

Article

Influence of the Geometric and Mechanical Characteristics on the Response of Homogeneous Isotropic Materials Under the Brazilian Splitting Test

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Abstract

The present paper concerns the study of the influence of some geometrical parameters, in particular the depth-to-height ratio, and some mechanical properties, in particular Young's modulus and Poisson's ratio, on the response of homogeneous isotropic specimens subjected to the Brazilian splitting test. For this purpose, principal stresses and strains are evaluated at the specimen barycenter through finite element analyses carried out in the ANSYS environment. The relevant analyses are performed by adopting systematic variations in both geometrical and mechanical parameters within suitably defined ranges. The obtained numerical results are compared with commonly adopted reference stresses from the literature, showing that the plane stress assumption is independent of material properties for low depth-to-height ratios, and it is generally not realistic under standard testing conditions. Furthermore, the obtained results prove that the tensile principal stress typically assumed in the literature overestimates the actual value. Moreover, the strain field analysis indicates that the validity of the plane strain hypothesis depends jointly on the geometrical and mechanical characteristics of the specimen. Finally, the study proposes a graphical approach for the estimation of the actual tensile principal stress starting from conventional literature-based values, accounting for specimen-specific parameters. Future work will include the numerical validation of the behavior of the principal stresses along the specimen axis, as well as the experimental validation of the numerical results on a specimen basis, for instance, by employing full-field, contactless displacement measurement techniques.

Keywords: Brazilian splitting test; stress analysis; strain analysis; homogeneous isotropic material; FEM model; numerical simulations



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1. Introduction

The splitting test (also referred to as the Brazilian test or indirect tensile test) is widely employed to experimentally determine the tensile strength of brittle materials, primarily in the fields of rock mechanics and of the mechanical characterization of cementitious composites. The knowledge of a reliable value of the tensile strength, especially in brittle materials, is crucial for the evaluation of structural safety; therefore, a critical analysis of the results obtained by the Brazilian test is extremely important in the field of structural engineering. The method [1,2] was first introduced in the early 1940s by the Brazilian engineer Fernando Lobo Carneiro to assess the tensile strength of concrete cylinders.

At that time, this parameter was crucial for evaluating the feasibility of replacing steel cylinders with concrete ones during the structural repositioning of a historical church in Rio de Janeiro. The resounding success at the international level of this test, related to both its efficacy and simple arrangement, led the scientists to call the splitting test a Brazilian one in honor of the nationality of Lobo Carneiro. Nowadays, all major international standards (see, e.g., [3–6]) incorporate this test, which plays a fundamental role in evaluating the mechanical properties of specimens extracted through coring from masonry or existing reinforced concrete structures. Mainly, this test is performed by compressing a cylindrical specimen along two opposite diametral generatrices. The specimen breaks along the plane containing the loaded generatrices, due to tensile stresses orthogonal to that plane.

In recent years, the Brazilian test has become increasingly widespread among scientists working in various fields of research. For instance, the nonuniform stress field induced in the specimen by the Brazilian test serves as the basis for the mechanical characterization of anisotropic materials, such as shale [7–9], and cementitious materials reinforced with artificial or natural fibers (see, e.g., respectively, [10] and [11,12]), which are of particular interest for both construction and extractive industries. Liu et al. [13] experimentally analyzed thermally treated granite specimens to identify the change in the fracture mechanisms, finding significant correlations between treatment temperature and fracture behavior. Zhang et al. [14], always in granite specimens, explored the enhancement of safety by using different resin types for repairing fractures. The paper by Li et al. [15] proposed the novelty of studying the fracture behavior of prismatic specimens subjected to the Brazilian test with supporting fixtures, using image processing techniques. Furthermore, Lei et al. [16] proposed a numerical model for shale specimens, building on Rocco's theoretical framework [17], and incorporating void and defect distributions characterized by CT-scan imaging. The experimental identification of mechanical parameters (Young's modulus, Poisson's coefficient, and tensile strength) by the Brazilian test has been the topic of recent papers (see, e.g., [18–23]).

From a theoretical point of view, the problem of determining the stress field arising in a circular disk subjected to two opposite concentrated forces applied on the circumference, representing the disk contour, has been addressed by many authors such as Timoshenko and Goodier in [24]. These results have been largely adopted in the literature (see, e.g., [25–33]) for the knowledge of the mechanical behavior of brittle and quasi-brittle materials. Furthermore, Jemiolo [34] investigated the strain and displacement fields within both plane stress and plane strain formulations, but the results are limited to the case of a circular disk and cannot be extended to the case of real 3D specimens.

Wei and Chau, in [35], published an interesting paper in which a mathematical formulation to determine the three-dimensional mechanical response of a cylindrical specimen under a Brazilian test is proposed. This formulation is based on modeling the displacement functions by the z -component of the Galerkin vector plus the z -component of the irrotational part of the Helmholtz decomposition vector. The proposed formulation is very interesting since it leads to a complete mathematical model of both the mechanical and kinematical behavior of the specimen, also investigating the role of the Poisson coefficient. Unfortunately, this formulation lacks from a physical point of view since, as reported in many graphs in the paper, the tensile stress acting along the axis of the specimen shows an asymptotic behavior when reaching the center of the basis of the cylindrical specimen with a limit value neither graphically identifiable (it seems to reach zero) nor reported or discussed in the text.

To overcome the critical issues of the existing formulations reported above, it would be useful to perform computational analyses capable of describing the behavior of the

specimen under the effective loading conditions. Such analyses have been adopted in the past, but the computational effort and time consumption were often unacceptable.

Nowadays, due to the huge computing power available even in consumer personal computers, in wider and wider application fields it is common to adopt numerical approaches to investigate real problems for which the theoretical solution is not available (see, e.g., [36–38]). Within these approaches, the so-called Finite Element Method (FEM) is the most widely adopted, both in the development of new theoretical solutions and in the improvement of existing ones.

It is noteworthy that the actual Brazilian test departs from the idealized models considered in previous studies for the following reasons. Experimentally, the test is performed using a loading frame with a fixed and a movable platen, the latter of which imposes either force or displacement on the specimen. Mechanically, this configuration is fully consistent with theoretical assumptions. From a kinematical perspective, however, the experimental configuration markedly differs from the theoretical models typically considered, primarily due to the absence of idealized boundary conditions. Specifically, in the theoretical formulation, no kinematical constraints are imposed at the specimen boundary, whereas in the actual test, the displacements of the lower generatrix are constrained to zero by the fixed platen of the testing machine. A further distinction concerns the adoption of plane stress or plane strain assumptions, which underlie the theoretical results reported in the cited studies. The elastic problem is a general three-dimensional problem with stress and strain fields coupled by means of the constitutive relation. Consequently, the plane stress state is an ideal assumption valid when the depth H of the specimen tends to zero, while the plane strain state assumption is valid when the depth tends to infinity.

Other authors (see, e.g., [32,33]) have proposed the use of numerical models for the evaluation of the stress field acting in a specimen under a Brazilian test, but in these papers, the influence of the depth/diameter ratio has not been investigated.

Yua and Yin propose in [39] an FEM-based numerical study of a cylindrical specimen under the Brazilian test, in which they state that the tensile stress acting on the base of the specimen is greater than that obtained by the relevant standards, and suggest a possible correction depending linearly on the ratio between the depth H of the specimen and its diameter D . The limitations of this study are related to two main features: (a) the study does not consider the influence of the mechanical parameters of the material (i.e., Young's modulus and Poisson ratio); (b) the analysis and the subsequent linear correction is limited to depth/diameter ratio interval between 0.5 and 1.25. These limitations stimulate the authors to explore a wider range for the depth/diameter ratio and to consider the influence of the mechanical parameters, as will be presented in the following sections.

A final remark is related to the role of the contact between the plates of the testing machine and the specimen. In the ideal model, the loading is represented by a concentrated force applied at a point on the contour, while in the real test, this condition can be considered valid over a generatrix only at the beginning of the test. Clearly, when increasing the applied force, the interaction between the moving platen and the specimen related to strain induced in the latter leads the applied force to be distributed over a finite area. This problem is discussed in many papers available in the literature (see, e.g., [19–23]), evidencing its influence on the stress field rather than on the strain one.

The principal aim of the present paper is to perform a coherent and reliable numerical analysis of a homogeneous elastic isotropic material specimen under a real Brazilian test by means of a suitable FEM model of the Brazilian splitting test carried out by ANSYS (2021 R1 release) software. Furthermore, the specific novelty of the study concerns the investigation of the role of the geometrical characteristics (in terms of the depth/diameter ratio) and that of the mechanical characteristics (in terms of Young's modulus and Poisson ratio) in the

stress and strain fields arising at the barycenter of the involved specimen. The numerical simulations have been performed by adopting a wide range of values for both geometrical and mechanical characteristics; these ranges have been identified considering the regulatory indications, as well as specific features of the materials (concrete and rocks) usually tested. The numerical results obtained allow us to confirm that: (1) the assumptions of plane stress state or plane strain state are not valid for the common values of the depth/diameter ratio assumed by the standards; (2) the material characteristics (especially Young's modulus) strongly influence the results for high values of the depth/diameter ratio. Furthermore, the findings allow for the identification of the analytical interpolating expression of both the ratio between the tensile principal stress and the corresponding theoretical one, as well as that of the ratio between the compressive principal stress and the corresponding theoretical one, giving the knowledge of the real stress and strain states at the barycenter of the specimen. Future developments are expected to extend the present study to the analysis of both stress and strain fields acting at the free surface of the specimen subjected to a Brazilian test, as well as to plan a wide campaign of experimental tests by adopting a full-field contactless approach to check of the overall behavior of the specimen.

2. Materials and Methods

As stated before, the principal aim of the present paper is to perform a coherent and reliable numerical analysis of a homogeneous elastic isotropic material specimen under a real Brazilian test. To this end, it is important to recall the main characteristics of the Brazilian test and to report the classical formulations that allow for the interpretation of the results, highlighting the critical issues that this study aims to address (Section 2.1).

In Section 2.2, the adopted numerical approach within the framework of the Finite Element Method will be described in detail, and all the parameters used to achieve the goal of the paper will be defined. Although the FEM is an approximate method, it allows for results that become increasingly consistent with the real behavior of the continuous solid as the mesh is refined.

2.1. Remarks on the Standard Brazilian Test

The Brazilian test (see Figure 1) [1,3–6] is an experimental test to determine the tensile strength of brittle materials such as rocks, concrete, bricks and ceramics. It offers an alternative to the direct tensile test, which is often difficult to carry out for specimens of such materials due to issues related to clamping the specimen and to the correct application of the load.

The test is based on the application of a diametrical compressive distributed load p_x to a cylindrical specimen (diameter D and depth H) by the plates of the testing machine supplemented by a system for recording the applied load. Depending on the geometry of the specimen and the applied load, tensile stresses develop in the central zone of the specimen. When the critical distributed load $p_{x,F}$ is reached, a crack appears and propagates along the diameter on which the load is applied, and the maximum load value $P_F = p_{x,F} H$ is recorded in correspondence of failure. Consequently, although the applied load is a compressive one, failure occurs due to tension, allowing the tensile strength of the material to be determined, even if indirectly.

The aim of the test is to determine the indirect tensile strength of the material by measuring the maximum load withstood by the test specimen before failure. This property is particularly important in the study of brittle materials, as they generally exhibit a tensile strength that is much lower than their compressive strength. Although this strength is often completely disregarded (non-tensile-resistant materials, NTRMs), it is advisable to

evaluate it to determine the safety margins to be adopted and, in any case, to carry out various detailed analyses (for example, crack propagation analysis).

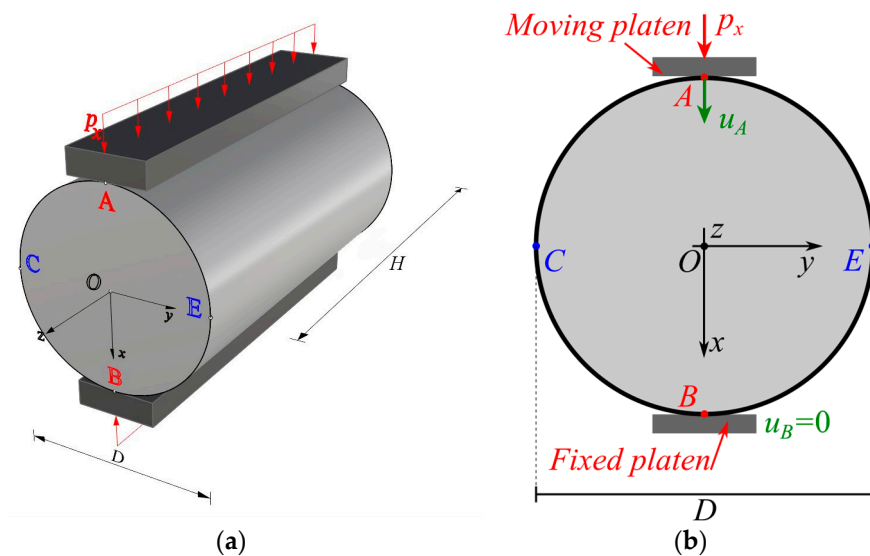


Figure 1. Schematic of the real Brazilian test: (a) 3D sketch; (b) front view.

Clearly, although the specimens are straight, cylindrical, elastic solids, the determination of the stress state is a 3D problem, and the analytical approach to the problem can be stated in terms of plane elasticity if the contact stresses are uniformly distributed along the z direction [34]. Hence, a three-dimensional elastic problem is usually converted into a two-dimensional problem, simplifying its mathematical treatment.

The general case of a disk subjected to a couple of opposite concentrated forces P acting along a chord has been studied, and the corresponding stress field has been determined under the assumption of a perfectly elastic material [7]. In this framework, the case of the cross-section of a specimen composed of homogeneous linear elastic material subjected to the Brazilian splitting test can be regarded as a special case in which the chord coincides with the vertical diameter (Figure 2) and $P = p_x H$.

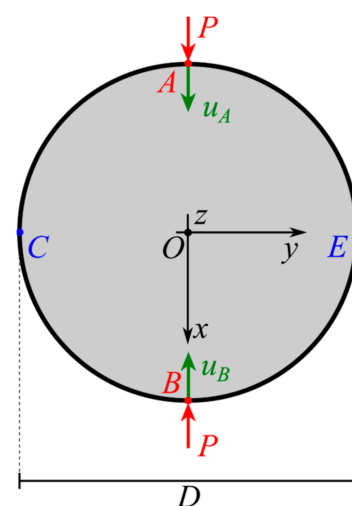


Figure 2. Theoretical model of the Brazilian test $|u_A| = |u_B|$.

The assumption of the plane stress state leads to the following expressions for the maximum and minimum values of stress at the center of the cross-section:

$$\sigma_y = \frac{2 P_F}{\pi D H} \quad (1)$$

$$\sigma_x = -\frac{6 P_F}{\pi D H} \quad (2)$$

where σ_y is the indirect tensile strength (MPa), $P_F = p_{x,F} H$ (N) is the acting load in correspondence of which the failure occurs and, as reported before, D (mm) and H (mm) are the diameter and the depth of the specimen, respectively.

The results derived from Brazilian tests under real conditions deviate from these predictions for three main reasons: (i) the experimental configuration departs from the theoretical model, as the displacement of the section centroid is not null; (ii) the specimen thickness (i.e., depth H) is generally non-negligible, violating the plane stress requirement; and (iii) the elastic equilibrium problem is intrinsically coupled, making it necessary to assess the influence of material properties (Young's modulus and Poisson's ratio) on the actual response. From the above remarks, it follows that, since the Brazilian test is adopted worldwide to gather mechanical information on material behavior, it is fundamental to analyze the influence of the real test conditions on the obtained results. To this aim, several numerical analyses based on the Finite Element Method have been conducted. Details of this procedure are described in the following subsection.

2.2. Numerical Approach Using the FEM Model

The kinematical and mechanical response of the specimen under the Brazilian splitting test, when modeled as a continuum three-dimensional elastic body, is, as is well known, not reachable. Consequently, it is necessary to refer to numerical simulations utilizing suitable approximate methodologies. The most effective methodology is related to the use of the Finite Element Method (FEM), widely used in scientific and professional fields, due to its easy implementation in technical software. In the present paper, the latter have been performed in the ANSYS Workbench environment, (2021 R1 release), under the hypothesis of linear elastic behavior of the involved materials, by varying the geometrical parameters, the distributed force p_x , the Young's modulus E , and the Poisson's ratio ν , recording stress components and the principal stresses and strains at the barycenter of the specimen. This point has been selected to avoid the influence of the boundary conditions on the results.

To reproduce as best as possible the real conditions of a Brazilian test, the numerical model geometry accounts for both the specimen under testing and the presence of two steel plates ($E = 210$ GPa, $\nu = 0.33$, see Figure 3a): one devoted to imposing the kinematically fully restrained constraint on the lower generatrix of the specimen and the other to applying the distributed load on the upper generatrix of the specimen. The presence of the plates implies the modeling of the contact between the plates and the specimen surface, as is described below. The adopted elements are 10-node isoparametric for solid discretization and 8-node 3D solid contact elements for surface-to-surface contact modeling.

Preliminary numerical tests have been performed to evaluate both the convergence velocity and the influence of the contact type (bonded or with friction) on the principal stresses at the barycenter, always considering that both penetration and gap are not allowed. The obtained results for $H/D = 1$ showed that, as was expected, the convergence velocity is largely greater in the case of bonded contact and that the influence on the principal stresses, as can be deduced by Figure 4, is about 1.5%. Results for other H/D ratios are analogous and are not reported for the sake of brevity. Consequently, the bonded contact has been adopted in numerical analyses. Further, in the contact model, penetration and gap are not allowed and the large deformation option is set to improve convergence in

cases with low stiffness. All analyses have been nonlinear due to contact and the large deformation option, but convergence has been gained in relatively few iterations.

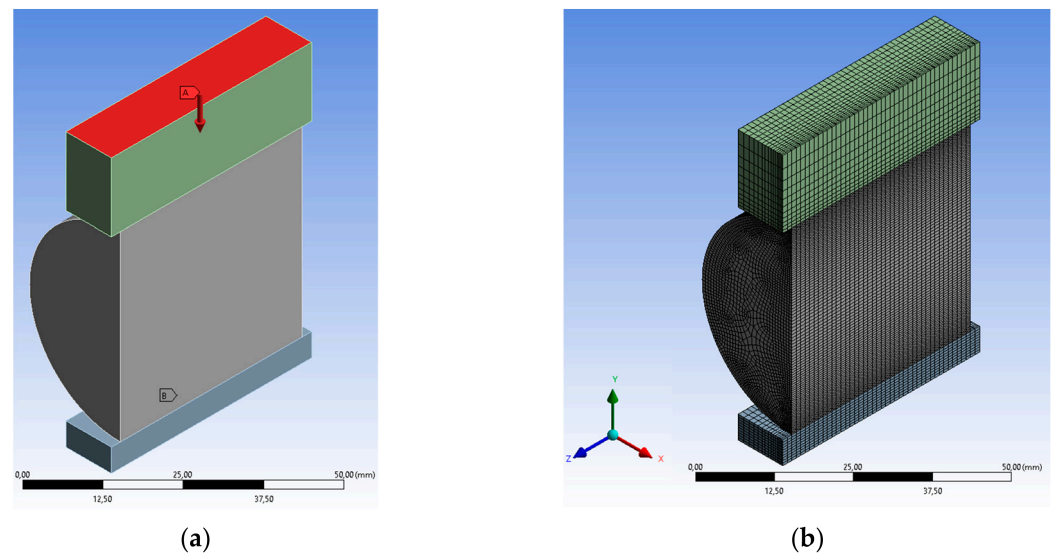


Figure 3. FEM model in ANSYS: (a) adopted geometry; (b) adopted mesh.

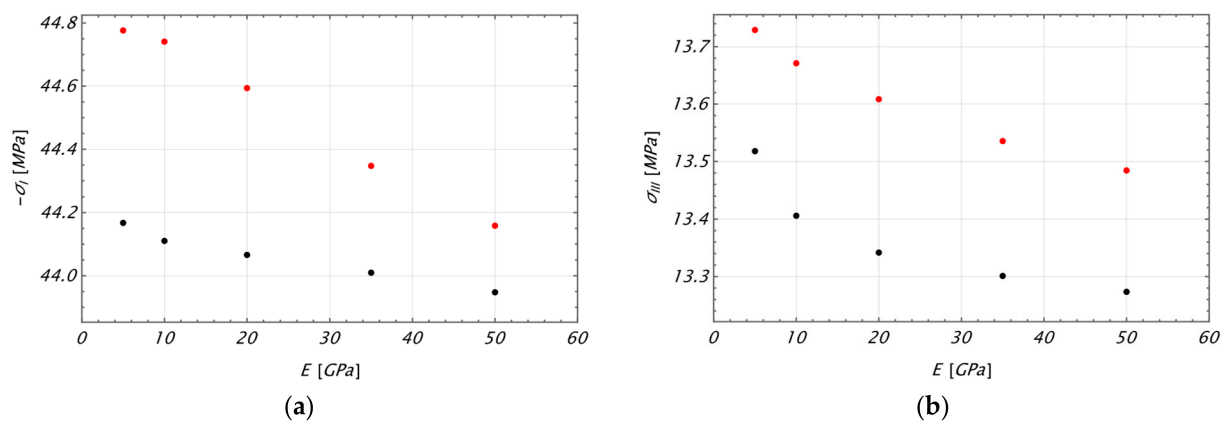


Figure 4. Comparison of the principal stresses vs. E ($H/D = 1$, $\nu = 0.25$ and $\bar{\sigma}_y = 1.5$ MPa) in the case of bonded contact (black dots) and friction (red dots, friction coefficient equal to 0.15): (a) $-\sigma_I$; (b) σ_{III} .

The mesh dimension was selected after suitable preliminary mesh independence investigations to find the best compromise between results and computational time. This mesh independence analysis has been conducted for the largest specimen dimensions considered ($H/D = 3$; $D = 0.1$ m) and with the lowest Young's modulus considered, applying the same numbers of elements for all other analyzed cases. The final mesh (Figure 3b) leads to a displacement value within 3% of that obtained by doubling the number of elements, presenting about 150,000 elements in total, with the same element distribution for all geometries. To reduce the computational effort, the geometric numerical model takes into account the symmetry of the real problem. Therefore, the adopted boundary conditions are as follows: (a) symmetry in the vertical diametral faces; (b) fixed support (all displacements are equal to zero) on the lower face of the lower platen (plane B in Figure 3a); and (c) surface-distributed force on the upper face of the upper platen (plane A in Figure 3a), with all contact modeled as bounded.

The steps performed in the numerical analyses are summarized below, while the values of the selected parameters are reported in the following section. For the assigned

depth-to-diameter ratio (H/D) and for the selected values of Young's modulus E and Poisson's ratio ν , the specimen diameter D was varied within the selected range. For each configuration, the corresponding distributed force applied to the upper plate was calculated based on a prescribed pressure acting at the center of the specimen's cross-section, as indicated by the relevant international standards (see Equation (1)). During each simulation, the stress field was recorded.

3. Results

The analyses reported in this section have been performed by suitably varying the H/D , Young's modulus, Poisson's ratio and the diameter, as shown in Table 1.

Table 1. Involved parameters and their corresponding adopted ranges.

Parameter	Range	Step	Unit
H/D	$0.25 \leq H/D \leq 3$	0.25	Dimensionless
E	$5 \leq E \leq 50$	5, 10, 20, 35, 50	GPa
ν	$0.1 \leq \nu \leq 0.4$	0.05	Dimensionless
D	$0.02 \leq D \leq 0.1$	0.01	m
$\bar{\sigma}_y$	$0.02 \leq \bar{\sigma}_y \leq 20$	0.02, 0.1, 0.5, 1, 2.5, 5, 10, 15, 20	MPa

The numerical tests have been performed analogously to those reported in [40], taking as constant the theoretical stress (hereafter indicated as $\bar{\sigma}_y$) acting at the center of the specimen's cross-section (which is related to the acting load, see Equation (1)). The expression of $\bar{\sigma}_y$ is given by Equation (1) and the selected range is determined by considering that the Brazilian test is usually performed for rocks and concrete samples.

3.1. Stress Analysis

The obtained results for the principal stresses (indicated as σ_I , σ_{II} and σ_{III} and ordered from the minimum to the maximum one) are presented in Figure 5 as a function of H/D for $\bar{\sigma}_y = 5$ MPa and $\nu = 0.25$; in this figure, only the cases corresponding to $E = 5, 20$, and 50 GPa are shown, with the remaining ones being intermediate and therefore omitted for the sake of clarity.

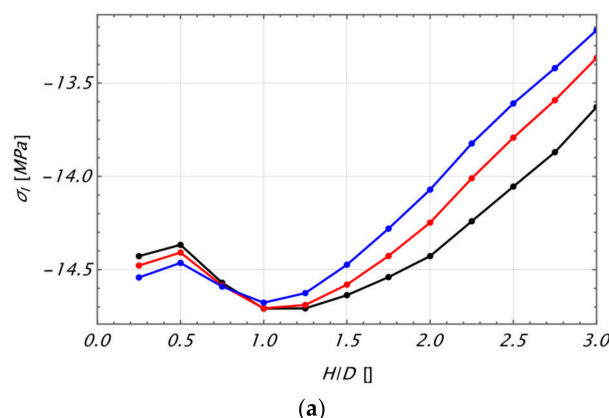


Figure 5. Cont.

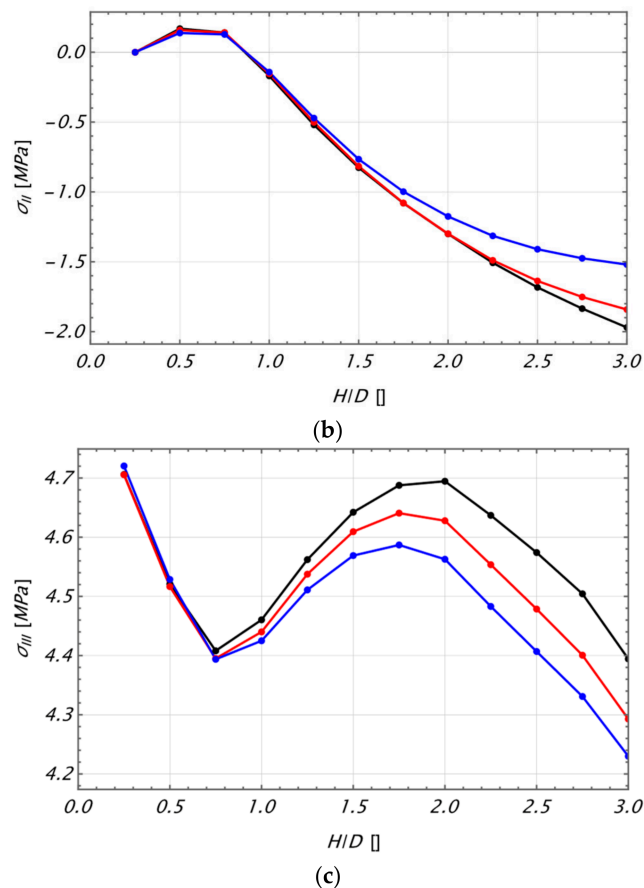


Figure 5. Principal stress vs. H/D for $\bar{\sigma}_y = 5$ MPa (black line: $E = 5$ GPa; red line: $E = 20$ GPa, blue line: $E = 50$ GPa): (a) σ_I ; (b) σ_{II} ; (c) σ_{III} .

The analysis of Figure 5 leads to the following remarks: (i) for low values of H/D ($H/D < 1$), the obtained results are independent from the Young's modulus; (ii) for values of $H/D \geq 1$, the obtained results clearly show a dependence on the Young's modulus, confirming that the elastic problem is coupled; and (iii) the behavior of all principal stress with H/D indicates that the stress relation with the geometric characteristics is highly nonlinear.

To check the influence of the acting load on the stress state, the analyses were repeated considering $E = 5$ GPa and $\nu = 0.25$ for different values of $\bar{\sigma}_y$ ($\bar{\sigma}_y = 5$ MPa, 10 MPa, 15 MPa). The obtained results for the tensile principal stress divided by the corresponding value of $\bar{\sigma}_y$ are reported in Figure 6.

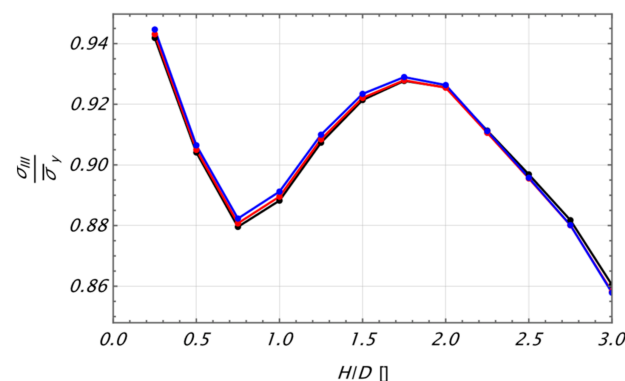


Figure 6. Comparison among the tensile principal stress $\sigma_{III}/\bar{\sigma}_y$ for $E = 5$ GPa and $\nu = 0.25$ (black line: $\bar{\sigma}_y = 5$ MPa; red line: $\bar{\sigma}_y = 10$ MPa, blue line: $\bar{\sigma}_y = 15$ MPa).

It should be emphasized that analogous results have been obtained for the other principal stresses, but they are not reported for the sake of brevity. The analysis of Figure 6 clearly indicates that the ratio between $\sigma_{III}/\bar{\sigma}_y$ is independent of $\bar{\sigma}_y$ and, consequently, the values of the principal stresses are linearly related with $\bar{\sigma}_y$.

The analysis of the stress acting at the barycenter of the specimen is completed by the definition of the well-known Mohr's circle of the stress state, reported in Figure 7 for $E = 5 \text{ GPa}$, $\nu = 0.25$ and H/D values equal to 0.25, one, two and three.

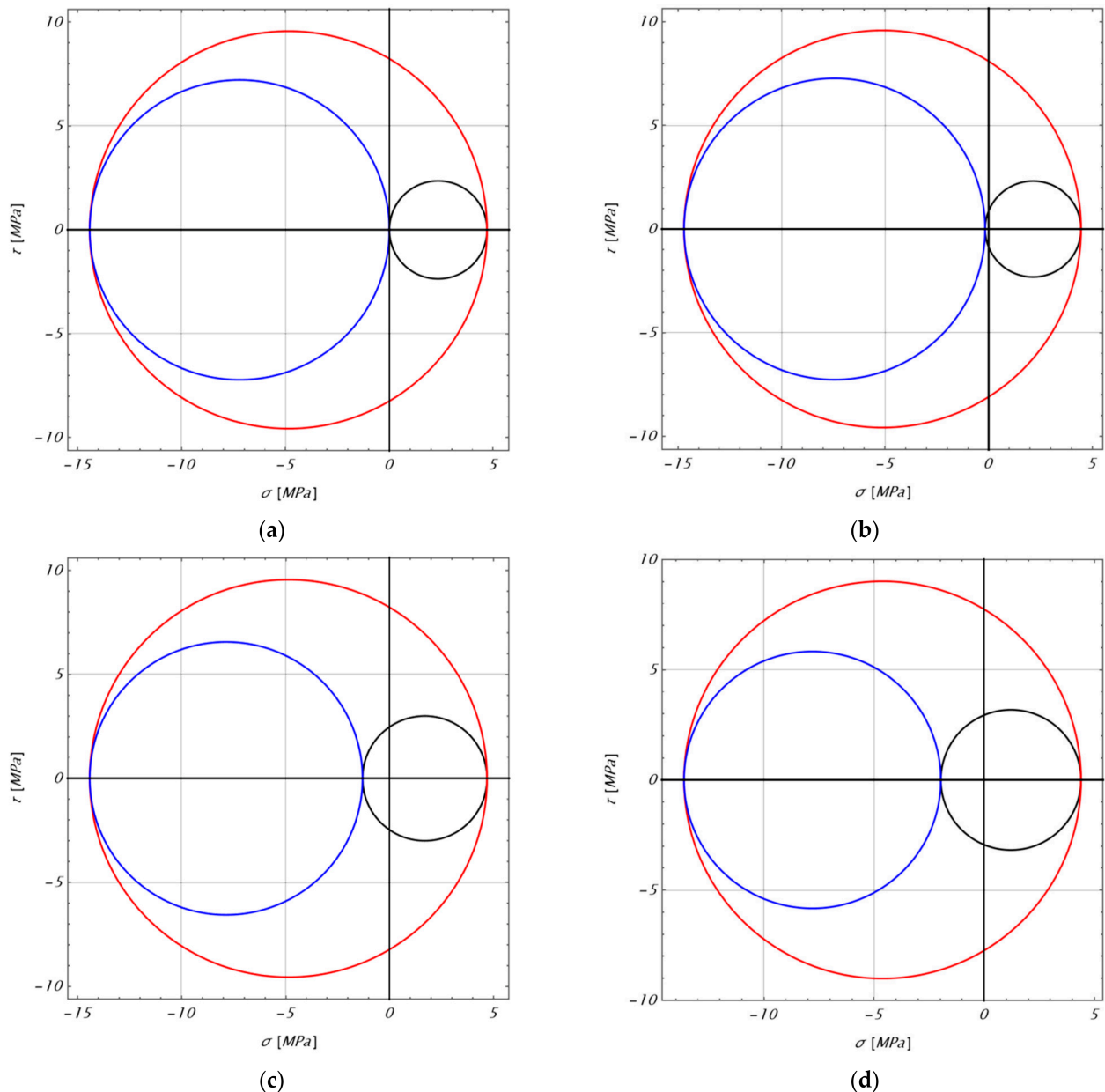


Figure 7. Mohr's stress circles for $\bar{\sigma}_y = 5 \text{ MPa}$ and $E = 5 \text{ GPa}$: (a) $H/D = 0.25$; (b) $H/D = 1.0$; (c) $H/D = 2.0$; (d) $H/D = 3.0$.

The analysis of Figures 5 and 7 clearly indicates that, as expected, the plane stress condition ($\sigma_{II} = 0$) is achieved only for $H/D \rightarrow 0$ (which is a theoretical condition). However, for $0 < H/D \leq 1$, the plane stress condition can be accepted. To further improve the analysis of the stress state in Figure 8, a comparison between the Mohr's circles obtained for $E = 5 \text{ GPa}$ and $E = 50 \text{ GPa}$ is reported, in the case of $H/D = 3.0$, $\nu = 0.25$ and $\bar{\sigma}_y = 5 \text{ MPa}$. The results for other values of the Poisson's coefficient are not

reported since they are coincident with those obtained for $\nu = 0.25$. The analysis of Figure 8 clearly shows that the influence of the Young's modulus on the Mohr's circles is negligible, although visible.

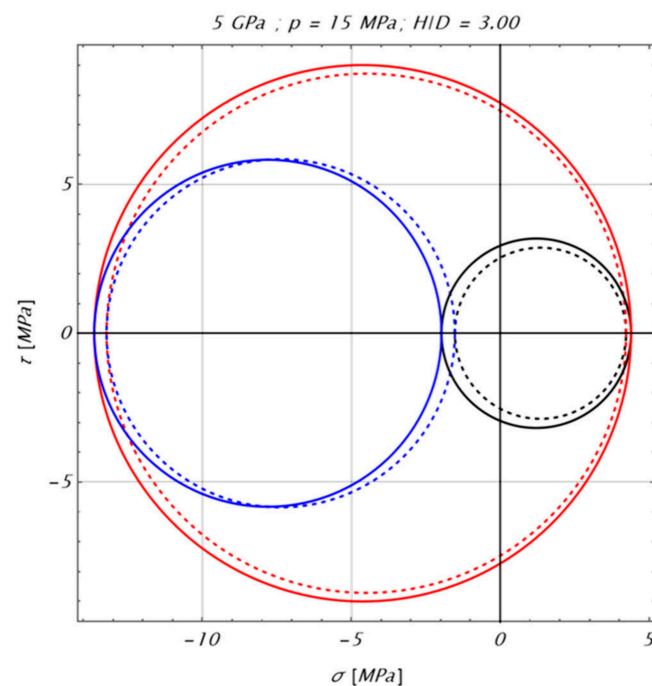


Figure 8. Comparison among the Mohr's stress circles for $\bar{\sigma}_y = 5$ MPa and $H/D = 3.0$: (continuous lines) $E = 5$ GPa; (dashed lines) $E = 50$ GPa.

Finally, in Figure 9, the τ_{max} acting at the barycenter of the specimen is reported vs. H/D , showing that the higher value of the tangential stress is achieved for $H/D < 0.5$, that is, when the plane stress condition, even if it is not mathematically attained, can be regarded as almost reached.

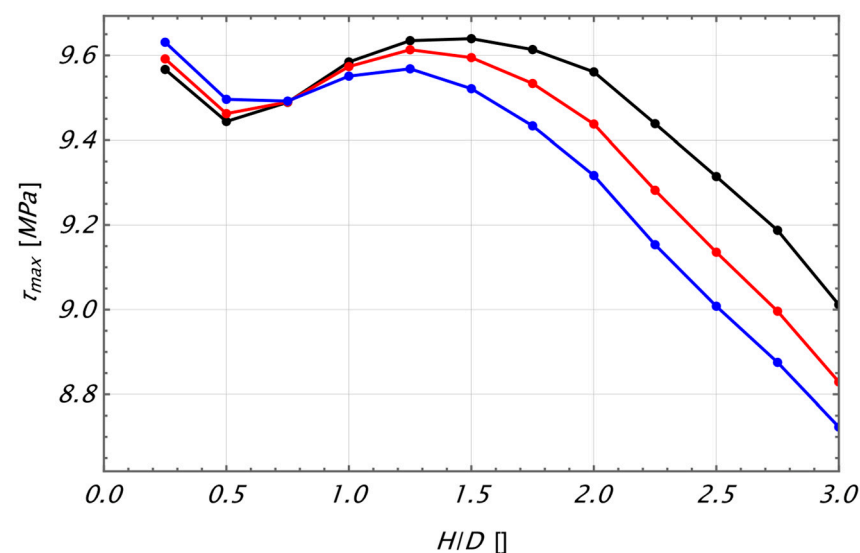


Figure 9. Maximum tangential stress vs. H/D : (black line) $E = 5$ GPa; (red line) $E = 20$ GPa; (blue line) $E = 50$ GPa.

3.2. Strain Analysis

The strain state acting at the barycenter of the specimen is characterized by the Mohr's circles reported in Figure 10 for $\bar{\sigma}_y = 5$ MPa, $E = 5$ GPa and $\nu = 0.25$. The influence of Poisson's coefficient can be verified by analyzing Figure 11 where, in the case of $H/D = 3$, $\bar{\sigma}_y = 5$ MPa and $E = 5$ GPa, the comparison between the Mohr's circles for $\nu = 0.10$ and those for $\nu = 0.25$ and $\nu = 0.40$ is reported. In Figure 10, for readability's sake, the comparison is made only for the outer circle. An examination of this figure clearly indicates that Poisson's coefficient mainly influences the tensile principal strain rather than the compressive one.

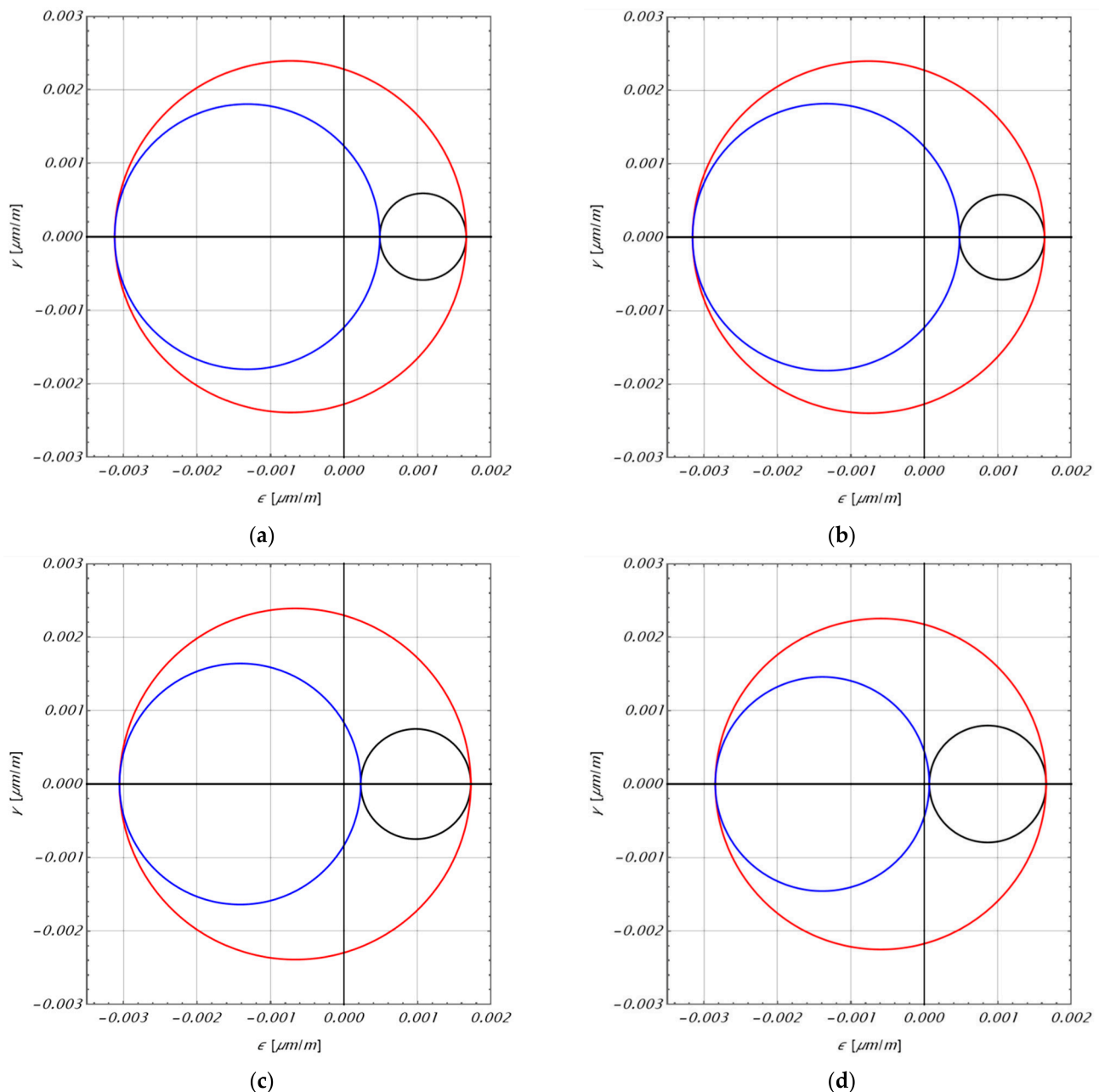


Figure 10. Mohr's strain circles for $\bar{\sigma}_y = 5$ MPa, $E = 5$ GPa, and $\nu = 0.25$: (a) $H/D = 0.25$; (b) $H/D = 1.0$; (c) $H/D = 2.0$; (d) $H/D = 3.0$.

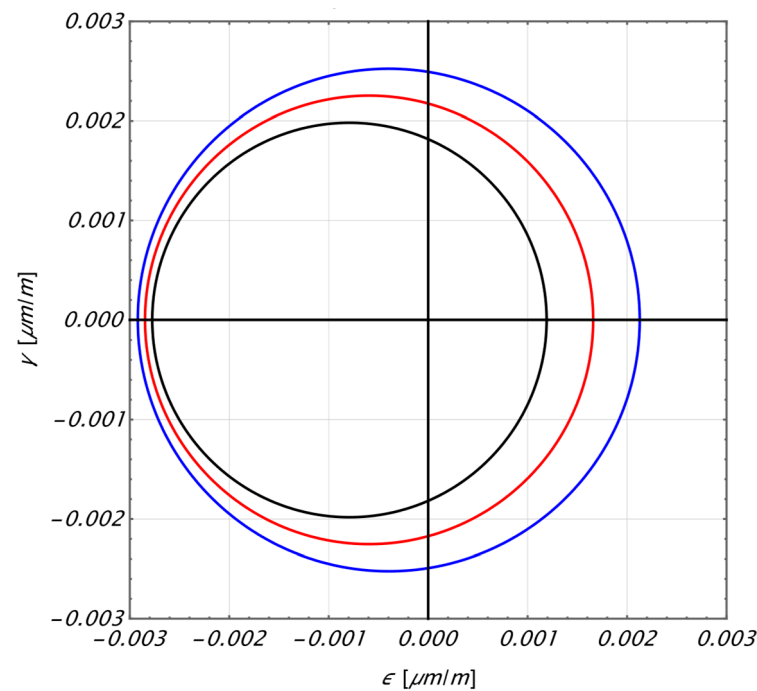


Figure 11. Comparison among the outer Mohr's strain circles for $\bar{\sigma}_y = 5$ MPa, $H/D = 3.0$ and $E = 5$ GPa; (black line) $\nu = 0.10$; (red line) $\nu = 0.25$; (blue line) $\nu = 0.40$.

Further, as expected, the influence of Poisson's coefficient on the strain state is meaningful; therefore, the hypothesis of plane strain is strongly influenced by the value of this coefficient. As an example, in Figure 12a, for $\nu = 0.25$, the Mohr's strain circles are sketched showing that the plane strain condition is achieved when $H/D > 3.0$. This remark must be associated with the examination of Figure 12b where the value of the middle principal strain ε_{II} (in the case of $H/D = 3.0$, $\bar{\sigma}_y = 5$ MPa and $E = 5$ GPa) is sketched vs. Poisson's coefficient. As can be deduced from the matched analysis of Figure 11, the hypothesis of plane strain strongly depends on both the H/D ratio and Poisson's coefficient.

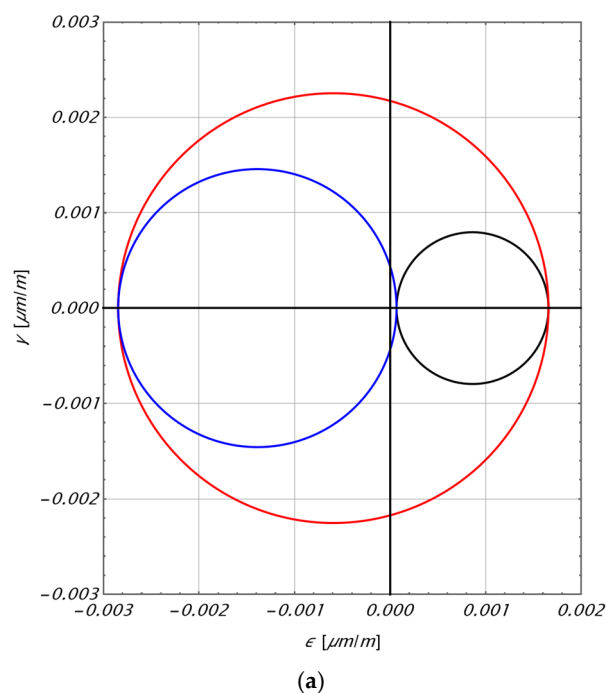


Figure 12. Cont.

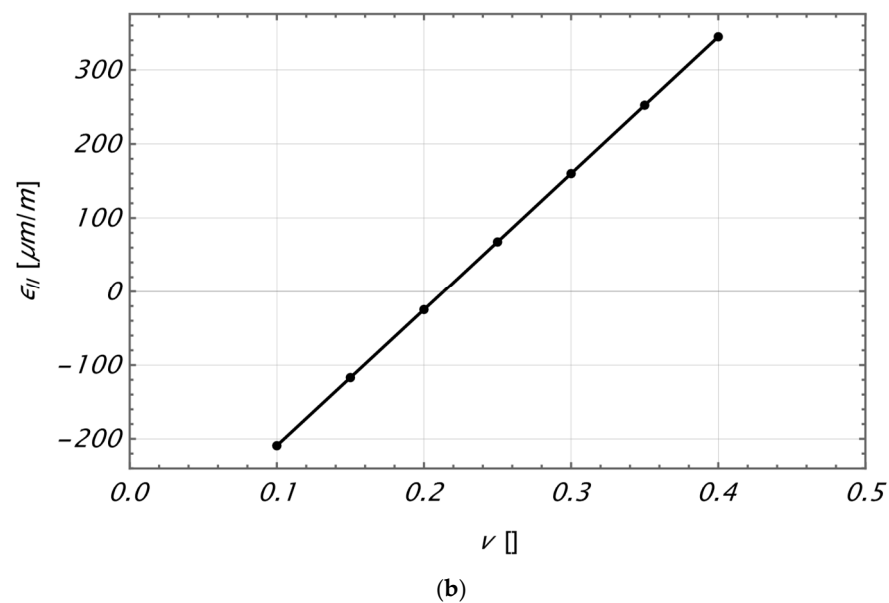


Figure 12. Influence of Poisson's coefficient on the plane strain hypothesis ($\bar{\sigma}_y = 5$ MPa, $H/D = 3.0$ and $E = 5$ GPa): (a) Mohr's strain circle for $\nu = 0.25$; (b) ε_{II} vs. ν .

3.3. Comparison with the Theoretical Results

An important point to be stressed relates to the accuracy of the theoretical results with respect to those obtained in the proposed analyses. To address this point, in Figure 13, the calculated ratio $\sigma_{III}/\bar{\sigma}_y$ is reported with colored dots vs. H/D for different values of the Young's modulus E . The analysis of this figure allows for the following remarks: (a) the calculated ratio tends to reach the unitary value only for the limit value of $H/D = 0$, which is perfectly coherent with the hypothesis assumed for the determination of $\bar{\sigma}_y$; (b) the ratio $\sigma_{III}/\bar{\sigma}_y$ is nonlinearly related with H/D ; (c) the ratio is always less than one, clearly indicating that the theoretical value of the stress acting at the barycenter of the specimen overestimates the real one; and (d) the ratio is strongly influenced by the Young's modulus E , especially for high values of H/D .

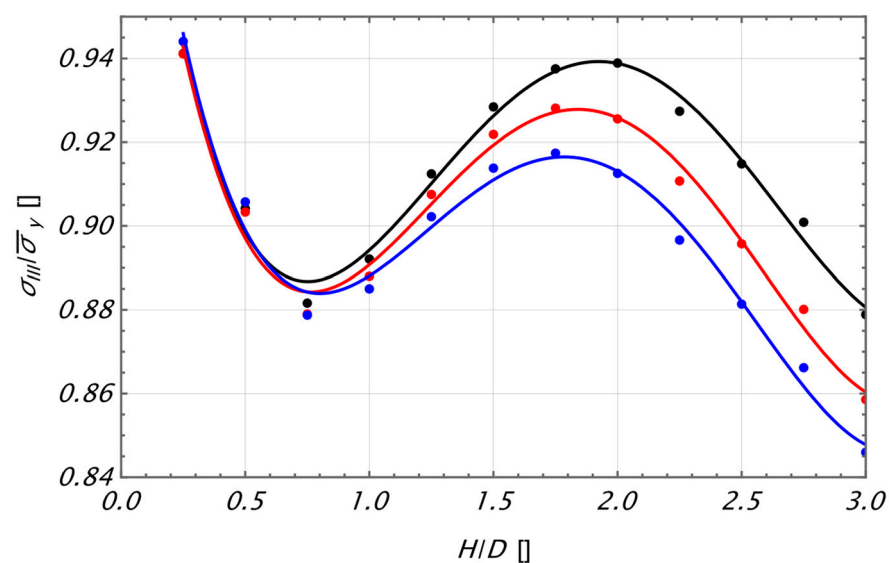


Figure 13. $\sigma_{III}/\bar{\sigma}_y$ vs. H/D ($\bar{\sigma}_y = 5$ MPa, $\nu = 0.25$) (black line and black circles: $E = 5$ GPa; red line and red circles: $E = 20$ GPa; blue line and blue circles: $E = 50$ GPa).

To allow for the correction of the results obtained in engineering practice, the numerical results have been fitted using the following polynomial equation:

$$f = a + b x + c x^2 + d x^3 + e x^4 \quad (3)$$

The graphs in continuous lines are also shown in Figure 13. The coefficients of the interpolating polynomial are reported in Table 2, where the R^2 value for each polynomial is also reported.

Table 2. Coefficients of the interpolating polynomials (The gray means that the colored cells are not meaningful).

Parameter	$-\sigma_I/\bar{\sigma}_x$			$\sigma_{III}/\bar{\sigma}_y$		
E [GPa]	5	20	50	5	20	50
a	0.94638	0.95311	0.963393	1.03516	1.03832	1.0411
b	0.05098	0.043656	0.027089	−0.48565	−0.50584	−0.50119
c	−0.021465	−0.022066	−0.018934	0.529883	0.54923	0.534348
d				−0.20782	−0.21973	−0.21448
e				0.0266258	0.028756	0.028297
R^2	0.973	0.974	0.975	0.979	0.979	0.981

To complete this point, as in the case of $\sigma_{III}/\bar{\sigma}_y$, in Figure 14, the calculated ratio $-\sigma_I/\bar{\sigma}_x$ is reported with colored dots vs. H/D for different values of the Young's modulus E , together with the lines of the corresponding interpolating polynomials. An examination of this figure leads, for the principal stress σ_I , to remarks analogous to those reported for the principal stress σ_{III} .

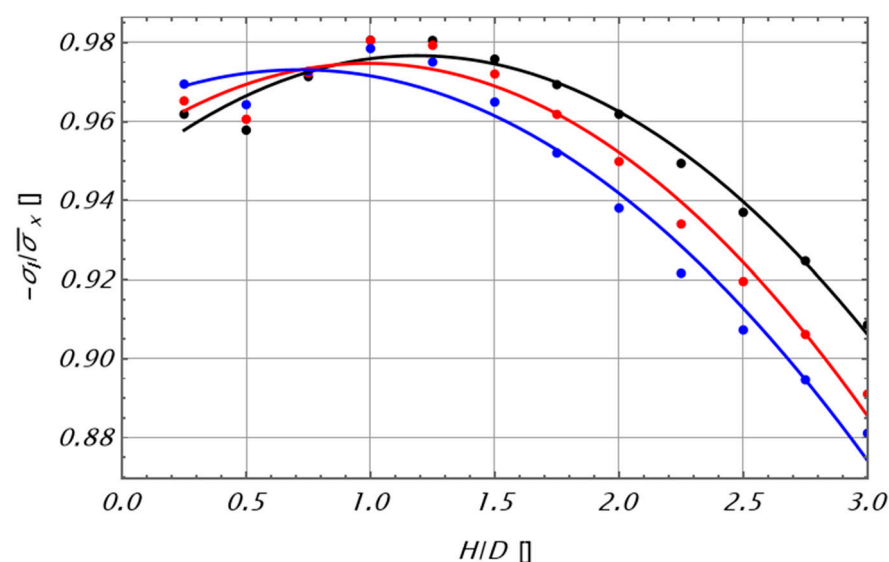


Figure 14. $-\sigma_I/\bar{\sigma}_x$ vs. H/D ($\bar{\sigma}_x = 5$ MPa, $\nu = 0.25$) (black line and black circles: $E = 5$ GPa; red line and red circles: $E = 20$ GPa; blue line and blue circles: $E = 50$ GPa).

The final point to be addressed is that, as it can be deduced from Equations (1) and (2), the absolute ratio between σ_x and σ_y is equal to three. To check this point, in Figure 15, the ratio $-\sigma_I/\sigma_{III}$ is reported vs. H/D for different values of E . Three main remarks can be made by examining this figure: (a) the ratio between these principal stresses is not constant, as indicated by the theoretical value; (b) the mathematical dependence with H/D is highly nonlinear; and (c) the influence of E is found but is negligible.

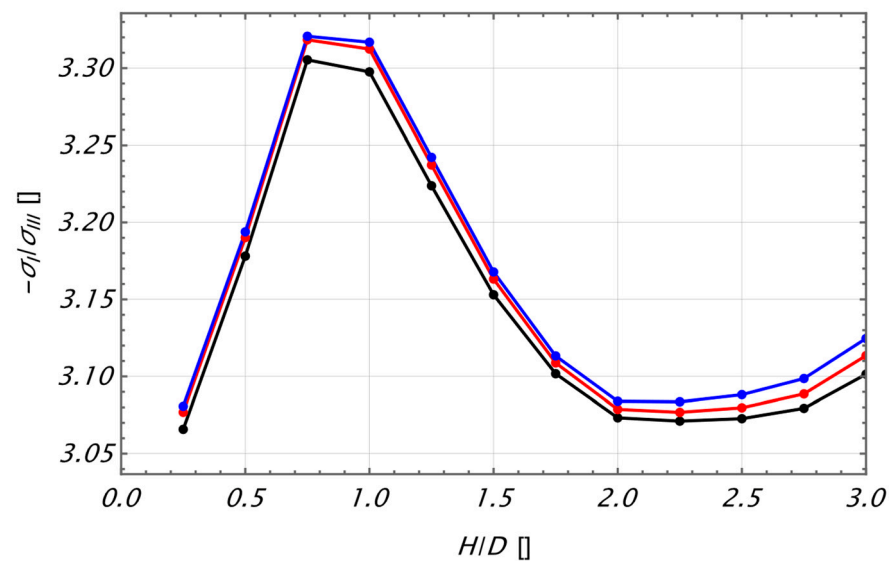


Figure 15. $-\sigma_I/\sigma_{III}$ vs. H/D ($\bar{\sigma}_y = 5$ MPa, $\nu = 0.25$) (black line: $E = 5$ GPa; red line: $E = 20$ GPa; blue line: $E = 50$ GPa).

The analysis of Figures 13–15 allows for us to draw the remark that, for the stress state, the depth-to-diameter ratio, H/D , always greatly influences the results, while the influence of the Young's modulus E is meaningful only for $H/D > 1$.

4. Conclusions

The paper investigated the influence of both geometrical parameters (in terms of the depth-to-height ratio, H/D) and mechanical properties (in terms of Young's modulus and Poisson's ratio) on the three-dimensional mechanical response of homogeneous isotropic specimens subjected to the Brazilian splitting test. The methodology adopted has been that of appropriate finite element simulations carried out in the ANSYS environment with systematic variations in both geometrical and mechanical parameters within ranges suitably defined considering the most common experimental cases in which the Brazilian test is adopted. To reach the goal of the paper, the principal stresses and strains evaluated at the specimen's barycenter have been calculated and compared with those proposed in the literature and in international standards which are obtained under the plane stress assumption (i.e., with the intermediate principal stress equal to zero). The performed analyses showed that: (a) the plane stress assumption is valid only for low depth-to-height ratios in which cases it can also be considered as independent of material properties; (b) the plane stress assumption is generally not realistic under standard testing conditions ($H/D > 1$); (c) the tensile principal stress assumed in the literature overestimates the real value; and (d) the stress state is greatly influenced by the depth-to-diameter ratio, H/D , while the influence of the Young's modulus E is limited to $H/D > 1$. Remark (c) implies a consequent thought regarding structural safety for which the graphs reported in the paper, together with the interpolating functions, allow for the opportune correction. The analysis of the strain field indicated that the validity of the plane strain hypothesis depends jointly on the H/D value of the specimen, as well as on both the Young's modulus and the Poisson's coefficient of the material constituting the specimen. Consequently, it can be affirmed that the plane strain hypothesis is not achieved for the common experimental cases in which the Brazilian test is adopted. Future works will include the numerical validation of the behavior of the principal stresses along the specimen axis, as well as the experimental validation of the numerical results on the specimen basis, for instance, by employing full-field, contactless displacement measurement techniques.

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