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Performance Limits of RIS-Assisted MIMO Systems in Nakagami- m Fading Environments

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Abstract

This work analyzes the ergodic capacity behavior of reconfigurable intelligent surface (RIS)-assisted multiple-input multiple-output (MIMO) systems with a finite and arbitrary number of antennas and RIS elements under Nakagami- m fading conditions. By combining Hadamard's determinant inequality with the Cauchy–Schwarz inequality, this work derives a dimensionally consistent closed-form upper bound on the ergodic capacity in terms of the Meijer G -function. Subsequently, it is demonstrated that at a high signal-to-noise ratio (SNR), a simplified expression for the capacity upper bound can be derived, enabling an analytical assessment of how the fading parameter influences the ergodic capacity. The study also explores the asymptotic behavior in the large-system regime, where the number of antennas or RIS elements tends to infinity. Monte Carlo (MC) simulations confirm the accuracy of the proposed bound and scaling laws.

Keywords: reconfigurable intelligent surface (RIS); multiple-input multiple-output (MIMO); Nakagami- m fading

1. Introduction

The surge in demand for ultra-high data rates, driven by bandwidth-intensive applications such as video streaming and the widespread use of mobile devices, has prompted the exploration of advanced wireless communication technologies [1,2]. Among the key enablers proposed to satisfy these requirements are millimeter-wave (mmWave) systems and massive multiple-input multiple-output (MIMO) architectures. While effective, these technologies often come with trade-offs in terms of increased hardware complexity and higher power consumption. Furthermore, they generally do not exploit the statistical nature of wireless propagation environments, which can significantly impact system performance [2].

To mitigate these challenges, reconfigurable intelligent surfaces (RISs) have emerged as a promising solution for environment-aware channel reconfiguration [3]. An RIS consists of a large number of low-power, nearly passive reflecting elements whose electromagnetic response can be dynamically adjusted through software control [4]. By tuning the phase shifts in the reflected signals, RIS technology enables programmable and adaptive wireless propagation environments [5,6]. RIS-assisted wireless communications have attracted substantial attention because of their ability to control the propagation environment with low-power, nearly passive reflecting elements. A comprehensive overview of the operating principles, channel models, implementation aspects, and open challenges of RIS-assisted communications was provided in [7]. Beyond conventional single-stream transmission, RISs have also been integrated with space–time coding schemes. For example, an RIS-assisted N -ary Alamouti scheme was investigated in [8], illustrating how configurable



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reflections can support enhanced spectral efficiency and communication reliability. Moreover, RIS technology has been considered for non-terrestrial networks, where it can facilitate coverage extension, energy-efficient transmission, and adaptive beamforming for satellite, aerial, and high-altitude-platform communications [9]. These developments motivate the characterization of the fundamental performance limits of RIS-assisted MIMO links under flexible fading models.

Several studies have investigated the capacity of RIS-assisted multi-antenna systems under different propagation and hardware assumptions. In [10], the capacity of RIS-assisted MIMO communication was characterized under narrowband and OFDM transmission, with emphasis on joint active and passive optimization. Deterministic LoS RIS-assisted MIMO channels were considered in [11], where upper and lower ergodic capacity bounds and high-SNR and large-RIS asymptotic results were derived. A closed-form ergodic capacity upper bound for an RIS-aided SISO system was obtained in [12]. Practical phase-dependent amplitude responses were subsequently incorporated in [13], whose main objective was to quantify the impact of non-ideal RIS hardware under Rayleigh-type fading.

Generally, Nakagami- m fading has also been studied despite being more mathematically involved than Rayleigh or Rician models because it offers greater modeling flexibility and unified analysis in a wide range of wireless propagation scenarios. Nakagami- m fading has been used to capture both mild and severe fading environments with a single unified model since Rayleigh fading is a special case for $m = 1$ while Rician fading can be approximated when $m > 1$. Examples include indoor and industrial environments, urban microcells, vehicle-to-infrastructure links, and aerial or elevated RIS deployments, where one hop may exhibit a strong specular component while the other remains scattering-dominated. In rich-scattering device-to-device or vehicle-to-vehicle scenarios, Rayleigh fading may still be an appropriate model. Such conditions are directly recovered from the present framework by setting $m = 1$, as mentioned. RIS-assisted systems under Nakagami- m fading have also attracted attention because this model can represent fading conditions that are more severe or milder than Rayleigh fading. In [14], however, the analysis was developed for a single-antenna RIS-assisted system and focused on scalar performance characterization. Thus, although that study establishes the relevance of Nakagami- m fading to RIS-assisted communications, it does not provide a finite-dimensional ergodic capacity characterization for a general multi-stream RIS-assisted MIMO channel. Table 1 summarizes the positioning of the present work with respect to representative studies.

Table 1. Comparison with representative RIS-assisted capacity studies.

Reference	System	Channel Model	RIS Model	Main Result	Asymptotic Analysis
[10]	MIMO	Narrowband/OFDM	Ideal	Capacity characterization	Limited
[11]	MIMO	Deterministic LoS	Ideal	Upper and lower bounds	High SNR and large RIS
[12]	SISO	Rayleigh-based	Ideal	Ergodic capacity upper bound	No general MIMO scaling
[13]	MIMO	Rayleigh-type	Practical amplitude–phase coupling	Ergodic capacity analysis	Large RIS
[14]	SISO	Nakagami- m	Ideal	Scalar performance analysis	Large RIS
This work	Finite- dimensional MIMO	Nakagami- m on both hops	Ideal	Closed-form upper bound	High SNR; large M , N , and L

Despite these advances, a finite-dimensional ergodic capacity bound for a general multi-stream RIS-assisted MIMO channel with Nakagami- m fading on both cascaded links, together with explicit high-SNR and large-system scaling laws, has not yet been comprehensively characterized.

This paper focuses on analyzing the ergodic capacity of RIS-assisted MIMO systems operating over Nakagami- m fading channels, considering an arbitrary but finite number of

antennas at both the transmitter and receiver (A related ergodic capacity analysis was presented in [13], where RIS-assisted MIMO systems were studied under a practical reflection model incorporating the coupling between the phase shift and the amplitude response of each RIS element. That work focuses mainly on the impact of non-ideal RIS hardware under Rayleigh-type fading. In contrast, the present work assumes ideal unit-amplitude reflection and investigates the effect of generalized Nakagami- m fading on the capacity of RIS-assisted MIMO channels. Since Rayleigh fading is recovered for $m = 1$, the proposed framework includes the Rayleigh case while also describing propagation conditions with more severe or milder fading. Furthermore, unlike [13], this work employs determinant and norm inequalities to derive a finite-dimensional ergodic capacity upper bound, obtain a tractable high-SNR approximation, and characterize the associated large-system scaling laws. Hence, the two studies are complementary: Ref. [13] emphasizes practical RIS amplitude–phase coupling, whereas this work focuses on generalized fading severity and its influence on the capacity limits). The key contribution lies in the development of a closed-form upper bound for the ergodic capacity of such channels. By combining Hadamard’s determinant inequality, the Cauchy–Schwarz inequality, the gamma distributions of channel-energy terms, and Mellin–Barnes integration, the analysis avoids the need to explicitly characterize the eigenvalue distribution of the cascaded channel. Moreover, a compact upper bound for the ergodic capacity is derived in the high-signal-to-noise-ratio (SNR) regime. This expression facilitates insights into how the fading parameter m influences the capacity of the system. The paper further explores the asymptotic capacity behavior in the large-antenna and RIS elements limits. Accordingly, the novelty lies in the joint consideration of arbitrary finite-dimensional MIMO, independent Nakagami- m fading on both RIS-assisted hops, a dimensionally consistent closed-form finite-dimensional capacity upper bound, and explicit high-SNR and large-system scaling characterizations.

2. System Model

In this work, a MIMO system comprising $M \geq 1$ transmit antennas and $N \geq 1$ receive antennas is studied, as depicted in Figure 1. To boost the communication performance, a reconfigurable intelligent surface (RIS) with L passive elements is strategically placed to assist the transmission between the transceiver ends. In other words, we consider an RIS-assisted MIMO system. The passive elements reflect the incident signal waves by applying controllable phase shifts. These phase adjustments are managed by a controller that communicates with the transmitter via a backhaul connection. We assume that the direct transmitter–receiver channel is severely attenuated by a large physical obstruction, such as a reinforced wall, a building corner, or another electromagnetically opaque object. Therefore, the received signal is dominated by the RIS-assisted cascaded link. This assumption represents blocked or non-line-of-sight deployments in which the direct-link power is substantially smaller than that of the reflected path and can be neglected for analytical purposes. The placement of the RIS close to the receiver does not contradict this assumption. In a user-side RIS deployment, the RIS may be installed around a corner, on the opposite side of an obstructing wall, or at a position having favorable visibility toward both the transmitter and the receiver, while the direct transmitter–receiver path remains blocked. The short RIS–receiver distance mainly reduces the path loss of the second hop and does not necessarily restore the blocked direct path. However, Nakagami- m fading for $m > 1$ can accurately model mild fading and LoS-dominant environments, and it is widely used as a statistical surrogate for Rician fading when closed-form analysis under Rician assumptions is intractable. (This capability is particularly relevant for RIS-assisted systems, where the RIS–user link often exhibits strong specular/LoS components, especially when the environment contains deterministic reflective paths).

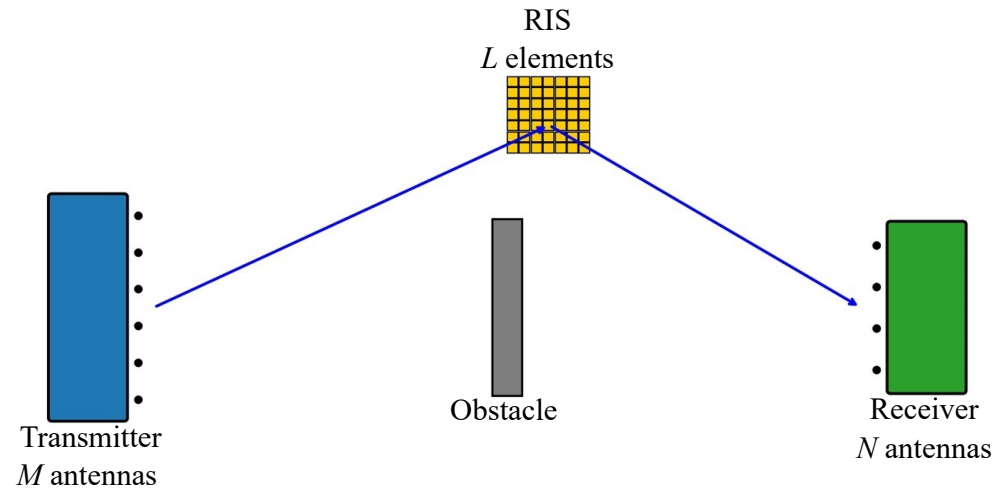


Figure 1. An RIS-assisted MIMO system with M transmit antennas, N receive antennas, and L reflecting elements.

We consider quasi-static block-fading channels and analyze a specific fading block. Let $\mathbf{H}_1 \in \mathbb{C}^{L \times M}$ and $\mathbf{H}_2 \in \mathbb{C}^{N \times L}$ be the channel between the transmitter and the RIS and the channel between the RIS and the receiver, respectively. The relationship between the $N \times 1$ received signal vector and the transmitted symbol vector can be expressed as follows

$$\mathbf{y} = \mathbf{G}\mathbf{x} + \mathbf{w}, \quad (1)$$

where $\mathbf{G} = \mathbf{H}_2\mathbf{\Theta}\mathbf{H}_1 \in \mathbb{C}^{N \times M}$, so that $\mathbf{G}\mathbf{G}^H \in \mathbb{C}^{N \times N}$, and $\mathbb{E}[\mathbf{x}\mathbf{x}^H] = \mathbf{Q}$ with $\mathbf{x}$ being the transmit symbol vector and $\mathbf{Q}$ being the transmit covariance matrix (If the direct link is non-negligible, the equivalent channel should be modeled as

$$\mathbf{G} = \mathbf{H}_d + \mathbf{H}_2\mathbf{\Theta}\mathbf{H}_1,$$

where $\mathbf{H}_d$ denotes the direct transmitter–receiver channel. In that case, the direct and RIS-assisted components interact coherently, and the capacity depends on their relative phases and powers. The analytical treatment of this more general model is beyond the scope of the present work). Also, $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_N)$ denotes the additive white Gaussian noise (AWGN) vector at the receiver. The matrix $\mathbf{\Theta} = \text{diag}(\alpha_1 e^{j\theta_1}, \dots, \alpha_L e^{j\theta_L}) \in \mathbb{C}^{L \times L}$ characterizes the reconfigurable beamforming matrix (RBM), representing the individual response of the L reflecting elements. Each element is described by a phase shift $\theta_l \in [0, 2\pi]$ and an amplitude coefficient $\alpha_l \in [0, 1]$. Following conventional assumptions supported by recent progress in low-loss metasurface designs [15], we consider ideal reflections with $\alpha_l = 1 \forall l$. In particular, the elements of $\mathbf{H}_1 = \{H_{1ij}\}$, $\mathbf{H}_2 = \{H_{2ij}\}$ are modeled as independent and identically distributed (i.i.d.), with phases uniformly distributed and magnitudes following a Nakagami- m probability density function (PDF). Thus, the PDFs of magnitudes $z_1 = |H_{1ij}|$ and $z_2 = |H_{2ij}|$ can be expressed as

$$p(z_1) = \frac{2}{\Gamma(m_1)} \left(\frac{m_1}{\Omega_1} \right)^{m_1} z_1^{2m_1-1} e^{-(m_1/\Omega_1)z_1^2}, \quad (2)$$

$$p(z_2) = \frac{2}{\Gamma(m_2)} \left(\frac{m_2}{\Omega_2} \right)^{m_2} z_2^{2m_2-1} e^{-(m_2/\Omega_2)z_2^2}, \quad (3)$$

where $m_i = \mathbb{E}^2[z_i^2] / \text{Var}[z_i^2]$ and $\Omega_i = \mathbb{E}[z_i^2]$ for $i = 1, 2$. Note that $\Gamma(\cdot)$ denotes the gamma function (The two channel matrices $\mathbf{H}_1$ and $\mathbf{H}_2$ are assumed to be mutually statistically independent. This assumption corresponds to the conventional independent-hop model, in

which the transmitter–RIS and RIS–receiver links experience distinct scattering processes. Although both links interact through the same RIS, the deterministic RIS response matrix Θ does not introduce statistical dependence between $\mathbf{H}_1$ and $\mathbf{H}_2$. Correlated two-hop propagation involving common scatterers or spatially coupled RIS elements is not considered in the present analysis).

3. Capacity Bound of RIS-Assisted MIMO Nakagami Fading Channels

This work assumes perfect knowledge of the channel state information (CSI) at the receiver and employs equal power distribution among the transmit antennas, which means that $\mathbf{Q} = \frac{P}{M}\mathbf{I}$, where P is the total power available for transmission. (Our assumption does not imply the absence of all CSI at the transmitter. Instead, the transmitter is assumed to have long-term statistical CSI, which is sufficient for the ergodic capacity analysis. Under statistical CSI, equal power allocation is known to be optimal for MIMO channels [16]). In such case, the ergodic capacity can be written as

$$C = \mathbb{E} \left[\log_2 \det \left(\mathbf{I}_N + \frac{P}{M\sigma^2} \mathbf{G}\mathbf{G}^H \right) \right], \quad (4)$$

where it is assumed that $M \geq N$. Following the same approach, the case $M < N$ can be resolved by invoking the property $\det(\mathbf{I} + \mathbf{A}\mathbf{B}) = \det(\mathbf{I} + \mathbf{B}\mathbf{A})$ for any matrices $\mathbf{A}$ and $\mathbf{B}$. The analysis relies on Hadamard’s determinant inequality, the Cauchy–Schwarz inequality, Mellin–Barnes integration, and Jensen’s inequality. For clarity, the analytical results are derived under the following assumptions: The two hops are mutually independent. The entries of each channel matrix are i.i.d. with uniformly distributed phases and Nakagami- m distributed magnitudes. The RIS has ideal unit-amplitude coefficients; the direct transmitter–receiver link is negligible. The receiver has perfect CSI; the transmitter uses equal power allocation based on long-term statistical CSI. The RIS phase-shift matrix is fixed and independent of the instantaneous channel realization. These assumptions delimit the scope of the derived bounds and scaling laws.

Proposition 1. *Let*

$$a \triangleq m_1 LM, \quad b \triangleq m_2 L, \quad \kappa \triangleq \frac{P\Omega_1\Omega_2}{M\sigma^2 m_1 m_2}. \quad (5)$$

Under the independent-hop Nakagami- m model, the ergodic capacity satisfies the following dimensionally consistent closed-form upper bound:

$$C \leq C_U \triangleq \frac{N}{\ln 2 \Gamma(a)\Gamma(b)} G_{2,4}^{4,1} \left(\frac{1}{\kappa} \left| \begin{array}{c} 0, 1 \\ 0, 0, a, b \end{array} \right. \right) \quad (6)$$

Proof. Please see Appendix A. $\square$

Corollary 1. *Let $r = \min(M, N)$. A second, simpler upper bound useful for establishing the large-system scaling laws is*

$$C \leq C_J \triangleq r \log_2 \left(1 + \frac{PNL\Omega_1\Omega_2}{\sigma^2 r} \right). \quad (7)$$

Proof. The result follows from the arithmetic–geometric mean inequality applied to the r nonzero eigenvalues of $\mathbf{G}\mathbf{G}^H$, Jensen’s inequality, and $\mathbb{E}\|\mathbf{G}\|_F^2 = NML\Omega_1\Omega_2$. $\square$

Remark 1. Under the statistical channel model adopted in this work, the ergodic capacity is invariant to any fixed RIS phase-shift matrix Θ that is independent of the instantaneous channel realizations. Specifically, the entries of $\mathbf{H}_1$ and $\mathbf{H}_2$ are assumed to be independent and identically distributed, with independent uniformly distributed phases and Nakagami- m distributed magnitudes. Since Θ is diagonal and unitary, its multiplication only introduces deterministic phase rotations, which do not alter the joint distribution of the cascaded channel. Therefore, the ergodic capacity, obtained by averaging over the channel distribution, is independent of the particular fixed RIS phase configuration. This result should not be interpreted as implying that RIS phase design is irrelevant in general. For a given channel realization, the instantaneous capacity generally depends on Θ , since the reflected components may combine constructively or destructively. Moreover, when the RIS phases are optimized using instantaneous channel state information, Θ becomes channel-dependent, and the distributional-invariance argument no longer applies. Hence, phase optimization may improve the instantaneous rate, outage performance, and reliability. The same qualification applies to finite-blocklength transmission, where the achievable rate depends not only on the average mutual information but also on the instantaneous channel conditions and channel dispersion. Therefore, RIS phase design may affect the finite-blocklength achievable rate and error probability. In the presence of channel estimation errors, adaptive phase design may still provide performance gains, although the coherent-combining gain is reduced and depends on the CSI quality and the robustness of the adopted phase-design method. The investigation of finite-blocklength transmission and imperfect-CSI-based RIS phase optimization is left for future work.

3.1. High-SNR Performance

Although the closed-form Meijer G-function expression is directly computable, it provides limited physical intuition regarding the capacity behavior. Therefore, we focus on the high-SNR regime, aiming to obtain a more tractable expression that offers clearer insights.

Proposition 2. For the upper bound in Proposition 1, the high-SNR approximation is

$$C_{U,HSNR} = \frac{N}{\ln 2} \left[\ln\left(\frac{P}{M\sigma^2}\right) + \psi(m_1 LM) + \psi(m_2 L) + \ln\left(\frac{\Omega_1 \Omega_2}{m_1 m_2}\right) \right]. \quad (8)$$

Proof. Please see Appendix B. $\square$

The high-SNR upper bound increases with the fading parameters m_1 and m_2 . In fact, the corresponding m_1 - and m_2 -dependent terms are $\psi(m_1 LM) - \ln(m_1)$ and $\psi(m_2 L) - \ln(m_2)$, respectively. Their derivatives are given by

$$LM\psi^{(1)}(m_1 LM) - \frac{1}{m_1} > 0$$

and

$$L\psi^{(1)}(m_2 L) - \frac{1}{m_2} > 0,$$

where $\psi^{(1)}(\cdot)$ is the trigamma function. Thus, for fixed average channel powers Ω_1 and Ω_2 , milder fading increases the capacity upper bound by reducing the probability of deep channel fades and concentrating the channel power more strongly around its mean.

It should be emphasized that, in the adopted i.i.d. Nakagami- m model, m_1 and m_2 describe only the severity of the amplitude fluctuations. They do not describe the number

of propagation paths, angular sparsity, spatial correlation, or the rank of the MIMO channel. Therefore, the monotonic increase with m_1 and m_2 does not imply that a physically less rich multipath environment necessarily provides a larger RIS beamforming or spatial-multiplexing gain. In realistic propagation environments, a strong dominant component may be accompanied by increased spatial correlation or rank deficiency, which can offset the benefit of reduced fading severity.

Moreover, by using

$$\psi(x) = \ln(x) - \frac{1}{2x} + \mathcal{O}(x^{-2}),$$

the fading-dependent terms satisfy

$$\psi(m_1 LM) - \ln(m_1) = \ln(LM) - \frac{1}{2m_1 LM} + \mathcal{O}((m_1 LM)^{-2})$$

and

$$\psi(m_2 L) - \ln(m_2) = \ln(L) - \frac{1}{2m_2 L} + \mathcal{O}((m_2 L)^{-2}).$$

Consequently, the capacity improvement obtained by increasing the fading parameters exhibits diminishing returns and becomes less significant as the number of antennas or RIS elements grows. A joint treatment of fading severity, multipath sparsity, and spatial correlation requires a more general geometry-based or correlated channel model and is left for future work.

3.2. Large-System Performance

The asymptotic statements are formulated as scaling laws rather than exact finite-dimensional equalities. This distinction is important because non-vanishing constant terms remain in the high-SNR expressions. Let $r = \min(M, N)$. Corollary 1 gives

$$C = \mathcal{O}\left[r \log_2\left(1 + \frac{NL}{r}\right)\right], \quad (9)$$

where the factors independent of M , N , and L have been absorbed into the $\mathcal{O}(\cdot)$ notation.

For fixed N and L with $M \rightarrow \infty$, one has $r = N$, and therefore

$$C = \mathcal{O}(1). \quad (10)$$

More specifically, the high-SNR form of the fading-dependent bound converges to the nonzero constant

$$C_{\text{U,HSNR}} \rightarrow \frac{N}{\ln 2} \left[\ln\left(\frac{PL\Omega_1\Omega_2}{\sigma^2 m_2}\right) + \psi(m_2 L) \right]. \quad (11)$$

Thus, increasing M under a fixed total-power constraint produces a saturation effect rather than an unbounded capacity gain.

For fixed M and L with $N \rightarrow \infty$, eventually $r = M$, and

$$C = \mathcal{O}(M \log_2 N). \quad (12)$$

Hence, when the number of transmit antennas is fixed, the growth with N is logarithmic rather than linear. Linear growth with the minimum array dimension arises when M and N increase jointly with a nonzero finite ratio. In particular, if $N/M \rightarrow \beta \in (0, \infty)$, then

$$C = \mathcal{O}(\min(M, N)). \quad (13)$$

Finally, for fixed M and N with $L \rightarrow \infty$, Corollary 1 yields

$$C = \mathcal{O}(r \log_2 L). \quad (14)$$

This establishes logarithmic growth with the RIS size while retaining the non-vanishing constant offset. The scaling law should therefore not be written as the exact equality $C = N \log_2 L$. The result also shows that the dominant growth term becomes independent of the Nakagami parameters, although the finite- L offset still depends on the fading statistics.

4. Numerical Results

All analytical curves in this section are evaluated using the corrected expressions in Proposition 1, Corollary 1, and Proposition 2. The Monte Carlo curves represent the ergodic capacity obtained from direct channel realizations, while the analytical curves represent either an upper bound or an asymptotic approximation, as identified in each figure. In this section, we evaluate the performance of an RIS-assisted MIMO system operating over Nakagami- m fading channels, utilizing both analytical derivations and Monte Carlo (MC) simulations. The transmitter and receiver are equipped with six and four antennas, respectively. The RIS comprises $L = 256$ reflecting elements. The system operates at a carrier frequency of 2 GHz with a bandwidth of 20 MHz. Each user transmits at a power level of $P = 20$ dBm, while the noise power at the receiver is set to $\sigma^2 = -110$ dBm. The fading parameters are set to $m_1 = m_2 = 2$. Unless stated otherwise, these parameters are held constant throughout the simulations. The MC results are obtained by averaging over 1000 independently generated channel realizations.

Figure 2 presents the ergodic capacity as a function of the first-hop fading parameter m_1 for different RIS sizes L . The capacity increases with m_1 because larger values correspond to milder fading and a lower probability of deep fades. Nevertheless, the incremental gain decreases as m_1 grows, consistently with the diminishing-return behavior predicted by the high-SNR analysis. The curves also become progressively flatter as L increases, showing that a larger RIS reduces the sensitivity of the bound to first-hop fading severity. The case $m_1 = 1$, corresponding to Rayleigh fading, is included as a reference.

Figure 3 plots the ergodic capacity versus the second-hop fading parameter m_2 for different RIS sizes L . The capacity increases with m_2 because larger values correspond to milder fading and a reduced probability of deep fades on the RIS–receiver link. The improvement becomes progressively smaller as m_2 increases, confirming the diminishing-return behavior predicted by the high-SNR analysis. Moreover, the curves become flatter as L grows, indicating that a larger RIS reduces the sensitivity of the bound to second-hop fading severity. Over the simulated parameter range, the analytical upper-bound curves remain close to the Monte Carlo results. This observation supports the practical tightness of the bound for the tested configurations; it is not intended as a claim of uniform tightness for every possible SNR, antenna configuration, or channel model. The case $m_2 = 1$, corresponding to Rayleigh fading on the RIS–receiver link, is also included.

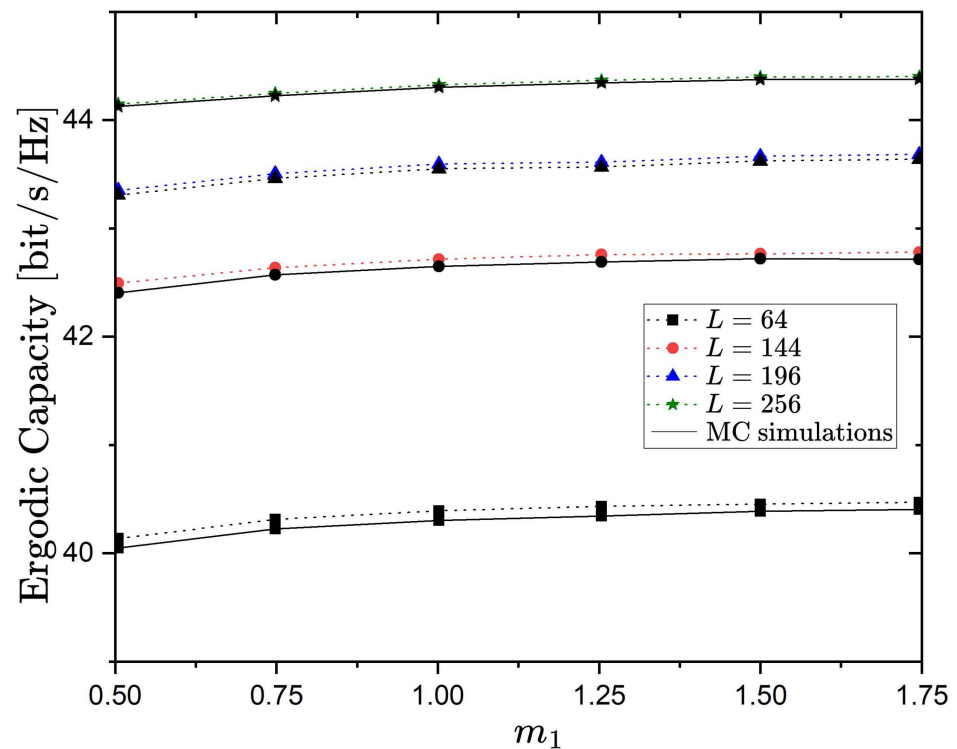


Figure 2. Ergodic capacity versus m_1 for different values of L .

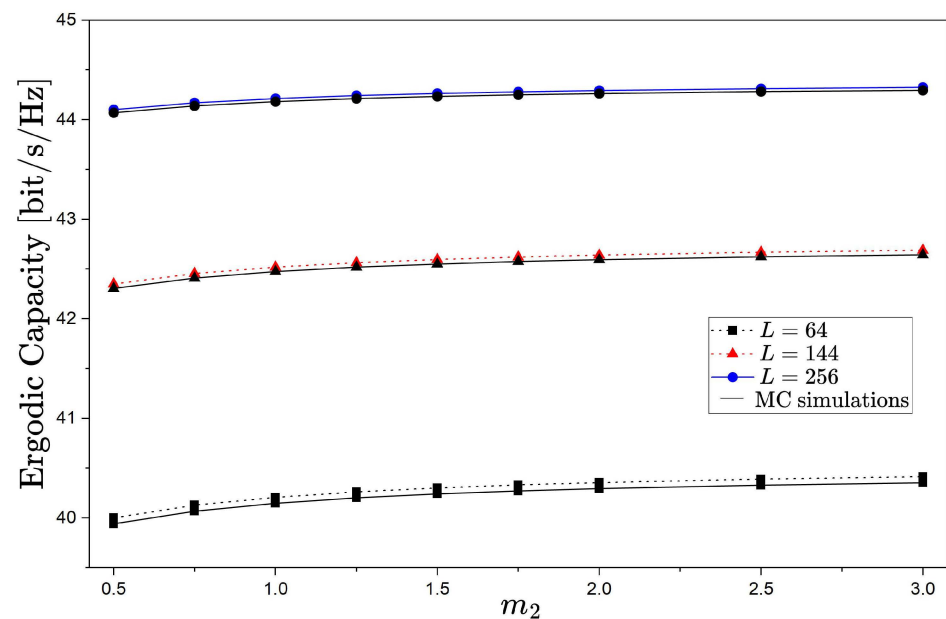


Figure 3. Ergodic capacity versus m_2 for different values of L .

Figure 4 shows the ergodic capacity upper bound versus SNR for different numbers of receive antennas N , together with the corresponding high-SNR approximation. The upper bound increases with both SNR and N . As expected, the high-SNR expression becomes increasingly accurate in the moderate-to-high-SNR region, whereas a visible discrepancy may remain at low SNR because the approximation replaces $\ln(1+x)$ by $\ln(x)$. Thus, the high-SNR expression should be interpreted as an asymptotic approximation to the upper bound rather than as an exact finite-SNR equality.

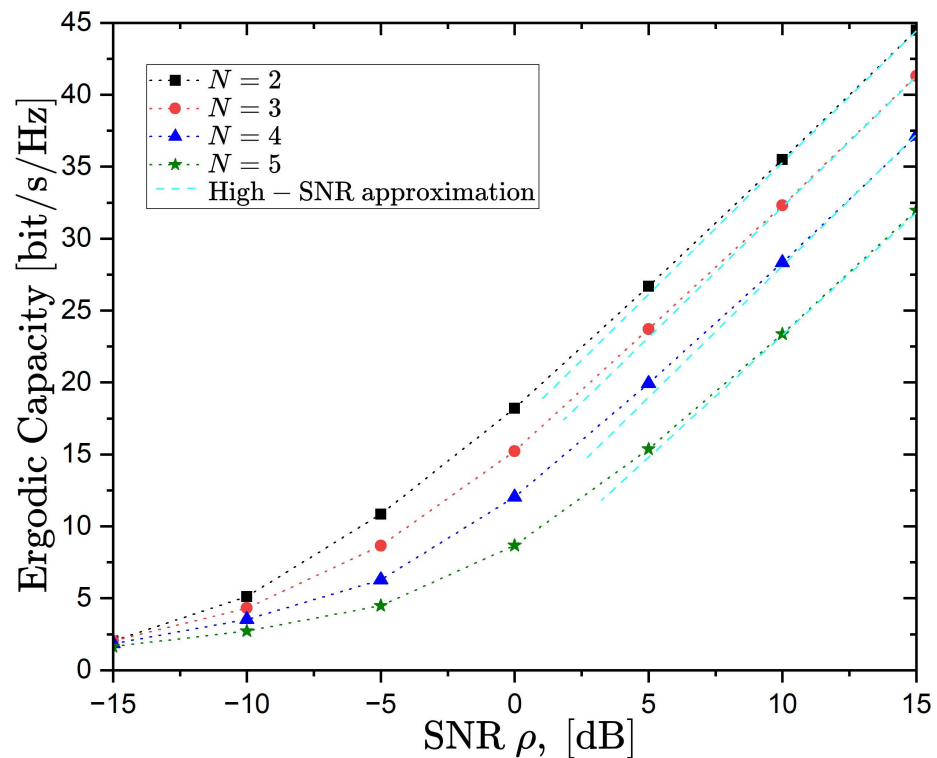


Figure 4. Ergodic capacity upper bound and high-SNR approximation versus SNR for different values of N .

Figure 5 presents the ergodic capacity versus SNR for four representative configurations, namely, $(M, N, L) = (4, 2, 64)$, $(6, 4, 144)$, $(8, 4, 256)$, and $(10, 6, 256)$. The capacity increases monotonically with SNR in every case. Configurations with larger receive dimensions and more reflecting elements generally provide higher capacity because they offer additional spatial observation dimensions and larger reflected-channel energy. Since more than one system parameter changes between the curves, the figure should be interpreted as a comparison of representative overall configurations rather than as an isolated assessment of a single dimension.

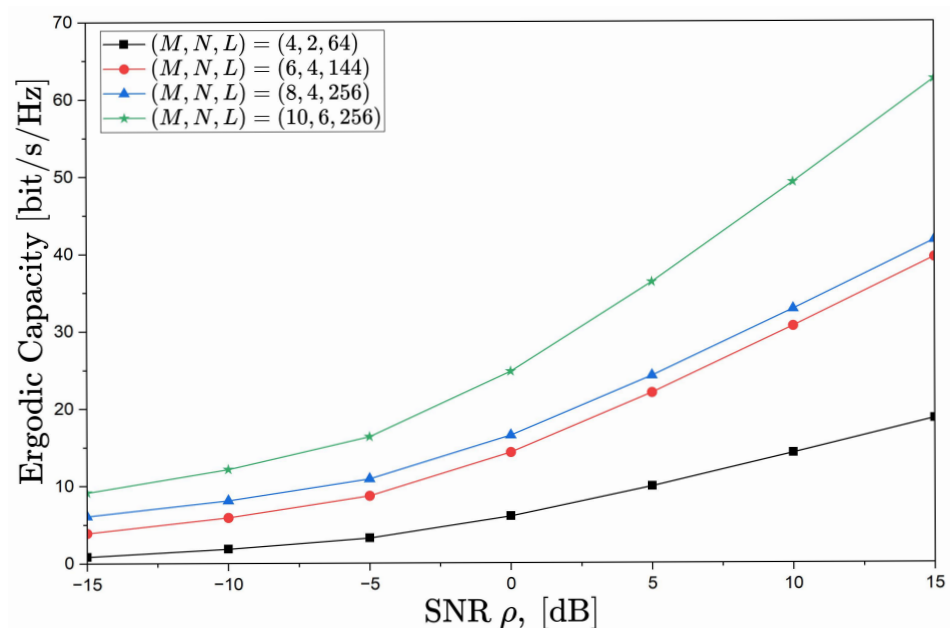


Figure 5. Ergodic capacity versus SNR for representative system dimensions.

Figures 6–8 illustrate the large-system trends derived in Section 3.2. When M increases while N and L remain fixed, the capacity initially increases and then approaches a finite limit, confirming the saturation induced by the fixed total transmit-power constraint. Figure 7 considers joint growth of M and N with a fixed nonzero ratio; in this regime, the capacity grows approximately linearly with $\min(M, N)$, as predicted by (13). This regime is distinct from fixed M with $N \rightarrow \infty$, for which (12) predicts logarithmic growth. Finally, equal multiplicative increases in L yield approximately equal additive gains, consistently with the logarithmic scaling in (14). The Monte Carlo curves remain close to the analytical upper bounds for the tested ranges. The comparison assesses the empirical gap between the exact simulated capacity and the upper bound, whereas Proposition 2 approximates the upper bound itself at high SNR, and Corollary 1 is used primarily to establish scaling orders. These three analytical quantities therefore serve different purposes and should not be interpreted as interchangeable exact capacity expressions.

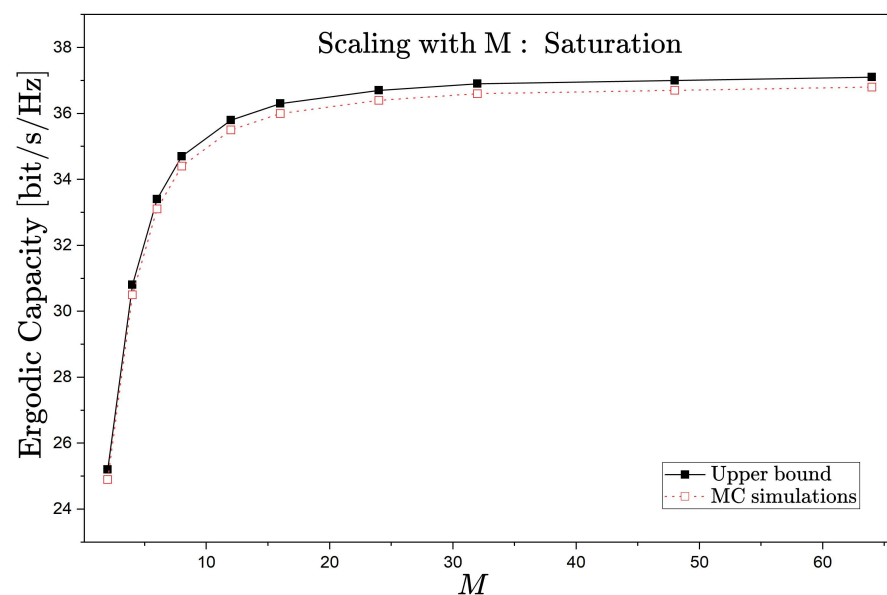


Figure 6. Ergodic capacity versus M , with N and L fixed.

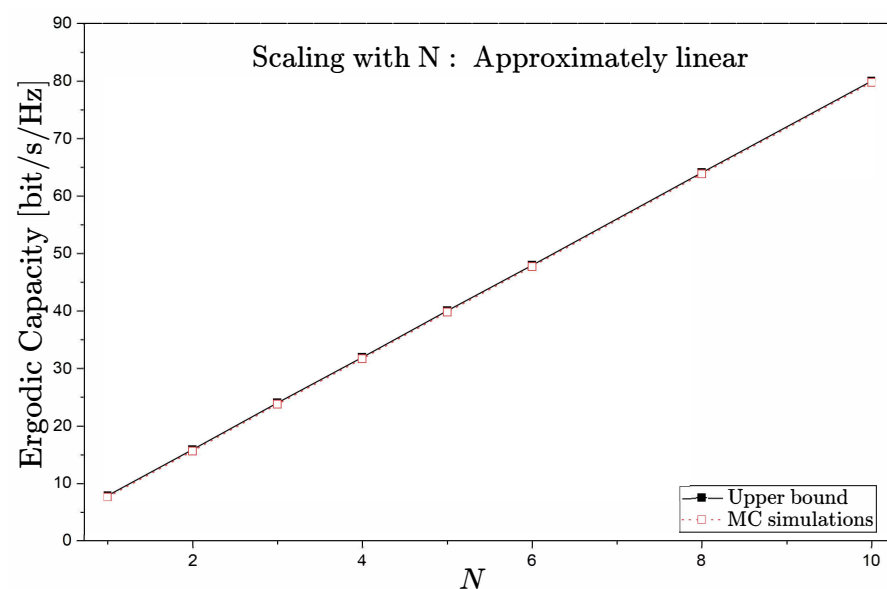


Figure 7. Ergodic capacity under joint growth of M and N with a fixed nonzero ratio, shown versus N .

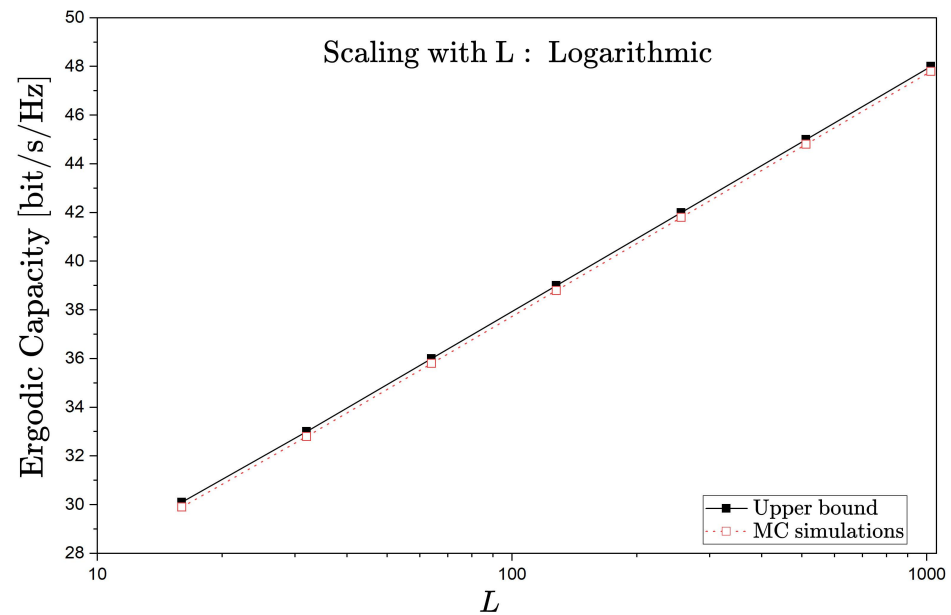


Figure 8. Ergodic capacity versus the number of RIS elements L .

5. Conclusions

This work investigated the ergodic capacity of RIS-assisted MIMO systems operating over Nakagami- m fading channels. By applying Hadamard's determinant inequality, the Cauchy–Schwarz inequality, and Mellin–Barnes integration, a dimensionally consistent closed-form upper bound was derived for arbitrary finite numbers of transmit antennas, receive antennas, and RIS elements without requiring the exact eigenvalue distribution of the cascaded channel. A tractable high-SNR approximation was also obtained, enabling the influence of the fading parameters and the system dimensions to be characterized analytically. Furthermore, the large-system analysis showed how the capacity scales with the numbers of antennas and RIS elements.

From a practical perspective, the proposed expressions provide low-complexity benchmarks for system dimensioning and deployment planning. They can be used to assess the potential benefit of increasing the number of antennas or RIS elements without relying exclusively on computationally intensive Monte Carlo simulations. The results also reveal the presence of diminishing returns in some asymptotic regimes, which is relevant when balancing capacity gains against hardware cost, control complexity, energy consumption, and physical deployment constraints. In addition, the analysis shows that a large number of RIS elements reduces the sensitivity of the capacity upper bound to the fading parameters, indicating that sufficiently large surfaces can partially mitigate severe small-scale fading. (The derived results assume perfect CSI at the receiver and should therefore be interpreted as fundamental performance benchmarks. In practice, the receiver estimates the equivalent cascaded channel from pilot observations. If

$$\mathbf{G} = \hat{\mathbf{G}} + \tilde{\mathbf{G}},$$

where $\hat{\mathbf{G}}$ denotes the channel estimate and $\tilde{\mathbf{G}}$ denotes the estimation error, the latter introduces residual interference and an effective SNR loss. Hence, imperfect CSI generally reduces the achievable rate relative to the perfect-CSI upper bound. Moreover, the pilot

overhead required for channel estimation reduces the useful data-transmission interval. A practical net rate may therefore be represented as

$$R_{\text{net}} = \left(1 - \frac{\tau_p}{T_c}\right) R_{\text{imp}},$$

where τ_p is the pilot duration, T_c is the coherence interval, and R_{imp} is an achievable rate under imperfect CSI.

The joint impact of channel-estimation errors, training overhead, spatial correlation, practical RIS amplitude–phase responses, and finite-resolution phase shifts is not captured by the current model. Extending the proposed framework to imperfect-CSI receivers and robust RIS-assisted MIMO design constitutes an important direction for future work.

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Appendix A. Proof of Proposition 1

The dimensions are $\mathbf{H}_1 \in \mathbb{C}^{L \times M}$, $\mathbf{\Theta} \in \mathbb{C}^{L \times L}$, and $\mathbf{H}_2 \in \mathbb{C}^{N \times L}$, so that $\mathbf{G} = \mathbf{H}_2 \mathbf{\Theta} \mathbf{H}_1 \in \mathbb{C}^{N \times M}$ and $\mathbf{G} \mathbf{G}^H \in \mathbb{C}^{N \times N}$.

Let $\rho = P/(M\sigma^2)$. By Hadamard's determinant inequality,

$$\det(\mathbf{I}_N + \rho \mathbf{G} \mathbf{G}^H) \leq \prod_{n=1}^N (1 + \rho [\mathbf{G} \mathbf{G}^H]_{nn}). \quad (\text{A1})$$

Let $\mathbf{h}_{2,n} \in \mathbb{C}^{1 \times L}$ denote the n th row of $\mathbf{H}_2$, and let $\mathbf{h}_{1,m} \in \mathbb{C}^{L \times 1}$ denote the m th column of $\mathbf{H}_1$. Then,

$$\begin{aligned} [\mathbf{G} \mathbf{G}^H]_{nn} &= \sum_{m=1}^M |\mathbf{h}_{2,n} \mathbf{\Theta} \mathbf{h}_{1,m}|^2 \\ &\leq \|\mathbf{h}_{2,n} \mathbf{\Theta}\|_2^2 \sum_{m=1}^M \|\mathbf{h}_{1,m}\|_2^2 \\ &= \|\mathbf{h}_{2,n}\|_2^2 \|\mathbf{H}_1\|_F^2, \end{aligned} \quad (\text{A2})$$

where Cauchy–Schwarz gives the inequality and the unitary property of $\mathbf{\Theta}$ gives the last equality.

Define

$$X \triangleq \|\mathbf{H}_1\|_F^2, \quad Y_n \triangleq \|\mathbf{h}_{2,n}\|_2^2. \quad (\text{A3})$$

Since the squared Nakagami- m magnitudes are gamma distributed and the channel entries are independent,

$$X \sim \text{Gamma}\left(a, \frac{\Omega_1}{m_1}\right), \quad Y_n \sim \text{Gamma}\left(b, \frac{\Omega_2}{m_2}\right), \quad (\text{A4})$$

where $a = m_1 LM$ and $b = m_2 L$. Moreover, X and Y_n are independent because $\mathbf{H}_1$ and $\mathbf{H}_2$ are mutually independent. Applying (A1) and (A2), taking logarithms and expectations, and using the identical distribution of the rows of $\mathbf{H}_2$ yields

$$\begin{aligned} C &\leq \sum_{n=1}^N \mathbb{E}[\log_2(1 + \rho XY_n)] \\ &= \frac{N}{\ln 2} \mathbb{E}[\ln(1 + \rho XY)], \end{aligned} \quad (\text{A5})$$

where Y is distributed as any Y_n .

Introduce the normalized variables $U = m_1 X / \Omega_1$ and $V = m_2 Y / \Omega_2$. Then $U \sim \text{Gamma}(a, 1)$, $V \sim \text{Gamma}(b, 1)$, and $\rho XY = \kappa UV$, with $\kappa = P\Omega_1\Omega_2 / (M\sigma^2 m_1 m_2)$. Hence,

$$C_U = \frac{N}{\ln 2 \Gamma(a) \Gamma(b)} \int_0^\infty \int_0^\infty \ln(1 + \kappa uv) u^{a-1} v^{b-1} e^{-u-v} du dv. \quad (\text{A6})$$

For $-\min(1, a, b) < c < 0$, the Mellin–Barnes representation

$$\ln(1 + z) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{\pi}{s \sin(\pi s)} z^{-s} ds \quad (\text{A7})$$

allows the integrations over u and v to be evaluated as gamma integrals. Therefore,

$$C_U = \frac{N}{\ln 2 \Gamma(a) \Gamma(b)} \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{\pi \Gamma(a-s) \Gamma(b-s)}{s \sin(\pi s)} \kappa^{-s} ds. \quad (\text{A8})$$

Using $\pi / [s \sin(\pi s)] = -\Gamma(s) \Gamma(-s)$ and the defining Mellin–Barnes integral of the Meijer G -function, (A8) becomes

$$C_U = \frac{N}{\ln 2 \Gamma(a) \Gamma(b)} G_{2,4}^{4,1} \left(\frac{1}{\kappa} \left| \begin{matrix} 0, 1 \\ 0, 0, a, b \end{matrix} \right. \right), \quad (\text{A9})$$

which proves (6). Importantly, the expectation form in (A5) is only an intermediate step. The final result is the closed-form Meijer G expression stated in Proposition 1.

Appendix B. Proof of Proposition 2

Starting from the equivalent expectation representation in (A5), at high SNR, $\ln(1 + \rho XY) = \ln(\rho XY) + o(1)$. Hence,

$$C_U = \frac{N}{\ln 2} [\ln \rho + \mathbb{E}(\ln X) + \mathbb{E}(\ln Y)] + o(1). \quad (\text{A10})$$

For a gamma random variable $Z \sim \text{Gamma}(k, \theta)$, $\mathbb{E}[\ln Z] = \psi(k) + \ln \theta$. Applying this identity to $X \sim \text{Gamma}(m_1 LM, \Omega_1 / m_1)$ and $Y \sim \text{Gamma}(m_2 L, \Omega_2 / m_2)$ gives

$$\begin{aligned} C_{U, \text{HSNR}} &= \frac{N}{\ln 2} \left[\ln \left(\frac{P}{M\sigma^2} \right) + \psi(m_1 LM) + \psi(m_2 L) \right. \\ &\quad \left. + \ln \left(\frac{\Omega_1 \Omega_2}{m_1 m_2} \right) \right], \end{aligned} \quad (\text{A11})$$

which proves (8).

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