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Type Synthesis of 5-DOF Hybrid (Parallel-Serial) Manipulators Designed from Open Kinematic Chains

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Abstract: The article proposes an approach for synthesizing hybrid (parallel-serial) manipulators with five degrees of freedom (5-DOF) using open kinematic chains. The method idea consists in taking an open kinematic chain, selecting a subchain within it, and replacing the subchain with a parallel mechanism. The article considers 5-DOF open chains and 3-DOF subchains, substituted for 3-DOF parallel mechanisms with the same motion pattern as the subchain. Thus, synthesized hybrid manipulators have a 3-DOF parallel part and a 2-DOF serial part. First, we grouped 26 structures of open chains with revolute and prismatic joints into five types and 78 subtypes. Next, for each type, we selected one subtype and presented several hybrid mechanisms that can correspond to it. We considered hybrid manipulators that included 3-DOF parallel mechanisms with planar, spherical, and other commonly used motion types. The suggested synthesis method is intuitive for a designer, and it does not need any mathematical formulations like screw theory or group theory approaches.

Keywords: type synthesis; 5-DOF hybrid (parallel-serial) mechanism; 3-DOF parallel mechanism; serial (open) kinematic chain; 3T2R and 3R2T motion patterns

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1. Introduction

Development of technologies implies creation of novel robotic, technological, medical, research, and other systems, which are usually based on effectively designed mechanisms. The first step in creating any mechanism is to synthesize its structural diagram, which specifies the types of links and joints used in the mechanism and their relative arrangement. This fundamental stage establishes the functional properties and required characteristics of the developed system. In this regard, the major task of mechanism type synthesis is designing its most optimal structural diagram according to the application.

Familiar methods of type synthesis can be classified into three categories [1]. The first category represents motion-based approaches, which focus on possible motions of the mechanism links. This category includes methods based on group theory [2], generalized function sets (G_F sets) [3,4], position and orientation characteristic sets (POC sets) [5], linear transformations [6], finite screws [7], and conformal geometric algebra [8]. The second category represents constraint-based methods, which consider constraints imposed on the mechanism links. This category comprises techniques that rely on instantaneous screws [9–12], virtual kinematic chains [13], Grassmann line geometry [14–17], and motion constraint generators [18,19]. The third category includes other methods, like the ones based on the mobility formulae [20], graph theory [21], and Denavit–Hartenberg parameters [22]. The papers [1,23] discuss most of these methods and provide a comprehensive bibliography with their applications.

Over the last few years, scholars have focused on studying hybrid manipulators. These mechanical systems include kinematic chains with a parallel structure (parallel-parallel mechanisms) or parallel and serial structures (parallel-serial mechanisms). Such manipulators have several advantages compared to conventional parallel or serial manipulators. One of the major advantages over the parallel manipulators is an increased workspace. This is especially useful in fields like machining (for processing large-sized parts) [24], agriculture (for planting and picking operations) [25], and medicine (for large movements of medical equipment relative to the patient) [26]. Compared to serial manipulators, hybrid ones possess a higher rigidity and motion accuracy, which is also important for precise applications [27].

Approaches for type synthesis of hybrid mechanisms are mainly based on the approaches mentioned above. There are few works devoted to systematic type synthesis of these mechanisms, and most studies focus on parallel-parallel structures. One of the first studies in this field was the work by Chakarov and Parushev [28], who used mobility formulae and Assur groups with linear drives. These groups were placed between the links of the primary open-chain mechanism and did not affect its mobility. This approach allowed displacing the drives of the primary mechanism from its joints to the prismatic joints of the structural groups. Campos et al. [29] developed a similar method based on Assur groups. The authors considered symmetrical hybrid manipulators obtained from a primary zero-DOF mechanism by "cutting" one of its links. Mobility formulae and Assur groups were also used by Alizade and Bayram [30], but the authors focused mainly on parallel-parallel manipulators.

In the synthesis approaches discussed above, the authors considered only the number of DOFs, but ignored the motion type of the output link. Other works examined the nature of these DOFs (rotational or translational). For example, Zeng and Fang [31] proposed an approach based on group theory and represented mechanisms as logical matrices. The method relies on performing some logical operations which can automate type synthesis, as the authors notice. The authors advanced their approach in works [32,33] and presented several novel hybrid mechanisms. Another study to be mentioned is by Shen et al. [34], who applied the POC sets to synthesize hybrid manipulators with three to five DOFs. The authors considered various combinations of parallel and serial kinematic chains and obtained many novel mechanisms.

The studies enumerated above suggested systematic approaches for type synthesis of hybrid manipulators with a different number of DOFs and different motion types. Among the variety of hybrid manipulators, mechanical systems with five DOFs take a special place. In most cases, the output link of such manipulators has either three translational and two rotational DOFs (a 3T2R motion pattern) [35] or three rotational and two translational DOFs (a 3R2T motion pattern) [36]. These manipulators are often used for applications that do not need an excessive rotational or translational freedom, or where this freedom is achieved by a separate drive. Such applications include surgical operations [37,38], pick-and-place tasks [39], machining [40,41], polishing [42], and welding [43].

There are several studies that considered the type synthesis of 5-DOF hybrid manipulators, and most works focused mainly on synthesizing parallel mechanisms. For example, Cao et al. [44,45] applied G_F sets to design 3T2R hybrid manipulators, whose parallel parts had three or four DOFs: the authors obtained over three hundred various mechanisms. Later, the authors used that method to synthesize 3R2T hybrid manipulators and also obtained numerous novel mechanisms [46]. The POC sets method, proposed above by Shen et al. [34], was applied by the authors in paper [47]: the authors developed a novel 5-DOF hybrid manipulator with a 2-DOF serial part and a 3-DOF parallel part. Xu et al. [48] applied screw theory techniques to design 5-DOF hybrid manipulators with 3- and 4-DOF parallel parts; later, in the works [49–51], the authors synthesize planar and spatial 3-DOF parallel mechanisms within various 5-DOF hybrid manipulators. Xie et al. [24] performed a research similar to [49] and considered type synthesis of 3-DOF 1T2R parallel mechanisms, which represented a parallel part of other 5-DOF hybrid manipulators. The authors' method relied on Grassmann line geometry.

In the current article, we introduce a novel approach for the type synthesis of 5-DOF hybrid manipulators based on open kinematic chains. The advantage of this approach over the existing ones is that it is more intuitive and does not require any profound mathematical formulations like other methods based on group theory, screw theory, POC sets, or G_F sets.

The rest of the paper has the following organization. Section 2 presents an original synthesis method. Section 3 shows structural diagrams of 5-DOF open kinematic chains, taken as primary mechanisms, and hybrid manipulators, designed using the suggested techniques. Section 4 discusses the obtained results and possible improvements of the presented approach. Section 5 recaps the entire study and mentions directions for future research.

2. Synthesis Method

In the current section, we will look at the proposed type synthesis method. The method idea consists in taking a 5-DOF open kinematic chain and transforming it into a hybrid mechanism. In particular, we select a 3-DOF subchain within the open chain and transform it into a parallel mechanism, so the synthesized hybrid manipulator has a 3-DOF parallel part and a 2-DOF serial part. We consider parallel parts with three DOFs because 3-DOF parallel mechanisms usually have a simple and symmetrical design, and they prevail among other lower-DOF parallel mechanisms [52].

As primary open kinematic chains, we only consider chains with revolute (R) and prismatic (P) joints: we can always decompose any multiple-DOF joint into a combination of R and P joints [13] (p. 28). In addition, open kinematic chains should satisfy the following conditions to avoid any local mobility within the chain [53]:

1. There are no coaxial P joints;
2. The greatest number of (coplanar) P joints is three;
3. There are no coaxial R joints;
4. The greatest number of R joints with parallel axes is three;
5. The greatest number of R joints with intersecting axes is three.

Conditions 1 and 2 guarantee there is no local translational motion. Conditions 3–5 guarantee there is no local rotational motion. We can find number n of different kinematic chains, which meet these conditions, using the following formula [54] (p. 4):

$$n = U(2,5) - \sum_{m=4}^{5} C(5,m) = 2^5 - \frac{5!}{4!(5-4)!} - \frac{5!}{5!(5-5)!} = 26, \quad (1)$$

where $U(2,5)$ is a number of permutations with unrestricted repetitions, equal to a number of 5-DOF chains with P and R joints; $C(5,m)$ is a number of combinations, equal to a number of 5-DOF chains with m P joints. The subtraction of terms $C(5,m)$ in Equation (1) corresponds to Condition 2 and allows us to exclude chains with four and five P joints.

Table 1 presents all 26 structures of open kinematic chains with R and P joints. Note that the same structure can have different joint arrangements. For example, the RRRRR chain can include subchains of planar, spherical, or other types.

Table 1. Possible structures of primary open kinematic chains.

(1) RRRRR	(2) PRRRR	(3) RPRRR	(4) RRPRR	(5) RRRPR	(6) RRRRP
(7) PPRRR	(8) PRPRR	(9) PRRPR	(10) PRRRP	(11) RPPRR	(12) RPRPR
(13) RPRRP	(14) RRPPR	(15) RRPRP	(16) RRRPP	(17) PPPRR	(18) PPRPR
(19) PPRRP	(20) PRPPR	(21) PRPRP	(22) PRRPP	(23) RPPPR	(24) RPRPP
		(25) RPPRP	(26) RRPPP		

Next, we use the enumerated open kinematic chains to synthesize hybrid manipulators as follows. We select a 3-DOF subchain within the 5-DOF primary kinematic chain and replace it with a 3-DOF parallel mechanism, which has the same motion pattern as the substituted subchain (it can also be possible that the parallel mechanism has a motion type different from the subchain—this situation is addressed in Section 4). For each 5-DOF kinematic chain, three possibilities exist to select a 3-DOF subchain, so there are $26 \times 3 = 78$ possible structures of hybrid mechanisms. Table 2 demonstrates all these structures, where the underline signifies the substituted subchain. The structures are classified according to the number of P and R joints in the primary open kinematic chain. There are four distinct types of structures: 5R, 4R1P, 3R2P, and 3P2R. Each type includes several subtypes: the 5R type has three subtypes, the 4R1P type has fifteen subtypes, and both 3R2P and 3P2R types have thirty subtypes.

Table 2. Possible structures of hybrid manipulators designed from open kinematic chains.

		5R Type			
(1) $\underline{\mathrm{RRR}}\mathrm{RR}$		(2) $\mathrm{R}\underline{\mathrm{RRR}}\mathrm{R}$		(3) $\mathrm{RR}\underline{\mathrm{RRR}}$	
		4R1P type			
(4) $\underline{\mathrm{PRR}}\mathrm{RR}$	(5) $\mathrm{P}\underline{\mathrm{RRR}}\mathrm{R}$	(6) $\mathrm{PR}\underline{\mathrm{RRR}}$	(7) $\underline{\mathrm{RPR}}\mathrm{RR}$	(8) $\mathrm{R}\underline{\mathrm{PRR}}\mathrm{R}$	(9) $\mathrm{RP}\underline{\mathrm{RRR}}$
(10) $\underline{\mathrm{RRP}}\mathrm{RR}$	(11) $\mathrm{R}\underline{\mathrm{RPR}}\mathrm{R}$	(12) $\mathrm{RR}\underline{\mathrm{PRR}}$	(13) $\underline{\mathrm{RRR}}\mathrm{PR}$	(14) $\mathrm{R}\underline{\mathrm{RRP}}\mathrm{R}$	(15) $\mathrm{RR}\underline{\mathrm{RPR}}$
(16) $\underline{\mathrm{RRR}}\mathrm{RP}$		(17) $\mathrm{R}\underline{\mathrm{RRR}}\mathrm{P}$		(18) $\mathrm{RR}\underline{\mathrm{RRP}}$	
		3R2P type			
(19) $\underline{\mathrm{PPR}}\mathrm{RR}$	(20) $\mathrm{P}\underline{\mathrm{PRR}}\mathrm{R}$	(21) $\mathrm{PP}\underline{\mathrm{RRR}}$	(22) $\underline{\mathrm{PRP}}\mathrm{RR}$	(23) $\mathrm{P}\underline{\mathrm{RPR}}\mathrm{R}$	(24) $\mathrm{PR}\underline{\mathrm{PRR}}$
(25) $\underline{\mathrm{PRR}}\mathrm{PR}$	(26) $\mathrm{P}\underline{\mathrm{RRP}}\mathrm{R}$	(27) $\mathrm{PR}\underline{\mathrm{RPR}}$	(28) $\underline{\mathrm{PRR}}\mathrm{RP}$	(29) $\mathrm{P}\underline{\mathrm{RRR}}\mathrm{P}$	(30) $\mathrm{PR}\underline{\mathrm{RRP}}$
(31) $\underline{\mathrm{RPP}}\mathrm{RR}$	(32) $\mathrm{R}\underline{\mathrm{PPR}}\mathrm{R}$	(33) $\mathrm{RP}\underline{\mathrm{PRR}}$	(34) $\underline{\mathrm{RPR}}\mathrm{PR}$	(35) $\mathrm{R}\underline{\mathrm{PRP}}\mathrm{R}$	(36) $\mathrm{RP}\underline{\mathrm{RPR}}$
(37) $\underline{\mathrm{RPR}}\mathrm{RP}$	(38) $\mathrm{R}\underline{\mathrm{PRR}}\mathrm{P}$	(39) $\mathrm{RP}\underline{\mathrm{RRP}}$	(40) $\underline{\mathrm{RRP}}\mathrm{PR}$	(41) $\mathrm{R}\underline{\mathrm{RPP}}\mathrm{R}$	(42) $\mathrm{RR}\underline{\mathrm{PPR}}$
(43) $\underline{\mathrm{RRP}}\mathrm{RP}$	(44) $\mathrm{R}\underline{\mathrm{RPR}}\mathrm{P}$	(45) $\mathrm{RR}\underline{\mathrm{PRP}}$	(46) $\underline{\mathrm{RRR}}\mathrm{PP}$	(47) $\mathrm{R}\underline{\mathrm{RRP}}\mathrm{P}$	(48) $\mathrm{RR}\underline{\mathrm{RPP}}$
		3P2R type			
(49) $\underline{\mathrm{PPP}}\mathrm{RR}$	(50) $\mathrm{P}\underline{\mathrm{PPR}}\mathrm{R}$	(51) $\mathrm{PP}\underline{\mathrm{PRR}}$	(52) $\underline{\mathrm{PPR}}\mathrm{PR}$	(53) $\mathrm{P}\underline{\mathrm{PRP}}\mathrm{R}$	(54) $\mathrm{PP}\underline{\mathrm{RPR}}$
(55) $\underline{\mathrm{PPR}}\mathrm{RP}$	(56) $\mathrm{P}\underline{\mathrm{PRR}}\mathrm{P}$	(57) $\mathrm{PP}\underline{\mathrm{RRP}}$	(58) $\underline{\mathrm{PRP}}\mathrm{PR}$	(59) $\mathrm{P}\underline{\mathrm{RPP}}\mathrm{R}$	(60) $\mathrm{PR}\underline{\mathrm{PPR}}$
(61) $\underline{\mathrm{PRP}}\mathrm{RP}$	(62) $\mathrm{P}\underline{\mathrm{RPR}}\mathrm{P}$	(63) $\mathrm{PR}\underline{\mathrm{PRP}}$	(64) $\underline{\mathrm{PRR}}\mathrm{PP}$	(65) $\mathrm{P}\underline{\mathrm{RRP}}\mathrm{P}$	(66) $\mathrm{PR}\underline{\mathrm{RPP}}$
(67) $\underline{\mathrm{RPP}}\mathrm{PR}$	(68) $\mathrm{R}\underline{\mathrm{PPP}}\mathrm{R}$	(69) $\mathrm{RP}\underline{\mathrm{PPR}}$	(70) $\underline{\mathrm{RPR}}\mathrm{PP}$	(71) $\mathrm{R}\underline{\mathrm{PRP}}\mathrm{P}$	(72) $\mathrm{RP}\underline{\mathrm{RPP}}$
(73) $\underline{\mathrm{RPP}}\mathrm{RP}$	(74) $\mathrm{R}\underline{\mathrm{PPR}}\mathrm{P}$	(75) $\mathrm{RP}\underline{\mathrm{PRP}}$	(76) $\underline{\mathrm{RRP}}\mathrm{PP}$	(77) $\mathrm{R}\underline{\mathrm{RPP}}\mathrm{P}$	(78) $\mathrm{RR}\underline{\mathrm{PPP}}$

As mentioned in the paragraph above Table 1, each structure of the primary open kinematic chain can characterize various mechanisms with different joint arrangements. This diversity allows us to synthesize various hybrid manipulators within the same subtype of Table 2. For example, in the $\underline{\mathrm{RRR}}\mathrm{RR}$ subtype, the parallel part, indicated by the underline, can represent a planar, spherical, or other parallel mechanism. In the next section, we will exemplify this situation and show several mechanisms corresponding to the same subtype.

3. Results of Method Application

In this section, we will show how to synthesize hybrid manipulators using the proposed approach. We will consider several examples for each of the four types given in Table 2 (5R, 4R1P, 3R2P, and 3P2R). For each type, we will look at one subtype and present various mechanisms that correspond to it. Other subtypes can be analyzed in the same manner. We will focus on open kinematic chains (hence, hybrid manipulators) whose output link has either 3T2R or 3R2T motion pattern: as we saw in Section 1, these are two typical motion patterns of 5-DOF mechanical systems. In this regard, we will consider primary open chains with the following joint arrangement [13] (p. 52):

1. For the 3T2R motion pattern, the axes of all R joints should remain parallel to a common plane;
2. For the 3R2T motion pattern, the axes of all R joints should intersect a common line, orthogonal to the axes of all P joints.

To ensure full-cycle mobility, the conditions above should be satisfied for any configuration of the open chain.

3.1. Type 5R, Subtype RRRRR

The RRRRR subtype (# 1 in Table 2) can correspond to different hybrid manipulators. For example, consider an RRRRR primary open kinematic chain, where the axes of the first three R joints intersect at a common point, whereas the axes of the remaining R joints are parallel (Figure 1a). In the figure, the red curve indicates the replaced subchain, and the blue curve—the remaining serial part. Figure 1b,c illustrate two hybrid manipulators developed from this chain. The manipulators have similar serial parts, but different parallel parts. In Figure 1b, the parallel part is a 3-RRR spherical mechanism [55]; in Figure 1c, the parallel part is a 3-RUS/S spherical mechanism [56] (U and S indicate a universal and a spherical joint, respectively). Both manipulators provide its output link with a 3R2T motion pattern, but different parallel parts affect the manipulator characteristics. The 3-RRR mechanism is overconstrained, so we expect the manipulator in Figure 1b to have higher stiffness and motion accuracy than the one in Figure 1c. On the other hand, the circular rail used in the 3-RUS/S parallel mechanism allows the manipulator in Figure 1c to rotate unlimitedly around the rail axis. Such an unlimited rotation is impossible in the 3-RRR mechanism, so the orientation workspace of the manipulator in Figure 1c can be greater than in Figure 1b.

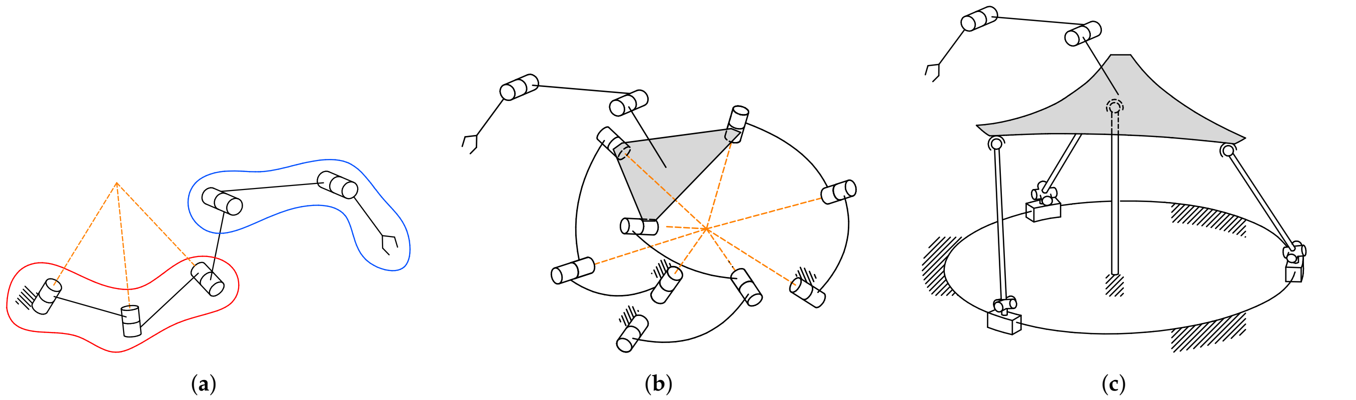


Figure 1. 5R type, RRRRR subtype (# 1 in Table 2): (**a**) primary open kinematic chain; (**b**,**c**) synthesized 5-DOF 3R2T hybrid manipulators with a 3-DOF spherical parallel mechanism.

An RRRRR primary open kinematic chain can also have a joint arrangement that leads to hybrid manipulators with a 3T2R motion pattern. For example, let the chain comprise two sets of joints with parallel axes (Figure 2a). Figure 2b,c show two hybrid manipulators synthesized from this chain. In Figure 2b, the parallel part is a 3-RRR planar mechanism [57]; in Figure 2c, the parallel part is a 3-RRP planar mechanism [58]. The 3-RRR mechanism includes only the revolute joints and has a simple design. The circular rail and unactuated prismatic joints of the 3-RRP mechanism make its design more cumbersome, but its output link can rotate unlimitedly about the rail axis, like the mechanism in Figure 1c. In addition, the 3-RRP planar mechanism is free of singular configurations [58].

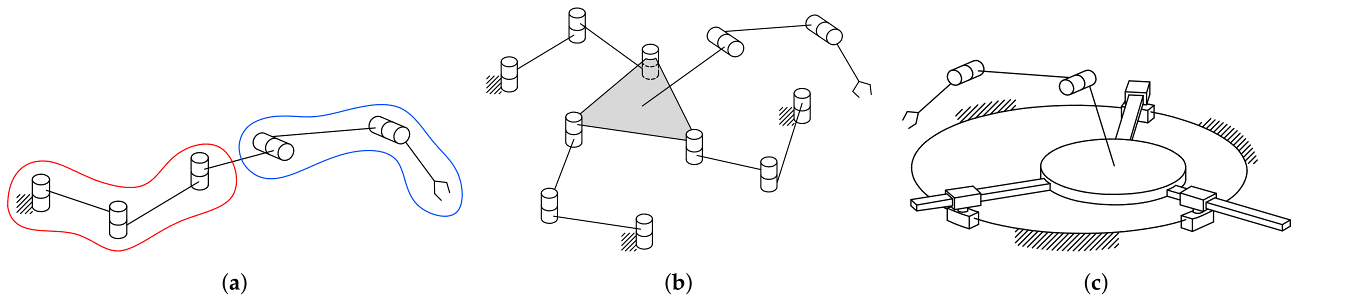
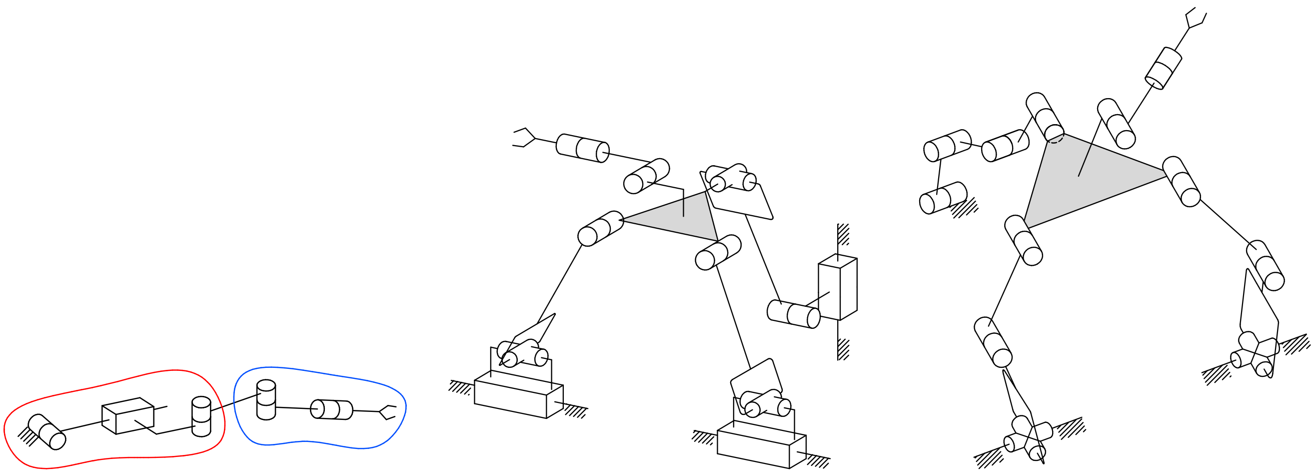


Figure 2. 5R type, RRRRR subtype (# 1 in Table 2): (**a**) primary open kinematic chain; (**b**,**c**) synthesized 5-DOF 3T2R hybrid manipulators with a 3-DOF planar parallel mechanism.

3.2. Type 4R1P, Subtype RPRRR

The next example considers hybrid manipulators that can be designed from the RPRRR subtype (# 7 in Table 2). As an example, we look at an RPRRR primary open chain with the following joint arrangement (Figure 3a). The axes of the first two R joints are orthogonal to each other and the axis of the P joint; the axis of the third R joint is parallel to the axis of the second R joint; the axis of the last R joint is orthogonal to the axis of the third R joint and intersects it as well as the axis of the first R joint. Figure 3b,c illustrate two hybrid manipulators designed from the open chain. Both manipulators include a 3-DOF RPR-equivalent parallel part with a 2R1T motion pattern [2] (ch. 8). The axes of the R joints in the serial part are arranged in such a way that the output link has a 3R2T motion pattern. In Figure 3b, the 3-DOF parallel mechanism has a 2-PUR/PRU structure, where the use of P joints placed on the base increases the mechanism stiffness [59]. On the other hand, the 3-DOF 2-URR/4R parallel mechanism depicted in Figure 3c does not have prismatic joints, and each of its branches includes a 3-RRR planar mechanism [60]. Such a structure makes the mechanism more compact, so it can fold to almost a planar state.



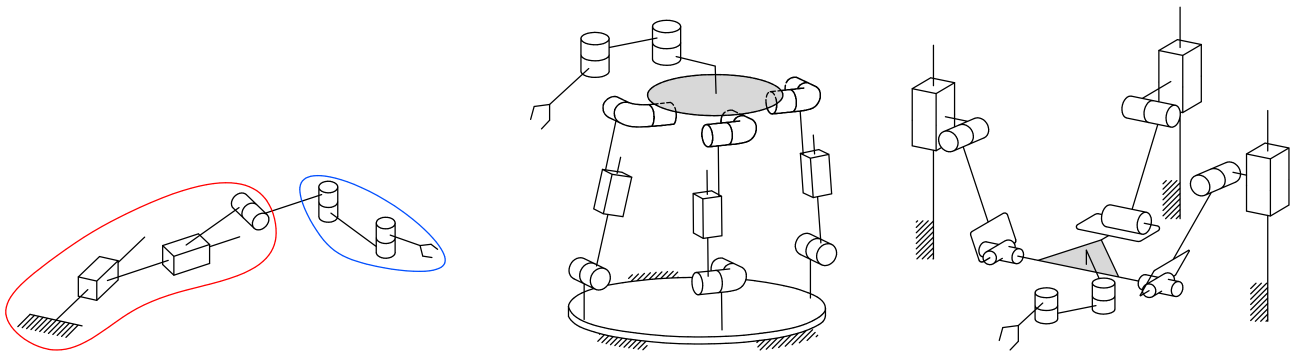
(**a**) (**b**) (**c**)

Figure 3. 4R1P type, RPRRR subtype (# 7 in Table 2): (**a**) primary open kinematic chain; (**b**,**c**) synthesized 5-DOF 3R2T hybrid manipulators with a 3-DOF RPR-equivalent parallel mechanism.

The RPRRR subtype can also give rise to hybrid manipulators with a 3T2R motion pattern. For example, if the RPR subchain corresponds to a planar motion and the axes of the remaining R joints are parallel, we can develop manipulators similar to Figure 2b,c.

3.3. Type 3R2P, Subtype PPRRR

This subsection examines hybrid manipulators that we can synthesize from the PPRRR subtype (# 19 in Table 2). As an example, we consider a PPRRR primary open chain, where the axis of the first R joint is parallel to the plane defined by the axes of the P joints, whereas the axes of the two remaining R joints are parallel to each other (Figure 4a). This open chain allows us to synthesize hybrid manipulators with a 3T2R motion pattern. Figure 4b,c show two such manipulators, where the parallel part is a 3-DOF PPR-equivalent parallel mechanism with a 2T1R motion pattern [13] (ch. 8). The parallel mechanism in Figure 4b has a 2-RPU/UPU structure [61], and the parallel mechanism in Figure 4c has a 2-PRU/PRC structure [62], where C indicates a cylindrical joint. If we compare these mechanisms, we see that the second one is overconstrained. This means it will be more challenging to manufacture and assemble this mechanism, but it will have higher stiffness if it is designed properly. All these properties will be inherent to the synthesized 5-DOF hybrid manipulators too.



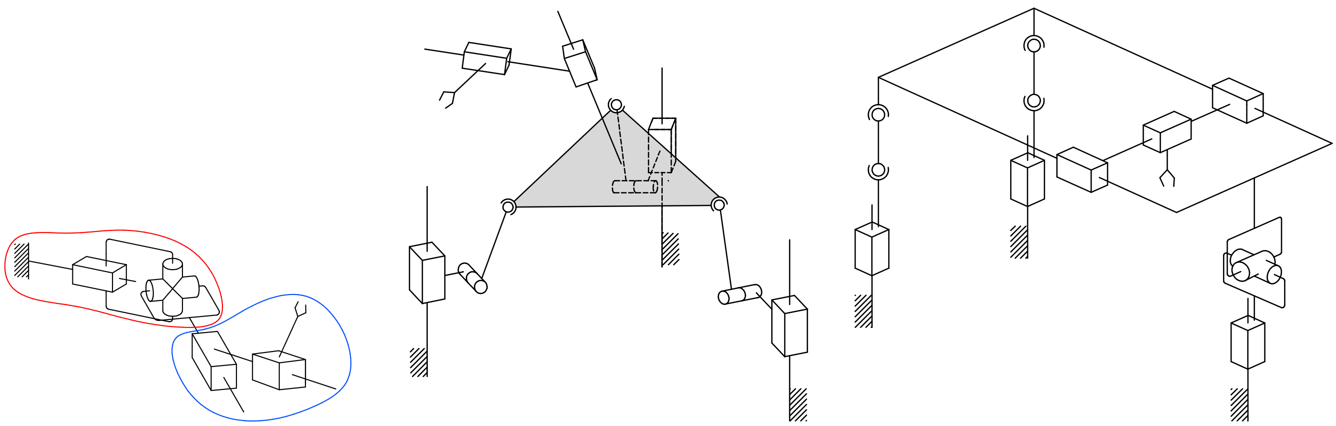
(**a**) (**b**) (**c**)

Figure 4. 3R2P type, PPRRR subtype (# 19 in Table 2): (**a**) primary open kinematic chain; (**b**,**c**) synthesized 5-DOF 3T2R hybrid manipulators with a 3-DOF PPR-equivalent parallel mechanism.

We can also use the PPRRR subtype to develop hybrid manipulators with a 3R2T motion pattern: the axes of the three R joints of the primary open chain should intersect at a common point.

3.4. Type 3P2R, Subtype PRRPP

The final example considers hybrid manipulators that correspond to the PRRPP subtype (# 64 in Table 2). Because 3P2R open kinematic chains include only two R joints, the developed hybrid manipulators can have only a 3T2R motion pattern. Figure 5a shows a primary open kinematic chain, where two R joints are equal to a single U joint. Using this chain, we can develop 3T2R hybrid manipulators with a 3-DOF PU-equivalent parallel part, which has a 2R1T motion pattern [2] (ch. 9). Figure 5b presents one such manipulator with a 3-PRS parallel mechanism [24]. The parallel mechanism has a symmetric design with prismatic joints placed on the base; the axes of all these P joints are parallel. We expect this manipulator to have a high stiffness and an elongated workspace along the direction of the P joints. It is known, however, that such a 3-PRS mechanism has parasitic motions: any rotation of the platform causes linear displacements of its center [63]. In contrast, a 3-DOF 2-SPS/PU parallel mechanism shown in Figure 5c does not have such a parasitic motion [64]. Orientation of the output link of the hybrid manipulator is determined directly by the PU branch of the 3-DOF parallel mechanism.



(**a**) (**b**) (**c**)

Figure 5. 3P2R type, PRRPP subtype (# 64 in Table 2): (**a**) primary open kinematic chain; (**b**,**c**) synthesized 5-DOF 3T2R hybrid manipulators with a 3-DOF PU-equivalent parallel mechanism.

4. Discussion

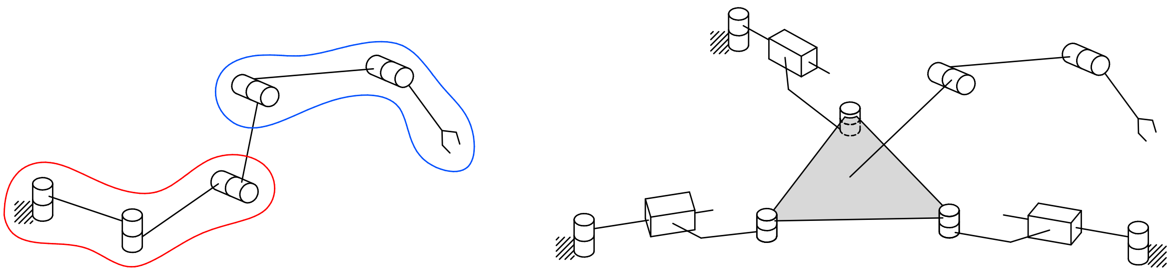
In the previous section, we showed how to apply the proposed method to synthesize 5-DOF hybrid manipulators from open kinematic chains. We focused on subtypes where the parallel part was near the base, because most practical applications mentioned in Section 1 use exactly these manipulators. However, some applications like machining [50] or manipulation [65] can also use hybrid manipulators with a parallel part near the output link. The synthesis method we introduced here is suitable for designing these manipulators too.

Although the suggested approach for type synthesis looks straightforward, it has never been proposed before. Unlike the studies mentioned in Section 1, this approach does not need any mathematical background used in those studies, so it can be more intuitive for a designer. Indeed, open kinematic chains usually include subchains with joints whose axes are parallel, orthogonal, or intersecting. Hence, it can often be possible to select a 3-DOF subchain and substitute it for a parallel mechanism with the same motion pattern.

In Section 3, we considered spherical, planar, RPR-equivalent, PPR-equivalent, and PU-equivalent parallel mechanisms—these are typical 3-DOF parallel mechanisms, which were the subject of numerous studies. There are two more widely used classes of 3-DOF parallel mechanisms we missed here: translational [66] and UP-equivalent [67], which can also be used in the proposed method. As we have seen in the examples, each class of 3-DOF parallel mechanisms can be represented by different mechanical structures with their pros and cons, which will affect the entire 5-DOF hybrid manipulator. Moreover, several possibilities exist to select a subchain within the same open kinematic chain. For example, we can transform the PRRPP open chain in Figure 5a into a hybrid manipulator that includes a PU-equivalent parallel mechanism (subtype PRRPP), a UP-equivalent parallel mechanism (subtype PRRPP), or an (inverted) PPR-equivalent parallel mechanism (subtype PRRPP). A designer should select the substituted subchain and the structure of the 3-DOF parallel mechanism according to the application.

Apart from diverse possibilities of selecting the substituted subchain, there is a non-unique choice of the actuation scheme of the 3-DOF parallel mechanism. A properly chosen actuation scheme should make the manipulator controllable in any general (non-singular) configuration [13] (p. 40). It is also preferable to have the actuated joints located close to the manipulator base and distributed evenly among the kinematic chains. Most of the parallel mechanisms presented in Section 3 meet these conditions: the actuated joints of all these mechanisms, except for Figure 4b, can be placed on the base, and all the mechanisms are controllable. Some 3-DOF parallel mechanisms, however, accept several actuation schemes. For example, the 3-RRR planar mechanism shown in Figure 2b can have actuators in the middle R joints. Such an actuator arrangement affects the singularity loci of this mechanism and its kinematic performance [68], and this actuation scheme can also be suitable for practical applications.

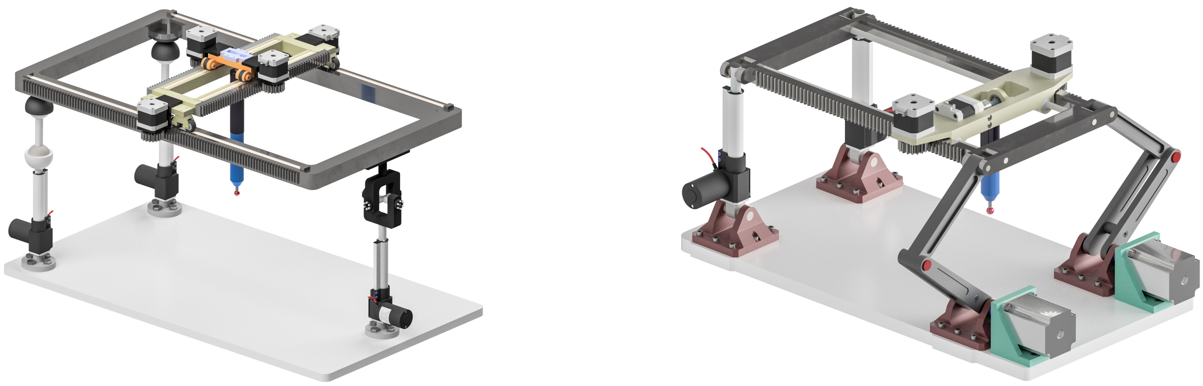
We should note that it is also possible to synthesize a hybrid manipulator whose output link repeats the motion pattern of the primary open kinematic chain, but the parallel part does not replicate the motion pattern of the substituted subchain. To demonstrate this situation, we consider an RRRRR subtype (# 1 in Table 2) and an open kinematic chain presented in Figure 6a. The chain has two sets of R joints with parallel axes, but in contrast to Figure 2c, the selected subchain does not correspond to a planar motion. Furthermore, it does not correspond to any 3-DOF motion type we have examined so far, and it is difficult to find a parallel mechanism that can replace this subchain. We can, however, substitute the subchain with another parallel mechanism. Figure 6b shows a synthesized hybrid manipulator with a 3-RPR planar parallel mechanism [57]. Although the motion pattern of this planar mechanism differs from the RRR subchain in Figure 6a, the output link of the hybrid manipulator has the same 3T2R motion pattern as the primary open kinematic chain. This mismatch between the motion patterns allows us to generate many novel hybrid manipulators that will keep motion patterns of their primary open chains. Developing this approach, however, will require techniques of screw theory or group theory, which are beyond the current paper.



(a) (b)

Figure 6. 5R type, RRRRR subtype (# 1 in Table 2): (**a**) primary open kinematic chain; (**b**) synthesized 5-DOF 3T2R hybrid manipulator, whose parallel part has a motion pattern different from the replaced subchain.

Following the proposed type synthesis method, we have designed and analyzed several novel hybrid manipulators with five DOFs and developed their virtual prototypes. Figure 7a shows one such manipulator, whose structural diagram corresponds to Figure 5c. We examined the kinematics, workspace, and singularities of this manipulator in works [35,69]. A relatively large and elongated workspace of the manipulator makes it suitable for processing or inspecting long-shaped objects with complex geometry. Figure 7b illustrates another 5-DOF 3T2R hybrid manipulator, which we developed and studied in papers [70,71]. The manipulator structure corresponds to either an RPRPR or an RRRPR subtype from Table 2, with the parallel part being a redundantly actuated 3-DOF 2-RPR/2-RRR planar parallel mechanism. A redundant chain makes the mechanism design symmetrical and suitable for applications like those listed above. Using different kinematic chains is a trade-off: a 4-RRR mechanism is less rigid than a 4-RPR one, while the latter is prone to architecture singularities [72]. In addition, the redundant actuation affects the manipulator performance [73], and it can also be used in other hybrid manipulators we presented in Section 3.



(a) (b)

Figure 7. Virtual prototypes of two 5-DOF 3T2R hybrid manipulators developed by the proposed method: (**a**) manipulator with a 3-DOF PU-equivalent parallel mechanism and a PP serial chain; (**b**) manipulator with a redundantly actuated 3-DOF planar parallel mechanism and a PR serial chain.

5. Conclusions

The article has proposed a novel approach for synthesizing 5-DOF hybrid manipulators from open kinematic chains. In contrast to other synthesis methods that rely on screw theory, group theory, POC sets, or G_F sets, the suggested approach does not need any extensive mathematical formulations, and it is more intuitive for a designer. The method idea is in taking a 5-DOF open kinematic chain, selecting a 3-DOF subchain, and replacing it with a 3-DOF parallel mechanism that has the motion pattern of this subchain. The choice of the substituted subchain and the parallel mechanism determines the performance of the synthesized hybrid manipulator, and it should be carried out by a designer according to the application.

In this paper, we have considered open kinematic chains with revolute (R) and prismatic (P) joints and found 26 possible structures of these chains, which we have classified into four types (5R, 4R1P, 3R2P, and 3P2R). For each type, there are three possibilities to select a subchain, so we have found 78 subtypes of hybrid manipulators. We have studied mechanisms with 3T2R and 3R2T motion patterns and presented several hybrid manipulators for each subtype. The synthesized hybrid manipulators include spherical, planar, RPR-equivalent, PPR-equivalent, and PU-equivalent parallel mechanisms.

In the future, we will aim to enhance the proposed synthesis method and consider the cases when the substituted subchain and the parallel mechanism have different motion patterns, while the output link of the designed hybrid manipulator keeps the motion pattern of its primary open kinematic chain. A further development of the current work includes its extension for manipulators with another number of DOFs. The proposed approach is certainly not limited to 5-DOF open chains and 3-DOF parallel mechanisms, so we will look at other combinations in the future. Another extension of the performed research is a comprehensive analysis of synthesized manipulators, such as the ones we presented here. The drive location, actuation redundancy, and joint and link design affect the manipulator workspace, singular configurations, and stiffness, so our forthcoming studies will aim to address these issues too.

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