

Article

Bilevel Trajectory Optimization for Vacuum-Based Pick-and-Place Operations: A Numerical Study

Georg Steinert *, Clemens Troll and Jens-Peter Majschak

Institute of Natural Materials Technology, Faculty of Mechanical Science and Engineering, TUD Dresden University of Technology, Bergstraße 120, 01069 Dresden, Germany; verarbeitungsmaschinen@tu-dresden.de (J.-P.M.)

* Correspondence: georg.steinert@tu-dresden.de; Tel.: +49-351-463-34348

Abstract

Motion optimization has a significant influence on performance and robustness of modern robotic handling systems. In this study, a pick-and-place operation, as can be found in many processing machines, serves as a representative use case to develop a novel method for motion optimization. Based on optimal control theory, the introduced method uses bilevel optimization simultaneously addressing process stability, favorable dynamic behavior and practical applicability. A process model for gripper load estimation established in the literature serves both as a basis for optimization and for evaluating the solution found. To establish a benchmark, a spline-based trajectory is generated. As this work proposes a theoretical approach, simulations based on the retrieved model serve as an evaluation basis of the resulting trajectories. The results show that a significant reduction in gripper load by approximately 60% was achieved compared to the reference motions. Eventually, requirements and limitations for application of the new method are discussed.

Keywords: pick-and-place; bilevel optimization; optimal control; motion optimization; vacuum gripper

1. Introduction

1.1. Robotic Handling Systems

Pick-and-place processes have become an indispensable part of modern production lines. They offer fast, safe, flexible and increasingly cost-effective solutions for machine loading and unloading, transport, handling and insertion processes. The accompanying increase in the availability of industrial robots is enhancing this trend toward the automation of manufacturing and, in particular, handling processes. An advantage of modern industrial robots is their flexibility in terms of motion path and trajectory, offering an easy way to improve process behavior through motion optimization.

1.2. Related Work

The field of robotic handling has attracted considerable research attention, with studies investigating various aspects of the handling process. While in [1] the mechanical structure of the robot is focused, refs. [2–4] examine the potential of alternative gripping methods and mechanisms. A review of gripping methods in robotics can be found in [5,6].

Modeling vacuum grippers is the subject of many publications, depending on the systems required to simulate and the objective of the simulation. In [7] or [8], the viscoe-

Academic Editor: Andrius Dzedzickis

Received: 19 May 2026

Revised: 26 June 2026

Accepted: 29 June 2026

Published: 30 June 2026

Copyright: © 2026 by the author. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

lastic behavior of a vacuum gripper is simulated using analytical spring-damper elements. VAN DER KRUK ET AL. extend this approach to include elastic properties of the object being grasped and the compliance of the robot and control system as spring-damper elements, resulting in a complex, coupled mechanical equivalent model. MAHLER ET AL. used a quasi-static spring model for simulating seal formation on objects with irregular geometries [9]. A segmentation of the perimeter of the suction cup allows to retrieve predictions for leakage. PHAM AND PHAM chose a different approach, establishing kinetostatic force balances at discrete points along the suction cup's circumference. They place particular emphasis on accounting for lateral loads using a linearized Colom friction model [10].

A considerable amount of research has been dedicated to the application of machine-learning techniques in pick-and-place processes. In [11] for example, the energy consumption of pick-and-place processes is examined and machine-learning-based methods for improvement are suggested. A machine-learning approach to generate trajectories for high accuracy and vibration reduction is introduced in [12]. Fundamentals of motion synthesis for pick-and-place tasks and robotics in general can be found in [13–15]. A review of optimization methods for robotic motions can be found in [16]. Optimal control theory offers great potential in this area. In [17], the application of optimal motion profiles for intermittent fluid transport offers significant improvement in achievable machine speed. BRÜCKMANN ET AL. show a novel approach to use optimal control to drastically reduce frame vibrations [18]. MASSARO ET AL. use optimal control to find minimum time and minimum distance solutions for a two-link planar manipulator performing a point-to-point motion-respecting dynamic and joint constraints as well as obstacles [19]. DOS SANTOS ET AL. extend this approach to a multi-link manipulator in 3D workspace considering moving obstacles [20]. But there are also other methods suitable for trajectory optimization. In [21] for example, a particles swarm optimization is used for various objectives such as minimal cycle time or minimal specific energy consumption while in [22] the use of established input shaping methods for residual vibration reduction is presented. An optimization with regard to the gripper load based on a model of a multi-suction-cup gripper can be found in [23], whereas the underlying optimization method is described in [24]. However, the authors focus on optimization of the temporal domain of the trajectory and assume the path to be predetermined. SAVKIV ET AL. introduce an approach for minimizing energy consumption of a pneumatic gripper system by varying the grippers orientation along a specified path [25]. Another significant contribution in the field of model-based motion optimization for pick-and-place processes using vacuum grippers is provided by AVIGAL ET AL. [26]. The authors introduce an experimental trained machine-learning model, based on a simple analytical model for the vacuum gripper. This model as well as the actuators dynamics are used as boundary conditions for a sequential optimization problem pursuing both minimal time and minimal sum-of-squared acceleration of the resulting trajectory.

1.3. Aim of the Paper

This paper introduces a new approach for motion optimization for pick-and-place processes based on optimal control theory. Particular attention is paid to the load on the gripper during the process. The reason for this is that contact losses caused by excessive gripper load are a common reason for limiting machine output rates. This study focuses on operations in processing machines, where little variations in terms of boundary conditions, handling object and environment occur. Section 2 will exploit all necessary restrictions, models and basic assumptions. Those strong restrictions allow the use of computationally intensive numerical methods which are also described in Chapter 2. The resulting trajectories are subject to Section 3. Sections 4 and 5 evaluate the results and show the expected benefits and limitations.

2. Materials and Methods

2.1. Use Case

This article examines an academic use case based on a typical handling task in the food or packaging industry. Figure 1 shows the relevant parts of the handling system. An industrial robot, a 3-axis-delta robot (1) is used to perform the handling task. The handling object (2) is provided by an input conveyor belt (3). The robot uses a vacuum suction gripper (4) to grasp the object and move it to the output conveyor belt moving in the opposite direction (5) where the object is released. As this use case is of a purely academic nature, some basic assumptions can be made to simplify the intended solution. Firstly, the robot and control system provide an ideal motion and do not restrict the solution space due to actuator limitations. Secondly, the orientation of the objects does not change during the process and thirdly, the center of mass of the handling objects is directly below the grasp point. For application on a practical use case, these assumptions must be reviewed. Where necessary, additional constraints resulting from the robot, gripper and handling object must be considered.

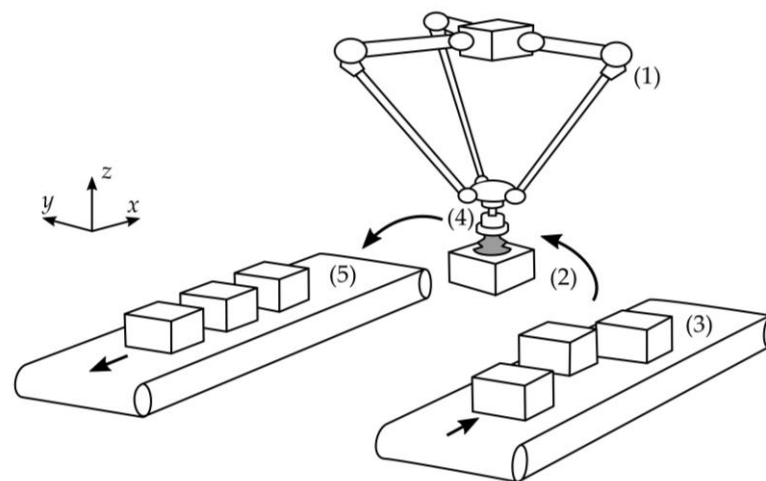


Figure 1. Sketch of the examined use case. An object is supplied by a conveyor belt. A robot equipped with a suction gripper picks the object and places it on a second conveyor belt running in the opposite direction.

In general, handling processes are cyclic processes in processing machines. This means that, in order to achieve high machine output rates, after releasing one object, the robot moves immediately towards another object to be picked. Hence, the process could be divided into four phases [27]:

- Reach phase (moving the gripper towards the object) Δt_{reach} ;
- Picking phase (attaching the object to the gripper) Δt_{pick} ;
- Transportation phase (moving gripper and the attached object towards the target position) Δt_{transp} ;
- Releasing phase (detaching the object from the gripper) Δt_{place} ;

The output rate n of a pick-and-place machine can therefore be calculated according to Equation (1):

$$n = \frac{1 \text{ product}}{\Delta t_{\text{reach}} + \Delta t_{\text{pick}} + \Delta t_{\text{transp}} + \Delta t_{\text{place}}} \quad (1)$$

The required duration for picking and placing can either be calculated by the volume to be evacuated and flooded and the respective flow rate or experimentally determined.

The ratio of Δt_{reach} and Δt_{transp} can be chosen depending on the gripper and robot used. In this use case, a ratio of:

$$b = \frac{\Delta t_{pick} + \Delta t_{transp} + \Delta t_{place}}{\Delta t_{reach} + \Delta t_{pick} + \Delta t_{transp} + \Delta t_{place}} = 0.6 \quad (2)$$

is predetermined. To define the handling task, further specifications must be made. Figure 2 shows the respective parameters which are quantified in Table 1.

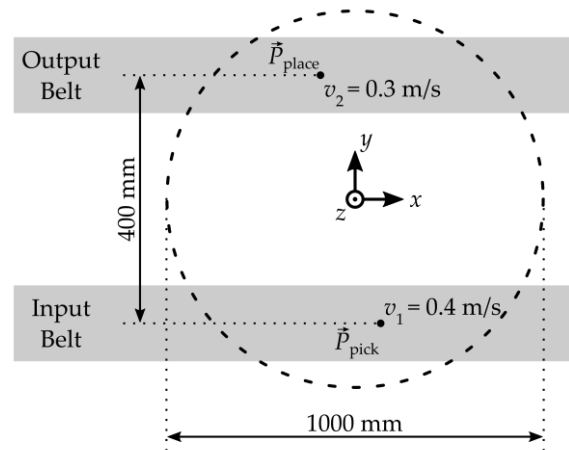


Figure 2. Top view of the use case. The pick position $\vec{P}_{pick}$ is located on the input belt, and the place position $\vec{P}_{place}$ is on the output belt. Both positions are located within the robot's workspace, which is simplified as a cylinder of constant diameter (dashed line) within the actual workspace. Furthermore, the conveyor velocities of both belts v_1 and v_2 are specified.

Table 1. Summary of parameters specifying the examined use case.

Parameter	Symbol	Value	Parameter	Symbol	Value
Picking position	$\vec{P}_{pick}$	[0.05; -0.2; 0] m	Belt velocity input	v_1	0.4 m/s
Placing position	$\vec{P}_{place}$	[-0.05; 0.2; 0.1] m	Belt velocity output	v_2	0.3 m/s
Time to pick	Δt_{pick}	0.03 s	Output rate	n	120 products per min
Time to place	Δt_{place}	0.03 s			

2.2. Motion Plan

The motion plan contains the kinematic restrictions to the desired trajectory [28]. The restrictions are either in the form of point restrictions or section restrictions on displacement, velocity and/or acceleration. Every motion that complies with the motion plan is considered to fulfill the motion task. For the presented use case, a motion plan can be derived, as shown in Figure 3.

The values of x_{pick} , y_{pick} , z_{pick} are determined by the picking position $\vec{P}_{pick}$ while x_{place} , y_{place} , z_{place} are determined by the picking position $\vec{P}_{place}$ respectively. The time values t_0 to t_5 are determined according to:

$$\begin{aligned} t_0 &= 0, & t_3 &= t_4 - \Delta t_{sync2}, \\ t_1 &= \Delta t_{pick}, & t_4 &= t_5 - \Delta t_{place}, \\ t_2 &= \Delta t_{pick} + \Delta t_{sync1}, & t_5 &= 60/n \cdot b, \end{aligned} \quad (3)$$

where the values of the synchronization time Δt_{sync1} and Δt_{sync2} are yet to be determined. The slope of the area restrictions in the x-function are defined by the belt velocity v_1 and v_2 , forcing a synchronous motion with the respective belt. While the y-function is set to be constant during Δt_{sync1} and Δt_{sync2} , the z-function specifies a translation of Δz_1 after the gripper evacuation and Δz_2 before gripper flooding. This results in a smooth lift off and takedown with no relative motion between belt and object in x- and y-direction. Otherwise, an object damage can result from the object grinding on the belt.

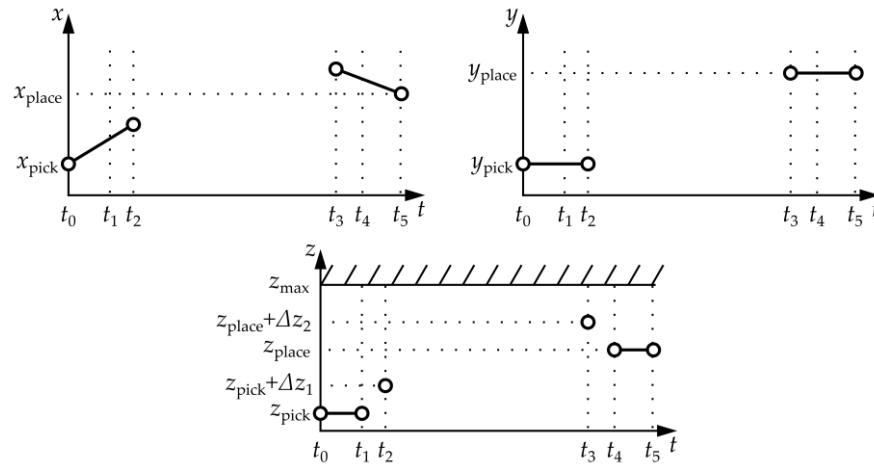


Figure 3. Motion plan specifying the kinematic constraints. Point constraints are indicated by circles, regional restrictions by lines. Unilateral boundary constraints, such as the maximum permissible z-value in this case, are represented by additional diagonal lines.

2.3. State of the Art Trajectory Synthesis

The aim of this chapter is to generate a spline-based trajectory, which will later be used as a reference for the optimal solution. In industrial applications, robotic motion is defined by a geometric and a temporal domain specified by a path and trajectory. These are determined separately [13,14]. Spline-based trajectory synthesis uses a finite number of control points in space that are connected by splines. There are different approaches to generate the mentioned control points. One way to consider multiple constraints on the path $\vec{p}(s)$ is the use of multiple connected segments, described by path primitives $\vec{p}_i(s)$ with s being the reference parameter such as the arc length and S_i referring to the arc length of the respective segment:

$$\vec{p}(s) = \begin{cases} \vec{p}_1(s) & s \in [0, S_1] \\ \vec{p}_2(s) & s \in [S_1, S_2] \\ \vdots & \\ \vec{p}_N(s) & s \in [S_{N-1}, S_N] \end{cases} \quad (4)$$

The control points are then placed evenly along the resulting path. For path primitives, elementary geometric paths such as linear or circular segments can be used [13]. These can be described through basic mathematical representation, depending on starting point P_0 and ending point P_1 and path length l . A linear primitive can be defined by:

$$\vec{p}_{\text{lin}}(s) = \vec{P}_0 + \frac{s}{l}(\vec{P}_1 - \vec{P}_0) \quad s \in [0, l]. \quad (5)$$

For more advanced path primitives, such as circular arcs, additional parameters, such as the plane normal, defined by vectors $\vec{e}_1$ and $\vec{e}_2$, radius R , center point C or the opening angle θ must be specified:

$$\vec{p}_{\text{circ}}(s) = \vec{C} + r \left(\cos\left(\frac{s}{R}\right) \vec{e}_1 + \sin\left(\frac{s}{R}\right) \vec{e}_2 \right) \quad s \in [0, l]. \quad (6)$$

More details on the calculation can be found in [13]. The exact combination of segments is at the user's discretion and allows a wide range of solutions, as long as the motion plan is fulfilled. An example can be found in [29]. Figure 4 shows two different paths, consisting of linear and circular segments, that are solutions to the path planning task.

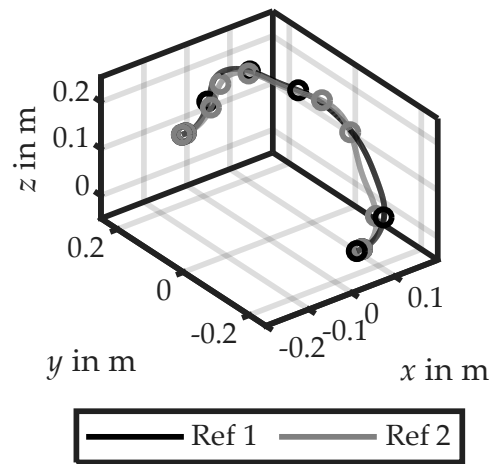


Figure 4. Two paths, consisting of manually assembled circular and linear path primitives. The borders of the segments are highlighted through circular markers. Both paths are consistent with the motion plan and are therefore suitable for the examined motion task.

Both paths shown in Figure 4, visibly smooth, show non-continuous curvature change at the segment boundaries. This will lead to acceleration jumps for arbitrary trajectories. To prevent this, the path is sampled and the resulting control points are smoothly connected using splines. In this case, quintic B-splines [30] are used, guaranteeing C^3 -continuity.

As a second step, a motion profile $s(t)$ must be found to specify the time-dependent behavior and the time derivatives of the resulting motion according to:

$$\begin{aligned}\vec{p}(t) &= \vec{p}(s(t)), \\ \dot{\vec{p}}(t) &= \vec{p}'(s(t)) \cdot \dot{s}(t), \\ \ddot{\vec{p}}(t) &= \vec{p}''(s(t)) \cdot \dot{s}(t) + \vec{p}'(s(t)) \cdot \ddot{s}(t).\end{aligned}\tag{7}$$

The required motion profile needs to take into account the respective path length and the speed boundary conditions defined by the belt speed. For this use case a “Double S Profile” was used. Explanations on the calculation and further detail as well as alternative suitable motion profiles can be found in [31]. Figure 5 shows the resulting functions of displacement s_i , velocity v_i , and acceleration a_i for the two presented paths in x -, y - and z -direction.

The generated trajectories are meant to reflect common industrial practice and are expected to result in relatively moderate dynamic process behavior. More advanced methods were introduced in Section 1, and generally involve the use of complex mathematical methods.

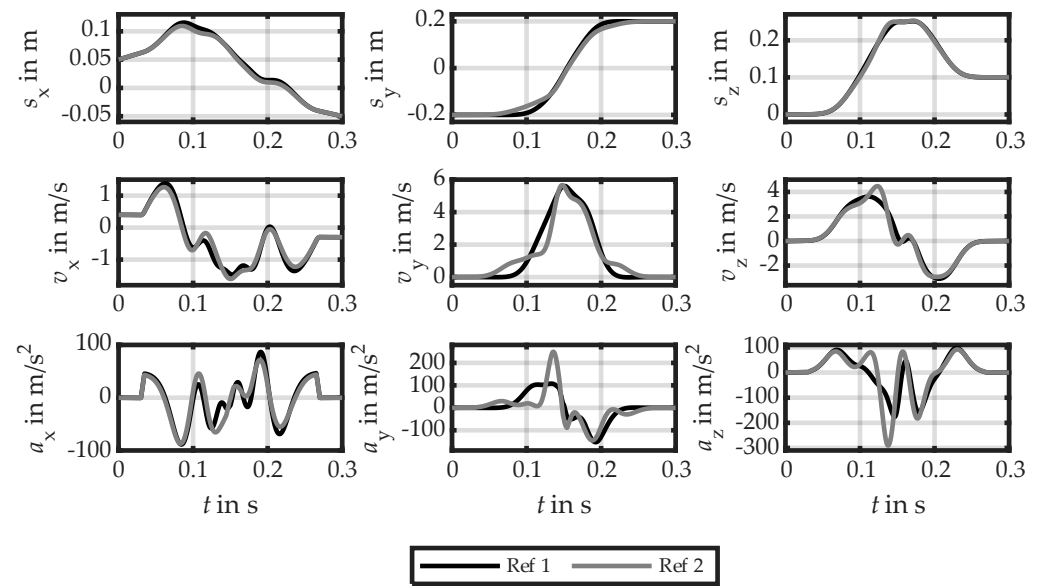


Figure 5. Time-dependent functions of displacement, velocity and acceleration for both spline-based generated reference motions. The trajectories are generated using a standard “Double S” motion profile.

2.4. Evaluation of a Motion in Regard to the Load on the Gripper

There are different kinds of failures that can occur during handling tasks, namely, the loss of contact during handling or slipping of the object relative to the gripper resulting in a loss of placement accuracy [23]. A disproportion of gripping force and object stability can furthermore lead to undesired damage of the object. Each of the mentioned failures can be attributed to excessive or insufficient retention force. There are many different approaches for simulating a gripper–object pairing (see Section 1). This work, however, focusses on the numerical optimization approach, and therefore a simple kinetostatic model as introduced in [32] or [26] is assumed to be sufficient. This model will later be used as a basis for optimization. The basic assumption of this model is, as Equation (8) states, that, for grasp stability, the retention force of the gripper F_{ret} must exceed the resulting force F_{res} at all times:

$$F_{\text{ret}} \geq F_{\text{res}}. \quad (8)$$

Figure 6 shows a model for a motion in x - and z -direction derived from [32]. This model must be expanded to represent a three-dimensional use case.

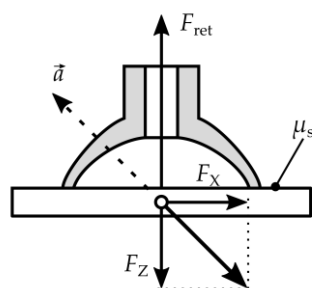


Figure 6. Standard model representing the state of a moving vacuum gripper derived from [32]. The movement of the gripper and object causes inertial forces which must be absorbed by the gripper through its retention force.

A vacuum gripper is basically able to exclusively generate a vertical force determined by the pressure difference between inside and outside the gripper Δp and the effective area of retention A_{eff} according to Equation (9):

$$F_{\text{ret}} = \Delta p \cdot A_{\text{eff}}. \quad (9)$$

The load on the gripper can be divided into static forces and kinetostatic forces, that sum up to the resulting force. Horizontal forces must be taken into account via the static friction coefficient μ_s between gripper and the object resulting from drawing the object towards the gripper. Deviating from [32], the resulting force considers the amount of the horizontal force:

$$F_{\text{res}} = F_g + F_z + \frac{1}{\mu_s} |F_{XY}|. \quad (10)$$

Otherwise, a, according to the definition of the reference frame, negative horizontal force would lead to a decreasing resulting force. In contrast, the force in vertical direction F_z does not require any special consideration. A downward acceleration causes the object to press against the gripper due to its inertia. This compressive load is not a critical factor for grasp stability. It can even allow us to compensate for higher horizontal forces. The static and kinetostatic forces can be calculated according to (11) to (13) respectively resulting from object mass m , constant of gravitation g , and the acceleration a_i affecting the gripper–object pair as a result of the motion:

$$F_g = m \cdot g, \quad (11)$$

$$F_z = m \cdot a_z, \quad (12)$$

$$F_{XY} = m \cdot \sqrt{a_x^2 + a_y^2}. \quad (13)$$

Using Equations (10)–(13), an explicit correlation between the acceleration resulting from the motion of the object and the resulting force at the gripper can be made:

$$F_{\text{res}} = m \left(g + \frac{1}{\mu_s} \sqrt{a_x^2 + a_y^2} + a_z \right). \quad (14)$$

This allows an approximation of expected load during the process. This basic model does not take into account leakage, deformation of the gripper and/or object or other dynamic effects. Therefore, large security coefficients are used in practical applications. Since this work focusses on comparing different trajectories, it is sufficient to assume the same simplified model approach. Since m and g are constants, a simplified objective function Q can be defined, that has the same local extrema as F_{res} :

$$Q = \frac{1}{\mu_s} \sqrt{a_x^2 + a_y^2} + a_z. \quad (15)$$

This objective function serves as a basis for motion optimization and, due to its independence from object mass, allows for a more general perspective. In the study's use case, a static friction coefficient of $\mu_s = 0.3$ is assumed, based on typical empirical values.

2.5. Bilevel Motion Optimization

To generate an optimal solution for the specified robotic task, a strategy for the motion synthesis must be developed with focus on the desired aim of the process task. Therefore, at first the properties of the problem are analyzed in terms of motion optimization:

- (1) The derived objective function describing the gripper load is a nonlinear function dependent on the acceleration profiles of the three motion components $x(t)$, $y(t)$, and $z(t)$.
- (2) The motion plans for the three motion components are coupled by the two introduced time steps Δt_{sync1} and Δt_{sync2} .
- (3) Assuming the unknown time steps Δt_{sync1} and Δt_{sync2} are fixed, then the three motion components can be synthesized and decoupled.

- (4) For each motion component only kinematic constraints are stated, so the dependencies are linear.
- (5) No dynamic process model is considered here, but this assumption only holds for robotic trajectories that do not significantly stimulate the dynamic behavior of the vacuum gripper–product–interaction.

One obvious approach to generate an optimal trajectory regarding minimizing the maximal vacuum gripper force could be the solution to an optimal control problem, in which all three motion components and their derivatives are state variables. By adding a slack variable s , which is also the objective function to be minimized, and simultaneously adding the constraint $\max(F_{res}) \leq s$ to the problem, one could calculate a corresponding optimal solution trajectory. However, this approach not only leads to a nonlinear optimal control problem in which a global extremum cannot be found with certainty but also has the disadvantage that no C^2 continuous motions can result from the structure of the optimization problem. Therefore, the following approach considers both the coupling of the three motion directions of the trajectory over the unknown time intervals Δt_{sync1} and Δt_{sync2} as well as the continuity of the resulting solutions. The applied approach consists of a bilevel optimization strategy, that is illustrated in Figure 7. The figure displays the proposed strategy for computing feasible and smooth robot trajectories. The method separates the problem into two nested optimization layers with distinct roles.

At the first level, a static optimization problem is solved in which the motion is parameterized by the two discrete synchronization time intervals. This upper-level problem operates on a low-dimensional parameter vector $\underline{p} = [\Delta t_{sync1}, \Delta t_{sync2}]$ and aims at minimizing the global objective function represented by the maximum vacuum gripper force.

At the second level, the parameters determined by the first level are used to formulate three independent optimal control problems, one for each trajectory dimension. For fixed timing parameters from the first level, time-continuous trajectories are computed by solving these optimal control problems. This lower-level formulation explicitly incorporates kinematic constraints and smoothness requirements.

The key advantage of this bilevel structure is the separation of objective minimization and admissible trajectory generation. While the upper level drives the minimization of the global objective function, the second-level optimal control problems restrict the feasible set to trajectories that satisfy predefined regularity and structural properties. In particular, by construction of the optimal control formulation, only C^2 -continuous trajectories are admitted, ensuring continuous position, velocity, and acceleration profiles. Consequently, the lower level does not merely evaluate candidate solutions but actively shapes the solution space of the upper-level optimization. This embedded optimal control layer guarantees kinematic feasibility and desired smoothness a priori, thereby preventing the upper-level optimizer from exploring physically infeasible or non-smooth trajectories. The gradient-based update of $\underline{p}$ iteratively refines the temporal parameters until a suitable stopping criterion is met, resulting in an optimal solution $\underline{p}_{opt}$ that simultaneously minimizes the objective function and satisfies the imposed smoothness and dynamic constraints.

The first-level optimization is implemented with a simple active-set-strategy since only two parameters are to be iterated, while for each parameter a lower and an upper bound are considered. These box constraints are necessary to guarantee kinematic useful motion plans for the second-level optimization.

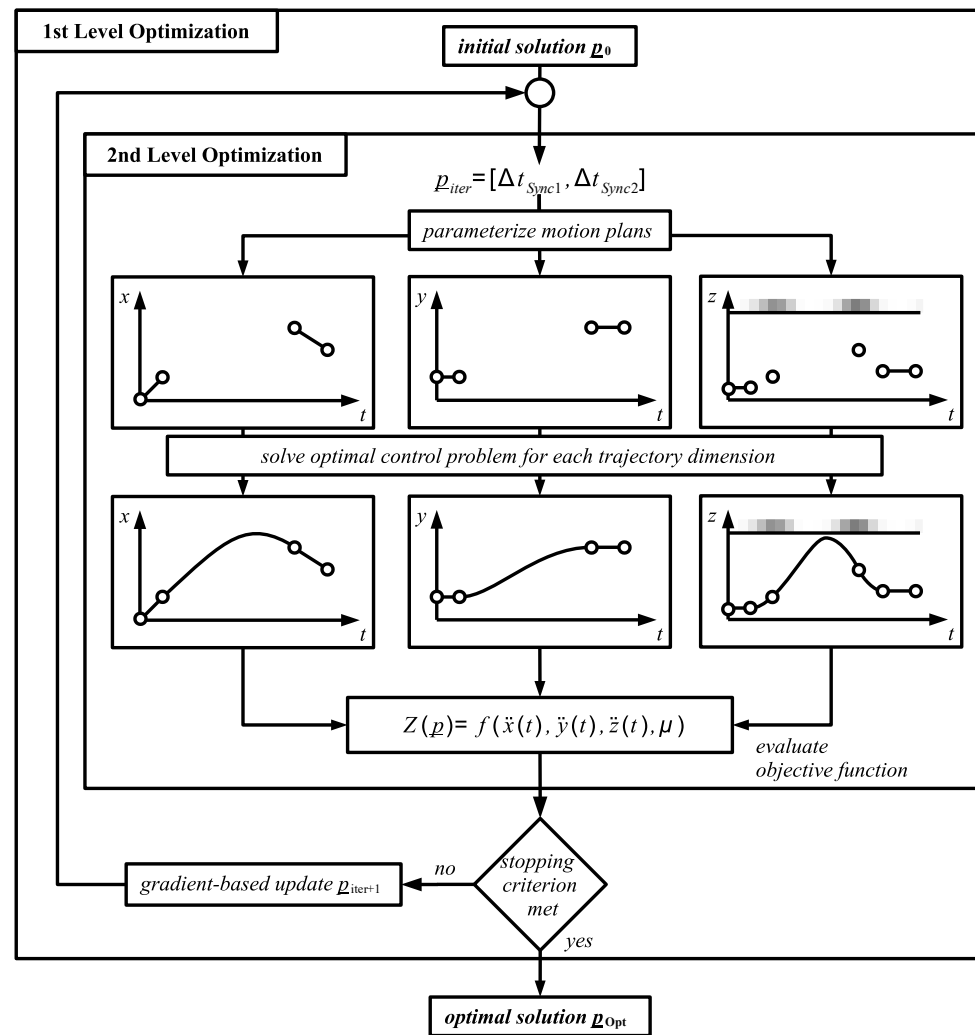


Figure 7. Bilevel optimization approach for optimizing the vacuum gripper trajectory. On the first level a static optimization problem, containing two discrete time intervals, is solved. Using these two time intervals three independent optimal control problems can be solved on the second level.

The second-level optimization consists of three independent optimal control optimizations for each dimension of the trajectory. In the following, the approach for formulating and solving the optimal control problems is explicated with no loss of generality for only one dimension. Since the perspective of the underlying problem is only kinematic, it is sufficient to consider the fundamental kinematic equations for describing the system's behavior. To generate C^2 continuous motion specifications, each trajectory dimension is described as a triple integrator system with $x_1(t) = s(t)$, $x_2(t) = v(t)$, $x_3 = a(t)$ and $u(t) = j(t)$, where $\underline{x}(t): [0, T] \rightarrow \mathbb{R}^3$ represents the state vector $\underline{x}(t) = [x_1(t), x_2(t), x_3(t)]^T$ and $u(t): [0, T] \rightarrow \mathbb{R}^1$ the control variable:

$$\dot{\underline{x}}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \cdot \underline{x}(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot u(t). \quad (16)$$

Additionally, lower $\underline{c}_l(t): [0, T] \rightarrow \mathbb{R}^3$ and upper $\underline{c}_u(t): [0, T] \rightarrow \mathbb{R}^3$ limits can be defined for each state variable along the time horizon in order to adhere to the specified motion plan:

$$\underline{c}_l(t) \leq \underline{x}(t) \leq \underline{c}_u(t). \quad (17)$$

The aim of the motion optimization is to determine optimal solutions $\underline{x}^*(t)$ and $u^*(t)$, that satisfy the system (16), the boundary conditions (17) and that minimize the following cost functional:

$$J = \frac{1}{2} \int_0^T u^2(t) dt \rightarrow \min. \quad (18)$$

The application of this cost functional leads to smooth C^2 continuous trajectories since discontinuities can only occur to the jerk as the control variable. Since no analytical solution to this problem can be stated in general, a numerical approach for approximating the solution of this optimal control problem is applied. Basically, numerical approaches for solving optimal control problems can be subdivided into direct and indirect approaches. Whereby indirect approaches evaluate the necessary conditions of the problem, that leads to the necessity of solving a boundary value problem, direct approaches transform the dynamic optimization problem into a static optimization problem by discretizing the time horizon. With increasing amount of discrete time steps, the solution converges to the unknown solution of the original optimal control problem.

For the considered formulation—the dynamics correspond to a triple integrator system—the analytical solution is known to consist of piecewise polynomial trajectories of degree five (e.g., [33]). These solutions may exhibit discontinuities in the third derivative. A direct collocation method using linear trial functions is well suited to represent such non-smooth behavior and is therefore chosen for solving the optimal control problem for each of the three trajectories' dimension.

Accordingly, the time horizon is discretized into K , increasing but not necessarily equally distributing points: $0 < t^1 < t^2 < \dots < t^K = T$. The discrete time points must be selected in such a way that they correspond to the specific requirements of the upper and lower conditions of the original optimal control problem. To accommodate the characteristics of the continuous optimization problem, which allows discontinuous solutions for the control variable, sectionally constant trial functions $\tilde{u}(t)$ of the control variable are applied to approximate the numerical solution, see Figure 8:

$$\tilde{u}(t) = u^k, \quad t^k \leq t \leq t^{k+1}, \quad k = 1 \dots K-1. \quad (19)$$

In addition, the state variables are approximated using sectionally linear trial functions for each state $\tilde{\underline{x}}(t) = [\tilde{x}_1(t), \tilde{x}_2(t), \tilde{x}_3(t)]^T$:

$$\tilde{\underline{x}}(t) = \underline{\tilde{x}}^k + \frac{\tilde{x}^{k+1} - \tilde{x}^k}{t^{k+1} - t^k} \cdot (t - t^k), \quad t^k \leq t \leq t^{k+1}, \quad k = 1 \dots K-1. \quad (20)$$

From the assumed boundary points of the trial functions, the unknown parameter vector $\underline{p} \in \mathbb{R}^{4 \cdot K-1 \times 1}$ of the static substitute optimization problem can be derived:

$$\underline{p} = [\tilde{x}_1^1, \tilde{x}_2^1, \tilde{x}_3^1, \tilde{u}^1, \dots, \tilde{x}_1^K, \tilde{x}_2^K, \tilde{x}_3^K]^T. \quad (21)$$

Thereby, the triple integrator system equation, formulated in Equation (16), can be discretized by applying the trapezoidal rule evaluated at the so-called collocation points in the middle of each interval $t^{k+1/2} = \frac{t^k + t^{k+1}}{2}$. This leads to three linear equations for each time interval:

$$\begin{aligned} 0 &= \tilde{x}_1^k - \tilde{x}_1^{k+1} + \frac{t^{k+1} - t^k}{2} \cdot (\tilde{x}_2^k - \tilde{x}_2^{k+1}), \\ 0 &= \tilde{x}_2^k - \tilde{x}_2^{k+1} + \frac{t^{k+1} - t^k}{2} \cdot (\tilde{x}_3^k - \tilde{x}_3^{k+1}), \quad k = 1 \dots K-1. \\ 0 &= \tilde{x}_3^k - \tilde{x}_3^{k+1} + \frac{t^{k+1} - t^k}{2} \cdot u^k, \end{aligned} \quad (22)$$

These equations can be assembled to the linear equality constraints $A_{eq} \cdot \underline{p} = \underline{0}$, with $A_{eq} \in \mathbb{R}^{3 \cdot (K-1) \times 4 \cdot K-1}$, of the static substitute optimization problem. The cost functional, formulated in Equation (18), can be approximated by evaluating the definite integral assuming the sectionally constant trial functions, which results in a quadratic expression of the following form:

$$J \approx \tilde{J} = \frac{1}{2} \underline{p}^T H \underline{p}. \quad (23)$$

Finally, the lower and upper limits for the state for the control variables in Equation (17) can be easily transformed to the following linear inequality constraints

$$A_{ineq} \cdot \underline{p} \leq \underline{0} \quad (24)$$

with $A_{ineq} \in \mathbb{R}^{4 \cdot K-1 \times 4 \cdot K-1}$.

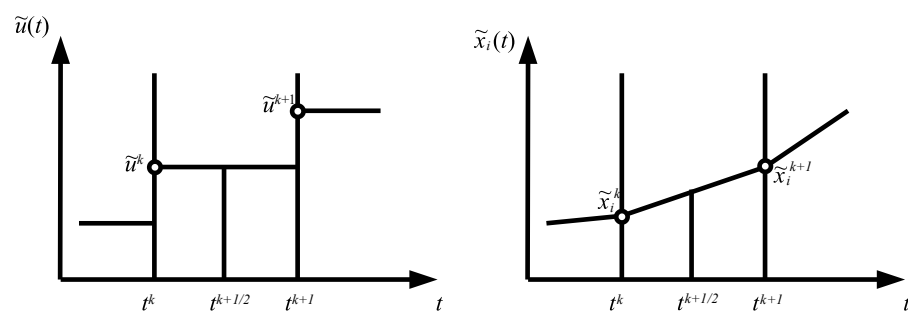


Figure 8. Sectionally constant trial functions for approximating the state variable and sectionally linear trial functions for approximating the state variables.

Hence, the continuous optimal control problem is fully transformed into a static substitute optimization problem, which above is a convex quadratic problem with linear constraints and therefore very efficient to solve numerically. Hence, each local optimal solution found is simultaneously a global optimal solution. But this only holds for the static substitute problem. In [34] it was shown that the numerical solution converges to the theoretical solution of the continuous optimal control problem. Therefore, a convergence analysis must be implemented, to choose a sufficient discretization.

The bilevel optimization problem can thus be divided into a high-dimensional convex quadratic program with linear constraints, which can be solved very efficiently, and a two-dimensional, non-convex, non-differentiable upper-level optimization problem. As an alternative to the chosen direct approach, other solution methods for bilevel optimization problems may be used (see [35]). For example, reformulating the problem as a mathematical program with equilibrium constraints (MPEC) would be permissible in this case. This would, however, replace the problem by a single-level high-dimensional optimization problem with complementarity constraints. Although theoretically possible, such a reformulation would destroy the favorable structure of the lower-level quadratic program and would likely lead to a significantly less efficient numerical solution.

3. Results

Before results can be calculated using the bilevel optimization approach, a convergence analysis must be performed to select the correct discretization for the numerical solution of the optimal control problems at the second level. In Figure 9, an exemplary analysis of the z-motion component is displayed. Based on the examined objective function values depending on the discretization, an approximation of these values is performed to a function that maps the expected convergence. Based on the parameter determination from the approxi-

mation, a theoretical limit value is obtained, which is assumed to be the theoretical limit value. Using this theoretical limit, a relative error can be derived. In this case a discretization of $K = 1000$ discrete points is applied for the second-level optimization.

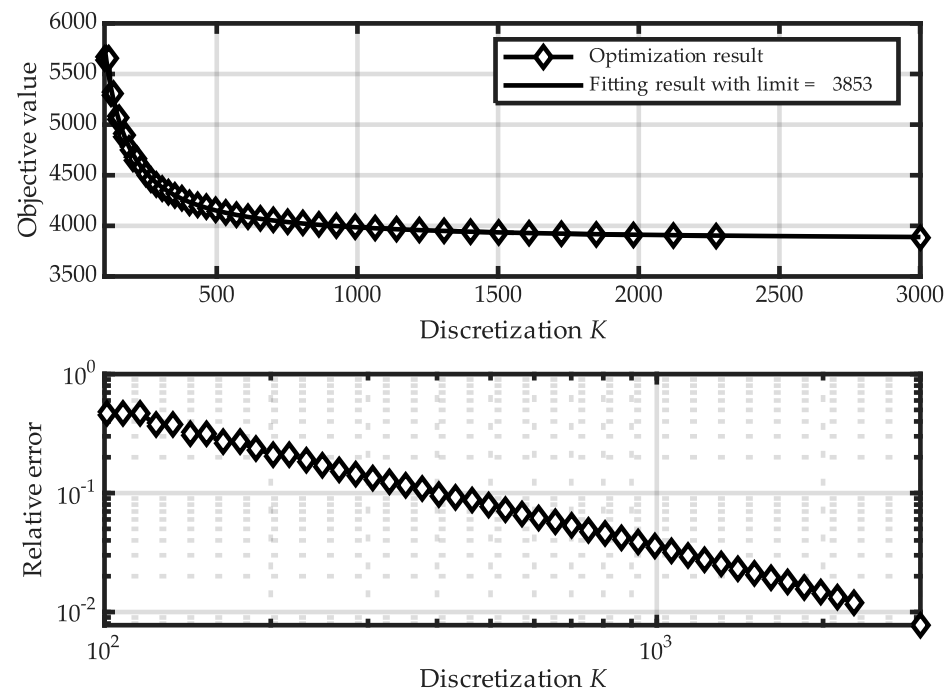
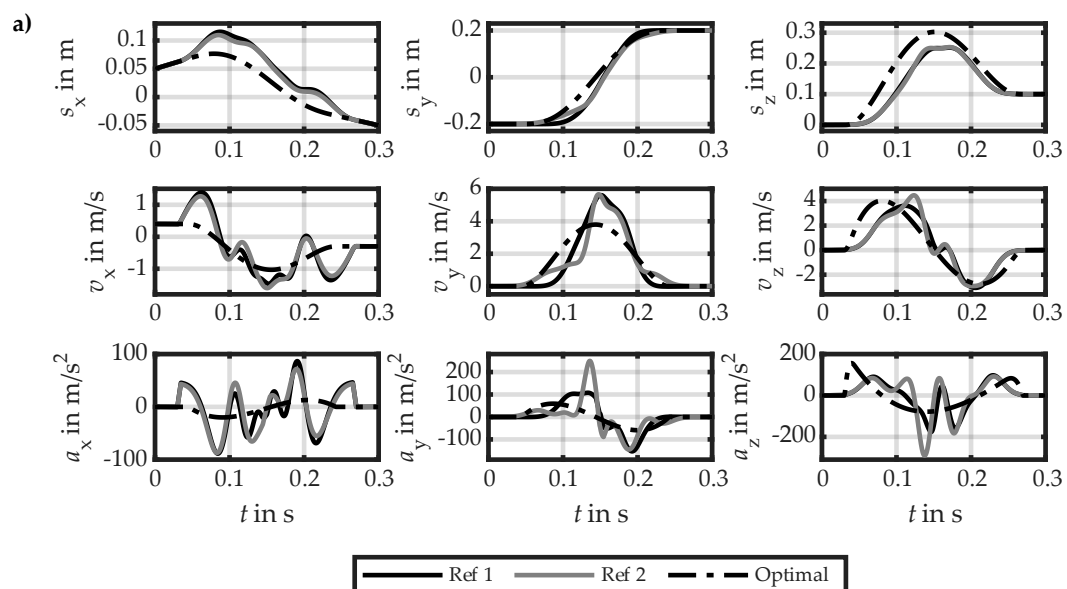


Figure 9. Exemplary convergence analysis of the z-motion component. Top: Representation of the objective function value as a function of discretization K . An approximation of the measurement results is shown in red. The limit value of the approximation is assumed to be the true objective function value. Bottom: Relative error regarding the limit of the approximation in double logarithmic representation.

The motion functions derived with the optimization method presented can be seen in Figure 10. A comparison with Figure 3 confirms that all three examined motions are in accordance with the motion plan and therefore suitable for performing the required task.



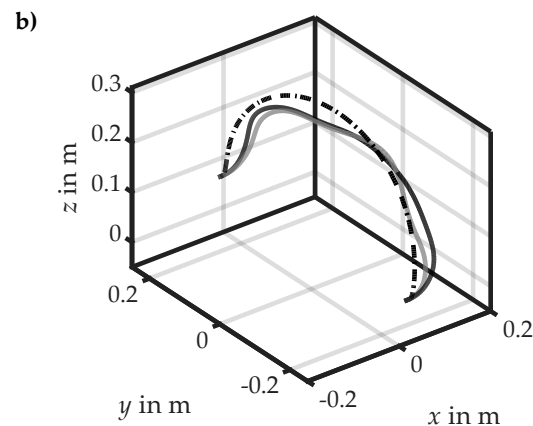


Figure 10. (a) Time-dependent motion functions for the derived optimal motion and the piecewise defined motions for reference. (b) Paths of the optimal motion compared to the reference motion paths.

Using the derived motion functions, a comparison of the objective function (s. Equation (15)) can be made. Figure 11 shows the results where the maximum values are marked, highlighting the points of maximum gripper load.

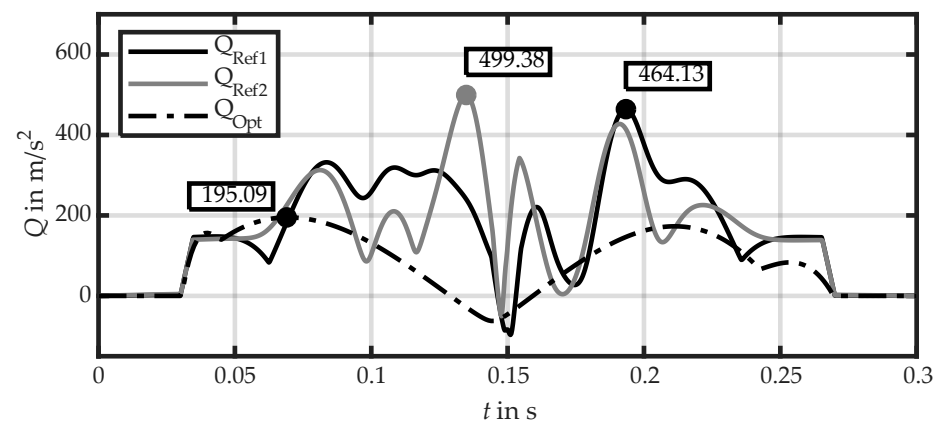


Figure 11. Objective function of the optimal and the reference motions representing the gripper load. The maximum values, indicating peak gripper load are marked for each examined method.

4. Discussion

The resulting trajectory shows a significant reduction in gripper load. A relative reduction of approximately 60% was achieved compared to the reference motions. The optimal motions are therefore expected to show improved process behavior, especially in terms of gripper contact failure. Depending on gripper, object, and handling task requirements, a reduction in vacuum level or motion time could be considered, leading to either improvement of energy consumption or machine output rate.

The resulting path differs significantly from the paths of the reference motions. Thus, the optimal solution has a maximum z -value that is approximately 20% higher, which seems counterintuitive at first glance. Furthermore, the severe curvature of the path is noticeable at the points where the gripper–object pairing leaves the input belt and meets the output belt. Considering the process model from Equation (14), these effects become plausible. Since horizontal accelerations have a more pronounced impact on the gripper load, the time required to complete the horizontal stroke needs to be as long as possible. To ensure this, the conveyor belts must be exited as quickly as possible, which results in a sharp curvature and a high apex. An additional notable feature is the smooth progression of the acceleration functions, showing few alternations around zero. Using the spline-based approach resulted in multiple acceleration peaks that may excite unfavorable dy-

dynamic effects in both the gripper–object pairing and machine structure. Since this approach depends heavily on the user’s background, skill and experience, a more effective solution is certainly conceivable. This highlights another advantage of the novel approach, where the user’s creative input is limited to accurately formulating the motion plan. The results are therefore targeted and reproducible. As can be seen, the intended behavior was achieved through bilevel optimization. This allowed, on one hand, to reduce gripper load (first-level optimization) and, on the other hand, to generate smooth C^2 -continuous motion functions (second-level optimization).

The introduced method might also bring some challenges for practical use. The calculation algorithm is computationally expensive which prevents the approach from online use. Advanced methods such as the use of characteristic curves [36] or the use of edge devices [37] could work around this problem.

5. Conclusions

This paper introduced a novel approach to generate optimal motions for robotic handling processes. The bilevel optimization ensures minimal expected gripper load while the trajectory, being C^2 -continuous, is expected to offer favorable dynamic behavior at the same time. The gripper load is approximated using a simple kinetostatic model of a vacuum gripper. This model is integrated in the optimization algorithm and serves preliminarily for evaluation of the results. Two reference motions have been generated using spline-based motion synthesis. Experimental evaluation will be the subject of future research and must be an essential preliminary step for the practical application of the method. Depending on the results of the validation experiments, further research may follow, such as the refinement of the process model to take into account the dynamic behavior of the gripper—for example due to micro slippage caused by shear stresses—or the use of multiple suction cups. Using a more complex process model may require different optimization methods or tools, to ensure convergence and reasonable solving duration. In particular, the examination of leakage would be of high interest. Systematic leakage caused by transmissive objects being grasped, as well as dynamic leakage during transportation result in a significant reduction in retention force. These effects might offer a valuable subject for further research. Furthermore, a comparison against advanced state-of-the-art trajectory optimization algorithms such as those introduced in [26] would provide a better perspective of the solution found. Another further step towards practical use would be the consideration of additional constraints resulting from actuator limits of the robot or obstacles within the working area.

Author Contributions: Conceptualization, G.S. and C.T.; methodology, G.S. and C.T.; software, C.T.; validation, C.T.; formal analysis, G.S.; investigation, G.S.; resources, G.S. and C.T.; data curation, C.T.; writing—original draft preparation, G.S. and C.T.; writing—review and editing, G.S., C.T. and J.-P.M.; visualization, G.S. and C.T.; supervision, J.-P.M.; project administration, J.-P.M.; funding acquisition, J.-P.M. All authors have read and agreed to the published version of the manuscript.

Funding: This project is co-funded by the European Union and co-financed from tax revenues on the basis of the budget adopted by the Saxon State Parliament.

Data Availability Statement: The raw data supporting the conclusions of this article will be made available by the authors on request.

Acknowledgments: During the preparation of this manuscript/study, the authors used generative AI (ChatGPT-5.2) for the purposes of code generation for minor tasks, e.g., plot generation or data extraction. Furthermore, the authors used DeepL to translate portions of the manuscript. The authors have reviewed and edited the output and take full responsibility for the content of this publication.

Conflicts of Interest: Author Clemens Troll was employed by the company Theegarten Pactec GmbH & Co. KG. The funders and the employer had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript; or in the decision to publish the results.

References

1. Cheng, M.-Y.; Liao, H.-C.; Lan, C.-C. A 4-DoF Pick-and-Place Robot with High Structural Rigidity and Pose Repeatability. *IEEE Access* **2025**, *13*, 101735–101744. <https://doi.org/10.1109/ACCESS.2025.3578357>.
2. Choi, H.; Park, T.; Hwang, G.; Ko, Y.; Lee, D.; Lee, T.; Park, J.-O.; Bang, D. Fabrication of Origami Soft Gripper Using On-Fabric 3D Printing. *Robotics* **2023**, *12*, 150. <https://doi.org/10.3390/robotics12060150>.
3. Hernandez, J.; Sunny, M.S.H.; Sanjuan, J.; Rulik, I.; Zarif, M.I.I.; Ahamed, S.I.; Ahmed, H.U.; Rahman, M.H. Current Designs of Robotic Arm Grippers: A Comprehensive Systematic Review. *Robotics* **2023**, *12*, 5. <https://doi.org/10.3390/robotics12010005>.
4. Kremer, F.; Nohooji, H.R.; Voos, H. Constrained trajectory optimization and force control for UAVs with universal jamming grippers. *Sci. Rep.* **2023**, *14*, 11968. <https://doi.org/10.1038/s41598-024-62416-1>.
5. Efendi, A.; Shao, Y.-H.; Huang, C.-Y. Technological development and optimization of pushing and grasping functions in robot arms: A review. *Measurement* **2025**, *242*, 115729. <https://doi.org/10.1016/j.measurement.2024.115729>.
6. Mykhailishyn, R.; Savkiv, V.; Maruschak, P.; Xiao, J. A Systematic Review on Pneumatic Gripping Devices for Industrial Robots. *Transport* **2022**, *27*, 201–231. <https://doi.org/10.3846/transport.2022.17110>.
7. Karako, Y.; Moriya, T.; Abe, M.; Shimakawa, H.; Shirahori, S.; Saitoh, Y. A Practical Simulation Method for Pick-and-Place with Vacuum Gripper. In *2017 56th Annual Conference of the Society of Instrument and Control Engineers of Japan (SICE)*; IEEE: Hoboken, NJ, USA, 2017; pp. 1351–1356. <https://doi.org/10.23919/SICE.2017.8105736>.
8. Gabriel, F.; Fahning, M.; Meiners, J.; Dietrich, F.; Dröder, K. Modeling of Vacuum Grippers for the Design of Energy Efficient Vacuum-Based Handling Processes. *Prod. Eng.* **2020**, *14*, 545–554. <https://doi.org/10.1007/s11740-020-00990-9>.
9. Mahler, J.; Matl, M.; Liu, X.; Li, A.; Gealy, D.; Goldberg, K. Dex-Net 3.0: Computing Robust Vacuum Suction Grasp Targets in Point Clouds Using a New Analytic Model and Deep Learning. In *2018 IEEE International Conference on Robotics and Automation (ICRA)*; IEEE: Hoboken, NJ, USA, 2018. <https://doi.org/10.1109/ICRA.2018.8460887>.
10. Pham, H.; Pham, Q.C. Critically Fast Pick-and-Place with Suction Cups. In *2019 International Conference on Robotics and Automation (ICRA)*; IEEE: Hoboken, NJ, USA, 2019. <https://doi.org/10.1109/ICRA.2019.8794081>.
11. Vodovozov, V.; Lehtla, M.; Raud, Z.; Semjonova, N.; Petlenkov, E. Managing Energy Consumption of Linear Delta Robots Using Neural Network Models. *Energies* **2024**, *17*, 4081. <https://doi.org/10.3390/en17164081>.
12. Wu, M.; Puppenyan, A.; Corves, B. Autogeneration and optimization of pick-and-place trajectories in robotic systems: A data-driven approach. *Robot. Comput.-Integr. Manuf.* **2026**, *97*, 103080. <https://doi.org/10.1016/j.rcim.2025.103080>.
13. Siciliano, B.; Sciavicco, L.; Villani, L.; Guseppe, O. *Robotics—Modelling, Planning and Control*; Springer: London, UK, 2009. <https://doi.org/10.1007/978-1-84628-642-1>.
14. Lorenz, M. Energy-Efficient Trajectory Planning for Robot Manipulators. Ph.D. Thesis, RWTH Aachen University, Aachen, Germany, 2021. <https://doi.org/10.18154/RWTH-2021-02421>.
15. Romero, S.; Valero, J.; García, A.V.; Rodríguez, C.F.; Montes, A.M.; Marín, C.; Bolaños, R.; Álvarez-Martínez, D. Trajectory Planning for Robotic Manipulators in Automated Palletizing: A Comprehensive Review. *Robotics* **2025**, *14*, 55. <https://doi.org/10.3390/robotics14050055>.
16. Zhao, Z.; Cheng, S.; Ding, Y.; Zhou, Z.; Zhang, S.; Xu, D.; Zhao, Y. A Survey of Optimization-based Task and Motion Planning: From Classical To Learning Approaches. *IEEE/ASME Trans. Mechatron.* **2024**, *30*, 2799–2825. <https://doi.org/10.1109/TMECH.2024.3452509>.
17. Troll, C.; Tietze, S.; Majschak, J.-P. Controlling Liquid Slosh by Applying Optimal Operating-Speed-Dependent Motion Profiles. *Robotics* **2020**, *9*, 18. <https://doi.org/10.3390/robotics9010018>.
18. Brückmann, N.; Mirz, C.; Hüsing, M.; Takeda, Y.; Corves, B. Optimal Control Approach for Trajectory Determination to Suppress Residual Frame Vibrations of Delta Robots in Pick and Place Tasks. *Mech. Mach. Theory* **2025**, *219*, 106302. <https://doi.org/10.1016/j.mechmachtheory.2025.106302>.
19. Massaro, M.; Lovato, S.; Bottin, M.; Rosati, G. An Optimal Control Approach to the Minimum-Time Trajectory Planning of Robotic Manipulators. *Robotics* **2023**, *12*, 64. <https://doi.org/10.3390/robotics12030064>.
20. Dos Santos, R.R.; Steffen, V.; Saramago, S.d.F.P. Robot path planning in a constrained workspace by using optimal control techniques. *Multibody Syst. Dyn.* **2007**, *19*, 159–177. <https://doi.org/10.1007/s11044-007-9059-1>.

21. Gruszka, L.; Barty's, M. A New Energy-Efficient Approach to Planning Pick-and-Place Operations. *Energies* **2022**, *15*, 8795. <https://doi.org/10.3390/en15238795>.
22. Shan, X.; Li, Y.; Liu, H.; Huang, T. Residual Vibration Reduction of High-Speed Pick-and-Place Parallel Robot Using Input Shaping. *Chin. J. Mech. Eng.* **2022**, *35*, 16. <https://doi.org/10.1186/s10033-022-00679-3>.
23. Lee, J.-E.; Sun, R.; Bylard, A.; Sentis, L. Grasp Failure Constraints for Fast and Reliable Pick-and-Place Using Multi-Suction-Cup Grippers. *IEEE Trans. Autom. Sci. Eng.* **2026**, *23*, 317–331. <https://doi.org/10.48550/arXiv.2408.03498>.
24. Lee, J.-E.; Sun, R.; Bylard, A.; Sentis, L. On the Performance of Jerk-Constrained Time-Optimal Trajectory Planning for Industrial Manipulators. *IEEE Trans. Autom. Sci. Eng.* **2024**, *14*, 8. <https://doi.org/10.48550/arXiv.2404.07889>.
25. Savkiv, V.; Mykhailysyshyn, R.; Duchon, F.; Mikhailishin, M. Modeling of Bernoulli Gripping Device Orientation when Manipulating Objects along the Arc. *Int. J. Adv. Robot. Syst.* **2018**, *15*, 2. <https://doi.org/10.1177/1729881418762670>.
26. Avigal, Y.; Ichnowski, J.; Cao, M.Y.; Goldberg, K. GOMP-ST: Grasp Optimized Motion Planning for Suction Transport. In *International Workshop on the Algorithmic Foundations of Robotics XV (WAFR 2022)*; Springer International Publishing: Cham, Switzerland, 2022; pp. 488–505. https://doi.org/10.1007/978-3-031-21090-7_29.
27. Gabriel, F.; Bobka, P.; Dröder, K. Model-Based Design of Energy-Efficient Vacuum-Based Handling Processes. *Sci. Procedia CIRP* **2020**, *93*, 538–543. <https://doi.org/10.1016/j.procir.2020.03.006>.
28. Troll, C. Synthese und Umsetzung Optimaler Bewegungen unter Berücksichtigung der Einstellbaren Maschinendrehzahl. Ph.D. Thesis, TU Dresden, Dresden, Germany, 2021.
29. Chen, Y.; Li, L. Predictable Trajectory Planning of Industrial Robots with Constraints. *Appl. Sci.* **2018**, *8*, 2648. <https://doi.org/10.3390/app8122648>.
30. Li, Y.; Huang, T.; Chetwynd, D.G. An approach for smooth trajectory planning of high-speed pick-and-place parallel robots using quintic B-splines. *Mech. Mach. Theory* **2018**, *126*, 479–490. <https://doi.org/10.1016/j.mechmachtheory.2018.04.026>.
31. VDI 2143; Part 1: Motion Rules for Cam Mechanisms and Motion Control Systems—Theoretical Fundamentals. Verein Deutscher Ingenieure e.V.: Düsseldorf, Germany, 2024.
32. Monkmann, G.; Hesse, S.; Steinmann, R.; Schunk, H. *Robot Grippers*; Wiley-VCH Verlag GmbH & Co. KGaA: Berlin, Germany, 2004.
33. Sinah, A.; Malo, P.; Deb, K. A Review on Bilevel Optimization from Classical to Evolutionary Approaches and Applications. *IEEE Trans. Evol. Comput.* **2018**, *22*, 2. <https://doi.org/10.1109/TEVC.2017.2712906>.
34. Flash, T.; Hogan, N. The Coordination of Arm Movements: An Experimentally Confirmed Mathematical Model. *J. Neurosci.* **1985**, *5*, 1688–1703. <https://doi.org/10.1523/jneurosci.05-07-01688.1985>.
35. Von Stryk, O. Numerische Lösung Optimaler Steuerungsprobleme: Diskretisierung, Parameteroptimierung und Berechnung der Adjungierten Variablen. Ph.D. Thesis, Technische Universität München, Munich, Germany, 1994.
36. Holowenko, O.; Troll, C.; Ihlenfeldt, S.; Majschak, J.-P. A Novel Adaptive Interpolation Approach for the Implementation of Operating-Speed-Dependent Motion Profiles in Processing Machines. *Eng. Sci. Technol. Int. J.* **2021**, *24*, 1308–1317. <https://doi.org/10.1016/j.jestch.2021.03.007>.
37. Holowenko, O.; Troll, C. Increasing the Motion Accuracy of a Servo-Controlled Mechanism at Variable Machine Speeds. In *Proceedings of the Elektromechanische Antriebssysteme 2023: 9. Fachtagung (VDE OVE)*; VDE Verlag: Berlin, Germany, 2023; pp. 333–339.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.