)

2.11. Singularities and Workspace Limitations

A very important aspect of any robot mechanism design is the consideration of possible singularities in the robot workspace. Though no mathematical analysis based on the robot Jacobian matrix is portrayed in this work, some details are visually observable.

Thus, regarding the SPM used for the hip joint, the occurrence of singularities is strictly possible at certain orientations, yet they are not near the intended workspace required for moving the leg in regard to locomotion. Furthermore, the workspace of the SPM is mainly limited by the internal Cardan (universal) joint, which limits the possible space of configurations, reducing the possibility to enter singularity poses.

Regarding the leg structure, Figures 15a and 18 depict the general layout. In essence, the knee motion is realised in the leg plane by a closed-loop triangle shape with axes $c_4, c_5, c_6$ marking the corner points and with the length of one side ($c_4$ to $c_6$) being altered via the slider mechanism. Importantly, by moving the slider, the shape of the triangle changes, which drives the manipulator towards singularities near its fully retracted and fully expanded pose. With the addition of the spring DOF, this triangle actually expands into a quadrilateral with the axes $c_3, c_4, c_5, c_6$. Due to the mechanical limitations of the mechanism—despite possibly coming close to such a pose in certain configurations—the manipulator is physically unable to enter a state of singularity, which would require the points of $c_4, c_5, c_6$ to be located on a straight line inside the leg plane. In such a case, e.g., if
the leg is theoretically extended beyond its mechanical joint ranges, this would result in the knee joint being able to perform infinitesimal rotatory motion even if the slider mechanism is locked. Hence, the knee joint linkage would lose its ability to resist externally applied torques, which might not be problematic while standing, but can be problematic while performing motions that are more dynamic. In general, the acceptable closeness to points of singularity may be evaluated depending on the actual motion and force requirements of the robot under certain conditions.

2.12. Materials

The real model was built mainly of 3D-printed parts, utilizing the Formlabs Form 2 stereolithography 3D printer [43]. For the model, Formlabs White Resin V4 was used, despite the material not being suitable for functional models, but considering the requirement for high-dimensional accuracy and precise fits, the resin was selected. In future iterations, more suitable materials and printing technology will be evaluated. Aluminium was used for connecting parts that had to withstand high stress over small surfaces or volumes. Furthermore, plastic was used for all other basic parts, e.g., rods, tubes, screws, nuts, bushings, etc. Steel was only used for very highly stressed parts, which applies for the linear slider bars, the support axis, the CV joints, and the actuators themselves. Table 4 shows the weights of the individual assemblies, which were based on the CAD data.

The actuator-to-total-robot-mass ratio \( r_{mass} \) reads:

\[
 r_{mass} = \frac{m_{actuators}}{m_{robot}} = \frac{3(3m_t + m_k)}{m_{robot}} = 0.388, \tag{4}
\]

which demonstrates a lightweight structural construction, as it only takes up around 61.2% of the total robot mass. Thus, the remaining relative amount of 38.8% is occupied by the mass of the actuators. Since it was the goal of the design to achieve low leg masses and inertias, while shifting the accumulated centre of mass (COM) into the torso as much as possible, this distribution is also depicted in Table 4 by the masses \( m_{torso} \) and \( m_{leg} \) and the according percentages of 66.71% for the accumulated amount of torso mass relative to the total robot mass and only 11.10% per leg, which shows the successful distribution of the total robot mass \( m_{robot} \). As one might note, the discrete grouping into torso and leg mass is obviously subjective to a certain degree, since there are multiple bodies that are incorporated in the creation of the virtual hip joint, separating torso and leg. Hence, the mass grouping is calculated by the following equations, which are sorted into leg and torso groups, depending on the amount of transmitted motion,

\[
m_{torso} = m_{ct} + 3(m_{bp} + m_{pc} + m_{pta} + m_{cc} + 3m_{pl} + m_{cor} + 0.5m_{mta}), \tag{5}
\]

\[
m_{leg} = m_{tp} + m_{dc} + m_{dta} + m_{dta} + m_{dl} + m_{sa} + 3m_{dl} + m_{cor} + 0.5m_{mta}. \tag{6}
\]

One result that is the direct consequence of the design is the position of the actual COM. Thus, for the default standing pose of the robot, as shown in the later figures, the height of the COM was \( h_{com} = 0.371 \) m. In comparison, the geometrical torso centre \( h_t \)—which is the centre point in the plane spanned by the centre points of the three SPMs—in the same pose was \( h_t = 0.380 \) m, which shows a very close distance between the total mass location and the geometric torso centre point. Importantly, the COM location lies within the torso and above the leg structures, which resembles the SLIP model. Noticeably, all information was gathered from the CAD model, which then was used to derive the masses, inertias, and COM locations for each rigid body assembly, required for the later simulations. Since a complete model of the robot does not exist in the current state of the research project, the approximation through this modelling approach was necessary. Yet, a comparison with the build prototype leg and the CAD data was performed. Therefore, the practical
measurement at the laboratory scale was $m_{\text{leg,real}}^* = 1156.6 \pm 0.5 \text{ g}$, which corresponds to the CAD-related equation:

$$m_{\text{leg}}^* = m_{\text{leg}} + \frac{m_{\text{torso}} - m_{\text{ct}}}{3} + m_{\text{ct}} = 1138.469 \text{ g}. \quad (7)$$

Hence, the error between the CAD approximation and real-world model was $\Delta m_{\text{leg}}^* = |m_{\text{leg,real}}^* - m_{\text{leg}}^*| \approx 18.131 \pm 0.5 \text{ g}$, which was considered small enough to render the COM calculation and the following simulations valid.

**Table 4.** Physical properties and mass distribution of the robot model with $N$ representing the number of repeated parts. The data were derived from the CAD model. Entries are ordered by function.

<table>
<thead>
<tr>
<th>Assembly</th>
<th>Variable</th>
<th>$N$</th>
<th>Mass $m$ (g)</th>
<th>$\frac{m}{m_{\text{robot}}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Torso) centre tray</td>
<td>$m_{\text{ct}}$</td>
<td>1</td>
<td>47.993</td>
<td>1.45</td>
</tr>
<tr>
<td>(SPM) base-platform</td>
<td>$m_{\text{bp}}$</td>
<td>3</td>
<td>478.649</td>
<td>43.26</td>
</tr>
<tr>
<td>(SPM) tool-platform</td>
<td>$m_{\text{tp}}$</td>
<td>3</td>
<td>56.411</td>
<td>5.10</td>
</tr>
<tr>
<td>(SPM) proximal link</td>
<td>$m_{\text{pl}}$</td>
<td>9</td>
<td>25.606</td>
<td>6.94</td>
</tr>
<tr>
<td>(SPM) distal link</td>
<td>$m_{\text{dl}}$</td>
<td>9</td>
<td>12.417</td>
<td>3.37</td>
</tr>
<tr>
<td>(Support) prox. Cardan part</td>
<td>$m_{\text{pc}}$</td>
<td>3</td>
<td>121.475</td>
<td>10.98</td>
</tr>
<tr>
<td>(Support) Cardan cross-ring</td>
<td>$m_{\text{cc}}$</td>
<td>3</td>
<td>20.551</td>
<td>1.86</td>
</tr>
<tr>
<td>(Support) distal Cardan part</td>
<td>$m_{\text{dc}}$</td>
<td>3</td>
<td>117.734</td>
<td>10.64</td>
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<tr>
<td>(CV) prox. transmission axis</td>
<td>$m_{\text{pl}}$</td>
<td>3</td>
<td>20.168</td>
<td>1.82</td>
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<tr>
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<td>3.92</td>
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<tr>
<td>(CV) dist. transmission axis</td>
<td>$m_{\text{pta}}$</td>
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<td>4.000</td>
<td>0.72</td>
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<tr>
<td>(CV) inner CV ring ball cage</td>
<td>$m_{\text{cvr}}$</td>
<td>6</td>
<td>36.432</td>
<td>3.29</td>
</tr>
<tr>
<td>(Leg) slider structure</td>
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<tr>
<td>(Leg) support axis</td>
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<td>47.363</td>
<td>4.28</td>
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<td>Torso assembly</td>
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</tr>
<tr>
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<td>368.315</td>
<td>11.10</td>
</tr>
<tr>
<td>Build prototype (CAD)</td>
<td>$m_{\text{leg}}^*$</td>
<td>1</td>
<td>1138.469</td>
<td>-</td>
</tr>
<tr>
<td>Build prototype (real)</td>
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<td>(measured) 1156.6 ± 0.5</td>
<td>-</td>
</tr>
<tr>
<td>Robot assembly</td>
<td>$m_{\text{robot}}$</td>
<td>1</td>
<td>3319.421</td>
<td>100.00</td>
</tr>
</tbody>
</table>

2.13. Topological, CAD, and Real Models

The topological model of the robot joint and body assembly is depicted in Figure 19. The multi-body representation of Figure 19 resembles the mechanical implementation of the robot, although in the numerical simulation, additional DOFs were allowed for some joints in the closed-loop structure responsible for the spring DOF. This was performed without altering the dexterity or behaviour of the mechanism and was required to prevent redundancy problems, avoiding an indeterminate description of the model.

Since the axis construction between the proximal and distal transmission axis contains two constant velocity (CV) joints, by connecting the inner CV rings that are joined prismatically to the middle transmission axis, no difference regarding the torque or velocity will occur between the input and output side of the system. Due to this property and the small mass and inertia impact of the involved components, the actual connection was not modelled physically as a measure to reduce the numerical expense of the simulation. Instead, the torque was directly injected into the distal transmission axis. Thus, currently, the influence of the configuration regarding the universal support joint was not considered. Hence, in a static pose, there is no drawback for this assumption considering the input and output velocity between the knee actuator and leg slider mechanism axis as identical. Yet, this does not hold true in the case that the velocity of the axes $c_1$, $c_2$ and $c_3$ is not zero; thus, the non-homogeneous input–output velocity of the centre joint has to be compensated by
the velocity or angular pose of the knee actuator axis. This property will become relevant in the context of implementing an actual control law.

![Diagram](image)

**Figure 19.** Multi-body system of the robot model with the non-overconstrained SPM. Red joints are actuated. The blue joint depicts the passively applied torque due to the integrated mechanical spring. Dotted areas show components that were not modelled in the simulated system. Axes $c_1 - c_6$ are displayed at the respective joints.

The final model of the CAD assembly and the build prototype of one leg according to the topological model of Figure 19 are shown in Figure 20a,b. The geometrical properties of the robot model are displayed in Table 5.

**Table 5.** Geometrical properties of the robot model and default values. Variables are depicted in the CAD model in Figure 20a.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hip radius</td>
<td>$r_h$</td>
<td>0.089</td>
<td>m</td>
</tr>
<tr>
<td>Foot radius</td>
<td>$r_f$</td>
<td>0.170</td>
<td>m</td>
</tr>
<tr>
<td>Torso height</td>
<td>$h_t$</td>
<td>0.380</td>
<td>m</td>
</tr>
<tr>
<td>Upper leg length</td>
<td>$l_{ul}$</td>
<td>0.260</td>
<td>m</td>
</tr>
<tr>
<td>Lower leg length</td>
<td>$l_{ll}$</td>
<td>0.254</td>
<td>m</td>
</tr>
<tr>
<td>Robot COM height</td>
<td>$h_{com}$</td>
<td>0.371</td>
<td>m</td>
</tr>
</tbody>
</table>
3. Experiments in Simulation

In this section, experiments with the robot model are performed in simulation, exploring certain characteristics of the robot design. The results discovered are shown and discussed in regard to the intended design goal of the robot model. Simulations were computed using MATLAB/Simulink by implementing a dynamic model in a rigid multi-body simulation. The mechanical system containing rigid bodies connected by joints was modelled using the MathWorks SimMechanics library. Additional features such as joint friction, joint limits, and environmental collisions were implemented manually and were further detailed in the previously published work [21], hence only discussed briefly here.

In general, the decision to model the system in Simulink was made in regard to extending the systemic representation to multiple domains in future works. Thus, including specific behaviours for friction, contacts, joints, actuators, and sensors was considered more appropriate—by the authors of this work—to apply in the Simulink environment than in other alternatively available, but dedicated robot simulation software.

3.1. Rigid Body Contact, Joint Range, and Friction Modelling

Regarding environmental contacts, these were modelled as a measure to represent the feet to floor collisions. The contact model includes a surface normal force $f_n$, implemented as the basic Kelvin–Voigt model, and a tangential force $f_t$. The tangential force alternates between two modes, one for a static contact and the other one for sliding. If the static friction force passes a certain threshold—related to the current applied normal force—the virtual contact breaks and goes into sliding mode. The concept of modelling the tangential contact force as a virtual and breakable spring–damper system was also implemented similarly in [44]. Hence, within the surface local reference frame, the equation:
\[ f_n = \begin{cases} \max(-k_c p_n - d_c \dot{p}_n, 0) & \text{if } p_n < 0 \\ 0 & \text{otherwise} \end{cases} \quad (8) \]

implements the normal force with the surface normal penetration depth component \( p_n \). The parameters \( k_c \) and \( d_c \) correspond to the contact stiffness and damping values, respectively. For the local tangential vector, the spring and damper parts are evaluated,

\[ f_{tk} = -k_c (p_t - c_t) \quad \text{and} \quad f_{td} = -d_c \dot{p}_t \quad (9) \]

and combined, yet depending on the static or dynamic friction expressed by parameter \( s_\mu \),

\[ s_\mu = \begin{cases} 0 & \text{if } |f_{tk} + f_{td}| > f_{t,max} \\ 1 & \text{otherwise} \end{cases} \quad (10) \]

Here, \( c_t \) corresponds to the tangential impact location that was initially detected at the moment that holds the condition \( p_n < 0 \). By defining the intermediate force \( f_t^* = s_\mu f_{tk} + f_{td} \), the tangential friction force vector follows with:

\[ f_t = \min(|f_t^*|, f_{t,max}) \cdot \frac{f_t^*}{|f_t^*|}. \quad (11) \]

The limiting force \( f_{t,max} \) depends on the current friction mode, thus:

\[ f_{t,max} = f_n \cdot \begin{cases} \mu_s & \text{if } |\dot{p}_t| < \varepsilon \\ \mu_d & \text{otherwise} \end{cases}. \quad (12) \]

Since the contact model is discontinuous, it does not account for a transition region, whether being utilised for the normal or for the tangential component, which is sometimes used as a measure to numerically stabilise the behaviour of the simulation due to the discrete course of the reaction forces at the moment of impact or the switch between static and dynamic friction. Thus, a very small simulation step size \( \Delta t \) for the model dynamics was selected to account for the sudden change of contact forces, yielding an adequately precise simulation. Since the model delivered quite stable results without any perceivable jitter in the contact points, it also worked well with larger simulation step sizes, which is possibly required in simulations that have to run in real-time. Still, very large step times in the region of 10 ms tended to show instabilities, requiring a more sophisticated implementation of the contact model in the future research. In comparison to the less-complex multi-body representation of the preliminary model, the simulation of the refined model was computationally more demanding for each step; thus, real-time execution will require more robust alternatives to the current contact expressions, possibly allowing for the selection of larger step sizes.

Joint limits were implemented similarly to the plane–point contacts of the feet; thus, by passing a certain angular threshold, the internal joint contact model acts and restrains joints to certain ranges. In general, the specific setup of the experiments yielded sudden peaks of relatively high joint torques that were exerted in the model in certain instances, as depicted in the experiments that are discussed later in this article. Thus, as a measure of stabilizing the simulation numerically, a linear transition between the active and inactive state of the limit behaviour was applied, as this addition showed noticeably more stable and robust results than with a discontinuous implementation. In this regard, with the joint stiffness \( k_r \) and the joint damping \( d_r \), the limit torque was constructed by a spring–damper system with the relative angular velocity \( \dot{\phi} \) and the angular position error \( \Delta \phi \), which corresponds to the angular penetration of the invalid joint range. The range controller was evaluated for the upper and lower limit of the joint; thus, the joint moves freely between the cases:
\[ \tau_r = \begin{cases} r_t(\Delta \phi_U) \cdot \max(k_r \cdot \Delta \phi_U, 0) - d_r \cdot \phi_U & \text{if } \Delta \phi_U > 0 \text{ (upper limit)} \\ r_t(\Delta \phi_L) \cdot \min(k_r \cdot \Delta \phi_L, 0) - d_r \cdot \phi_L & \text{if } \Delta \phi_L < 0 \text{ (lower limit)} \end{cases} \]

which requires the deviations \( \Delta \phi_U = \phi_U - \phi \) and \( \Delta \phi_L = \phi_L - \phi \), with the limits \((\phi_U, \phi_L)\), and the actual angular position \( \phi \). The scaling parameter \( r_t \) for the transition region is calculated by:

\[ r_t(\Delta \phi) = \frac{\min(|\Delta \phi|, r_{t,\text{lim}})}{r_{t,\text{lim}}} \]

with the transition range \( r_{t,\text{lim}} \). The implementation is equally valid for forces at linear joint DOFs.

Friction inside the joints was modelled additionally as an important property of the system as a measure to approximate a realistic behaviour in certain experiments. By default, the SimMechanics Toolbox does not account for energy loss due to friction or other reasons; thus, any constrained and passively moving system will not stop oscillating. Therefore, friction was included to allow the robot model to come to rest after some time in the experiments that are depicted in the following sections. In this sense, with the static and dynamic coefficients \( \mu_f \) and \( d_f \), friction inside the joint axes is expressed by:

\[ \tau_f = \begin{cases} -\text{sgn}(\omega)(\mu_f \cdot f_{R,\perp} + |\omega| \cdot d_f) & \text{if } |\omega| > 0 \\ 0 & \text{otherwise} \end{cases} \]

As one might note, the employed friction implementation depends on the calculated joint reaction forces \( f_{R,\perp} \), orthogonal to the motion axis, rotating at angular velocity \( \omega \). A very small value was selected for parameter \( \mu_f \) in order to increase friction at very high joint loads, yet not being intrusive in the equation, possibly yielding motion by itself. Thus, the explicit description of the robot dynamics is fundamentally required, since an overconstrained system results in indeterminate joint reaction forces, yielding incorrect friction torques. Future work will require a more sophisticated friction model based on experimentation with the real-world model as a measure to better approximate the actual behaviour.

### 3.2. Setup and Spring Behaviour

As a basis for the following experiments, the mechanical design and the corresponding mathematical relationship, covering the spring slot joint motion and the internal spring behaviour, has to be discussed in more detail. Thus, Figure 21 shows the mechanical layout of the spring implementation.

![Figure 21](image-url)
The variable $\varphi_{spr}$ depicts the angle between the symmetrical centre lines between the tool-platform and upper leg. In the case $\varphi_{spr} = 0$, the upper leg is oriented along the vertical axis of the manipulator tool-platform. The relative rotatory motion between the parts is only limited by the mechanical collision occurring at the lower and upper sides of the spring slot, with the specific values $\varphi_{spr,min} = -8^\circ$ and $\varphi_{spr,max} = +8^\circ$; thus, the upper leg is able to freely rotate in a $16^\circ$ range. By observation of the geometry, the distance $d_{spr}$ between the pulley points guiding the tensioned Dynema cord is always equal to the elongation of the spring $\Delta x_{spr}$ and marks the base side of an isosceles triangle with the side legs $r$. By the rule of cosines, it follows the internal spring elongation $\Delta x_{spr}$ as a function of radius $r$ and angle $\gamma$,

$$\Delta x_{spr} = \sqrt{2r^2(1 - \cos \gamma)},$$

with $r = 0.035$ m being a constant of the static part geometry, resembling the placement of the pulleys, and $\gamma$ being the angle between $\varphi_{spr,min} = -8^\circ$ and the current $\varphi_{spr}$. The contraction force that is exerted by the spring onto the Dynema cord pulls the lower spring slot side of the tool-platform against the upper leg, respectively the distal Cardan part. Both parts rotate around the joint axis $c_3$; thus, the relative motion between the tool-platform and upper leg can be expressed by a virtual torque $\tau$ that is applied on axis $c_3$, which was actually performed in simulation as a measure to implement the spring in the multi-body topology. Hence, by observing Figure 21, the torque $\tau$ is related by:

$$\tau = f_{\perp}r \quad \text{with} \quad f_{\perp} = f \cos \alpha.$$

From geometric considerations, it can be derived that it holds that:

$$\frac{\pi}{2} = \alpha + \beta \quad \text{and} \quad \pi = \gamma + 2\beta,$$

with $\beta$ being the angle between the base and legs of the isosceles triangle. After rearranging, it holds that:

$$\alpha = \frac{\gamma}{2} \quad \text{with} \quad \gamma = \varphi_{spr,min} - \varphi_{spr};$$

thus, it follows with the constant spring stiffness $k_{spr}$,

$$\tau = k_{spr}\left(x_0 + \sqrt{2r^2(1 - \cos \gamma)}\right)r \cos \frac{\gamma}{2},$$

which yields the virtual torque $\tau$ as a function of the current pose-dependent spring slot distance angle $\gamma$ and the selected initial spring elongation $x_0$.

### 3.3. Drop Experiment Simulation

Figure 22 shows snapshots of a free-fall experiment performed in simulation, which are more detailed in Table 6. This experiment explored the difference concerning the behaviour of the robot for a configuration with compliant springs in contrast to a model with extremely stiff springs, basically rendering the spring slot joint around axis $c_3$ immovable. In both scenarios, the robot was dropped from a height of $h_{A,t0} = 68$ cm and $h_{B,t0} = 48$ cm, resulting in an initial distance of $d_{A,t0} = 30$ cm and $d_{B,t0} = 10$ cm between the foot tips and floor. Letters A and B refer to different initial heights, from which the robot was dropped; hence, in total, four experiments were carried out. It was intentional by the specific setup of A and B that in Case A, the spring slot DOF reached its limit, resulting in an internal mechanical collision, while in B, the leg spring was able to catch the fall without internal collision.
Figure 22. Drop test simulation states A–E of the compliant robot configuration in chronological order captured from the simulation framework and motion vector of the torso, depicting Setup A.

Table 6. Characteristic drop test states of Figure 22.

<table>
<thead>
<tr>
<th>#</th>
<th>State Description</th>
<th>( h_t ) (m)</th>
<th>Time ( t ) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Release, free-fall</td>
<td>0.680</td>
<td>0.000</td>
</tr>
<tr>
<td>B</td>
<td>Ground contact</td>
<td>0.376</td>
<td>0.252</td>
</tr>
<tr>
<td>C</td>
<td>Max. spring deflection</td>
<td>0.275</td>
<td>0.300</td>
</tr>
<tr>
<td>D</td>
<td>Spring contraction</td>
<td>0.381</td>
<td>0.399</td>
</tr>
<tr>
<td>E</td>
<td>Apex after lift-off</td>
<td>0.492</td>
<td>0.549</td>
</tr>
</tbody>
</table>

For the setup, the compliant spring configuration included a height strength spring with stiffness \( k_{spr} = 9 \, \text{N/mm} \) and an static elongation of \( x_0 = 20 \, \text{mm} \) in the robot non-disturbed pose, which was also the resting pose after the robot stopped oscillating a few seconds after the drop event. The spring parameters were selected to be able to carry the robot mass, resulting in a minimal spring deflection with \( \phi_{spr} = -8^\circ \) with a constant spring force of \( f_{spr} = 180 \, \text{N} \), which can be observed in the static standing, respectively resting, pose of the robot, as shown, e.g., in Figure 20a.

On the contrary, the stiff configuration employed a fixed connection between the tool-platform and distal Cardan part in the simulation; therefore, the spring DOF of the leg mechanism was blocked, resembling the virtual inclusion of an extremely stiff spring. The only compliance of the remaining system stemmed from the joint limit and collision implementations, which featured state-dependent and breakable spring damper systems.

Since controller influence should be avoided in the test scenario as only the passive dynamics of the springs were of interest for this setup, all actuator axes were locked for this purpose; therefore, the reaction torques \( \tau_R \) resembled the motor torques necessary to keep the joints’ axes in a fixed angular position. Table 7 displays the general simulation settings and the parameters used for the simulation, including the parameters used for the Kelvin–Voigt contact model regarding the surface normal reaction force calculation, the friction vales for surface tangential forces, the spring–damper values for the internal mechanical joint limits, and the joint friction calculation.

Figure 23a shows the simulated torso height \( h_t \), the spring slot angle \( \phi_{spr} \in [-8^\circ, +8^\circ] \), the internal spring reaction force \( f_{spr} \), and the absolute ground reaction forces \( |f_{grf}| \). Figure 23b shows the joint actuator torques \( \tau_{R,k} \) for the knee actuator and \( \tau_{R,j} \) for the base actuators with \( j = 1, 2, 3 \). The specific location of the actuator axes inside the robot topology is depicted in Figure 19. The connection between index \( j \) and the actual spatial location of the actuator inside the base-platform is shown in Figure 3a and also in Figure 4b. For readability and due to the symmetrical layout of the robot, only data from one leg is displayed. Both figures only refer to Experiment A. Slow-motion videos of the stiff and compliant experiment for Setup A are included in the Supplementary Materials. Figure 24 shows key aspects of Setup B over time. The complete diagrams for Setups A and B are accessible at [45] and via the Supplementary Materials.
Table 7. Characteristic modelling parameters, used for the drop test simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floor contact stiffness</td>
<td>( k_c )</td>
<td>20,000</td>
<td>N/m</td>
</tr>
<tr>
<td>Floor contact damping</td>
<td>( d_c )</td>
<td>200</td>
<td>N s/m</td>
</tr>
<tr>
<td>Contact static friction</td>
<td>( \mu_s )</td>
<td>0.8</td>
<td>–</td>
</tr>
<tr>
<td>Contact dynamic friction</td>
<td>( \mu_d )</td>
<td>0.7</td>
<td>–</td>
</tr>
<tr>
<td>Simulation step time</td>
<td>( \Delta t )</td>
<td>( 2.5 \times 10^{-5} )</td>
<td>s</td>
</tr>
<tr>
<td>Simulink solver</td>
<td>–</td>
<td>ode2 (Heun)</td>
<td>–</td>
</tr>
<tr>
<td>Joint limit stiffness</td>
<td>( k_r )</td>
<td>12,000</td>
<td>Nm/rad</td>
</tr>
<tr>
<td>Joint limit damping</td>
<td>( d_r )</td>
<td>30</td>
<td>Nm s/rad</td>
</tr>
<tr>
<td>Joint friction coefficient</td>
<td>( \mu_f )</td>
<td>0.01</td>
<td>–</td>
</tr>
<tr>
<td>Joint friction damping</td>
<td>( d_f )</td>
<td>( 10^{-4} )</td>
<td>Nm s/rad</td>
</tr>
<tr>
<td>Torso height</td>
<td>( h_{A}^0 )</td>
<td>0.68</td>
<td>m</td>
</tr>
<tr>
<td>Torso height</td>
<td>( h_{B}^0 )</td>
<td>0.48</td>
<td>m</td>
</tr>
<tr>
<td>Fall distance</td>
<td>( d_{A}^0 )</td>
<td>0.30</td>
<td>m</td>
</tr>
<tr>
<td>Fall distance</td>
<td>( d_{B}^0 )</td>
<td>0.10</td>
<td>m</td>
</tr>
<tr>
<td>Spring stiffness</td>
<td>( k_{spr} )</td>
<td>9</td>
<td>N/m</td>
</tr>
<tr>
<td>Initial spring deflection</td>
<td>( x_0 )</td>
<td>0.020</td>
<td>N/mm</td>
</tr>
</tbody>
</table>

**Figure 23.** (a) Free-fall simulation profiles of Setup A of the robot model for both stiff and compliant configurations. The red line shows the time of the internal collision. The grey background visualises states of no contact between the robot feet and the ground. (b) Reaction torques of the fixed actuator axis of one leg, captured from the drop simulation of Setup A for both stiff and compliant configurations. Internal collisions occur at the red line.
Drop Experiment Results

Since the robot was dropped from an unnatural height in Setup A, the included spring cannot fully reduce the collision impact; therefore, at event $e_0$ (red line), the Cardan part hits the upper spring slot surface at $\varphi_{spr} = +8^\circ$, which results in a high ground reaction force. The impact can be observed at $t \approx 0.3$ s in all diagrams, including the motor torques. In normal operation, this should be avoided, which also implies the future addition of damping elements to prevent similar situations. In Setup B, which is discussed later, the spring slot angle tops at $\varphi_{spr} = +3.5^\circ$, before recovering its initial position.

It should be pointed out here that the reaction torques of the actuators recorded in the diagrams in general exceed the continuous torque limits of the actuators employed for the actual robot. The drop test therefore serves as the depiction of the leg elasticity behaviour, since the actual real-world robot was only designed to perform legged locomotion with the aid of compliant leg structures, which does not include drops of certain heights.

Regarding the robot model in its resting position after settling down from the drop experiment, considering all velocities to be zero, the actual actuator torques are substantially smaller than for the drop test and lie within the available actuator power range. Table 8 lists these values for the static robot pose. As one can observe, it holds that $|\tau_{b1}| \approx 2|\tau_{b2}|$ and $|\tau_{b1}| \approx 2|\tau_{b3}|$ in the static robot pose, which suggests that the current pose or hip orientation is not optimal in regard to an even load distribution. Thus, as a subject of future investigations, the robot pose in regard to an optimal torque distribution on the actuators will be considered, also accounting for the possible exploitation of the leg redundancy.

Table 8. Actuator torques necessary to keep the robot in a static standing pose and resulting joint loads, not accounting for the possible real-world stick–slip effects of the robot joints.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\tau_c$ (mNm)</th>
<th>$\tau_{b1}$ (mNm)</th>
<th>$\tau_{b2}$ (mNm)</th>
<th>$\tau_{b3}$ (mNm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>151</td>
<td>425</td>
<td>-205</td>
<td>-210</td>
</tr>
</tbody>
</table>

As one can observe in Figures 23b and 24, it becomes obvious that the compliant configuration reduces the collision impact in direct comparison to the stiff configuration, which contrarily comes to a resting position sooner due to the lack of major oscillatory components. For the analysis, the actual reduction of collision-induced forces and torques...
can be obtained by inspecting only the first ground collision of the falling robot, since later impacts do not start from the same systemic state. Furthermore, internal collisions of the leg itself with the upper and lower spring slot surface produce sudden torque loads that are applied on the actuators.

Thus, in the following, torques $\tau_{\text{peak,compl}}$ and $\tau_{\text{peak,stiff}}$ of those events—producing high loads exerted on the actuator axes in the simulation—are compared for both stiff and compliant configurations. Peak torques are abbreviated as $\tau_c$ and $\tau_s$. For reference, Table 9 shows the notation used for different events. Figure 25 visually depicts peak torque events of both configurations (stiff and compliant) and both experiment setups (A and B). As for the evaluation, the following Equation (21) is used, computed at the highest peaks at the moment in time of certain events,

$$\kappa(\tau_c, \tau_s) = 1 - \frac{\tau_{\text{peak,compl}}}{\tau_{\text{peak,stiff}}},$$

which thus expresses the reduction of the load torques. The final comparison is then presented in Figure 26, with an alternative numerical depiction of all data points in the Supplementary Materials.

Table 9. Notation for different cases, respectively events, of the drop test experiment.

<table>
<thead>
<tr>
<th>Torque Description</th>
<th>Torque</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compliant robot configuration</td>
<td>$\tau_c$</td>
</tr>
<tr>
<td>Stiff robot configuration</td>
<td>$\tau_s$</td>
</tr>
<tr>
<td>Initial foot–ground contact</td>
<td>$\tau_{IL}$</td>
</tr>
<tr>
<td>Upper spring slot collision ($\varphi_{spr} = +8^\circ$)</td>
<td>$\tau_{UL}$</td>
</tr>
<tr>
<td>Lower spring slot collision ($\varphi_{spr} = -8^\circ$)</td>
<td>$\tau_{UL}$</td>
</tr>
</tbody>
</table>

Figure 25. Impact torques for both Experiment Setups A and B exerted in the four actuator axes due to different events, e.g., ground impacts or internal spring slot collisions. The last row depicts the points in time of the individual events, which thus can be picked up individually in Figures 23b and 24.

Regarding Setup A, one can observe two impacts on the robot system within a short time interval, namely the initial ground impact at $t = 0.252s$, yielding the reaction torque peaks $\tau_{UL}^A$ and $\tau_{UL}^I$, and a second peak at $t = 0.297s$, which results from the subsequent internal spring slot collision, which applies only to the compliant configuration, yielding the peak torque $\tau_{UL}^C$. Thus, Figure 26 shows the single peak $\tau_{UL}^I$ from the stiff configuration.
compared against both subsequent peaks $\tau_{A}^cI$ and $\tau_{A}^cU$ of the compliant configuration. As one can observe, the first impact shows a substantial reduction of torques, exerted into the joints, while the subsequent impact actually shows a negative reduction for the base joints. Thus, this yields the conclusion that the subsequent impact of the internal spring slot collision exerts a higher load onto the joints than the initial ground impact in the stiff configuration. The general conclusion from this result is that internal collisions of this kind should be avoid, which is either possible by preventing the robot from experiencing situations with undesired falling distances, by changing the properties of the internal spring, or by altering the actual effect of the mechanical collisions via damping elements.

Figure 26. Impact reduction ratio between the stiff and compliant configuration for each actuator. Higher percentages correspond to a better impact reduction.

One should note here that the torques obtained by the simulation highly depend on the stiffness and damping parameters of the internal spring slot joint limit, which represents the collision. Thus, the results of the second impact may change with different model parameters. For this simulation, the parameters were selected to only allow for a minor penetration of the geometric shapes due to the stiff joint settings, which was additionally accomplished by a small transition region of $r_{lim}=0.3^\circ$. Furthermore, the simulation was a simplification, as being a rigid multi-body simulation, not accounting for any elasticity inside the parts itself. In this regard, the results are an indicator for potentially critical joint loads in similar impact conditions, demanding for suitable damping near the joint limits in the real robot model as a measure to prevent the system from taking damage. In addition, the spring forces possibly apply a very high stress onto certain points in the 3D-printed parts, which should be taken with caution.

In comparison, Setup B shows a different outcome. Due to the lower initial height of the drop in this setup, only one impact occurs at the ground collision in both the stiff and compliant configuration at $t=0.147$ s. In this setup, the reduction of impact forces shows a substantial improvement over the impact situation without springs with around 75% for the base joints and 95% for the knee joint. The only caveats of the spring setup can be observed at the collision of the spring slot joint in recovering its undisturbed pose at $t=0.351$ s with $q_{spr}=-8^\circ$, yielding an internal collision that induces sudden torques $\tau_{B}^cL$ in the joints, which is immediately followed by a short airtime of the robot model. The comparison of the torques $\tau_{B}^cL$ with the torques $\tau_{B}^sI$, as depicted in Figure 26, shows that the subsequent internal collision in the compliant configuration—in recovering the undisturbed resting pose of the spring slot joint—injects torques into the base joints that are similar to the ones from the ground impact observed in the purely stiff configuration. As a consequence, again, the handling of internal collisions due to the current implementation of the spring mechanism may demand additional constructive measures to increase the damping near the joint limits.

Importantly and as shown by Figure 26, different events that produce peak torques were compared against the peak torques $\tau_{A}^cI$ and $\tau_{A}^cU$. The diagram thus expresses the amount of load reduction, which is important regarding the possible damaging of the
actuators or the case that in certain situations, the torque capabilities of the actuators may be exceeded, yielding possibly the loss of control of the robot system. However, as observable in Figure 26, the impact reduction regarding the base actuators in general is promising regarding the first ground contact, yet subsequent internal collisions may be problematic. Thus, this may allow for improvements, possibly allowing a single impact situation to be broken down and distributed on subsequent impacts, lowering the load introduced into the mechanical structure and the actuators. On the other hand, the knee actuator in general experiences the biggest reduction of peak torques in both Setups A and B, which shows that the compliance-based shock absorbance is fundamentally of high significance for the robot model, yet requires intricate adjustments to yield a suitable impact reduction in different situations.

3.4. Measurement of the Virtual SLIP Spring Stiffness

For the measurement of the virtual spring stiffness $k_v$, depicted in Figure 27b, the simulation setup displayed in Figure 27a was employed. As it turned out by inspection of the measurement data from the drop test performed earlier, the previous setup is not suitable for this kind of calculation, since minor stick–slip effects of the foot to ground contacts introduce jitter into the reaction force profiles; hence, a new test setup was created.

![Diagram](image)

**Figure 27.** (a) Setup for the virtual spring stiffness measurement. Green shows the undisturbed pose, red the compressed pose after exerting force $f_t$. In Detail A, the constraint foot displacement is shown. (b) Geometric representation of the virtual spring between undisturbed and maximal compressed robot pose.

In the simulation setup, a force $f_t$ is exerted in the vertical direction onto the robot torso, pushing the robot against the floor. Surface tangential ground reaction forces are lowered in this setup by setting $\mu_s$ to zero, disabling static friction, and setting $\mu_d = 0.005$, enabling a low-velocity-dependent dynamic friction force. Hence, the influence of ground friction forces that will occur due to the horizontal motion of the foot tips over the leg compression sequence is kept small. The presence of $\mu_d \neq 0$ is actually important in the simulation setup as a damping component, mitigating potential oscillations in the otherwise frictionless rigid body simulation model. A minor foot displacement $\Delta r_f$ will occur due to the one-DOF constraint of the spring mechanism and can be observed in Detail A in Figure 27a. To avoid any disturbances due to the dynamics of the system, the increase of $f_t$ was performed slowly; thus, the model reached the mechanical spring slot angle limit after a time span of approximately 120 s; therefore, the results can be considered as captured from a system in static equilibrium. For the record, due to the different initial robot poses depending on the foot radius $r_f$, each individual setup starts and ends at a
different point in time; thus, the comparison of different simulations is expressed as the relationship between different measurement signals in the following, independent of the actual simulation time.

Spring Stiffness Measurement Results

The virtual spring stiffness $k_v(x_v)$ as a nonlinear function of the virtual leg compression can be easily calculated by the relationship of the projected leg compression $\Delta x_v$ and projected ground reaction forces $f_v$ in the direction of the virtual spring DOF; see Figure 27b. Considering the ground reaction force vector $f_{grf}$ and the foot to torso vector $d_R$, the projected force amount $f_v$ is calculated by:

$$f_v = f_{grf} \cdot \frac{d_R}{|d_R|},$$

which also has to satisfy the virtual spring force equation:

$$f_v = k_v(x_v + \Delta x_v),$$

that can be rearranged as:

$$k_v = \frac{f_v - f_{v0}}{\Delta x_v} \text{ with } f_{v0} = k_v x_{v0}. \quad (24)$$

The initial force $f_{v0}$ of the virtual spring is captured from the moment that the spring slot joint angle starts to increase from its resting value of $\phi_{spr} = -8^\circ$. Table 10 displays the characteristic values of the simulation experiment.

Table 10. Push down experiment parameters, depicting the initial default robot pose at the transition from the static pose at $h_{t,max}$ to beginning compression.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default ground reaction force $</td>
<td>f_{grf0}</td>
<td>$</td>
</tr>
<tr>
<td>Initial spring threshold $f_{v0}$</td>
<td>15.638</td>
<td>N</td>
</tr>
<tr>
<td>Max. torso height $h_{t,max}$</td>
<td>0.380</td>
<td>m</td>
</tr>
<tr>
<td>Min. torso height $h_{t,min}$</td>
<td>0.280</td>
<td>m</td>
</tr>
<tr>
<td>Initial foot radius $r_f$</td>
<td>0.170</td>
<td>m</td>
</tr>
<tr>
<td>Max. horizontal foot displacement $\Delta r_f$</td>
<td>0.013</td>
<td>m</td>
</tr>
</tbody>
</table>

Figure 28a depicts simulated values over the increasing virtual leg compression, starting in the current default robot pose with $r_f$ according to Table 10. Thus, as the torso force $f_t$ increases, the profile of the geometrical variables, namely the elongation $\Delta x_{spr}$ and the spring slot angle $\phi_{spr}$—depicted in the upper plot and related to the internal spring—and the measured ground reaction forces $f_{grf}$ yield the virtual stiffness $k_v$, as shown in the lower plot. Forces $f_{grf}$ are not displayed as they simply correspond to $1/3$ of the vertical force $f_t$. 
Figure 28. (a) Measurements of the characteristic values over the virtual compression $\Delta x_{\text{spr}}$. The upper plot is related to the internal spring behaviour, the lower plot to the resulting virtual spring. (b) Profiles over time, which shows the required build-up of forces before the robot starts to alter its pose. The red line marks the point of transition.

As one can observe in the lower plot of Figure 28a, the virtual spring acts as a reasonable linear compression spring with an increase of stiffness between approximately $k_v \approx 100$ N/m and $k_v \approx 110$ N/m in relation to the constant high-stiffness internal leg spring as the leg becomes compressed and features an initial threshold force $f_{v0} = 15.638$ N that has to be exceeded before any compression starts. Thus, the corresponding behaviour of the internal linear spring in comparison to the virtual one shows a noticeable gain, yet still does not respond with a major nonlinearity.

A conclusion that can be extracted from this behaviour is the possible employment of a nonlinear internal leg spring, as a measure to create possibly linear or customised virtual spring profiles. In general, the relationship between the virtual and internal spring forces—also depending on the current leg pose—will be important in any possible future addition of force control that targets the specific interaction forces between the feet and the environment, which becomes increasingly important in comparison to position control, especially in the context of legged locomotion in unknown environments.

One aspect of the spring implementation that should not be overlooked is the actual internal reaction of the spring due to the external application of contact forces at the robot feet. Specifically, the resulting internal spring reaction force and the torque resulting between the tool-platform and upper leg at axis $c_3$ are crucial for the feasibility of the system. Importantly, if the spring is too stiff, no compliance will be possible, as the resulting spring torque at axis $c_3$, in series with the SPM, might exceed the torque limits of the actuators inside the base-platform. In this case, the tool-platform would start to change its orientation due to overloaded actuators, before the spring-related torque $\tau_{\text{spr}}$ threshold—required to be exceeded before any spring slot motion starts—in axis $c_3$ will be topped by the external torque load. Thus, the electric actuators might be capable of delivering high torques for the short amount of time near or at zero velocity without overheating, yet a different spring profile should be employed to reduce the transmitted spring torque, or—in other words—enable the internal spring to deform with less required external contact forces. Figure 28b depicts the build-up of forces over time in the experiment until the point in time is reached—marking the force-related threshold—at which the robot starts to change its pose.
Consequently, the resulting threshold values $f_{v0}$ and stiffness $k_v$ of the experiment can be altered easily by selecting a different internal spring; thus, one might simply adjust $k_{spr}$ and $x_0$ to produce a different outcome. Importantly, the spring implementation in the robot presented can optionally, but not necessarily be configured by the selection of the spring parameters in a way so that a threshold force is required before any compliant behaviour will be observed in the robot structure. Moreover, a nonlinear spring profile may be considered to yield an always-compliant behaviour, yet still preventing the robot spring slot from hitting its upper joint limit.

In this regard, the specific relationship between the internal spring and the virtual spring stiffness is considered in the following. In the current setup, the transfer ratio $g$ between the stiffness of the internal spring and the virtual spring as a function of the leg compression,

$$g(r_f, \Delta x_v) = \frac{k_{spr}}{k_v}$$

is depicted in Figure 29a and compared against the spring ratios based on different initial poses, respectively values, for the foot radius $r_f$. Since minor jitter was observable in the measurement data—as the data were captured from a very slow-moving, yet not totally static system—the graphs depicted in the figure are the ninth-degree polynomial curve fittings for the original data. For the record, only the ratio $g$ was smoothed, since the noise of each component necessary for calculating this value culminates in a jittery result, without being visually pronounced in the actual individual measurement signals. Table 11 shows the root-mean-squared error (RMSE) and the coefficient of determination ($R^2$) based on the applied polynomial curve fitting. In general, by the results, the curves can be considered to approximate the original data sufficiently. For completeness, all data signals are provided as MATLAB table-format data in the Supplementary Materials.

**Table 11.** Fitting quality of the polynomial curves for ratio $g$, depicted in Figure 29a.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_f$ (m)</td>
<td>0.000 0.025 0.050 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275</td>
</tr>
<tr>
<td>RMSE ($\times 10^{-4}$)</td>
<td>6.24 7.16 8.40 8.90 28.87 4.06 11.68 13.38 10.95 5.26 17.36 46.30</td>
</tr>
</tbody>
</table>

Regarding Figure 29a, the ratio $g$ clearly depends on the initial foot radius $r_f$, which shows that with a lower radius $r_f$, the stiffness $k_v$ increases. The reduction of $k_v$ in comparison to the internal spring stiffness $k_{spr}$ is defined by $k_v = g^{-1}k_{spr}$ and yields a factor of approximately $g^{-1} = \frac{1}{90}$ to $g^{-1} = \frac{1}{160}$ in the mostly constant region of Figure 29a. Considering the initially high values of $g$ at the start of certain values of $r_f$, the stiffness of the virtual spring decreases substantially with extremely high values of $r_f$. This behaviour is based on the one-DOF constraint of the leg mechanism, which in turn requires large horizontal displacements of the feet for the corresponding height variation of the robot torso. The displacement $\Delta r_f$ is depicted in Figure 29a and states that the lowest horizontal shift is found approximately at an initial foot radius below the floor-projected hip centre point. It should be noted that high displacements $\Delta r_f$ are not considered as design drawbacks in general, because in the context of locomotion, it is required to create a horizontal motion of the robot torso, which, in this regard, may potentially be exploited as a useful mechanical property in future investigations. Furthermore, in the real application, the displacements $\Delta r_f$ can by altered directly by reorienting the hip and knee joints, which—in contrast to this experiment—are not kept completely fixed. Thus, inverse kinematics can be employed until the leg has reached a suitable pose for the compression phase inside a possible locomotion cycle.

An intuitive depiction of the experiment is also featured in Figure 29b, which shows the force profiles for the virtual force $f_v$ and the actual push down force $f_t$ over virtual compression $x_v$ and overall torso height $h_v$. Fundamentally, the plot on the right side of
Figure 29b depicts the overall compliance behaviour of the complete robot system, which shows a reasonable linear profile, yet slightly different stiffness values for each individual starting pose.

![Figure 29a](image1.png) ![Figure 29b](image2.png)

Figure 29. (a) Transfer ratio $g$ and corresponding foot displacement $\Delta r_f$ over the virtual leg compression $\Delta x_v$. Text annotations on the graphs depict the initial foot radius $r_f$ of the specific setup. (b) Force profiles during the push down phase regarding the virtual compression and the geometrical torso height. Text annotations in the plot on the right side mark only the first and last element with steps of 0.025 m between neighbouring graphs.

4. Real-World Prototype Experiment

In this final experiment section, the actual real-world prototype of the robot leg is investigated. Thus, the experiment setup and procedure are explained in the following, but are also contained in the filmed video footage included in the Supplementary Material. Figure 30 shows the build prototype model, connected with the centre tray to a camera tripod; thus, the leg cannot collide with its environment. Noticeably, the curved contour of the centre tray that is slotted into the base-platform turned out to be a very rigid connection, which is a promising aspect of the overall design regarding a stable and stiff robot torso in the complete robot model that will be built in the future development. Since—for now—only the knee actuation concept and the spring DOF were inspected closely, the base actuators were kept inactive in this early experiment. Future work will expand the real-world experiments, but this requires the complete implementation of sensor, actuator, and controller communication in real-time hardware, which is currently in process. Consequently, the robot motion was based on open-loop control, and the torque signal for the knee actuator was controlled manually through a user in this experiment. Due to the inactive base actuators, the tool-platform was able to rotate freely, which was
also supported by the low-friction plastic plain bushings in the central Cardan-type support structure. Thus, as one can observe in the pictures and video files, the tool-platform was restrained with strong polypropylene ropes, pulling the individual tool axes towards the base-platform, which kept the orientation of the tool-platform reasonably fixed in the experiment.

![Robotics Figure 30](image)

Figure 30. Experiment setup. The prototype leg is attached to a camera tripod, which prevents collisions with the environment during motion. An attachment plate connects the tripod and centre tray.

Figure 31 shows different poses of the robot leg during its motion, taken from the filmed sequence. Thus, in the retracted pose, as shown in Figure 31a, the robot foot is able to physically hit the hip base-platform, which might yield a suitable overall dexterity that potentially enables the robot to stand up and sit down by itself in the future development. The lower limit of the slider is also located close to this point. During the expanding motion of the slider, the leg swings until both the slider and the lower leg experience an internal surface collision. This is also depicted in Figure 31b. This point marks the end of the pure kinematic motion range. Hence, further moving the slider deforms the closed-loop structure of the upper leg that is fixed due to the contracting force of the internal spring. Consequently, the motion of the slider from this point on works against the internal leg spring, which further stretches out the leg, while simultaneously, the upper leg starts to rotate inside the spring slot. This motion ends at the actual upper limit of the slider range. Figure 31c depicts the situation.
Actually, due to the manual control of the motor torque during the experiment, the knee actuator was powerful enough to break the epoxy connection between the two parts that together create a form fitting with the motor shaft. Thus, a limit detection sensor and a precise linear encoder will be required as further additions inside the leg mechanism to safely integrate a closed-loop control architecture.

Included also in the video is the kinematic motion demonstration of the spring mechanism DOF, as depicted in Figure 32. Fundamentally, the mechanism exhibits two motion cases. First, with the blocked thread DOF of the slider screw, there exists the pure angular motion of the spring slot joint around axis $c_3$. This is depicted in Figure 32a,b. Second, after the surface contact between the lower leg and slider near the knee joint axis, further motion requires the simultaneous utilisation of both the spring slot DOF and the slider thread DOF. This case is shown in Figure 32c,d. Fundamentally, the leg structure contains a four-bar closed-loop kinematic structure with one side—referring to the slider—being adjustable in its length. After the surface collision explained above, one joint of the four-bar linkage basically becomes fixed, which reduces the system to a three-bar linkage. Obviously, any additional motion of this structure requires altering the length of on side of the triangle, which is possible due to the slider with its thread DOF. Both motion cases are demonstrated in the video in the Supplementary Materials.

Importantly, the real-world model presented in this section is a physical realisation of an ideal rigid-body model that was considered in the simulation. Thus, the real-world build of the robot leg may show systemic behaviours or hidden dynamics that were not captured by the mathematical representation of the robot. Consequently, alongside the goal of bringing the real and virtual model closer together by a system identification approach, further analysis and the possible exploitation of these hidden and unmodelled properties might actually enhance [46] the robot performance during more advanced control objectives.
Figure 32. (a,b) depict the spring slot motion with a fixed thread DOF, thus fixing the position of the slider. In (c,d) the slider and lower leg are kept fixed relative to each other. Any further slider motion requires the additional motion of the spring slot DOF that also acts against the internal spring.

5. Discussion

A central aspect of the presented robot design was the spring implementation. As one can observe from the results of the experiments provided, which were meant to explore the elasticity behaviour of the robot legs, the robot showed a quite untypical spring profile. In contrast to the simple SLIP model, the current spring implementation in this robot needs a threshold force $f_{\text{c0}}$ to be applied before any spring motion is achieved. It is part of future work if this is actually a positive property, but for now, the robot system behaves as a hybrid system, which—depending on the internal spring properties—may yield an entirely stiff robot that is able to also store and release energy from the locomotion cycle if the initial threshold force is exceeded. This ability may become beneficial as to gate the robot system from behaving compliantly before actually transitioning to a more dynamic and fast motion. One should note here that it is obviously possible to reduce the spring stiffness to a degree, so that the system does not yield a stiff leg behaviour, but this will more likely require a different stiffness profile as a measure to prevent the robot from collapsing and therefore avoiding sudden internal spring slot collisions. A general result that was revealed by the experiments was the possible drawback of the spring mechanism of experiencing discrete changes of the system behaviour due to the hard internal collisions of the spring slot joint, which were able to yield sudden torque load peaks on the actuators. This might be circumvented by more suitable damping at the joint limits or the future employment of custom spring stiffness profiles, which are able to prevent these contact situations.

Regarding the current robot setup, the experiments showed that there existed regions in which the lateral displacement of the feet was kept small while compressing the leg. Furthermore, the resulting spring stiffness behaved reasonable linearly and constant in an interval that was considered to be small enough to ensure stability in the sense of being able to keep the robot system from losing its controllably. An important note to make here is the ability of the robot leg to tilt itself around the virtual axis between the hip and foot due to its inherent redundant design. The resulting behaviour was not examined in this proposal, but considering small or zero displacement of the leg tip based on a suitable initial pose, the orientation of the leg will not alter the spring behaviour. The reason for this property is the design of the spring DOF, which was constructed as a mechanism that changes the distance between the foot and hip, respectively torso, without being dependent on the initial rotation of the leg itself.
As a general result, it was shown that the basic robot design was successful in regard to providing a mechanical structure that was able to carry itself with the actuators selected while still providing a suitable amount of headroom for more dynamic motion, which will be examined in the future. The mechanical realisation was successful regarding the real-world prototype, which is important to prevent the experiments performed from being purely theoretical in nature, thus allowing the results discovered from the simulations to be applied to the real-world model. However, one has to be careful about transferring the results from the rigid body simulation onto the real robot model, which—based on the materials selected—features a slight elasticity inherent to the parts themselves. Likely, this will yield more deviations between the simulation and real-world model with higher general load forces that are exerted onto the robot system. Consequently, under certain unusual conditions that are not considered as typical for the intended motion profile of the robot, the real model will bend instead of actually delivering extreme reaction forces and loads as would be the case in a rigid body simulation. Despite that there will be a certain gap between the current simulation model and the real-world prototype, the simulations performed give insights into the general mechanical behaviour of the robot model. Consequently, these simulations are relevant to offer the possibility to observe the response of the system for different spring settings and form the basis for the future testing and validation of actual control algorithms. To summarise, Table 12 gives an overview of the main findings and properties of the robot presented.

Table 12. Summary of the main results and properties of the robot discovered in this work.

<table>
<thead>
<tr>
<th>Resulting Robot Properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Combined serial–parallel 4-DOF actuation scheme per leg.</td>
<td>• Lightweight, compact, and dense mechanical model with a non-overconstrained topology.</td>
</tr>
<tr>
<td>• Radially symmetrical, omnidirectional layout.</td>
<td>• Possible hybrid compliant behaviour, with the threshold value gating stiff from the compliant state.</td>
</tr>
<tr>
<td>• High load carrying capacity of the central support joint, reducing stress on the SPM linkages.</td>
<td>• Reasonable linear resulting spring profile of the robot model.</td>
</tr>
<tr>
<td>• Redundant leg kinematics, possibly allowing for optimised actuator load and agility.</td>
<td>• Feasibility of the mechanical system shown via the real-world model.</td>
</tr>
</tbody>
</table>

6. Conclusion and Future Work

In this paper, a three-legged robot was presented, covering the concepts and realisations of the mechanical design, while targeting a lightweight, agile, and omnidirectional robot that matches the well-known SLIP model. The robot design was then examined by rigid multi-body simulations, and a real-world prototype model of one leg was shown. The robot features a parallel joint actuation for the hip joints with the knee actuator connected in series, yet still being located outside of the moving leg structure. Thus, this design makes it possible to place all 12 actuators in an interlocking arrangement inside the robot torso, concentrating the robot mass successfully in a narrow central volume, yielding agile legs with low mass and inertias.

As a continuation of the previous work, the general robot design focussed on 3D printing and a compound design, which allow for highly optimised shapes and a dense geometric arrangement. With the realisation of specifically shaped parts, the mechanism thereby implements a virtual supporting central hip joint as an extension to the typical spherical parallel manipulator unit, absorbing high loads and relieving the outer parallel structure from the necessity to also withstand forces that are not part of the intended degrees of freedom of the spherical hip joint. Thus, the actuation design yields a mechanical layout of the robot that features a non-overconstrained hybrid serial–parallel structure. Being not overconstrained is a key element of the hip joint design as this ensures the specifically intended and decoupled force and torque load distribution between the parts of mechanical
structure, the linkages, and the actuators. Hence, this yields compact and thin linkages that are only required to transfer the hip joint torque components. Furthermore, with the Cardan-type layout of the support joint, the distant actuation of the knee joint becomes possible while also incorporating the spring degree of freedom onto a shared axis—used for the connection of the tool-platform and the support joint—hence successfully combining multiple aspects of the robot functionality with minimal constructive overhead.

A further design aspect that was considered important was the mechanical redundancy of the four-DOF leg design, which was based on the spherical mechanism design of the hip joints, and is part of future investigations, as this may be an option to optimize the joint load distribution on the individual actuators.

The last aspect of the robot design regards the spring implementation as a central aspect of enabling energetically efficient locomotion, while the actual mechanical realisation was based on a four-bar closed-loop linkage that exploits the geometry of the Cardan-type hip support joint. Specifically, the presented mechanism thereby uses one axis simultaneously for the Cardan joint and for the spring DOF and also includes one bar that changes its length as the knee joint is actuated by the integrated slider mechanism.

As a final conclusion, the robot presented employed several closely interwoven mechanical structures as a measure to incorporate multiple target properties. Since the resulting mechanical structure is of a highly complex nature, this might also result in constraints or drawbacks in comparison to more simple robot designs. Regarding the intricate layout of the mechanism, future work thus will contain analyses based on the kinematical and Jacobian description of the robot model, including the examination regarding singularities, performance criteria, conditioning evaluation over the robot workspace, and the robot’s dynamic behaviour during motion. Therefore, the simulation model will be updated to better approximate the real prototype, which also includes attention to several communication and hardware components—e.g., sensors and actuators—which will additionally influence the behaviour of the real-world model and possibly impose certain restrictions, unlike in the simulation. In this regard, future iterations of the robot model will feature more robust and stiff 3D-printed parts, which should further close the gap between the simulation and real-world model as an important measure to reliably deploy control algorithms tested in simulation on the real model. Thus, future research will further investigate the locomotion capabilities of the robot model and consequently focus on real-world controller implementations and experiments as a measure to further explore the potential of the mechanical design itself for the purpose of legged locomotion.

**Supplementary Materials:** The following Supplementary Materials are available online at https://www.mdpi.com/article/10.3390/robotics11020039/ and also at [45]:

- **Video S1:** Robot Prototype by David Feller. One video including multiple sections: (1) The filmed sequence of the actuated knee motion with additional comments. (2) The filmed closed-loop leg kinematics demonstration. (3) The drop experiment simulation from Section 3.3 for both stiff and compliant configurations. (4) A filmed non-actuated mechanical visualisation of the real-world hip manipulator based on the general SPM build for the preliminary robot model. Additional clips of the preliminary model being actuated based on joint space PID control through commands via the real-time-capable hardware interface dSpace ds1104 [47];
- **Data S1:** Free Fall Experiment Data. Four diagrams for the drop experiment simulation, showing the complete force/torque and profile plots for Setups A and B. One table including numerical data of the impact torques and reduction ratios for the drop experiment simulation;
- **Data S2:** Push Down Experiment Data. Simulation measurement signals and calculated signals of the push down experiment from Section 3.4 exported as txt files from MATLAB table data.

**Author Contributions:** Conceptualisation, D.F.; methodology, D.F.; software, D.F.; validation, D.F.; formal analysis, D.F.; investigation, D.F.; resources, D.F.; data curation, D.F.; writing—original draft preparation, D.F.; writing—review and editing, D.F.; visualisation, D.F.; supervision, D.F. and C.S.; project administration, D.F.; funding acquisition, D.F. All authors have read and agreed to the published version of the manuscript.
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